Geometry
几何

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GRE 冲分班数学

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To Begin With

QR Mathematical Convention 3

All figures are assumed to lie in a plane unless otherwise indicated.

QR Mathematical Convention 4

Geometric figures are not necessarily drawn to scale.

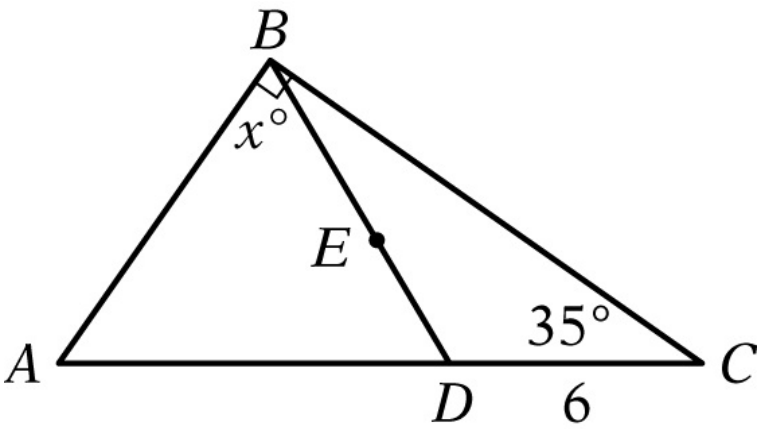
例

- Can not assume that quantities such as lengths and are as they appear in a figure
- Can not assume that angle measures such as lengths and are as they appear in a figure
- Can assume all geometric objects are in the relative positions shown.

Rely on Your Geometric Reasoning, not Estimating or Comparing Quantities By Eyesight

用几何推理做题!

Which of the following statements Must Be right?



1. Points A, D, and C are distinct. Point D lies between points A and C, and the line containing them is straight.
2. The length of line segment AD is less than the length of line segment AC.
3. ABC, ABD, and DBC are triangles.
4. Point E lies on line segment BD.
5. Angle ABC is a right angle, as indicated by the small square symbol at point B.
6. The length of line segment DC is 6, and the measure of angle C is 35 degrees.
7. The measure of angle ABD is x degrees,

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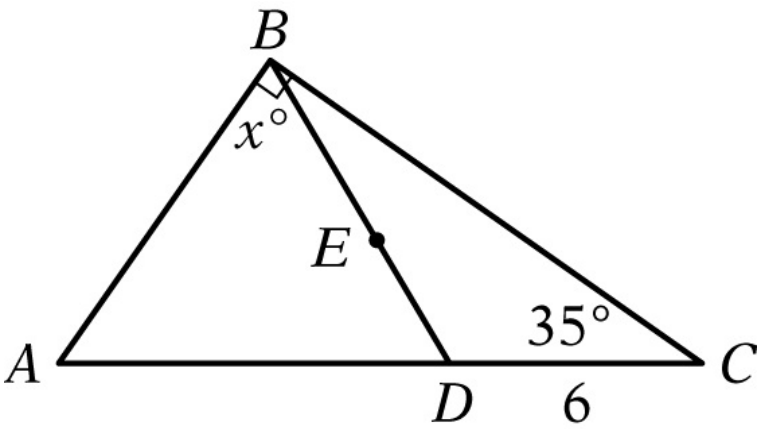

Answer: **They all must be right!**

1. Points A, D, and C are distinct. Point D lies between points A and C, and the line containing them is straight.
2. The length of line segment AD is less than the length of line segment AC.
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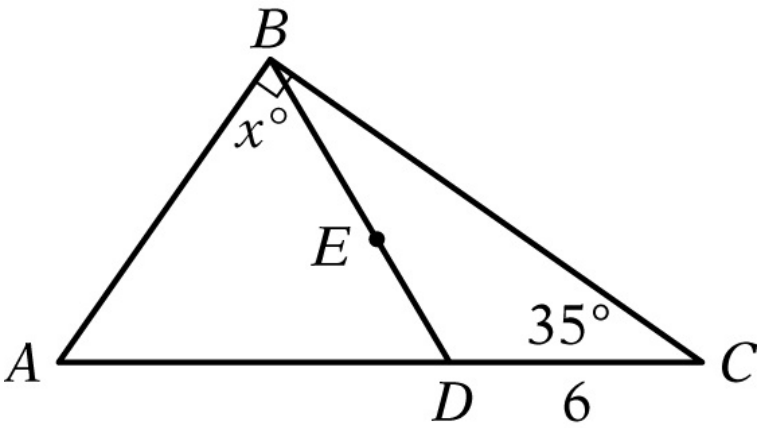


1. The length of line segment AD is greater than the length of line segment DC.
2. The measures of angles BAD and BDA are equal.
3. The measure of angle is less than x degrees.
4. The area of triangle ABD is greater than the area of triangle DBC.

