Linear Algebra

part 1

Vector and Vector space

Def: $X \in \Re^n$ is a vector with n entries where $x_i \in \Re$ is the i^{th} entry :

$$X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} \in \Re^n$$

Examples

In [2]:

```
# first we need to import numpy
import numpy as np

# zero-vector
np.zeros((3,1))
```

Out[2]:

```
array([[0.],
       [0.],
       [0.]])
```

In [3]:

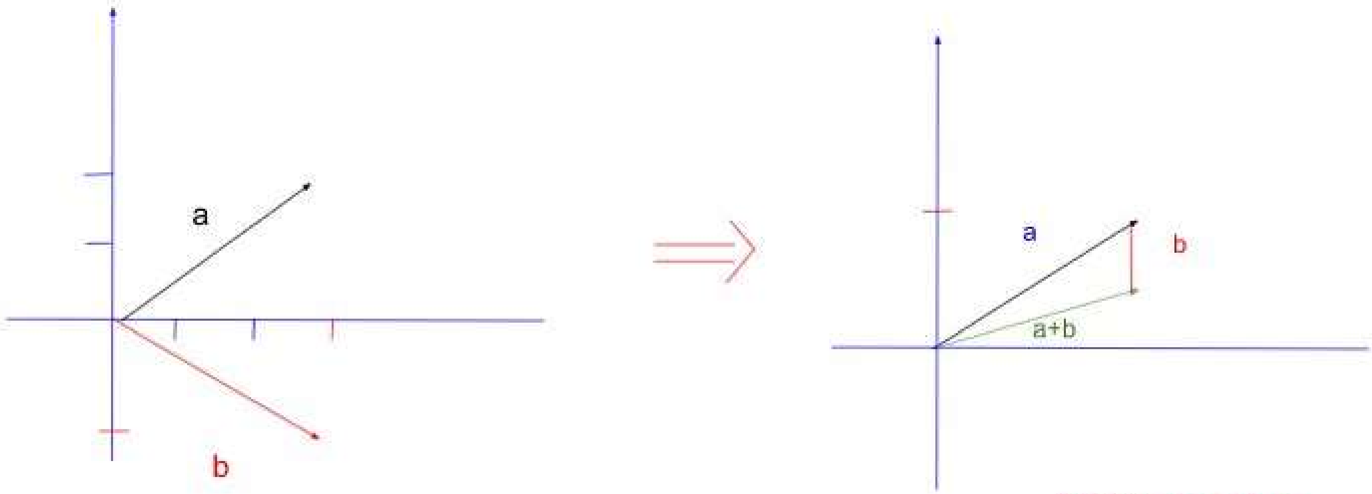
```
# unit vector
np.ones((4,1))
```

Out[3]:

```
array([[1.],
       [1.],
       [1.],
       [1.]])
```

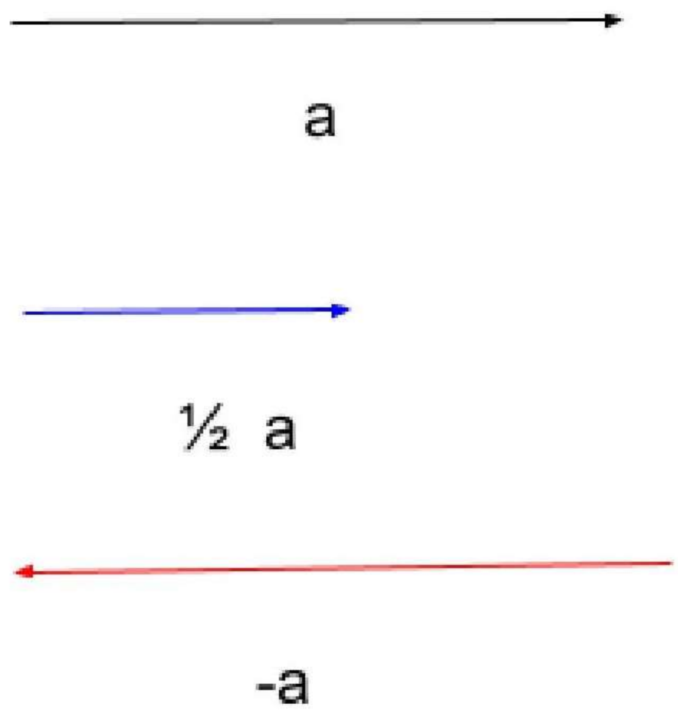
The two basic vector operations are scalar multiplication and vector addition:

