Andrew Sweeting: Dynamic Product Positioning

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Motivation

- Sweeting formulates a dynamic investment and repositioning game between radio stations that are choosing formats (for multiple stations in a given market) and whether or not to be active.
- Radio station owners (firms) make money from selling ads (derived from heterogenous listening behavior of different demographic groups), and may enjoy scope economies from owning many stations.
  - There are many rivals in each market, a ton of formats and a lot of demographic variables that influence listening behavior.
  - Key state variables include station formats and characteristics, market prices, and demographics.
- Challenge: need **very large** state space to capture the features of the market that are likely to affect how industry evolves.
- Solution: CCP estimation + approximate dynamic programming

Context

- Radio stations make money by selling ads that play between songs (or other content).
    - The legislation proposed that for music stations with revenues above some cap, fees would be determined as a percentage of advertising revenues, and would not depend on the exact amount of music that the station played.
- However, stations that provide primarily talk programming (or sports) were exempt.
    - Therefore, radio stations might be prompted to switch formats (to non-music).
- Tension is that music formats are **much** more popular, especially with lucrative demographic groups $\rightarrow$ a reason not to switch

Two Questions

- Why dynamic model?
  1. Analyzing variety requires model with differentiated products.
  2. If firms must pay significant sunk costs to develop new products or reposition existing ones, you a dynamic model to predict policy change will impact industry evolution.
- Why radio? Several nice features...
  1. The industry has many local markets, each with its own local stations.
  2. Distinct and well-defined programming categories (formats) facilitate discrete choice approach.
  3. Excess demand for station licenses in most markets (spectrum constraints) $\rightarrow$ can ignore entry/exit decisions and focus on repositioning across formats.

Data

- Data for 102 local radio markets (U.S. cities), observed bi-annually, 2002-2005
  - 2,375 stations and 16,566 station-market period observations.
- Choices: Arbitron's estimates of station audience shares, based on listening by people aged 12 and above, in the spring and fall of each year.
  - Turn audience shares into market shares for estimating demand by assuming that every person in the market could listen to the radio for up to 6 hours per day (average listening is around 2.5 hours per day) and by using Arbitron's estimate of how many people listened to the radio in a given market-quarter.
- Revenues: BIAfn estimates (proprietary formula) for 96% of station-years in the sample between 2002 and 2004.
- Demographics: Measure each market's population in 18 demo groups, product of 3 age categories (12-24, 25-49, and 50 plus), 3 ethnic categories (black, white, and Hispanic) and gender (U.S. Census).

Data: Summary Statistics

TABLE I

FORMAT AGGREGATION[a]

Aggregated Format (* = music format)	BIAfn Format Categories	Number of Station-Qtrs FM	AM	Mean Audience Share	% of Listeners in Demographic Male	12–24	25–49	Black	Hisp.
1. AC/CHR*	Adult Contemporary, Contemporary Hit Radio	3,299	47	5.3%	39.1	28.4	49.8	13.1	15.4
2. Rock, AOR/Classic Rock*	Rock, AOR/Classic Rock	2,597	6	4.7%	70.5	25.6	63.0	2.3	8.9
3. Country*	Country	1,799	223	6.3%	47.4	15.1	44.2	2.4	5.0
4. Urban*	Urban, Gospel	1,172	609	4.5%	43.5	29.3	50.2	83.6	5.0
5. News/Talk	News, Talk, Sports	267	2,656	3.2%	60.7	3.7	38.1	7.5	5.1
6. Spanish	Spanish-language	562	460	2.2%	51.4	20.0	59.9	0.8	91.3
7. Other Programming	Oldies, Easy Listening, Variety, Classical, Jazz, Big Band, Religious (non-Gospel)	1,873	911	3.2%	47.1	6.8	36.4	9.9	10.2

[a]Demographic percentages based on Arbitron's *Radio Today* publications 2003–2006 and author's calculations.

- Aggregate BIAfn categories into seven active formats
- Assume that stations in the first four formats would have to pay fees under new rules.

Data: Summary Statistics

TABLE II

SUMMARY STATISTICS

Variable	Observations (market-periods)	Mean	Std. Dev.	Min.	Max.
Market Characteristics					
Number of stations in market	714	23.2	5.8	11	38
Number of different station owners in market	714	9.4	3.2	3	18
Population (12 and above, in millions)	714	0.73	0.75	0.06	3.62
Proportion of population black	714	0.11	0.12	0.002	0.340
Proportion of population Hispanic	714	0.09	0.10	0.005	0.583
Combined listening to sample stations (% of total market[a])	714	35.0	3.3	22.8	45.9
BIAfn estimated annual market advertising revenues (2002–2005, \$ m.)	408	53.5	66.0	3.9	403.7
	Observations (station-periods)				
Station Characteristics (note: statistics exclude Dark stations)					
Market share	16,481	1.5%	1.3%	0.0%	10.5%
BIAfn Estimated advertising revenues (station-year 2002–2004, \$ m.)	6,413	2.4	3.7	0.003	45.6
Dummy for station located outside market boundaries	16,481	0.06	0.24	0	1
Dummy for AM band	16,481	0.30	0.46	0	1
Proportion of market covered by signal (out of market stations excluded)[b]	15,464	0.79	0.36	0.002	1.1
Dummy for station that has imputed market share in at least one quarter	16,481	0.14	0.35	0	1
Dummy for station that switches formats before next quarter	14,120	0.032	0.18	0	1
Market share of a switching station	454	1.0%	0.9%	0.0%	6.3%

[a] The total market includes the outside good of not listening to radio and listening to noncommercial and commercial nonsample stations. Market definition allows for each individual to spend up to 6 hours listening to the radio between 6am and midnight.