Answer: **They are all not necessarily right!**

Lines and Angles

Presentation Overview for Lines and Angles

Lines

Congruent line segments

用几何推理做题!

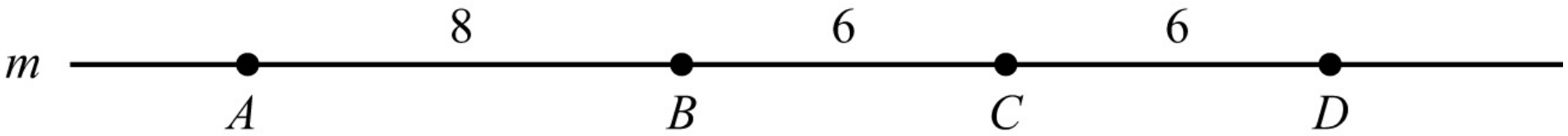


图: BC and CD are congruent line segments.

定义

Line segments that have equal lengths are called congruent line segments .

congruent

/kən rooənt, käNG rooənt/

(of figures) identical in form; coinciding exactly when superimposed.

全等：相同，叠加的时候完全重合

Angles

Opposite Angles
对角相等

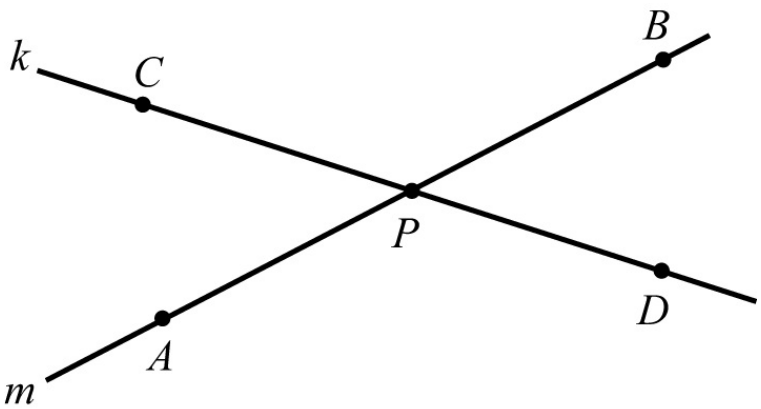


图: $\angle APC$ and $\angle BPD$ are opposite angles; So are $\angle CPB$ and $\angle DPA$.

定义

Opposite angles have equal measure, and angles that have equal measure are called congruent angles. Hence, opposite angles are congruent.

Acute,Right, Obtuse Angles

锐角 直角 钝角

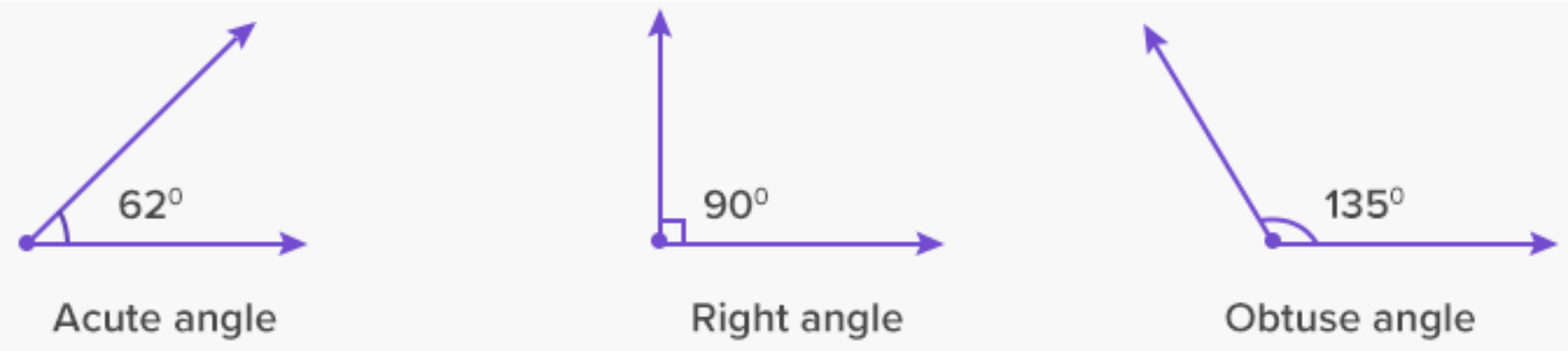


图: BC and CD are congruent line segments.

