Introduction to Quantitative Reasoning of GRE

GRE 数学小白必看

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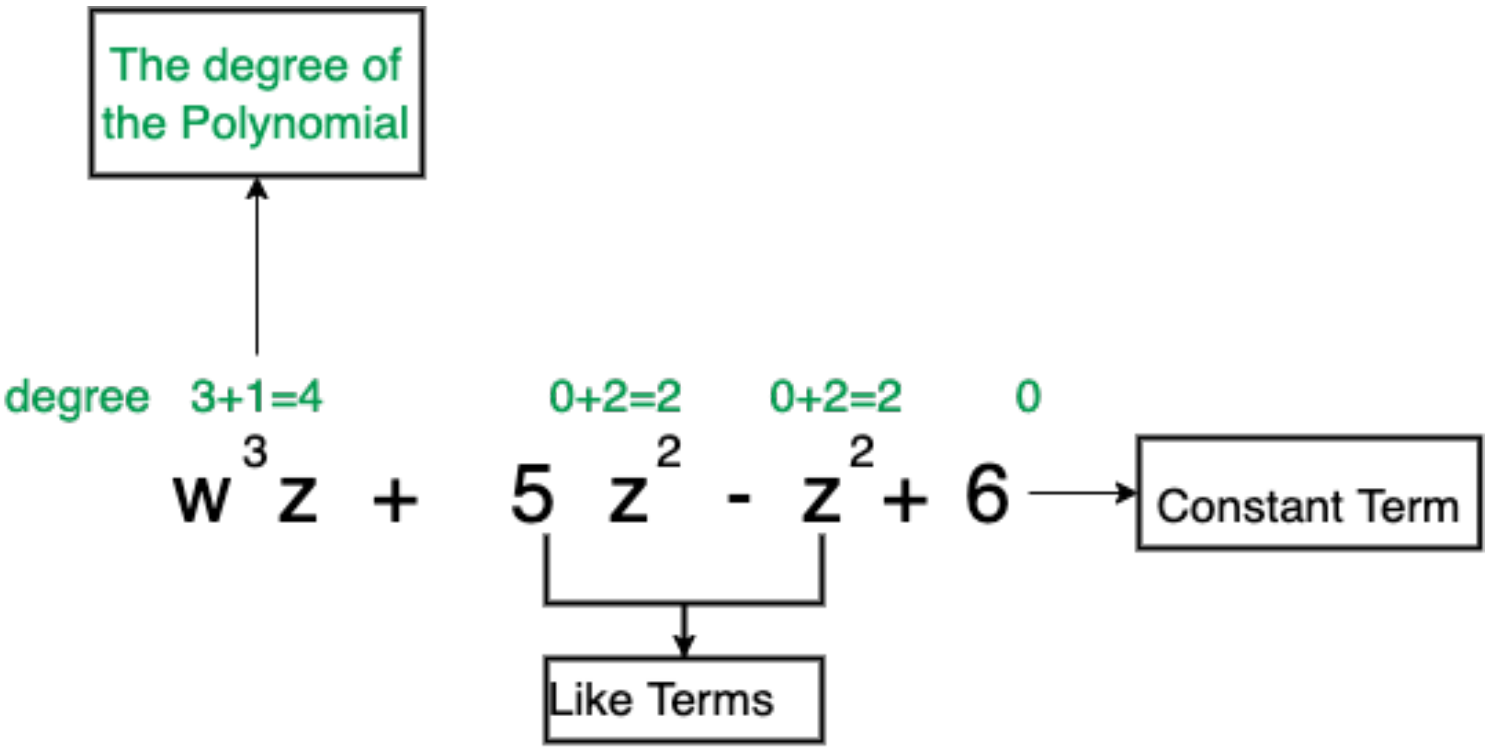
Algebra Expressions

Presentation Overview for Algebra Expressions

Terminologies of Algebra
代数专业名词



- Like Terms 同类项
- The Degree of a Polynomial 多项式的次数

A Real QR Problem!

The expression $x^4 + 2x^2y^2 + 9y^4$ is equivalent to which of the following?

- $(x^2 + 3y^2)^2$
- $(x^2 + 3y^2)(x^2 - 3y^2)$
- $(x^2 + 3y^2 + xy)^2$
- $(x^2 + 2xy + 3y^2)(x^2 - 2xy + 3y^2)$
- $(x^2 + 2xy - 3y^2)(x^2 - 2xy - 3y^2)$

图: 10-Sec3-19

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凑中间项的系数

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图: 10-Sec3-19

凑中间项的系数 Answer **D**

Coordinate Geometry

Presentation Overview for Coordinate Geometry

To Begin With

QR Mathematical Convention 2

When coordinate systems, such as and number lines, are shown with scales, you should read, estimate, or compare quantities by sight or by measurement, according to the corresponding scales.

象限的英文怎么说?

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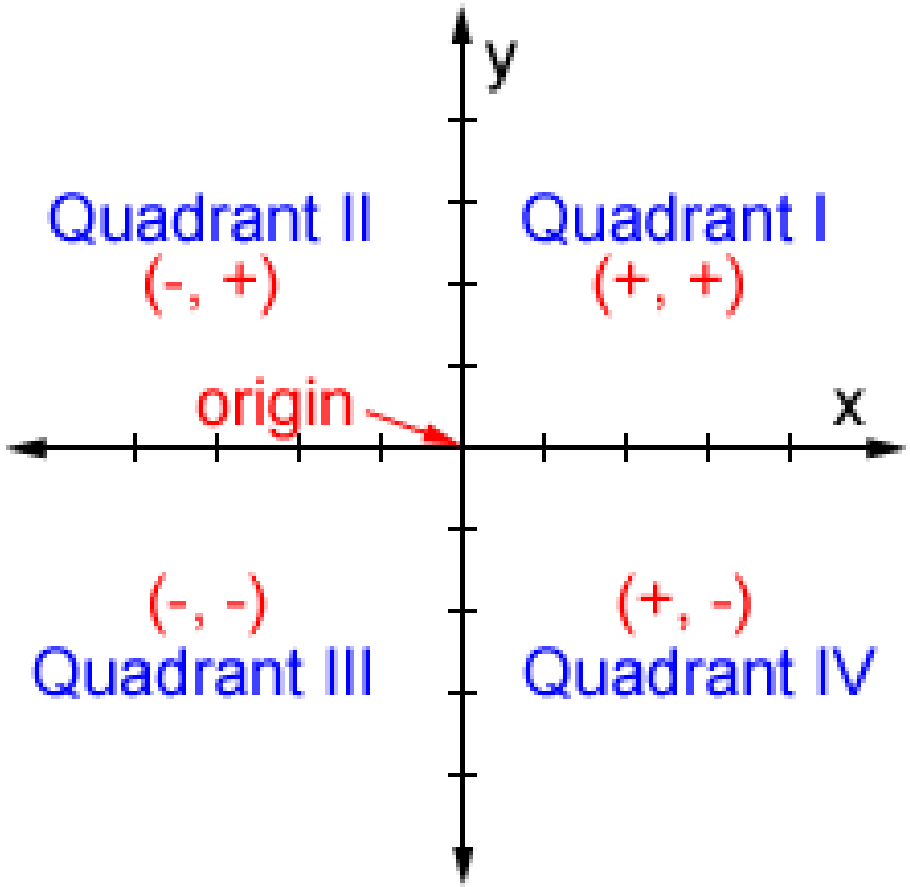


Linear Problems

Presentation Overview for Linear Problems

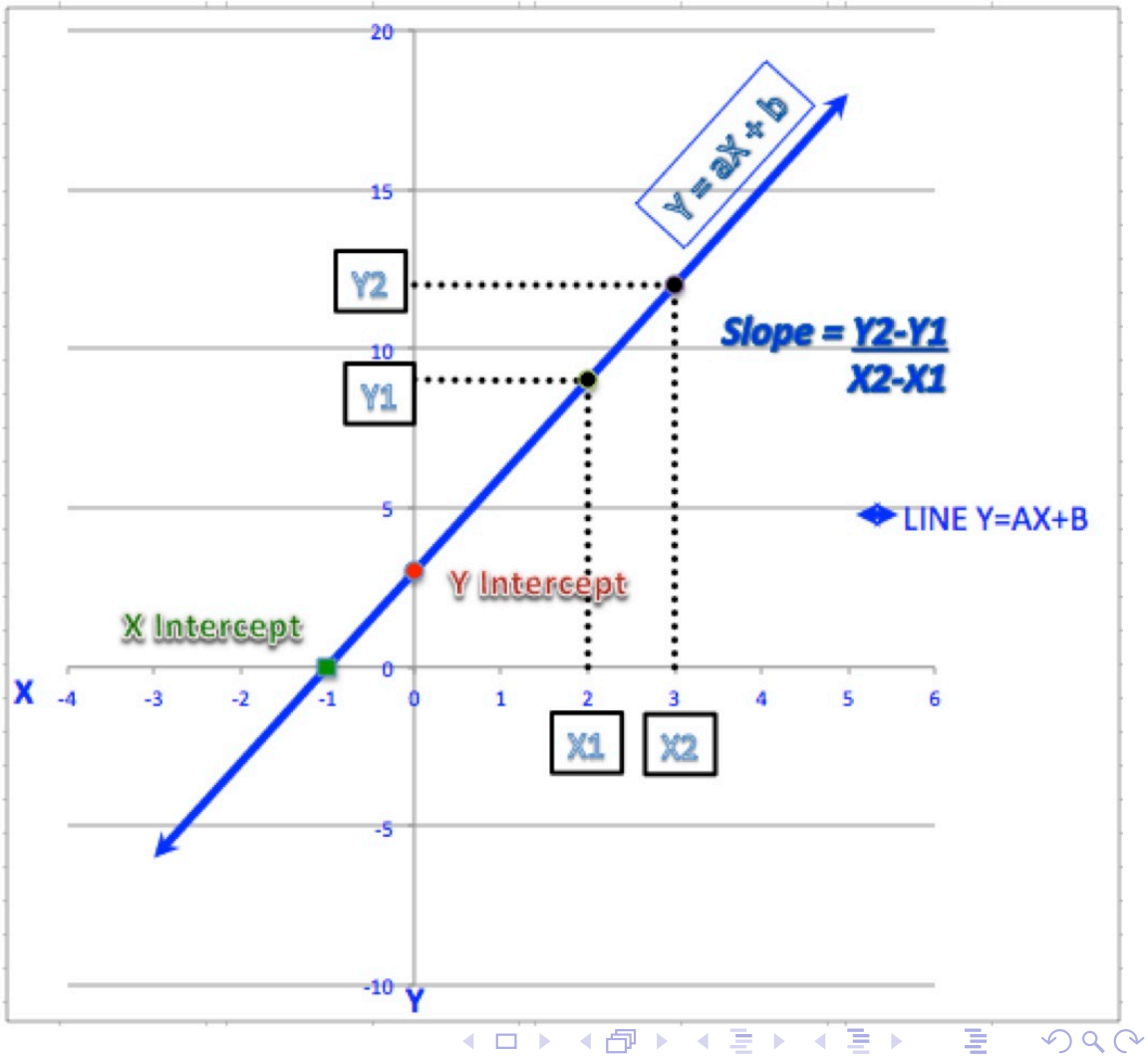
Linear Function

Slope and Intercepts
斜率和截距

定义

The graph of a linear equation of the form $y = mx + b$ is a straight line in the $xy - plane$, where m is called the slope of the line and b is called the y-intercept.