[b] Signal coverage is defined relative to the market population and I cap it at 1.1 to address outliers that appeared to distort the demand estimates.

Overview

- Ericson and Pakes (1995) style dynamic game in which firms
  1. Own a portfolio of radio stations across potentially several formats
  2. Earn advertising revenues based on listener shares from different demo groups (that differ in value)
  3. Can choose to re-position up to one station each period (mostly in response to changing demos)
  4. Pay a cost to do so, but may exploit scope economies from aligning portfolio
- Need to
  1. Specify demand/revenue system (how firms/stations earn money)
  2. Formulate game structure
  3. Develop solution and estimation approaches

Set Up

- Radio station owners (firms) o in markets m play an infinite horizon discrete time game over periods t.
- Markets have exogenous characteristics X_{mt} (demographics, population) that evolve as AR(1) process.
- A firm o owns a set of stations S^o, with differing characteristics, both observed X_{st} and unobserved ξ_{st}, as well as formats F_{st}.
- Full collection of state variables is denoted $\mathcal{M}_{jot}$.

List of State Variables

TABLE A-I

STATE VARIABLES

	Evolution	Information?	Observed?
Market Variables			
Population in $d = 1, \ldots, 18$ mutually exclusive groups	Ethnic group size evolves with growth rates	Public	Observed
Growth rates for black, Hispanic and white populations	AR(1), i.i.d. innovations	Public	Observed, AR(1) process estimated
Advertising prices per listener	Fixed	Public	Estimated
Station Variables (for firm's own stations)			
Format	Changes with choice	Public	Observed
Changed Format in Previous Period	Changes with choice	Public	Observed
Station Quality:			
observed characteristic component (e.g., based on signal coverage, station location)	Fixed	Public	Characteristics observed, quality coefficients estimated
band	Fixed	Public	Observed (interaction with format estimated)
ξ_{st}	AR(1), i.i.d. innovations	Public	Implied by demand estimates AR(1) process estimated
Station Variables (for each station owned by competitors)			
Owner	Fixed	Public	Observed
Format	Changes with choice	Public	Observed
Changed Format in Previous Period	Changes with choice	Public	Observed
Station Quality:			
observed characteristic component (e.g., based on signal coverage, station location)	Fixed	Public	Characteristics observed, quality coefficients estimated
band	Fixed	Public	Observed (interaction with format estimated)
ξ_{st}	AR(1), i.i.d. innovations	Public	Estimated
Choice-Specific Payoff Shocks			
ε for each choice for each firm	i.i.d. across firms, choice & time	Private	No, scale estimated

Timing

3.2. *Timing*

Within each period t, the timing of the game is as follows:

1. each firm o observes the current state, $\mathcal{M}_{j,o,t}$;

2. each firm o pays fixed costs for each of its active stations. The cost of operating a station is reduced by θ^C when a station operates another station in the same format, creating a total cost saving of $C(\mathcal{M}_{j,o,t})\theta^C$, where $C(\mathcal{M}_{j,o,t})$ is simply a count of how many stations it operates in formats where it has multiple stations. Given my specification of repositioning costs, only θ^C, and not the level of fixed costs, is identified, so I will proceed treating fixed costs as equal to zero;

3. each firm o observes the private information shocks ε_{ot} to its format choices, and makes its format choice a_{ot};

4. each firm receives advertising revenues $\sum_{s \in S^o} R_s(\mathcal{M}_{j,o,t}|\gamma)$, where γ are the parameters of the demand and revenue models, pays repositioning costs $W_o(a_{ot})\theta^W$, and receives the payoff shock $\varepsilon_{ot}(a_{ot})$;

5. $\mathcal{M}_{j,o,t}$ evolves to the state in the next period, reflecting firms' format choices, and the stochastic evolution of station qualities and the growth rates of the white, black, and Hispanic populations.

Product Market Competition (Listener Demand)

- Stations make money by selling ads, the price of which depend on listener shares.
- Key idea: allow different demographic groups to have heterogeneous programming tastes, and for advertisers to place different values on listeners with different demographics
  - Because stations in the same market-format can have quite different market shares, allow differences to be explained by several observable variables (AM-band, signal coverage, etc.)
- Demand (listener shares) derived from listening utility, given by

$$u_{ist} = \gamma_i^R + X_{st}\gamma^S + F_{st}(\bar{\gamma}^F + \gamma_D^F D_i) + \xi_{st} + \varepsilon_{ist}^L \tag{1}$$

or more compactly

$$u_{ist} = \delta_{st}(F_{st}, X_{st}, \gamma^S, \bar{\gamma}^F, \xi_{st}) + \gamma_i^R + F_{st}\gamma_D^F D_i + \varepsilon_{ist}^L \tag{2}$$

- Estimation (fitting listener shares) follows NFXP approach of Berry et al. (1995).

