Robust tube-MPC based lane keeping system for autonomous driving vehicles

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ABSTRACT

This paper proposes a novel framework for lane keeping system for autonomous driving vehicles. The method presented in this paper guarantees stability of the vehicle in presence of bounded disturbances which includes the road curves, banking angle and changes in the longitudinal velocity of the vehicle along the banked road curves. A linear parameter varying (LPV) mathematical model is utilized to define the dynamics of the ego vehicle with the longitudinal velocity of the vehicle as the time varying parameter. For a bounded velocity range, a robust stabilizing feedback gain is pre-computed using linear matrix inequalities (LMI) and semi-definite programming (SDP) techniques. A robust tube based model predictive controller (RTMPC) is implemented utilizing the disturbance invariant set and the computed robust stabilizing gain to guarantee feasibility and stability of the controller. Simulation & results displays better performance and stability of the system for the proposed controller compared to clipped LQR (CLQR) controller.

CCS CONCEPTS

• **Theory of computation → Semidefinite programming**; • **Computer systems organization → Robotic control**;

KEYWORDS

Vehicle dynamics & control, LMI, Robust model predictive control, Lane keeping system, LPV systems, Autonomous vehicles.

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1 INTRODUCTION

In last few years, there has been an increasing awareness about automotive safety systems [5] & tremendous amount of improvement has been made in the field of advanced driver assistance system (ADAS) & autonomous driving vehicles. Autonomous driving vehicles epitomizes the need for safe, comfortable & energy efficient passenger transportation system and many automotive companies are striving for accomplishing this task. In the last decade, there has been significant contribution in the domain of active safety system/ADAS & autonomous driving. According to [21], it is predicted that autonomous driving would mitigate the possibility of collision, traffic accidents for about more than 90%. Development & advancement of autonomous vehicle modules such as automatic emergency braking (AEB), adaptive cruise control (ACC), forward collision warning system (FCW) etc. serves as the harbingers for complete fruition of autonomous driving vehicles.

Among the currently existing modules, lane keeping system (LKS) is one of the most important technology in development of autonomous driving vehicle. The main functionality of the lane keeping system is to control the steering of the vehicle (drive by wire) to follow a particular lane in highways [16]. The task is simple when the vehicle speed is less & roads are less curvy, but however the task becomes daunting under the conditions such as vehicle traveling at high speed, more curvy roads, satisfaction of vehicle physical limits, safety & comfort constraints and robustness against any disturbances. According to [18], the crash rate in road curves are about 1.5 to 4 times higher than straight roads and also, the report accentuates the fact that, at times the driver's decision making becomes spurious while making judgments of the road geometry and conditions required to adjust the vehicle dynamics. Under such situation, an autonomous robust lane keeping control module would obviate any possible mishap to occur.

Several research has been conducted on lane keeping system for an autonomous vehicle. In the work proposed in [23], the driver preview characteristics is considered and an artificial potential field method is contrived for trajectory decision with two evaluation indexes of safety and maneuverability. In [12], a control law is proposed using Lyapunov theory of stability to bilinear matrix inequalities (BMI) which uses bounds in the control input and minimizes the reachable set of the vehicle by considering the road curvature as an exogenous input to the model. In [1], a linear time variant prediction model is utilized, which estimates the cornering stiffness of bicycle model using recursive least square method and employs an adaptive model predictive control. In the work proposed in [22], a new state feedback control is implemented by introducing the integral of lateral offset error to reduce the ripple in the yaw rate & also, a multi-rate Kalman filter is implemented to estimate lane information and predict the virtual lane whenever the actual lane information is momentarily unavailable. In [13], MPC based vehicle lateral motion control for lane keeping is implemented for constant longitudinal velocity vehicle model.

The paper is organized follows. Section 2 expounds the vehicle dynamics model and the vehicle constraints considered for control design. In Section 3, Robust tube MPC formulation along with the disturbance rejection controller design is discussed. The paper is ended with results & simulation and conclusion in Section 3.3 & 5 respectively.

2 VEHICLE MODELING

2.1 Vehicle dynamics

The vehicle dynamics model used in this study is the lateral bicycle dynamics model [16], where the state variables are expressed in terms of position & orientation error with respect to road. The model includes the influence of road curvature & banking angle which are considered as exogenous input/disturbances to the system. The state space equation for the model is defined as

