CSC418 Assignment 2 – PartA

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Question 1.

(a) Let points x = (2,3), y = (1,2), z = (3, -1) and x' = (10, 8), y' = (8, -4), z' = (2, 0)

Then, $\text{let A} = [\text{x}, \text{y}, \text{z}] = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}, \text{A}' = \begin{bmatrix} 10 & 8 & 2 \\ 8 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix},$

$\text{Suppose affined transformation M} = \begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix}, \text{Then}$

$\text{MA} = \text{A}' =>$

$$\begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2a+3c+e & a+2c+e & 3a-c+e \\ 2b+3b+f & b+2d+f & 3b-d+f \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 8 & 2 \\ 8 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \text{M} = \begin{bmatrix} 0 & 2 & 4 \\ 8 & 4 & -20 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) 3 points required for a general 2D Homography

2 points required for a regid 2D transform.

(c) Centroid preserved.

$\text{Suppose M} = \begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix} \text{ is affine transformation,}$

triangle has verteces $(\text{x}_1, \text{y}_1), (x_2, y_2), (\text{x}_3, \text{y}_3)$ Then:

$\text{centroid C} = \left(\frac{\text{x}_1+\text{x}_2+\text{x}_3}{3}, \frac{\text{y}_1+\text{y}_2+\text{y}_3}{3}\right)$

after transform , three points are $(\text{ax}_1 + \text{cy}_1 + \text{e}, \text{bx}_1 + \text{dy}_1 + \text{f})$

, $(\text{ax}_2 + \text{cy}_2 + \text{e}, \text{bx}_2 + \text{dy}_2 + \text{f})$, $(\text{ax}_3 + \text{cy}_3 + \text{e}, \text{bx}_3 + \text{dy}_3 + \text{f})$

$\therefore \text{centroid C}' = \left(\frac{a(\text{x}_1+\text{x}_2+\text{x}_3)+c(\text{y}_1+\text{y}_2+\text{y}_3)+3e}{3}, \frac{b(\text{x}_1+\text{x}_2+\text{x}_3)+d(\text{y}_1+\text{y}_2+\text{y}_3)+3f}{3}\right)$

$= \text{MC}$

$\therefore$ centroid preserved.

Ortho-center does not preserved.

Suppose triangle ABC, where A=(0, 0), B = (0, 1), C = (1, 0), ortho-center is A

$\text{M} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is affine transformation}$

then after transformation, A'=(0, 0), B' = (1, 1), C' = (1, 0), ortho-center is C,

while MA = A' $\neq$ C

∴ Ortho-center does not preserved.

Question 2.

(a) In real camera, the function of lens is to reflect light and converge it to the image. And it can also help to produce a sharp image with finite opening.

Focal length affects the size of image and the depth of field.

The aperture affect the image's brightness and depth of field.

(b) $\mathrm{w} = -\frac{(-1,2,1)-(2,1,3)}{||(-1,2,1)-(2,1,3)||} = (-\frac{3\sqrt{14}}{14}, \frac{\sqrt{14}}{14}, -\frac{2\sqrt{14}}{14})$, let $\mathrm{t} = (0,1,0)$

$$\mathrm{u} = \frac{\mathrm{t}\times\mathrm{w}}{||t\times w||} = \frac{\left(-\frac{2\sqrt{14}}{14}, 0, \frac{3\sqrt{14}}{14}\right)}{||(-\frac{2\sqrt{14}}{14}, 0, \frac{3\sqrt{14}}{14})||} = (-\frac{2\sqrt{13}}{13}, 0, \frac{3\sqrt{13}}{13})$$

$$\mathrm{v} = \frac{\mathrm{w}\times\mathrm{u}}{||w\times u||} = (\frac{\sqrt{5}}{10}, \frac{13\sqrt{5}}{30}, \frac{\sqrt{5}}{15})$$

$$\therefore \mathrm{M_{wc}} = \begin{bmatrix} u^T \\ v^T \\ w^T \end{bmatrix} = \begin{bmatrix} -\frac{2\sqrt{13}}{13} & 0 & \frac{3\sqrt{13}}{13} \\ \frac{\sqrt{5}}{10} & \frac{13\sqrt{5}}{30} & \frac{\sqrt{5}}{15} \\ -\frac{3\sqrt{14}}{14} & \frac{\sqrt{14}}{14} & -\frac{2\sqrt{14}}{14} \end{bmatrix}$$