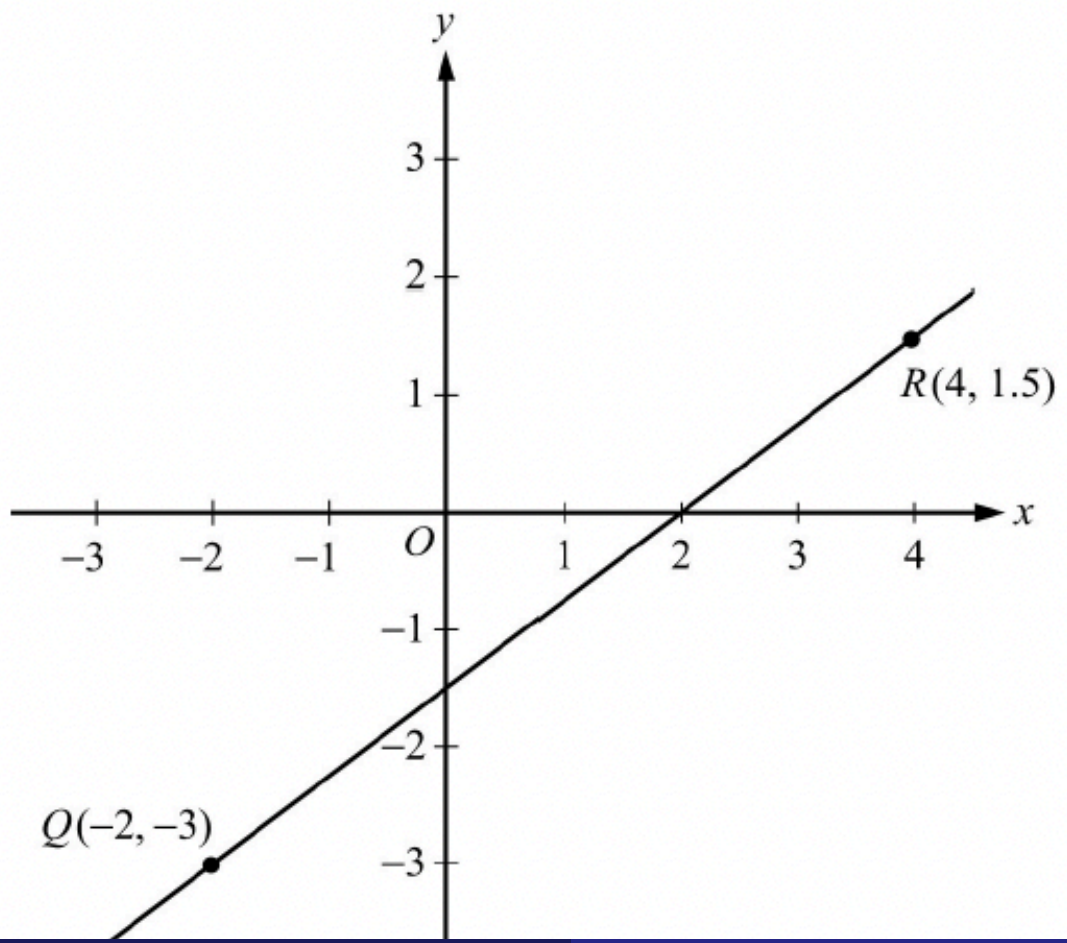
The x-intercepts of a graph are the x-coordinates of the points at which the graph intersects the x-axis.



Have a try!

两点确定一条直线

og-p385-2.8.1 Below shows the graph of the line through the points Q(−2, −3) and R(4, 1.5).



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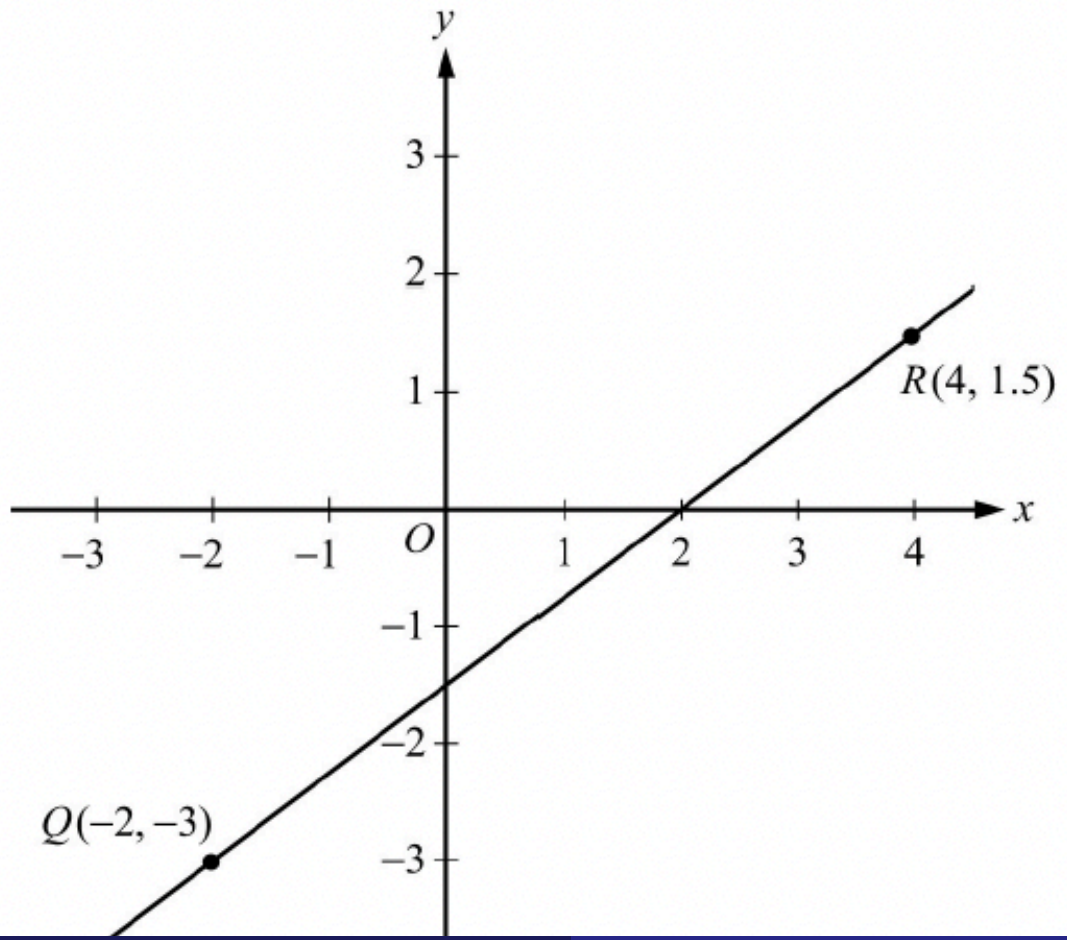

$y = 0.75x - 1.5$

看图 Drawn to scale

$slope = \frac{1.5-(-3)}{4-(-2)} = \frac{4.5}{6} = \frac{3}{4} = 0.75$

$y - intercept =$
$(-3) - 0.75\ times(-2) = -1.5$

A Real QR Problem!

A line in the xy -plane has the equation $y = mx + 6$, where m is a constant and $3 \le m \le 4$. Which of the following values could be the x -intercept of the line?

Indicate all such values.

☐ -3 ☐ -2 ☐ $-\frac{7}{4}$ ☐ $-\frac{5}{4}$ ☐ $\frac{5}{4}$ ☐ $\frac{7}{4}$ ☐ 2 ☐ 3

图: 6-Sec3-18

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图: 6-Sec3-18

$-2 \le x \le -1.5$

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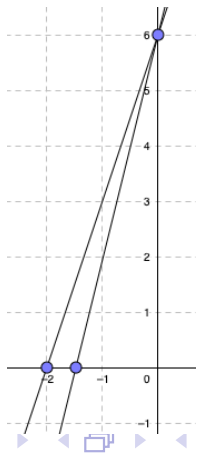
图: 6-Sec3-18

$-2 \le x \le -1.5$

Answer **BC** $-2; -\frac{7}{4}$



A Real QR Problem!

In the xy -plane, a triangular region is enclosed by the x -axis, the y -axis, and the line with equation $2x - y + k = 0$, where k is a positive constant. For which of the following values of k is the area of the triangular region greater than 1 and less than 4?

- ⬭ 0
- ⬭ 1
- ⬭ 2
- ⬭ 3
- ⬭ 4

图: 9-Sec2-10

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- ⬭ 1
- ⬭ 2
- ⬭ 3
- ⬭ 4

图: 9-Sec2-10

$$4 \leq k \leq 2$$

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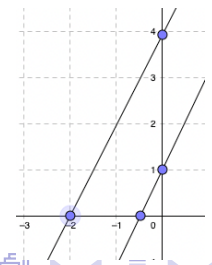
- ○ 0
- ○ 1
- ○ 2
- ○ 3
- ○ 4

图: 9-Sec2-10

$4 \leq k \leq 2$

Answer **D** $k = 3$



The Relation of Slopes for Parallel or Perpendicular
平行或垂直直线斜率关系

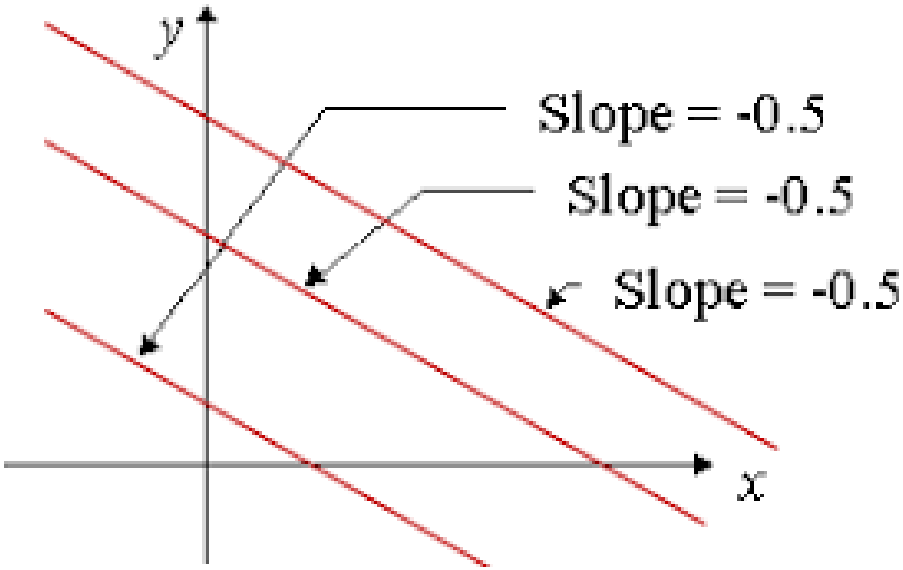


图: Two lines are parallel if their slopes are equal.

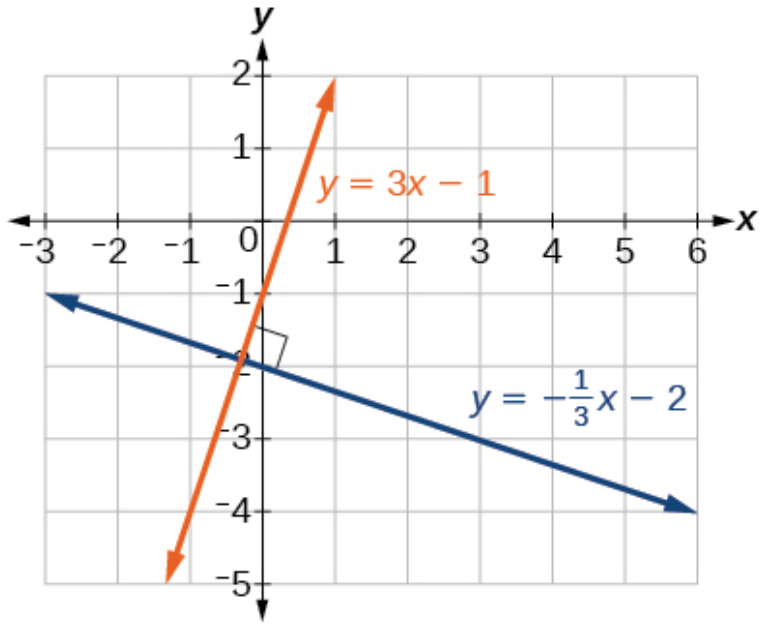
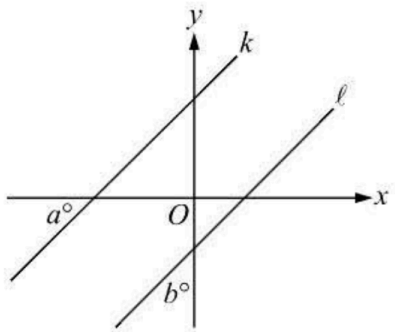


图: Two lines are perpendicular if their slopes are negative reciprocals of each other.

A Real QR Problem!



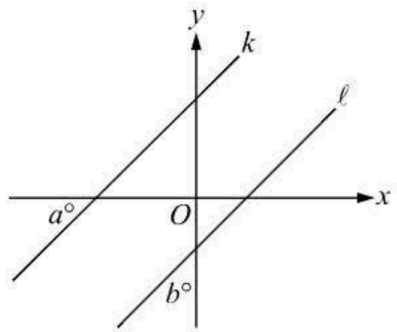
Lines k and ℓ lie in the xy -plane and are parallel.

Quantity A	Quantity B
a	b

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

图: 6-Sec3-7

A Real QR Problem!



