Problem_Saet_2

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Link to the GitHub

The link to myh GitHub repository is https://github.com/FYlee39/Stats-506/tree/main/PS2.

Problem 1

a.

Version one:

```
#' Using for loop to implement the game
#'
#' @param n number of dice to roll
#' @return win total winnings
play_dice_v1 <- function(n){
  win <- -2 * n # cost 2 to play a roll
  roll_results <- sample(1: 6, n, replace=TRUE)
  for (single in roll_results){
    if (single == 3 | single == 5){
      win <- win + 2 * single
    }
  }
  return(win)
}

play_dice_v1(10)
```

```
[1] 2
```

Version two:

```r
#' Using built-in R vectorized functions to implement the game
#'
#' @param n number of dice to roll
#' @return win total winnings
play_dice_v2 <- function(n){
  win <- -2 * n # cost 2 to play a roll
  roll_results <- sample(1: 6, n, replace=TRUE)
  win_index <- (roll_results == 3) | (roll_results == 5)
  roll_results[win_index] <- 2 * roll_results[win_index]
  roll_results[!win_index] <- 0
  win <- win + sum(roll_results)
  return(win)
}

play_dice_v2(10)
```

```
[1] -4
```

Version three:

```r
#' Using table to implement the game
#'
#' @param n number of dice to roll
#' @return win total winnings
play_dice_v3 <- function(n){
  win <- -2 * n # cost 2 to play a roll
  roll_results <- table(sample(1: 6, n, replace=TRUE))
  winning <- 6 * ifelse(!is.na(roll_results['3']), roll_results['3'], 0) +
    10 * ifelse(!is.na(roll_results['5']), roll_results['5'], 0)
  win <- win + winning[[1]]
  return(win)
}

play_dice_v3(10)
```

```
[1] 16
```

Version four:

```r
#' Using lapply to implement the game
#'
#' @param n number of dice to roll
#' @return win total winnings
play_dice_v4 <- function(n){
  win <- -2 * n # cost 2 to play a roll
  roll_results <- sample(1: 6, n, replace=TRUE)
  #' Get winning point of given roll
  #'
  #' @param x one roll result
  #' @return point the point gain from this rolling
  get_points <- function(x){
    point = 0
    if (x == 3 | x == 5){
      point = x * 2
    }
    return(point)
  }

  winning <- sum(sapply(roll_results, get_points))
  win <- win + winning
  return(win)
}

play_dice_v4(10)
```

```
[1] 10
```

b.

Test for version one:

```r
play_dice_v1(3)
```

```
[1] 4
```

```r
play_dice_v1(3000)
```

```
[1] 1570
```

Test for version two:

```r
play_dice_v2(3)
```

```
[1] 0
```

```r
play_dice_v2(3000)
```

```
[1] 1920
```

Test for version three:

```r
play_dice_v3(3)
```

```
[1] 10
```

```r
play_dice_v3(3000)
```

```
[1] 1946
```

Test for version four:

```r
play_dice_v4(3)
```

```
[1] -6
```

```r
play_dice_v4(3000)
```

```
[1] 1936
```

c.

To demonstrate the same result, one needs set the same seed before each sampling. For 3:

```r
set.seed(09152024)
play_dice_v1(3)
```

```
[1] 4
```

```r
set.seed(09152024)
play_dice_v2(3)
```

```
[1] 4
```

```r
set.seed(09152024)
play_dice_v3(3)
```

```
[1] 4
```

```r
set.seed(09152024)
play_dice_v4(3)
```

```
[1] 4
```

For 3000:

```r
set.seed(09152024)
play_dice_v1(3000)
```

```
[1] 2344
```

```r
set.seed(09152024)
play_dice_v2(3000)
```

```
[1] 2344
```

```r
set.seed(09152024)
play_dice_v3(3000)
```

```
[1] 2344
```

```r
set.seed(09152024)
play_dice_v4(3000)
```

```
[1] 2344
```

d.

For low input (1,000):

```r
library(microbenchmark)
microbenchmark(v1=play_dice_v1(1000), v2=play_dice_v2(1000),
               v3=play_dice_v3(1000), v4=play_dice_v4(1000))
```

```
Unit: microseconds
 expr    min      lq     mean  median      uq    max neval
   v1  270.1  278.30  314.281  283.50  306.65 2618.2   100
   v2  123.6  134.25  144.758  144.20  152.75  188.7   100
   v3  342.4  372.40  395.363  392.40  410.05  502.8   100
   v4 1234.8 1252.75 1322.515 1273.55 1349.60 3442.4   100
```

For large input (100,000):

```r
library(microbenchmark)
microbenchmark(v1=play_dice_v1(100000), v2=play_dice_v2(100000),
               v3=play_dice_v3(100000), v4=play_dice_v4(100000))
```

```
Unit: milliseconds
 expr      min        lq      mean    median        uq      max neval
   v1  27.2613  33.70495  41.18404  37.99515  42.32890 120.0446   100
   v2  10.9449  11.42630  12.06080  12.08950  12.35245  19.0951   100
   v3  12.6773  13.75390  14.39522  14.53115  15.00865  18.7380   100
   v4 131.5864 162.37990 183.64667 184.07045 196.48575 274.2992   100
```

From two experiments, one can find that among these four function, the implementation using built-in R vectorized functions is the fastest. Mean while, the function using `sapply` is the slowest.

e.

This game is unfair, to defend the decision using a Monte Carlo simulation, the version two will be used.

```
sum <- 0
n <- 100000
for (i in 1: n){
  sum <- sum + play_dice_v2(1)
}
sample_mean <- sum / n
sample_mean
```

```
[1] 0.6645
```

Since the sample mean is greater than zero, one can argue that this is not a fair game.

Problem 2

a

```
raw_data <- read.csv('cars.csv', header=TRUE, sep=',',
                     col.names=c('Height',
                                 'Length',
                                 'Width',
                                 'Driveline',
                                 'Type',
                                 'Hybird',
                                 'Gears',
                                 'Transmission',
                                 'City_mpg',
                                 'Fuel_type',
                                 'Highway_mpg',
                                 'Classification',
                                 'ID',
                                 'Make',
                                 'Model_year',
                                 'Year',
                                 'horsepower',
```

```r
                                  'Torque'))
head(raw_data)
```

```
  Height Length Width         Driveline
1    140    143   202   All-wheel drive
2    140    143   202 Front-wheel drive
3    140    143   202 Front-wheel drive
4    140    143   202   All-wheel drive
5    140    143   202   All-wheel drive
6     91     17    62   All-wheel drive
                                          Type Hybird Gears
1         Audi 3.2L 6 cylinder 250hp 236ft-lbs   True     6
2 Audi 2.0L 4 cylinder 200 hp 207 ft-lbs Turbo   True     6
3 Audi 2.0L 4 cylinder 200 hp 207 ft-lbs Turbo   True     6
4 Audi 2.0L 4 cylinder 200 hp 207 ft-lbs Turbo   True     6
5 Audi 2.0L 4 cylinder 200 hp 207 ft-lbs Turbo   True     6
6        Audi 3.2L 6 cylinder 265hp 243 ft-lbs   True     6
                    Transmission City_mpg Fuel_type Highway_mpg
1 6 Speed Automatic Select Shift       18  Gasoline          25
2 6 Speed Automatic Select Shift       22  Gasoline          28
3                 6 Speed Manual       21  Gasoline          30
4 6 Speed Automatic Select Shift       21  Gasoline          28
5 6 Speed Automatic Select Shift       21  Gasoline          28
6                 6 Speed Manual       16  Gasoline          27
          Classification                         ID Make   Model_year Year
1 Automatic transmission           2009 Audi A3 3.2 Audi 2009 Audi A3 2009
2 Automatic transmission      2009 Audi A3 2.0 T AT Audi 2009 Audi A3 2009
3    Manual transmission         2009 Audi A3 2.0 T Audi 2009 Audi A3 2009
4 Automatic transmission 2009 Audi A3 2.0 T Quattro Audi 2009 Audi A3 2009
5 Automatic transmission 2009 Audi A3 2.0 T Quattro Audi 2009 Audi A3 2009
6    Manual transmission           2009 Audi A5 3.2 Audi 2009 Audi A5 2009
  horsepower Torque
1        250    236
2        200    207
3        200    207
4        200    207
5        200    207
6        265    243
```

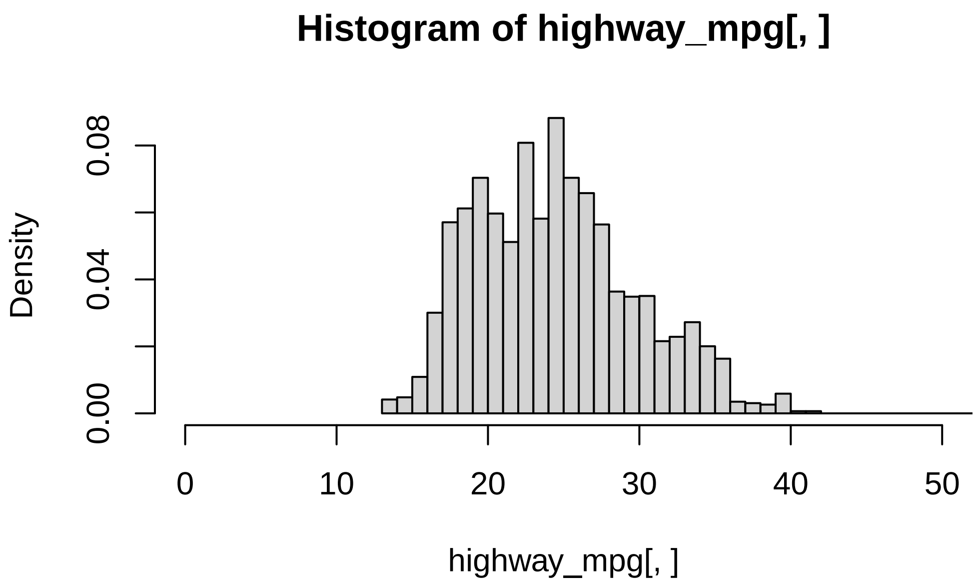
b.

```r
gasoline_data <- raw_data[raw_data['Fuel_type'] == 'Gasoline', ]
```

