The Independent-Samples *t* Test

Chapter 11

Quick Test Reminder

- One person = Z score
- One sample with population standard deviation = Z test
- One sample no population standard deviation = single t-test
- One sample test twice = paired samples t

Independent Samples t-Test

> Used to compare two means in a between-groups design (i.e., each participant is in only one condition)

- Remember that dependent t (paired samples) is a repeated measures or within-groups design

Between groups design

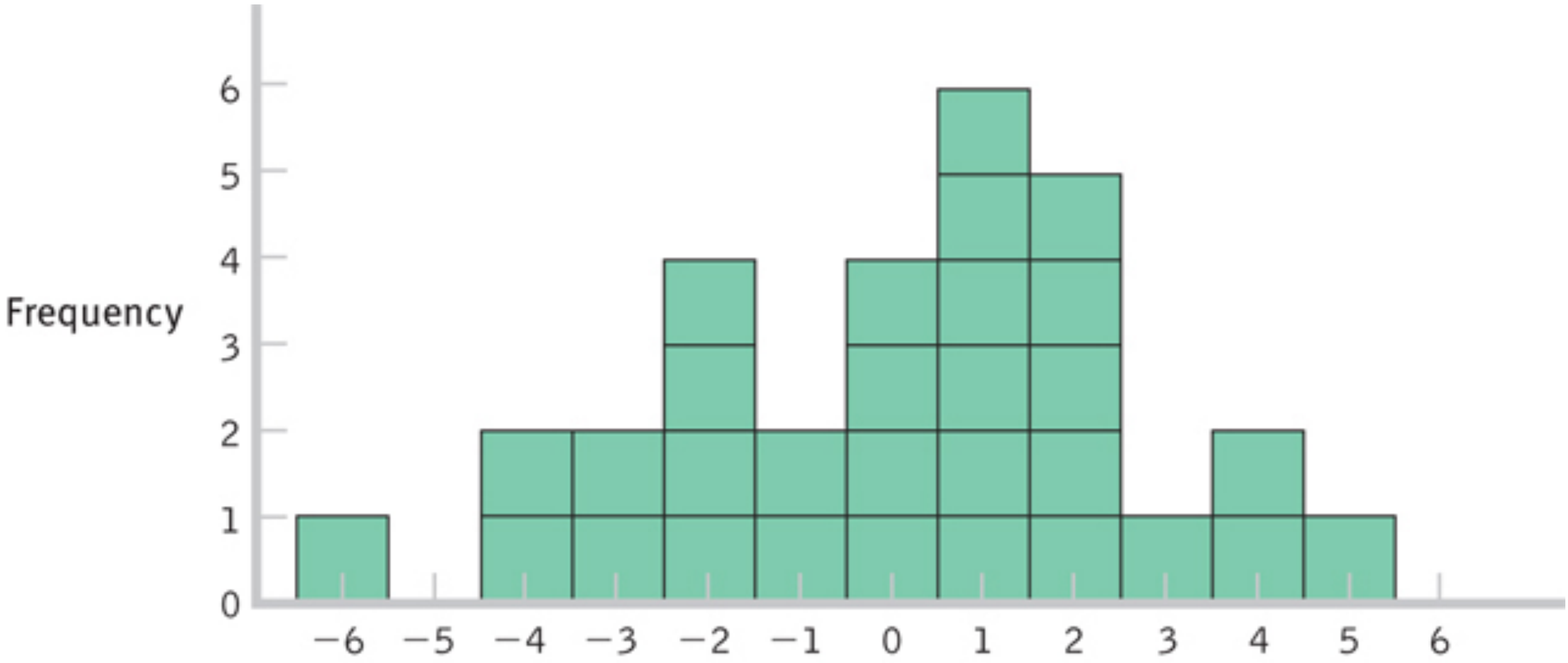
> In between groups, your sets of participant's scores (i.e. group 1 versus group 2) have to be *independent*