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- ○ Quantity A is greater.
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- ○ The relationship cannot be determined from the information given.

图: 6-Sec3-7

a °+ b °= 90 °

A Real QR Problem!

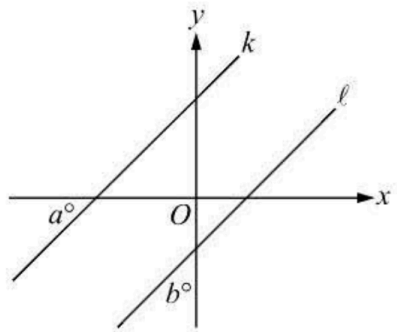

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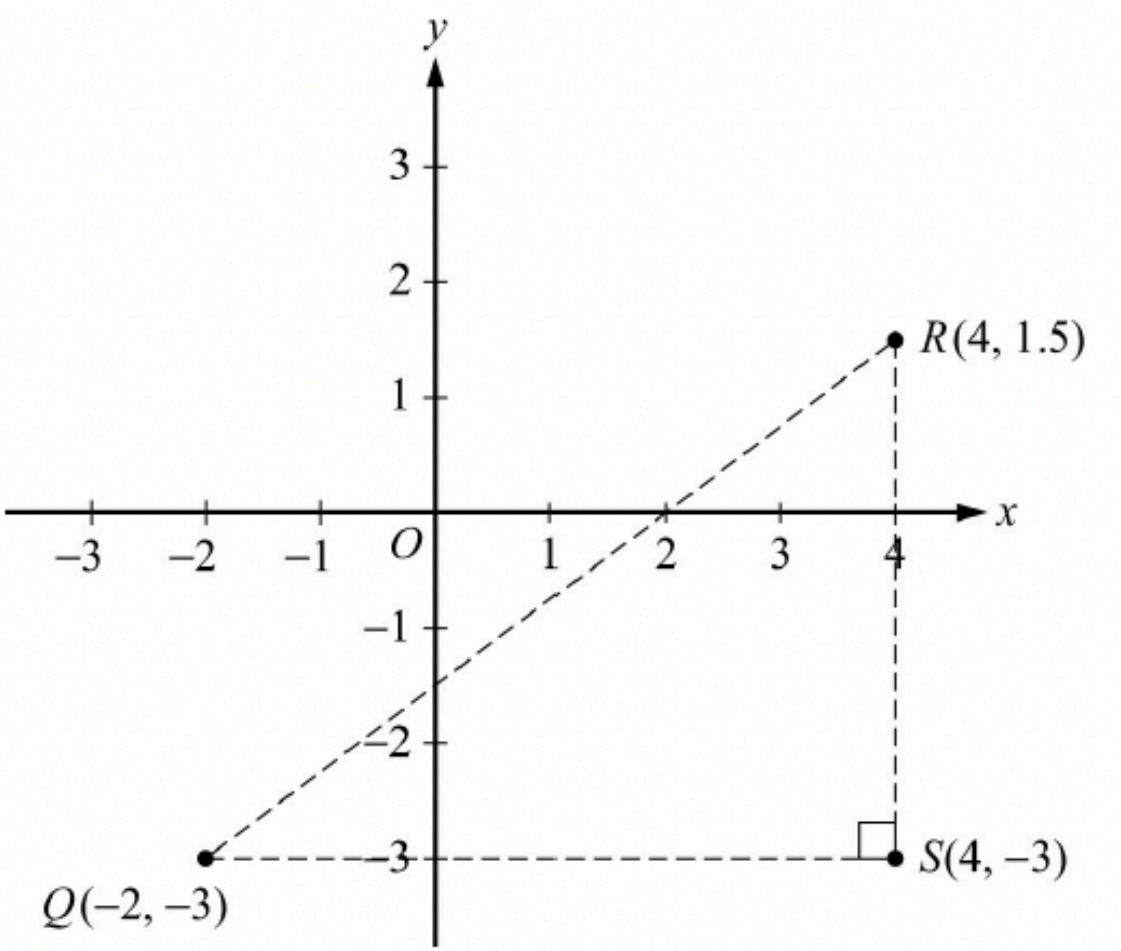
图: 6-Sec3-7

a °+ b °= 90 °

Answer **D** The relationship cannot be determined from the information

Calculating the Distance Between Two Points
两点间距离



$$
\begin{aligned}
&QR \\
&= \sqrt{QS^2 + RS^2} \\
&= \sqrt{(x_1 - x_2^2) + (y_1 + y_2)^2} \\
&= \sqrt{6^2 + 4.5^2} \\
&= 7
\end{aligned}
$$

Linear Equations in One Variable

Equivalent Equations
等价方程

定义

Two equations that have the same solutions are called equivalent equations.

例

$x+1=2$ and $2x+2=4$

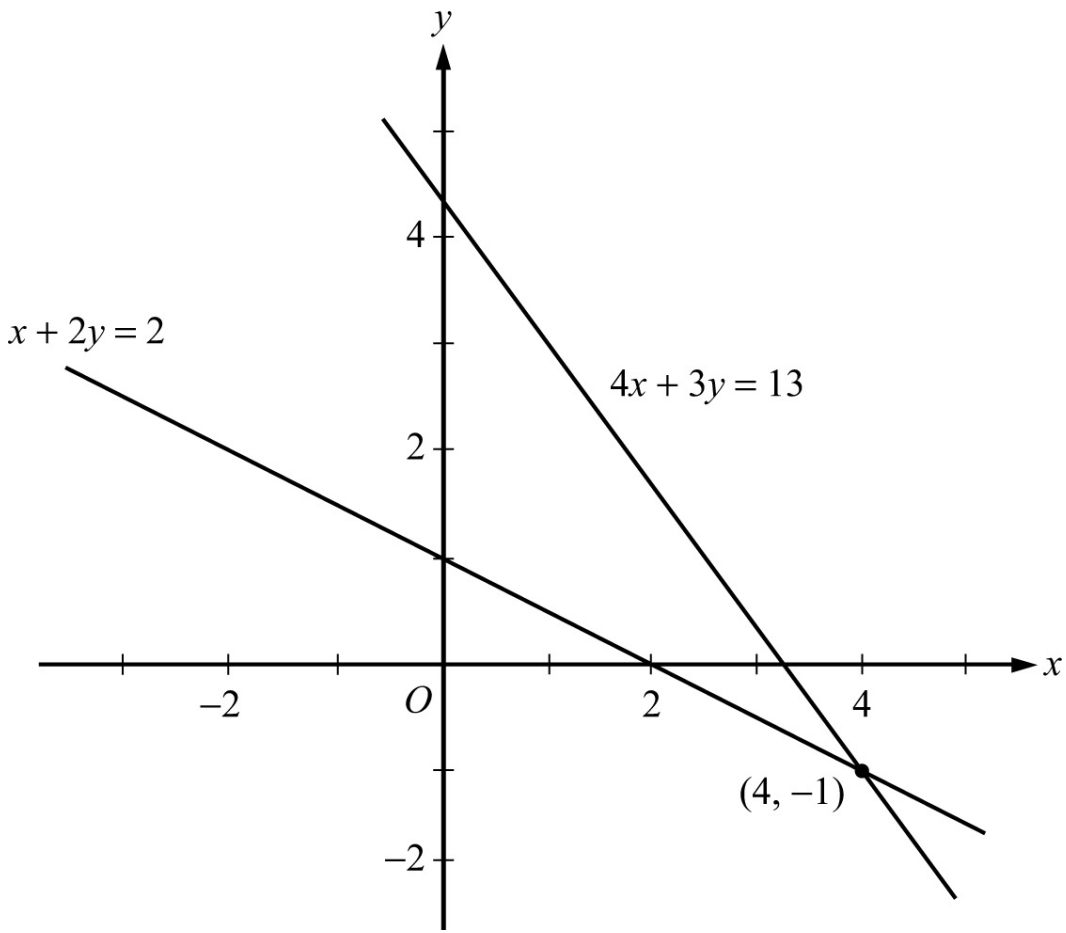
Linear Equations in Two Variable

Solution For Linear Equations in Two Variables

交点就是 Solution

$$4x + 3y = 13$$
$$x + 2y = 2$$

$$y = -\frac{3}{4} + \frac{13}{4}$$
$$y = -\frac{1}{2}x + 1$$

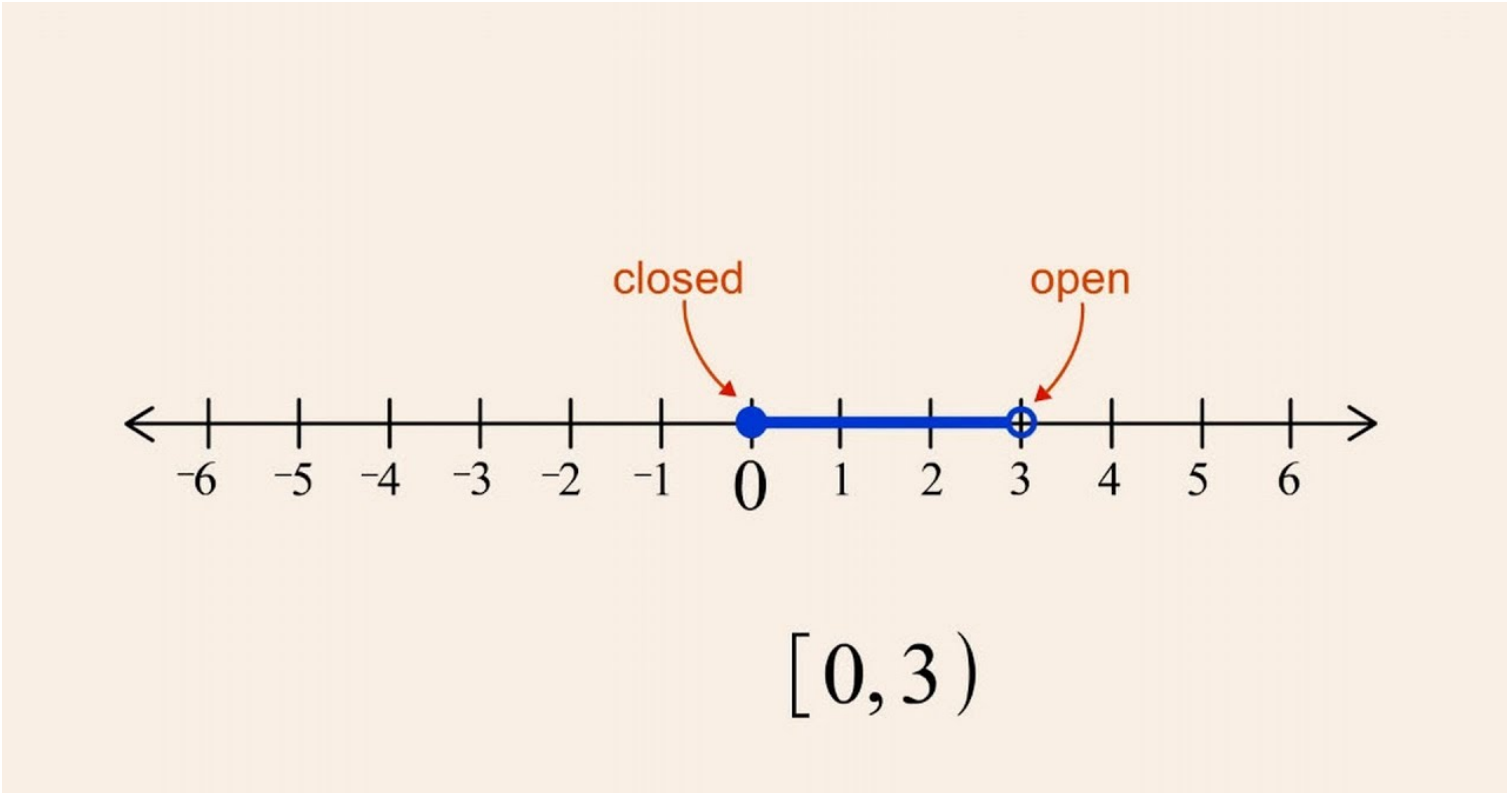


Solving Linear Inequalities

Solution Set
线性不等式的解集

定义

To solve an inequality means to find the set of all values of the variable that make the inequality true. This set of values is also known as the solution set of an inequality.



Equivalent Inequalities
等价不等式

定义

Two inequalities that have the same solution set are called equivalent inequalities.