c.

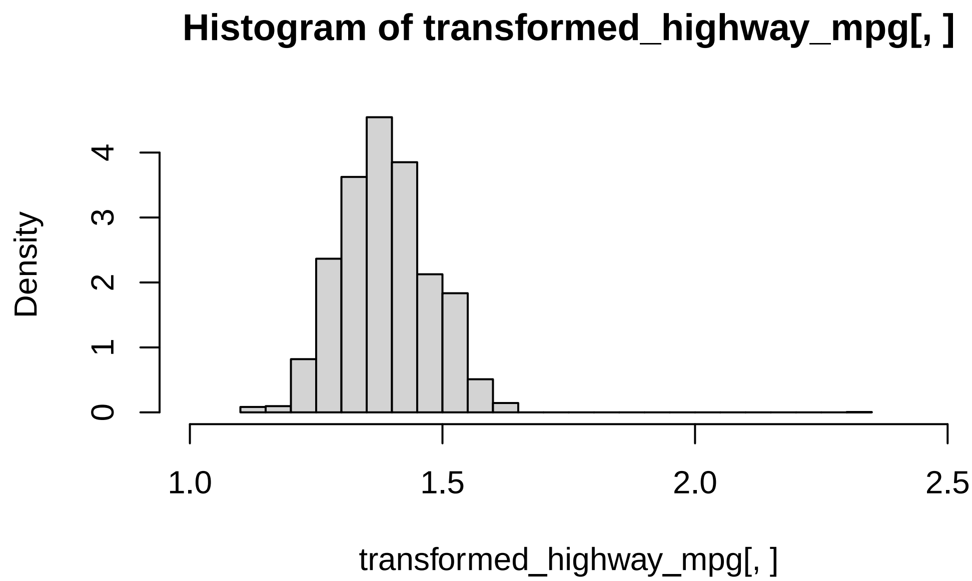
The original data distribution is:

```r
highway_mpg <- gasoline_data['Highway_mpg']
hist(highway_mpg[,], breaks = 200, probability = TRUE, xlim = c(0, 50))
```



Since the data are all positive and the distribution has a right skew with a long tail, a log transformation would likely be the best choice. Then update the data in the data frame.

```r
transformed_highway_mpg <- log10(highway_mpg)
hist(transformed_highway_mpg[,], breaks = 20, probability = TRUE, xlim = c(1, 2.5))
```




```
gasoline_data['Highway_mpg'] <- transformed_highway_mpg
```

d.

```
gasoline_data$Year <- as.factor(gasoline_data$Year)
d_lm <- lm(Highway_mpg~Torque + horsepower
           + Height + Length + Width
           + Year, data=gasoline_data)
summary(d_lm)
```

```
Call:
lm(formula = Highway_mpg ~ Torque + horsepower + Height + Length +
    Width + Year, data = gasoline_data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.23782 -0.04076 -0.00180  0.04297  1.05035

Coefficients:
```

```
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.523e+00  9.625e-03 158.236  < 2e-16 ***
Torque      -9.964e-04  2.934e-05 -33.956  < 2e-16 ***
horsepower   4.012e-04  3.033e-05  13.227  < 2e-16 ***
Height       1.759e-04  1.501e-05  11.719  < 2e-16 ***
Length       1.509e-05  1.177e-05   1.282  0.19980    
Width       -3.788e-05  1.205e-05  -3.144  0.00168 ** 
Year2010    -9.473e-03  9.015e-03  -1.051  0.29342    
Year2011    -1.055e-03  9.000e-03  -0.117  0.90665    
Year2012     1.742e-02  9.071e-03   1.921  0.05485 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0613 on 4582 degrees of freedom
Multiple R-squared:  0.5638,    Adjusted R-squared:  0.563 
F-statistic: 740.3 on 8 and 4582 DF,  p-value: < 2.2e-16
```

From the coefficient of torque, which is -9.964155×10^{-4}, meaning for each additional unit of torque, highway MPG will decrease by -9.964155×10^{-4} while holding other variables constant. The coefficient is significant with a p-value less than 2×10^{-16}, indicating that the relationship is statistically significant.

