

# Optimizing Controller Gains and Time Constants for Dynamic Response Tests on a Virtual Distillation Column

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In Final Form: December 9, 2022

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## Abstract

In this work, we optimized the temperature, pressure, and liquid level controllers in a virtual distillation column for the Aniline-Butanol separation process. The controllers were tuned using the Ziegler-Nichols closed-loop tuning method for both P- and PI-control, giving values for the controller gain ( $K_c$ ) and time constant ( $\tau_c$ ), and were evaluated based on setpoint tracking and disturbance attenuation. As expected, the P-controller was not able to reach the new setpoint during setpoint tracking, and was not able to settle back down to the setpoint during disturbance attenuation. The PI-controller was able to reach the new setpoint during setpoint tracking, however it tended to oscillate the temperature and overshoot the liquid level responses; the PI-controller attenuated disturbances quickly at the cost of minor oscillations, settling back down to the setpoint around 30 seconds after the disturbance was introduced. The Ziegler-Nichols tuning method yields promising results for control systems which can track setpoints and attenuate disturbances, however strictly using this method can also yield dynamic responses that oscillate or otherwise deviate from the setpoint, an important consideration for uses in operations that require high levels of precision. The Ziegler-Nichols tuning method can provide a starting point for more intensive closed-loop tuning methods, such as the Kappa-Tau or Lambda tuning methods.

## 1 Introduction

### Transfer Functions

Process control is a complex field with applications in automobile computer systems, audio processing systems, and chemical reaction systems to name a few. Process control dynamics are difficult to determine in some instances, such as chemical reaction systems: equipment and chemicals need to be procured and sensors need to be calibrated, and setting up these systems requires time and labor. Simulation programs are often utilized to bypass these issues to approximate how the real system will react under certain situations, such as setpoint changes and varying operating conditions. The basis of this experiment lies in process control theory, which gives us a systematic way of determining the speed and stability of a process's response to input changes.

At the heart of process control theory lies the transfer function, denoted as  $G(s)$ , which is derived by taking the Laplace transform of the governing equation for the system. Transfer functions allow us to relate the output of the system (such as outlet temperature, concentration, etc.) to a specific input (such as changes in inlet tem-

perature, molar flowrate, etc.), and combining transfer functions can fully describe the dynamics of a system.

The control of a system is evaluated based on its ability to track setpoints and attenuate input disturbances, which can be described by the combination of the system's transfer functions. The general closed-loop transfer function for setpoint tracking takes the form

$$G_{SP}(s) = \frac{G_p G_f G_c}{1 + G_p G_f G_c G_m} \quad (1)$$

where  $G_p$ ,  $G_f$ ,  $G_c$ , and  $G_m$  are the separate transfer functions for the physical process, final element/valve, controller, and sensor, respectively. The general closed-loop transfer function for disturbance response takes the form

$$G_{DR}(s) = \frac{G_d}{1 + G_p G_f G_c G_m} \quad (2)$$

where  $G_d$  is the transfer function of an input disturbance. For simplicity, we will assume that the final element/valve and sensor have negligible dynamics ( $G_f = G_m = 1$ ) with no time-delay. The transfer function for the physical process,  $G_p$ , was assumed to be first-order.

The controller transfer function,  $G_c$ , plays a crucial role in the dynamics and stability of a system – a first-order system can be transformed into a higher-order system just due to the implementation of a controller, allowing its response to oscillate. Therefore, optimizing the controller to both track setpoints and attenuate disturbances in a quick yet stable way is imperative.

## P- and PI-Control

A majority of process control systems utilize a Proportional-Integral-Derivative (PID) controller, which changes how much control it applies relative to the error between a control variable and its setpoint. Proportional (P) control applies control based solely on the magnitude of the error, which offers a stable and non-oscillatory method for control, however it is not able to completely reach new setpoints or totally attenuate disturbances. Proportional-Integral (PI) control applies control based on the magnitude of the error and the sum of previous errors, stopping only when both the magnitude and sum of errors is zero; this offers generally quicker responses and can completely reach new setpoints and attenuate disturbances.

Optimizing P- and PI-controllers requires the proper controller gain ( $K_c$ ) and time constant ( $\tau_c$ ) for all controllers in a system, and can be obtained by analyzing the dynamic response of the system after being subject to known input step changes. Tuning methods, such as the Ziegler-Nichols closed-loop tuning method studied in this work, can then optimize the controller for quick and stable responses with minimal oscillations.