$$\dot{x} = \begin{bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2C_{\alpha f} l_f}{I_z} \end{bmatrix} \delta + \begin{bmatrix} 0 \\ \frac{-2C_{\alpha f} l_f - 2C_{\alpha r} l_r}{mV_x} - V_x \\ 0 \\ \frac{-2C_{\alpha f} {l_f}^2 - 2C_{\alpha r} {l_r}^2}{I_z V_x} \end{bmatrix} \dot{\psi}_{des} + \begin{bmatrix} 0 \\ g \\ 0 \\ 0 \end{bmatrix} sin\phi$$
$$+ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-2C_{\alpha f} - 2C_{\alpha r}}{mV_x} & \frac{2C_{\alpha f} + 2C_{\alpha r}}{mV_x} & \frac{-2C_{\alpha f} l_f + 2C_{\alpha r} l_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_{\alpha f} l_f + 2C_{\alpha r} l_r}{I_z V_x} & \frac{2C_{\alpha f} l_f - 2C_{\alpha r} l_r}{I_z} & \frac{-2C_{\alpha f} {l_f}^2 - 2C_{\alpha r} {l_r}^2}{I_z V_x} \end{bmatrix} x \tag{1}$$

Where I_z and m denotes the inertia & mass of the vehicle respectively, $C_{\alpha f}$, $C_{\alpha r}$ denotes the cornering stiffness of front & rear tire respectively, l_f and l_r are the distances of the front & rear tire with respect to the center of gravity of the vehicle, V_x denotes the longitudinal velocity of the vehicle. The state space vector is denoted as $x = [e_1\ \dot{e}_1\ e_2\ \dot{e}_2]^T$, where e_1, e_2 are the lateral position error & yaw angle error respectively, $\dot{e}_1$, $\dot{e}_2$ denotes its derivatives respectively and δ denotes the input, which is the steering angle of the vehicle. $\dot{\psi}_{des}$ is the desired vehicle yaw rate (defined by the road geometry), which is determined by the relation, $\dot{\psi}_{des} = V_x \gamma$, where γ denotes the road curvature. ϕ denotes the banking angle of the road & for small angle of ϕ, $sin\phi \approx \phi$. g denotes the acceleration due to gravity.

The standard error model (1) is extended with an input integrator [13], thus the state variable vector is modified to $x = [e_1\ \dot{e}_1\ e_2\ \dot{e}_2\ \delta]^T$ and the state space model is compactly expressed with the relation

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t) + B^w(t)\,w(t) \tag{2}$$

The state space model is linear parameter varying (LPV) model due to consideration that the velocity is subject to a bounded variation. $A(t)$, $B(t)$, $B^w(t)$ denotes the system matrix, input matrix & disturbance matrix for the LPV system. The disturbance and input vectors are denoted as $w(t) = [\gamma\ \phi]^T$ and $u(t) = \dot{\delta}$ respectively. The numerical values for the model parameters are obtained from [1] listed in Table 1

Table 1: Model parameters for the vehicle

Parameter	Symbol	Value (SI unit)
Vehicle mass	m	$2023(kg)$
Distance from C.G. to front axle	l_f	$1.265(m)$
Distance from C.G. to rear axle	l_r	$1.9(m)$
Cornering stiffness of front tires	$C_{\alpha f}$	$81000(N/rad)$
Cornering stiffness of rear tires	$C_{\alpha r}$	$95000(N/rad)$
Yaw-plane rotational inertia	I_z	$6286(kg\text{-}m^2)$
Acceleration due to gravity	g	$9.81(m/s^2)$

2.2 Vehicle constraints

The set of system constraints considered are the state constraints, input constraints, disturbance constraints & model parameter constraints. The state, input & disturbance constraints are defined by design specifications which include safety, comfort, performance & practical limitations of the system. These constraints are expressed as polytopic set constraints defined as

$$\mathbb{X} := \{x \in \mathbb{R}^n | A_x x \leq b_x\} \tag{3}$$

$$\mathbb{U} := \{u \in \mathbb{R}^m | A_u u \leq b_u\} \tag{4}$$

$$\mathbb{W} := \{w \in \mathbb{R}^l | A_w w \leq b_w\} \tag{5}$$

The numerical bounds for the state constraint set $\mathbb{X}$ is defined as,

$$\mathbb{X} := \begin{cases} e_1^{min} \leq e_1 \leq e_1^{max},\ e_1^{max} = -e_1^{min} = 0.35\ (m) \\ \dot{e}_1^{min} \leq \dot{e}_1 \leq \dot{e}_1^{max},\ \dot{e}_1^{max} = -\dot{e}_1^{min} = 0.85\ (m/s) \\ e_2^{min} \leq e_2 \leq e_2^{max},\ e_2^{max} = -e_2^{min} = 0.095\ (rad) \\ \dot{e}_2^{min} \leq \dot{e}_2 \leq \dot{e}_2^{max},\ \dot{e}_2^{max} = -\dot{e}_2^{min} = 0.25\ (rad/s) \\ \delta^{min} \leq \delta \leq \delta^{max},\ \delta^{max} = -\delta^{min} = 0.075\ (rad) \end{cases} \tag{6}$$

