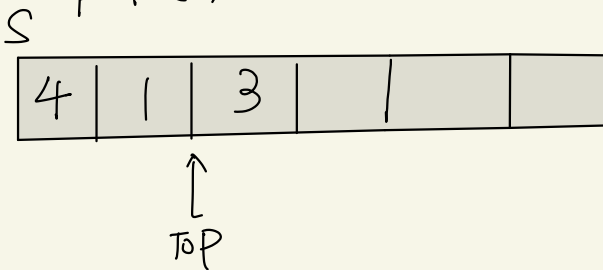
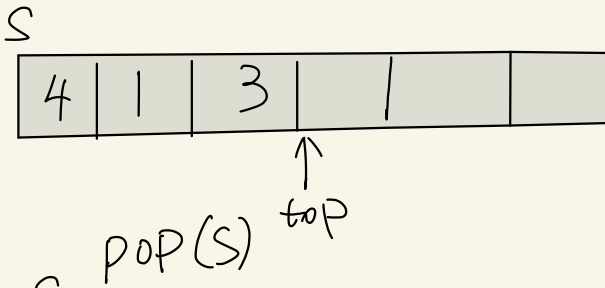
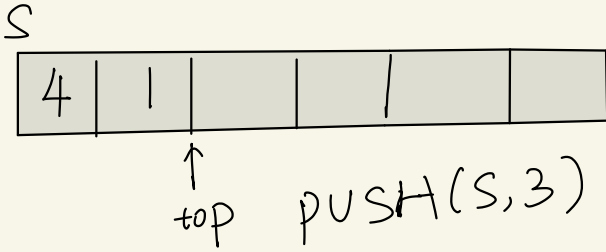
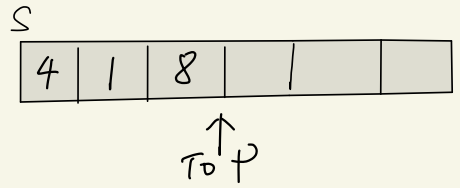


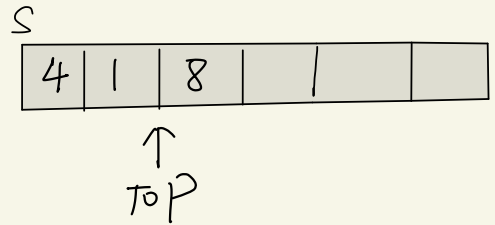
PUSH(S, 1)



PUSH(S, 8)



POP(S)



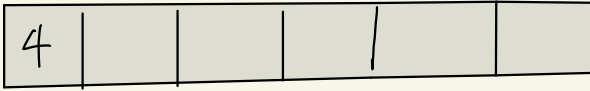
10.1.2 the TOPs pointer of the two stacks starts at two ends of the array.

10.1.3 head
Q tail



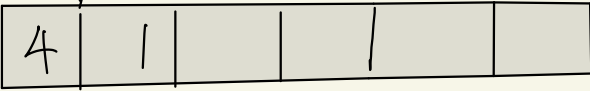
ENQUEUE(Q, 4)

Q head tail



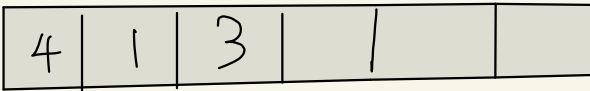
ENQUEUE(Q, 1)

Q head tail



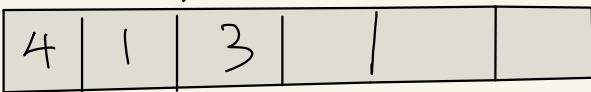
ENQUEUE(Q, 3)

Q head tail

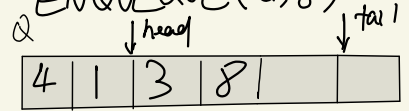


DEQUEUE(Q)

Q head tail

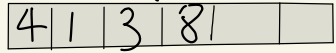


ENQUEUE(Q, 8)



