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Driver assistance for collision avoidance by constrained MPC

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Abstract: A driver assistance control algorithm for a vehicle with an active front steering system is proposed in this paper. The control algorithm is designed so as to optimize the turning performance and realize the collision avoidance simultaneously under the constraints on the front wheel turn angle and the lateral tire force. The control algorithm is developed based on a model predictive control law for tracking a time-varying reference signal. The proposed control algorithm is reduced to a convex quadratic programming problem.

Keywords: vehicle control, collision avoidance, yaw rate control, model predictive control, constraints.

1. INTRODUCTION

Recently, various control methodologies that realize collision avoidance have been developed for a vehicle with an active front steering (AFS) system based on model predictive control (MPC). MPC can be considered an effective means for collision avoidance control since the method can naturally utilize the predictive information about obstacles. For example, MPC algorithms in which collision avoidance condition is described as state constraints have been proposed in References [1], and the methods based on the artificial potential field have been shown in References [2-4]. In these literatures, the collision avoidance control algorithms for autonomous vehicles have been developed.

On the other hand, driver assistance control methods have been studied in Reference [5, 6]. By using the methods in these literatures, when the control action of a human driver is insufficient for collision avoidance, the steering angle is corrected automatically so as to prevent collisions. In the methods of these literatures, the AFS is used only for collision avoidance and has not been used to improve turning performance of the vehicle. However, the collision avoidance performance is highly affected by the turning performance. Hence, it is expected to develop a control methodology which optimizes the turning performance and realizes the collision avoidance simultaneously.

In this paper, we propose a driver assistance control algorithm for a vehicle with AFS that simultaneously achieves yaw rate control and collision avoidance. To this end, firstly, we construct a yaw rate control algorithm based on MPC in Reference [7]. The control algorithm is designed so that the vehicle yaw rate tracks a time-varying reference yaw rate rapidly under the constraints on the front wheel turn angle and lateral tire force. Then a condition for collision avoidance is derived as a state constraint. By incorporating the collision avoidance constraint into the yaw rate control algorithm, we derive a control algorithm which optimizes turning performance and achieves the obstacle avoidance.

Notations: For a vector $x \in \mathbf{R^n}$, we define $\|x\|_2 := (x^T x)^{1/2}$. For a vector $x \in \mathbf{R^n}$ and a positive definite matrix $P \in \mathbf{R^{n\times n}}$, we define $\|x\|_P := (x^T P x)^{1/2}$. For a symmetric matrix P, $P > 0$ ($P \geq 0$) implies that P is a positive-definite (semidefinite) matrix. For vectors $u, v \in \mathbf{R^n}$, the notation $u < v$ ($u \leq v$) implies $u^{(i)} < v^{(i)}$ ($u^{(i)} \leq v^{(i)}$), $\forall i = 1, \cdots, n$. For vectors $u, v \in \mathbf{R^n}$, the notation $|u| < v$ ($|u| \leq v$) implies $|u^{(i)}| < v^{(i)}$ ($|u^{(i)}| \leq v^{(i)}$), $\forall i = 1, \cdots, n$.

2. VEHICLE MODEL

Let us consider the system shown in Fig.1. G is the center of gravity. V is the vehicle velocity, δ_f is the front wheel turn angle, F_{Yi} is the lateral force of the ith tire. l_f and l_r are the distance between the center of gravity and the front and rear axis. β is the vehicle side slip angle, γ is the vehicle yaw rate. We denote the slip angle of the ith wheel as α_i. In the following we assume that vehicle velocity V is constant and $|\beta| \ll 1$ and $|\delta_f| \ll 1$. In this case, the vehicle dynamics can be described by the following differential equations [8].

$$\begin{aligned} MV(\dot{\beta} + \gamma) &= F_{Yf} + F_{Yr} && (1) \\ I_z \dot{\gamma} &= l_f F_{Yf} - l_r F_{Yr} && (2) \end{aligned}$$