The numerical bounds for the input constraint set $\mathbb{U}$ is defined as

$$\mathbb{U} := \left\{ \dot{\delta}^{min} \leq \dot{\delta} \leq \dot{\delta}^{max},\ \dot{\delta}^{max} = -\dot{\delta}^{min} = 0.163\ (rad/s) \right. \tag{7}$$

The numerical bounds for the disturbance constraint set $\mathbb{W}$ is defined as

$$\mathbb{W} := \begin{cases} \gamma^{min} \leq \gamma \leq \gamma^{max},\ \gamma^{max} = -\gamma^{min} = 0.01\ (1/m) \\ \phi^{min} \leq \phi \leq \phi^{max},\ \phi^{max} = -\phi^{min} = 0.0873\ (rad) \end{cases} \tag{8}$$

The longitudinal velocity V_x for the LPV system is constrained to a closed set such that $V_x \in [V_x^{min}, V_x^{max}]$

$$V_x^{min} \leq V_x \leq V_x^{max},\ \text{where,} \begin{cases} V_x^{max} = 17\ (m/s) \\ V_x^{min} = 14\ (m/s) \end{cases} \tag{9}$$

From [16], the numerical bound for the system (2) is expressed with the relation

$$
\begin{aligned}
&\underline{A} \leq A \leq \overline{A}, \text{ where, } \begin{cases} \overline{A} = A(t)|_{V_x = V_x{}^{max}} \\ \underline{A} = A(t)|_{V_x = V_x{}^{min}} \end{cases} \\
&\underline{B} \leq B \leq \overline{B}, \text{ where, } \begin{cases} \overline{B} = B(t)|_{V_x = V_x{}^{max}} \\ \underline{B} = B(t)|_{V_x = V_x{}^{min}} \end{cases} \\
&\underline{B^w} \leq B^w \leq \overline{B^w}, \text{ where, } \begin{cases} \overline{B^w} = B^w(t)|_{V_x = V_x{}^{max}} \\ \underline{B^w} = B^w(t)|_{V_x = V_x{}^{min}} \end{cases}
\end{aligned}
\tag{10}
$$

The state space equation & constraints described in (2), (3), (4), (5), (10) are discretized to a discrete-time model using zero order hold (ZOH) method with a sample time $T_s = 0.025(s)$. The resulting discrete-time state space model is expressed with

$$x^+ = A_d(k)\, x(k) + B_d(k)\, u(k) + B_d^w(k)\, w(k) \tag{11}$$

The model uncertainty in (11) is polytopic in nature with

$$[A_d(k)\, B_d(k)] \in \Omega_d = \mathbf{Co}\{[\underline{A}_d \underline{B}_d], [\overline{A}_d \overline{B}_d]\} \tag{12}$$

The disturbance in the system (11) induced due to the presence of polytopic model uncertainty is decomposed to a LTI model term & an external disturbance term [8], which is expressed as

$$x^+ = \tilde{A}_d x(k) + \tilde{B}_d u(k) + \tau(k) + B_d^w(k)\, w(k) \tag{13}$$

Where the matrices $\tilde{A}_d$, $\tilde{B}_d$ are defined as

$$\tilde{A}_d := \frac{1}{J}\sum_{i=1}^{J} A_d^{(j)},\ \tilde{B}_d := \frac{1}{J}\sum_{i=1}^{J} B_d^{(j)} \tag{14}$$

$A_d^{(j)}$ & $B_d^{(j)}$ denote the jth vertex of the polytopic model set Ω_d and J denotes the total number of vertices for the polytopic set Ω_d. The external disturbance term in (13) is defined as

$$\tau(k) := (A_d(k) - \tilde{A}_d)x(k) + (B_d(k) - \tilde{B}_d)u(k) \tag{15}$$

The disturbance (15) is bounded & is contained in the polytopic set

$$
\begin{aligned}
\Gamma := \{(A_d(k) - \tilde{A}_d)x + (B_d(k) - \tilde{B}_d)u|\ \forall k \in \mathbb{Z}_+, \\
[A_d(k)\, B_d(k)] \in \Omega_d, (x, u) \in \mathbb{X} \times \mathbb{U}\}
\end{aligned}
\tag{16}
$$

In order to account for the total disturbance present in (13), a conservative bound for the disturbance terms is computed with interval arithmetic method [14]. The additive disturbance term $B_d^w(k)\, w(k) + \tau(k)$ in (13) is condensed to $d(k)$, such that $d(k) \in \mathbb{D} \subset \mathbb{R}^n$, where $\mathbb{D} := [\underline{d}, \overline{d}] \in \mathcal{I}^n$ is a multidimensional interval set. The set $\mathbb{D}$ serves as an over-approximation set for the disturbance terms. Under this consideration, the modified governing difference equation is compactly expressed with

$$x^+ = \tilde{A}_d x(k) + \tilde{B}_d u(k) + d(k) \tag{17}$$

3 ROBUST TUBE MPC

3.1 Preliminaries

The uncertain discrete time system defined by (17) is subject to state, control & disturbance constraints defined by closed, compact set defined as $\mathbb{X} \subset \mathbb{R}^n$, $\mathbb{U} \subset \mathbb{R}^m$, $\mathbb{D} \subset \mathbb{R}^n$ respectively & $0 \in \operatorname{int}(\mathbb{X})$, $0 \in \operatorname{int}(\mathbb{U})$, $0 \in \operatorname{int}(\mathbb{D})$. Let $\mathbf{u} = \{u_0, u_1, \ldots u_{N-1}\}$ and $\mathbf{d} = \{d_0, d_1, \ldots d_{N-1}\}$ denote the input sequence and disturbance sequence respectively. The solution for the system at time $j \in \mathbb{Z}_+$ is denoted by the relation $\xi(j; x, \mathbf{u}, \mathbf{d})$, where at time 0, $\xi(0; x, \mathbf{u}, \mathbf{d}) = x$ and the state trajectory is denoted as $\mathbf{x} = \{x_0, x_1, \ldots x_N\}$. A nominal system (reference) is defined for (17) without including the disturbance term as

$$\overline{x}^+ = \tilde{A}_d \overline{x}(k) + \tilde{B}_d \overline{u}(k) \tag{18}$$

The solution for the nominal system is denoted by the relation $\overline{\xi}(j; x, \mathbf{u})$, where $\overline{\mathbf{u}} = \{\overline{u}_0, \overline{u}_1, \ldots \overline{u}_{N-1}\}$ is the input sequence for nominal system and the nominal state trajectory is denoted as $\overline{\mathbf{x}} = \{\overline{x}_0, \overline{x}_1, \ldots \overline{x}_N\}$, where $\overline{x}_0 = x$. Let $\mathcal{K} \in \mathbb{R}^{m \times n}$ denote the disturbance rejection gain, such that $\tilde{A}_{\mathcal{K}} = \tilde{A}_d + \tilde{B}_d \mathcal{K}$ is Schur stable, then the disturbance invariant set $\mathcal{Z}$ for (17), such that $x^+ = \tilde{A}_{\mathcal{K}} x + d$ is defined as

$$\tilde{A}_{\mathcal{K}} \mathcal{Z} \oplus \mathbb{D} \subseteq \mathcal{Z} \tag{19}$$

The disturbance invariant $\mathcal{Z}$ implemented is a minimal robust positive invariant set (mRPI) such that the conservativeness is reduced. The mRPI set $\mathcal{Z}$ is defined as

$$\mathcal{F}_\infty := \sum_{i=1}^{\infty} \tilde{A}_{\mathcal{K}}^i \mathbb{D} \tag{20}$$

An ϵ-outer approximation of $\mathcal{F}_\infty$ set proposed in [20] is utilized to compute the mRPI set in finite number of iterations. From [3], it is definite that if $\mathcal{Z}$ is disturbance invariant for the system $x^+ = \tilde{A}_{\mathcal{K}} x + d$ if the initial state satisfies the property $x_0 \in \overline{x}_0 \oplus \mathcal{Z}$ & if the feedforward input is $u_i = \overline{u}_i + \mathcal{K}(x_i - \overline{x}_i), \forall i \in \mathbb{Z}_+$ then $x_i^+ \in \overline{x}_i^+ \oplus \mathcal{Z}, \forall i \in \mathbb{Z}_+$ and $\forall d \in \mathbb{D}$. Therefore, the state feedback policy defined, maintains the states of the uncertain system (17) close to the nominal system (18).

3.2 Problem formulation

The proposed finite time optimal control problem (FTOCP) is formulated from ideas demonstrated in [3], [9]. In order to guarantee recursive feasibility, constraint satisfaction & robust asymptotic stability of $\mathcal{Z}$, the initial state $\overline{x}_0$ is modeled as a decision variable such that $x \in \overline{x}_0 \oplus \mathcal{Z}$, where x is the current state of the system defined by (17). The decision variables for the FTOCP $\overline{\mathbb{P}}_N(x)$ are $\{\overline{x}_0, \overline{\mathbf{u}}\}$ & constraint space for $\overline{\mathbb{P}}_N(x)$ are tightened than the actual constraint space. This eradicates the uncertainty from the optimization problem. The FTOCP $\overline{\mathbb{P}}_N(x)$ is defined as

$$V_N^*(x) := \min_{\overline{x}_0, \overline{\mathbf{u}}} \{V_N(\overline{x}_0, \overline{\mathbf{u}})|\ \overline{\mathbf{u}} \in \mathcal{U}_N(\overline{x}_0), x \in \overline{x}_0 \oplus \mathcal{Z}\} \tag{21}$$

$$
\begin{aligned}
(\overline{x}_0^*(x), \overline{\mathbf{u}}^*(x)) := &\operatorname*{argmin}_{\overline{x}_0, \overline{\mathbf{u}}} \{V_N(\overline{x}_0, \overline{\mathbf{u}})|\ \overline{\mathbf{u}} \in \mathcal{U}_N(\overline{x}_0), \\
&x \in \overline{x}_0 \oplus \mathcal{Z}\}
\end{aligned}
\tag{22}
$$

Where the cost function $V_N(\cdot)$ in (21), (22) is defined as

$$V_N(\overline{x}_0, \overline{\mathbf{u}}) := V_f(\overline{x}_N) + \sum_{i=0}^{N-1} \mathcal{L}(\overline{x}_i, \overline{u}_i) \tag{23}$$

The cost function is positive definite $V_N : \mathbb{R}^{n \times Nm} \mapsto \mathbb{R}_+$ with stage cost $\mathcal{L}(\cdot, \cdot)$ defined by weighted $(Q \succ 0, R \succ 0)$ ℓ_2-norm as

$$\mathcal{L}(\overline{x}_i, \overline{u}_i) := \|\overline{x}_i\|_Q^2 + \|\overline{u}_i\|_R^2 \tag{24}$$

and terminal cost $V_f(\cdot)$ defined by $(P \succ 0)$ ℓ_2-norm

$$V_f(\overline{x}_N) := \|\overline{x}_N\|_P^2 \tag{25}$$

The tightened state, control & terminal state constraint for the FTOCP $\overline{\mathbb{P}}_N(x)$ is defined as

$$\overline{x}_i \in \overline{\mathbb{X}} \triangleq \mathbb{X} \ominus \mathcal{Z}, \forall i \in \{0, 1, 2, \dots N\} \tag{26}$$

$$\overline{u}_i \in \overline{\mathbb{U}} \triangleq \mathbb{U} \ominus \mathcal{K}\mathcal{Z}, \forall i \in \{0, 1, 2, \dots N-1\} \tag{27}$$

$$\overline{x}_N \in \overline{\mathbb{X}}_f \subset \mathbb{X} \ominus \mathcal{Z} \tag{28}$$

$\overline{\mathbb{X}}_f$ is the terminal constraint set for FTOCP $\overline{\mathbb{P}}_N(x)$. The controllable set $\mathcal{U}_N(\overline{x}_0)$ is defined as

$$\begin{aligned}\mathcal{U}_N(\overline{x}_0) := \{\overline{\mathbf{u}} | \, \overline{u}_i \in \overline{\mathbb{U}}, \overline{\xi}\,(j; \overline{x}_0, \overline{\mathbf{u}}) \in \overline{\mathbb{X}}, \forall i \in \{0, \dots N-1\}, \\ \overline{\xi}\,(N; \overline{x}_0, \overline{\mathbf{u}}) \in \overline{\mathbb{X}}_f\}\end{aligned} \tag{29}$$

The region of attraction $\mathbb{X}_N$ for the value function $V_N^*(\cdot)$ is defined as

$$\mathbb{X}_N \triangleq \{x \in \mathbb{R}^n | \, \mathcal{U}_N(\overline{x}_0) \neq \emptyset, \exists \overline{x}_0, \text{such that } x \in \overline{x}_0 \oplus \mathbb{Z}\} \tag{30}$$

The terminal cost $V_f(\cdot)$ and the constraint set $\overline{\mathbb{X}}_f$ is designed such that the axioms defined in [2] are satisfied. The terminal cost $V_f(\cdot)$ is formulated using the terminal weight matrix P computed in section (3.3). The terminal set is formulated by computing the maximal positive invariant set O_∞ defined for $\tilde{A}_{\mathcal{K}}, \overline{\mathbb{X}}$ [4]. The solution of the FTOCP $\overline{\mathbb{P}_N^*}(x)$ yields the optimal initial state $\overline{x_0^*}(x)$ and optimal control sequence $\overline{\mathbf{u}}^*(x) = \{\overline{u_0^*}, \overline{u_1^*}, \dots \overline{u_{N-1}^*}\}$. The optimal state trajectory is obtained from $\overline{\xi}\left(j; \overline{x_0^*}(x), \overline{\mathbf{u}}^*(x)\right), \forall j \in \{0, 1, 2, \dots N\}$, which is $\overline{\mathbf{x}}^*(x) = \{\overline{x_0^*}, \overline{x_1^*}, \dots \overline{x_N^*}\}$. The receding horizon control policy applied to the system (17), when the current state is x is defined by the implicit model predictive control law

$$\kappa_N^*(x) := \overline{u_0^*} + \mathcal{K}(x - \overline{x_0^*}(x)) \tag{31}$$

The controller achieves robust exponential stability by convergence to the disturbance invariant set $\mathcal{Z}$, thereby the controller guarantees stability in Lyapunov sense [3].

