Question1

A

last 5 rows of demeaned arithmetic returns

	SPY	AAPL	EQIX
Date			
2024-12-27	-0.011492	-0.014678	-0.006966
2024-12-30	-0.012377	-0.014699	-0.008064
2024-12-31	-0.004603	-0.008493	0.006512
2025-01-02	-0.003422	-0.027671	0.000497
2025-01-03	0.011538	-0.003445	0.015745

standard deviations of demeaned arithmetic returns

SPY	0.008077
AAPL	0.013483
EQIX	0.015361

B

last 5 rows of demeaned log returns

	SPY	AAPL	EQIX
Date			
2024-12-27	-0.011515	-0.014675	-0.006867
2024-12-30	-0.012410	-0.014696	-0.007972
2024-12-31	-0.004577	-0.008427	0.006602
2025-01-02	-0.003392	-0.027930	0.000613
2025-01-03	0.011494	-0.003356	0.015725

standard deviations of demeaned log returns

SPY	0.008078
AAPL	0.013446

EQIX 0.015270

Question 2

A

251862.4969

B

Normal distribution result:

100SPY: VaR = $811.49, ES = $1,017.65

200AAPL: VaR = $958.43, ES = $1,201.91

150EQIX: VaR = $2,905.43, ES = $3,643.52

Portfolio: VaR = $3,829.45, ES = $4,802.28

T and Gaussian Copula result:

100SPY: VaR = $721.24, ES = $1,085.82

200AAPL: VaR = $998.56, ES = $1,506.49

150EQIX: VaR = $3,328.41, ES = $5,040.52

Portfolio: VaR = $4,252.85, ES = $6,218.95

Historical simulation result:

100SPY: VaR = $871.06, ES = $1,078.74

200AAPL: VaR = $1,068.56, ES = $1,438.12

150EQIX: VaR = $3,638.52, ES = $4,710.38

Portfolio: VaR = $4,570.61, ES = $6,053.84

C

1. The normal distribution with exponentially weighted covariance method assumes that returns follow a normal distribution and places greater weight on more recent data using an exponential weighting scheme. This approach is effective in capturing

recent market trends and volatility changes. However, it may underestimate tail risks because normal distributions do not fully account for extreme events. In the results, this method produces the lowest VaR and ES estimates for both individual stocks and the portfolio, suggesting that it may not adequately capture extreme downside risk.

2. The t-distribution with a Gaussian copula method allows for heavier tails in the return distribution, which better accounts for extreme price movements. The Gaussian copula models dependencies between assets, capturing potential joint downturns more effectively. This method generally results in higher VaR and ES values than the normal distribution approach, reflecting its ability to capture greater downside risk. The difference is especially notable for EQIX and the overall portfolio, where the ES is significantly higher than in the normal case, indicating that this method better accounts for extreme losses.

3. The historical simulation method does not make parametric assumptions about return distributions. It relies on historical data to estimate risk, making it particularly useful for capturing real-world extreme events and nonlinear dependencies. This method produces the highest portfolio VaR but a slightly lower ES compared to the t-distribution approach. The results indicate that historical simulation is effective in incorporating past tail events, but its accuracy is dependent on the length and representativeness of historical data.

Question 3

A

iv: 0.3351(33.51%)

B

B.

$$C = S\phi(d_1) - Xe^{-rT}\phi(d_2)$$

$$d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$\Delta = \frac{dc}{ds} = \phi(d_1) = 0.6659$$

$$v = \frac{dc}{d\sigma} = S\sqrt{T}\phi(d_1) = 5.6407$$

$$\Theta = \frac{dc}{dt} = -\frac{S\phi(d_1)\sigma}{2\sqrt{T}} - rXe^{-rT}\phi(d_2) = -5.5446$$

When implied volatility increases 1% option price change $= v \times 0.01 = 0.056407$

$C_1 = 3$ when implied volatility increase 1% $\sigma = 0.3351 + 0.01 = 0.3451$

$$d_1 = \frac{\ln(\frac{31}{30}) + (0.1 + \frac{0.3451^2}{2}) \cdot 0.25}{0.3451 \cdot \sqrt{0.25}} = 0.4212 \qquad d_2 = 0.2562$$

$C_2 = 3.0564$ $\quad c_2 - C_1 = 0.0564$

C

$$d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = 0.4287 \qquad d_2 = 0.2612$$

$$P = Xe^{-rT}\phi(-d_2) - S\phi(-d_1) = 1.2593$$

$$C + Xe^{-rT} = 3 + 30 \cdot e^{-0.1 \cdot 0.25} = 32.2593$$

$$P + S = 1.2593 + 31 = 32.2593$$

$$C + Xe^{-rT} = P + S$$

Put - call Parity holds.