定义

- An angle with measure less than 90° is called an acute angle.
- An angle with a measure of 90° is called a right angle.
- an angle with measure between 90° and 180° is called an obtuse angle.

Parallel Lines

Parallel Lines

平行线同位角相等，内错角之和为 180 度

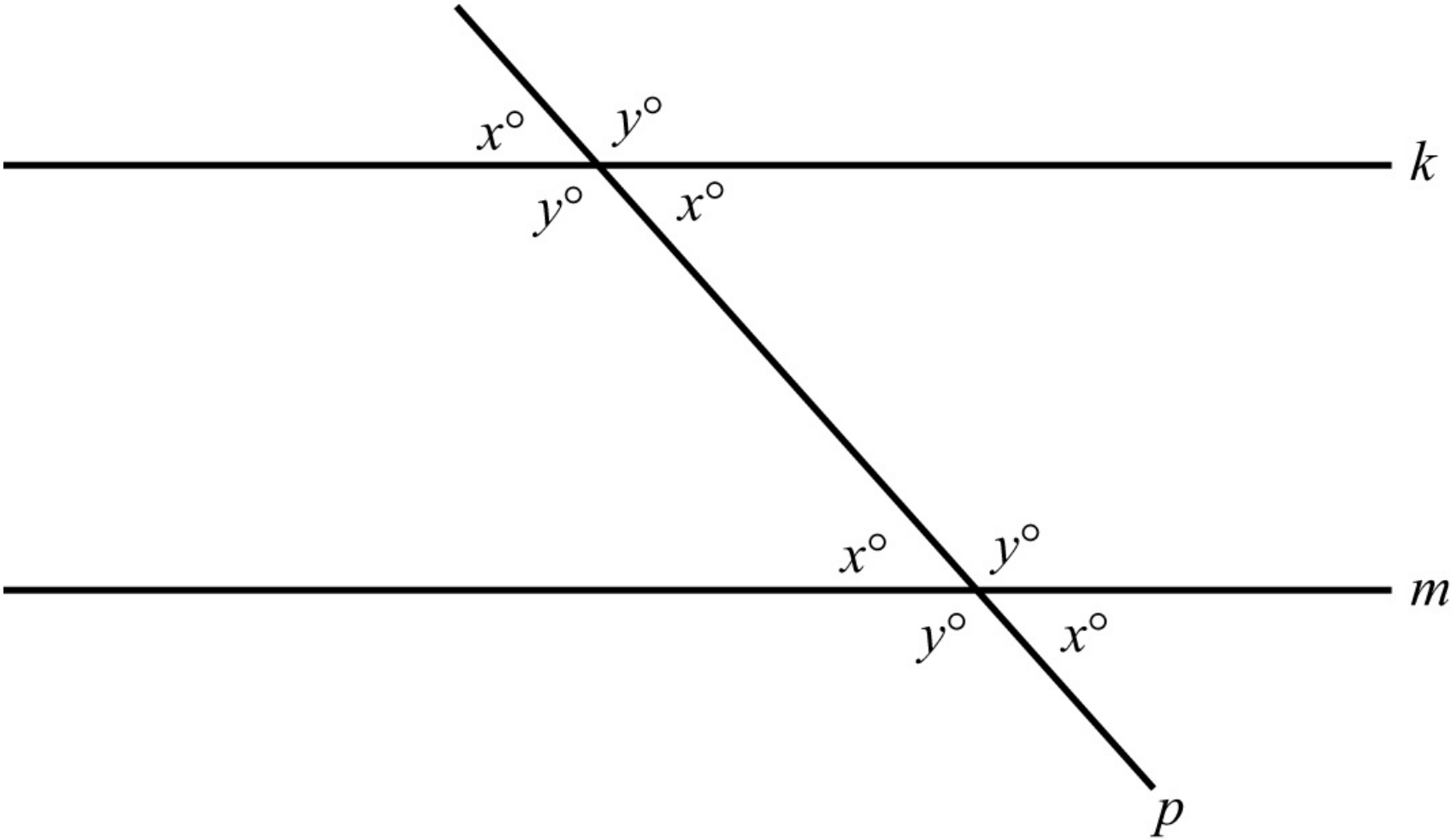


图: $k \parallel m$

Triangles

