

# MATHS PAPERS

August 29, 2024

## DISCRETE

1. **Assertion(A):**  $a, b, c$  are in  $A.P.$  if and if only if  $2b = a + c$ .  
**Reason(R):** The sum of first  $n$  natural numbers is  $n^2$ .
  - (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
  - (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
  - (c) Assertion (A) is true but Reason (R) is false.
  - (d) Assertion (A) is false but Reason (R) is true.
2. How many terms are there in  $A.P.$  whose first and fifth term are  $-14$  and  $2$ , respectively and the last term is  $62$ .
3. Which term of the  $A.P.$  :  $65, 61, 57, 53, \dots$  is the first negative term?

## POLYNOMIAL EQUATIONS

4. Find the sum and product of the roots of the quadratic equation  $2x^2 - 9x + 4 = 0$ .
5. Find the discriminant of the quadratic equation  $4x^2 - 5 = 0$  and hence comment on the nature of roots of the equation.

6. If one zero of polynomial  $p(x) = 6x^2 + 37x - (k - 2)$  is reciprocal of the other, then find the value of  $k$ .
7. Find the value of ' $p$ ' for which one root of the quadratic equation  $px^2 - 14x + 8 = 0$  is 6 times the other.

## TRIGNOMETRY

8. Evaluate

$$2 \sec^2 \theta + 3 \csc^2 \theta - 2 \sin \theta \cos \theta \text{ if } \theta = 45^\circ. \quad (1)$$

9. If

$$\sin \theta - \cos \theta = 0, \text{ then find the value of } \sin^4 \theta + \cos^4 \theta. \quad (2)$$

10. Prove that

$$\frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A \quad (3)$$

11. Prove that

$$\sec A (1 - \sin A) (\sec A + \tan A) = 1. \quad (4)$$

12. A straight highway leads to the foot of a tower. A man standing on the top of the 75m high tower observes two cars at angles of depression of  $30^\circ$  and  $60^\circ$ , which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. use  $(\sqrt{3} = 1.73)$ .
13. From the top of a 7m building, the angle of elevation of the top of a cable tower is  $60^\circ$  and the angle of depression of its foot is  $30^\circ$ . Determine the height of the tower.

## GEOMETRY

14. From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the center of the circle bisects the angle between the two tangents.
15. Two concentric circles are of radii 5cm and 3cm. Find the length of the chord of the larger circle which touches the smaller circle.
16. In a  $\triangle PQR$ ,  $N$  is a point on  $PR$ , such that  $QN \perp PR$ . If  $PN \times NR = QN^2$ , prove that  $\angle PQR = 90^\circ$ .

17. In the given figure,  $\triangle ABC$  and  $\triangle DBC$  are on the same base  $BC$ .  $AD$  intersects  $BC$  at  $O$ . prove that  $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$

