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Moving Obstacle Avoidance for a Mobile Robot

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***Abstract*— This paper presents the near time-optimal motion planning method for moving obstacle avoidance. We decomposed the robot motion into three phases: approach, contact, and detachment phase. The constraints of each phase for a feasible collision-free robot motion were described by three necessary and sufficient conditions and one sufficient condition. We formulated the near time-optimal motion planning as optimization problem with inequality constraints. Simulations present the efficiency of the results by comparing them with two widely used approaches: reactive and path-velocity decomposed approach.**

I. Introduction

In mobile robot system, navigation with collision avoidance for a moving obstacle is a fundamental problem. Many collision avoidance techniques have been developed over twenty years. It is an important issue for the robot to move to its goal as early as possible while the collision avoidance is guaranteed.

Most researches of the collision avoidance for a moving obstacle belong to short-term planning. They make the robot reactively change its heading angle and velocity to cope with an obstacle around in each planning time. Originated from the behavior-based approach [1], obstacle avoidance using the repulsive vector such as potential field [2] and vector field histogram [3] has been developed. In dynamic window approach [4], velocity of the next step maximizing the objective function is chosen among the admissible velocity range allowing the robot to stop safely. The objective function includes the forward velocity of the robot and the distance from the obstacle. A fuzzy logic is also used in planning the robot's reaction of each time step in cope with the obstacle [5]. We refer to these short-term planning as the *reactive approach*.

In another techniques [6, 7], the robot motion is planned in path-velocity decomposed way. The path is a geometric specification of a curve in the configuration space and the velocity is the speed when traveling along the path. The path is planned not to overlap with static obstacles. Along the previously planned path, the velocity is planned with guaranteeing that the robot does not collide with moving obstacles. The computational complexity is reduced by decomposing the problem into path planning and velocity planning. This method can consider the time-optimality in a fixed path, but global time-optimality issue cannot be considered.

Randomized approach also has been studied in collision-free motion planning [8]. It plan the piecewise collision-free motions at random instead of considering all possible avoidance motion. It guarantees the probabilistic completeness, but not time-optimality.

In this paper, we dealt with long-term planning for collision-free motion. Reif and Sharir [9] analyzed the computational complexity of motion planning in the presence of moving obstacles. They proved the 3-D dynamic movement problem is PSPACE-hard. Specially, they also proved that the *asteroid avoidance problem* can be solved in polynomial time. The asteroid avoidance problem is defined as the movement of a convex polyhedron by translation with a maximum speed bound in the presence of the polyhedral obstacles which have known translational trajectories but cannot rotate. Han and Bang [10] solved the near time-optimal robot motion with collision avoidance of a circular obstacle. They transformed this dynamic problem into a static one by considering the sweeping area of the obstacle during a predicted conflict time as a static obstacle. This approximation results in a loss of robot motion.

We formulated near time-optimal collision-free robot motion planning to the optimization problem with four parameters. In section II, we state the problem to be solved and decompose the robot motion into three phases. Section III gives the formulation of the time-optimal problem and section IV approximates the constraints of one of three phases in order to reduce the complexity. In section V, simulation results are presented with comparison between

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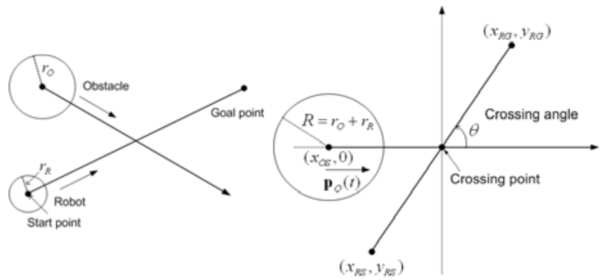


(a) (b)

Fig. 1. The movement of a circular robot in (a) is reduced to the movement of a point robot in (b). The coordinates is constructed as shown in (b).

results of three approaches, the reactive approach, the path-velocity decomposed approach, and near time-optimal result. Section VI gives a discussion and section VII finally concludes the paper.