Presentation Overview for Triangles

Equilateral Triangles

Equilateral

/ ēkwə ladərəl, ekwə ladərəl/

(of figures) having all its sides of the same length..
等边：所有边长相等

Equilateral Triangles

等边三角形：内角均为 60 度

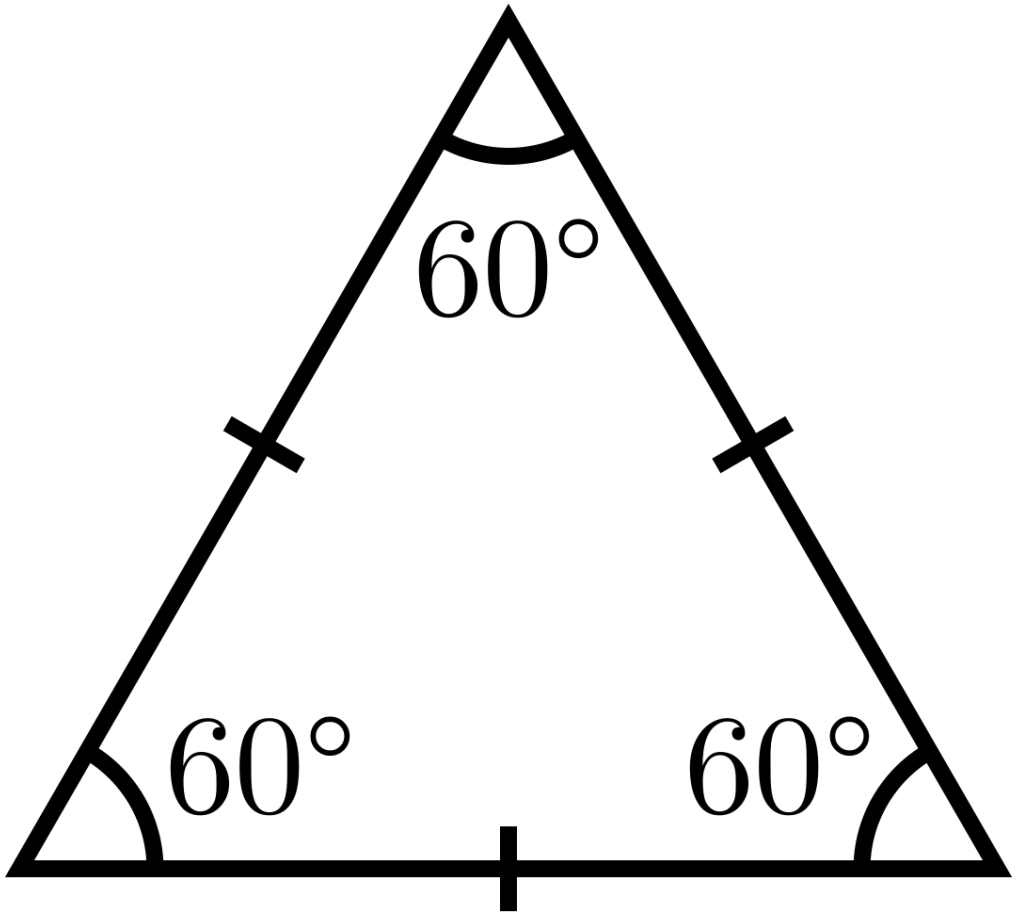


图: The measures of the three interior angles of such a triangle are equal, and each measure is 60°.

