

CSC418 A2 Part a

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1.

(a)

$$P1=(0, 0.2828,1)$$

$$P2=(0.1818, 0.5143,1)$$

$$P3=(0.0952, 0.6734,1)$$

$$P4=(-0.1053, 0.4466,1)$$

$$L_{12}=p1 \times p2=(-0.2315,0.1818,-0.05141304)$$

$$L_{23}=p2 \times p3=(-0.1591,-0.0866,0.07346276)$$

$$L_{34}=p3 \times p4=(0.2268,-0.2005,0.11342534)$$

$$L_{41}=p4 \times p1=(0.1638,0.1053,-0.02977884)$$

The vanishing points are where l_{12} and l_{34} intersect, l_{23} and l_{41} intersect

$$l_{12} \times l_{34}=(0.010312,0.014597,0.051835) \text{ or } (1.9895,2.8161,1)$$

$$l_{23} \times l_{41}=(-0.0051568,0.0072954,-0.0025682) \text{ or } (2.00798,-2.84072,1)$$

then the points are $(1.9895,2.8161)$ and $(2.00798,-2.84072)$

(b)

Line $l_1(x_1, y_1, z_1)+t(a, b, c)$

Line $l_2(x_2, y_2, z_2)+t(a, b, c)$

Camera stay at origin facing z-axis:

$$(A_1, B_1)=\left(\frac{x_1+t*a}{z_1+t*c}, \frac{t*b}{z_1+t*c}\right)$$

$$(A_2, B_2)=\left(\frac{x_2+t*a}{z_2+t*c}, \frac{t*b}{z_2+t*c}\right)$$

Since $t \rightarrow \infty$

$$(A_1, B_1) \rightarrow \left(\frac{t*a}{t*c}, \frac{t*b}{t*c}\right)$$

$$(A_2, B_2) \rightarrow \left(\frac{t*a}{t*c}, \frac{t*b}{t*c}\right) \text{ since } x_1 + \infty = \infty \text{ so does } x_2, z_1 \text{ and } z_2$$

Thus $(A_1, B_1) = (A_2, B_2)$

(c)

Consider vanishing points are $(1.9895, 2.8161)$ and $(2.00798, -2.84072)$

If $r_1 \cdot r_2 = 0$ then they are orthogonal

If rays passing through these points are orthogonal

Then $v_1 \cdot v_2 = 0$

But $v_1 \cdot v_2 \approx -4$ which is not equal to 0 thus rays passing through these points are not orthogonal

(d)

Distance from origin to v1 is $\sqrt{1.9895^2 + 2.8161^2} = 3.4480$

Distance from origin to v2 is $\sqrt{2.00798^2 + (-2.84072)^2} = 3.4787$

Distance between v1 to v2 $\sqrt{(2.00798 - 1.9895)^2 + (-2.84072 - 2.8161)^2} = 5.65685$

So we got $3.4480^2 = d^2 + f^2$

$$3.4787^2 = (5.65685 - d)^2 + f^2$$

Equation2 - equation1

Then $0.21265 = 31.9999 - 11.3137d$

Then $11.3137d = 31.78730192$

$$d = 2.80963$$

thus we got $f^2 = 3.4480^2 - 2.80963^2$

$$f = 1.99868$$

thus focal length is 1.99868

(e) since vanishing line is parallel to the 3D rectangle, the normal of the rectangle is also the normal for the plane intersecting the vanishing line and camera so the normal is the cross product of v1 and v2

$$\begin{aligned} v1 \times v2 &= (1.9895, 2.8161, 1) \times (2.00798, -2.84072, 1) \\ &= (5.65682, 0.01848, -11.3063) \end{aligned}$$

2.

Since it is rotate the curve about the y-axis

Which means $x^2 + z^2 = 1$

Thus we set another variable v thus we get equation

$$X(t, v) = (\sin(2\pi t) + 2t) * \sin v$$

$$Y(t, v) = t^2$$

$$Z(t, v) = (\sin(2\pi t) + 2t) * \cos v$$

Where $0 \leq t \leq 5$ $0 \leq v \leq 2\pi$

3.

