

Unit 1 Test

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3. Problem set

$$1) A^2 - 3I \quad A = \begin{pmatrix} 1 & -1 & -2 \\ 0 & 3 & -1 \\ 4 & 0 & 2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} -7 & -4 & -5 \\ -4 & 9 & -5 \\ 12 & -4 & -4 \end{pmatrix}$$

$$A^2 - 3I = \begin{pmatrix} -7 & -4 & -5 \\ -4 & 9 & -5 \\ 12 & -4 & -4 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} -10 & -4 & -5 \\ -4 & 6 & -5 \\ 12 & -4 & -7 \end{pmatrix}$$

$$\text{Ans } \begin{pmatrix} -10 & -4 & -5 \\ -4 & 6 & -5 \\ 12 & -4 & -7 \end{pmatrix}$$

$$2) \begin{cases} x + 4y + z = 1 \\ -x - 2y + 2z = 3 \\ 2x - z = 4 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 4 & 1 \\ -1 & -2 & 2 \\ 2 & 0 & -1 \end{vmatrix} = 18 \neq 0$$

$$\Delta_x = \begin{vmatrix} 1 & 4 & 1 \\ 3 & -2 & 2 \\ 4 & 0 & -1 \end{vmatrix} = 54$$

$$\Delta_2 = \begin{vmatrix} 1 & 4 & 1 \\ -1 & -2 & 3 \\ 2 & 0 & 4 \end{vmatrix} = 36$$

$$x = \frac{\Delta_x}{\Delta} = \frac{54}{18} = 3$$

$$\boxed{\begin{aligned} x &= 3 \\ y &= -1 \\ z &= 2 \end{aligned}}$$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 3 & 2 \\ 2 & 4 & -1 \end{vmatrix} = -18$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-18}{18} = -1$$

$$z = \frac{\Delta_z}{\Delta} = \frac{36}{18} = 2$$

$$3) B = \begin{pmatrix} 2 & 1 & -1 \\ 4 & 3 & -2 \\ 0 & 1 & 4 \end{pmatrix}$$

$$|B| = \begin{vmatrix} 2 & 1 & -1 & | & 2 & 1 \\ 4 & 3 & -2 & | & 4 & 3 \\ 0 & 1 & 4 & | & 0 & 1 \end{vmatrix}$$

$$|B| = 2 \cdot 3 \cdot 4 + 1 \cdot (-2) \cdot 0 + (-1) \cdot 4 \cdot 1 - 0 \cdot 3 \cdot (-1) -$$
$$- 1 \cdot (-2) \cdot 2 - 4 \cdot 4 \cdot 1 = 8$$

$$|B| = 8$$

$$4) \quad y \cdot \begin{pmatrix} 3 & 1 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$$

$$y = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix} \cdot \frac{1}{3 \cdot 5 - 1 \cdot (-2)} \cdot \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$$

$$y = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix} \cdot \begin{pmatrix} \frac{5}{17} & \frac{-1}{17} \\ \frac{2}{17} & \frac{3}{17} \end{pmatrix} = \begin{matrix} 0 \cdot \frac{5}{17} + 2 \cdot \frac{2}{17} & 0 \cdot \frac{-1}{17} + 2 \cdot \frac{3}{17} \\ 3 \cdot \frac{5}{17} + (-4) \cdot \frac{2}{17} & 3 \cdot \frac{-1}{17} + (-4) \cdot \frac{3}{17} \end{matrix}$$

$$y = \begin{pmatrix} \frac{4}{17} & \frac{6}{17} \\ \frac{4}{17} & -\frac{15}{17} \end{pmatrix}$$