Isosceles Triangles

Isosceles

/ī säsə lēz/

(of a triangle) having two sides of equal length.
等腰三角形：两边长相等

Isosceles Triangles

等腰三角形：内角均为 60 度

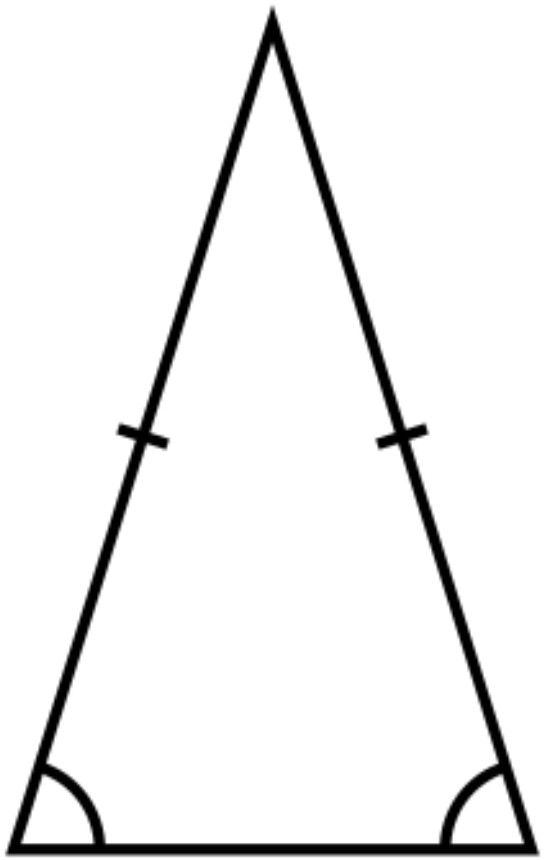


图: Congruent sides suggest congruent angles.

定理 (两角相等互推两边相等)

If a triangle has two congruent sides, then the angles opposite the two congruent sides are congruent. The converse is also true.

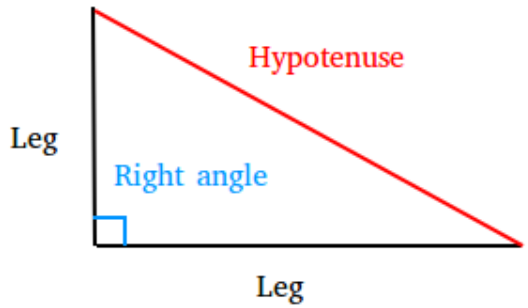
定理 (Law Of Sines 正弦定理)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Right Triangles

hypotenuse

/hī pätn (y)oos/



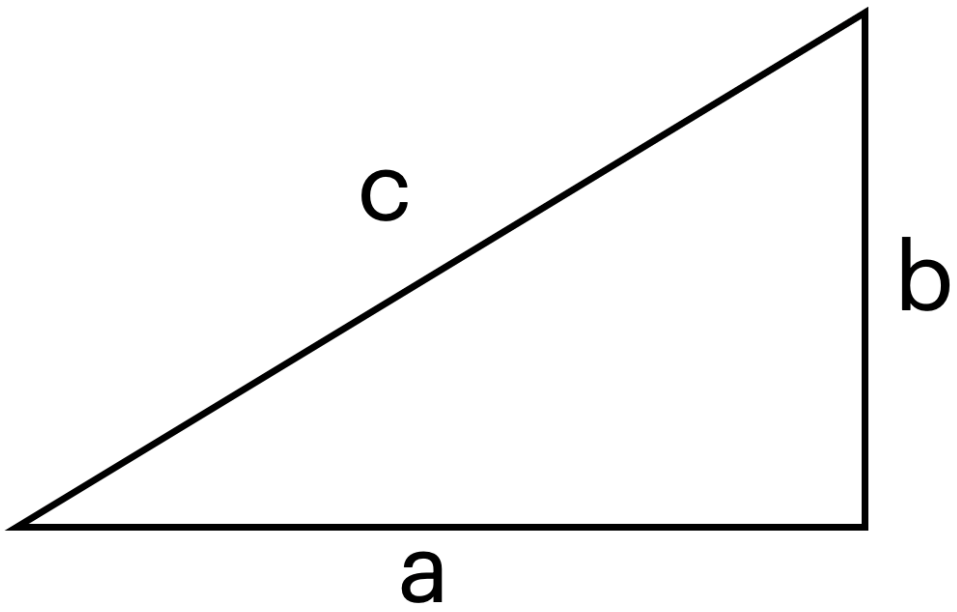
the longest side of a right triangle, opposite the right angle.