e.

```r
e_lm <- lm(Highway_mpg ~ horsepower * Torque +
           + Height + Length + Width, data=gasoline_data)
summary(e_lm)
```

```
Call:
lm(formula = Highway_mpg ~ horsepower * Torque + +Height + Length + 
    Width, data = gasoline_data)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.23084 -0.03530 -0.00158  0.03424  1.06415 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)        1.673e+00  6.746e-03 247.952  < 2e-16 ***
```

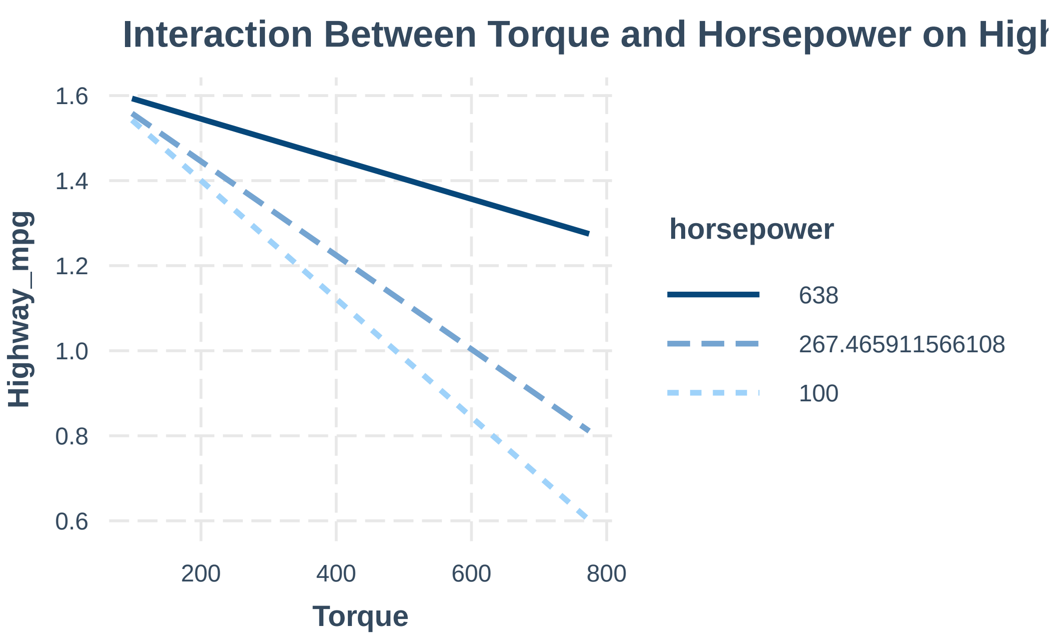
```
horsepower        -7.294e-05  3.359e-05  -2.172   0.0299 *
Torque            -1.562e-03  3.352e-05 -46.618  < 2e-16 ***
Height             1.273e-04  1.414e-05   9.002  < 2e-16 ***
Length             1.042e-05  1.102e-05   0.946   0.3443
Width             -4.992e-05  1.129e-05  -4.422 9.98e-06 ***
horsepower:Torque  1.710e-06  6.139e-08  27.864  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05747 on 4584 degrees of freedom
Multiple R-squared:  0.6165,    Adjusted R-squared:  0.616
F-statistic:  1228 on 6 and 4584 DF,  p-value: < 2.2e-16
```

For the three different horsepower values, they will be the min, mean and the max value.

```r
library(interactions)

interact_plot(e_lm, pred=Torque, modx=horsepower, data=gasoline_data,
              modx.values=c(min(gasoline_data$horsepower),
                              mean(gasoline_data$horsepower),
                              max(gasoline_data$horsepower)),
              main.title="Interaction Between Torque and Horsepower on Highway MPG")
```



f.

```r
design_matrix <-  model.matrix(Highway_mpg ~ Torque + horsepower
           + Height + Length + Width
           + Year, data=gasoline_data)

Y <- gasoline_data$Highway_mpg

design_matrix_t <- t(design_matrix)

XTX_inv <- solve(design_matrix_t %*% design_matrix)

XTY <- design_matrix_t %*% Y

beta_hat <- XTX_inv %*% XTY

beta_hat
```

```
                     [,1]
(Intercept)  1.523037e+00
Torque      -9.964155e-04
horsepower   4.012067e-04
Height       1.758848e-04
Length       1.509263e-05
Width       -3.788045e-05
Year2010    -9.473036e-03
Year2011    -1.055492e-03
Year2012     1.742184e-02
```

```r
manual_coef <- setNames(as.vector(beta_hat), names(d_lm$coefficients))
all.equal(manual_coef, d_lm$coefficients)
```

```
[1] TRUE
```

The result is True, which shows that the manual result is the same with the `lm` result.