DEQUEUE(Q)

Q head tail



10.1-4

QUEUE-FULL(head, tail)

if tail == Q.Length,

head = 1

return true

if Q.head = Q.tail + 1

return true

else

return false

DEQUEUE-UNDERFLOW(Q, x)

if QUEUE-EMPTY(Q.head, Q.tail)

error "underflow"

else

DEQUEUE(Q, x)

QUEUE-EMPTY(head, tail)

if tail == head

return true

else

return false

ENQUEUE-OVERFLOW(Q, x)

if QUEUE-FULL(Q.head, Q.tail)

error "overflow"

else

ENQUEUE(Q, x)

10.1-5 x

10.1-6

ENQUEUE(A, B, x)
while not STACK-EMPTY(A)

y = POP(A)

B.PUSH(y)

$O(n)$

A.PUSH(x)

while not STACK-EMPTY(B)

y = POP(B)

A.PUSH(y)

DEQUEUE(A)

return A.POP()

$O(1)$

10.1-7

PUSH(A, B, x)

while not A.QUEUE-EMPTY()

y = A.DEQUEUE()

B.ENQUEUE(y)

$O(n)$

A.ENQUEUE(x)

while not B.QUEUE-EMPTY()

y = B.DEQUEUE()

A.ENQUEUE(y)

POP(A)

return A.DEQUEUE()

10.2 -1

INSERT(L, x)

x.next = L.head

L.head = x O(1)

DELETE(L, x)

y = L.head

while y != nil

if y.next == x

y.next = x.next

return

y = y.next

10.2.2

PUSH(SL, X)

INSERT(SL, X) $O(1)$

POP(SL)

result = SL.head

SL.head = SL.head.next

return result $O(1)$

10.2-3

ENQUEUE(SL, X)

SL.tail.next = X

X.next = nil

SL.tail = X

$O(1)$

DEQUEUE(SL)

result = SL.head

SL.head = SL.head.next

return result.

$O(1)$

10.2-4

$L.nil.key = x.key$ [?] check k cannot be nil?

10.2-5

SL:

INSERT(SL, x) $O(1)$

DELETE(SL, x) $O(n)$

SEARCH(SL, x) $O(n)$

CL:

INSERT(SL, x) $O(1)$

DELETE(SL, x) $O(1)$

SEARCH(SL, x) $O(n)$

10.2-6

circular array with sentinel

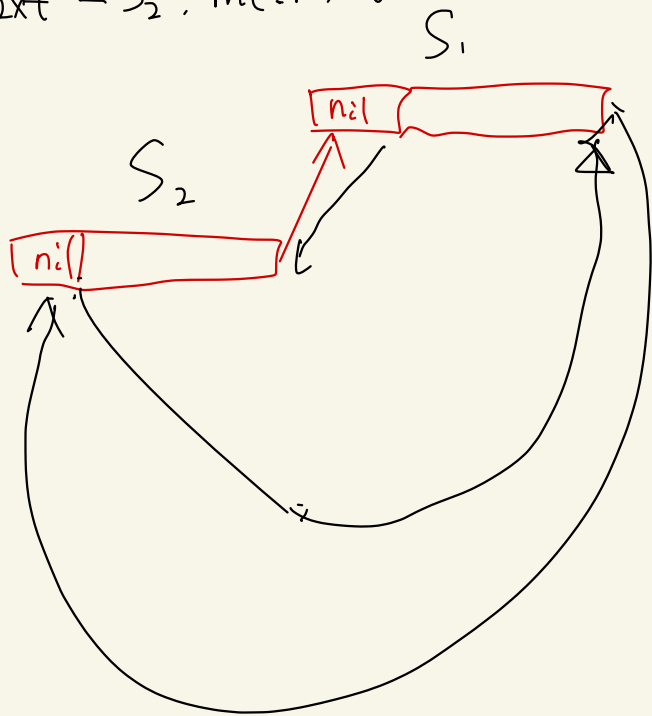
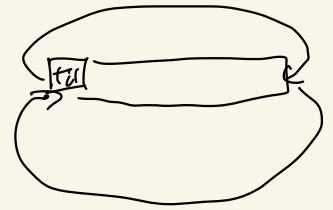
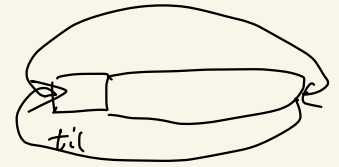
$UNION(S_1, S_2)$

