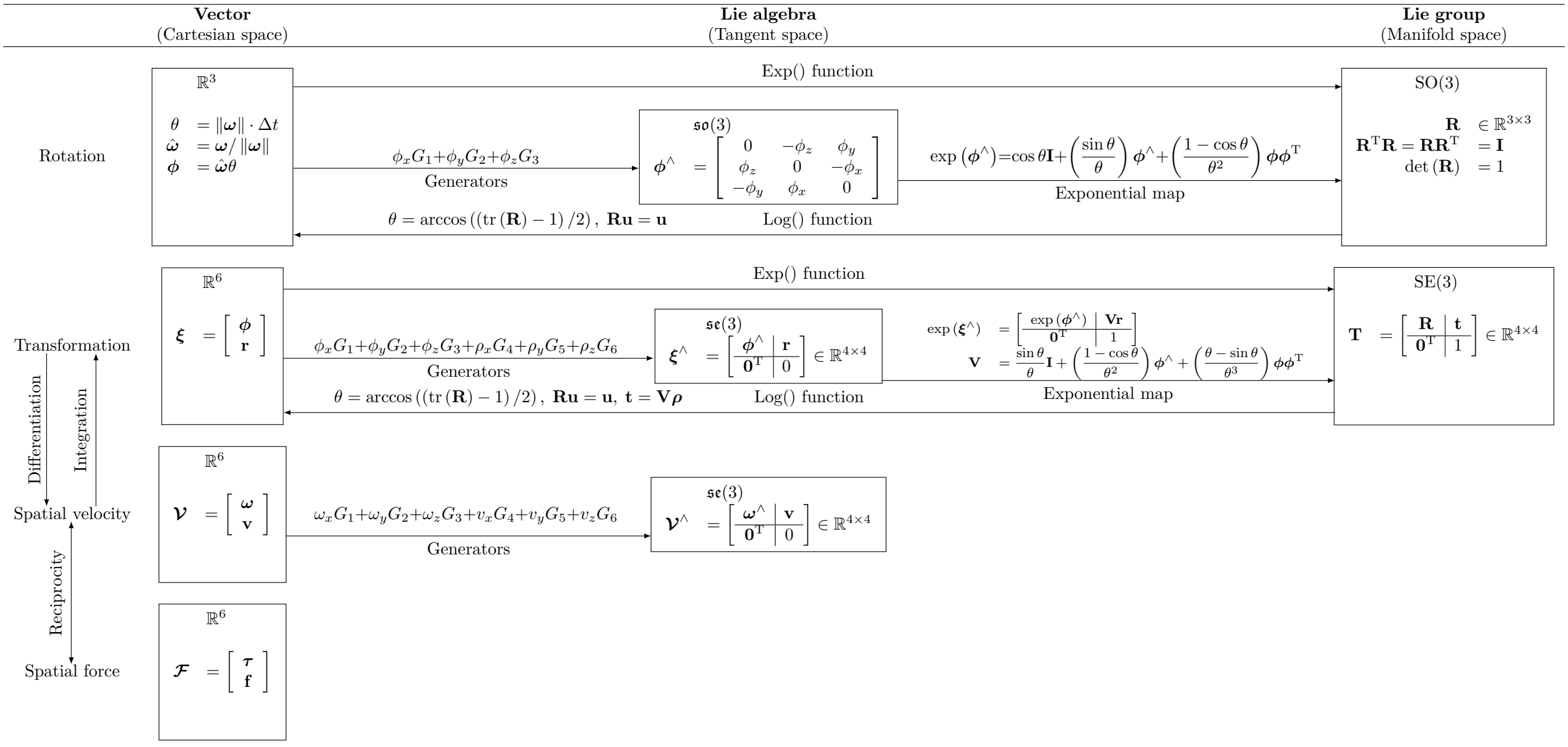
Vector
(Cartesian space)
Lie algebra
(Tangent space)
Lie group
(Manifold space)
Rotation
$\mathbb{R}^3$
$\theta = \|\boldsymbol{\omega}\| \cdot \Delta t$
$\hat{\boldsymbol{\omega}} = \boldsymbol{\omega} / \|\boldsymbol{\omega}\|$
$\boldsymbol{\phi} = \hat{\boldsymbol{\omega}}\theta$
Exp() function
$\phi_x G_1 + \phi_y G_2 + \phi_z G_3$
Generators
$\mathfrak{so}(3)$
$\boldsymbol{\phi}^\wedge = \begin{bmatrix} 0 & -\phi_z & \phi_y \\ \phi_z & 0 & -\phi_x \\ -\phi_y & \phi_x & 0 \end{bmatrix}$
$\exp(\boldsymbol{\phi}^\wedge) = \cos\theta\mathbf{I} + \left(\frac{\sin\theta}{\theta}\right)\boldsymbol{\phi}^\wedge + \left(\frac{1-\cos\theta}{\theta^2}\right)\boldsymbol{\phi}\boldsymbol{\phi}^\mathrm{T}$
Exponential map
SO(3)
$\mathbf{R} \in \mathbb{R}^{3\times3}$
$\mathbf{R}^\mathrm{T}\mathbf{R} = \mathbf{R}\mathbf{R}^\mathrm{T} = \mathbf{I}$
$\det(\mathbf{R}) = 1$
$\theta = \arccos\left((\mathrm{tr}(\mathbf{R}) - 1)/2\right),\ \mathbf{R}\mathbf{u} = \mathbf{u}$
Log() function
Transformation
Differentiation
Integration
$\mathbb{R}^6$
$\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\phi} \\ \mathbf{r} \end{bmatrix}$
Exp() function
$\phi_x G_1 + \phi_y G_2 + \phi_z G_3 + \rho_x G_4 + \rho_y G_5 + \rho_z G_6$
Generators
$\mathfrak{se}(3)$
