

Introduction to Quantitative Reasoning of GRE

GRE 数学小白必看

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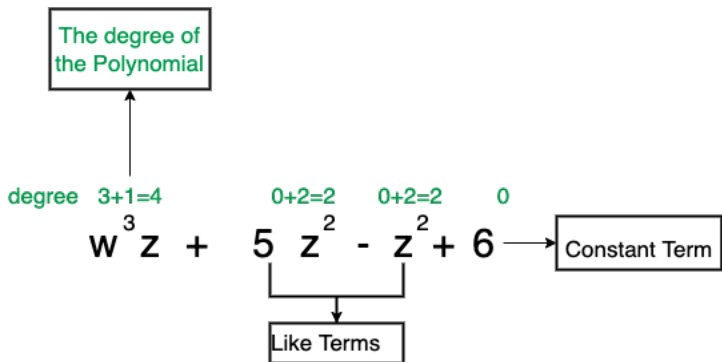
Algebra Expressions

Presentation Overview for Algebra Expressions

- ① Algebra Expressions
- ② Coordinate Geometry
- ③ Linear Problems
- ④ Quadratic Problems
- ⑤ Piecewise-Defined Function
- ⑥ Reflecting, Shifting and Stretching of Functions
- ⑦ Applications

Terminologies of Algebra

代数专业名词



- Like Terms 同类项
- The Degree of a Polynomial 多项式的次数

A Real QR Problem!

The expression $x^4 + 2x^2y^2 + 9y^4$ is equivalent to which of the following?

☐ $(x^2 + 3y^2)^2$

☐ $(x^2 + 3y^2)(x^2 - 3y^2)$

☐ $(x^2 + 3y^2 + xy)^2$

☐ $(x^2 + 2xy + 3y^2)(x^2 - 2xy + 3y^2)$

☐ $(x^2 + 2xy - 3y^2)(x^2 - 2xy - 3y^2)$

图: 10-Sec3-19

凑中间项的系数 Answer **D**

Coordinate Geometry

Presentation Overview for Coordinate Geometry

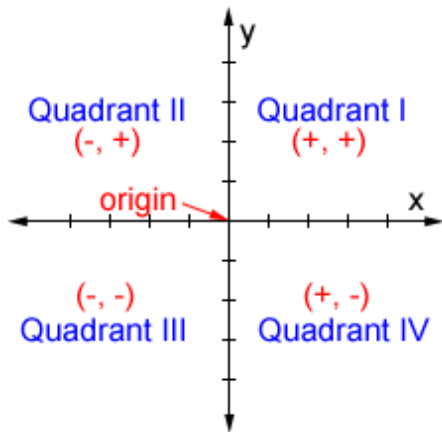
- ① Algebra Expressions
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To Begin With

QR Mathematical Convention 2

When coordinate systems, such as and number lines, are shown with scales, you should read, estimate, or compare quantities by sight or by measurement, **according to the corresponding scales**.

象限的英文怎么说？



Linear Problems

Presentation Overview for Linear Problems

① Algebra Expressions

② Coordinate Geometry

③ Linear Problems

- Linear Function

- Linear Equations in One Variable

- Linear Equations in Two Variable

- Solving Linear Inequalities

- Linear Inequalities In Two Variable

④ Quadratic Problems

⑤ Piecewise-Defined Function

Linear Function

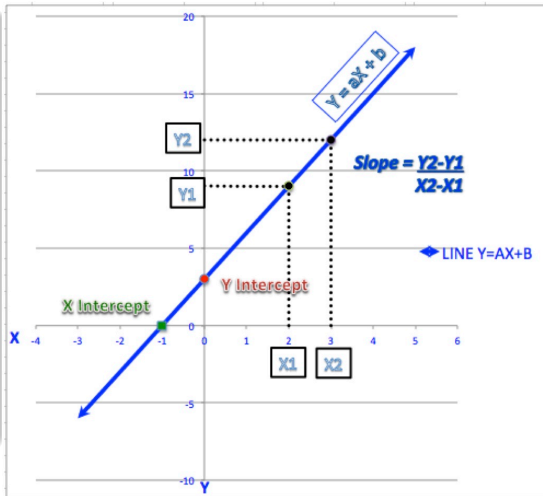
Slope and Intercepts

斜率和截距

定义

The graph of a linear equation of the form $y = mx + b$ is a straight line in the xy - plane, where m is called the **slope** of the line and b is called the **y-intercept**.