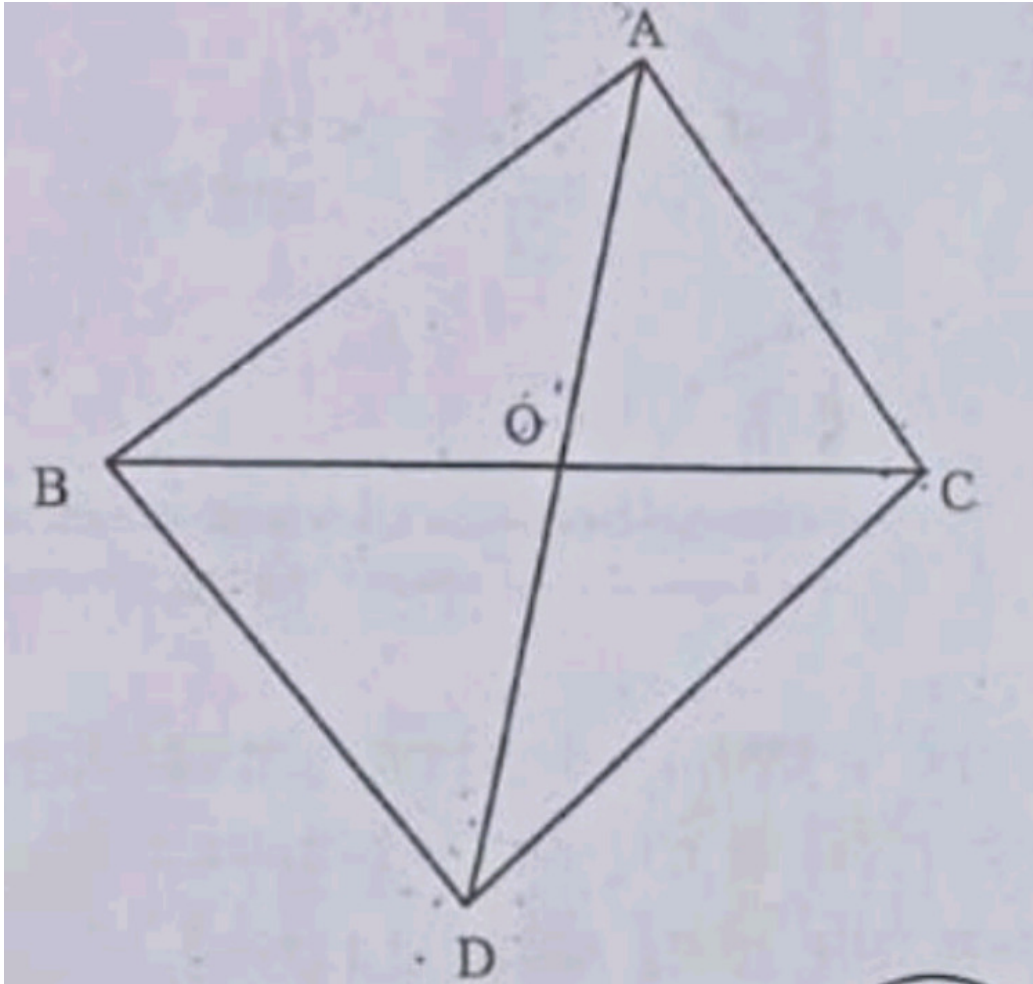


Figure 1: Image 1

18. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10cm and its base is of radius 3.5cm, find the total surface area of the article.

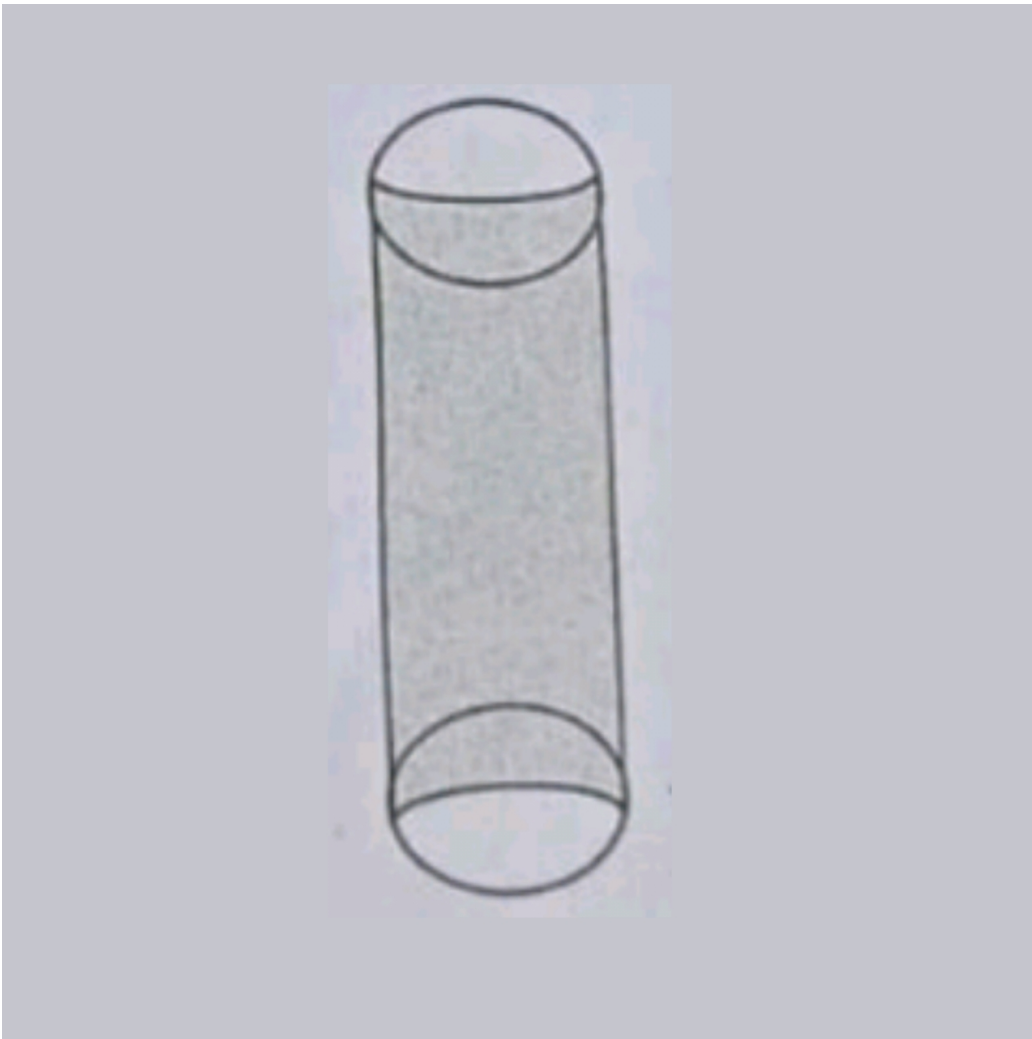


Figure 2: Image 2

19. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking. After survey, it was decided to build