3.3 Disturbance rejection controller design

The disturbance rejection controller $\mathcal{K}$ determines the robustness of the tube MPC as it shapes the disturbance invariant set $\mathcal{Z}$, which implicitly bounds the nominal system state constraint $\overline{\mathbb{X}}$ & input constraint $\overline{\mathbb{U}}$. If the spectral radius [9], $\rho(A_{\mathcal{K}}) \ll 1$, then vol($\mathcal{Z}$) would be small & this would yield larger tightened state & input constraints for the nominal problem. However, placing the eigenvalues of $A_{\mathcal{K}}$ close to origin would require large gain $\mathcal{K}$, which would shrink the size of $\overline{\mathbb{U}}$. In this paper, we propose a semidefinite programming (SDP) method to compute the appropriate gain which balances the tradeoff between design of nominal state constraints $\overline{\mathbb{X}}$ & input constraints $\overline{\mathbb{U}}$ and also deals with the model uncertainty present in the system. The idea is to solve the robust linear state feedback problem [11] using SDP & linear matrix inequalities (LMI). The state feedback policy $u_i = \mathcal{K}x_i$ is designed for the system (11) without the exogenous disturbance & an upper bound of worst case infinite horizon quadratic cost is minimized [6], which is defined by the cost function

$$J_\infty := \max \sum_{i=0}^{\infty} \|x_i\|_Q^2 + \|u_i\|_R^2, \text{where, } Q > 0, R > 0 \tag{32}$$

The infinite horizon quadratic cost function is upper bound by a quadratic function, $J_\infty \le \|x_i\|_P^2, \forall i \in \mathbb{Z}_+$ with $P > 0$ iff, it holds for all possible models in Ω_d which satisfies the condition

$$\|x_{i+1}\|_P^2 - \|x_i\|_P^2 \le -\|x_i\|_Q^2 - \|u_i\|_R^2, \forall i \in \mathbb{Z}_+ \tag{33}$$

Inserting the feedback law $u_i = \mathcal{K}x_i$ for (11) in (33) without the exogenous disturbance yields the matrix inequality,

$$(A_d(k) + B_d(k)\mathcal{K})^T P(A_d(k) + B_d(k)\mathcal{K}) - P \le -Q - \mathcal{K}^T R\mathcal{K} \tag{34}$$

From [17], a congruence transformation is implemented such as $W = P^{-1}$ & $\mathcal{K} = \mathcal{G}P^{-1}$ and by Schur complement, (34) is transformed to LMI form. The LMI holds for all models, but due to polytopic model uncertainty, only the vertices are considered [17], which is expressed as

$$\begin{bmatrix} W & (A_d^i W + B_d^i \mathcal{G})^T & W & \mathcal{G}^T \\ (A_d^i W + B_d^i \mathcal{G}) & W & 0 & 0 \\ W & 0 & Q^{-1} & 0 \\ \mathcal{G} & 0 & 0 & R^{-1} \end{bmatrix} \ge 0 \tag{35}$$

Where (35) holds $\forall i \in \text{ext}(\Omega_d)$. The SDP problem is defined as maximization of trace of matrix W, which is defined by the following optimization problem.

$$\begin{aligned} \max_{W, \mathcal{G}} \quad & \text{Tr}W \\ \text{subject to} \quad & W \ge 0, (35) \end{aligned} \tag{36}$$

The solution of this optimization problem yields $P = W^{-1}$ and $\mathcal{K} = \mathcal{G}P^{-1}$,which are the worst case cost matrix & the disturbance rejection controller gain respectively that guarantees asymptotic stability of the mismatch system [15].

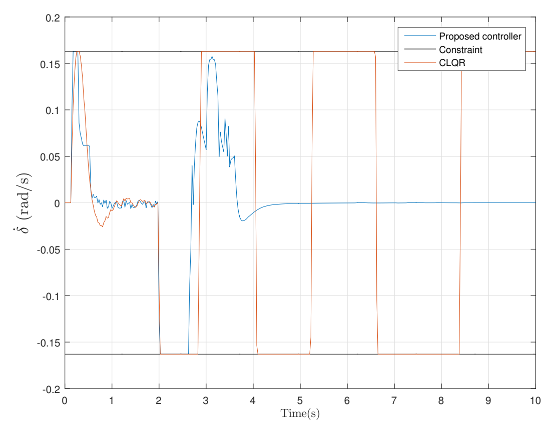

Figure 1: Input steering rate ($\dot{\delta}$)

