

$$p = \frac{n}{\sum_{i=1}^n x_i} = \frac{1}{\bar{x}} \rightarrow \text{MLE}$$

$$E(x) = \frac{1}{p}$$

$$a) \quad x \sim B(n, p) \rightarrow p(x=x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

$$L(p) = \prod_{i=1}^n \binom{n}{x_i} \cdot p^{x_i} \cdot (1-p)^{n-x_i} = \binom{n}{x_i} \cdot p^{\sum_{i=1}^n x_i} \cdot (1-p)^{n - \sum_{i=1}^n x_i}$$

$$\downarrow$$

$$\ln L(p) = \ln \left( \binom{n}{x_i} \right)^n + \sum_{i=1}^n x_i \ln p + \left( n - \sum_{i=1}^n x_i \right) \cdot \ln(1-p)$$

$$\downarrow$$

$$\frac{d \ln L(p)}{dp} = \sum_{i=1}^n x_i \cdot \frac{1}{p} + \left( n - \sum_{i=1}^n x_i \right) \cdot \frac{-1}{(1-p)} = 0$$

$$\sum_{i=1}^n x_i (1-p) - p \cdot \left( n - \sum_{i=1}^n x_i \right) = 0 \Rightarrow \sum_{i=1}^n x_i - p \cdot \sum_{i=1}^n x_i - pn^2 + p \cdot \sum_{i=1}^n x_i = 0$$

$$\sum_{i=1}^n x_i - pn^2 = 0$$

$$p = \frac{\sum_{i=1}^n x_i}{n \cdot n} \Rightarrow \hat{p} = \frac{\bar{x}}{n} \rightarrow \text{MLE}$$

$$E(x) = n \cdot p$$