COMP3121 Assignment 3

Question 1

* 1. Since we already know the last two element and , and the sequence is beautiful, we know the last three elements in the subsequence satisfy the relationship, or . If we denote the third last element by .

Hence, we can find the value of by . Then we may use binary search to find the position of in the original sequence, which is strictly increasing by definition. This binary search cost if total number of elements is n. Thus, total time cost of our algorithm is since finding value of costs constant time.

* 1. According to question 1.1, we know that we can find the third last element in a beautiful sequence if we already know the last two. So we may apply DP and denote be the length of the longest beautiful sequence with the last two elements being the and element in the original sequence, respectively. Then we may dynamically update the entries using:

, if we can find an element that is ;

Otherwise, .

We update the entries in increasing order of both and requiring, and initial entries are given as for all , since all beautiful sequences should have at least 3 elements so the second last element must be indexed 2 or greater. But we set this value to be 2 since later on we may use and as previous two elements. Notice a sequence with length 2 is not considered beautiful but can be a prefix of a beautiful sequence. Finally, we find the maximum entry in T, which costs to secure the length of the global longest sequence.

Since we update elements and each time we use time to do binary search (as we discussed before). So total time cost is .

* 1. Once we find the max length, we may trace back using the relationship between neighboring elements in a beautiful sequence. Since when we go over all elements in to find the maximum length we also have the corresponding indices , at hand, we can back-trace for the 3rd last element using binary search, and the 4th last, and so on and so forth. Each search costs and we need to search for elements, less than the total number of the original sequence. Thus we can rebuild the longest beautiful sequence in time.