Demand: Revenues per Listener

- Listener utility determines shares, but revenue comes from advertising.
- The advertising revenue that a station s receives for a listener with demographics D_d is given by
$$r_{st}(D_d) = \gamma_m(1 + Y_{st}\gamma^Y)(1 + D_d\gamma_d)$$
where γ_m is a market-year fixed effect and γ_d allows advertiser valuations to vary with listener demographics.
- Estimation matches observed to predicted revenues via NLLS.
- Y includes number of other stations that firm has in same format, number of other stations in same format, and a dummy for format switch in previous period.
- Total station revenues $R_{st}(\mathcal{M}_{jot}|\gamma)$ are then the sum, over the 18 demographic groups, of the number of listeners multiplied by these prices.
- The number of listeners from each group comes from the demand system.

Per Period (Flow) Profits

- The flow profit of firm o in period t can be written

$$\pi_{ot}(a_{ot}, \mathcal{M}_{jot}, \theta, \gamma) + \theta^{\varepsilon}\varepsilon_{ot}(a_{ot}) = \sum_{s \in S_o} R(\mathcal{M}_{jot}, \gamma) + \beta C_o(a_{ot})\theta^C - W_o(a_{ot})\theta^W + \theta^{\varepsilon}\varepsilon_{ot}(a_{ot}) \tag{3}$$

  where $\beta C_o(a_{ot})\theta^C$ captures scope economies (reduced fixed costs) and $W_o(a_{ot})\theta^W$ is the repositioning cost.
- Key parameters of interest are γ's (static) and θ's (dynamic)
  - Static parameters can be recovered from demand system and revenue regression
  - Dynamic parameters inferred from actions $\rightarrow$ need to account for future implications (continuation values)

Value Functions

- Assume firms play stationary MPE, with strategies $\Gamma_o : (\mathcal{M}_{jot}, \varepsilon_{ot}) \rightarrow a_{ot}$.
- Firm o's value in state $(\mathcal{M}_{jot}, \varepsilon_{ot})$ when it employs an an optimal strategy (and all rivals follow Γ) is given by:

$$
\begin{aligned}
& V_o^{\Gamma}(\mathcal{M}_{jot}, \varepsilon_{ot}) \\
& \qquad = \max_{a \in A_o(\mathcal{M}_{jot})} \Big[\pi(a, \mathcal{M}_{jot}) + \theta^{\varepsilon} \varepsilon_{ot}(a) \\
& \qquad + \beta \int \bar{V}_o^{\Gamma}(\mathcal{M}_{jot+1}) g(\mathcal{M}_{jot+1} | a, \Gamma_{-o}, \mathcal{M}_{jot}) d\mathcal{M}_{jot+1} \Big]
\end{aligned}
\tag{4}
$$

where $g(\cdot)$ is transition kernel (given a and Γ_{-o}) and $\bar{V}_o^{\Gamma}(\mathcal{M}_{jot+1})$ is the integrated value function (IVF)

$$\bar{V}_o^{\Gamma}(\mathcal{M}_{jot}) = \int V_o^{\Gamma}(\mathcal{M}_{jot}, \varepsilon_{ot}) f(\varepsilon_{ot}) d\varepsilon_{ot} \tag{5}$$

CCPs, CSVFs and IVFs

- T1EV ε's provide analytic solutions for CCPs (strategies)

$$P^{\Gamma_o}(a, \mathcal{M}_{jot}, \Gamma_{-o}) = \frac{\exp\left(\frac{v_o^{\Gamma}(a, \mathcal{M}_{jot}, \Gamma_{-o})}{\theta^{\varepsilon}}\right)}{\sum_{a' \in A_o(\mathcal{M}_{jot})} \exp\left(\frac{v_o^{\Gamma}(a', \mathcal{M}_{jot}, \Gamma_{-o})}{\theta^{\varepsilon}}\right)} \tag{6}$$

where $v_o^{\Gamma}(a, \mathcal{M}_{jot}, \Gamma_{-o})$ is the choice specific VF (CSVF)

$$\begin{aligned} & v_o^{\Gamma}(a, \mathcal{M}_{jot}, \Gamma_{-o}) \\ & \qquad = \pi(a, \mathcal{M}_{jot}) \\ & \qquad + \beta \int \bar{V}_o^{\Gamma}(\mathcal{M}_{jot+1}) g(\mathcal{M}_{jot+1} | a, \Gamma_{-o}, \mathcal{M}_{jot}) d\mathcal{M}_{jot+1} \end{aligned} \tag{7}$$

Estimation Approach

- Estimation matches observed choices to those in data

$$P^{\Gamma_o}(a, \mathcal{M}_{jot}, \Gamma_{-o}) = \frac{\exp\left(\frac{v_o^{\Gamma}(a,\mathcal{M}_{jot},\Gamma_{-o})}{\theta^{\varepsilon}}\right)}{\sum_{a' \in A_o(\mathcal{M}_{jot})} \exp\left(\frac{v_o^{\Gamma}(a',\mathcal{M}_{jot},\Gamma_{-o})}{\theta^{\varepsilon}}\right)} \tag{8}$$

- Key challenges
  - Computing v_f^{Γ}, integrating over next period's states, continuous states/massive state space.
- Solution
  - CCP estimation paired with a parametric policy iteration (Benitez-Silva et al., 2000) implementation of an approximate dynamic programming approach (Bellman et al., 1963; Judd, 1998; Keane and Wolpin, 1994)

