Arithmetic
算术

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To Begin With

QR Mathematical Convention 1

Any number in QR is a real number.
Imaginary Numbers are out of scope of QR.

Integers

Presentation Overview for Integers

Even v.s. Odd

The Big Question

Can negative numbers be odd or even?

For example, is -2 Even?

Can negative numbers be odd or even?
For example, is -2 Even?

YES!

Even v.s. Odd

奇偶运算: 看剩下的尾巴

定义

x is an odd number if $x = 2k + 1$, where $k = ... - 2, -1, 0, 1, 2, ...$

x is an even number if $x = 2k$, where $k = ... - 2, -1, 0, 1, 2, ...$

Facts about Odd and Even Numbers

- *odd* $\pm$ *even* $=$ *odd* $\quad (2k_1 + 1) \pm 2k_2 = 2(k_1 \pm k_2) + \mathbf{1}$
- *odd* $\pm$ *odd* $=$ *even* $\quad (2k_1 + 1) \pm (2k_2 + 1) = 2(k_1 \pm k_2)$
- *even* $\pm$ *even* $=$ *even* $\quad 2k_1 \pm 2k_2 = 2(k_1 \pm k_2)$

- *odd* $\times$ *even* $=$ *even* $\quad (2k_1 + 1) \times 2k_2 = 2(2k_1 k_2 + k_2)$
- *odd* $\times$ *odd* $=$ *odd* $\quad (2k_1 + 1) \times (2k_2 + 1) = 2(2k_1 k_2 + k_1 + k_2) + \mathbf{1}$
- *even* $\times$ *even* $=$ *even* $\quad 2k_1 \times 2k_2 = 4k_1$

Have a Try!

用奇偶运算算如下题目 (看尾巴)；注意读题（“must be”)；

If a and b are both positive integers, and a −b and a/b are even, which of following must be an odd integer?

- (A) $\frac{a}{2}$
- (B) $\frac{b}{2}$
- (C) $\frac{(a+b)}{2}$
- (D) $\frac{(a+2)}{2}$
- (E) $\frac{(b+2)}{2}$

Have a Try!

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If a and b are both positive integers, and a −b and a/b are even, which of following must be an odd integer?

A $\frac{a}{2}$

B $\frac{b}{2}$

C $\frac{(a+b)}{2}$

D $\frac{(a+2)}{2}$

E $\frac{(b+2)}{2}$

$\because$ a - b is even

$\therefore$ a is even and b is even;

Or, a is odd and b is odd;

$\because$ a/b is even

$\therefore$ a $=$ 2kb;

$\therefore$ a/2 is even

b/2 is even or odd

Have a Try!

用奇偶运算算如下题目 (看尾巴)；注意读题（“must be”）；

If a and b are both positive integers, and a $-$b and a$/$b are even, which of following must be an odd integer?

A $\frac{a}{2}$
B $\frac{b}{2}$
C $\frac{(a+b)}{2}$
D $\frac{(a+2)}{2}$
E $\frac{(b+2)}{2}$

$\because$ a - b is even
$\therefore$ a is even and b is even;
Or, a is odd and b is odd;

$\because$ a$/$b is even
$\therefore$ a $=$ 2kb;
$\therefore$ a$/$2 is even
b$/$2 is even or odd

A $\frac{a}{2}$ even
B $\frac{b}{2}$ even or odd
C $\frac{(a+b)}{2}$ even or odd
D $\frac{(a+2)}{2}$ even $+$ 1 $=$ odd
E $\frac{(b+2)}{2}$ even or odd $+$ 1 $=$ even or odd

Answer **D**

A Real QR Problem!

用奇偶运算算如下题目 (看尾巴)；注意读题（“must be”)；

If p is an even integer, which of the following must be an odd integer?

- ○ $\frac{3p}{2}$
- ○ $\frac{3p}{2}+1$
- ○ $\frac{3p^2}{2}$
- ○ $\frac{3p^2}{2}+1$
- ○ p^3

图: 2-Sec1-9

A Real QR Problem!

用奇偶运算算如下题目 (看尾巴)；注意读题（“must be”)；

If p is an even integer, which of the following must be an odd integer?

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图: 2-Sec1-9

$\because p = 2k$
$\therefore \frac{p}{2} = k$, which is even or odd.
$\therefore \frac{p^2}{2} = 2k^2$, which is even.

A Real QR Problem!

用奇偶运算算如下题目 (看尾巴)；注意读题（“must be”）；

If p is an even integer, which of the following must be an odd integer?

