

Matrix Algebra and Determinants – More Practice Exercises (Optional)

1. Given the matrices

$$A = \begin{pmatrix} -2 & 1 \\ 0 & 5 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 3 \\ 5 & -5 \\ 2 & 6 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 4 \\ 11 & -5 \\ 7 & 6 \end{pmatrix},$$

find (a) $3A + 2B$; (b) $A - 3B + 2C$.

2. Find $A + A^T$, given $A = \begin{pmatrix} 1 & 0 & 6 \\ 4 & -2 & 0 \\ -5 & 7 & 3 \end{pmatrix}$

3. The matrix $A = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix}$.

Find a matrix X such that it satisfies the equation

(a) $5A + \frac{1}{2}X = 0$; (b) $3A + 2X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

4. Multiply:

(a) $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 2 & -6 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & 1 \\ 4 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 4 & 5 \\ 6 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 1 & 2 & -1 \\ 2 & 0 & -1 & 1 \\ 1 & -3 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 0 \\ 1 & -3 \\ 4 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 3 & 1 & 2 \\ 1 & 4 & -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \\ 1 & -1 \\ 2 & 3 \end{pmatrix}$

5. Find $C = AB - BA$ if $A = \begin{pmatrix} -1 & 2 & 1 \\ 2 & 0 & 4 \\ 4 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 4 & 2 \\ 1 & 2 & 5 \end{pmatrix}$.

6. Find the determinants:

(a) $\begin{vmatrix} 2 & 5 & 0 & 4 \\ 1 & 7 & 0 & 2 \\ 3 & 8 & 1 & 6 \\ 4 & 9 & 3 & 8 \end{vmatrix}$ (b) $\begin{vmatrix} 7 & 6 & 3 & 7 \\ 3 & 5 & 7 & 2 \\ 5 & 4 & 3 & 5 \\ 5 & 6 & 5 & 4 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & -4 & -1 & 2 \\ 4 & 3 & -2 & -1 \end{vmatrix}$ (d) $\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix}$

Answers:

1. (a) $\begin{pmatrix} -8 & 9 \\ 10 & 5 \\ 13 & 24 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 7 & 10 \\ 11 & -2 \end{pmatrix}$

2. $\begin{pmatrix} 2 & 4 & 1 \\ 4 & -4 & 7 \\ 1 & 7 & 6 \end{pmatrix}$

3. (a) $\begin{pmatrix} 10 & 0 \\ -20 & -30 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 0 \\ -3 & -4 \end{pmatrix}$

4. (a) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 9 & 6 \\ 0 & 13 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 7 \\ 5 & 10 \\ 11 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 18 & 19 \\ 19 & 27 \end{pmatrix}$

5. $\begin{pmatrix} -3 & 3 & 2 \\ -6 & 6 & 0 \\ -8 & 3 & -3 \end{pmatrix}$

6. (a) 0 (b) -10 (c) 900 (d) 394