斜边：直角三角形直角对边

leg

直角边

The Pythagorean Theorem
勾股定理



$$c^2 = a^2 + b^2$$

Pythagorean

/ī säsə lēz/

relating to or characteristic of the Greek philosopher Pythagoras or his ideas.
毕达哥拉斯

45°-45°-90° Triangle

边长比为 1:1:$\sqrt{2}$

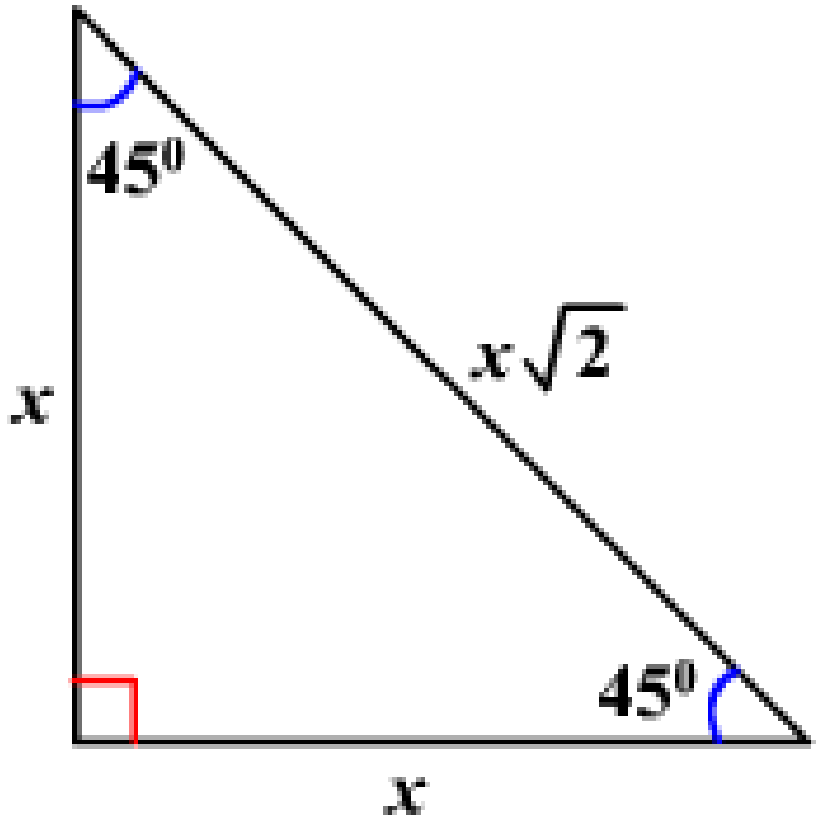
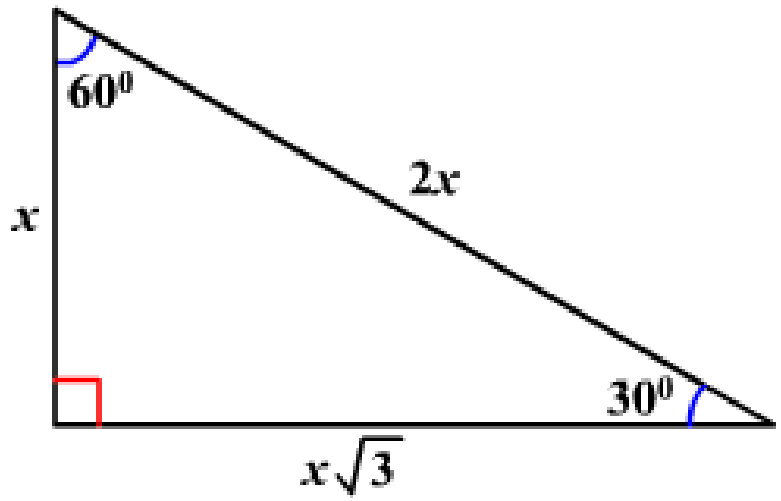


图: Isosceles Right Triangle

30°-60°-90° Triangle

边长比为 $1:\sqrt{3}:2$



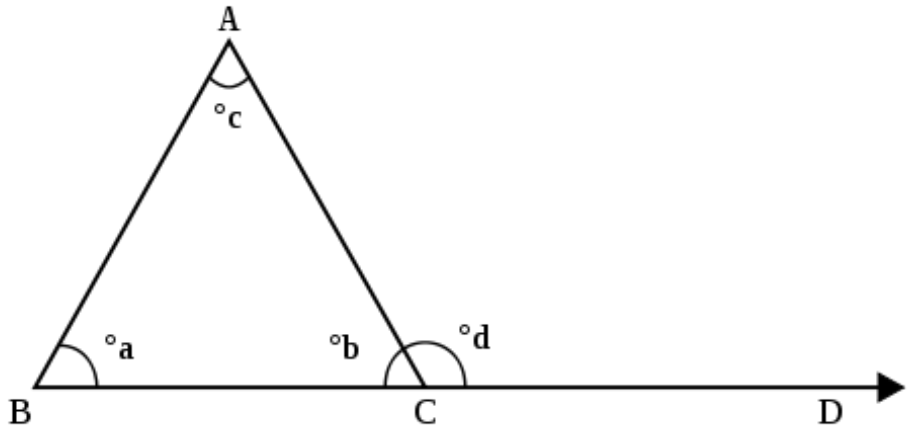
The Triangle Inequalities
三角不等式

定理 (两边之和大于第三边；两边之差小于第三边)

