1) addition $Z_{18} = \{x \in \mathbb{Z} : 0 \le x \le 17\}$

let $a, b, c \in Z_{18}$

Closure $(a + b) \% 18 \in Z_{18}$

Assoc: $[(a+b)\%18 + c] \% 18 = [a + (b + c)\%18] \% 18$

mod 18 subtracts away the greatest possible multiple of 18, this will have the same effect regardless of summation order

Ident $a \% 18 = (a + 0) \% 18$

$0 \in Z_{18}$

Inv $a + (-a) = 0 \rightarrow -a = 18 - a \in Z_{18}$

Yes

Mult: ident $(a \cdot 1) \% 18 = a \% 18$

inv $a \cdot x \% 18 = 1$

$x \notin Z_{18}$

No

2-ident) Find i s.t. $\forall w \in W.(gcd(w, i) = w)$

$w \mid w$ (trivially)

$0 \mid w$

$0 = i$

assoc. let $a, b, c \in W$

$gcd(gcd(a,b), c)$ $gcd(a, gcd(b, c))$

let $gcd(a,b) = x$ let $gcd(b,c) = y$

$gcd(x, c) = gcd(y, a)$

closure $gcd(a,b) \in W$ trivially

inv $gcd(a, -a) = 0$

- a DNE

3) $\gcd(10946, 19838)$

$= \gcd(10946, 8892)$

$\gcd(8892, 2054)$

$\gcd(2054, 626)$

$\gcd(626, 26)$

$\gcd(26, 0)$

$\therefore$ 26

9) MI

19 % 35

gcd (19, 35)

= gcd (35, 19) residle 19 = 1x19 + 0x35

= gcd (19, 16) residle 16 = −1x19 1x35

= gcd (16, 3) residle 3 = 19 − 16

1x19 − 1x(−1x19 + 1x35)

2x19 − 1x35

gcd (3, 1) residue 1 = 1x16 − 5x3

1x(−1x19+1x35) − 5x(2x19 − 1x35)

1x35 − 1x19 − 10x19 + 5x35

−11x19 + 6x35

−11 → 35 − 11

24

19 · 24 = 456 % 35 =

1 ✓

5) $6x \%23 = 3$

23 46 69

26 49 72 ↖

6.12

$x = 12$

$7x \%13 = 11$

13 26 39 52 65

24 37 50 63

7.9

$x = 9$

$5x \%11 = 7$

11 22 33

18 27 40

5.8 = 40

$x = 8$