II. Problem Overview

A. Problem Statement

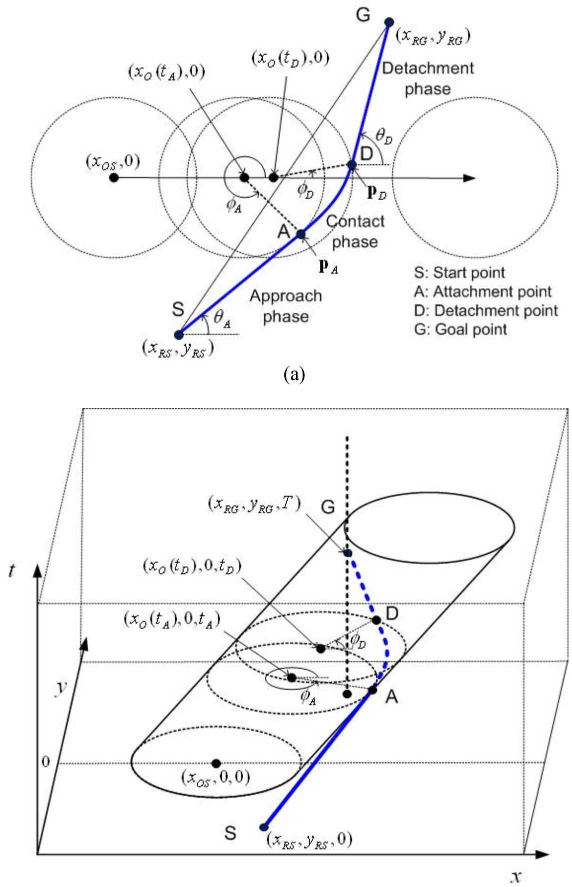
Given a moving obstacle with a specified motion, the purpose is to plan the robot motion so that the obstacle and robot are collision-free and the arrival time of the robot is minimized. The circular obstacle is assumed to move with a constant velocity. We consider a simplified model in which the circular robot has a maximum speed v_R and is permitted to instantaneously change its speed and direction. That is, the robot is omnidirectional and has a infinite acceleration.

We consider the case when the obstacle blocks the way of the robot as shown in Fig. 1(a). If the radius of the robot r_R is given to that of the obstacle r_O, the robot is considered to be a point as shown in Fig. 1(b). In order to easily formulate the problem, we construct the coordinates as shown in Fig. 1(b). The origin is set as the crossing point between the moving line of the obstacle and the straight line from start to goal of the robot. The x axis is set as the moving direction of the obstacle. In this coordinates, the start point and goal point of the robot is represented as (x_{RS}, y_{RS}) and (x_{RG}, y_{RG}), respectively. The position of the obstacle at the initial time is represented as $(x_{OS}, 0)$. The obstacle motion is assumed as follows.

$$\mathbf{p}_O(t) = (x_O(t), y_O(t)) = (x_{OS} + v_O t, 0) \tag{1}$$

B. Robot Motion Decomposition

In this section, we explain that the time-optimal robot motion is decomposed into three phases as shown in Fig. 2(a). The obstacle motion can be depicted as an oblique cylinder in the configuration-time space (C-T space), which is the configuration space augmented of the time dimension as shown in Fig. 2(b). We can plan the time-optimal robot motion by drawing a line from $(x_{RS}, y_{RS}, 0)$ to (x_{RG}, y_{RG}, T) so that the arrival time T is minimized. Here, this line should not penetrate the oblique cylinder (obstacle) in C-T space because of the collision-free condition and the slope of this line should not be lesser than the minimum slope $1/v_R$ because of the maximum speed constraint.



(b)

Fig. 2. Three phases of the robot motion (a) in the 2-D space (b) in 3-D configuration-time space (C-T space)

Reif and Sharir [9] proved that a robot has a collision-free movement from $(x_{RS}, y_{RS}, 0)$ to (x_{RG}, y_{RG}, T) if and only if the robot has sequences of direct movements and contact movements from $(x_{RS}, y_{RS}, 0)$ to (x_{RG}, y_{RG}, T) in *asteroid avoidance problem*. A direct movement is a movement with a constant velocity vector and a contact movement is a movement in which the robot moves on the boundary of the obstacle.

Similarly, in our problem, the robot motion is decomposed into three phases, which are the *approach phase* from the start point (S) to the attachment point (A), the *contact phase* from the attachment point (A) to the detachment point (D), and the *detachment phase* from the detachment point (D) to the goal point (G). In the detachment point, the robot should move straightly to the goal with its maximum speed in order to make the robot reach the goal in the earliest time. In the contact phase, the robot follows the boundary of the obstacle. We consider the approach phase symmetrically as the detachment phase in order to easily discriminate the reachability to the attachment point. That is, we make the robot move straightly to the goal with its maximum speed,.