$$a-b<c<a+b$$

Exterior Angle of Triangles

外角等于相对应内对角之和



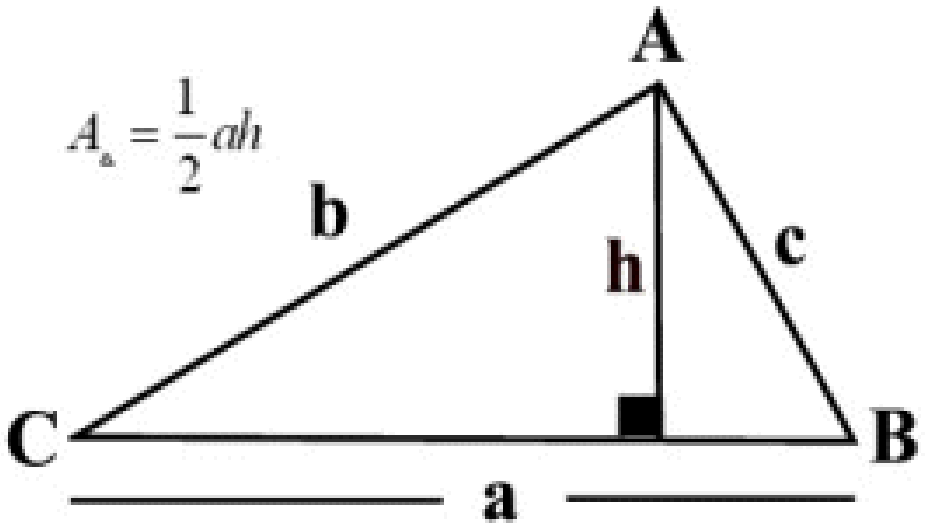
定理

$$d = a + c$$

The Area of a Triangle

The Area of a Triangle

底乘高除以二



Congruent Triangles

SSS, SAS, ASA Congruence

边边边 边角边 角边角 全等

定理 (Side-Side-Side Congruence)

If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

定理 (Side-Angle-Side Congruence)

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

定理 (Angle-Side-Angle Congruence)

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

What about AAS?**Yes!**

Similar Triangles

Scale Factor Of Similarity
相似比例

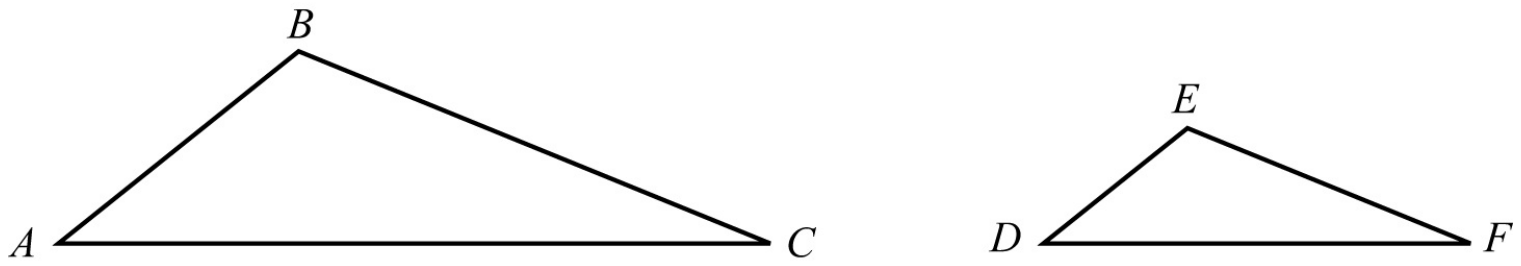


图: Two similar triangles

定义

More precisely, two triangles are similar if their vertices can be matched up so that the corresponding angles are congruent or, equivalently, the lengths of the corresponding sides have the same ratio, called the scale factor of similarity.