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图: 2-Sec1-9

$\because p = 2k$
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Answer **D** 请把 5 个选项大小排序

Divisor and the Greatest Common Divisor

Definitions & Examples For A Divisor(Factor)

约数（因数）

定义

When integers are multiplied, each of the multiplied integers is called a factor or divisor of the resulting product

例

- $(2)(3)(10) = 60$, so 2,3, and 10 are factors of 60.
- The integers 4, 15, 5, and 12 are also factors of 60.
- $(-2)(-30) = 60$. The negatives of the positive factors are also factors of 60.
- 0 is not a factor of any integer except 0.

推论

Every integer a is divisible by the trivial divisors, 1 and a.

定义

The greatest common divisor (or greatest common factor) of two nonzero integers c and d is the greatest positive integer that is a divisor of both c and d.

例

The least common multiple of 30 and 75 is 150.

- The positive divisors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.
- The positive divisors of 75 are 1, 3, 5, 15, 25, and 75.
- The common positive divisors of 30 and 75 are 1, 3, 5, and 15.
- The greatest of these is 15.

The Big Question

Is there a better way to find the gcd of two nonzero integers c and d?

Prime v.s. Composite

Prime V.s. Composite
质数 V.s. 合数

定义

A prime number is an integer greater than 1 that has only two positive divisors: 1 and itself.

定义

An integer greater than 1 that is not a prime number is called a composite number.

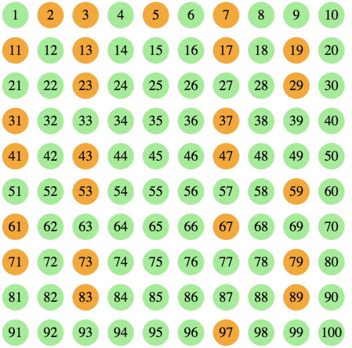

图: There are 25 prime numbers which are less than 100.

Divisible Rules

定理 (Divisible by 2)

The last digit is even (0, 2, 4, 6, or 8). Thus, Any even number who is greater than 2 is not a prime.

定理 (Divisible by 3)

The Sum of digits if divisible by 3. For example, 12, 36, 93, 102.

定理 (Divisible by 5)

The last digit is 0 or 5.

Prime Factorization
质数分解

定理 (Prime Factorization)

Every integer greater than 1 either is a prime number or can be uniquely expressed as a product of factors that are prime numbers, or prime divisors

例

- $12 = 2^2 \cdot 3$
- $81 = 3^3$
- $3398 =$

定理 (Prime Factorization)

Every integer greater than 1 either is a prime number or can be uniquely expressed as a product of factors that are prime numbers, or prime divisors

例

- $12 = 2^2 \cdot 3$
- $81 = 3^3$
- $3398 = 2 \cdot 13^2$
- $1155 =$

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例

- $12 = 2^2 \cdot 3$
- $81 = 3^3$
- $3398 = 2 \cdot 13^2$
- $1155 = 3 \cdot 5 \cdot 7 \cdot 11$

First Try!

If y is the smallest positive integer such that 3150 multiplied by y is the square of an integer, then y must be?

定理 (Prime Factorization)

Every integer greater than 1 either is a prime number or can be uniquely expressed as a product of factors that are prime numbers, or prime divisors

例

- $12 = 2^2 \cdot 3$
- $81 = 3^3$
- $3398 = 2 \cdot 13^2$
- $1155 = 3 \cdot 5 \cdot 7 \cdot 11$

First Try!

If y is the smallest positive integer such that 3150 multiplied by y is the square of an integer, then y must be?

$3150 = 2 \cdot 3^2 \cdot 5^2 \cdot 7$

Prime Factorization
质数分解

定理 (Prime Factorization)

Every integer greater than 1 either is a prime number or can be uniquely expressed as a product of factors that are prime numbers, or prime divisors

例

- $12 = 2^2 \cdot 3$
- $81 = 3^3$
- $3398 = 2 \cdot 13^2$
- $1155 = 3 \cdot 5 \cdot 7 \cdot 11$

First Try!

If y is the smallest positive integer such that 3150 multiplied by y is the square of an integer, then y must be?

$3150 = 2 \cdot 3^2 \cdot 5^2 \cdot 7$

$y \cdot 2 \cdot 3^2 \cdot 5^2 \cdot 7 = x^2$, in which x and y are positive integers.

The smallest y $= 2 \cdot 7 =$ **14**

A Real QR Problem!

What is the greatest prime factor of $3^{100} - 3^{97}$?

