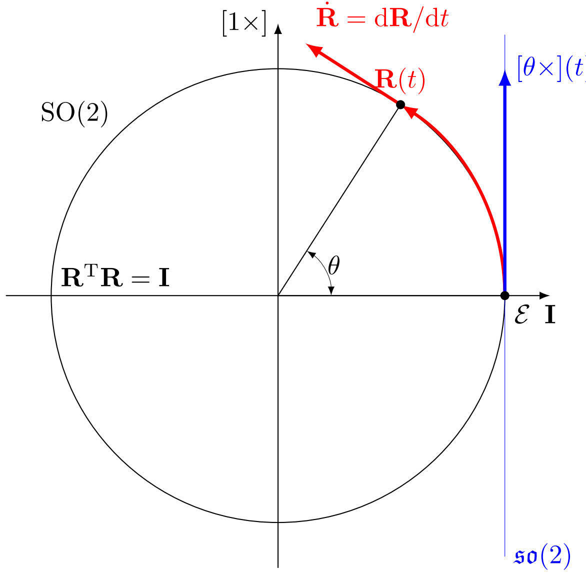
SO(2) : The exponential map



$*$ Write ODE and integrate :

$$\begin{aligned} \mathbf{R}^{\mathrm{T}}\dot{\mathbf{R}} &= [\omega\times] \Rightarrow \dot{\mathbf{R}} = \mathbf{R}\cdot[\omega\times] \\ \mathbf{R}(t) &= \mathbf{R}_0 \cdot \exp\left([\omega\times]\, t\right) \end{aligned}$$

$*$ If $\mathbf{R}_0 = \mathbf{R}(0) = \mathbf{I}$ and $[\omega\times]\, t = [\theta\times] = [1\times]\cdot\theta$

$$\mathbf{R}(t) = \exp\left([\omega\times]\, t\right) = \exp\left([\theta\times]\right)$$

$*$ Taylor expansion, with $[1\times]^0 = \mathbf{I}$ and $[1\times]^3 = -[1\times]$

$$\begin{aligned} \exp\left([\theta\times]\right) &= \mathbf{I} + [1\times]\cdot\theta + \left([1\times]\cdot\theta\right)^2/2! + \left([1\times]\cdot\theta\right)^3/3! + \cdots \\ &= \mathbf{I}\cdot\left(1 - \theta^3/3! + \cdots\right) + [1\times]\cdot\left(\theta - \theta^2/2! + \cdots\right) \\ &= \mathbf{I}\cdot\cos(\theta) + [1\times]\cdot\sin(\theta) \\ &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \end{aligned}$$