The attachment and detachment point are represented by two parameters, time and angle, respectively. As shown in Fig. 2, let t_A and ϕ_A ($0 \le \phi_A < 2\pi$) be the time and angle of the attachment point, and let t_D and ϕ_D ($0 \le \phi_D < 2\pi$) be those of the detachment point. Then attachment point $\mathbf{p}_A$ and detachment point $\mathbf{p}_D$ are represented as follows.

$$\mathbf{p}_A = (x_A, y_A) = (x_O(t_A) + R\cos\phi_A, y_O(t_A) + R\sin\phi_A) \quad (2)$$

$$\mathbf{p}_D = (x_D, y_D) = (x_O(t_D) + R\cos\phi_D, y_O(t_D) + R\sin\phi_D) \quad (3)$$

Planning the time-optimal collision-free robot motion is reduced to finding these two points, the attachment point and the detachment point. In other words, we have to find four parameters, t_A, ϕ_A, t_D, and ϕ_D which minimize the arrival time of the robot. Here, these parameters have a restricted possible range because of the maximum speed bound constraint and the collision-free condition.

III. Requirements for Time-Optimal Collision-Free Motion

A. Approach Phase

In a approach phase, the robot moves straightly to the attachment point $\mathbf{p}_A$ with the maximum speed v_R. In order to reach the attachment point $\mathbf{p}_A$ at the attachment time t_A, the robot may wait for some time on the start point. Its motion is represented as follows.

$$\mathbf{p}_R(t) = (x_R(t), y_R(t)) = \begin{cases} (x_{RS}, y_{RS}) & for\ 0 \le t < t_d \\ (x_{RS} + v_R(t - t_d)\cos\theta_A, y_{RS} + v_R(t - t_d)\sin\theta_A) & for\ t_d \le t < t_A \end{cases} \quad (4)$$

$$\text{where } t_d = t_A - \frac{\sqrt{(x_A - x_{RS})^2 + (y_A - y_{RS})^2}}{v_R} \quad (5)$$

where θ_A is the angle from the start point to the attachment point as shown in Fig. 2(a) and t_d is delay time on the start point.

In this approach phase, the robot motion should satisfy two conditions. First, the robot can reach the attachment point at the time t_A. For some values of t_A and ϕ_A, the robot cannot reach $\mathbf{p}_A$ even if it moves with its maximum speed. It is called *reachability condition*. Second, the obstacle and robot should not collide each other, which is called *collision-free condition*.

The reachability condition is satisfied when the distance between the start point and the attachment point is smaller than the maximum distance the robot can move in attachment time t_A (i.e, $\sqrt{(x_A - x_{RS})^2 + (y_A - y_{RS})^2} \le v_R t_A$). It is reduced to

$$(x_{OS} + v_O t_A + R\cos\phi_A - x_{RS})^2 + (R\sin\phi_A - y_{RS})^2 - (v_R t_A)^2 \le 0 \quad (6)$$

The collision-free condition is represented by

$$\|\mathbf{p}_R(t) - \mathbf{p}_O(t)\| \ge R \quad for \quad 0 \le t < t_A \quad (7)$$

If the start point of the robot does not exist in the swept area by the obstacle (i.e., $|y_{RS}| \ge R$), the robot is always collision-free during $0 \le t \le t_d$. Then, (7) becomes

$$\begin{aligned}(x_{RS} + v_R(t - t_d)\cos\theta_A - x_{OS} - v_O t)^2 \\ + (y_{RS} + v_R(t - t_d)\sin\theta_A)^2 - R^2 \ge 0 \quad for \quad t_d \le t < t_A\end{aligned} \quad (8)$$

Let the left side of the inequality be $f_A(t)$, which is a second order polynomial function with a positive coefficient of the leading term. In addition, $f_A(t_A)$ is zero because the robot contacts the obstacle at attachment time t_A. Therefore $f_A(t)$ is always positive in $t < t_A$ if and only if $\dot{f}_A(t_A) \le 0$. The condition (8) is reduced to

$$v_R\cos(\theta_A - \phi_A) - v_O\cos\phi_A \le 0 \quad (9)$$

Note that θ_A is also a function of two parameters, t_A and ϕ_A.