$\boldsymbol{\xi}^\wedge = \left[\begin{array}{c|c} \boldsymbol{\phi}^\wedge & \mathbf{r} \\ \hline \mathbf{0}^\mathrm{T} & 0 \end{array}\right] \in \mathbb{R}^{4\times4}$
$\exp(\boldsymbol{\xi}^\wedge) = \left[\begin{array}{c|c} \exp(\boldsymbol{\phi}^\wedge) & \mathbf{V}\mathbf{r} \\ \hline \mathbf{0}^\mathrm{T} & 1 \end{array}\right]$
$\mathbf{V} = \frac{\sin\theta}{\theta}\mathbf{I} + \left(\frac{1-\cos\theta}{\theta^2}\right)\boldsymbol{\phi}^\wedge + \left(\frac{\theta-\sin\theta}{\theta^3}\right)\boldsymbol{\phi}\boldsymbol{\phi}^\mathrm{T}$
Exponential map
SE(3)
$\mathbf{T} = \left[\begin{array}{c|c} \mathbf{R} & \mathbf{t} \\ \hline \mathbf{0}^\mathrm{T} & 1 \end{array}\right] \in \mathbb{R}^{4\times4}$
$\theta = \arccos\left((\mathrm{tr}(\mathbf{R}) - 1)/2\right),\ \mathbf{R}\mathbf{u} = \mathbf{u},\ \mathbf{t} = \mathbf{V}\boldsymbol{\rho}$
Log() function
Spatial velocity
$\mathbb{R}^6$
$\boldsymbol{\mathcal{V}} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix}$
$\omega_x G_1 + \omega_y G_2 + \omega_z G_3 + v_x G_4 + v_y G_5 + v_z G_6$
Generators
$\mathfrak{se}(3)$
$\boldsymbol{\mathcal{V}}^\wedge = \left[\begin{array}{c|c} \boldsymbol{\omega}^\wedge & \mathbf{v} \\ \hline \mathbf{0}^\mathrm{T} & 0 \end{array}\right] \in \mathbb{R}^{4\times4}$
Reciprocity
Spatial force
$\mathbb{R}^6$
$\boldsymbol{\mathcal{F}} = \begin{bmatrix} \boldsymbol{\tau} \\ \mathbf{f} \end{bmatrix}$