Estimation via Forward Simulation

- Before going through the PPI-ADP approach, let's see how you would estimate the model using more "conventional" methods.
- A powerful approach to estimating dynamic games with a rich state space involves approximating the value function via forward simulation based on initial first-stage estimates of the conditional choice probabilities (Hotz et al., 1994).
- Here, we are essentially constructing the NPV directly, using the more "cumbersome" version of the Bellman equation.
- Estimation is based on the inequalities, or moment inequalities, that are implied by the equilibrium assumption that each firm's actual strategy, reflected in its estimated conditional choice probabilities, should result in a higher value than any alternative, given the strategies of other players.
- Sweeting considers two approaches: one based on Bajari et al. (2007) (BBL) and the other on Pakes et al. (2007) and Pakes et al. (2015) (POB, PPHI).

Estimation via Forward Simulation

As a firm's payoffs are linear in the parameters, a firm's value when it uses strategy Γ_o and other firms use strategies Γ^*_{-o} can be expressed as

$$\begin{aligned} V_o(\mathcal{M}_{j,o,t} | \Gamma_o, \Gamma^*_{-o}, \theta) \\ &= \mathbf{V}_{o,\Gamma_o,\Gamma^*_{-o}} \theta \\ &= \mathbf{R}_{o,\Gamma_o,\Gamma^*_{-o}} - \theta^W \mathbf{W}_{o,\Gamma_o,\Gamma^*_{-o}} + \theta^C \mathbf{C}_{o,\Gamma_o,\Gamma^*_{-o}} + \theta^\varepsilon \boldsymbol{\varepsilon}^F_{o,\Gamma_o,\Gamma^*_{-o}}, \end{aligned}$$

where $\mathbf{R}_{o,\Gamma_o,\Gamma^*_{-o}} = E_{o,\Gamma_o,\Gamma^*_{-o}} \sum_{t'=0}^{\infty} \beta^{t'} \sum_{s \in S^o} R_s(\mathcal{M}_{o,t+t'} | \gamma)$, the expected discounted sum of future revenues, and $\mathcal{M}_{o,t+t'}$ is o's state at time $t+t'$. $\mathbf{W}$ (expected discounted repositioning costs), $\mathbf{C}$ (economies of scope), and $\boldsymbol{\varepsilon}$ are defined similarly. The equilibrium restrictions used in estimation are that

$$(20) \qquad V_o(\mathcal{M}_{j,o,t} | \Gamma^*_o, \Gamma^*_{-o}, \theta) - V_o(\mathcal{M}_{j,o,t} | \Gamma^a_o, \Gamma^*_{-o}, \theta) \geq 0 \quad \forall \Gamma^a_o, \mathcal{M}_{j,o,t},$$

where Γ^* are equilibrium strategies and Γ^a_o is a particular alternative policy.

Estimation via Forward Simulation

Empirical implementation involves constructing estimates of **R**, **W**, **C**, and $\boldsymbol{\varepsilon}$ based on observed policies (i.e., first-stage estimates of the conditional choice probabilities, and the demand and revenue models) and a finite number of alternatives (the ones used are detailed in the Appendix) using forward simulation. I consider two estimators of the parameters: the one proposed by BBL,

$$(21) \qquad \widehat{\theta^{\mathrm{BBL}}} = \arg\min_{\theta} \sum_{o} \sum_{\forall a} \max\{(\mathbf{V}_{o,\Gamma_o^a,\Gamma_{-o}^*} - \mathbf{V}_{o,\Gamma_o^*,\Gamma_{-o}^*})\theta, 0\}^2,$$

and one that follows the moment-inequality approach of PPHI which finds the set of parameters that satisfy the following linear moment inequalities:

$$(22) \qquad \widehat{\theta^{\mathrm{PPHI}}} \text{ is the set of } \theta \quad \text{where} \quad \frac{1}{O}\sum_{o}(\mathbf{V}_{o,\Gamma_o^*,\Gamma_{-o}^*} - \mathbf{V}_{o,\Gamma_o^a,\Gamma_{-o}^*})\theta \geq 0 \quad \forall \Gamma_o^a.$$

Estimation via Forward Simulation

- The difference between these estimators is that the PPHI estimator uses the equilibrium implication that observed policies should do better **on average** whereas the BBL estimator uses the fact that the equilibrium implies that they should do better **in every state**.
- In a similar way to POB's comparison between a moment-based entry rate matching estimator and a likelihood-based estimator, the PPHI estimator sacrifices information.
- Because of averaging, it may be more robust to approximation errors arising from either forward simulation or inaccuracies in the first-stage choice probabilities that result from either specification bias (a parametric specification is imposed) or the finite sample.

Role of Parametric Policy Iteration

- Okay, back to the PPI-based ADP approach.
- Why is it helpful here?
    - A key issue is that BBL is an *estimation* technique.
    - It will not help you *at all* when it comes time to do counterfactuals.
- As such, Sweeting also employs a PPI-based estimator that connects more seamlessly with the underlying DDP.
- However, it is an *approximate* approach, so it may not perform well.
    - So Sweeting compares all three (Really. I am not joking!)
- I will first present the solution approach, and then show how it is used in estimation.