- 3
- 5
- 7
- 11
- 13

图: 6-Sec3-20

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- 5
- 7
- 11
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图: 6-Sec3-20

$$
\begin{aligned}
&3^{100} - 3^{97} \\
&= 3^{97} \cdot (3^3 - 1) \\
&= 3^{97} \cdot 26 \\
&= 3^{97} \cdot 2 \cdot 13
\end{aligned}
$$

3 , 2 and 11 are the prime factors.

A Real QR Problem!

What is the greatest prime factor of $3^{100} - 3^{97}$?

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- 5
- 7
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&= 3^{97} \cdot 2 \cdot 13
\end{aligned}
$$

3 , 2 and 11 are the prime factors.

Answer **E**

Find GCD with Prime Factorization
用质数分解找公因数

What is the gcd of 168 and 96?

1. Prime Factorization of 168:
   $168 = 2^3 \cdot 3 \cdot 7$
2. Prime Factorization of 96:
   $96 = 2^5 \cdot 3$
3. The gcd equals the products of the common factors with smaller exponent.
   取公因子的最小指数
   $gcd(168, 96) = 2^3 \cdot 3 = \mathbf{24}$

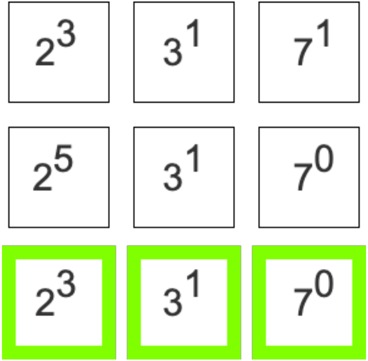

图: The Factors with the smaller exponents

Have a try!

先 Prime Factorization，然后找 Common Factors

What is the gcd of 42 and 56?

Have a try!

先 Prime Factorization，然后找 Common Factors

What is the gcd of 42 and 56?

1. Prime Factorization of 42: $42 = 3 \cdot 2 \cdot 7$

Have a try!

先 Prime Factorization，然后找 Common Factors

What is the gcd of 42 and 56?

1. Prime Factorization of 42: $42 = 3 \cdot 2 \cdot 7$
2. Prime Factorization of 56: $56 = 2^3 \cdot 7$

What is the gcd of 42 and 56?

1. Prime Factorization of 42: $42 = 3 \cdot 2 \cdot 7$
2. Prime Factorization of 56: $56 = 2^3 \cdot 7$
3. $gcd(42, 56) = 2 \cdot 7 = \mathbf{14}$

A Real QR Problem!

n and q are different positive integers.

Quantity A	Quantity B
The greatest common factor of n and q	The greatest common factor of $201n+2q$ and $100n+q$

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

图: 7-Sec2-7

Answer

$A = gcd(n, q)$
$\therefore n = k_1 \cdot gcd(n, q)$ and
$q = k_2 \cdot gcd(n, q)$

$$B = gcd(201n + 2q, 100n + q)$$

$$201n + 2q$$
$$= 67 \cdot 3n + 2q$$
$$= gcd(n, q)(67 \cdot 3k_1 + 2k_2)$$

$$100n + q$$
$$= 2^2 \cdot 5^2 n + q$$
$$= gcd(n, q)(2^2 \cdot 5^2 k_1 + k_2)$$

$$B = gcd(201n + 2q, 100n + q)$$
$$= gcd(n, q) \cdot$$
$$gcd(67 \cdot 3k_1 + 2k_2, 2^2 \cdot 5^2 k_1 + k_2)$$

Answer

$A = gcd(n, q)$
$\therefore\ n = k_1 \cdot gcd(n, q)$ and
$q = k_2 \cdot gcd(n, q)$

$$B = gcd(201n + 2q, 100n + q)$$

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$$= 67 \cdot 3n + 2q$$
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$$100n + q$$
$$= 2^2 \cdot 5^2 n + q$$
$$= gcd(n, q)(2^2 \cdot 5^2 k_1 + k_2)$$

$$B = gcd(201n + 2q, 100n + q)$$
$$= gcd(n, q) \cdot$$
$$gcd(67 \cdot 3k_1 + 2k_2, 2^2 \cdot 5^2 k_1 + k_2)$$

What if $67 \cdot 3k_1 + 2k_2$ and $2^2 \cdot 5^2 k_1 + k_2$ are different primes?
$\because\ gcd(67 \cdot 3k_1 + 2k_2, 2^2 \cdot 5^2 k_1 + k_2) \geq 1$
$\therefore\ B \geq A$