B. Contact Phase

In a contact phase, the robot moves with contacting the obstacle until the detachment time t_D. Its motion is represented as follows

$$\mathbf{p}_R(t) = (x_O(t) + R\cos\phi(t), y_O(t) + R\sin\phi(t)) \quad for \quad t_A \le t \le t_D \quad (10)$$

where $\phi(t_A) = \phi_A$ and $\phi(t_D) = \phi_D + 2n\pi,\ n \in \mathbb{N}$. $\phi(t)$ represents the angle from the origin of the obstacle to the robot position.

In this contact phase, the collision-free condition does not need to be considered because the robot always maintain at the distance of R from the obstacle. On the other hand, the reachability condition should be considered. It should guarantee that the speed of the robot does not exceed the maximum speed v_R while it moves to the detachment point.

$$\|\dot{\mathbf{p}}_R(t)\| \le \mathrm{v_R} \quad for \quad t_A \le t \le t_D \quad (11)$$

It is reduced to

$$\sqrt{{v_O}^2 + R^2\dot{\phi}(t)^2 - 2v_O R\dot{\phi}(t)\sin\phi(t)} \le \mathrm{v_R} \quad for \quad t_A \le t \le t_D \tag{12}$$

The reachability condition of the contact phase is satisfied if and only if there exists a continuous function $\phi(t)$ such that $\phi(t_A) = \phi_A$, $\phi(t_D) = \phi_D + 2n\pi,\ n \in \mathbb{N}$ which satisfies (12)

C. Detachment Phase

In a detachment phase, the robot moves from the detachment point $\mathbf{p}_D$ straightly to the goal with a maximum speed v_R . Its motion is represented as follows.

$$\mathbf{p}_R(t) = (x_D + v_R(t-t_D)\cos\theta_D,\, y_D + v_R(t-t_D)\sin\theta_D) \quad for\ t_D < t \le T \tag{13}$$

where θ_D is the angle from the detachment point to the goal as shown in Fig. 2(a) and T is the arrival time of the robot which is the objective function to be minimized.

$$T = t_D + \frac{\sqrt{(x_{RG}-x_D)^2 + (y_{RG}-y_D)^2}}{v_R} \tag{14}$$

In this detachment phase, the robot and the obstacle should be collision-free. It is represented as follows.

$$\|\mathbf{p}_R(t) - \mathbf{p}_O(t)\| \ge R \quad for \quad t > t_D \tag{15}$$

The case that the robot collides with the obstacle after it arrives at the goal can be excluded if we assume that the goal point does not exist in the swept area by the obstacle (i.e., $|y_{RG}| \ge R$). The collision-free condition becomes

$$\begin{aligned}&(v_O t_D + R\cos\phi_D + v_R(t-t_D)\cos\theta_D - v_O t)^2 \\ &\quad + (R\sin\phi_D + v_R(t-t_D)\sin\theta_D)^2 - R^2 \ge 0 \quad for\ t_D < t \le T\end{aligned} \tag{16}$$

Let the left side of the inequality be $f_D(t)$, which is a second order polynomial function with a positive coefficient of the leading term. Because $f_D(t_D) = 0$, $f_D(t)$ is always positive in $t > t_D$ if and only if $\dot{f}_D(t_D) \ge 0$, which is reduced to

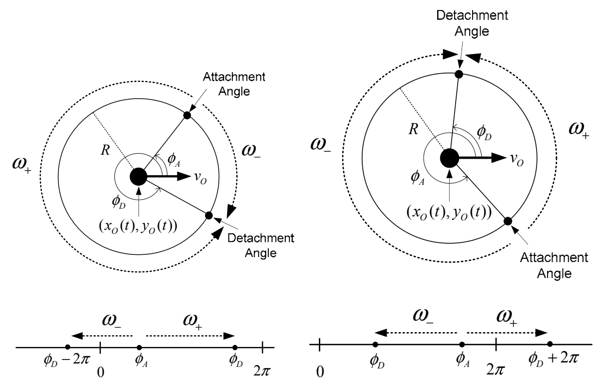
$$v_R\cos(\theta_D - \phi_D) - v_O\cos\phi_D \ge 0 \tag{17}$$

Note that θ_D is also a function of two parameters t_D and ϕ_D .