Value Function Approximation

- Assume value function can be approximated by a parametric linear function of K functions (ϕ) of the state variables

$$V_o^{P^*}(\mathcal{M}_{jot}) \approx \sum_{k=1}^{K} \lambda_k \phi_{ko}(\mathcal{M}_{jot})$$

- Solving the value function now requires finding K λ coefficients rather than N values.

Value Function Approximation

- Stacking the equations for each of the N selected states in matrix form, the following equations should hold for equilibrium strategies P^*:

$$\Phi\lambda = \tilde{\pi}(P^*) + \beta E_{P^*}\Phi\lambda$$

  where Φ is the matrix of the functions of the state variables and $E_{P^*}\Phi$ is a matrix with element (j, k):

$$E_{P^*}\Phi_{j,k} = \int \phi_{ko}(\mathcal{M}_{hot+1})g(\mathcal{M}_{hot+1}|P^*, \mathcal{M}_{jot})d\mathcal{M}_{hot+1}$$

- Nice trick: for over-identified case $(N > K)$, $\hat{\lambda}^P$ can be computed by OLS.

$$\widehat{\lambda^P} = \left((\Phi - \beta E_P\Phi)'(\Phi - \beta E_P\Phi)\right)^{-1}(\Phi - \beta E_P\Phi)'\widetilde{\pi}(P).$$

Parametric Policy Iteration (for solving game)

The *parametric policy iteration* procedure (Benitez-Silva et al. (2000)) now consists of iterating several steps. Before the procedure begins, I calculate Φ for the N selected states, which include the observed states and duplicates of these states where the features that can vary over time are perturbed. The variables in the approximation include measures of revenues, firm and rival characteristics, and opportunities to increase revenues by switching formats. For the counterfactual, the model is solved market-by-market, so no restrictions that the approximating function has to be the same across markets are imposed. For an iteration i, the following steps are used:

1. calculate $\widetilde{\pi}(P^i)$ and $E_{P^i}\Phi$;
2. create matrices $(\Phi - \beta E_{P^i}\Phi)$ and use (17) to calculate $\widehat{\lambda^{P^i}}$;
3. use $\widehat{\lambda^{P^i}}$ to calculate the choice-specific value functions for each choice for each firm and the multinomial logit formula (9) to calculate updated probabilities P';
4. if the maximum absolute difference between P' and P^i is sufficiently small (I use a tolerance of 1e−6), the procedure stops and $\widehat{\lambda^{P^i}}$ is saved as λ^*; otherwise, $P^{i+1} = \psi P' + (1-\psi)P^i$, and iteration $i+1$ begins at step 1. I use $\psi = 0.1$.

This procedure solves for the conditional choice probabilities in the observed states (and the chosen duplicates). However, to perform the counterfactual, I need to simulate the model forward to states that will not have been included in this selection. Therefore, in each future period, λ^* is used to solve for equilibrium choice probabilities. Full details of this procedure are in Appendix B of the Supplemental Material.

Estimation via PPI

- Ok, now on to estimation!
- Estimation of the model proceeds in two main stages.
- The first stage involves estimation of (i) the listener demand model and the process for the unobserved (ξ) component of station quality; (ii) the revenue function; (iii) an initial guess of firms' conditional choice probabilities; and (iv) the transition process governing demographics.
- The preferred estimates come from an estimator that combines parametric approximation of the value function with a pseudo-likelihood procedure that follows Aguirregabiria and Mira (2007).
- This procedure follows a similar iterative procedure to the one used to solve the model, with an added estimation step, but the choice probabilities of other firms (P_{-o}) are held constant at initial first-stage estimates.

Estimation nests the solution (for the focal firm) inside a PML routine

In the iterated procedure, the following steps are followed in iteration i, where the current guess of the structural parameters is θ^i and the current guess of o's choice probabilities is P_o^i (see Appendix C of the Supplemental Material for full details):

1. calculate $\widetilde{\pi}(P_o^i, \theta^i)$ and $E_{P^i}\Phi$;
2. create matrices $(\Phi - \beta E_{P^i}\Phi)$ and use (17) to calculate $\lambda^i\widehat{(\theta^i, P_o^i)}$;
3. use $\lambda^i\widehat{(\theta^i, P_o^i)}$ to approximate the future value from making each choice a:

$$\begin{aligned} \text{(18)} \qquad & \mathrm{FV}(a, \mathcal{M}_{j,o,t}, P_o^i, P_{-o}) \\ & = \sum_{k=1}^{K} \left\{ \left(\int \phi_{ko}(\mathcal{M}_{h,o,t+1}) g(\mathcal{M}_{h,o,t+1} | P_o^i, P_{-o}, \mathcal{M}_{j,o,t}, a)\, d\mathcal{M}_{h,o,t+1} \right) \right. \\ & \quad \left. \times \lambda_k^i\widehat{(\theta^i, P_o^i)} \right\} \end{aligned}$$

(this is not quite the same as the choice-specific value function defined before, which also included current revenue and repositioning costs associated with action a);

4. estimate the structural parameters θ' using a pseudo-likelihood estimator where the probability that an action a is chosen is

$$\text{(19)} \qquad \frac{\exp\left(\frac{\mathrm{FV}(a, \mathcal{M}_{j,o,t}, P_o^i, P_{-o}) - W_o(a)\theta^{W\prime} - \beta C_o(a)\theta^{C\prime}}{\theta^{\varepsilon\prime}} \right)}{\sum_{a' \in A_{ot}} \exp\left(\frac{\mathrm{FV}(a, \mathcal{M}_{j,o,t}, P_o^i, P_{-o}) - W_o(a')\theta^{W\prime} - \beta C_o(a)\theta^{C\prime}}{\theta^{\varepsilon\prime}} \right)},$$

where current revenues drop out because they are common across choices.