Answer

$A = gcd(n, q)$
$\therefore n = k_1 \cdot gcd(n, q)$ and
$q = k_2 \cdot gcd(n, q)$

$$B = gcd(201n + 2q, 100n + q)$$

$201n + 2q$
$= 67 \cdot 3n + 2q$
$= gcd(n, q)(67 \cdot 3k_1 + 2k_2)$

$100n + q$
$= 2^2 \cdot 5^2 n + q$
$= gcd(n, q)(2^2 \cdot 5^2 k_1 + k_2)$

$B = gcd(201n + 2q, 100n + q)$
$= gcd(n, q) \cdot$
$gcd(67 \cdot 3k_1 + 2k_2, 2^2 \cdot 5^2 k_1 + k_2)$

What if $67 \cdot 3k_1 + 2k_2$ and $2^2 \cdot 5^2 k_1 + k_2$ are different primes?
$\because gcd(67 \cdot 3k_1 + 2k_2, 2^2 \cdot 5^2 k_1 + k_2) \geq 1$
$\therefore B \geq A$

Answer **D:** The relationship cannot be determined from the information given.

Have Another Try!

$$A = gcd(n, q)$$

$$B = gcd(201n + 3q, 96n + 81q)$$

Have Another Try!

$A = gcd(n, q)$

$B = gcd(201n + 3q, 96n + 81q)$

$B \geq 3 \cdot gcd(n, q) > A$
Answer **B:** Quantity B is greater

Multiple and Least Common Multiple

Definitions & Examples For A Multiple
倍数

定义
We say that an integer is a multiple of each of its factors and that an integer is divisible by each of its divisors.

例
- 25 is a multiple of only six integers: 1, 5, 25, and their negatives.
- The list of positive multiples of 25 has no end: 25, 50, 75, 100, ...; likewise, every nonzero integer has infinitely many multiples.
- 1 is not a multiple of any integer except 1 and -1.
- 0 is a multiple of every integer.

A Real QR Problem!

Quantity A	Quantity B
The number of multiples of 3 between 1 and 10,000	The number of multiples of 7 between 1 and 23,000

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

图: 2-Sec1-1

A Real QR Problem!

Quantity A	Quantity B
The number of multiples of 3 between 1 and 10,000	The number of multiples of 7 between 1 and 23,000

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

图: 2-Sec1-1

x: Multiples of 3 between 1 and 10, 000

$x = 3k_1$, where $k_1 = 1, 2, ...333$

$x_{max} = 9,999$

$\mathrm{A} = 333 \quad 333 \cdot 7 = 2,331 > 2300$

A Real QR Problem!

Quantity A	Quantity B
The number of multiples of 3 between 1 and 10,000	The number of multiples of 7 between 1 and 23,000

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

图: 2-Sec1-1

x: Multiples of 3 between 1 and 10, 000

$x = 3k_1$, where $k_1 = 1, 2, ...333$

$x_{max} = 9,999$

A $= 333$ $\quad 333 \cdot 7 = 2,331 > 2300$

Answer **A**

Definitions & Examples For The Least Common Multiple

lcm（最小公倍数）

定义

The least common multiple of two nonzero integers c and d is the least positive integer that is a multiple of both c and d.

例

The least common multiple of 30 and 75 is 150.

- the positive multiples of 30: 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 390, 420, 450, ...
- the positive multiples of 75: 75, 150, 225, 300, 375, 450, ...
- The common positive divisors of 30 and 75 are 1, 3, 5, and 15.
- The common positive multiples of 30 and 75: 150, 300, 450, ...

How can find the lcm of two nonzero integers?

How can find the lcm of two nonzero integers?

Find the gcd!

gcd v.s. lcm
最大公因数 v.s. 最小公倍数

$$lcm(c,d) = \frac{|\ c \cdot d\ |}{gcd(c,d)}$$

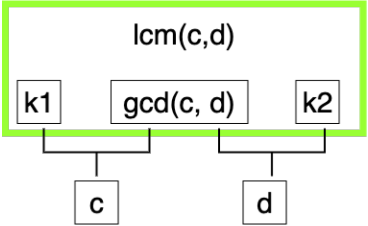
$$|\ c\ | = k_1 \cdot gcd(c,d)$$

$$|\ d\ | = k_2 \cdot gcd(c,d)$$

$$lcm(c,d) = \frac{|\ c \cdot d\ |}{gcd(c,d)}$$

$$= \frac{[k_1 \cdot gcd(c,d)] \cdot [k_2 \cdot gcd(c,d)]}{gcd(c,d)}$$

$$= k_1 \cdot gcd(c,d) \cdot k_2$$



The Big Question

What is the lcm of -12 and 5?