In our study, we used Part 1 of the Process Control Virtual Laboratory (PCVL) created by the Chemical Engineering department at the University of Colorado, Boulder<sup>2, 3</sup> to obtain the process gain ( $K_p$ ) and time constant ( $\tau_p$ ) for the temperature, pressure, and liquid level responses to 10%, 20%, and 30% input step changes in the heat duty, valve lift, and pump speed. These values were then used as the foundation for finding the ultimate controller gain ( $K_{cu}$ ) and ultimate period ( $P_u$ ) in Part 2 of the PCVL, allowing us to use the Ziegler-Nichols tuning method and develop tuned P- and PI-controllers.

## 2 Results and Discussion

The temperature, pressure, and liquid level underwent step responses due to 10%, 20%, and 30% increases in heat duty, valve lift, and pump speed, respectively. Their responses were recorded and the data was fitted using least-squares regression in Scipy's `scipy.optimize.curve_fit` function.

### Temperature Step Responses

The heat duty, originally at 500 kW, was increased to 550 kW, 600 kW, and 650 kW, and the step responses can be

seen below in Fig. (1) below. The average process gain was  $K_p = 0.1901 \pm 0.001 \text{ kW/K}$ , and the average time constant was  $\tau_p = 89.1091 \pm 1.5597 \text{ sec}$ .

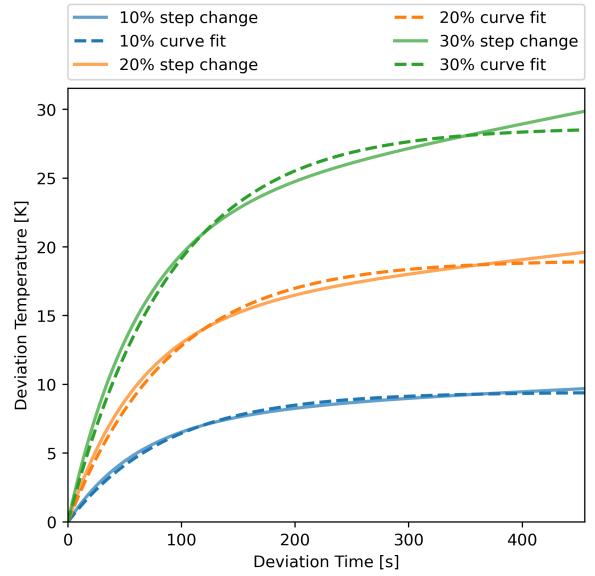


Figure 1: Dynamic temperature response due to input step changes in heat duty.  $K_p = 0.1901 \pm 0.001 \text{ kW/K}$ ,  $\tau_p = 89.1091 \pm 1.5597 \text{ sec}$ .

### Pressure Step Responses

The valve lift, originally at 0.5 (unitless), was increased to 0.55, 0.6, and 0.65, and the step responses can be seen below in Fig. (2) below. The average process gain was  $K_p = -53.8447 \pm 1.8287 \text{ kPa}^{-1}$ , and the average time constant was  $\tau_p = 41.1038 \pm 1.4903 \text{ sec}$ .

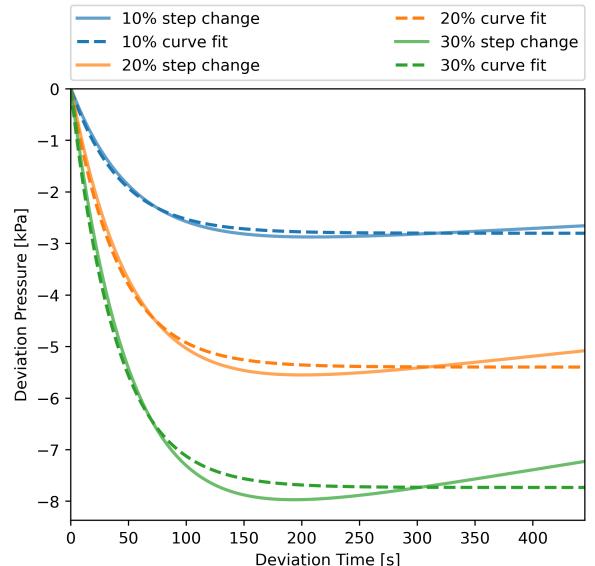


Figure 2: Dynamic pressure response due to input step changes in valve lift.  $K_p = -53.8447 \pm 1.8287 \text{ kPa}^{-1}$ ,  $\tau_p = 41.1038 \pm 1.4903 \text{ sec}$ .