where M is the vehicle mass, I_z is the moment of the inertia around the center of gravity. Moreover, $F_{Yf} := F_{Y1} + F_{Y2}$, $F_{Yr} := F_{Y3} + F_{Y4}$. When $|\beta|$, $|l_f \gamma / V|$ and $|d_f \gamma/(2V)|$ are sufficiently small, the slip angles of the wheels can be approximated by $\alpha_f := \alpha_1 = \alpha_2 = -\beta - l_f\gamma/V + \delta_f$ and $\alpha_r := \alpha_3 = \alpha_4 = -\beta + l_r\gamma/V$. F_{Yi} is proportional to α_i when $|\alpha_i|$ is sufficiently small. Hence, we use the lateral tire force model described by $F_{Yi} = C_i \alpha_i$. Moreover, we assume that $C_f := C_1 = C_2$, $C_r := C_3 = C_4$. In this case, the vehicle dynamics can be described by

$$\dot{x}_p = \bar{A}_p x_p + \bar{B}_p u \qquad (3)$$

We discretize the system (3) using the zero-order hold with the sampling period T_s and obtain

$$x_p(k+1) = A_p x_p(k) + B_p u(k). \qquad (4)$$

We choose the controlled output as

$$\gamma(k) = C_p x_p(k), \qquad (5)$$

† Takuya Matsumoto is the presenter of this paper.

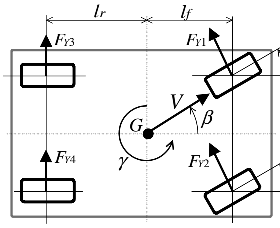

Fig. 1 Vehicle model

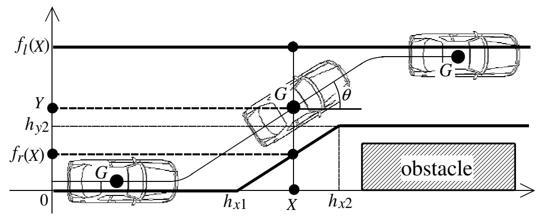

Fig. 2 Collision avoidance scenario

where $C_p := [0, 1]$. The vehicle model (4), (5) will be used for yaw rate controller design in Section 4.

Fig. 2 shows the coordinate system. (X, Y) denotes the position of the center of gravity G, and θ denotes the yaw angle of the vehicle. Clearly, the relation $\dot{Y} = V \sin(\beta + \theta)$ holds. Hence, when $|\beta + \theta|$ is sufficiently small, the following relation holds.

$$\dot{x}_d = \bar{A}_d x_d + \bar{B}_d x_p, \tag{6}$$

where $x_d := [Y, \theta]^T$ and

$$\bar{A}_d := \begin{bmatrix} 0 & V \\ 0 & 0 \end{bmatrix}, \ \bar{B}_d := \begin{bmatrix} V & 0 \\ 0 & 1 \end{bmatrix}.$$

From (3) and (6), we obtain

$$\dot{x}_s = \bar{A}_s x_s + \bar{B}_s u, \tag{7}$$

where $x_s := [x_d^T, x_p^T]^T$.

We discretize (7) using the zero-order hold with the sampling period T_s and obtain

$$x_s(k+1) = A_s x_s(k) + B_s u(k), \tag{8}$$

The dynamical model (8) will be used to derive a collision avoidance condition in Section 5.

3. OVERVIEW OF THE PROPOSED ASSIST CONTROL SYSTEM

In this paper, we consider the collision avoidance control problem of a vehicle with an AFS system. Fig. 2 shows a typical scenario of the collision avoidance in this paper. We assume that the vehicle velocity V is constant. We design a controller which satisfies the following requirements.

- The vehicle yaw rate γ tracks the reference yaw rate rapidly under the constraints on the front wheel turn angle and the lateral tire force.
- When the reference yaw rate produced by a human driver is insufficient for collision avoidance, the controller corrects the vehicle yaw rate automatically to avoid the collision.

We assume that the constraint on the front wheel turn angle is given as

$$|u(k)| \leq \bar{u}, \ \forall k \geq 0, \tag{9}$$

where $\bar{u}$ is a positive scalar.

Also, the constraint on the lateral tire force is assumed to be given as

$$|F_{Yf}(k)| \leq \bar{F}, \ \forall k \geq 0, \tag{10}$$

where $\bar{F}$ is a positive scalar. From (**??**), the condition (10) can be viewed as a constraint on the input and the states. In this paper, for simplicity, we will not consider the constraint on the lateral force of the rear wheels. However, it is possible to incorporate the constraint into the proposed control algorithm.

Then we derive a collision avoidance condition as a state constraint. We assume that the condition for collision avoidance is given as follows (see Fig. 2).