4 RESULTS AND SIMULATION

The proposed RTMPC was implemented & tested in MATLAB/ Simulink environment & results were compared with clipped LQR (CLQR) controller. The prediction horizon for the proposed controller was $N = 7$. The state penalizing matrix ($Q > 0$) & input penalizing matrix ($R > 0$) are defined as $Q = \text{diag}(25, 25, 1, 1, 10)$ & $R = 12$. The disturbance rejection gain ($\mathcal{K}$) & the terminal cost

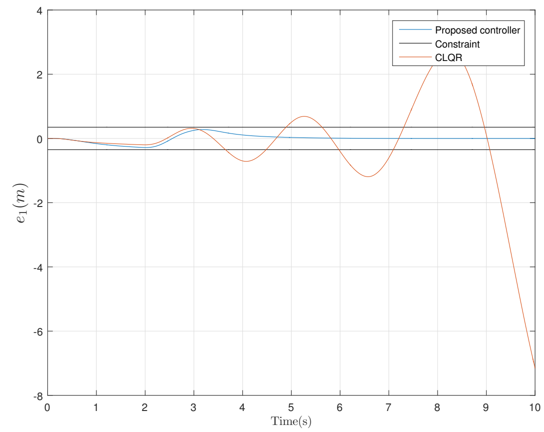

Figure 2: Lateral position error (e_1)

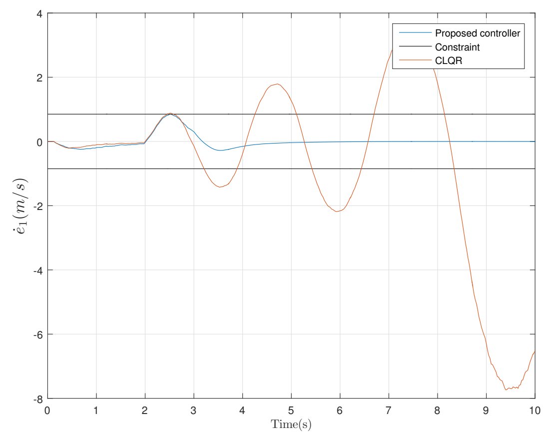

Figure 3: Lateral position error rate ($\dot{e}_1$)

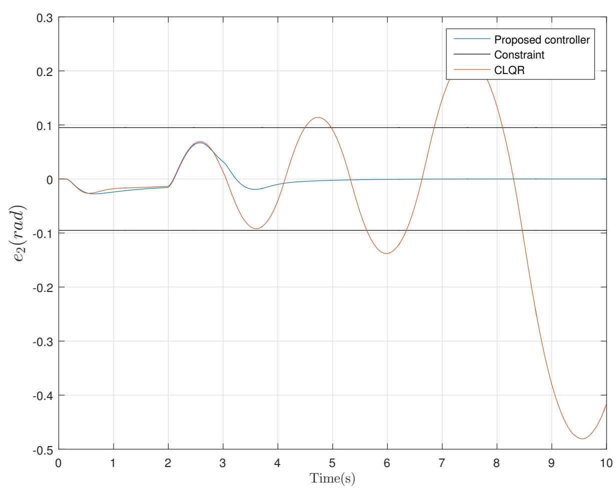

Figure 4: Yaw error (e_2)

matrix ($P \succ 0$) computed from the SDP optimization problem were solved using YALMIP [7] toolbox.

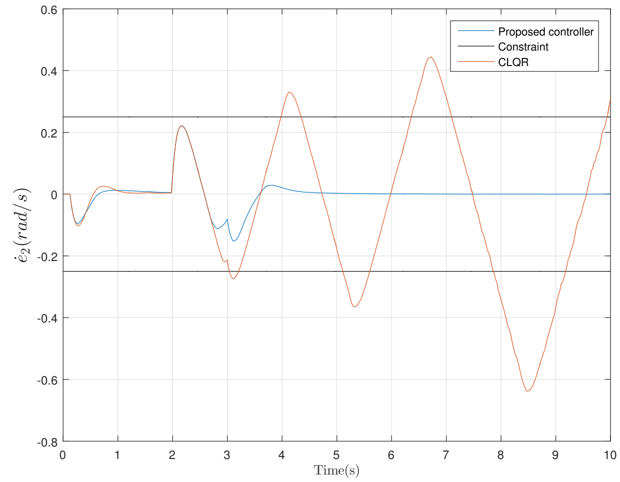

Figure 5: Yaw error rate ($\dot{e}_2$)

For CLQR problem, the respective gain & cost functions were computed by solving the discrete time algebraic Riccati equations (DARE).