## Liquid Level Step Responses

The pump speed, originally at 0.66 L/s, was increased to 0.726 L/s, 0.792 L/s, and 0.858 L/s, and the step responses can be seen in Fig. (3) below. The average process gain was  $K_p = -174.6542 \pm 14.5275 \text{ L}/(\text{s } \%)$ , and the average time constant (though not theoretically valid) was  $\tau_p = 1790.947 \pm 162.0745 \text{ sec}$ .

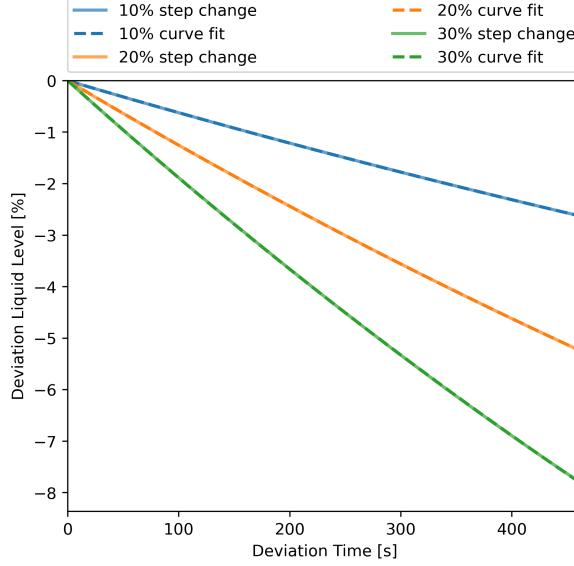


Figure 3: Dynamic pressure response due to input step changes in valve lift.  $K_p = -174.6542 \pm 14.5275 \text{ L}/(\text{s } \%)$ ,  $\tau_p = 1790.947 \pm 162.0745 \text{ sec}$ .

## P-Controller Gains

The ultimate gains,  $K_{cu}$ , for the P-controller were found by trial-and-error, incrementally increasing  $K_c$  until the dynamic response exhibited continued oscillations. The ultimate gains were then tuned using the Ziegler-Nichols method to obtain  $K_{ct}$ . The resulting tuned gains are listed below in Table (1).

Table 1: Tuned gains for each response due to input step changes.

Response	$K_{ct}$
Temperature [kW/K]	131
Pressure [ $\text{kPa}^{-1}$ ]	-0.01375
Liquid Level [L/(s %)]	-10

The values for  $K_{ct}$  were then applied to their respective controller, and the distillation column was then subjected to input step changes. The first step change had a 20% increase in molar flow rate of Butanol, a 20% decrease in the mole fraction of Butanol, and a 20% increase in the inlet temperature. The second step change had 30% changes in each of the listed inlet conditions. The

temperature step response can be seen in Fig. (4), with the others being available in the appendix.

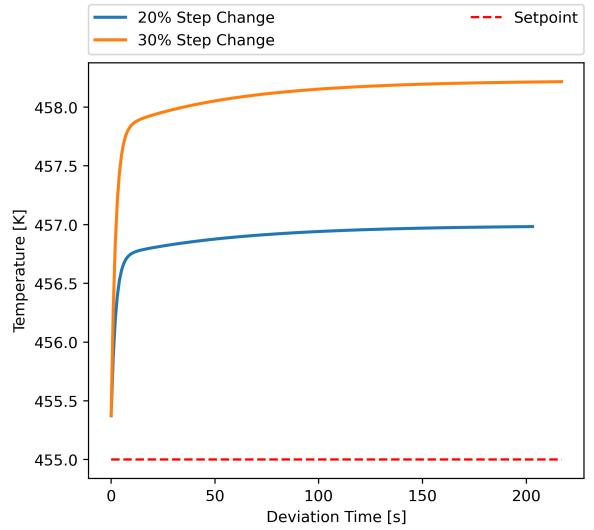


Figure 4: Temperature step response with a P-controller after being subject to 20% and 30% input step changes.

