$$\ln L(p\ldots$$

$$\frac{d\ln L(p)}{d(\lambda)} = -n + \sum_{i=1}^{n} x_i \cdot \frac{1}{\lambda} = 0 \Rightarrow \lambda n = \sum_{i=1}^{n} x_i \quad /:n$$

$$\boxed{\Rightarrow \hat{\lambda} = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x} \longrightarrow \text{אומד נראות מקסימלית}}$$

ב) $X \sim \exp(\theta)$ $\qquad f(x) = \theta e^{-\theta x}$

$$L(f(x)) = \prod_{i=1}^{n} \theta e^{-\theta x_i} = \theta e^{-\theta x_1} \cdot \theta e^{-\theta x_2} \cdot \ldots\ldots \cdot \theta e^{-\theta x_n} = \theta^n e^{-\theta \sum_{i=1}^{n} x_i}$$

$$\downarrow$$

$$\ln(f(x)) = \ln \theta^n - \theta \sum_{i=1}^{n} x_i = n \ln \theta - \theta \sum_{i=1}^{n} x_i$$

$$\downarrow$$

$$\frac{d\ln L(f(x))}{d\theta} = n \cdot \frac{1}{\theta} - \sum_{i=1}^{n} x_i = 0 \Rightarrow \boxed{\theta = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{\bar{x}} \longrightarrow \text{אומד נראות מקסימלית}}$$

$$\downarrow$$

$$\frac{1}{\bar{x}} = \frac{1}{E(x)} \Rightarrow \bar{x} = E(x)$$

ג) $X \sim G(p)$

$$p(X=x) = p(1-p)^{x-1}$$

$$L(p) = \prod_{i=1}^{n} p(1-p)^{x_i-1} = p(1-p)^{x_1-1} \cdot p(1-p)^{x_2-1} \cdot \ldots\ldots \cdot p(1-p)^{x_n-1} = p^n(1-p)^{\sum_{i=1}^{n} x_i - n}$$

$$\downarrow$$

$$\ln L(p) = \ln p^n + \ln(1-p)^{\sum_{i=1}^{n} x_i - n} = n \ln p + \left(\sum_{i=1}^{n} x_i - n\right) \cdot \ln(1-p)$$

$$\downarrow$$

(נגזרת פנימית)

$$\frac{d\ln L(p)}{dp} = \frac{n}{p} + \left(\sum_{i=1}^{n} x_i - n\right) \cdot \frac{-1}{(1-p)} = 0$$

$$n(1-p) - p \cdot \left(\sum_{i=1}^{n} x_i - n\right) = 0$$

$$n - \cancel{np} + \cancel{np} - p \cdot \left(\sum_{i=1}^{n} x_i - n\right) = 0$$