

Complex Analysis Reference Sheet

Stuyvesant Class of 2022

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Complex Calculus, Mr. Stern

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1 THE COMPLEX NUMBER SYSTEM

1.1 The Algebra of Complex Numbers

Definition 1.1 (Complex Numbers). The *complex numbers* are the set

$$\mathbb{C} := \{ [(a, b)] \mid a, b \in \mathbb{R} \}$$

where

$$[(a, b)] := \begin{cases} a & b = 0 \\ (a, b) & b \neq 0 \end{cases}$$

Definition 1.2 (Adding Complex Numbers).

1.2 The Geometry of Complex Numbers

1.2.1 Möbius Transformations and the Riemann Sphere

2 COMPLEX FUNCTIONS

2.1 The Complex Exponential

2.2 Complex Trigonometry

2.3 The Argument Functions and Complex Logarithm

3 TOPOLOGY OF \mathbb{C}

3.1 Neighborhoods, Open and Closed Sets

3.1.1 Accumulation Points and the Closure of a Set

3.2 Connectedness and Compactness

3.3 Sequences in \mathbb{C} , Limits of Sequences

3.4 Limits of Functions, Continuity

3.4.1 Continuous Images of Connected and Compact Sets

4 COMPLEX DIFFERENTIATION

4.1 Differentiability and Analyticity

4.1.1 The Cauchy-Riemann Equations

4.2 Rules for Derivatives

4.3 Conformal Maps

5 COMPLEX INTEGRATION

5.1 Real Integrals of Complex Functions

5.2 Contours and Contour Integration

5.2.1 Contour Parameterization

5.3 Introduction to Cauchy's Theorem

5.3.1 Tile-Centered Regions and the "Not-Too-Bad" Condition

5.4 Cauchy's Integral Formula