## PI-Controller Gains and Time Constants

The ultimate gains for the PI-controller were obtained similarly to the ones for the P-controller, and the resulting oscillation periods,  $P_u$ , were fitted using the expression  $f(x) = A \cos(\omega t)$ , where  $A$  and  $\omega$  were numerically optimized. Temperature oscillation data and its corresponding fitted cosine wave are shown in Fig. (5).

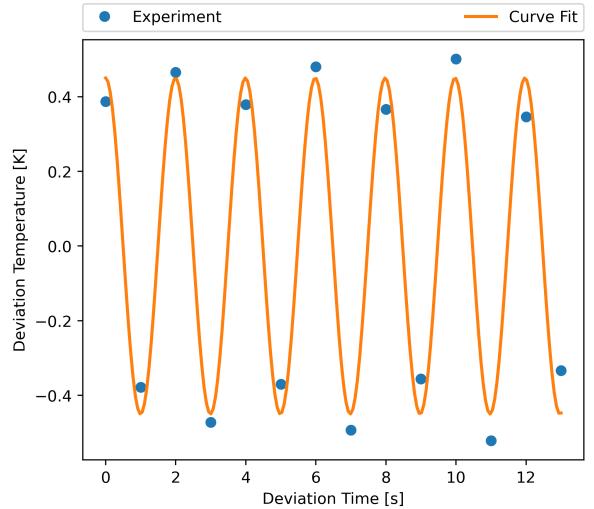


Figure 5: Temperature oscillations and corresponding fitted cosine wave. In this instance,  $A = 0.45 \text{ K}$  and  $\omega = 3.15 \text{ s}^{-1}$ .

Using the Ziegler-Nichols method, the values for  $K_{cu}$  and  $P_u$  were used to obtain  $K_{ct}$  and the controller time constant,  $\tau_c$ .

Table 2: Tuned gains and time constants for each response due to input step changes.

Response	$K_{ct}$	$\tau_c$
Temperature	119.1 (kW/K)	1.66 s
Pressure	-0.0132 (kPa <sup>-1</sup> )	1.67 s
Liquid Level	-9.1 (L(s %))	1.67 s

As before, these values of  $K_{ct}$  and  $\tau_c$  were applied to their respective controller and the distillation column was subject to the same 20% and 30% input step changes as with the P-controller. The temperature step response can be seen in Fig. (6), with the others being available in the appendix.

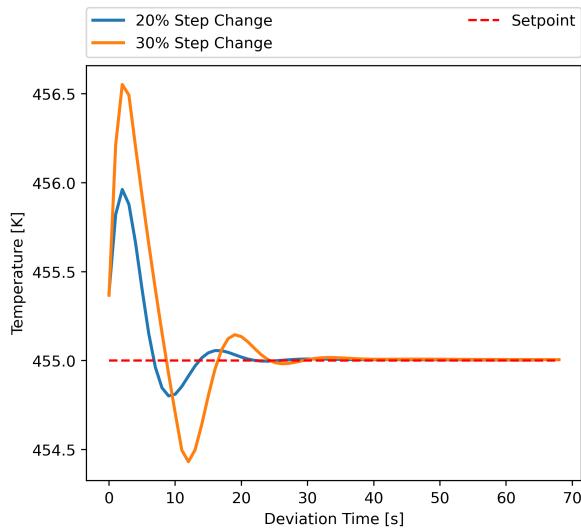


Figure 6: Temperature step response with a PI-controller after being subject to 20% and 30% input step changes.

## Setpoint Tracking

The controllers were then tested on their setpoint tracking capabilities. With the same  $K_{ct}$  and  $\tau_c$  (for the PI-controllers) as before, the setpoint for temperature was increased from 455K to 500K, the setpoint for pressure was increased from 74.6kPa to 76 kPa, and the setpoint for liquid level was increased from 17.6% to 20%. The setpoint tracking of temperature and liquid level can be seen in Fig. (8), with the figure for pressure setpoint tracking being available in the appendix.