- Remember independence is the assumption that my scores are completely unrelated to your scores

Quick Distributions Reminder

- Z = Distribution of scores
- Z = distribution of means (for samples)
- t = distribution of means (for samples with estimated standard deviation)
- t = distribution of mean differences between paired scores (for paired samples with estimated standard deviation)
- t = distribution of differences between means (for two groups independent t)

Distribution of Differences Between Means



Hypothesis Tests & Distributions

TABLE 11-1. Hypothesis Tests and Their Distributions

We must consider the appropriate comparison distribution when we choose which hypothesis test to use.

Hypothesis Test	Number of Samples	Comparison Distribution
z test	one	Distribution of means
Single-sample *t* test	one	Distribution of means
Paired-samples *t* test	two (same participants)	Distribution of mean difference scores
Independent-samples *t* test	two (different participants)	Distribution of differences between means

Let's talk about Standard Deviation

Test	Standard Deviation	Standard deviation of distribution of … (standard error)
Z	σ (population)	σ_M
Single t	s (sample)	s_M
Paired t	s (sample on difference scores)	s_M
Independent t	s group 1 s group 2 s_{pooled}	$s_{difference}$

Let's talk about Standard Deviation

$$s^2 = \frac{\sum (X - M)^2}{N - 1}$$

Variance = same for all tests, but paired t is on difference scores

$$s_M^2 = \frac{s^2}{N}$$

Standard error = same for paired and single t

Take the square root for standard deviation of these

Let's talk about Standard Deviation

$$s^2 = \frac{\sum (X - M)^2}{N - 1}$$

Variance = same for all tests, but paired t is on difference scores

$$s^2_{pooled} = \left(\frac{df_X}{df_{total}}\right) s^2_X + \left(\frac{df_Y}{df_{total}}\right) s^2_Y$$

$$s^2_{difference} = s^2_{M_X} + s^2_{M_Y}$$

This section is for independent t only

Take the square root for standard deviation of these

Let's talk about test statistics

Test type	Formula
Z	$\frac{M - \mu_M}{\sigma_M}$
Single t	$\frac{M - \mu_M}{s_M}$
Paired t	$\frac{M}{s_M}$
Independent t	$\frac{M - M}{s_{difference}}$

Additional Formulae

$$t = \frac{(M_X - M_Y) - (\mu_X - \mu_Y)}{s_{difference}}$$

$$t = \frac{M_X - M_Y}{s_{difference}}$$

Let's talk about *df*

Test type	***df***
Single sample	N – 1
Paired samples t	N – 1
Independent t	N – 1 + N – 1

Steps for Calculating Independent Sample *t* Tests

> Step 1: Identify the populations, distribution, and assumptions.
> Step 2: State the null and research hypotheses.
> Step 3: Determine the characteristics of the comparison distribution.
> Step 4: Determine critical values, or cutoffs.
> Step 5: Calculate the test statistic.
> Step 6: Make a decision.

Let’s work some examples!

> Let’s work some examples: chapter 11 docx on blackboard.

Assumptions

Assumption	Solution
Normal distribution	N >=30
DV is scale	Nothing – do nonparametrics
Random selection (sampling)	Random assignment to group

Step 2

- List the sample, population, and hypotheses
  - Sample: group 1 versus group 2
  - Population: those groups mean difference will be 0 (u – u = 0)

Step 2

- Now, we can list those as group 1 versus group 2 in our R and N
    - Should also help us distinguish between independent t and dependent t
- R: group 1 =/ OR > OR < group 2
- N: group 1 = OR <= OR >= group 2
    - Watch the order!