How to prove similarity? **AA!**

vertices

The plural noun of vertex
顶点点的复数

Quadrilaterals

Presentation Overview for Quadrilaterals

quadrilateral

/ kwädrə ladərəl, kwädrə latrəl/

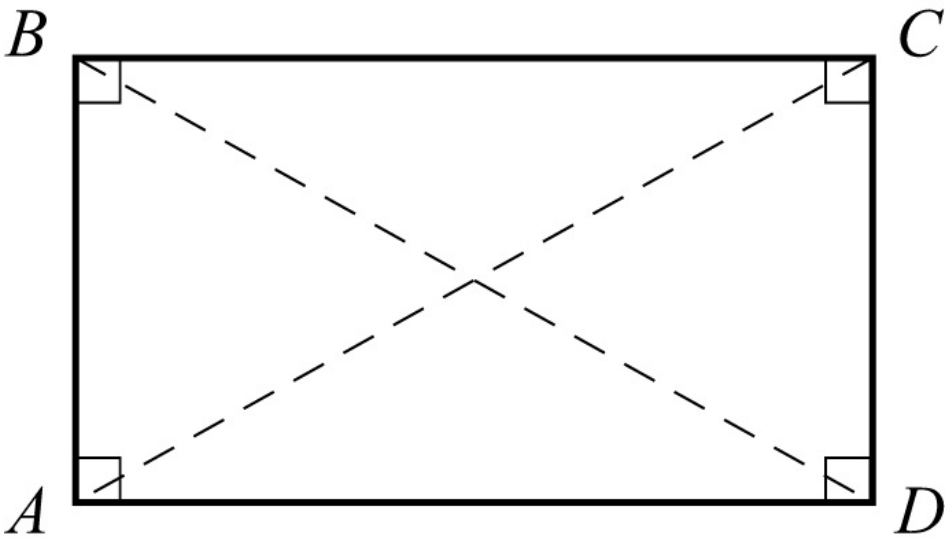
a four-sided figure.
四边形

Rectangle

Rectangle
矩形

定义

A quadrilateral with four right angles is called a rectangle. Opposite sides of a rectangle are parallel and congruent, and the two diagonals are also congruent.
A rectangle with four congruent sides is called a square.



$$Area: \ A = base \cdot height$$

Parallelogram

parallelogram

/ perə lelə ram/

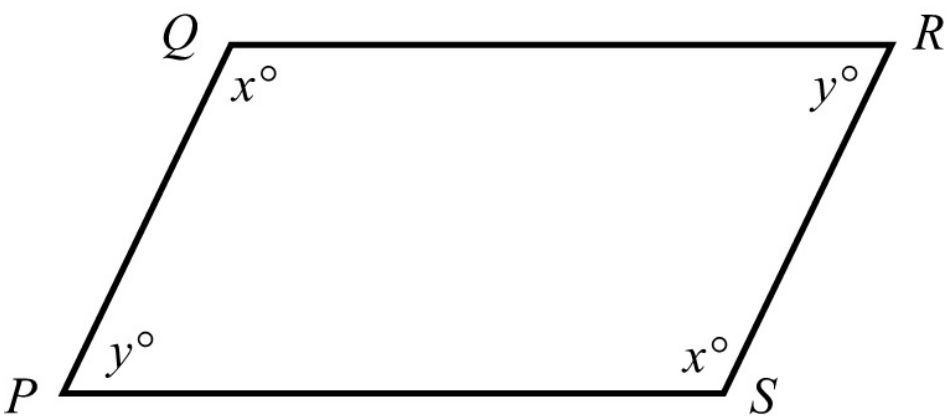
a four-sided plane rectilinear figure with opposite sides parallel.

平行四边形

Parallelogram
平行四边形

定义

A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram. In a parallelogram, opposite sides are congruent and opposite angles are congruent.
Note that all rectangles are parallelograms.



$$Area: \ A = base \cdot height$$

Trapezoid

Polygons

Presentation Overview for Polygons

Circles

Presentation Overview for Circles

Radius, Diameter, And Chord

Central Angle, Arc, Circumference and Area

Tangent Lines

Inscribed In Or Circumscribed About The Polygon

Concentric Circles

Three-Dimensional Figures

Presentation Overview for Three-Dimensional Figures

Rectangular Solid(Right Rectangular Prism)

Circular Cylinder And Right Circular Cylinder

1 Min Break

Questions? Comments?