例

$-3x + 5 \leq 17$ and $-3x \leq 12$

The Big Question

How to we find Equivalent Inequalities

Addition and Subtraction in Linear Inequalities

不等式两边同加减一个数，不等式仍成立

定理 (Rule 1)

When the same constant is added to or subtracted from both sides of an inequality, the direction of the inequality is preserved and the new inequality is equivalent to the original.

例

- $-3x + 5 \leq 17$ and $-3x \leq 12$
- $72x \geq 81$ and $72x - 81 \geq 0$

Multiplying or Dividing in Linear Inequalities
正同负反

定理 (Rule 2)

When both sides of the inequality are multiplied or divided by the same nonzero constant, the direction of the inequality is preserved if the constant is positive but the direction is reversed if the constant is negative.

例

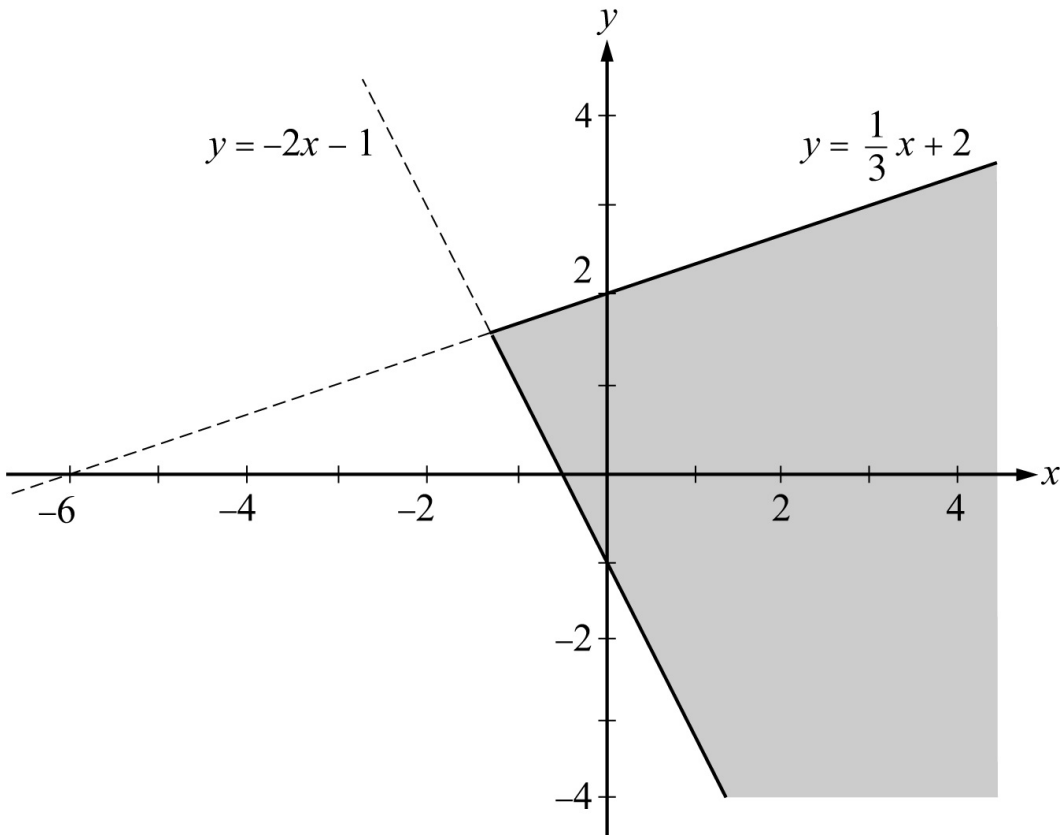
$$\begin{aligned} -3x + 15 &\leq 17 \\ -3x &\leq 12 \\ 3x &> 12 \\ x &> 4 \end{aligned}$$

Linear Inequalities In Two Variable

Solution Set For Linear Inequalities in Two Variables

$$x - 3y \geq -6$$
$$2x + y = \geq -1$$

$$y \leq \frac{1}{3} + 2$$
$$y \geq -2x + 1$$



Quadratic Problems

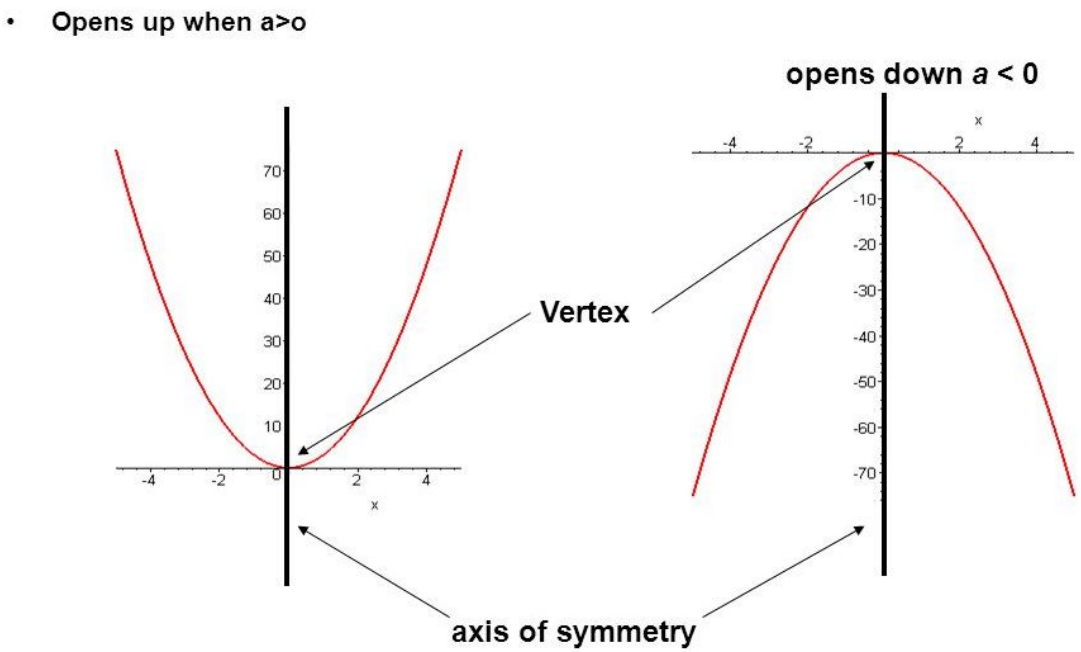
Presentation Overview for Quadratic Problems

Quadratic Function

The Opening and Vertex of a Parabola
抛物线开口和顶点

定义

The graph of a quadratic equation of the form $y = ax^2 + bx + c$, where a, b, and c are constants and $a \neq 0$, is a parabola. The symmetric axis is $x = -\frac{2a}{b}$



A Real QR Problem!

Which of the following could be a portion of the graph of $y = (x+2)^2 - 5$ in the xy-plane