5. if the maximum absolute difference between θ' and θ^i and between P_o' (based on (19)) and P_o^i is less than 1e−4, the procedure stops. Otherwise, the algorithm returns to step 1 using $P_o^{i+1} = \psi P_o' + (1-\psi)P_o^i$ and $\theta^{i+1} = \psi\theta' + (1-\psi)\theta^i$ where $\psi = 0.1$.

Standard errors are calculated using a bootstrap where markets are resampled (20 replications).

Results: Taste Parameters

TABLE III

ESTIMATES OF FORMAT TASTE PARAMETERS[a]

	Mean Tastes	γ^σ	Age 25–49	Age 50 plus	Female	Black	Hispanic
Radio Listening	−15.8753	16.7749	2.1404	7.7839	2.8197	3.6171	2.8563
	(0.9224)	(1.3946)	(0.2046)	(0.7247)	(0.2098)	(0.3303)	(0.4419)
Format Interactions (AC/CHR excluded)							
Rock	0.6575	–	0.2813	−0.4054	−1.2581	−1.8814	−0.8157
	(0.0580)		(0.0087)	(0.0158)	(0.0043)	(0.0292)	(0.0402)
Country	−0.1187	–	0.4847	1.3039	−0.4428	−1.9726	−1.2733
	(0.0646)		(0.0095)	(0.0175)	(0.0046)	(0.0330)	(0.0524)
Urban	−1.2040	–	−0.7373	−1.0188	−0.3996	3.9375	0.6158
	(0.0832)		(0.0393)	(0.0497)	(0.0086)	(0.0406)	(0.0501)
News/Talk	−1.2918	–	1.6979	3.1485	−1.1171	−0.7935	−1.1071
	(0.1443)		(0.0080)	(0.0144)	(0.0049)	(0.0275)	(0.0385)
Other Programming	−0.9883	–	1.0958	2.4600	−0.5384	−0.4204	−0.2916
	(0.0650)		(0.0079)	(0.0150)	(0.0049)	(0.0275)	(0.0385)
Spanish	−2.7945	–	1.0300	1.1111	−0.3649	−0.5138	3.9489
	(0.1955)		(0.0264)	(0.0506)	(0.0163)	(0.1519)	(0.1694)

[a] 16,481 observations, GMM objective function 2.90e−12, standard errors in parentheses. Mean tastes will reflect valuations of a white male aged 12–24.

Quality Parameters

TABLE IV

ESTIMATES OF STATION QUALITY PARAMETERS[a]

AM * AC/CHR or Rock	−0.7781
	(0.3174)
AM * Country	−0.4538
	(0.1776)
AM * Urban	−0.3523
	(0.1296)
AM * News/Talk	−0.0806
	(0.1658)
AM * Other	−0.3811
	(0.1196)
AM * Spanish	−0.2714
	(0.1593)
Signal Coverage	1.5938
(for stations located in the market)	(0.1004)
FM * Signal Coverage	0.6057
	(0.1068)
Small Station Dummy	−0.4277
(shares imputed for some quarters)	(0.0580)
Out of Market Dummy	−0.5082
	(0.0834)
Transition Process for Unobserved Quality	
ρ^{ξ}	0.8421
	(0.0058)
$\sigma_{\nu^{\xi}}$	0.3132
	(0.0020)
Effect of Format Switch on Unobserved Quality	−0.0501
	(0.0103)

[a] 16,481 observations, GMM objective function 2.90e−12. Time coefficients not reported. Std. errors in parentheses. AM * AC/CHR and *Rock combined due to small number of observations.

Revenue Parameters

TABLE V

PARAMETER ESTIMATES FOR THE REVENUE FUNCTION[a]

	(1)	(2)
Demographics		
Female	0.1797	0.1917
	(0.0368)	(0.0374)
Age 12–24	−0.5811	−0.5883
	(0.1075)	(0.1084)
Age 50+	−0.4531	−0.4572
	(0.0577)	(0.0581)
Black	−0.1964	−0.1961
	(0.0148)	(0.0155)
Hispanic	−0.1593	−0.1596
	(0.0159)	(0.0159)
Station Characteristics and Competition		
Number of stations with same owner in format	–	0.0064
		(0.0047)
Number of other stations in format	–	−0.0019
		(0.0018)
Format switch in previous quarter	–	−0.1045
		(0.0279)
R^2 (compared to a model with only market-year fixed effects)	0.3182	0.3208

[a]4,483 annual station observations (observations with imputed shares excluded). Market-year coefficients not reported. Standard errors corrected for imprecision in the demand parameters.

