



$$\begin{cases} A = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} R_1 \\ 0 \end{bmatrix} \end{cases}$$

$$\begin{cases} \mathbf{p} = A\hat{\mathbf{x}} = Q_1 Q_1^T \mathbf{b} \end{cases}$$

$$\begin{cases} \boldsymbol{\epsilon} = A\hat{\mathbf{x}} - \mathbf{b} = -Q_2 Q_2^T \mathbf{b} \end{cases}$$

$$\begin{cases} Q_1 Q_1^T \text{ gives projection onto } C(A) \end{cases}$$

$$\begin{cases} Q_2 Q_2^T \text{ gives projection onto } N(A^T) \end{cases}$$

$$\text{rank}(Q_1 Q_1^T) = n$$

$$\text{rank}(Q_2 Q_2^T) = m - n$$