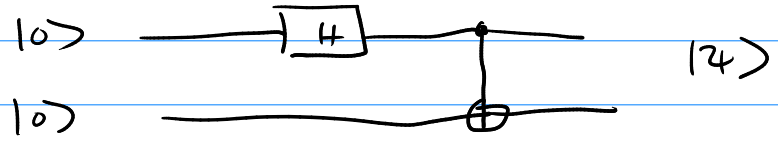
Two qubit state

$$(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)$$

$$\equiv \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a\begin{pmatrix} c \\ d \end{pmatrix} \\ b\begin{pmatrix} c \\ d \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$$

Quantum circuit example

init state



$$|0\rangle \otimes |0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

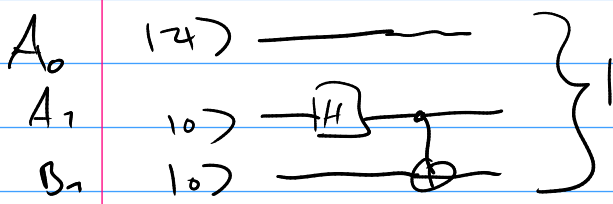
$$= \frac{1}{\sqrt{2}}(|0\rangle \times |0\rangle + |1\rangle \otimes |1\rangle)$$

$$\xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |1\rangle)$$

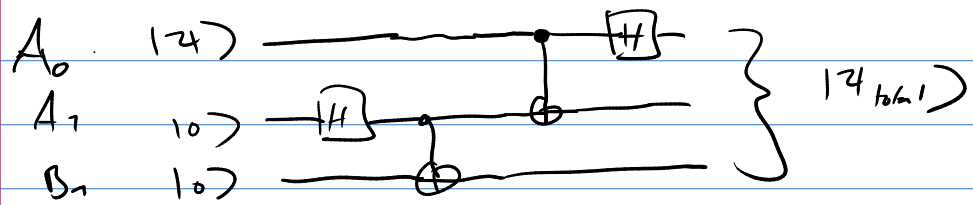
$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \text{(Bell state)}$$

$$= |\Psi\rangle$$

Q - Teleport



$$|\psi_{total}\rangle = |\psi\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

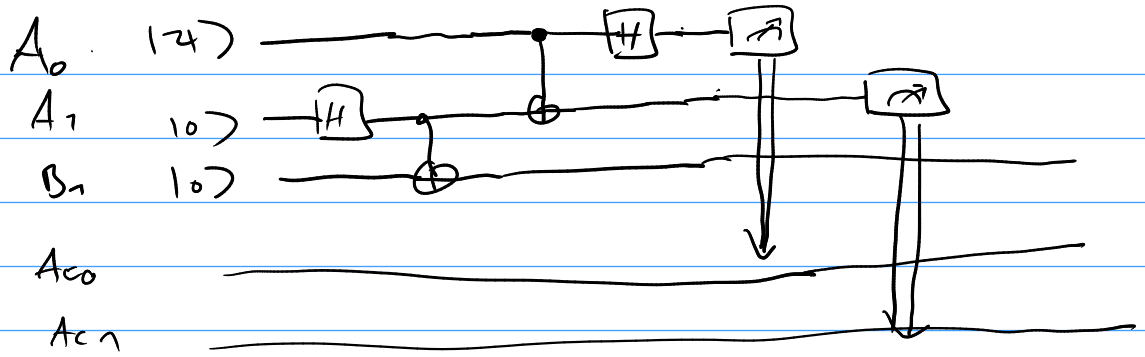


$|\psi\rangle = a|0\rangle + b|1\rangle$

$$|\psi\rangle(|00\rangle + |11\rangle) = (a|000\rangle + b|100\rangle + a|011\rangle + b|111\rangle)$$

$$\xrightarrow{CNOT_{A_0 A_1}} (a|000\rangle + b|110\rangle + a|011\rangle + b|101\rangle)$$

$$\xrightarrow{H_{A_0}} a|000\rangle + a|100\rangle + b|010\rangle - b|110\rangle$$
$$+ a|011\rangle + a|111\rangle + b|001\rangle - b|101\rangle$$



$m = (0,0)$:

$a|000\rangle + a|100\rangle + b|010\rangle - b|110\rangle$

$+a|011\rangle + a|111\rangle + b|001\rangle - b|101\rangle$

$A_{C0} = A_{C1} = 0$

$\rightarrow$ $a|00\rangle + b|01\rangle$

$= |0\rangle \otimes |+\rangle$ ✓

$A_{C0}, A_{C1} = 0,1$

$\rightarrow$ $a|011\rangle + b|010\rangle$

$\xrightarrow{X_{B1}}$ $a|010\rangle + b|011\rangle$

$= |01\rangle \otimes |+\rangle$ ✓

$A_{C0}, A_{C1} = 1,0$

$\rightarrow$ $a|100\rangle - b|101\rangle$

$\xrightarrow{Z_{B1}}$ $a|100\rangle + b|101\rangle$

$= |10\rangle \otimes |+\rangle$ ✓

$A_{C0} = A_{C1} = 1$

$\rightarrow$ $a|111\rangle - b|110\rangle$

$\xrightarrow{X_{B1}}$ $a|110\rangle - b|111\rangle$

$\xrightarrow{Z_{B1}}$ $a|110\rangle + b|111\rangle$

$= |11\rangle \otimes |+\rangle$ ✓