In the case for pressure, the PI-controller performed much better than the P-controller, as the PI-controller was able to get the response to completely reach the new setpoint and the P-controller tapered off before reaching the setpoint, as expected. In the case for temperature and liquid level, however, their responses tended to oscillate (temperature) and overshoot the setpoint (liquid level). Increasing their values of  $\tau_c$  appeared to fix this issue, however this would not be adhering strictly to the Ziegler-Nichols method.

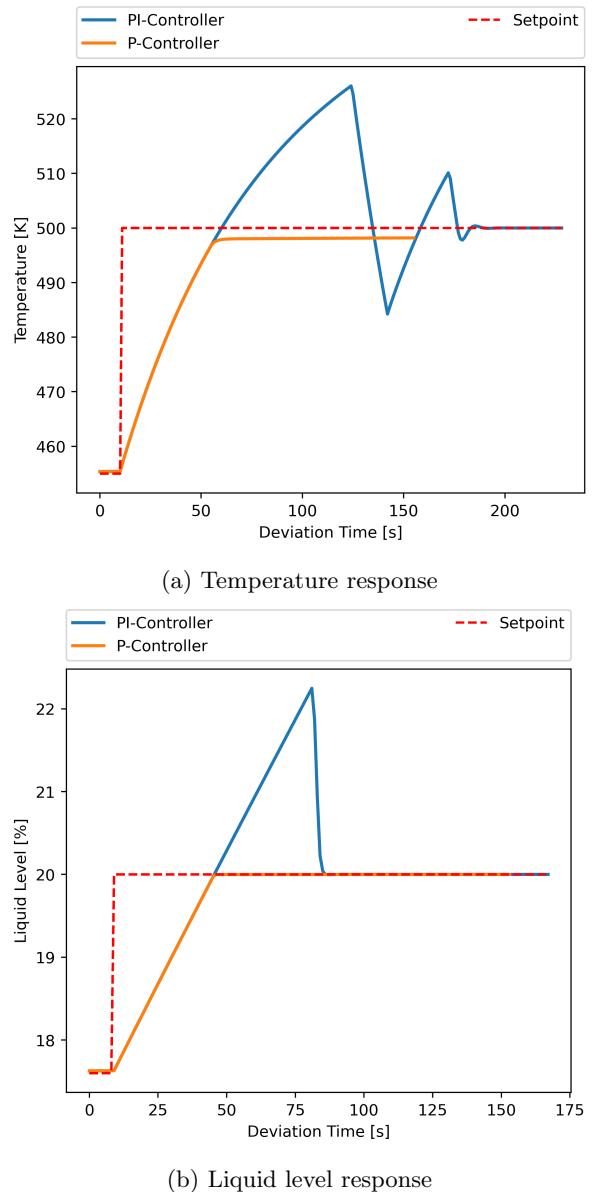


Figure 8: Temperature and liquid level responses for setpoint tracking.

The Ziegler-Nichols tuning resulted in considerable sources of error in the temperature and liquid level setpoint tracking responses, and oscillations in all the disturbance attenuation responses indicates an overall under-damped control system. Though generally quicker than critically-damped or over-damped systems, an under-damped system can introduce unwanted artifacts after even minor disturbances or changes in setpoints. In some instances, like the liquid level setpoint tracking, the PI-controller settled near the setpoint almost twice as long as it took the P-controller.

The cons that the PI-controller has is largely outweighed by its innate ability to return to the setpoint. Given a new setpoint or disturbance, the PI-controller will eventually cause the response to go to the setpoint,

however the oscillations must still be considered, especially for operations that are highly sensitive to changes in operating conditions.

### 3 Conclusion

The Ziegler-Nichols tuning method for control systems offers a simple way of obtaining working controller gains and time constants. Though these gains and time constants work in the sense that they reduce the error in both P- and PI-controllers, they have consistently caused the system to become under-damped (and therefore oscillatory) for the PI-controller. The Ziegler-Nichols tuning method does succeed in creating a stable control system, and can provide a starting point for future tuning using more intensive closed-loop tuning methods, such as the Kappa-Tau or Lambda tuning methods.