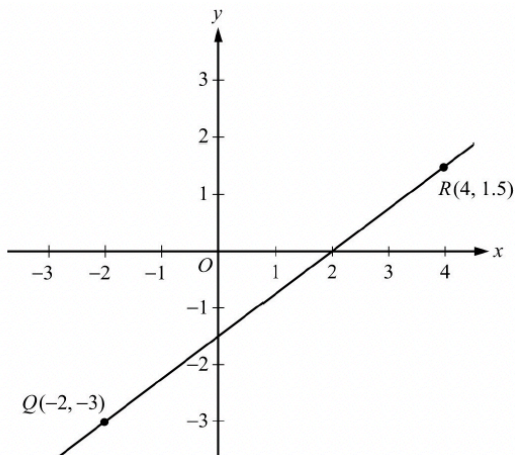
The x-intercepts of a graph are the **x-coordinates** of the points at which the graph intersects the x-axis.



Have a try!

两点确定一条直线

og-p385-2.8.1 Below shows the graph of the line through the points $Q(-2, -3)$ and $R(4, 1.5)$.



$$y = 0.75x - 1.5$$

看图 Drawn to scale

$$\text{slope} = \frac{1.5 - (-3)}{4 - (-2)} = \frac{4.5}{6} = \frac{3}{4} = 0.75$$

$$y - \text{intercept} = (-3) - 0.75 \text{ times } (-2) = -1.5$$

A Real QR Problem!

A line in the xy -plane has the equation $y = mx + 6$, where m is a constant and $3 \leq m \leq 4$. Which of the following values could be the x -intercept of the line?

Indicate all such values.

- ☐ -3 ☐ -2 ☐ $-\frac{7}{4}$ ☐ $-\frac{5}{4}$ ☐ $\frac{5}{4}$ ☐ $\frac{7}{4}$ ☐ 2 ☐ 3

图: 6-Sec3-18

$$-2 \leq x \leq -1.5$$

Answer **BC** -2 ; $-\frac{7}{4}$



A Real QR Problem!

In the xy -plane, a triangular region is enclosed by the x -axis, the y -axis, and the line with equation $2x - y + k = 0$, where k is a positive constant. For which of the following values of k is the area of the triangular region greater than 1 and less than 4?

- ☐ 0
- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4

图: 9-Sec2-10

$$4 \leq k \leq 2$$

Answer **D** $k = 3$



The Relation of Slopes for Parallel or Perpendicular

平行或垂直直线斜率关系

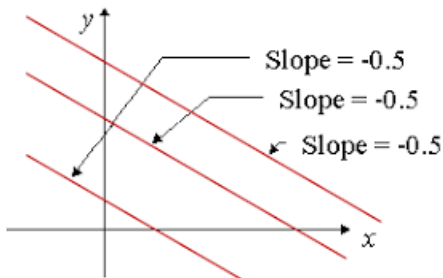


图: Two lines are parallel if their slopes are equal.

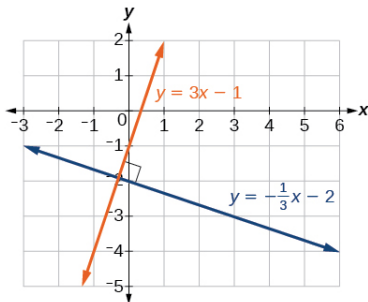
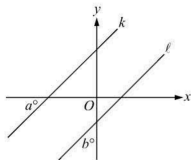


图: Two lines are perpendicular if their slopes are negative reciprocals of each other.

A Real QR Problem!



Lines k and l lie in the xy -plane and are parallel.

Quantity A

a

Quantity B

b

- ☐ Quantity A is greater.
- ☐ Quantity B is greater.
- ☐ The two quantities are equal.
- ☐ The relationship cannot be determined from the information given.