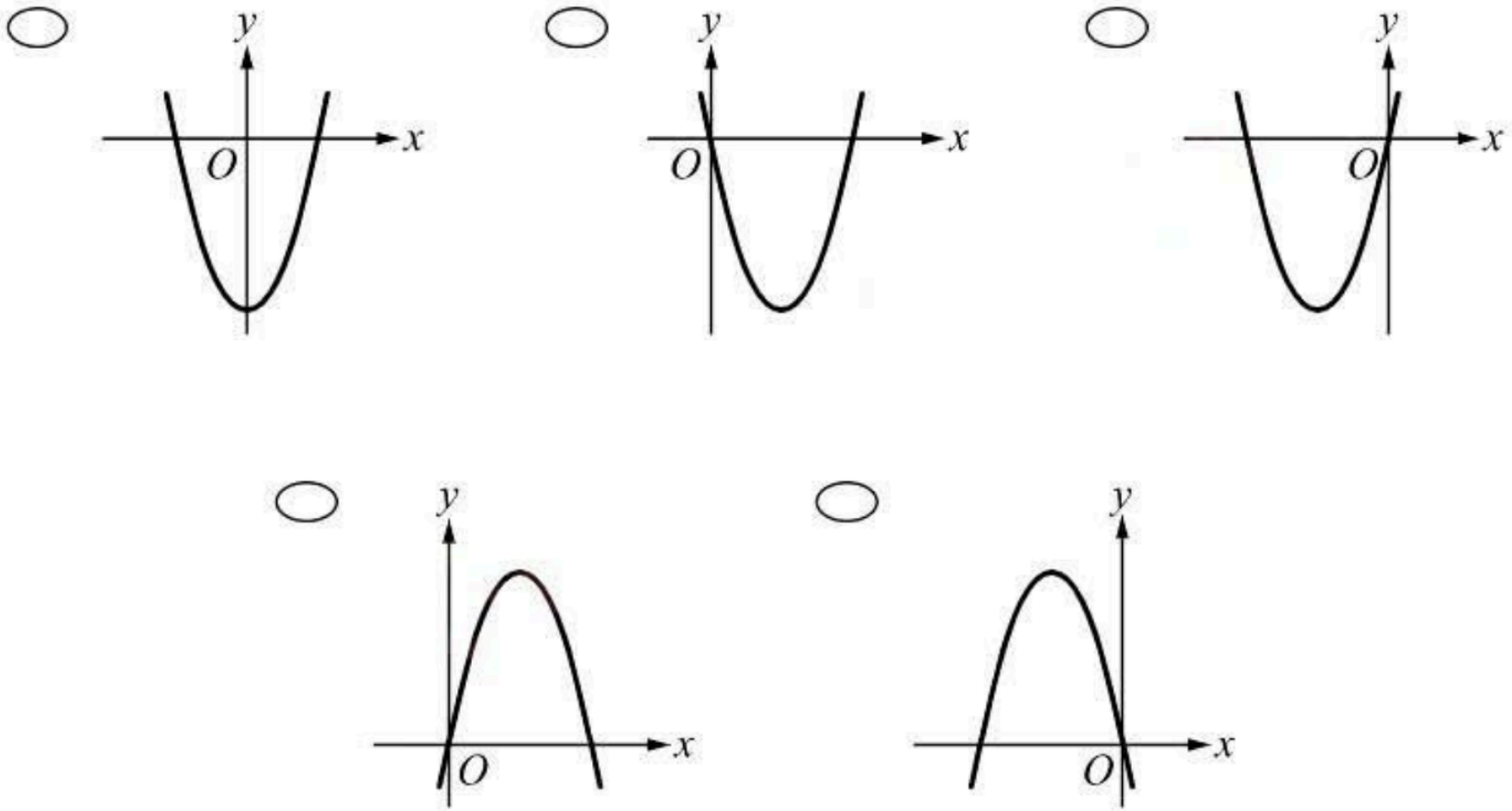


图: 6-Sec3-11

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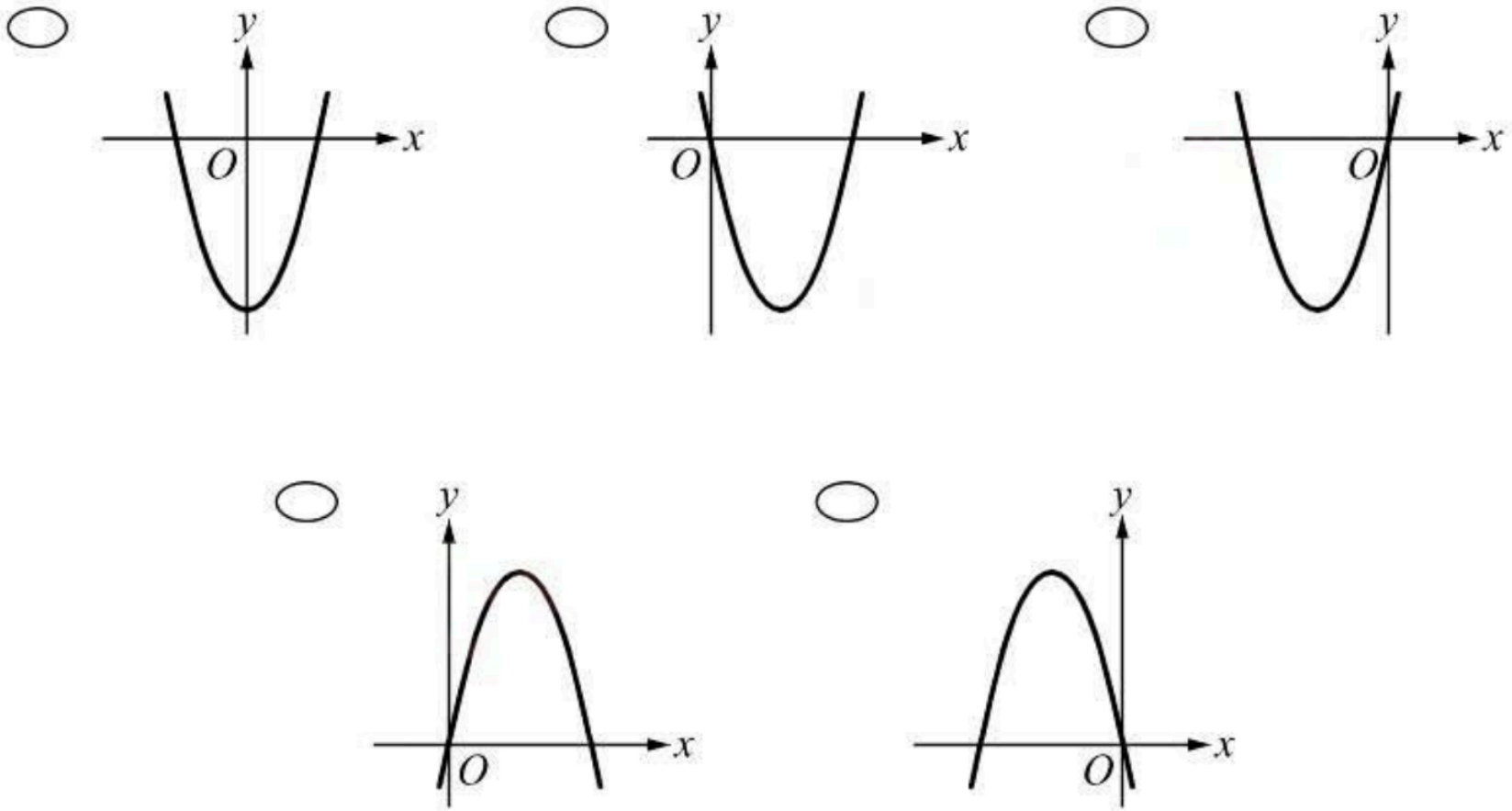

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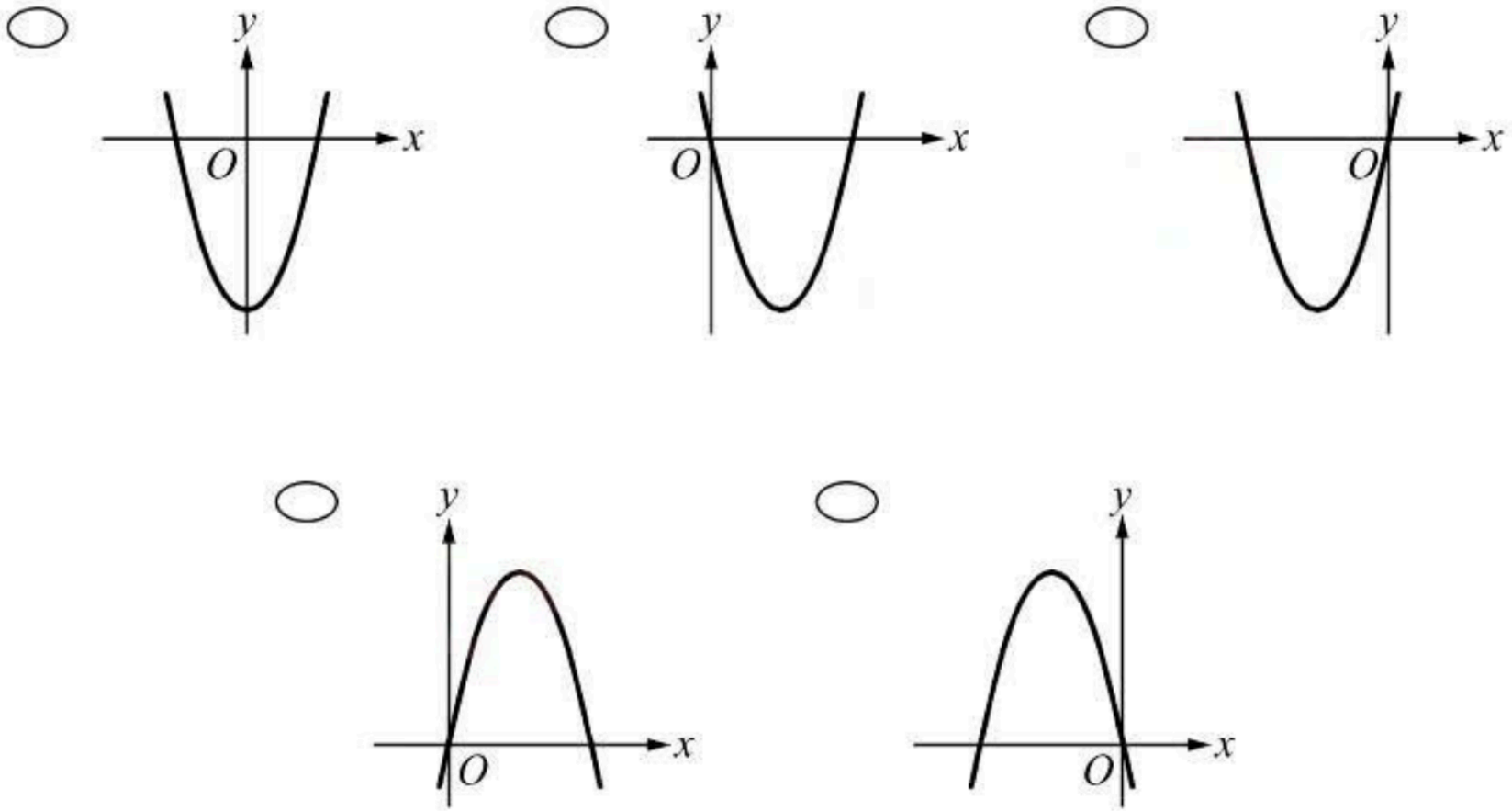

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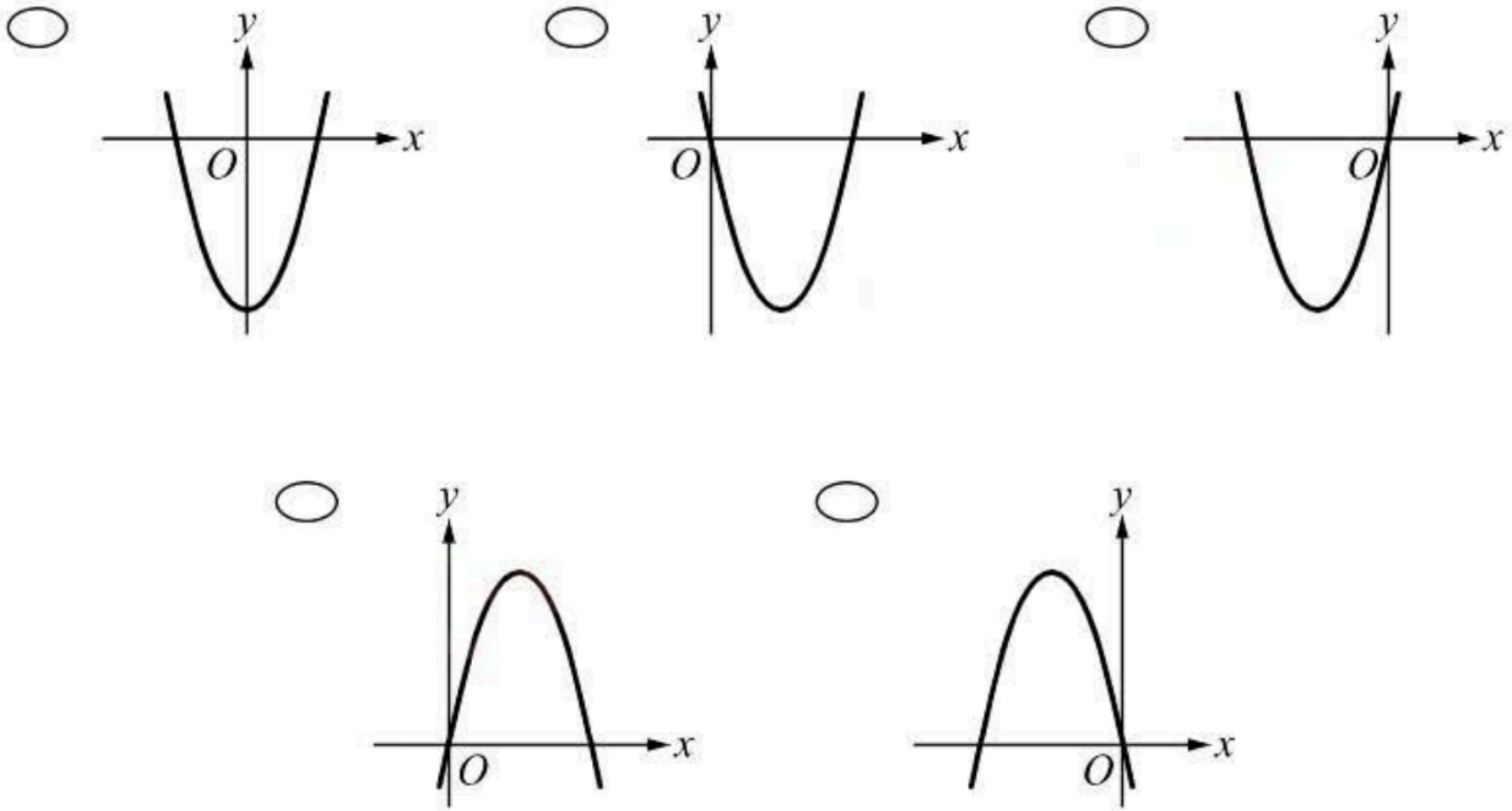

图: 6-Sec3-11

Have a try!

In the range of $-3/4 < x < -1/2$, what is the least possible value of x?

A. x
B. $x+3$
C. x^2-3x
D. x^3-x
E. x^4

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A. x
B. $x+3$
C. x^2-3x
D. x^3-x
E. x^4

A. $x<0$
B. $x+3>0$
C. $x^2-3x>0$ since $x=\frac{4}{3}$ is the symmetric axis and the opening is upward
D. $x^3-x=x(x^2-1)>0$ since $x^2<1$
E. $x^4>0$

Answer **A**

Solving Quadratic Equations By the Quadratic Formula Or Factoring

因式分解（配方法

定理 (一元二次方程公式)

For $y = ax^2 + bx + c$, *the solutions is*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

例

$$2x^2 - x - 6 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{1 \pm 7}{4} = 2 \text{ or } -\frac{3}{2}$$

例

$$2x^2 - x - 6 = 0$$

$$2(x^2 - 2 \cdot \frac{1}{4}x + \frac{1}{16}) - \frac{49}{8} = 0$$

$$2(x - \frac{1}{4})^2 - \frac{49}{8} = 0$$

$$(x - \frac{1}{4})^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm\frac{7}{4}$$

$$x = 2 \text{ or } -\frac{3}{2}$$

Have a try!

Parabola 和 X 轴交点就是 Solution

og-p390-2.8.5 Consider the line whose equation is $y = x^2 - 2x - 3$. Find the solution when $y = 0$.

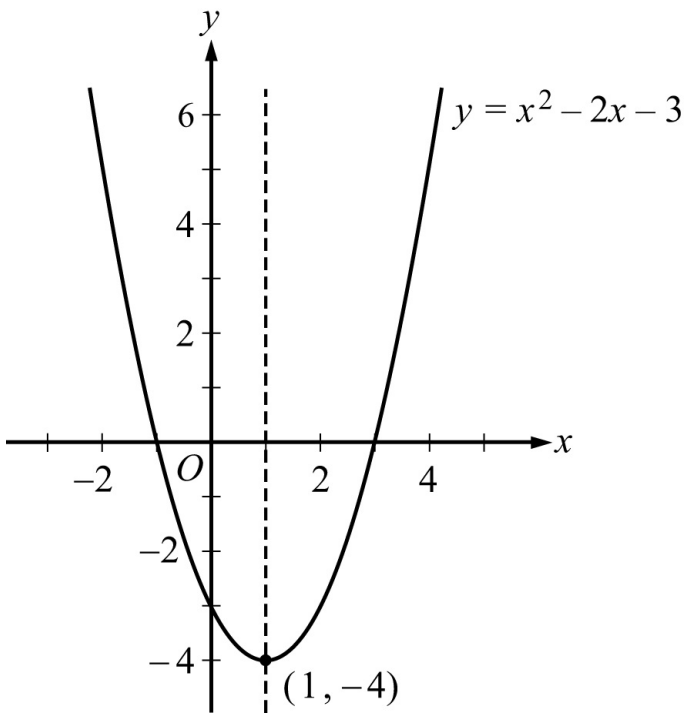
Have a try!