Vector addition



Addition of two Vectors

vector scalar-multiplication



In [9]:

```
a = np.array([1, 2, 3]) # first vector

b = np.array([4, 5, 6])# second vector

print('addition :',a+b) # addition of two vectors
print('multiply by scalar:',2*a)# scale vector a by 2
```

```
addition : [5 7 9]
multiply by scalar: [2 4 6]
```

Vector space

Def: A linear vector space X is a set of vectors that satisfies the following condition:

- $x_1, x_2 \in X$ then $x_1 + x_2 \in X$
- $x_1 + x_2 = x_2 + x_1$
- $(x_1 + x_2) + x_3 = x_1 + (x_2 + x_3)$
- $\exists 0 \in X$
- $\forall x_1 \in X \exists - x_1$ such that $x_1 + (-x_2) = 0$
- multiply by scalar a: $if x_1 \in X$ then $ax_1 \in X$
- a, b are two scalares : $a(bx_1) = abx_1$
- $(a + b)x_1 = ax_1 + bx_2$
- $a(x_1 + x_2 = ax_1 + bx_2$
- for any $x_1 \in X : 1x_1 = x_1$

Example for a vector space : $\Re^2$ **because all the above conditions are satisfies**

" the set of all polynomial of degree n does not form a vector space" because the closure property does not hold

Linear independence

$\{x_i\}$ are **linearly dependent** if there exist n scalars $a_1, a_2, \ldots, a_n$ and at least one which is nonzero such that $a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0$.

In case that $a_i = 0$ then $\{x_i\}$ is set of **linearly independent vectors**. In other words " there is no vector in the set can be written as a linear combination of the other vectors

In [24]:

```
# Example
import numpy as np

x_1 = np.array([1, -1, -1]).reshape(-1,1)
x_2 = np.array([1, 1, -1]).reshape(-1,1)
for i in range(-10,10,2):
     print('Does x_1 and x_2 linearly independent:', np.all(x_2 == -i*x_1))
```

```
Does x_1 and x_2 linearly independent: False
Does x_1 and x_2 linearly independent: False
Does x_1 and x_2 linearly independent: False
Does x_1 and x_2 linearly independent: False
Does x_1 and x_2 linearly independent: False
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Does x_1 and x_2 linearly independent: False
Does x_1 and x_2 linearly independent: False
Does x_1 and x_2 linearly independent: False
```

In [23]:

```
# another example
x_1=np.array([[1],[-1],[2]])
x_2=np.array([[0],[3],[1]])
x_3=np.array([[2],[1],[5]])
print('Does x_1 and x_2 linearly dependent:', np.all(x_3 == 2*x_1+x_2))
#linearly dependent  because we can write x_3 as linear combiniation of x_1 and x_2
```

```
Does x_1 and x_2 linearly dependent: True
```

In [ ]:

```

```

Linear combination assume that X is a vector space and $x_1, x_2, \dots, x_n \in X$, $a_1, a_2, \dots, a_n$ scalars then

$\sum_{i=1}^{n} a_i x_i$ is a linear combination of x_i

Example

In [13]:

```
a_1, a_2 = 2,4 # two scalar
x_1 , x_2 = np.array([[2],[3],[5]]), np.array([[6], [7],[8]])# two vectors
print('shape of x_1 is:',x_1.shape)

print('shape of x_2 is:',x_2.shape)

# linear combination of x_1 and x_2
a_1*x_1+a_2*x_2
```

```
shape of x_1 is: (3, 1)
shape of x_2 is: (3, 1)
```

Out[13]:

```
array([[28],
       [34],
       [42]])
```

In [ ]:

```

```

subspace

A suset S of a vector space X is called a subspace of X if S is itself a vector space under the addition and scalar multiplication :

- S is closed under vector addition
- S is closed under vector multiplication
- $0 \in S$

Examples

1. Zero vector is subspace of $\Re^2$
2. The line through the origin in $\Re^2$ is subspace of $\Re^2$
3. Any line not pass through the origin in $\Re^2$ is subspace of $\Re^2$

The span of a set of vectors

consider a set of vectors $x_1, x_2, \dots, x_n$. The set of all linear combination of this set is called the **span**

Examples

1.

Suppose $\{x_1, x_2\}$ where $x_1 = [1, 3]^T$ and $x_2 = [2, 5]^T$ then the span of $x_1, x_2 = a_1[2, 3]^T + a_2[2, 5]^T$ and the new vector would be span in $\{x_1, x_2\}$ for a random choose of e.g. $a_1 = 1, a_2 = 3.$

2.

$x_1 = \begin{bmatrix} 3 \\ -2 \\ -4 \\ 5 \end{bmatrix}$, is not the span of $x_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ and $x_3 = \begin{bmatrix} 2 \\ -2 \\ -3 \\ 1 \end{bmatrix}$

In [ ]: