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# A Real-Time Capable Model Predictive Approach to Lateral Vehicle Guidance

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*Abstract*— **The paper at hand proposes a real-time capable approach to combined trajectory planning and control. One single prediction model is used to plan a feasible trajectory and to perform lateral guidance of the vehicle at the same time. Nonlinear model predictive control (NMPC) methods are applied to solve the optimal control problem, which incorporates environmental constraints leading to a model predictive planning and control approach (MPPC). Experiments are conducted utilizing a rapid prototyping system. The analysis shows the versatile application range of the developed algorithm, such as challenging emergency evasive maneuvers as well as automated steering as an approach to lateral guidance of the vehicle in general.**

## I. INTRODUCTION

The application of advanced driver assistance systems (ADAS) is particularly motivated by the aspect to enhance safety. Active safety systems are control systems for vehicle stability and collision avoidance starting with anti-lock braking systems (ABS) and electronic stability control (ESC) to prevent over- and/or understeer by controlling the yaw dynamics of the ego vehicle. To further reduce the number of accidents in road traffic, systems that perform automated collision avoidance maneuvers are developed. Autonomous braking systems for example incorporate sensor data of the environment to detect whether an emergency braking maneuver has to be performed. Even though obstacle vehicles are detected in an early stage, hidden obstacles may suddenly move in the direction of the ego vehicle, causing a critical situation. At higher velocities an evasive maneuver is preferred compared to a sole braking intervention. This is due to the fact that the last-point-to-steer is located after the last-point-to-brake. This means that in some critical cases the imminent collision can be avoided only by an evasive maneuver, whereas a sole braking intervention could just mitigate the collision. The application of an ADAS to support the driver's steering intervention in critical situations is motivated by the fact that the average driver often fails to guide the vehicle during the performed evasive maneuver.

In collision avoidance systems path following approaches are widely used. The main idea of these approaches can be summarized as calculating a static collision avoidance path in a first step and guiding the vehicle along this path in a second step [1], [2], [3]. A major disadvantage of these approaches is that the application is limited due to the predefined characteristics of the planned path, which cannot be applied to different and more complex situations regarding for example multiple obstacles.

A more general approach to collision avoidance originating from mobile robotics offers the elastic band introduced in [4]. In contrast to path following approaches the elastic band adapts to the current situation. The authors of [5] and [6] apply this idea to collision avoidance for vehicles utilizing sole steering intervention. The transition from path to trajectory planning is performed by the inclusion of time as it is proposed in [7], also originating from mobile robotics applications. This leads to the concept of online trajectory optimization with *Timed Elastic Bands* considering kinodynamic constraints, as it is applied to emergency maneuvers [8] and autonomous driving [9].

The authors of [10] consider vehicle dynamics within their approach, leading to the concept of trajectory planning with model predictive control (MPC). Similarly in [11] a simplified vehicle model is used.

All the mentioned approaches above tackle the problem of path or trajectory planning and have in common, that an underlying controller is necessary to realize the planned trajectory. The approaches in [12] and [13] are rather motivated from control perspective, predominantly focusing on the problem of control. Therein a high level trajectory planner is utilized to generate a trajectory, which is then tracked by a low level model predictive controller. The authors of [14] apply an MPC approach utilizing a double track model to follow a reference trajectory.

In previous work related to MPC the two problems of trajectory planning and control have been united to a combined planning and control approach [15]. The separation of planning and control is abolished, utilizing the same prediction model for planning and control. This method differs from the approaches mentioned above and is in further course referred to as model predictive planning and control (MPPC). The choice of a suitable vehicle dynamics model is crucial to the accuracy and real-time capability of this approach. Especially the application of a double track model has shown to be challenging with respect to a real-time application. For this reason a suboptimal MPPC approach is applied [16]. But the developed approach suffers from the low prediction horizon due to the amount of needed samples.

This paper focuses on a MPPC approach to lateral guidance of a vehicle. MPPC enables a general formulation of

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the planning and control problem, valid in different and complex situations, facilitating a broad application of the developed algorithm, e.g. lateral guidance of the vehicle in terms of automated driving. In this paper emergency evasive maneuvers, representing a major challenge to stable vehicle guidance, have been chosen to show the benefits and capabilities of the developed approach. The explicit consideration of vehicle dynamics leads to a solution, which inherently guarantees feasibility of the planned trajectory. As a consequence of the experience gained from MPPC utilizing a double track model, a model reduction is proposed.

The remainder of this paper is organized as follows. In Section II the vehicle dynamics model is described. Section III presents the developed model predictive planning and control approach. The results of the proposed approach are discussed in Section IV and Section V concludes this paper.

## II. Prediction model

As the prediction model is used for both planning and control the choice is made regarding the trade-off between model accuracy and computational effort. To predict future states of the vehicle within the presented approach, a nonlinear single-track model is considered. Fig. 1 shows a schematic sketch of the applied nonlinear single-track model. The earth-, vehicle- and tire coordinate frame are denoted $E$, $F$ and $R$ respectively. The vehicle mass $m$ and the moment of inertia $J_z$ are assumed to be located in the vehicles center of gravity.

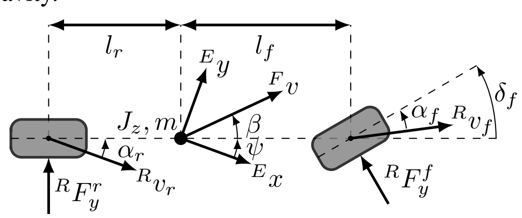


Fig. 1: Nonlinear single-track model.

The model can be represented as a nonlinear state-space model

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), u(t)), \quad \mathbf{x}(t_0) = \mathbf{x}_0 \tag{1}$$

with states

$$\mathbf{x} = \begin{bmatrix} {}^F\beta & {}^E\dot{\psi} & {}^E\psi & {}^Ex & {}^Ey \end{bmatrix}^T \tag{2}$$

and control variable $u = \delta$. Steering wheel angle $\delta$ is calculated from wheel angle $\delta_f$ with steering ratio $i_w$:

$$\delta = i_w \delta_f. \tag{3}$$

Tire dynamics are modeled using a simplified version of the magic tire formula introduced in [17].

## III. Model Predictive Planning and Control

In this section a nonlinear model predictive control approach is extended by integrating objectives for trajectory planning, leading to a concept for combined planning and control. Therefore constraints given by the vehicle environment are incorporated into the formulated optimal control problem, which is repeatedly solved by the application of direct multiple shooting. The numerical solution of the discretized optimal control problem is then calculated by an optimization algorithm.

### A. Nonlinear Model Predictive Planning and Control

The task of combined planning and control can be formulated as an optimal control problem. Utilizing the direct multiple shooting method, the control variables are discretized in a first step

$$u(t) = \tilde{u}_k, \quad t \in [t_k, t_{k+1}] \tag{4}$$

and step width $\Delta T = t_{k+1} - t_k$ for a fixed prediction horizon $T_p$ is determined. Furthermore a parameterized state trajectory $\tilde{\mathbf{x}}$, representing initial values for each interval, is added to the optimization variables. Thereby an initial value problem has to be solved in each interval $[t_k, t_{k+1}]$

$$\dot{\mathbf{x}}_k(t) = \mathbf{f}(\mathbf{x}_k(t), \tilde{u}_k), \tag{5a}$$

$$\mathbf{x}_k(t_k) = \tilde{\mathbf{x}}_k. \tag{5b}$$

The state trajectory $\mathbf{x}_k(t; \tilde{\mathbf{x}}_k; \tilde{u}_k)$ as the solution of the initial value problem with initial values $\tilde{\mathbf{x}}_k$ and control variables $\tilde{u}_k$ is calculated by numerical integration of

$$\mathbf{x}_k(t_{k+1}, \tilde{\mathbf{x}}_k, \tilde{u}_k) = \int_{t_k}^{t_{k+1}} \mathbf{f}(\tilde{\mathbf{x}}_k, \tilde{u}_k) \, \mathrm{d}t. \tag{6}$$

To cover the interdependency of the initial values $\tilde{\mathbf{x}}_k$ in form of the vehicle dynamics model an equality constraint

$$\tilde{\mathbf{x}}_{k+1} - \mathbf{x}_k(t_{k+1}, \tilde{\mathbf{x}}_k, \tilde{u}_k) = 0 \tag{7}$$

is introduced and added to the optimal control problem. The optimization parameters are

$$B := \{\tilde{\mathbf{x}}_1, \tilde{u}_1, \tilde{\mathbf{x}}_2, \tilde{u}_2, \ldots, \tilde{\mathbf{x}}_{n-1}, \tilde{u}_{n-1}, \tilde{\mathbf{x}}_n\}. \tag{8}$$

with each state and control input assigned to one point in time.

The discretized optimal control problem, given as

$$B^* = \arg\min_B \; f(B), \tag{9a}$$

$$\tilde{\mathbf{x}}_0 - \mathbf{x}_0 = 0, \tag{9b}$$

$$\tilde{\mathbf{x}}_{k+1} - \mathbf{x}_k(t_{k+1}, \tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k) = 0, \tag{9c}$$

$$\mathbf{g}(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k) \geq 0. \tag{9d}$$

with inequality constraints $\mathbf{g}$ can then be solved by methods of nonlinear optimization.

The optimization algorithm is chosen to match the requirements for real-time performance. Therefore a soft constrained solver is chosen, that relies on a nonlinear least-squares objective function. The hard constraints of the optimal control problem are transformed into an optimization problem without constraints, by substituting the constraints into the objective function $f(B)$. This is done using the piecewise continuous and differentiable soft constraint function $e_\Gamma$ with bound $z_e$, which penalizes the violation of a constraint:

$$e_\Gamma(z, z_e) = \begin{cases} 0 & z \le z_e \\ (z - z_e)^2 & z > z_e \end{cases}. \tag{10}$$

The objective function $f(B)$ then combines the various requirements demanded to perform a successful combined planning and control step.

### B. Objective Function

The objectives are defined in a way that the generated trajectory matches the requirements to avoid a collision by performing a stable maneuver without departing from the road. Therefore the environment of the ego vehicle is modeled focusing on the road limits and obstacles. To prevent that the ego vehicle departs from the road two inequality constraints for the left and right road limit are introduced. Assuming an ego vehicle of a width of $2.5\,\mathrm{m}$, the costs increase quadratically as the center of gravity is approaching $y_{min} = 1.25\,\mathrm{m}$ and $y_{max} = 6.25\,\mathrm{m}$ respectively, indicating that the vehicle touches the left or right road boundary at $y_{right} = 0\,\mathrm{m}$ and $y_{left} = 7.5\,\mathrm{m}$ respectively. To complete the modeling of the environment obstacle vehicles are incorporated. This is implemented by adjusting the road boundaries utilizing two sigmoid functions

$$f(^E x) = \begin{cases} y_0 + \frac{y_\infty}{1+\mathrm{e}^{-(^E x - x_{obs} - \Delta x)}} & ^E x < x_{obs} \\ y_0 + y_\infty - \frac{y_\infty}{1+\mathrm{e}^{-(^E x - x_{obs} + \Delta x)}} & ^E x \ge x_{obs} \end{cases}. \tag{11}$$

This corresponds to a deformation of the road boundaries, as it is illustrated in Fig. 2. Parameters $y_0$ and $y_\infty$ represent

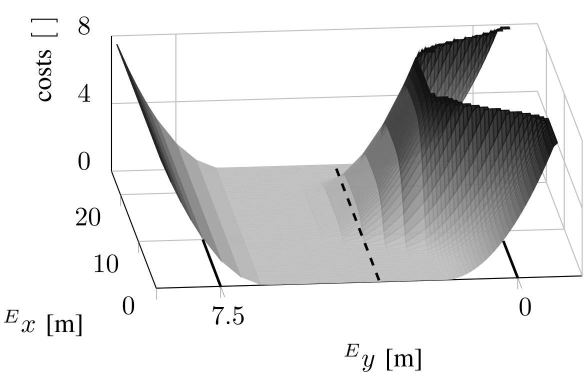


Fig. 2: Illustrative example of environment modeling including obstacle vehicle on the right lane and road boundaries.

the displacement of the sigmoid function along the $y$-axis and the final value respectively. The safety distance to the obstacle vehicle at $x_{obs}$, which should be obtained during the maneuver, is set by parameter $\Delta x$. Considering the road boundaries together with the aspect of collision avoidance yields the environmental constraints

$$\mathbf{\Gamma}_{r,e_\Gamma}(\tilde{\mathbf{x}}_k) = e_\Gamma(-\tilde{y}_k, -\max(y_{min}, f(\tilde{x}_k))), \tag{12}$$

$$\mathbf{\Gamma}_{l,e_\Gamma}(\tilde{\mathbf{x}}_k) = e_\Gamma(\tilde{y}_k, \min(y_{max}, f(\tilde{x}_k))). \tag{13}$$

The total costs for the ego vehicle environment responsible for collision avoidance and road departure avoidance are resulting from the combination of the left and right side

$$\mathbf{\Gamma}_{env,e_\Gamma}(\tilde{\mathbf{x}}_k) = \mathbf{\Gamma}_{r,e_\Gamma}(\tilde{\mathbf{x}}_k) + \mathbf{\Gamma}_{l,e_\Gamma}(\tilde{\mathbf{x}}_k)\,, \tag{14}$$

resulting in the following term:

$$f_{env}(B) = \sum_{k=1}^{n} \mathbf{\Gamma}_{env,e_\Gamma}(\tilde{\mathbf{x}}_k)\,. \tag{15}$$

To ensure that vehicle dynamics are satisfied during the process of trajectory planning and control, the equality constraint (7) is incorporated in the objective function. By solving the integral in (6) using the explicit forward Euler method inequality constraint (7) is reformulated as

$$\tilde{\mathbf{x}}_{k+1} - \tilde{\mathbf{x}}_k - \mathbf{f}(\tilde{\mathbf{x}}_k, \tilde{u}_k)\Delta t = \mathbf{0}. \tag{16}$$

The constraint is then formulated in a weighted quadratic form:

$$\begin{aligned} h(\tilde{\mathbf{x}}_k, \tilde{\mathbf{x}}_{k+1}, \tilde{u}_k) = (\tilde{\mathbf{x}}_{k+1} - \tilde{\mathbf{x}}_k - \mathbf{f}(\tilde{\mathbf{x}}_k, \tilde{u}_k)\Delta t)^T \cdot \\ \cdot\, \mathbf{P}(\tilde{\mathbf{x}}_{k+1} - \tilde{\mathbf{x}}_k - \mathbf{f}(\tilde{\mathbf{x}}_k, \tilde{u}_k)\Delta t). \end{aligned} \tag{17}$$

Weighting matrix $\mathbf{P}$ is a diagonal matrix, penalizing the deviation between the planned states $\tilde{\mathbf{x}}$ and the solutions $\mathbf{x}$ obtained by solving the vehicle state space model

$$\mathbf{P} = \begin{bmatrix} \gamma_{\beta,sys} & 0 & 0 & 0 & 0 \\ 0 & \gamma_{\dot{\psi},sys} & 0 & 0 & 0 \\ 0 & 0 & \gamma_{\psi,sys} & 0 & 0 \\ 0 & 0 & 0 & \gamma_{x,sys} & 0 \\ 0 & 0 & 0 & 0 & \gamma_{y,sys} \end{bmatrix}. \tag{18}$$

A Lagrange term as an additional objective is added, forcing the dynamic system to operate at minimum costs and to consider energy optimality

$$f_Q(B) = \sum_{k=1}^{n-1} \Big[ \left( \tilde{\mathbf{x}}_k^T \mathbf{Q} \tilde{\mathbf{x}}_k + \gamma_\delta \tilde{u}_k^2 \right) \Big]. \tag{19}$$

Weighting matrix $\mathbf{Q}$ is a diagonal matrix chosen as follows:

$$\mathbf{Q} = \begin{bmatrix} \gamma_\beta & 0 & 0 & 0 & 0 \\ 0 & \gamma_{\dot{\psi}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

penalizing side slip angle $\beta$ and yaw rate $\dot{\psi}$. Adding the yaw angle is not necessary, since this term tends in the same direction of generating a stable maneuver. Furthermore a penalization of the position is not desired, so that in total the last three diagonal elements of $\mathbf{Q}$ are zero. The final state is particularly weighted by adding a Mayer term to the objective function, aiming at minimal driving dynamics and a stable final state with the vehicle aligned to the road

$$f_R(B) = \tilde{\mathbf{x}}_n^T \mathbf{R} \tilde{\mathbf{x}}_n\ , \tag{20}$$

$$\mathbf{R} = \begin{bmatrix} \gamma_{\beta,n} & 0 & 0 & 0 & 0 \\ 0 & \gamma_{\dot{\psi},n} & 0 & 0 & 0 \\ 0 & 0 & \gamma_{\psi,n} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_{y,n} \end{bmatrix} . \tag{21}$$

By weighting the $y$-position of the last state a lane guidance function is realized, aiming at the behavior to return to the right lane.

The entire objective function composed of the aforementioned objectives is

$$\begin{aligned} f(B) = &\tilde{\mathbf{x}}_n^T \mathbf{R} \tilde{\mathbf{x}}_n + \\ &+ \sum_{k=1}^{n-1} \left[ \left( \tilde{\mathbf{x}}_k^T \mathbf{Q} \tilde{\mathbf{x}}_k + \gamma_\delta \tilde{u}_k^2 \right) \right] + \\ &+ \sum_{k=1}^{n-1} \left[ h(\tilde{\mathbf{x}}_k, \tilde{\mathbf{x}}_{k+1}, \tilde{u}_k) \right] + \\ &+ \sum_{k=1}^{n} \mathbf{\Gamma}_{env,e_\Gamma}(\tilde{\mathbf{x}}_k) \ . \end{aligned} \tag{22}$$

Note that all terms are quadratic yielding a least-squares formulation, such that efficient solvers can be applied.

## IV. Results

The assistance system is designed to take over control in critical situations. Therefore an algorithm is developed to trigger the steering intervention in case that the last point to steer is reached. The algorithm takes the vehicle dynamics into account and decides, whether the planned trajectory is feasible to avoid an imminent collision within the bounds of driving dynamics. The number of obstacles does not restrict the performance of the algorithm in terms of generalizability. If the optimization problem is infeasible or the constraints are violated no valid solution can be found, meaning that in the current situation no feasible driving maneuver exists. The algorithm is implemented in a C++ MPC framework utilizing an efficient nonlinear least-squares implementation of the MPC problem presented in [18], which highly exploits the sparsity structure of the formulated problem and which enables the real-time application of the developed algorithm. The prediction horizon is set to $T_p = 2.5\,\mathrm{s}$ with a computation time for one MPPC step of less than $50\,\mathrm{ms}$.

### A. Simulation results

In a first test scenario two static obstacle vehicles are placed in front of the ego vehicle. The ego vehicle, driving at a speed of $100\,\frac{\mathrm{km}}{\mathrm{h}}$, is forced to perform a double lane change maneuver to avoid a collision. Fig. 3 shows the resulting maneuver, whereas the upper figure shows the position of the ego vehicle and the planned trajectory at chosen time instances and the lower figure depicts the time series of the actual driven trajectory.

As soon as the first obstacle vehicle appears within the prediction horizon $T_p$ the system plans a trajectory avoiding the collision. This point in time is denoted as $I_v$. The planned steering angle $\delta_{ref}$ intends to initiate a swerving maneuver

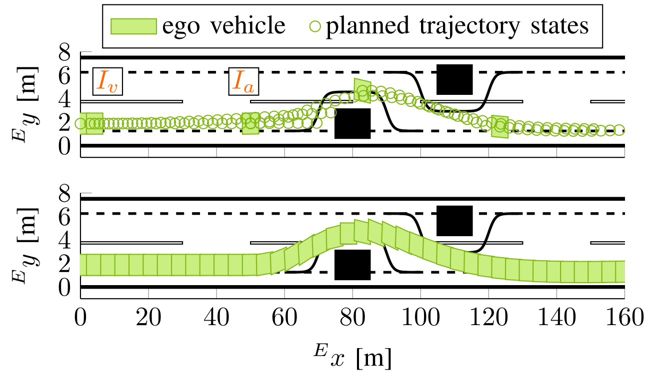


Fig. 3: Simulation result of double lane change scenario.

around the obstacle. But the system is not activated unless the last-point-to-steer (at time instance $I_a$) is detected, so that the actual steering angle cannot follow the reference until this point in time (see Fig. 4). The dynamic quantities

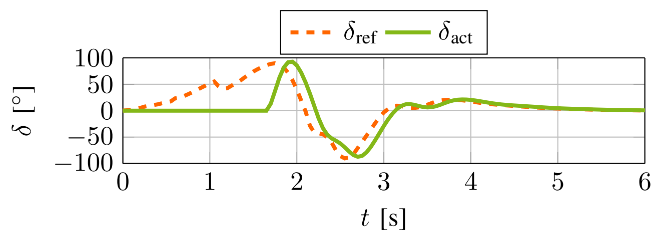


Fig. 4: Control input of simulated double lane change scenario.

are depicted in Fig. 5. The lateral acceleration shows that the maneuver reaches the limits of driving physics, whereas the depicted phase plane indicates a stable maneuver.

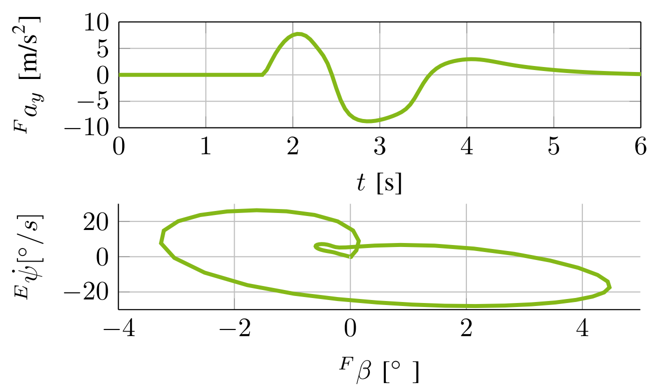


Fig. 5: Dynamic quantities of simulated double lane change scenario.

In a second test scenario the behavior of the system with respect to a dynamic obstacle is tested. The obstacle vehicle drives at a speed of $100\,\frac{\mathrm{km}}{\mathrm{h}}$ towards the ego vehicle, which is also moving at a speed of $100\,\frac{\mathrm{km}}{\mathrm{h}}$. It is assumed that the obstacle trajectory is known. Fig. 6 shows the result of MPPC in case of an approaching vehicle. The developed algorithm takes the moving obstacle into account and the planned trajectory is adjusted at an early stage yielding a stable and collision free maneuver.

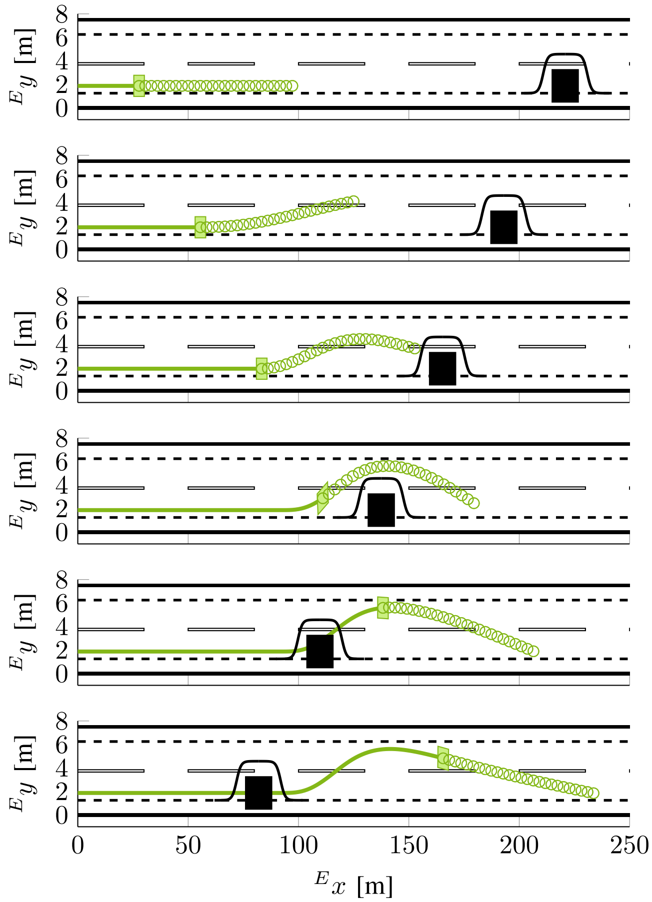


Fig. 6: Emergency evasive maneuver with approaching obstacle vehicle driving at a speed of $100\,\frac{\text{km}}{\text{h}}$.

## B. Driving simulator studies

The following experiments are conducted on a fixed based driving simulator as shown in Fig. 7. Simulations of the approach are performed on a commonly used dSPACE rapid prototyping system, which offers a sufficiently high computation power. The steering wheel of the driving simulator is actuated by an electric motor, whereas the longitudinal guidance is performed by the driver.

Fig. 7: Fixed based driving simulator for rapid prototyping.

In the previous test scenarios the position of each obstacle is known right from the start and before the system has to intervene to avoid a collision. To increase the criticality of the scenario a static obstacle vehicle is appearing in front of the ego vehicle at $t_{appear} \approx 1\,\text{s}$ before the collision. This scenario covers for example the case of a concealed obstacle, which is not perceived by the vehicle sensors. The assistance system is forced to immediately plan a feasible trajectory avoiding the imminent collision. The position of the second obstacle vehicle on the left lane is assumed to be known at the same time. The experiment is driven at a manually controlled speed of approx. $100\,\frac{\text{km}}{\text{h}}$. The results of this scenario are depicted in Fig. 8. Right after

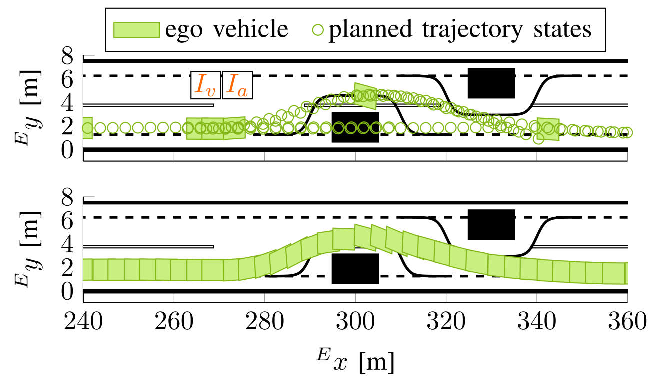


Fig. 8: Result of double lane change scenario with suddenly appearing obstacle. Both obstacle vehicles are static.

the obstacle vehicle appears a critical situation is detected forcing the system to intervene. A collision free trajectory is planned immediately reacting on the suddenly changed situation. Note that time instance $I_v$ at which the obstacle is perceived by the vehicle sensors is immediately followed by time instance $I_a$ at which the system is activated to perform an evasive maneuver. Still a feasible trajectory is generated and a stable maneuver is performed. The criticality

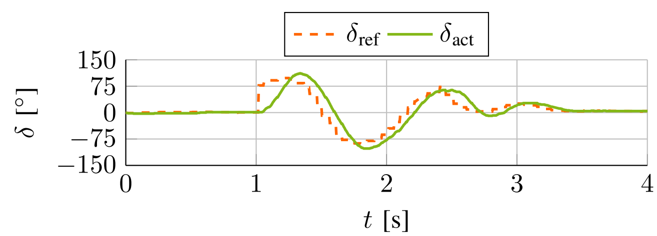


Fig. 9: Control input of double lane change scenario with suddenly appearing obstacle.

of the situation is emphasized by the fact that during the performed emergency evasive maneuver lateral accelerations up to $9.8\,\frac{\text{m}}{\text{s}^2}$ are reached. Still the perfomed maneuver is stable as the phase plane shows (see Fig. 10).

The versatile capabilities of the developed algorithm, due to the general formulation of the trajectory planning and control problem will be demonstrated by means of an application example to automatic steering in a race track scenario. The algorithm is not intended to yield a time optimal result in terms of a fast lap time, but it shows the versatility of the algorithm related to automated lateral vehicle guidance. To cover curve shaped roads a curvilinear coordinate system is used. The results, depicted in Fig. 11, clearly show the advantages of the presented approach. Especially the smoothness of the planned trajectory driven at highway speed has to

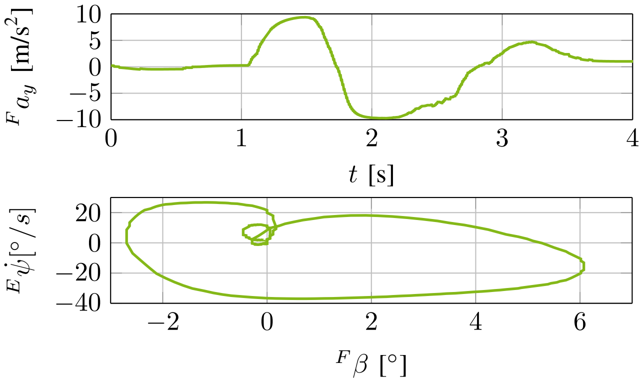


Fig. 10: Dynamic quantities of double lane change scenario with suddenly appearing obstacle.

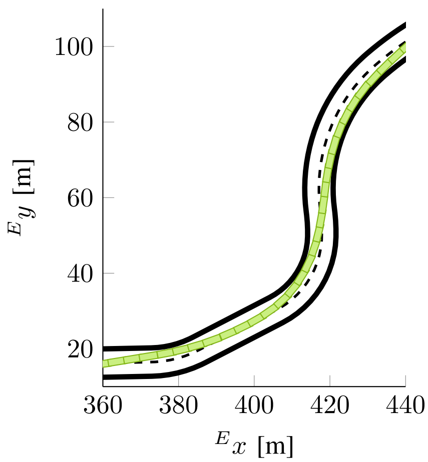


Fig. 11: Automated steering along a curved road.

be mentioned. The ego vehicle is cutting curves to realize energy optimality by applying as few steering interventions as possible, leading to a trajectory close to the racing line. As lateral guidance is one of the major aspects for automated cars the results prove that the presented approach is capable to serve as a basis for automated driving functions.

## V. Conclusion

This contribution presents an approach to lateral guidance of vehicles utilizing a MPPC approach, which combines trajectory planning and control within one single step. The novelty, which separates the developed approach from others, lies in the fact that the same model is used for planning and control. Therefore constraints on the vehicle environment are incorporated into the resulting optimal control problem, which is solved with direct multiple shooting as a nonlinear model predictive control method. The range of application is not limited to any predefined situation, as the MPPC approach is capable to adapt on different complex scenarios. The analysis of the developed algorithm shows the versatile application range, as it is applied to challenging emergency evasive maneuvers and to an example of automated steering. Further work will deal with the enhancement of the current real-time capable approach to enable braking intervention.


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