D

Delta Normal Approximation:

VaR = $5.3951

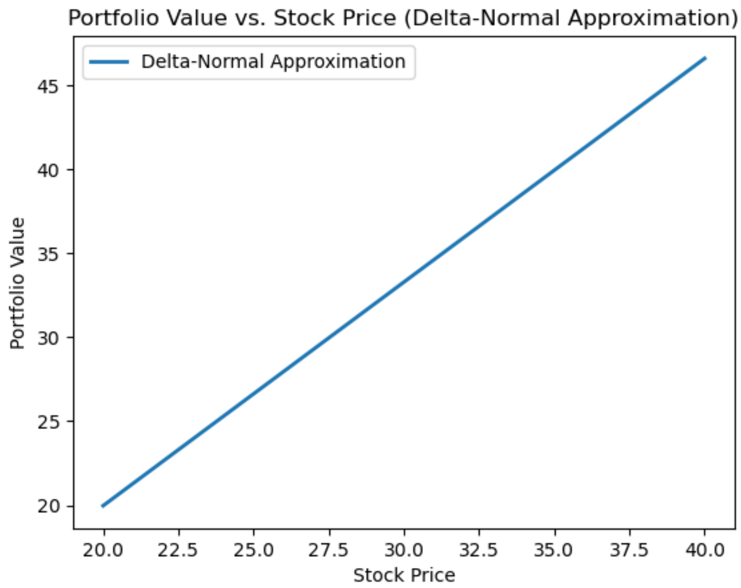
ES = $6.6030

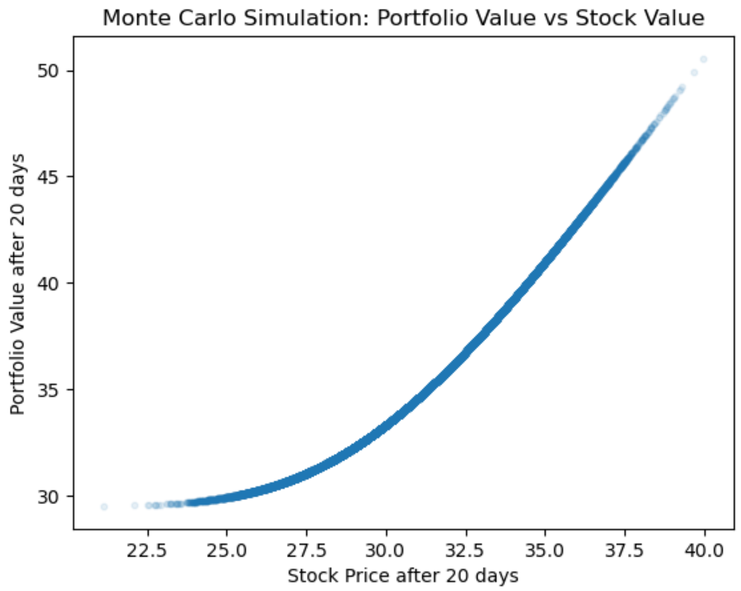
Monte Carlo Simulation:

VaR = $4.2574

ES = $4.7117

E





The differences between the Delta-Normal Approximation and Monte Carlo Simulation methods primarily stem from their assumptions and how they model risk.

1. The Delta-Normal Approximation assumes that portfolio returns are normally distributed and relies on a linear approximation of the portfolio's sensitivity to changes in the stock price (Delta). This method is computationally efficient and easy to implement but does not capture the non-linear behavior of options well, especially as stock prices deviate significantly from their initial values. The graph for this method shows a linear relationship between portfolio value and stock price, highlighting its assumption of linearity. However, this method may underestimate risk

in situations where options exhibit strong convexity, such as near expiration or when volatility is high.

2. The Monte Carlo Simulation, on the other hand, models a broader range of possible stock price movements and captures the non-linear effects of options more accurately. By simulating many possible future stock price paths, this method accounts for the gamma-driven convexity of the option portfolio, making it more suitable for pricing derivatives. The graph for Monte Carlo simulation shows a curved relationship between portfolio value and stock price, reflecting the option's non-linear payoff structure. This method is more computationally intensive but provides a more accurate estimate of risk when options play a significant role in the portfolio.

In terms of risk estimates, the Delta-Normal Approximation yields a higher VaR ($5.3951 vs. $4.2574) and ES ($6.6030 vs. $4.7117) than Monte Carlo Simulation. This discrepancy suggests that the normal approximation may overestimate risk when applied to portfolios with significant option exposure, where actual risk is lower due to the non-linearity of option payoffs. Conversely, if the portfolio contained only linear assets (e.g., stocks and bonds), the Delta-Normal method would likely be more accurate.

3. Overall, the Delta-Normal Approximation is useful for quick risk estimates but may not be reliable when the portfolio contains options with significant convexity. Monte Carlo Simulation provides a more robust risk assessment by incorporating non-linearity but comes at a higher computational cost. The choice between the two depends on the complexity of the portfolio and the computational resources available.