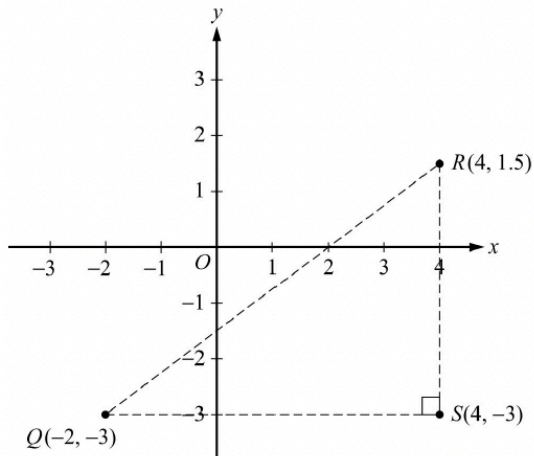
图: 6-Sec3-7

$$a^{\circ} + b^{\circ} = 90^{\circ}$$

Answer D The relationship cannot be determined from the information

Calculating the Distance Between Two Points

两点间距离



QR

$$\begin{aligned} &= \sqrt{QS^2 + RS^2} \\ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{6^2 + 4.5^2} \\ &= 7 \end{aligned}$$

Linear Equations in One Variable

Equivalent Equations

等价方程

定义

Two equations that have the **same** solutions are called equivalent equations.

例

$$x + 1 = 2 \text{ and } 2x + 2 = 4$$

Linear Equations in Two Variable

Solution For Linear Equations in Two Variables

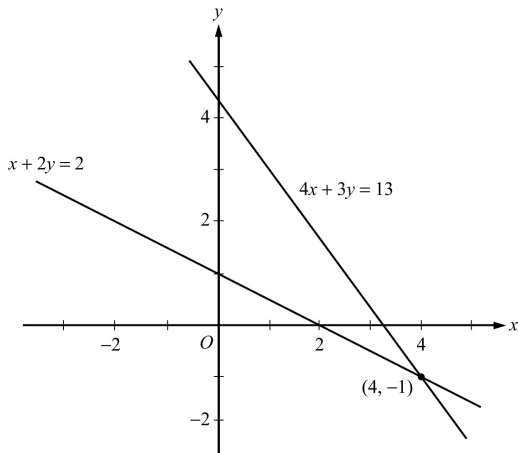
交点就是 Solution

$$4x + 3y = 13$$

$$x + 2y = 2$$

$$y = -\frac{3}{4}x + \frac{13}{4}$$

$$y = -\frac{1}{2}x + 1$$



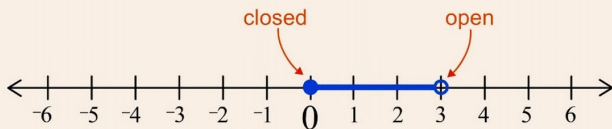
Solving Linear Inequalities

Solution Set

线性不等式的解集

定义

To solve an inequality means to find the set of all values of the variable that make the inequality true. This set of values is also known as the **solution set** of an inequality.



$$[0, 3)$$

Equivalent Inequalities

等价不等式

定义

Two inequalities that have the **same** solution set are called equivalent inequalities.

例

$$-3x + 5 \leq 17 \text{ and } -3x \leq 12$$

How to we find Equivalent Inequalities

Addition and Subtraction in Linear Inequalities

不等式两边同加减一个数，不等式仍成立

定理 (Rule 1)

When the same constant is added to or subtracted from both sides of an inequality, the direction of the inequality is preserved and the new inequality is equivalent to the original.

例

- $-3x + 5 \leq 17$ and $-3x \leq 12$
- $72x \geq 81$ and $72x - 81 \geq 0$

Multiplying or Dividing in Linear Inequalities

正同负反

定理 (Rule 2)

*When both sides of the inequality are multiplied or divided by the same nonzero constant, the direction of the inequality is **preserved if the constant is positive** but the direction is **reversed if the constant is negative**.*

例

$$-3x + 15 \leq 17$$

$$-3x \leq 12$$

$$3x > 12$$

$$x > 4$$

Linear Inequalities In Two Variable

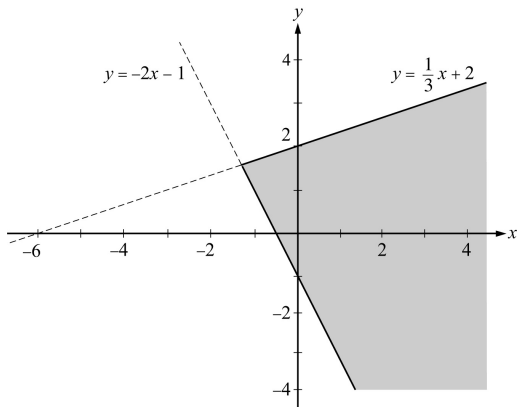
Solution Set For Linear Inequalities in Two Variables

$$x - 3y \geq -6$$

$$2x + y \geq -1$$

$$y \leq \frac{1}{3}x + 2$$

$$y \geq -2x + 1$$



Quadratic Problems

Presentation Overview for Quadratic Problems

① Algebra Expressions

② Coordinate Geometry

③ Linear Problems

④ Quadratic Problems

Quadratic Function

Solving Quadratic Equations By the Quadratic Formula Or Factoring

Graphing Circles

⑤ Piecewise-Defined Function

⑥ Reflecting, Shifting and Stretching of Functions

Quadratic Function

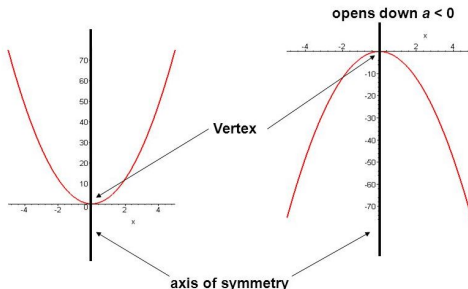
The Opening and Vertex of a Parabola

抛物线开口和顶点

定义

The graph of a quadratic equation of the form $y = ax^2 + bx + c$, where a , b , and c are constants and $a \neq 0$, is a **parabola**. The symmetric axis is $x = -\frac{2a}{b}$

- Opens up when $a > 0$



A Real QR Problem!

Which of the following could be a portion of the graph of $y = (x + 2)^2 - 5$ in the xy -plane?

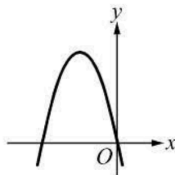
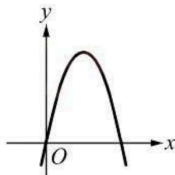
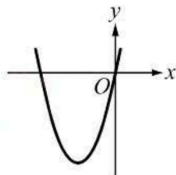
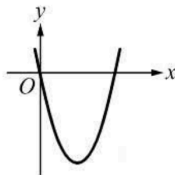
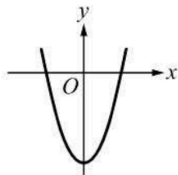


图: 6-Sec3-11

Have a try!

In the range of $-3/4 < x < -1/2$, what is the least possible value of x ?

Ⓐ x

Ⓑ $x + 3$

Ⓒ $x^2 - 3x$

Ⓓ $x^3 - x$

Ⓔ x^4

Ⓐ $x < 0$

Ⓑ $x + 3 > 0$

Ⓒ $x^2 - 3x > 0$ since $x = \frac{4}{3}$ is the symmetric axis and the opening is upward

Ⓓ $x^3 - x = x(x^2 - 1) > 0$ since $x^2 < 1$

Ⓔ $x^4 > 0$

Answer **A**

Solving Quadratic Equations By the Quadratic Formula Or Factoring

Solving Quadratic Equations

一元二次方程公式 因式分解

定理 (一元二次方程公式)

For $y = ax^2 + bx + c$, the solutions is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

例

$$2x^2 - x - 6 = 0$$

$$\begin{aligned} x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)} \\ &= \frac{1 \pm 7}{4} = 2 \text{ or } -\frac{3}{2} \end{aligned}$$

因式分解 (配方法)

例

$$2x^2 - x - 6 = 0$$

$$2\left(x^2 - 2 \cdot \frac{1}{4}x + \frac{1}{16}\right) - \frac{49}{8} = 0$$

$$2\left(x - \frac{1}{4}\right)^2 - \frac{49}{8} = 0$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm \frac{7}{4}$$

$$x = 2 \text{ or } -\frac{3}{2}$$

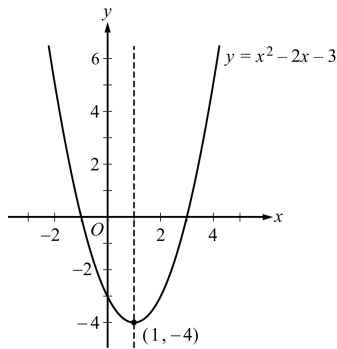
Have a try!

Parabola 和 X 轴交点就是 Solution

og-p390-2.8.5 Consider the line whose equation is $y = x^2 - 2x - 3$. Find the solution when $y = 0$.

$$(x - 3)(x + 1) = 0$$

Answer $x = -1$ or 3



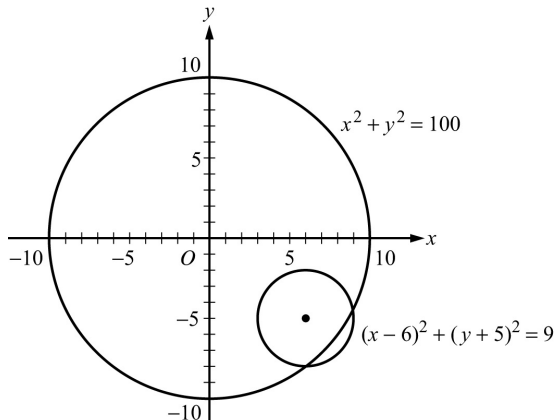
Graphing Circles

Circles

到定点距离都相等

定理 (圆)

$(x - a)^2 + (y - b)^2 = r^2$ is a circle with its center at the point (a, b) and with radius $r > 0$.



Piecewise-Defined Function

Presentation Overview for Piecewise-Defined Function

- ① Algebra Expressions
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Piecewise-Defined Function

$$y = \begin{cases} -x & x \leq 0 \\ x & x \geq 0 \end{cases}$$

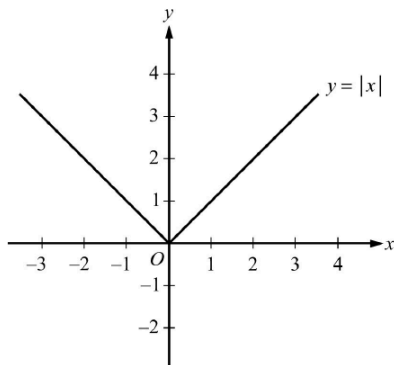


图: $y = |x|$

Reflecting, Shifting and Stretching of Functions

Presentation Overview for Reflecting, Shifting and Stretching of Functions

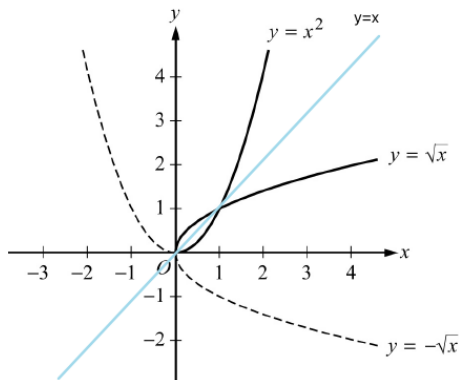
- ① Algebra Expressions
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- ⑥ Reflecting, Shifting and Stretching of Functions
 - Reflecting Functions
 - Shifting Functions
 - Stretching Functions

Reflecting Functions

Reflecting Functions about $y = x$

定理 (关于 $y = x$ 镜像对称: 调换 xy)

The inverse functions are the reflection of each other about $y = x$



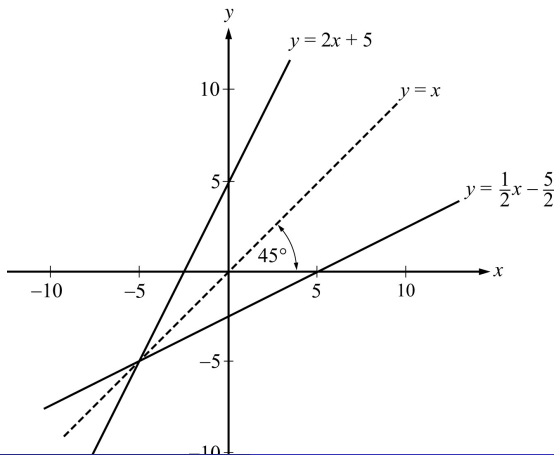
Have a try!

x 和 y 对调

og-p390-2.8.4 Consider the line whose equation is $y = 2x + 5$. Find the equation that is reflection of $y = 2x + 5$ about $y = x$.

$$x = 2y + 5$$

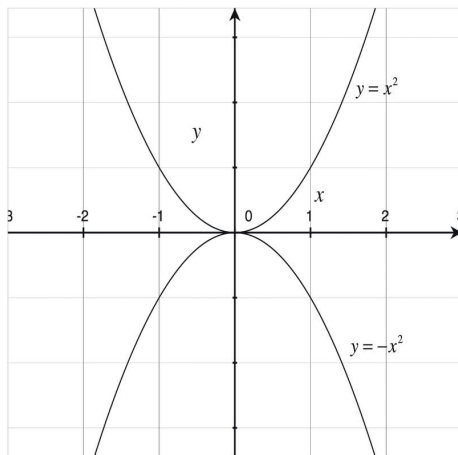
Answer $y = \frac{1}{2}x - \frac{5}{2}$



Reflecting Functions about x — axis

定理 (关于 x 轴镜像对称: 函数右边加负号)

In general, for any function h , the graph of $y = -h(x)$ is the reflection of the graph of $y = h(x)$ about the x -axis.