Dynamic Parameter Estimates

TABLE VI

PARAMETER ESTIMATES FROM THE DYNAMIC MODEL

Specification	(1) Modified Procedure P-Likelihood	(2) Modified Procedure Moments	(3) Iterated Procedure P-Likelihood
Costs of Move to Active Format ($ m.)			
Markets with pop. 1 m. +	2.524	1.897	1.467
	(0.380)	(0.441)	(0.409)
* Recent Format Switch	−0.001	−0.001	−0.000
	(0.098)	(0.110)	(0.065)
Markets with pop. 0.25–1 m.	0.669	0.446	0.388
	(0.157)	(0.167)	(0.102)
* Recent Format Switch	0.077	−0.213	−0.044
	(0.082)	(0.145)	(0.067)
Markets with pop. < 0.25 m.	0.233	0.069	0.135
	(0.099)	(0.048)	(0.063)
* Recent Format Switch	0.030	−0.0310	−0.017
	(0.023)	(0.105)	(0.054)
* Revenue of Switching Station (all markets)	0.034	0.116	0.022
	(0.168)	(0.211)	(0.118)
Additional Cost of Move to Active From Dark ($ m.)			
Markets with pop. 1 m. +	−0.501	−0.445	−0.291
	(0.220)	(0.183)	(0.095)
Markets with pop. 0.25–1 m.	−0.255	−0.495	−0.147
	(0.166)	(0.098)	(0.069)
Markets with pop. < 0.25 m.	−0.061	−0.078	−0.035
	(0.091)	(0.032)	(0.042)

Dynamic Parameter Estimates

TABLE VI

PARAMETER ESTIMATES FROM THE DYNAMIC MODEL

Specification	(1) Modified Procedure P-Likelihood	(2) Modified Procedure Moments	(3) Iterated Procedure P-Likelihood
Cost of Moving From Active to Dark ($ m.)			
Markets with pop. 1 m. +	2.704	3.126	1.572
	(0.744)	(0.964)	(0.503)
Markets with pop. 0.25–1 m.	0.636	1.636	0.369
	(0.230)	(0.582)	(0.130)
Markets with pop. $<$ 0.25 m.	0.199	0.654	0.115
	(0.054)	(0.322)	(0.022)
Revenue of Switching Station	1.108	3.105	0.642
	(1.330)	(1.506)	(1.003)
Economies of Scope			
Markets with pop. 1 m. +	0.134	−0.046	0.078
	(0.099)	(0.072)	(0.075)
Markets with pop. 0.25–1 m.	0.026	0.102	0.015
	(0.018)	(0.021)	(0.010)
Markets with pop. $<$ 0.25 m.	−0.006	−0.014	−0.033
	(0.012)	(0.010)	(0.029)
Scale of ε's			
Markets with pop. 1 m. +	0.517	0.408	0.300
	(0.180)	(0.063)	(0.087)
Markets with pop. 0.25–1 m.	0.144	0.091	0.083
	(0.055)	(0.037)	(0.026)
Markets with pop. $<$ 0.25 m.	0.050	0.014	0.028
	(0.014)	(0.007)	(0.009)

Alternative Estimation Aproaches

- To assess robustness and accuracy of approximation approach, Sweeting also estimates a (simplified) version of the model using two forward simulation approaches (BBL and Moment Inequalities (PPHI)).
- The MI approach yields bounds, while the BBL approach provides point estimates.

TABLE VII

ESTIMATES BASED ON FORWARD SIMULATION

	Repositioning Cost	Scope Economy	Scale of Payoff Shock
Markets With Pop. > 1 m.			
Moment inequality bounds (PPHI)	[2.194, 11.031]	[−0.081, 0.056]	[0.074, 1.729]
95% CI	[0.652, 13.877]	[−0.126, 0.102]	[0.011, 1.793]
BBL point estimate	18.668	0.337	3.771
std. error	(1.765)	(0.048)	(0.342)
proportion of inequalities violated	24.4%	–	–
Markets With Pop. 0.25 m.–1 m.			
Moment inequality bounds (PPHI)	[0.464, 3.421]	[−0.071, 0.031]	[0.015, 0.549]
95% CI	[0.232, 4.035]	[−0.082, 0.042]	[0, 0.568]
BBL point estimate	3.046	0.013	0.630
std. error	(0.190)	(0.006)	(0.043)
proportion of inequalities violated	10.0%	–	–
Markets With Pop. < 0.25 m.			
Moment inequality bounds (PPHI)	[0.230, 1.541]	[−0.022, 0.008]	[0.005, 0.251]
95% CI	[0.081, 1.690]	[−0.27, 0.014]	[0, 0.258]
BBL point estimate	2.148	0.011	0.455
std. error	(0.242)	(0.004)	(0.051)
% of BBL inequalities violated	22.4%	–	–

Counterfactuals

- Finally, Sweeting performs counterfactuals aimed at understanding the potential impact of the proposed fee legislation on format choices.
- He assumes fees were imposed as an unanticipated shock in Fall 2004, and, after re-solving model, simulates markets forward 40 periods from that date.
- He presents two sets of results.
  1. What would happen to the number of music stations and music audiences with no fees, 10% fees, and 20% fees based on the estimates in column (1) of Table VI.
  2. How the effects of a 10% fee vary with some of the model's parameters, such as the level of repositioning costs and the degree of heterogeneity in listener tastes.