What is the lcm of -12 and 5?

$$lcm(c, d) = \frac{|\ c \cdot d\ |}{gcd(c, d)}$$

How to find the lcm or the gcd of a negative integer and a positive integer

Ignore the negative signs of c and d when it comes to the lcm or the gcd!
The lcm and the gcd are both positive!

Find LCM with Prime Factorization
用质数分解找公倍数

What is the lcm of 168 and 96?

1. Prime Factorization of 168:
   $168 = 2^3 \cdot 3 \cdot 7$
2. Prime Factorization of 96:
   $96 = 2^5 \cdot 3$
3. The lcm equals the products of all factors with larger exponent.
   取所有因子的最大指数
   $lcm(168, 96) = 2^5 \cdot 3 \cdot 7 = \mathbf{672}$

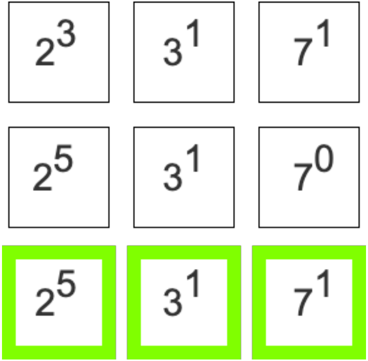


图: The Factors with the larger exponents

Have a try!

先 Prime Factorization，然后找 Factors with larger exponents

What is the lcm of 24 and 78?

Have a try!

先 Prime Factorization，然后找 Factors with larger exponents

What is the lcm of 24 and 78?

1. Prime Factorization of 24: $24 = 2^3 \cdot 3$

Have a try!

先 Prime Factorization，然后找 Factors with larger exponents

What is the lcm of 24 and 78?

1. Prime Factorization of 24: $24 = 2^3 \cdot 3$
2. Prime Factorization of 78: $56 = 2 \cdot 3 \cdot 13$

What is the lcm of 24 and 78?

1. Prime Factorization of 24: $24 = 2^3 \cdot 3$
2. Prime Factorization of 78: $56 = 2 \cdot 3 \cdot 13$
3. *lcm*$(24, 78) = 2^3 \cdot 3 \cdot 13 = \mathbf{312}$

Have a try!

If M is the least common multiple of 90, 196, and 300, which of the following is NOT a factor of M?

- (A) 600
- (B) 700
- (C) 900
- (D) 2100
- (E) 4900

Have a try!

If M is the least common multiple of 90, 196, and 300, which of the following is NOT a factor of M?

Ⓐ 600

Ⓑ 700

Ⓒ 900

Ⓓ 2100

Ⓔ 4900

$$90 = 2 \cdot 3^2 \cdot 5$$
$$196 = 2^2 \cdot 7^2$$
$$300 = 2^2 \cdot 3 \cdot 5^2$$
$$lcm(90, 196, 300)$$
$$= 2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2$$

Ⓐ $600 = 2^3 \cdot 3 \cdot 5^2$

Ⓑ $700 = 2^2 \cdot 5^2 \cdot 7$

Ⓒ $900 = 2^2 \cdot 3^2 \cdot 5^2$

Ⓓ $2100 = 2^2 \cdot 3 \cdot 5^2 \cdot 7$

Ⓔ $4900 = 2^2 \cdot 5^2 \cdot 7^2$

Answer **A**

Quotient and Remainder

定义

For any integer a and any positive integer n, there exist unique integers q and r such that $0 \leq r < n$ and $a = qn + r$.
The value q is the quotient of the division.
The value r is the remainder

例

- $100 \div 45 = 2 \cdots 10$
- $24 \div 2 = 12 \cdots 0$
- $(-32 \div 3 = -11 \cdots 1$

Modulus 7

$-7 \mod 7 = 0$	$0 \mod 7 = 0$	$7 \mod 7 = 0$
$-1 \mod 7 = 6$	$1 \mod 7 = 1$	$8 \mod 7 = 1$
$-2 \mod 7 = 5$	$2 \mod 7 = 2$	$9 \mod 7 = 2$
$-3 \mod 7 = 4$	$3 \mod 7 = 3$	$10 \mod 7 = 3$
$-4 \mod 7 = 3$	$4 \mod 7 = 4$	$11 \mod 7 = 4$
$-5 \mod 7 = 2$	$5 \mod 7 = 5$	$12 \mod 7 = 5$
$-6 \mod 7 = 1$	$6 \mod 7 = 6$	$13 \mod 7 = 6$

A Real QR Problem!