IV. Approximation of Contact Phase

In a contact phase, we approximate the angle function $\phi(t)$ as the first order Taylor series expansion as follows.

$$\phi(t) = \phi_A + \omega(t - t_A) \tag{18}$$



(a) $\phi_A \le \phi_D$ (b) $\phi_A > \phi_D$

Fig. 3. The angular speed of the contact phase can be selected in two choices, ω_+ and ω_- . While the obstacle moves a constant speed v_O , the robot follows it with the angular speed ω_+ (counterclockwise direction) or ω_- (clockwise direction).

where ω is a angular speed. Actually, ω is selected in two choices, the positive value and the negative value, that make the robot follows the obstacle in a clockwise direction and a counter clockwise direction, respectively. In Fig. 3, these two choices are depicted.

$$\begin{aligned}&\omega_+ = \frac{\phi_D - \phi_A}{t_D - t_A}, \quad \omega_- = \frac{\phi_D - 2\pi - \phi_A}{t_D - t_A} \quad \text{if } \phi_A \le \phi_D \\ &\omega_+ = \frac{\phi_D + 2\pi - \phi_A}{t_D - t_A}, \quad \omega_- = \frac{\phi_D - \phi_A}{t_D - t_A} \quad \text{if } \phi_A > \phi_D\end{aligned} \tag{19}$$

The reachability condition (12) is reduced as follows.

$${v_O}^2 + R^2{\omega_+}^2 - 2v_O R\omega_+\sin(\phi_A + \omega_+(t-t_A)) \le \mathrm{v_R}^2\ for\ t_A \le t \le t_D \tag{20}$$

$${v_O}^2 + R^2{\omega_-}^2 - 2v_O R\omega_-\sin(\phi_A + \omega_-(t-t_A)) \le \mathrm{v_R}^2\ for\ t_A \le t \le t_D \tag{21}$$

If either of condition (20) or (21) is satisfied, then the robot reaches the detachment point following the obstacle with a constant angular speed. Because the left sides of the conditions are a twice continuously differentiable function, the conditions can be represented as inequalities. The condition (20) is reduced to

$$\begin{aligned}&v_O + R\omega_+ - v_R \le 0 \text{ if } \phi_A \le \frac{3\pi}{2} + 2n\pi \le \phi_A + \omega_+(t_D - t_A),\ n = 0 \text{ or } 1 \\ &\begin{cases}{v_O}^2 + R^2{\omega_+}^2 - 2v_O R\omega_+\sin\phi_A - \mathrm{v_R}^2 \le 0 \\ {v_O}^2 + R^2{\omega_+}^2 - 2v_O R\omega_+\sin\phi_D - \mathrm{v_R}^2 \le 0\end{cases} \quad \text{otherwise}\end{aligned} \tag{22}$$

The condition (21) is similarly reduced to

$$\begin{aligned}&v_O - R\omega_- - v_R \le 0 \text{ if } \phi_A + \omega_-(t_D - t_A) \le \frac{\pi}{2} + 2n\pi \le \phi_A,\ n = 0 \text{ or } 1 \\ &\begin{cases}{v_O}^2 + R^2{\omega_-}^2 - 2v_O R\omega_-\sin\phi_A - \mathrm{v_R}^2 \le 0 \\ {v_O}^2 + R^2{\omega_-}^2 - 2v_O R\omega_-\sin\phi_D - \mathrm{v_R}^2 \le 0\end{cases} \quad \text{otherwise}\end{aligned} \tag{23}$$

TABLE I

FORMULATION FOR NEAR-OPTIMAL COLLISION-FREE MOTION PLANNING

Objective	minimize $f(t_D,\phi_D)$	refer to (14)
subject to		
Approach phase		
Reachability	$g_{AR}(t_A,\phi_A)\le 0$	refer to (6)
Collision-free	$g_{AC}(t_A,\phi_A)\le 0$	refer to (9)
Contact phase		
Reachability	$\begin{cases} \text{if } h^0_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \text{ or } h^1_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 & g^M_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \\ \text{otherwise} & \begin{cases} g^A_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \\ g^D_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \end{cases} \end{cases}$	refer to (22)
	or	
	$\begin{cases} \text{if } h^0_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \text{ or } h^1_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 & g^M_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \\ \text{otherwise} & \begin{cases} g^A_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \\ g^D_{CR+}(t_A,\phi_A,t_D,\phi_D)\le 0 \end{cases} \end{cases}$	refer to (23)
Detachment phase		
Collision-free	$g_{DC}(t_D,\phi_D)\le 0$	refer to (17)

where $h^0_{CR+}(t_A,\phi_A,t_D,\phi_D)=(\phi_A-3\pi/2)(\phi_A+\omega_+(t_D-t_A)-3\pi/2)$

