

Learning Parity

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1 Problem Brief

Given an unknown parity function

$$f : \{0, 1\}^n \rightarrow 0, 1,$$

we want to find a function g that behaves closely to f .

1.1 Application

Modeling for finding "relevant" subsets.

2 Information Theory Perspective

Given the input bitstring is " $x = x_1x_2 \cdots x_n$ ", and the parity function is

$$f(x) = x_{i_1} + x_{i_2} + \cdots + x_{i_k} \pmod{2}.$$

(Short notice that we actually substitute " k " for " n " in the original problem brief).

Let $B = \{x_{i_1}, x_{i_2}, \cdots, x_{i_k}\}$

Notation Definition . Here we specify a notation:

For a input x of length n , we use

$$\text{Parity}(x(S)) = \sum_{i \in S} x_i \pmod{2}.$$

Specifically, if $S = B$, then $\text{Parity}(x(S)) = \text{Parity}(x(B)) = f(x)$

We examine that for a random subset of bits T (the indices of the bits), where $|T| = |B| = k$, if we take s uniform random samples of inputs $x^{(1)}, x^{(2)}, \cdots, x^{(s)}$, what's the probability that it behaves exactly the same as the parity function.

We define event A_s for a given subset S as above: $\forall i \in \mathbb{Z} \cap [1, s]$, we have $\text{Parity}(x(S)) = f(x)$; and because of the property of binary addition, $P(A_s) = \frac{1}{2^s}$, where each input x has a probability of $1/2$ of behaving the same as the parity function.