For a positive integer n, when $2n+3$ is divided by 11, the remainder is 3. What is the remainder when $n+15$ is divided by 11 ?

- ○ 0
- ○ 1
- ○ 2
- ○ 3
- ○ 4

图: 1-Sec2-20

Answer

$\because (2n+3) \mod 11 = 3$
$\therefore 2n \mod 11 = 0$
$\therefore n \mod 11 = 0$
$\therefore (n+15) \mod 11 = 15 \mod 11 = 4$

Answer

$\because (2n+3) \mod 11 = 3$
$\therefore 2n \mod 11 = 0$
$\therefore n \mod 11 = 0$
$\therefore (n+15) \mod 11 = 15 \mod 11 = 4$

Answer **E:** $(n+15) \mod 11 = 4$

A Real QR Problem!

When the positive integer t is divided by 5, the remainder is 3, and when t is divided by 6, the remainder is 2.

Quantity A	Quantity B
t	38

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

图: 3-Sec2-6

Answer

$\because t \mod 5 = 3 \therefore t = 5k_1 + 3$
$\because t \mod 6 = 2 \therefore t = 6k_2 + 2$
$\therefore 6k_2 + 2 = 5k_1 + 3$
$\therefore k_2 = \frac{5k_1+1}{6}$

$\therefore (5k_1 + 1)$ is a multiple of 6.

$5k_1 + 1$	k_1	k_2	**t**
6	1	1	8
36	7	6	38 答案已确定
66	13	11	68

表: Several Possible values for k_1, k_2 and t

Answer **D:** The relationship cannot be determined from the information given.

If r is the remainder when 3^n is divided by 10, where n is a positive integer, which of the following is NOT a possible value of r?

- 1
- 3
- 5
- 7
- 9

图: 4-Sec1-11

Answer
看个位

n	3^n	$r = 3^n \mod 10$
1	3	3 B
2	9	9 E
3	27	7 D
4	81	1 A
5	243	3 B
⋮	⋮	⋮

Answer **C:** 5 is Not a possible values for r.
What if $r = 7^n$?

1 Min Break

Questions? Comments?

Initiate The Sonic Mode!

Fractions

Presentation Overview for Fractions

1. Integers
2. Fractions
3. Exponents and Roots
4. Decimal
5. Ratio
6. Percent

Numerator, Denominator, The Common Denominator and The Mixed Number

Numerator V.s. Denominator　分子 v.s. 分母

A fraction is a number of the form where c and d are integers and $d \neq 0$.
The integer c is called the numerator of the fraction.
The integer d is called the denominator.

Common Denominator　公分母

The common denominator is a common multiple (ideally the least common multiple) of the denominators of two fractions.
The common denominator of $\frac{1}{3}$ and $\frac{2}{5}$ is 15.

The Mixed Number

Mixed Number　带分数

A mixed number consists of an integer part and a fraction part, where the fraction part has a value between 0 and 1.

The mixed number $4\frac{3}{8}$ means $4 + \frac{3}{8}$.

Reciprocal

Reciprocal 倒数

The reciprocal of $\frac{c}{d}$ is $\frac{d}{c}$, where both c and d are non-zero numbers.

Exponents and Roots

Presentation Overview for Exponents and Roots

Exponents

The Power of Negative Numbers

- A negative number raised to an even power is always positive.
- A negative number raised to an odd power is always negative.

奇负偶正

$(-5)^{2n+1} < 0$, where

$n = \ldots, -1, 0, 1, \ldots$

$(-5)^{2n} > 0$, where

$n = \ldots, -1, 0, 1, \ldots$

Have a try!

注意到底是指数到底是减还是放在分母里

$a^m + a^n =$	$a^m \div a^n =$	$(a^m)^n =$
$a^m - a^n =$	$a^m \cdot b^m =$	$(a^{m/n} =$
$a^m \cdot a^n =$	$a^m \div b^m =$	$a^{-r} =$

表: Complete the equations

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表: Complete the equations

$a^m + a^n = a^m(1 + a^{n-m})$	$a^m \div a^n = a^{n-m}$	$(a^m)^n = a^{mn}$
$a^m - a^n = a^m(1 - a^{n-m})$	$a^m \cdot b^m = (ab)^m$	$a^{m/n} = \sqrt[n]{a^m}$
$a^m \cdot a^n = a^{m+n}$	$a^m \div b^m = (\frac{a}{b})^m$	$a^{-r} = \frac{1}{a^r}$

表: Complete the equations

Roots

Roots

开根号

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$(\sqrt[n]{a})^n = a$$

Odd Roots V.s. Even Roots

- A negative number raised to an even power is always positive.
- A negative number raised to an odd power is always negative.