$h^1_{CR+}(t_A,\phi_A,t_D,\phi_D)=(\phi_A-7\pi/2)(\phi_A+\omega_+(t_D-t_A)-7\pi/2)$

$h^0_{CR-}(t_A,\phi_A,t_D,\phi_D)=(\phi_A+\omega_-(t_D-t_A)-\pi/2)(\phi_A-\pi/2)$

$h^1_{CR-}(t_A,\phi_A,t_D,\phi_D)=(\phi_A+\omega_-(t_D-t_A)-5\pi/2)(\phi_A-5\pi/2)$

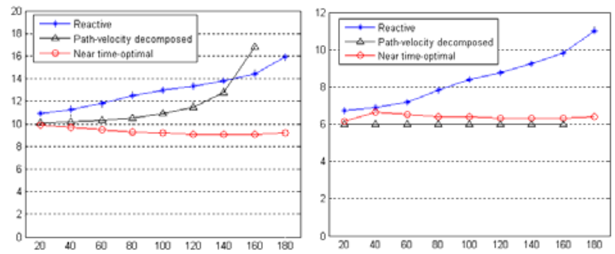
Note. This formulation is for the case when $|y_{RS}|\ge R$ and $|y_{RG}|\ge R$.

By approximating $\phi(t)$ which determines the robot motion of the contact phase, near time-optimal collision-free motion is formulated as shown in Table I. The constraints of the formulation are represented as inequalities. They are for the case when the start and goal of the robot does not exist in the swept area by the obstacle (i.e., $|y_{RS}|\ge R$ and $|y_{RG}|\ge R$). In other case, which rarely occurs, collision-free conditions of the approach phase and contact phase can be used as the forms (7) and (15), the original ones before being reduced.

V. SIMULATION RESULTS

We carried out simulation experiments with three different approaches: reactive, path-velocity decomposed, and near time-optimal approach. We implemented the behavior-based approach as the representative of the reactive approach. It was first introduced by Arkin [1] and widely used for a long period of time. In this approach, when the robot detects an obstacle using embedded sensors, it moves with the velocity of the combination of the attractive vector to the goal and the repulsive vector from obstacles. The path-velocity decomposed approach was implemented by the collision map method [7].

We performed simulations with varying the crossing angle θ between the moving direction of the obstacle and the direction from the start to the goal of the robot. The



(a) Arrival time (b) Path length

Fig. 4. The result of three different approaches with varying the crossing angle θ

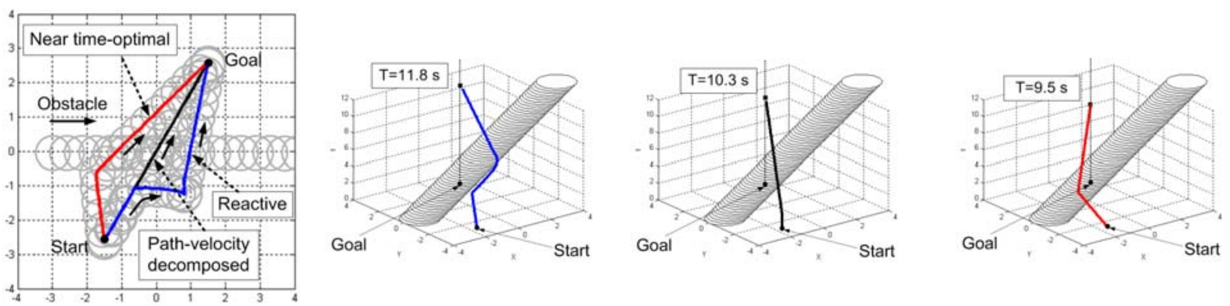
*start: ($-3.00\cos\theta$ m, $-3.00\sin\theta$ m), goal: ($3.00\cos\theta$ m, $3.00\sin\theta$ m)
initial point of obstacle: (-3.00m, 0.00m), speed of obstacle: 0.70m/s
radius of obstacle and robot: 0.50m, maximum speed of robot: 0.70m/s

arrival times and traveled path lengths of the robot are shown in Fig. 4. It is natural that the near-time optimal results have the lowest arrival time among three approaches. The arrival times of the reactive approach and the fixed path approach increased with the crossing angle. The path-velocity decomposed approach did not change the path from the straight line between the start to goal point that is the shortest path. The near-time optimal motion had larger path length than the shortest path length because it purposed the earlier arrival time at the expense of the path length. The reactive approach had the largest path length. Note that there were no results of the path-velocity decomposed approach in the case when the crossing angle is 180°. In this case, the obstacle came from the goal of the robot and the robot could not avoid the collision without changing its path.