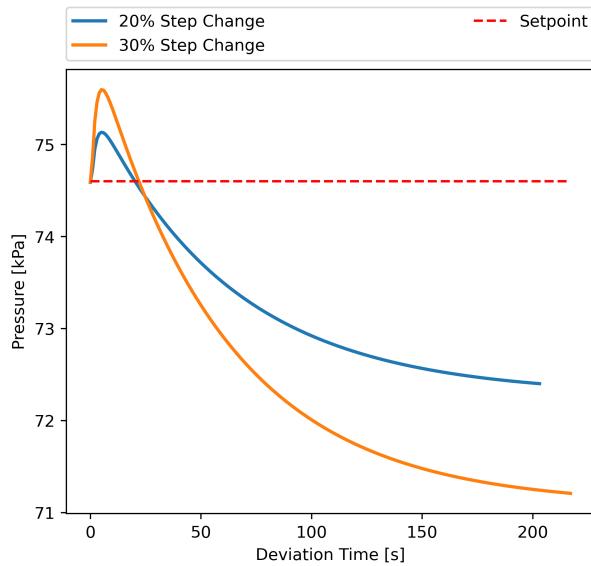
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### References

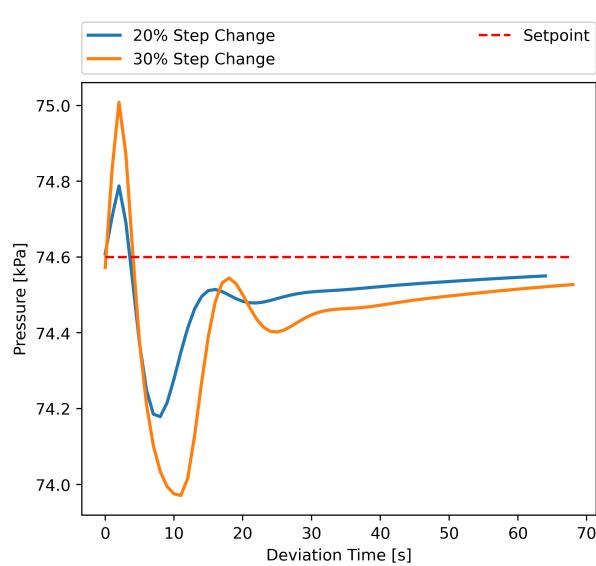
- [1] Ellis, George. Control System Design Guide: Using Your Computer to Understand and Diagnose Feedback Controllers. Butterworth-Heinemann, 2016.
- [2] Rowe, Neil Hendren and Scott. Distillation Lab Pt. 1, <https://virtual-labs.learncheme.com/single-stage-distillation/part-1/>.
- [3] Rowe, Neil Hendren and Scott. Distillation Lab Pt. 2, <https://virtual-labs.learncheme.com/single-stage-distillation/part-2/>.

# Appendix

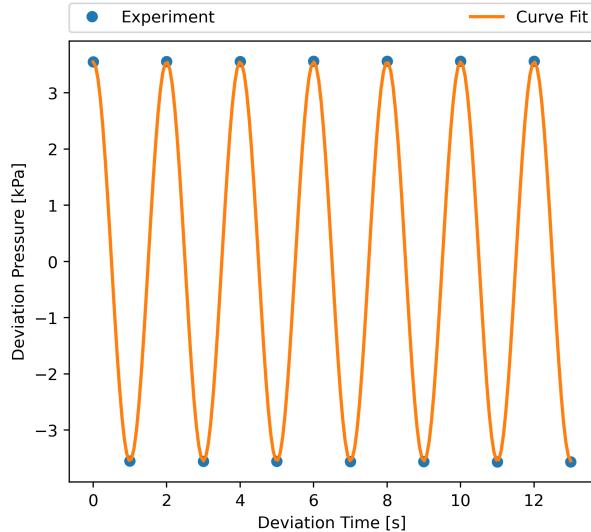
## Pressure Figures



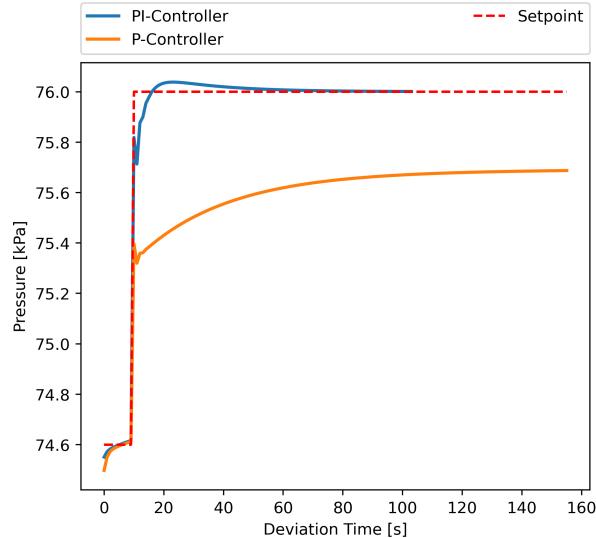
(a) P-control, 20% and 30% input step changes.