(a)

$$x_1(t) = x_2(u)$$

$$y_1(t) = y_2(u)$$

$$z_1(t) = z_2(u)$$

$$3 \sin(2\pi t) = u$$

$$3 \cos(2\pi t) = 0$$

$$2\sqrt{2} = \sqrt{u^2 - 1}$$

$$u^2 = 9$$

$$u = \pm 3$$

$$\text{when } u = 3 \quad 3 \sin(2\pi t) = 3 \quad 3 \cos(2\pi t) = 0$$

$t = 0.25 + k$ where k is an integer

point(3,0, $2\sqrt{2}$)

when $u = -3$ $3 \sin(2\pi t) = -3$ $3 \cos(2\pi t) = 0$

$t = -0.25 + k$ where k is an integer

point(-3,0, $2\sqrt{2}$)

(b)

$$f_1'(t) = (6\pi \cos(2\pi t), -6\pi \sin(2\pi t), 0)$$

$$f_2'(u) = (1, 0, \frac{u}{\sqrt{u^2 - 1}})$$

$$\text{when } t=0.25+k \quad f_1'(0.25+k)=(0, -6\pi, 0)$$

$$f_2'(3)=(1, 0, \frac{3\sqrt{2}}{4})$$

$$\text{when } t=-0.25+k \quad f_1'(-0.25+k)=(0, 6\pi, 0)$$

$$f_2'(-3)=(1, 0, -\frac{3\sqrt{2}}{4})$$

(c) when $t=0.25+k$

$$(1, 0, \frac{3\sqrt{2}}{4}) \times (0, -6\pi, 0) = (\frac{9\sqrt{2}}{2}\pi, 0, 6\pi)$$

$$\text{Unit normal vector} = (\frac{3\sqrt{17}}{17}, 0, \frac{2\sqrt{34}}{17})$$

when $t=-0.25+k$

$$(0, 6\pi, 0) \times (1, 0, -\frac{3\sqrt{2}}{4}) = (-\frac{9\sqrt{2}}{2}\pi, 0, -6\pi)$$

$$\text{Unit normal vector} = (-\frac{3\sqrt{17}}{17}, 0, -\frac{2\sqrt{34}}{17})$$

(d) Equation

$$\nabla(p) = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z} \right)$$

$$\text{Normal vector} = (2x, 2y, -2z)$$

(e)

Thus for point(3,0, $2\sqrt{2}$)

$$\text{Normal vector} = (6, 0, -4\sqrt{2})$$

$$\text{Normal unit vector} = (\frac{3\sqrt{17}}{17}, 0, -\frac{2\sqrt{34}}{17})$$

For point (-3,0, $2\sqrt{2}$)

$$\text{Normal vector} = (-6, 0, -4\sqrt{2})$$

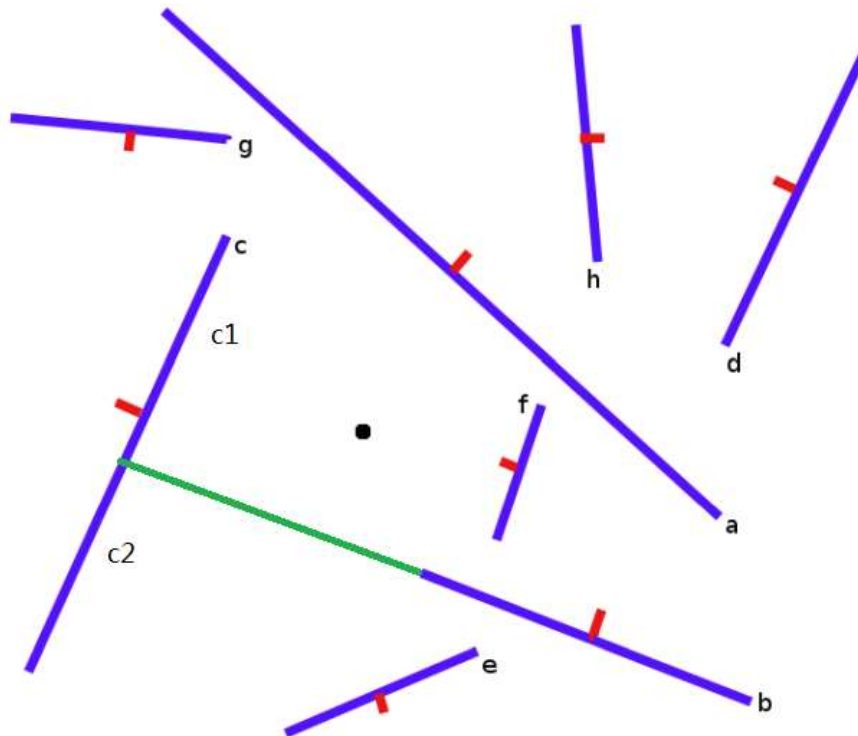
Normal unit vector= $(-\frac{3\sqrt{17}}{17}, 0, -\frac{2\sqrt{34}}{17})$

They are the same in (c).

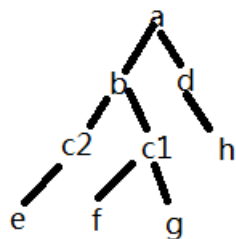
4.

(a) yes, it is possible since plane a c e h are all facing away the camera, they all can be removed by back-culling.

(b)



The BSP will be



(c)

The tree will be traversed in the following order d h a c2 e b g c1 f

If we consider back face culling then the order will be d b g f