Parabola 和 X 轴交点就是 Solution

og-p390-2.8.5 Consider the line whose equation is $y = x^2 - 2x - 3$. Find the solution when $y = 0$.

$(x-3)(x+1) = 0$

Answer $x = -1$ *or* 3



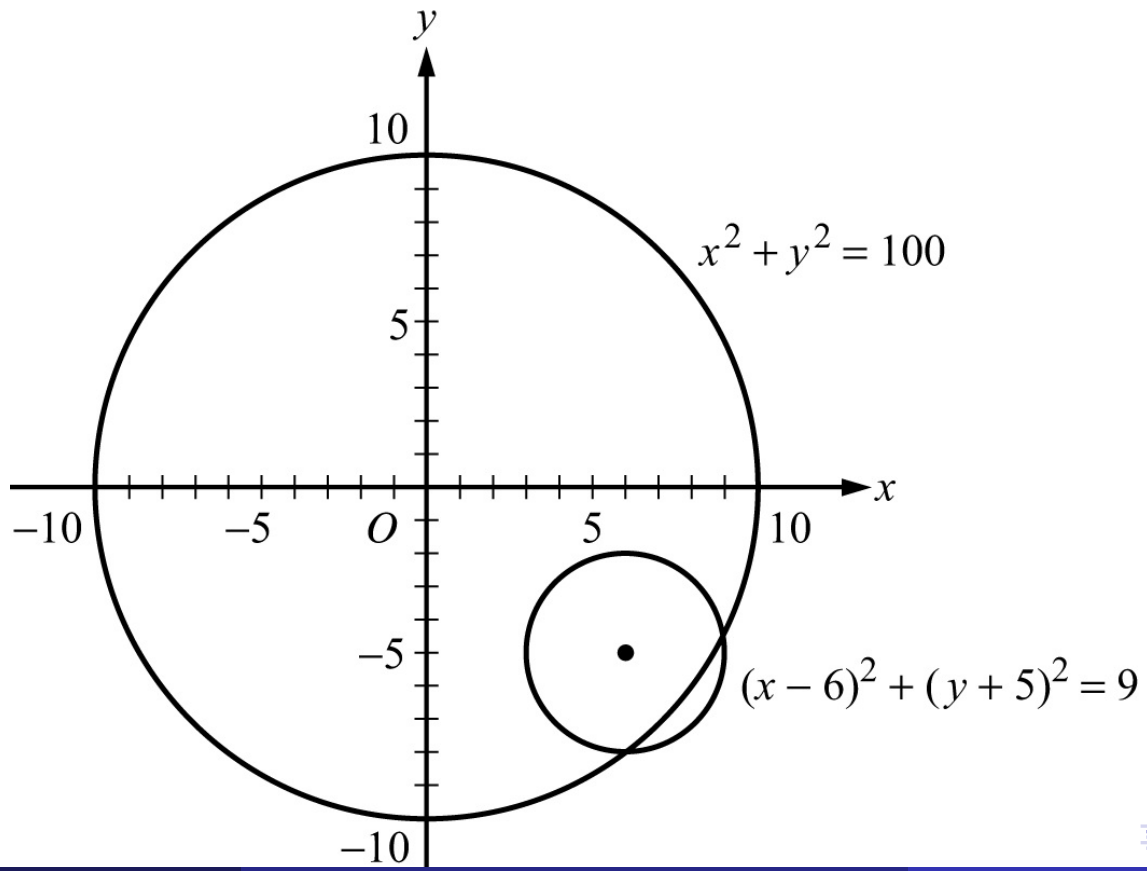
Graphing Circles

Circles

到定点距离都相等

定理 (圆)

$(x-a)^2+(y-b)^2=r^2$ is a circle with its center at the point (a,b) and with radius $r>0$.



Piecewise-Defined Function

Presentation Overview for Piecewise-Defined Function

Piecewise-Defined Function

$$y = \begin{cases} -x & x \leq 0 \\ x & x \geq 0 \end{cases}$$

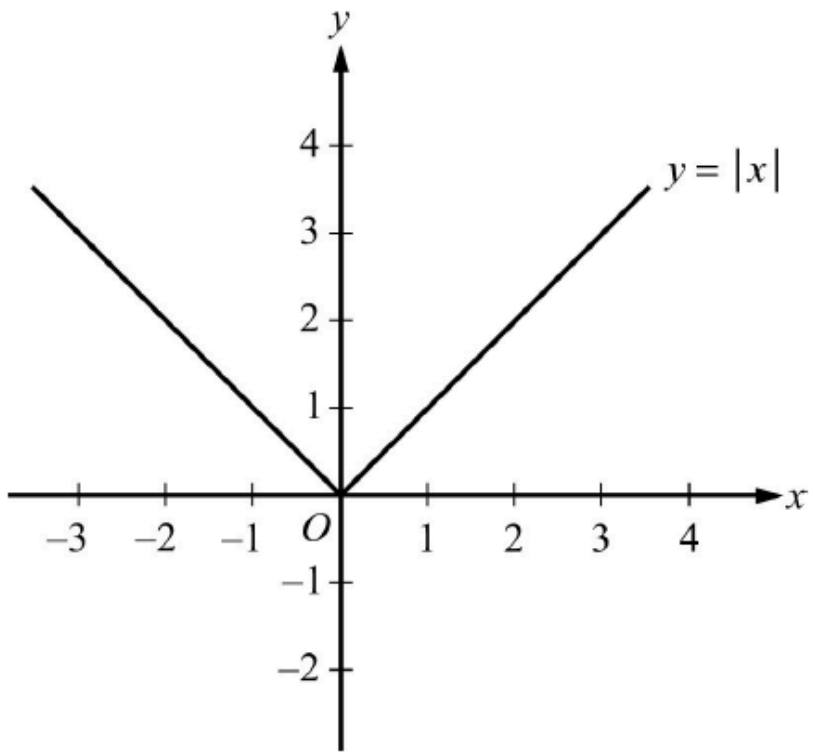


图: $y = |x|$

Reflecting, Shifting and Stretching of Functions

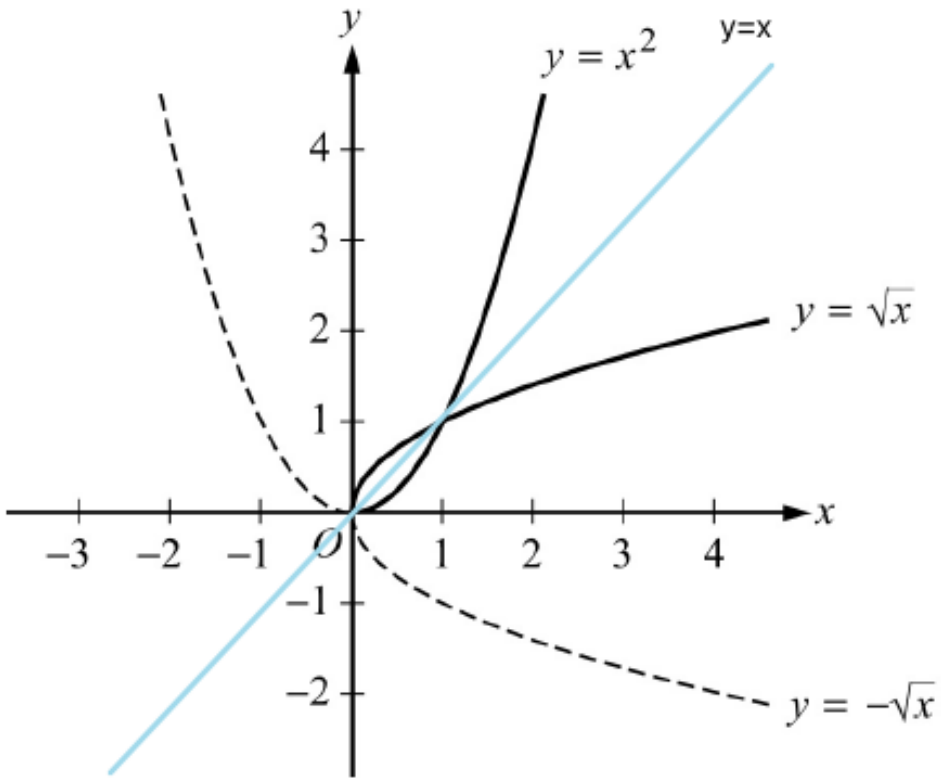
Presentation Overview for Reflecting, Shifting and Stretching of Functions

Reflecting Functions

Reflecting Functions about $y = x$

定理 (关于 $y = x$ 镜像对称：调换 xy)

The inverse funtions are the reflection of each other about $y = x$



Have a try!

x 和 y 对调

og-p390-2.8.4 Consider the line whose equation is $y = 2x + 5$. Find the equation that is reflection of $y = 2x + 5$ about $y = x$.

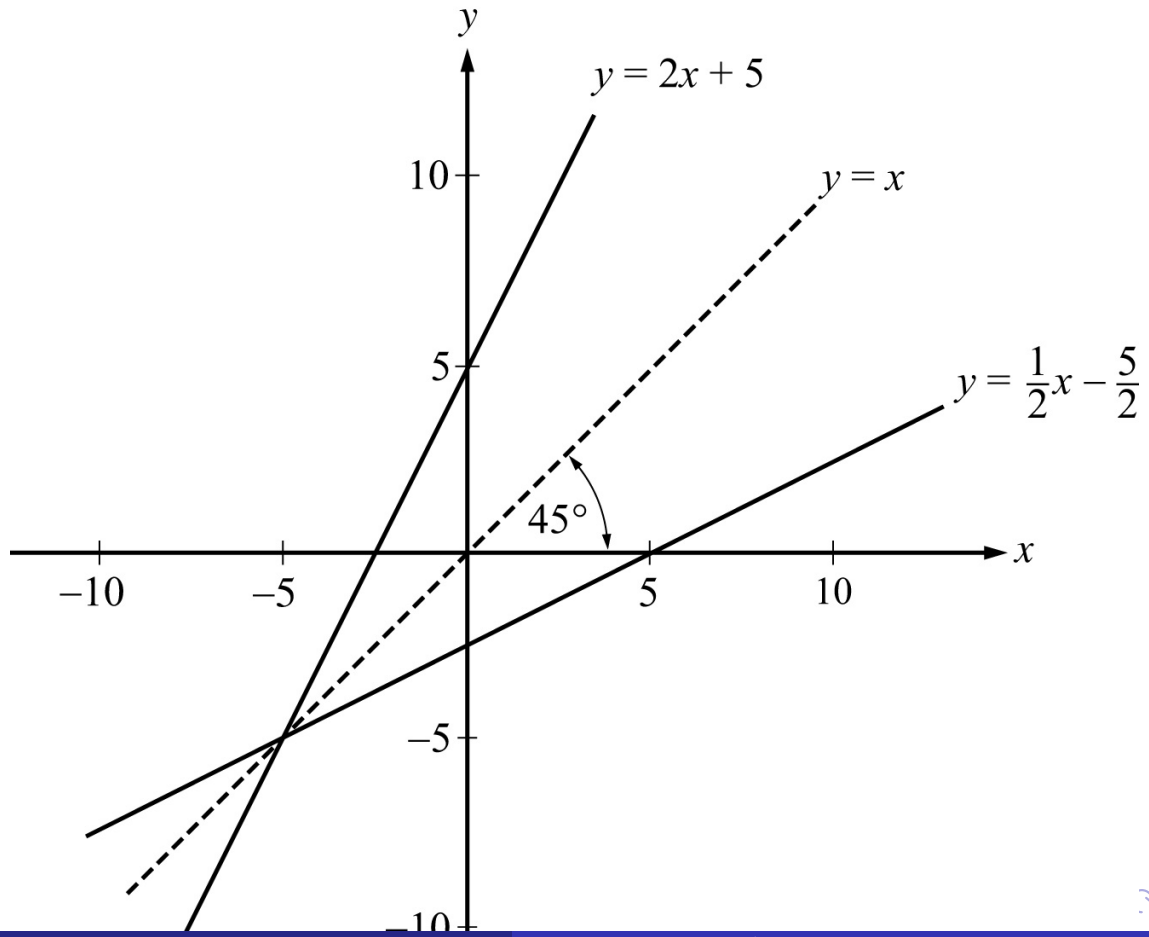
Have a try!

x 和 y 对调

og-p390-2.8.4 Consider the line whose equation is $y = 2x + 5$. Find the equation that is reflection of $y = 2x + 5$ about $y = x$.