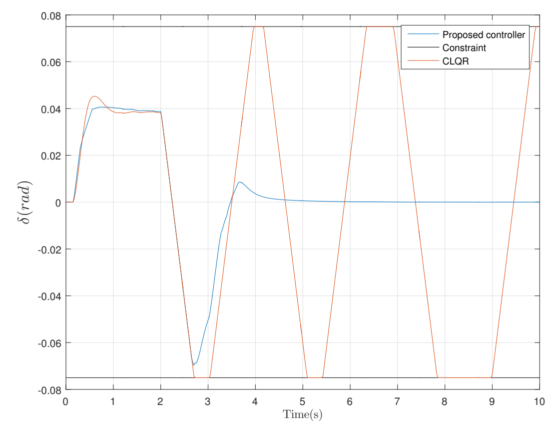

Figure 6: Steering angle (δ)

The disturbance invariant set $\mathbb{Z}$ computation and other set operations were implemented using PnPMPC toolbox [19] & MPT toolbox [10]. The longitudinal velocity was assumed to follow a uniform distribution, $V_x \sim U(V_x^{min}, V_x^{max})$ as illustrated in Figure 9 such that the robustness of proposed controller is verified under conditions of skidding, where the velocity shifts abruptly.The CLQR is designed such that physical limitations of the vehicle steering angle(δ) & steering rate($\dot{\delta}$) are satisfied. The considered desired path for simulation is illustrated in Figure 7 & Figure 8. From Figure 1, it is evident that CLQR controller input sequence exhibits oscillatory behavior between the input constraint bounds whereas the proposed controller performs much better & also stabilizes the system, which is illustrated in Figure 2, Figure 3, Figure 4, Figure 5, Figure 6. It is clear that CLQR fails to stabilize the vehicle & runs into unstable behavior, however the proposed controller stabilizes the vehicle and respects all the system constraints.

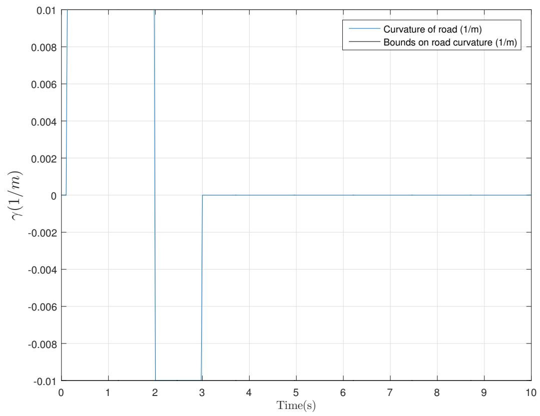

Figure 7: Curvature of road (γ)

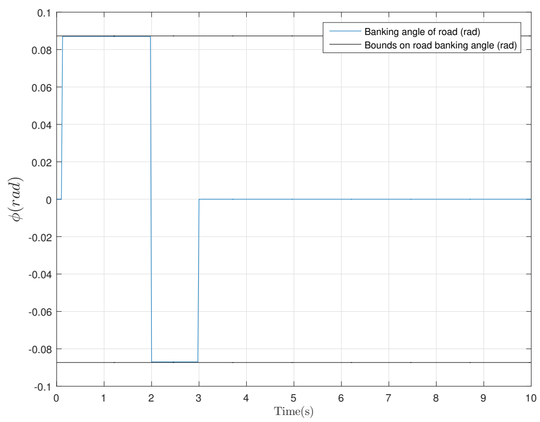

Figure 8: Banking angle of road (ϕ)

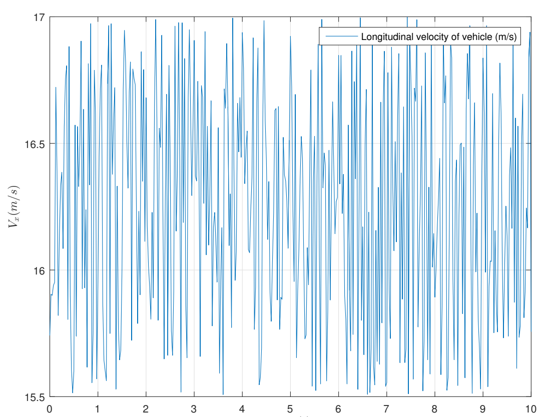

Figure 9: Longitudinal velocity of vehicle (V_x)

5 CONCLUSION

This paper proposes a robust model predictive control method for lane keeping system & a comparison is made with an ad-hoc control (CLQR) method. The simulation expounds the result that the proposed controller stabilizes the system whereas the CLQR fails to stabilize the system. Further studies may focus on inclusion of measurement noise, state estimation techniques & inclusion of traffic participants in scene.

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