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} Y \leq \begin{bmatrix} -f_r(X) \\ f_l(X) \end{bmatrix}, \tag{11}$$

where $f_r(X)$ and $f_l(X)$ are functions of X. We assume that the collision avoidance is achieved when the center of gravity satisfies the above constraint.

Fig. 3 shows the block-diagram of the proposed control system. The block-diagram consists of the vehicle model, the controller and a driver. In this figure, $f := [f_l, f_r]^T$. Also, γ_{ref} denotes the reference yaw rate produced by a driver. The controller is basically designed so that the vehicle yaw rate tracks the reference yaw rate rapidly. In this control system, the driver tries to determine the reference yaw rate γ_{ref} so that the collision avoidance is achieved. If the control action of the driver is insufficient for collision avoidance, the controller corrects the front wheel turn angle automatically so that the collision avoidance is achieved based on the information about the shape of the obstacle which is measured by the computer vision. The objective of this paper is to develop such a control system.

In the numerical example in Section 6, the following look-ahead driver model will be used as a human driver model.

$$\gamma_{\mathrm{ref}}(k) = k_f(Y_{\mathrm{pref}}(k) - Y_p(k)) \tag{12}$$

where k_f is a positive constant and $Y_p(k) = Y(k) + L\theta(k)$. L is the look-ahead distance. $Y_{\mathrm{pref}}(k)$ is a reference value for $Y_p(k)$. In Section 6, we will use $Y_{\mathrm{pref}}(k) = (f_l(X_p(k)) + f_r(X_p(k)))/2$, where $X_p(k) := X(k) + L$.

4. YAW RATE CONTROL BASED ON CONSTRAINED MPC

In this section, we show a yaw rate control law under the constraints on the front wheel turn angle and the lateral tire force. In the control problem considered in this paper, the reference yaw rate is produced by a human driver. Hence, the value of the reference yaw rate changes

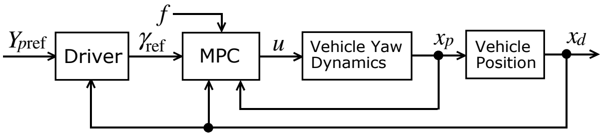

Fig. 3 Proposed control system with a driver

depending on the driving situation. Therefore, we design a yaw rate control law for tracking a time-varying reference signal based on [7]. In this section, we consider the following problem.

Problem 1: Consider the system (4), (5). For a given reference yaw rate $\gamma_{\rm ref}(k) \in \mathbf{R}$, design a control law

$$u(k) = \mathcal{K}(x_p(k), \gamma_{\rm ref}(k)) \tag{13}$$

that minimizes the deviation between $\gamma_{\rm ref}(k)$ and $\gamma(k)$ at each sampling time and achieves $\lim_{k\to\infty} \gamma(k) = \bar{\gamma}$ if $\gamma_{\rm ref}(k) = \bar{\gamma}, \forall k \geq T_f$ under the constraints (9), (10), where $\bar{\gamma}$ is a constant vector and T_f is a positive constant.

To eliminate steady state error, we insert the following integrator into the feedback system.

$$x_c(k+1) = x_c(k) + e(k) \tag{14}$$

$$e(k) = w(k) - \gamma(k) \tag{15}$$

where $x_c \in \mathbf{R}$ is a controller state and $w(k) \in \mathbf{R}$ is a virtual reference signal.

The prediction model of the system (4), (5), (14) and (15) is obtained as follows.

$$x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k) + Ew(k|k) \tag{16}$$

$$e(k+i|k) = Cx(k+i|k) + w(k|k) \tag{17}$$

where $x := [x_p^T, x_c]^T$, $C := [-C_p,\ 0]$. $x(k+i|k)$ denotes the state at time $k+i$ predicted at time k.

For the system (16) and (17), we make the following assumption.

Lemma 1: There exist matrices $\Pi \in \mathbf{R^3}$ and $\Gamma \in \mathbf{R}$ that satisfy

$$\Pi = A\Pi + B\Gamma + E,\ 0 = C\Pi + 1. \tag{18}$$

From (16)–(18), the error system can be derived as

$$\xi(k+i+1|k) = A\xi(k+i|k) + Bv(k+i|k), \tag{19}$$

$$e(k+i|k) = C\xi(k+i|k), \tag{20}$$

where $\xi(k+i|k) := x(k+i|k) - \Pi w(k|k)$ and $v(k+i|k) := u(k+i|k) - \Gamma w(k|k)$.