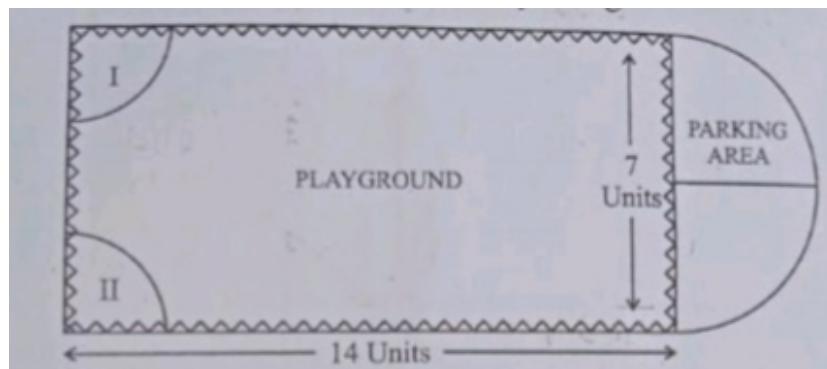


Figure 3: Image 3

rectangular playground, with a semi-circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats. Based on the above information, answer the following questions:

- (a) What is the total perimeter of the parking area?
- (b)
  - i. What is the total area of parking and the two quadrants?
  - ii. What is the ratio of area of playground to the area of parking area?
- (c) Find the cost of fencing the playground and parking area at the rate of ₹2 per unit.

- (d) Two schools ' $P$ ' and ' $Q$ ' decided to award prizes to their students for two games of Hockey ₹ $x$  per student and Cricket ₹ $y$  per student. School ' $P$ ' decided to award a total of ₹9,500 for the two games to 5 and 4 students respectively; while school ' $Q$ ' decided to award ₹7,370 for the two games to 4 and 3 students respectively. Based on the given



Figure 4: Image 4

information, answer the following questions:

- i. Represent the following information in algebraically (in terms of  $x$  and  $y$ ).
- ii. A. What is the prize amount for hockey?  
B. Prize amount on which game is more and by how much?
- iii. What will be the total prize amount if there are 2 students each from two games?

20. Jagadish has a field which is in the shape of a right angled triangle  $AQC$ . He wants to leave a space in the form of a square  $PQRS$  inside the field for growing wheat and the remaining for growing vegetables (*as shown in the figure*). In the field, there is a pole marked as  $O$ . Based on the above information,

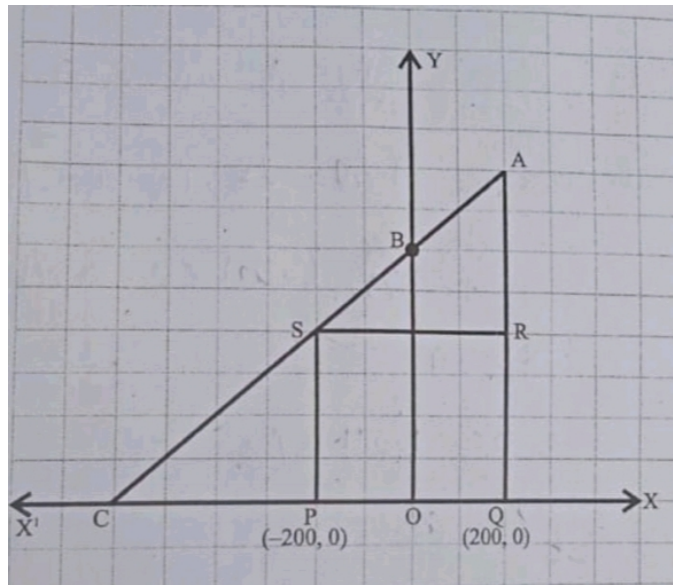


Figure 5: Image 5

answer the following questions:

- (a) Taking  $O$  as origin, coordinates of  $P$  are  $(-200, 0)$  and of  $Q$  are  $(200, 0)$ .  $PQRS$  being a square, what are the coordinates of  $R$  and  $S$ ?
- (b)
  - i. What is the area of square  $PQRS$ ?
  - ii. What is the length of diagonal  $PR$  in square  $PQRS$ ?
- (c) If  $S$  divides  $CA$  in the ratio  $K : 1$ , what is the value of  $K$ , where point  $A$  is  $(200, 800)$ ?

## PROBABILITY

21. If a fair coin is tossed twice, find the probability of getting 'atmost one head'.



## NUMBER SYSTEM

22. Two numbers are in the ratio 2 : 3 and their *LCM* is 180. What is the *HCF* of these numbers?
23. Prove that  $\sqrt{5}$  is an irrational number.

## STATISTICS

24. The monthly expenditure on milk in 200 families of a Housing Society is given below:

Monthly Expenditure ( <i>in</i> )	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of Families	24	40	33	$x$	30	22	16	7

Find the value of  $x$  and also, find the median and mean expenditure on milk.