Projection matrix from world to camera is:

$$\begin{bmatrix} M_{wc} & -e \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2\sqrt{13}}{13} & 0 & \frac{3\sqrt{13}}{13} & -2 \\ \frac{\sqrt{5}}{10} & \frac{13\sqrt{5}}{30} & \frac{\sqrt{5}}{15} & -1 \\ -\frac{3\sqrt{14}}{14} & \frac{\sqrt{14}}{14} & -\frac{2\sqrt{14}}{14} & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) w = (0, 0, -1), u = (-1, 0, 0), v = (0, 1, 0)

$$\therefore \mathrm{M_{wc}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathrm{p_c} = M_{wc}p = (-p_x, p_y, -p_z)$$

$$\because \frac{1}{f} = \frac{1}{p_z-d} + \frac{1}{d}$$

$$\therefore \text{focal length } \mathrm{f} = \frac{(\mathrm{p_z}-\mathrm{d})\mathrm{d}}{p_z}$$

$$\mathrm{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{pmatrix} -p_x \\ p_y \\ -p_z \end{pmatrix} = \left(\frac{\mathrm{p_x f}}{p_z}, -\frac{\mathrm{p_y f}}{p_z}, f, 1\right)$$

$$=> \text{2D point on image} = \left(\frac{p_x f}{p_z}, -\frac{p_y f}{p_z}\right) = \left(\frac{p_x(p_z-d)d}{p_z^2}, -\frac{p_y(p_z-d)d}{p_z^2}\right)$$

(d) If view direction is orthogonal to direction vector b =(bx, by, bz), then these two lines would still parallel in image. Otherwise, they would not be parallel in image. They would converge at "vanishing point"

If two lines are parallel:

Direction vector (bx, by, bz) is orthogonal to view direction (0, 0, 1)

- ⇨ 0 bx + 0 by + 1 bz = 0;
- ⇨ bz = 0

If two lines are not parallel:

They would converge on "vanishing point", which is a point on line f(x) = (x, 0, d).

Question 3.

(a) normal vector = $\nabla f(x, y, z)$

$$= \left(2x - \frac{2xR}{\sqrt{x^2+y^2}}, 2y - \frac{2yR}{\sqrt{x^2+y^2}}, 2z\right)$$

(b) Let p = (x', y', z'), then:

tangent plane at p is:

$\nabla f(x', y', z') \cdot (x - x', y - y', z - z') = 0$

Where $\nabla f(x', y', z') = \left(2x' - \frac{2x'R}{\sqrt{x'^2+y'^2}}, 2y' - \frac{2y'R}{\sqrt{x'^2+y'^2}}, 2z'\right)$

$$=> \left(2x' - \frac{2x'R}{\sqrt{x'^2+y'^2}}\right)(x - x') + \left(2y' - \frac{2y'R}{\sqrt{x'^2+y'^2}}\right)(y - y') + 2z'(z - z') = 0$$

(c) Apply $q(\lambda)$ into f(x, y, z):

$$f(q(\lambda)) = \left(R - \sqrt{Rcos\lambda^2 + Rsin\lambda^2}\right)^2 + r^2 - r^2 = 0$$

$\therefore q(\lambda)\, lies\ on\ the\ surface$

(d) Tangent vector = $\frac{dq(\lambda)}{d\lambda} = (-R\sin\lambda, R\cos\lambda, 0)$

(e) normal vector at $q(\lambda)$ = $\nabla f(q(\lambda)) = (0, 0, 2r)$

normal vector at $q(\lambda)$ $\perp$ Tangent plane

$\because \left(\text{normal vector at } q(\lambda)\right) \cdot \left(\text{Tangent vector at } q(\lambda)\right) = (-R\sin\lambda, R\cos\lambda, 0) \cdot (0, 0, 2r) = 0$

$\therefore \left(\text{normal vector at } q(\lambda)\right) \perp \left(\text{Tangent vector at } q(\lambda)\right)$

$\because q(\lambda)\, lies\ in\ tangent\ plane$

$\therefore$ Tangent vector at $q(\lambda)$ $lies\ in\ tangent\ plane$

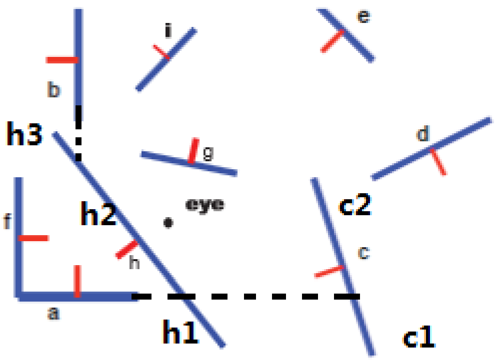
Question 4.

(a) Yes

a would be excluded since it is blocked by h

f would be excluded since it is blocked by h
d would be excluded since it is blocked by c
e would be excluded since it is blocked by g
I would be excluded since it is blocked by g

(b)



The BST is :

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                    a
               /         \
          b                   c1
       /     \             /
     f         c2        h1
   /         /    \
 h3        g        d
         /   \        \
        I     h2       e
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(c) $c_1 \to h_1 \to a \to f \to h_3 \to b \to d \to e \to c_2 \to i \to g \to h_2$