$S_2.nil.prev.next = S_1.nil$

$S_2.nil.next.prev = S_1.nil.prev$

$S_1.nil.prev = S_2.nil.prev$

$S_1.nil.prev.next = S_2.nil.next$



10.2-7

REVERSE(SL)

if SL.head == nil
return

x = SL.head

next = x.next

x.next = nil

while next != nil

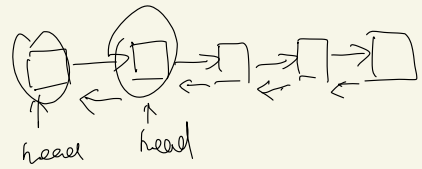
nextNext = next.next

next.next = x

x = next

next = nextNext

SL.head = x



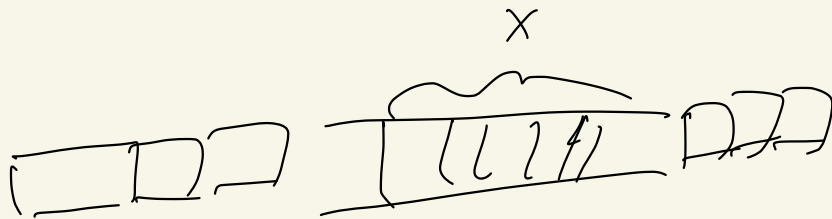
$O(n)$

10.3-1

prev	7	1	2	3	4	5	6
key	nil	¹ 3	³ 4	⁴ 8	⁵ 9	⁶ 5	⁷ 11
next	2	3	4	5	6	7	1

nil 22 4 13 1 7 4 4 10 8 7 13 19 10 16 5 13 19 6 16 22

10.3-2

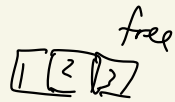


FREE-OBJECT(A, X, offset)

for $i = 0$ to offset

$$A[i+X] = \text{free}$$

$$\text{free} = i+X$$



ALLOCATE-OBJECT(A, offset)

if free == nil

error "out of space"

else

$$x = \text{free}$$

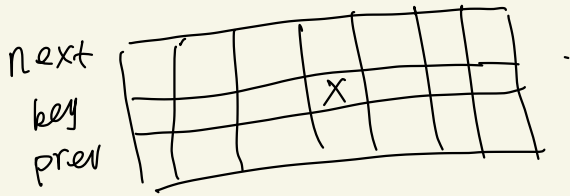
$$\text{free} = \text{free} - \text{offset}$$

return X

10.3-3

we only need
the next for
the singly-linked list

10.3-4 FIFO



ALLOCATE()

if $\text{next}[0] == \text{next.length}$
error "out of space" $O(1)$

return $\text{next}[0] + 1$

FREE-OBJECT(X) $O(n)$

for $i = x$ to $\text{next}[0]$

$\text{key}[x] = [x+1]$

$\text{next}[x] = \text{next}[x+1]$

$\text{prev}[x] = \text{prev}[x+1]$

$\text{next}[0] = \text{next}[0] - 1$

$\text{prev}[\text{next}[0]] = 1$

10.3.5

COMPACTIFY-LIST (L, F)