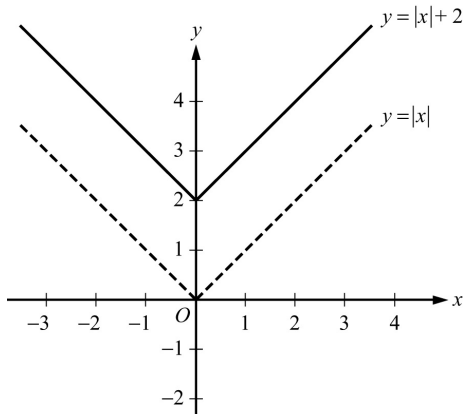


Shifting Functions

Shifting Functions Upward or Downward

定理 (上下平移: 函数右边加常数项)

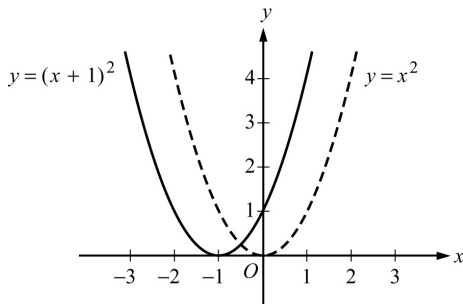
- The graph of $h(x) + c$ is the graph of $h(x)$ shifted upward by c units.
- The graph of $h(x) - c$ is the graph of $h(x)$ shifted downward by c units.



Shifting Functions to the Left or Right

定理 (左右平移: 在 x 上加減)

- The graph of $h(x + c)$ is the graph of $h(x)$ shifted to the **left** by c units.
- The graph of $h(x - c)$ is the graph of $h(x)$ shifted to the **right** by c units.

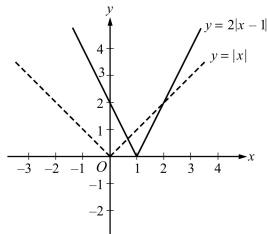
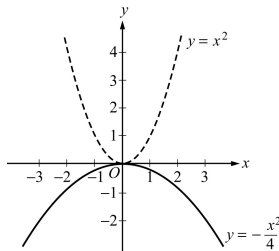


Stretching Functions

Stretching or Shrinking Functions

定理 (增大或者缩小开口: 函数乘常数项)

- The graph of $ch(x)$ is the graph of $h(x)$ stretched vertically by a factor of c if $c > 1$.
- The graph of $ch(x)$ is the graph of $h(x)$ shrunk vertically by a factor of c if $c < 1$.



Applications

Presentation Overview for Applications

- ① Algebra Expressions
- ② Coordinate Geometry
- ③ Linear Problems
- ④ Quadratic Problems
- ⑤ Piecewise-Defined Function
- ⑥ Reflecting, Shifting and Stretching of Functions
- ⑦ Applications
 - Average, Mixture, Rate, and Work Problems
 - Interest

Translate from Words to an Arithmetic or Algebraic Representation

Average, Mixture, Rate, and Work Problems

Average Problems: Have A Try!

求平均

Ellen has received the following scores on 3 exams: 82, 74, and 90. What score will Ellen need to receive on the next exam so that the average (arithmetic mean) score for the 4 exams will be 85 ? $\frac{82+74+90+x}{4} = 85$

Answer: **94**

A Real QR Problem!

A scientist conducted an experiment and collected three measurements. Each measurement was an integer. The range of the three measurements was 2 and the least value was 1. Which of the following values could be the average (arithmetic mean) of the measurements collected for the experiment?

Indicate all such values.

☐ $\frac{4}{3}$ ☐ $\frac{5}{3}$ ☐ 2 ☐ $\frac{7}{3}$ ☐ 3

图: 6-Sec3-13

- ∴ The range is 2 and the least value is 1
- ∴ Two of three integers must be 1 and 3. The rest one could be 1, 2, or 3.
- ∴ The sum of the measurements could be 5, 6, and 7.

Answer **BCD**: $\frac{5}{3}$; 2; $\frac{7}{3}$

Interest

Simple Interest v.s. Compound Interest

单利 复利

1 Min Break

Questions? Comments?