

						Tangent space		Exponential map		
						Lie algebra	Cartesian			
Lie group \mathcal{M}, \circ		size	dim	$\mathcal{X} \in \mathcal{M}$	Constraint	$\boldsymbol{\tau}^\wedge \in \mathfrak{m}$	$\boldsymbol{\tau} \in \mathbb{R}^m$	$\text{Exp}(\boldsymbol{\tau})$	Comp.	Action
n -D vector	$\mathbb{R}^n, +$	n	n	$\mathbf{v} \in \mathbb{R}^n$	$\mathbf{v} - \mathbf{v} = \mathbf{0}$	$\mathbf{v} \in \mathbb{R}^n$	$\mathbf{v} \in \mathbb{R}^n$	$\mathbf{v} = \exp(\mathbf{v})$	$\mathbf{v}_1 + \mathbf{v}_2$	$\mathbf{v} + \mathbf{x}$
circle	S^1, \cdot	2	1	$\mathbf{z} \in \mathbb{C}$	$\mathbf{z} * \mathbf{z} = 1$	$i\theta \in i\mathbb{R}$	$\theta \in \mathbb{R}$	$\mathbf{z} = \exp(i\theta)$	$\mathbf{z}_1 \mathbf{z}_2$	$\mathbf{z} \mathbf{x}$
Rotation	$SO(2), \cdot$	4	1	\mathbf{R}	$\mathbf{R}^\top \mathbf{R} = \mathbf{I}$	$[\theta]_\times \in \mathfrak{so}(2)$	$\theta \in \mathbb{R}$	$\mathbf{R} = \exp([\theta]_\times)$	$\mathbf{R}_1 \mathbf{R}_2$	$\mathbf{R} \mathbf{x}$
Rigid motion	$SE(2), \cdot$	9	3	$\mathbf{M} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{bmatrix}$	$\mathbf{R}^\top \mathbf{R} = \mathbf{I}$	$\begin{bmatrix} [\theta]_\times & \boldsymbol{\rho} \\ 0 & 0 \end{bmatrix} \in \mathfrak{se}(2)$	$\begin{bmatrix} \boldsymbol{\rho} \\ \theta \end{bmatrix} \in \mathbb{R}^3$	$\exp\left(\begin{bmatrix} [\theta]_\times & \boldsymbol{\rho} \\ 0 & 0 \end{bmatrix}\right)$	$\mathbf{M}_1 \mathbf{M}_2$	$\mathbf{R} \mathbf{x} + \mathbf{t}$
3-sphere	S^3, \cdot	4	3	$\mathbf{q} \in \mathbb{H}$	$\mathbf{q}^* \mathbf{q} = 1$	$\boldsymbol{\theta}/2 \in \mathbb{H}_p$	$\boldsymbol{\theta} \in \mathbb{R}^3$	$\mathbf{q} = \exp(\mathbf{u}\boldsymbol{\theta}/2)$	$\mathbf{q}_1 \mathbf{q}_2$	$\mathbf{q} \mathbf{x} \mathbf{q}^*$
Rotation	$SO(3), \cdot$	9	3	\mathbf{R}	$\mathbf{R}^\top \mathbf{R} = \mathbf{I}$	$[\boldsymbol{\theta}]_\times \in \mathfrak{so}(3)$	$\boldsymbol{\theta} \in \mathbb{R}^3$	$\mathbf{R} = \exp([\boldsymbol{\theta}]_\times)$	$\mathbf{R}_1 \mathbf{R}_2$	$\mathbf{R} \mathbf{x}$
Rigid motion	$SE(3), \cdot$	16	6	$\mathbf{M} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{bmatrix}$	$\mathbf{R}^\top \mathbf{R} = \mathbf{I}$	$\begin{bmatrix} [\boldsymbol{\theta}]_\times & \boldsymbol{\rho} \\ 0 & 0 \end{bmatrix} \in \mathfrak{se}(3)$	$\begin{bmatrix} \boldsymbol{\rho} \\ \boldsymbol{\theta} \end{bmatrix} \in \mathbb{R}^6$	$\exp\left(\begin{bmatrix} [\boldsymbol{\theta}]_\times & \boldsymbol{\rho} \\ 0 & 0 \end{bmatrix}\right)$	$\mathbf{M}_1 \mathbf{M}_2$	$\mathbf{R} \mathbf{x} + \mathbf{t}$