$i = j = 1$

$x = L.head$

$y = F.head$

while $i < n$ and $j < m-n$

while $x \leq n$

$x = x.next$

$i = i + 1$

while $y \geq n+1$

$y = y.next$

$j = j + 1$

$y.prev.next = x$

$x.prev.next = y$

$temp = x.key$

$x.key = y.key$

$y.key = temp$

$temp = x.next$

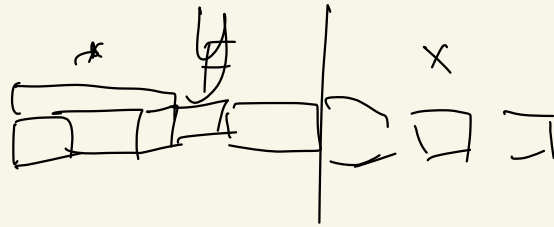
$x.next = y.next$

$y.next = temp$

$temp = x.prev$

$x.prev = y.prev$

$y.prev = x.prev$

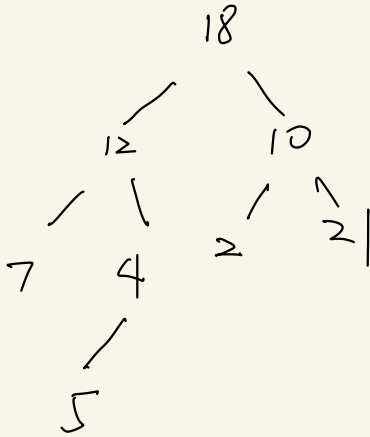


loop-invariant:

$1, \dots, i$ are located $\leq n$

$1, \dots, j$ are located $\geq n+1$

10.4 - 1



10.4 - 2 $O(n)$

PRINT-BINARY-TREE(

if root != nil

print (root. key)

PRINT-TREE-RECURSIVE (root. left)

PRINT-TREE-RECURSIVE (root. right)

10.4-3 Stack : Depth-First Queue : Breadth-First.
PRINT-BINARY-TREE - NONE-RECURSIVE (root)

S = new stack

S.push (root)

while ! STACK-EMPTY(S)

node = S.pop

print (node, key)

if node.left != nil

S.push (node.left)

if node.right != nil

S.push (node.right)

10.4-4 $O(n)$

PRINT-TREE-RECURSIVE (root)

PRINT (root, key)

if root.left-child != nil

PRINT-TREE-RECURSIVE (root.left-child)

if root.right-sibling != nil

PRINT-TREE-RECURSIVE (root.right-sibling)

10. 4-5
PRINT-BINARY-TREE (T)

PRINT-BINARY-TREE(T)