Step 3

> List the descriptive statistics

	Group 1	Group 2
Mean		
SD		
N		
df		
Spooled		
Sdifference		

Step 3

> Get the mean

- summary(*dataset*)

> Get the sd

- sd(*dataset$column,* na.rm = T)

> Get N

- length(*dataset$column*)

```
> summary(dataset)
     grumps           pew       
 Min.   :1.000   Min.   : 3.00  
 1st Qu.:2.000   1st Qu.: 4.75  
 Median :2.500   Median : 6.50  
 Mean   :3.125   Mean   : 6.50  
 3rd Qu.:4.250   3rd Qu.: 8.25  
 Max.   :6.000   Max.   :10.00  
```

```
> sd(dataset$grumps, na.rm = T)
[1] 1.726888
> sd(dataset$pew, na.rm = T)
[1] 2.44949
```

```
> length(dataset$grumps)
[1] 8
> length(dataset$pew)
[1] 8
```

Step 3

- Get Spooled (evil!)
- spooled = sqrt(((n1-1)*sd1^2 + (n2-1)*sd2^2) / (n1+n2 - 2))

```
> spooled = sqrt( ((n1-1)*sd1^2 + (n2-1)*sd2^2) / (n1+n2 - 2))
> spooled
[1] 2.119215
```

Step 3

- > Get Sdifference (less evil)
- > sdifference = sqrt((spooled^2/n1 + spooled^2/n2))

```
> sepooled = sqrt((spooled^2/n1 + spooled^2/n2))
> sepooled
[1] 1.059607
```

Step 4

> Since we are dealing with two groups, we have two *df* … but the *t* distribution only has one df?

- So add them together!
- *df* total = (n-1) + (n-1)

Step 4

> Figure out the cut off score, $t_{critical}$

> Less test:

- qt(.05, df, lower.tail = T)

> Greater test:

- qt(.05, df, lower.tail = F)

> Difference test:

- qt(.05/2, df, lower.tail = T)

May also be .01 – remember to read the problem.

Step 5

> Find t_{actual}

```
t.test(data$column,
       data$column,
       paired = F,
       var.equal = T,
       alternative = "less" OR "greater" OR
           "two.sided",
       conf.level = .95 OR .99)
```

Step 5

> Stop! Make sure your mean difference score, df, and hypothesis all match.

```
        Two Sample t-test

data:  dataset$grumps and dataset$pew
t = -3.1851, df = 14, p-value = 0.006613
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -5.647632 -1.102368
sample estimates:
mean of x mean of y 
    3.125     6.500 
```

Step 6

- Compare step 4 and 5 – is your score more extreme?
  - Reject the null
- Compare step 4 and 5 – is your score closer to the middle?
  - Fail to reject the null

Steps for Calculating CIs

> The suggestion for CI for independent *t* is to calculate the CI around the mean difference (M – M).

- This calculation will tell you if you should reject the null – remember you do NOT want it to include 0.
- Does not match what people normally do in research papers (which is calculate each M CI separately).

Confidence Interval

> Lower = $M_{difference} - t_{critical}$*SE

> Upper = $M_{difference} + t_{critical}$*SE

> A quicker way!

- Use t.test() with a TWO tailed test to get the two tailed confidence interval.
- The r script effsize will give you each mean CI separately (how to interpret?).

Effect Size

> Used to supplement hypothesis testing

> Cohen's d:

$$d = \frac{(M_X - M_Y) - (\mu_X - \mu_Y)}{s_{pooled}}$$

Effect Size

```
> d.indt(m1 = 3.125, m2 = 6.50, sd1 = sd1, sd2 = sd2, n1 = n1, n2 = n2, a = .05, k = 2)
M1 = 3.12, SD = 1.73, SE = 0.61, 95%CI[1.68 - 4.57]
M2 = 6.50, SD = 2.45, SE = 0.87, 95%CI[4.45 - 8.55]
t(14) = -3.19, p < .01, d = -1.59, 95%CI[-2.71 - -0.43]
>
```

- Remember, t(df) = t, p = p-value, d = d
- SE = standard error for each group, NOT Sdifference.
- Each CI here is calculated with df
  of the individual groups, not the total.