$x = 2y + 5$

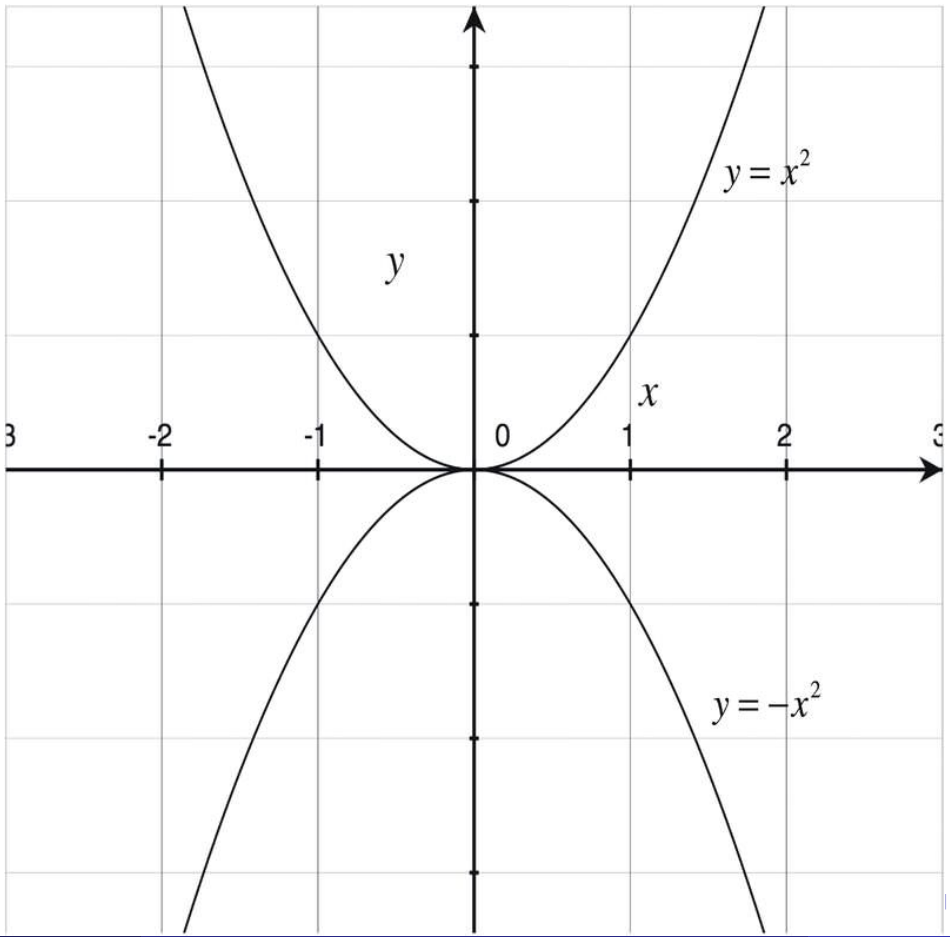
Answer $y = \frac{1}{2}x - \frac{5}{2}$



Reflecting Functions about $x-$ *axis*

定理 (关于 x 轴镜像对称：函数右边加负号)

In general, for any function h, the graph of $y = -h(x)$ *is the reflection of the graph of* $y = h(x)$ *about the x-axis.*

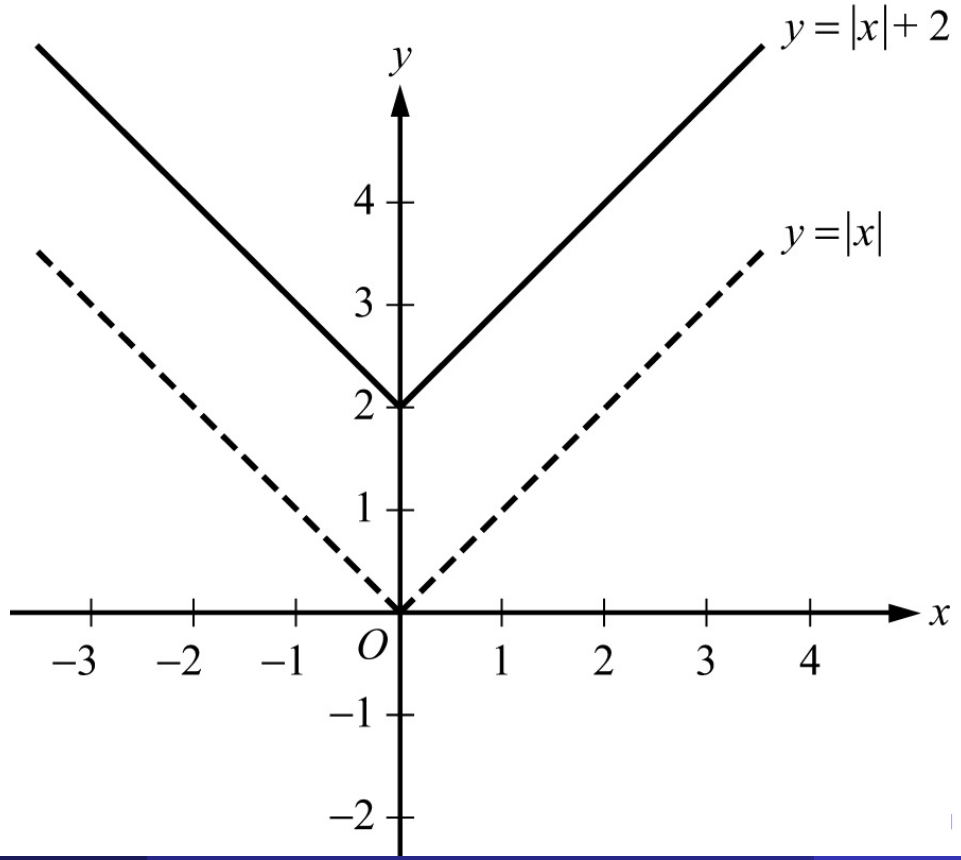


Shifting Functions

Shifting Functions Upward or Downward

定理 (上下平移：函数右边加常数项)

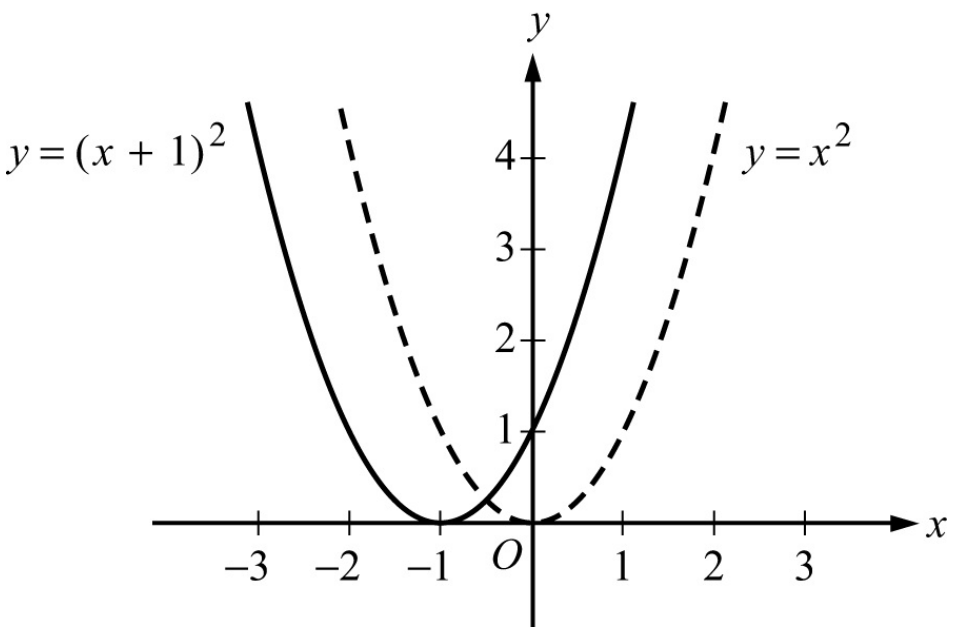
- *The graph of $h(x) + c$ is the graph of $h(x)$ shifted upward by c units.*
- *The graph of $h(x) - c$ is the graph of $h(x)$ shifted downward by c units.*



Shifting Functions to the Left or Right

定理 (左右平移：在 x 上加减)

- *The graph of $h(x+c)$ is the graph of $h(x)$ shifted to the left by c units.*
- *The graph of $h(x-c)$ is the graph of $h(x)$ shifted to the right by c units.*

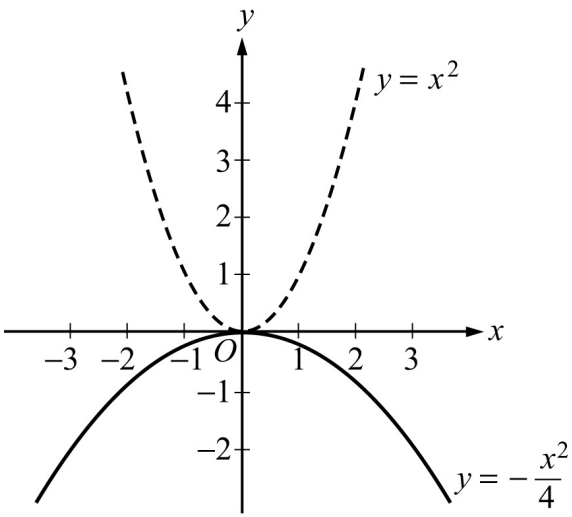


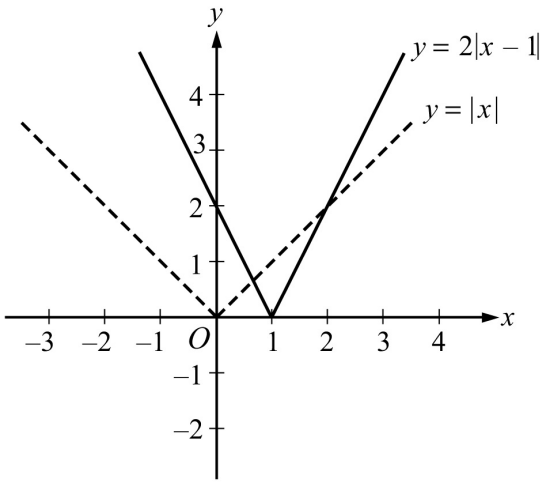
Stretching Functions

Stretching or Shrinking Functions

定理 (增大或者缩小开口：函数乘常数项)

- *The graph of $ch(x)$ is the graph of $h(x)$ stretched vertically by a factor of c if $c > 1$.*
- *The graph of $ch(x)$ is the graph of $h(x)$ shrunk vertically by a factor of c if $c < 1$.*





Applications

Presentation Overview for Applications

Translate from Words to an Arithmetic or Algebraic Representation

Average, Mixture, Rate, and Work Problems

Average Problems: Have A Try!
求平均

og-p375-2.7.4 Ellen has received the following scores on 3 exams: 82, 74, and 90. What score will Ellen need to receive on the next exam so that the average (arithmetic mean) score for the 4 exams will be 85 ?

Average Problems: Have A Try!

求平均

og-p375-2.7.4 Ellen has received the following scores on 3 exams: 82, 74, and 90. What score will Ellen need to receive on the next exam so that the average (arithmetic mean) score for the 4 exams will be 85 ?

$\frac{82+74+90+x}{4} = 85$

Average Problems: Have A Try!
求平均

og-p375-2.7.4 Ellen has received the following scores on 3 exams: 82, 74, and 90. What score will Ellen need to receive on the next exam so that the average (arithmetic mean) score for the 4 exams will be 85 ?

$\frac{82+74+90+x}{4} = 85$

Answer: **94**

A Real QR Problem!

A scientist conducted an experiment and collected three measurements. Each measurement was an integer. The range of the three measurements was 2 and the least value was 1. Which of the following values could be the average (arithmetic mean) of the measurements collected for the experiment?

Indicate all such values.

☐ $\frac{4}{3}$ ☐ $\frac{5}{3}$ ☐ 2 ☐ $\frac{7}{3}$ ☐ 3

图: 6-Sec3-13

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图: 6-Sec3-13

∵ The range is 2 and the least value is 1

∴ Two of three integers must be 1 and 3. The rest one could be 1, 2, or 3.