$X = T$. root

```
prev = x.parent // nil
```

while $x \neq \text{nil}$

```
if prev == x.parent
```

```
PRINT(x.key)
```

prev = x

if $x.\text{left} \neq \text{nil}$

$$x = x.\text{left}$$

else if $x.\text{right} \neq \text{nil}$

$$x = x.\text{right}$$

else

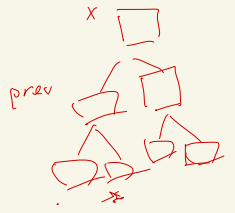
$$x = x_{\text{parent}}$$

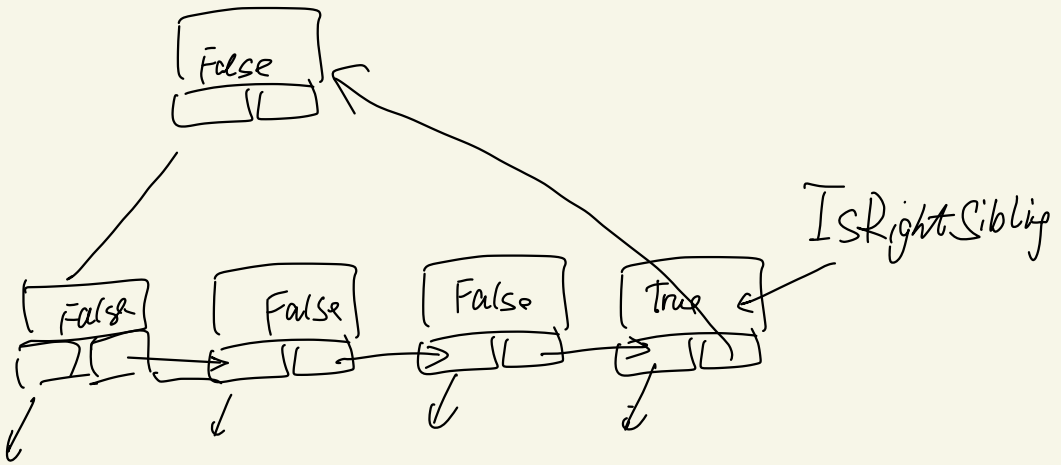
if prev == x.left and x.right != nil

$$\text{prev} = x$$

$x = x$. right

else

$$prev = x$$
$$x = x.\text{parent}$$




```
PARENT( node )  
while !node.IsRightSibling  
    node = node.next  
  
return node.next
```

10-1

	unsorted, singly linked	sorted, singly linked	unsorted, doubly linked	sorted, doubly linked
SEARCH(L, k)	$O(n)$	$O(n)$	$O(n)$	$O(n)$
INSERT(L, x)	$O(1)$	$O(n)$	$O(1)$	$O(n)$
DELETE(L, x)	$O(n)$	$O(n)$	$O(1)$	$O(1)$
SUCCESSOR(L, x)	$O(n)$	$O(1)$	$O(n)$	$O(1)$
PREDECESSOR(L, x)	$O(n)$	$O(n)$	$O(n)$	$O(1)$
MINIMUM(L)	$O(n)$	$O(1)$	$O(n)$	$O(1)$
MAXIMUM(L)	$O(n)$	$O(n)$	$O(n)$	$O(1)$

10-2.

α.
Sorted Double Linked list

MAKE-HEAP ()
L = new Double linked list $O(1)$
return L

INSERT (L, x)

node = L.head

next = node.next

while next != nil and next.key ≤ x.key

node = next

next = node.next

x.prev = node

x.next = node.next

node.next.prev = x

node.next = x

MINIMUM(L) $O(1)$
return L.head

EXTRACT-MIN(L) $O(n)$

L.nil.next = L.head.next

L.head.prev = L.nil

temp = L.head

L.head = L.head.next

return temp.

RIGHT(i)

return 2i+1

LEFT(i)

return 2i

PARENT(i)

return $\lfloor i/2 \rfloor$

$O(n)$

Union (L_1, L_2)

L = new double linked list $O(N)$

$x = L_1 \cdot \text{head}$

$y = L_2 \cdot \text{head}$

while $x \neq \text{nil}$ or $y \neq \text{nil}$

if $y == \text{nil}$ or $y \cdot \text{key} \geq x \cdot \text{key}$

$L \cdot \text{insert}(x)$

$x = x \cdot \text{next}$

if $x == \text{nil}$ or $x \cdot \text{key} > y \cdot \text{key}$

$L \cdot \text{insert}(y)$

$y = y \cdot \text{next}$

return L

b. Unsorted lists $O(1)$

MAKE-HEAP()

return new Double-linked list L

Assumes that the two lists are already a Min-Heap.

UNION(L_1, L_2)

10.3 a.

Since both algorithms return correctly. Thus, they return the same answer.