It is clear that the constraint (9) in the prediction time can be rewritten as

$$\Psi_u u(k+i|k) \leq \theta_u,\ \forall i \geq 0, \tag{21}$$

where $\Psi_u := [-1,\ 1]^T$ and $\theta_u := [\bar{u},\ \bar{u}]^T$.

Also, the constraint (10) can be rewritten as

$$\Psi_{xx} x(k+i|k) + \Psi_{xu} u(k+i|k) \leq \theta_x,\ \forall i \geq 0, \tag{22}$$

where $\Psi_{xx} := 2C_f[-1,\ 1]^T[-1,\ -l_f/V,\ 0]$, $\Psi_{xu} := 2C_f[-1,\ 1]^T$, and $\theta_x := \bar{F}[1,\ 1]^T$.

Moreover, we make the following assumption.

Assumption 1: The steady state value of the reference signal satisfies $\Psi_u \Gamma \bar{\gamma} \leq \theta_u$ and $(\Psi_{xx}\Pi + \Psi_{xu}\Gamma)\bar{\gamma} \leq \theta_x$.

The above assumption ensures that the control signal u and the state x satisfy the constraints (21) and (22) in the steady state.

In this paper, we adopt the following cost function[7].

$$J(k) := \sum_{i=0}^{H_s-1} \{\|\xi(k+i+1|k)\|_Q^2 + \|v(k+i|k)\|_R^2\} + \|\xi(k+H_s|k)\|_P^2 + \|\Pi(w(k|k) - \gamma_{\rm ref}(k|k))\|_M^2 \tag{23}$$

Q, R, M and P are positive definite matrices. The design condition of the matrix P is given in [7].

The model predictive control problem in this section is formulated as follows.

Problem 2: For a given reference signal $\gamma_{\rm ref}(k|k)$, solve $\min_{\tilde{v}, w(k|k)} J(k)$ subject to (19), (21) and (22), where $\tilde{v} := [v(k|k)^T, \cdots, v(k+H_s-1|k)^T]^T$ is a control sequence.

We propose the following MPC algorithm.

Algorithm 1: (Yaw rate control)

- Step 1 (Initialization): Set $k = 0$ and $x_c(0|0) = 0$.
- Step 2: Measure $x_p(k|k)$.
- Step 3: Solve $\min_\chi J(k)$ subject to (19), (21) and (22), where $\chi := [\tilde{v}^T, w(k|k)]^T$
- Step 4: Apply $u(k|k) = v(k|k) + \Gamma w(k|k)$ to the plant (4), (5).
- Step 5: Compute $x_c(k+1)$ by (14) and (15).
- Step 6: Set $k \leftarrow k+1$ and go to Step 2.

In the following, we briefly explain Algorithm 1. Here we assume that $\gamma_{\rm ref}(k|k) = \bar{\gamma}, \forall k \geq 0$. In Algorithm 1, the open-loop optimal control sequence $\tilde{v}$ and the virtual reference signal $w(k|k)$ are computed so that the function $J(k)$ is minimized at Step 3. Then the control signal $u(k|k)$ is applied to the plant at Step 4. By repeating this process, the cost function $J(k)$ decreases monotonically[7]. The optimization problem at Step 3 is a convex quadratic optimization problem.

5. COLLISION AVOIDANCE CONTROL

In this section, we derive a collision avoidance condition. Then, we incorporate the derived condition into Algorithm 1. By this means, we develop a control algorithm which optimizes the turning performance and realizes the collision avoidance.

From the assumption, the vehicle velocity V is constant. We assume that $X(0) = 0$. In this case, the relation

$$X(k) = VT_s k \tag{24}$$

holds. By substituting (24) into (11), we obtain

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} Y(k) \leq \begin{bmatrix} -\tilde{f}_r(k) \\ \tilde{f}_l(k) \end{bmatrix}, \tag{25}$$

where $\tilde{f}_l(k) := f_l(VT_s k)$ and $\tilde{f}_r(k) := f_r(VT_s k)$.