(b) PI-control, 20% and 30% input step changes.



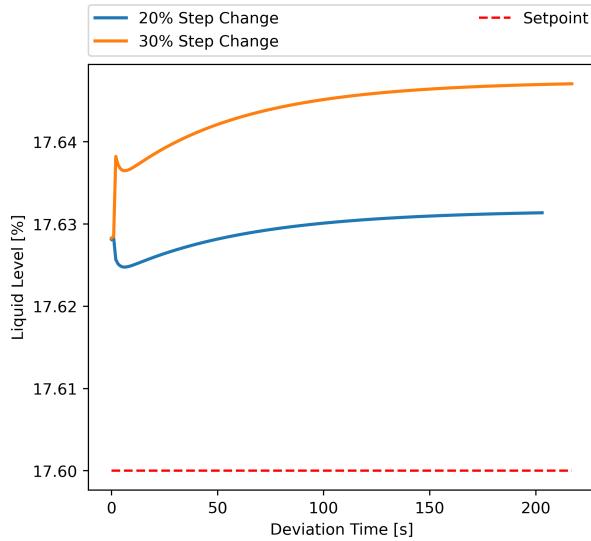
(c) Pressure oscillations and cosine wave.



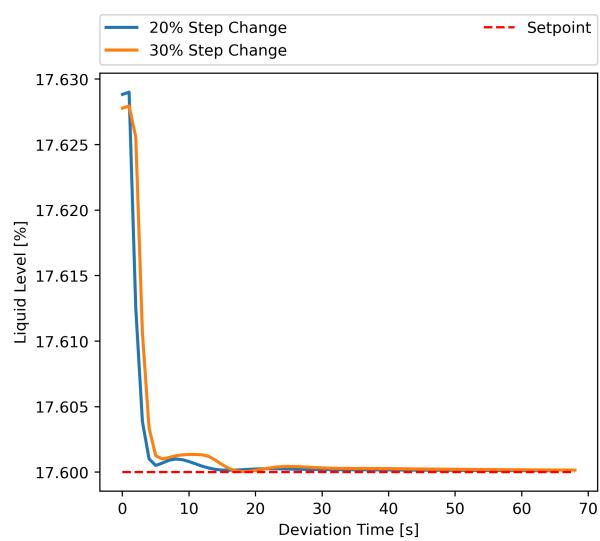
(d) Pressure responses for setpoint tracking.

Figure 9: Pressure Responses for setpoint tracking, disturbance attenuation, and oscillations at ultimate gain.

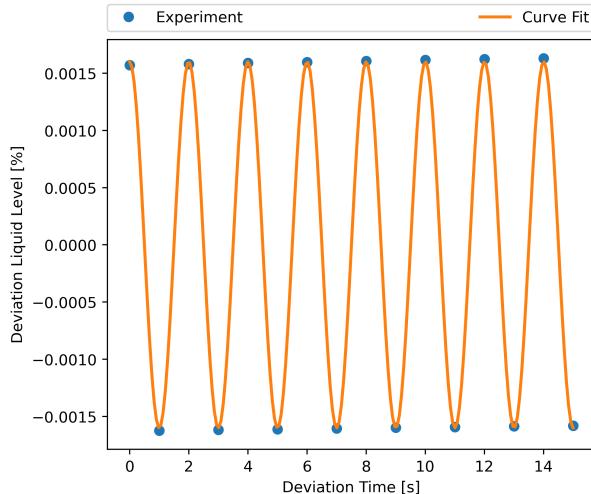
## Liquid Level Figures



(a) P-control, 20% and 30% input step changes.



(b) PI-control, 20% and 30% input step changes.



(c) Liquid level oscillations and cosine wave.

Figure 10: Liquid Level Responses for setpoint tracking, disturbance attenuation, and oscillations at ultimate gain.