COMPACT-LIST-SEARCH'(L, n, k, t)

has t iteration for the for loop. Thus the number of iterations are at least n .

b. $O(t + E(x_t))$

↓ ↓
for loop while loop

c. $E(x_t) \leq \sum_{r=1}^n (1 - r/n)^t$

C.25 $E(x) = \sum_{i=0}^{\infty} i \cdot \Pr\{x = i\}$

$$= \sum_{i=0}^{\infty} i (\Pr\{x \geq i\} - \Pr\{x \geq i+1\})$$

$$= \sum (i-1) (\Pr\{x \geq i-1\} - \Pr\{x \geq i\})$$

$$+ i (\Pr\{x \geq i\} - \Pr\{x \geq i+1\})$$

$$= \sum_{i=1}^{\infty} \Pr\{x \geq i\}$$

Assume k 's order m

$$m \geq n \quad \text{or} \quad m < n$$

$$x_t \in [0, n-1]$$

$$\begin{aligned} E[x_t] &= \sum_{d=0}^{n-1} d \Pr\{x_t = d\} \\ &= \sum_{d=1}^{n-1} \Pr\{x_t \geq d\} \end{aligned}$$

$$\Pr\{x_t \geq d\} = (1 - d/n)^t$$

$$\begin{aligned} E[x_t] &= \sum_{d=1}^{n-1} (1 - d/n)^t \\ &\leq \sum_{r=1}^n (1 - r/n)^t \end{aligned}$$

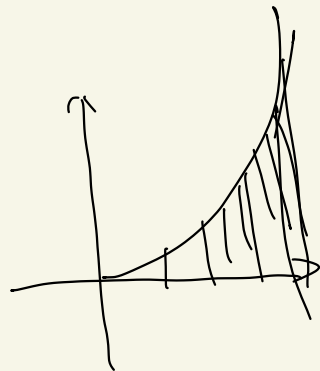
\downarrow
 $n=1$ equal

$$d. \sum_{r=0}^{n-1} r^t \leq n^{t+1}/(t+1)$$

$$\frac{1}{t+1} \cdot \cancel{(t+1)} n^{t+1}$$

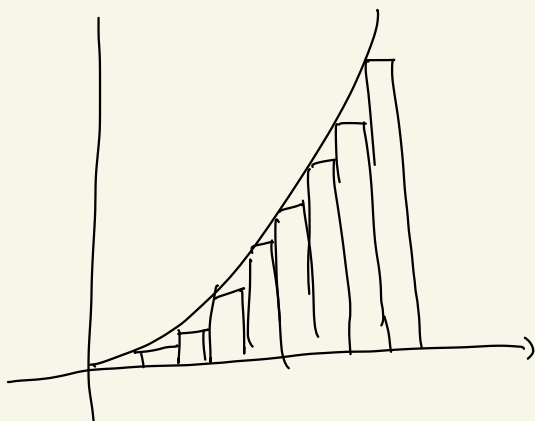
$$f(x) = \frac{x^{t+1}}{t+1}$$

$$f'(x) = x^t$$



$$d. \sum_{r=0}^{n-1} r^t = \int_0^{n-1} \lfloor r^t \rfloor dr$$

since the $y=r^t$ is convex function,



$$\begin{aligned} \sum_{r=0}^{n-1} r^t &= \int_0^n \lfloor r^t \rfloor dr \leq \int_0^n r^t dr \\ &= \frac{r^{t+1}}{t+1} \Big|_0^n \leq \frac{n^{t+1}}{t+1} \end{aligned}$$

$$\begin{aligned} Q. E[x^t] &\leq \sum_{r=1}^n \left(1 - \frac{r}{n}\right)^t \\ &= \frac{1}{n^t} \sum_{r=1}^n (n-r)^t \\ &= \frac{1}{n^t} \sum_{x=0}^{n-1} x^t \\ &\leq \frac{1}{n^t} \cdot \frac{n^{t+1}}{t+1} = \frac{n}{t+1} \end{aligned}$$

$$f. O(t + n/t + 1) = O(t + n/t)$$

$$g. O(t) = O(t + n/t)$$

\swarrow runtime of COMPACT-LIST-SEARCH¹
 runtime of COMPACT-LIST-SEARCH(L, n, k)

$$\Rightarrow O(t) = O(\sqrt{n})$$

h, when there are repeated key values in the list, they decrease the probability that the index will skip. In a extreme situation when all keys are repeated, line 2 to line 7 will never be executed