Figure 5 and 6 shows the motions of the robot in the case when the crossing angle is 60° and 160°, respectively. In the reactive approach, the robot first moved to the goal until it met the obstacle. When it detected the obstacle it reactively changed the path to avoid obstacle. In the path-velocity decomposed approach, the robot waited that the obstacle first passed by and glanced off the obstacle without changing its path. It is the time-optimal motion along the fixed path, the shortest one. In the near time-optimal result, the robot first moved obliquely from the direction of the goal in order to arrive in earlier time.

VI. DISCUSSION

The near time-optimal motion was efficient, compared to results of the other methods as shown in Fig. 4. It is useful especially for the case when the crossing angle is large, i.e., the robot and the obstacle move towards each other. In Fig. 4(a), no significant changes occurred in the near-optimal arrival time with varying the crossing angle while the other approaches yield inefficient results as the crossing angle increases. In Fig. 6(a) and (b), the robot retraced after it detects the obstacle. It caused to increase not only the arrival time but also the path length. This phenomenon is the



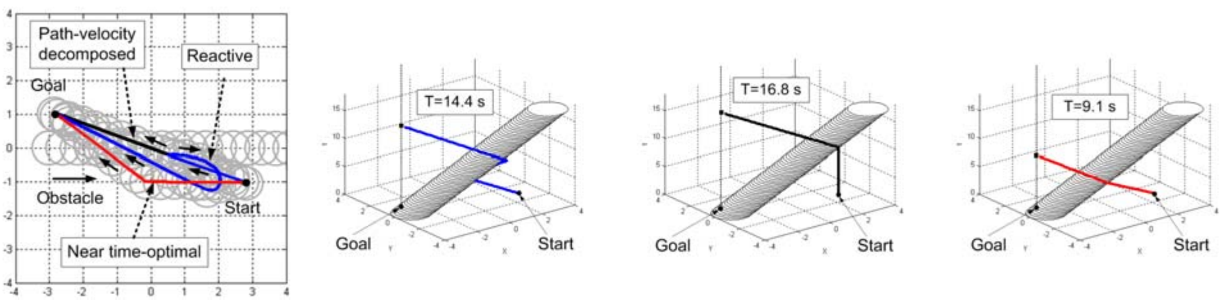
(a) Paths of each approach result

(b) motion of reactive approach result represented in C-T space

(c) motion of path-velocity decomposed approach result represented in C-T space

(d) motion of near time-optimal result represented in C-T space

Fig. 5. Simulation results in the case when the crossing angle is 60°



(a) Paths of each approach result

(b) motion of reactive approach result represented in C-T space

(c) motion of path-velocity decomposed approach result represented in C-T space

(d) motion of near time-optimal result represented in C-T space

Fig. 6. Simulation results in the case when the crossing angle is 160°

inevitable limitation of short-term planning. The reactive approach is inappropriate when the obstacle motion is known a priori. In Fig 6(a) and (c), the robot waited for a long time in order to let the obstacle go first. In this case, the path-velocity decomposed approach yields significantly inefficient result even if it finds the time-optimal result in the fixed path. In Fig. 6(a) and (d), the robot arrived in much earlier time by moving a long way around.

Even though the time-optimal motion is useful, it has not been widely used or studied because of its computational complexity. We formulated the near-time optimal problem with four parameters and reduced the constraints to a few inequalities. That is, the motion planning problem becomes the optimization problem in which various optimization techniques can be applied.

VII. CONCLUSIONS

In this paper, we presented a method to find the near time-optimal motion to cope with a moving obstacle. It is long-term planning with considering the path and velocity simultaneously, so it yielded uniformly efficient results in various cases. The computational burden for solving this problem was reduced by formulating it as the optimization problem with inequality constraints of four parameters.

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