For odd order roots, there is exactly one root for every number n, even when n is negative.
For even order roots, there are exactly one positive and one negative roots for every positive number n and no roots for any negative number n.

开偶次方两个根

One Cube roots for 64: $\sqrt[3]{64} = 4$
Two Fourth roots for 64: $\pm\sqrt[2]{64} = \pm 8$

Have a try!

The function f(x) is defined for each positive three-digit integer n by $f(n) = 2^x 3^y 5^z$, where x, y, and z are hundreds, tens, and units digits of n, respectively. If m and v are three-digit positive integers such that f(m) $=$ 9f(v), then what is the value of m $-$v?

Have a try!

The function f(x) is defined for each positive three-digit integer n by $f(n) = 2^x 3^y 5^z$, where x, y, and z are hundreds, tens, and units digits of n, respectively. If m and v are three-digit positive integers such that f(m) = 9f(v), then what is the value of m —v?

$m = 100x_1 + 10y_1 + z_1$ and $v = 100x_2 + 10y_2 + z_2$
$\because f(m) = 9f(v) \quad \therefore 2^{x_1}3^{y_1}5^{z_1} = 9 \cdot 2^{x_2}3^{y_2}5^{z_2}$
$\therefore 2^{x_1}3^{y_1}5^{z_1} = 2^{x_2}3^{y_2+2}5^{z_2}$

By the uniqueness of Prime Factorization,
$x_1 = x_2, \quad y_1 = y_2 + 2, \quad z_1 = z_2$
$m - v = (100x_1 + 10y_1 + z_1) - (100x_2 + 10y_2 + z_2) = 10(y_1 - y_2) = 20$

Answer **20**

Have a try!

Which of the following inequalities has a solution set that, when graphed in the number line, is a single line segment of finite length?

- Ⓐ $x^4 \geq 16$
- Ⓑ $x^3 \leq 27$
- Ⓒ $x^2 \geq 16$
- Ⓓ $2 \leq |x| \leq 5$
- Ⓔ $2 \leq 3x + 4 \leq 6$

Have a try!

Which of the following inequalities has a solution set that, when graphed in the number line, is a single line segment of finite length?

- (A) $x^4 \geq 16$
- (B) $x^3 \leq 27$
- (C) $x^2 \geq 16$
- (D) $2 \leq |x| \leq 5$
- (E) $2 \leq 3x + 4 \leq 6$

- (A) $x^4 \geq 16$ has two roots but is with infinite length
- (B) $x^3 \leq 27$ has one roots but is with infinite length
- (C) $x^2 \geq 16$ has two roots but is with infinite length
- (D) $2 \leq |x| \leq 5$ two segments of finite length
- (E) $2 \leq 3x + 4 \leq 6$ a single segment of finite length

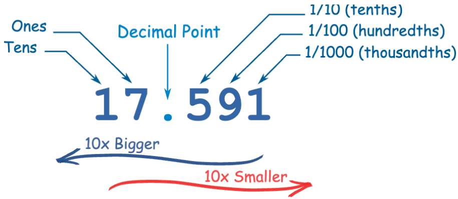
Answer **E**

Decimal

Presentation Overview for Decimal

Terminating and Repeating Decimal

Terminating and Repeating Decimal
终止小数循环小数



Terminating Decimal

- $\frac{3}{8} = 0.375$
- $\frac{259}{40} = 6475$

Repeating Decimal

- $\frac{25}{12} = 2.08333\ldots$
- $\frac{15}{14} = 1.0\overline{714285}$

A Real QR Problem!

What fraction is equivalent to the repeating decimal $0.0\overline{1}$?

$$\frac{\boxed{}}{\boxed{}}$$

图: 7-Sec2-17

A Real QR Problem!

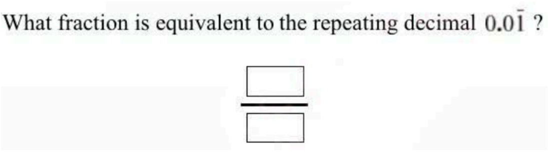

What fraction is equivalent to the repeating decimal $0.0\overline{1}$?

图: 7-Sec2-17

$$
\begin{aligned}
&0.0\overline{1} \\
&= \frac{1}{10} \times 0.\overline{1} \\
&= \frac{1}{10} \times \frac{1}{3} \times 0.\overline{3} \\
&= \frac{1}{10} \times \frac{1}{3} \times \frac{1}{3} \\
&= \frac{1}{90}
\end{aligned}
$$

A Real QR Problem!