Counterfactuals

TABLE VIII

EVOLUTION OF THE NUMBER OF MUSIC STATIONS UNDER DIFFERENT PERFORMANCE RIGHTS FEES[a]

	Music Stations			Music Listening			Nonmusic Listening		
Fee Level:	0%	10%	20%	0%	10%	20%	0%	10%	20%
Period prior to introduction (Fall 2004)	713	713	713	0.254	0.254	0.254	0.095	0.095	0.095
+1 period	714	701	693	0.254	0.250	0.246	0.094	0.097	0.101
	(3.7)	(4.6)	(4.6)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.004)
+5 periods	715	682	626	0.253	0.243	0.228	0.094	0.103	0.110
	(6.1)	(7.2)	(7.1)	(0.003)	(0.003)	(0.005)	(0.002)	(0.003)	(0.004)
+10 periods (5 years)	717	665	595	0.253	0.238	0.220	0.095	0.106	0.116
	(8.0)	(9.8)	(9.8)	(0.004)	(0.004)	(0.006)	(0.003)	(0.003)	(0.005)
+20 periods (10 years)	716	651	582	0.252	0.237	0.220	0.096	0.106	0.117
	(10.1)	(11.5)	(11.8)	(0.004)	(0.005)	(0.006)	(0.003)	(0.003)	(0.003)
+40 periods (20 years)	720	652	578	0.253	0.237	0.219	0.096	0.107	0.118
	(10.4)	(10.9)	(12.0)	(0.004)	(0.005)	(0.005)	(0.003)	(0.004)	(0.004)

[a]Standard deviations across 10 simulations in parentheses. Results based on sample of 51 markets. Music listening measured as the average combined market share of music stations across markets.

Counterfactuals

TABLE IX

EVOLUTION OF THE NUMBER OF MUSIC STATIONS AND MUSIC AUDIENCES UNDER 10% FEES FOR DIFFERENT ASSUMPTIONS ON THE STRUCTURAL PARAMETERS RELATIVE TO EVOLUTION WITH NO FEES (FALL 2004 INDEXED TO 1)[a]

	True Parameters ("Base case")		Less Taste Heterogeneity		Higher Repositioning Costs		Higher Repositioning Costs and High σ^{ε}	
Column:	(1) Stations	(2) Music Audiences	(3) Stations	(4) Music Audiences	(5) Stations	(6) Music Audiences	(7) Stations	(8) Music Audiences
Fall 2004	1	1	1	1	1	1	1	1
+1 period	0.981	0.984	0.941	0.950	0.986	0.992	0.986	0.993
	(0.006)	(0.005)	(0.010)	(0.012)	(0.002)	(0.003)	(0.004)	(0.002)
+5 periods	0.954	0.961	0.875	0.895	0.967	0.978	0.971	0.980
	(0.010)	(0.009)	(0.014)	(0.013)	(0.006)	(0.006)	(0.008)	(0.008)
+10 periods	0.930	0.941	0.810	0.851	0.940	0.959	0.952	0.964
(5 years)	(0.015)	(0.010)	(0.017)	(0.015)	(0.010)	(0.009)	(0.014)	(0.012)
+40 periods	0.906	0.937	0.801	0.841	0.910	0.942	0.918	0.945
(20 years)	(0.014)	(0.010)	(0.018)	(0.016)	(0.011)	(0.009)	(0.016)	(0.009)

[a] Results based on sample of 51 markets. Standard deviations across 10 simulations in parentheses. Music listening measured as the average combined market share of music stations across markets.

References

Aguirregabiria, V. and P. Mira (2007). Sequential estimation of dynamic discrete games. *Econometrica 75*(1), 1–53.

Bajari, P., C. L. Benkard, and J. Levin (2007). Estimating dynamic models of imperfect competition. *Econometrica 75*(5), 1331–1370.

Bellman, R., R. Kalaba, and B. Kotkin (1963). Polynomial approximation–a new computational technique in dynamic programming: Allocation processes. *Mathematics of Computation 17*(82), 155–161.

Benitez-Silva, H., G. Hall, G. J. Hitsch, G. Pauletto, and J. Rust (2000). A comparison of discrete and parametric approximation methods for continuous-state dynamic programming problems. Working Paper: Yale University.

Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile prices in market equilibrium. *Econometrica 63*(4), 841–890.

Ericson, R. and A. Pakes (1995). Markov-perfect industry dynamics: A framework for empirical work. *Review of Economic Studies 62*(1), 53–82.

Hotz, V. J., R. A. Miller, S. Sanders, and J. Smith (1994). A simulation estimator for dynamic models of discrete choice. *Review of Economic Studies 61*(2), 265–289.

Judd, K. L. (1998). *Numerical methods in economics*. MIT press.

Keane, M. P. and K. I. Wolpin (1994). The solution and estimation of discrete choice dynamic programming models by simulation and interpolation: Monte Carlo evidence. *Review of Economics and Statistics 76*(4), 648–672.

Pakes, A., M. Ostrovsky, and S. Berry (2007). Simple estimators for the parameters of discrete dynamic games (with entry/exit examples). *The RAND Journal of Economics 38*(2), 373–399.

Pakes, A., J. Porter, K. Ho, and J. Ishii (2015). Moment inequalities and their application. *Econometrica 83*(1), 315–334.