∴ The sum of the measurements could be 5, 6, and 7.

A Real QR Problem!

A scientist conducted an experiment and collected three measurements. Each measurement was an integer. The range of the three measurements was 2 and the least value was 1. Which of the following values could be the average (arithmetic mean) of the measurements collected for the experiment?

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图: 6-Sec3-13

∵ The range is 2 and the least value is 1

∴ Two of three integers must be 1 and 3. The rest one could be 1, 2, or 3.

∴ The sum of the measurements could be 5, 6, and 7.

Answer **BCD:** $\frac{5}{3}; 2; \frac{7}{3}$

Mixture Problems: Have A Try!

求混合比例

og-p376-2.7.5 A mixture of 12 grams of vinegar and oil is 40 percent vinegar, where all of the measurements are by weight. How many grams of oil must be added to the mixture to produce a new mixture that is only 25 percent vinegar?

Mixture Problems: Have A Try!

求混合比例

og-p376-2.7.5 A mixture of 12 grams of vinegar and oil is 40 percent vinegar, where all of the measurements are by weight. How many grams of oil must be added to the mixture to produce a new mixture that is only 25 percent vinegar? $\frac{12\times 0.4}{12+x} = 25\%$

Mixture Problems: Have A Try!

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og-p376-2.7.5 A mixture of 12 grams of vinegar and oil is 40 percent vinegar, where all of the measurements are by weight. How many grams of oil must be added to the mixture to produce a new mixture that is only 25 percent vinegar? $\frac{12\times 0.4}{12+x} = 25\%$

Answer: **7.2 grams**

Rate Problems: Have A Try!

求速率

og-p376-2.7.6 In a driving competition, Jeff and Dennis drove the same course at average speeds of 51 miles per hour and 54 miles per hour, respectively. If it took Jeff 40 minutes to drive the course, how long did it take Dennis?

Rate Problems: Have A Try!

求速率

og-p376-2.7.6 In a driving competition, Jeff and Dennis drove the same course at average speeds of 51 miles per hour and 54 miles per hour, respectively. If it took Jeff 40 minutes to drive the course, how long did it take Dennis?

$$d = r_J t_J = 51\text{mile/h} \times \frac{40\text{min}}{60\text{min/h}} = 34\text{miles}$$

$$t_D = \frac{d}{r_d} = \frac{34\text{mile}}{54\text{mile/h}} \times 60\text{min/h} \approx 37.8\text{min}$$

Rate Problems: Have A Try!

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Answer: **37.8** mins

Rate Problems: Have A Try!

求速度

Six machines, each working at the same constant rate, together can complete a certain job in 12 days. How many additional machines, each working at the same constant rate, will be needed to complete the job in 8 days?

Rate Problems: Have A Try!

求速度

Six machines, each working at the same constant rate, together can complete a certain job in 12 days. How many additional machines, each working at the same constant rate, will be needed to complete the job in 8 days?

$$w = x \cdot r \cdot t = 6 \cdot r \cdot 12$$

$$x\prime = \frac{w}{r \cdot t\prime} = \frac{6 \cdot r \cdot 12}{r \cdot 8} = 9$$

Rate Problems: Have A Try!

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Is 9 the final answer?

Rate Problems: Have A Try!

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Is 9 the final answer?

Answer: **3 Additional Machines**

A Real QR Problem!

A bicycle is traveling at a constant rate such that the wheels rotate 72 degrees per 0.1 second. If each wheel of the bicycle has a diameter of 26 inches, how many inches does the bicycle travel in 2 seconds?

○ 52 ○ 52π ○ 104 ○ 104π ○ 396

图: 7-Sec3-8

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图: 7-Sec3-8

$$degree = r \cdot t = \frac{72^\circ}{0.1\text{s}} \cdot 2\text{s} = 1440^\circ$$

$$circumference = 2\pi \cdot radius = 26\pi \text{ inch}$$

$$distance = \frac{degree}{360^\circ} \cdot circumference = \frac{1440^\circ}{360^\circ} \cdot 26\pi \text{ inch} = 104\pi \text{ inch}$$

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Answer **D:** 104π

Work Problems: Have A Try!

求速率

og-p377-2.7.7 A batch of computer parts consists of n identical parts, where n is a multiple of 60. Working alone at its constant rate, machine A takes 3 hours to produce a batch of computer parts. Working alone at its constant rate, machine B takes 2 hours to produce a batch of computer parts. How long will it take the two machines, working simultaneously at their respective constant rates, to produce a batch of computer parts?

Work Problems: Have A Try!

求速率

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$$r_A = \frac{w}{t_A} = \frac{1}{3}$$
$$r_B = \frac{w}{t_B} = \frac{1}{2}$$
$$t_{A+B} = \frac{w}{r_A + r_B} = \frac{1}{\frac{1}{3} + \frac{1}{2}} = \frac{6}{5} = 1.2\text{h}$$

Work Problems: Have A Try!

求速率

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Answer: **1.2** h

Interest

Simple Interest v.s. Compound Interest
单利 复利

Simple Interest : $V = P(1 + \frac{rt}{100})$

Compound Interest : $V = P(1 + \frac{r}{100})^t$

- P: the principal 本金
- r: the simple annual interest rate of r percent 年利率
- t: t years 时间（年）
- V: the value V of the investment at the end of t years 最终金额

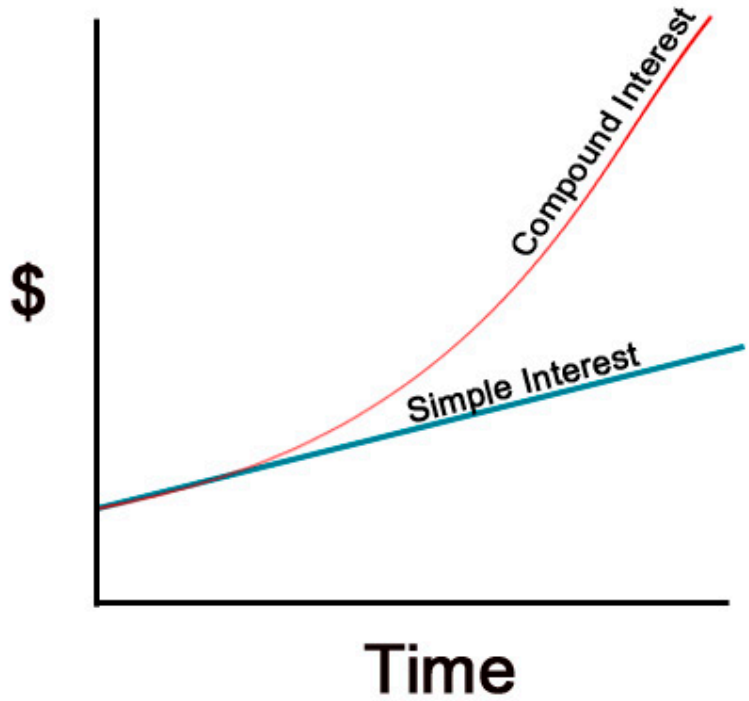

图: Compound Interest follow a exponential curve

Compounded Quarterly Or Monthly
复利的结算周期

Compound Interest :

$$V = P(1 + \frac{r}{100n})^{nt}$$

- P: the principal 本金
- r: the simple annual interest rate of r percent 年利率
- t: t years 时间（年）
- V: the value V of the investment at the end of t years 最终金额
- n: the times of compounding interest into the principal

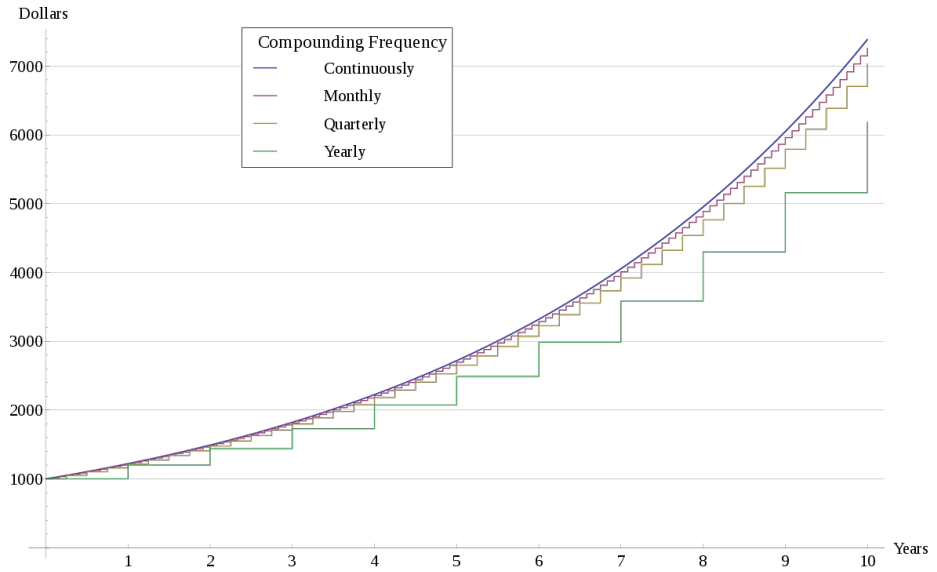

图: compound interest continuously, monthly, quarterly and yearly

Interest Problems: Have A Try!

分清楚单利复利

og-p379-2.7.10 If $ 10,000 is invested at a simple annual interest rate of 6 percent, what is the value of the investment after half a year?

Interest Problems: Have A Try!

分清楚单利复利

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$$
\begin{aligned}
V &= P(1 + \frac{rt}{100}) \\
&= \$1000(1 + 0.06(\frac{1}{2})) \\
&= \$10,300
\end{aligned}
$$

Interest Problems: Have A Try!

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&= \$10,300
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$$

Answer: **$ 10,300**

Interest Problems: Have A Try!

注意近似要求

og-p379-2.7.11 If an amount P is to be invested at an annual interest rate of 3.5 percent, compounded annually, what should be the value of P so that the value of the investment is $ 1,000 at the end of 3 years? (Give your answer to the nearest dollar.)

Interest Problems: Have A Try!

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$$
\begin{aligned}
V &= P(1+\frac{r}{100})^t \\
&= P(1+0.035)^3 \\
&= \$1000 \\
gP &= \frac{\$1000}{(1+0.035)^3} \\
&\approx \$902
\end{aligned}
$$

Interest Problems: Have A Try!

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&= P(1+0.035)^3 \\
&= \$1000 \\
gP &= \frac{\$1000}{(1+0.035)^3} \\
&\approx \$902
\end{aligned}
$$

Answer: **$ 902**

A Real QR Problem!

On a one-year loan of \$50,000, the interest charged for the first month is d dollars per \$1,000 loaned and the interest charged for each of the remaining 11 months is n dollars per \$1,000 loaned.

Quantity A	Quantity B
The total interest charged for the first 4 months	$50(d+3n)$ dollars

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

图: 4-Sec1-6

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图: 4-Sec1-6

Money Loaned!!

Answer **C:** The two quantities are equal

A Real QR Problem!

An organization will loan an amount of \$100,000 that will be paid back over 10 years with a loan payment at the end of each year according to a graduated payment plan, as follows. Each year the payment will consist of $\frac{1}{10}$ of the amount loaned, plus interest. For each of the first 2 years, the interest will be 8 percent of the amount loaned. For each of the next 2 years, the interest will be 12 percent of the amount loaned. For each of the last 6 years, the interest will be 16 percent of the amount loaned. What is the total amount of interest that will be paid?

- ◯ \$80,000
- ◯ \$96,000
- ◯ \$120,000
- ◯ \$136,000
- ◯ \$160,000

图: 4-Sec3-11

A Real QR Problem!

An organization will loan an amount of \$100,000 that will be paid back over 10 years with a loan payment at the end of each year according to a graduated payment plan, as follows. Each year the payment will consist of $\frac{1}{10}$ of the amount loaned, plus interest. For each of the first 2 years, the interest will be 8 percent of the amount loaned. For each of the next 2 years, the interest will be 12 percent of the amount loaned. For each of the last 6 years, the interest will be 16 percent of the amount loaned. What is the total amount of interest that will be paid?

- ◯ \$80,000
- ◯ \$96,000
- ◯ \$120,000
- ◯ \$136,000
- ◯ \$160,000

图: 4-Sec3-11

$$(0.08 \times 2 + 0.12 \times 2 + 0.16 \times 7) \times \$100,000 = \$136,000$$

Amount Loaned

Answer **D:** \$136,000

1 Min Break

Questions? Comments?