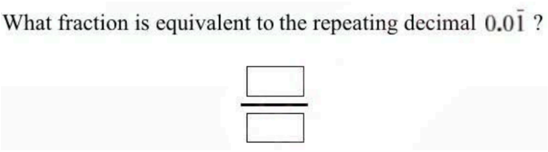


图: 7-Sec2-17

$$
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&= \frac{1}{10} \times \frac{1}{3} \times 0.\overline{3} \\
&= \frac{1}{10} \times \frac{1}{3} \times \frac{1}{3} \\
&= \frac{1}{90}
\end{aligned}
$$

Answer $\frac{1}{90}$ what about $0.0\overline{7}$, $0.8\overline{7}$, or $3.1\overline{52}$?

Rational Numbers v.s. Irrational Numbers

Rational Numbers v.s. Irrational Numbers
有理数无理数

定理 (Rational Numbers)

Every rational number can be expressed as a terminating or repeating decimal.

定理 (Irrational Numbers)

Every irrational number can be expressed as a non-terminating or non-repeating decimal.

Rational Numbers

- 6.67384 $\times$ 10—11
- 6.626070040 $\times$ 10—34

Irrational Numbers

- $2.718281828459045\ldots$
- $\sqrt{2} = 1.41421356237\ldots$
- $3.14159265358979323 8\ldots$

Ratio

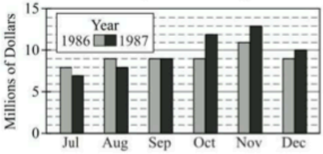
Presentation Overview for Ratio

Th ratio $r : s : t$:

r *to* s *to* t

Questions 14 to 16 are based on the following data.

Amount Spent for all Magazines



Sales for Three Leading Magazines

	Number of Copies Sold July 1–Dec. 31 1987	Percent Change in Number of Copies Sold from July 1–Dec. 31 1986 to July 1–Dec. 31 1987	Number of Copies Sold Dec. 1–Dec. 10 1987
Magazine *A*	2,518,776	+1.5	151,300
Magazine *B*	1,391,792	+6.3	110,313
Magazine *C*	614,399	−3.5	41,234
Total	4,524,967	+2.0	302,847

For which of the months, July through December, was the ratio $\frac{\text{amount spent for 1987}}{\text{amount spent for 1986}}$ greatest?

- July
- August
- September
- October
- November

图: 9-Sec1-14

Answer

分子大，分母小，看相差的条相比分母的占比

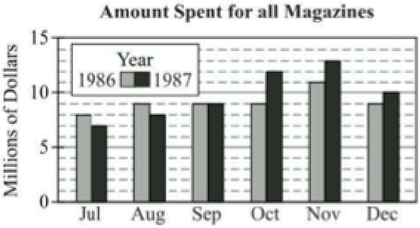


图: 9-Sec1-14

For Jul, Aug, Sep, $\frac{amount\ \ spent\ \ for 1987}{amount\ \ spent\ \ for 1986} \leq 1$;
For Oct, Nov, Dec, $\frac{amount\ \ spent\ \ for 1987}{amount\ \ spent\ \ for 1986} > 1$.
Oct have the greatest difference but the least denominator.

Answer **D:** October

Percent

Presentation Overview for Percent

Have a try!

注意“Nearest Whole Percent”

4. A merchant made a profit of $5 on the sale of a sweater that cost the merchant $15. What is the profit expressed as a percent of the merchant’s cost?

Give your answer to the nearest whole percent.

[ ] %

图: og-p205-4

Have a try!

注意“Nearest Whole Percent”

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[ ] %

图: og-p205-4

$$\frac{5}{15} = 33.\overline{3}\%$$

Have a try!

注意“Nearest Whole Percent”

4. A merchant made a profit of \$5 on the sale of a sweater that cost the merchant \$15. What is the profit expressed as a percent of the merchant’s cost?

Give your answer to the <u>nearest whole percent</u>.

[ ] %

图: og-p205-4

$\frac{5}{15} = 33.\overline{3}\%$

Answer **33**

A Real QR Problem!

In a survey of 1,400 college students, the ratio of women interviewed to men interviewed was 4 to 3. If 63 percent of the women and 48 percent of the men said they preferred product A to product B, what was the number of women and men combined who said they preferred product A to product B?

- ○ 762
- ○ 770
- ○ 780
- ○ 792
- ○ 817

图: 8-Sec1-10

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图: 8-Sec1-10

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图: 8-Sec1-10

Answer **D**

1 Min Break

Questions? Comments?