It is clear that Y can be expressed as $Y = C_a x_a$, where $C_a := [1, 0, 0, 0, 0]$ and $x_a := [x_d^T, x^T]^T$. Hence, the condition for collision avoidance in the prediction time is given as

$$\Psi_c x_a(k+i|k) \leq \theta_c(k+i|k). \tag{26}$$

From (8), (14) and (15), the collision avoidance constraint in the prediction horizon $0 \leq i \leq H_s - 1$ can be expressed as

$$\Lambda_5 \chi \leq \varphi_5. \tag{27}$$

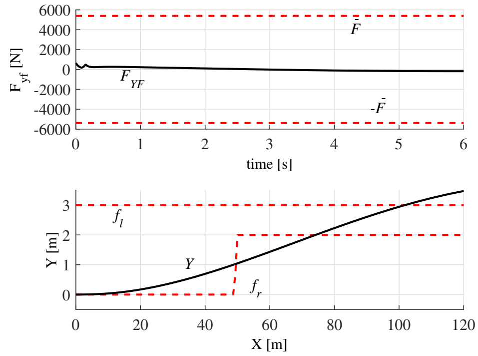

Fig. 4 Response of F_{FY} (upper) and trajectory of the vehicle (bottom) in the case of Algorithm 1

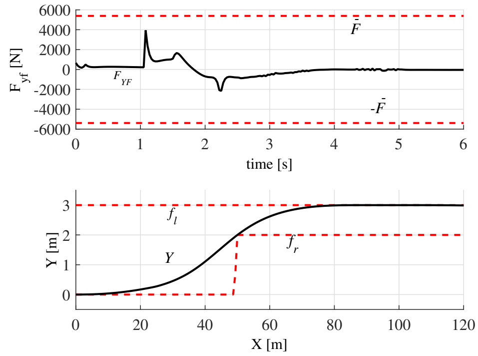

Fig. 5 Response of F_{FY} (upper) and trajectory of the vehicle (bottom) in the case of Algorithm 2

In the following, we propose a control algorithm which optimizes the yaw rate tracking performance and realizes the obstacle avoidance.

Algorithm 2: (Yaw rate control with collision avoidance)

- Step 1: Set $k = 0$ and $x_c(0|0) = 0$.
- Step 2: Measure $x_p(k|k)$, $\tilde{f}_l(k+i|k)$ and $\tilde{f}_r(k+i|k)$, for $i = 0, \cdots, H_s$.
- Step 3: Solve $\min_\chi J(k)$ subject to (19), (21), (22) and (27).
- Step 4: Apply $u(k|k) = v(k|k) + \Gamma w(k|k)$ to the plant (8).
- Step 5: Compute $x_c(k+1)$ by (14) and (15).
- Step 6: Set $k \leftarrow k+1$ and go to Step 2.

6. NUMERICAL EXAMPLE

The values of the parameters of the vehicle are M =1100kg, I_z=3760kgm^2, l_f=1.36m, l_r=1.36m, N_i=2695N, C_f=60000N/rad and C_r=60000N/rad. The vehicle velocity is V =80km/h. The parameters of the constraints on the front wheel turn angle and the lateral tire force are $\bar{u} = 0.6$rad and $\bar{F} = 5390$N. The sampling time is $T_s = 0.03$s. The matrix P is designed with $Q = \mathrm{diag}[0.1, 0.1, 0.001]$, $R = 1$ and $\rho = 0.5755$. The control horizon is $H_s = 40$. The matrix M in the cost function is $M = P + 0.1I$. The look-ahead distance of the driver model is $L = 20$m. Also, we chose $k_f = 0.01$. The parameters of the obstacle are $h_{x1} = 49$m, $h_{x2} = 50$m, $h_{y2} = 2$m and $h_{yl} = 3$m. Also, the initial states are $x_p(0) = 0$ and $x_d(0) = 0$.

Figures 4–5 show the simulation results. In the case of Algorithm 1, the control action of the driver model is too weak. As a result, the driver model fails to achieve collision avoidance. On the other hand, in the case of Algorithm 2, the proposed control system succeeds in performing collision avoidance.

7. CONCLUSIONS

We have proposed a driver assistance algorithm for collision avoidance based on constrained model predictive control. The proposed control algorithm realizes vehicle yaw rate control and collision avoidance simultaneously under the constraints on the front wheel turn angle and the lateral tire force. The proposed control algorithm is reduced to a convex quadratic programming problem. The effectiveness of the proposed control algorithm has been evaluated through a numerical example.

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