Universidad Rey Juan Carlos

Proyecto Tesis Doctoral

Relación entre el espectro de señales cardiacas y la dinámica espacio temporal de las fuentes cardiacas

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Resumen

inicio del resumen

Abstract

In this Doctoral Dissertation .....

Índice general

Resumen I

Abstract III

Acrónimos y abreviaturas XI

1. Introducción 1
- 1.1. Motivación y Estado del Arte . . . 1
- 1.2. Objetivos . . . 1
- 1.3. Metodología . . . 1
- 1.4. Estructura . . . 2

I Métodos 3

2. Modelos Bioeléctricos 5
- 2.1. introducción . . . 5
  - 2.1.1. Anatomía y fisiología de las celular . . . 5
- 2.2. Modelo Electrofisiología . . . 5
  - 2.2.1. Modelo Celular . . . 5
  - 2.2.2. Modelo de Tejido . . . 5
- 2.3. Sistemas de medida . . . 5
  - 2.3.1. sistema de electrodos . . . 6
  - 2.3.2. Volumen equivalente de sistema de electrodos . . . 6
- 2.4. Conclusiones . . . 6

3. Definición del Espectro 7
- 3.1. Introducción . . . 7
  - 3.1.1. Señal . . . 7
  - 3.1.2. Vector . . . 7

3.1.3. Campo Vectorial 7
3.2. Autocorrelación 7
3.2.1. Autocorrelation and spectrum of cardiac sources 7
3.2.2. Autocorrelation and spectrum of cardiac signals 9
3.3. Métodos estimación del espectro 10
3.4. Frecuencia Dominante 10
3.5. Concluciones 10

II Análisis y simulaciones 11

4. Análisis 13
4.1. Introducción 13
4.2. Modelos de fuentes de 2 orden 14
4.3. Función de distribución de tiempos 15
4.4. Lead Fields 15
4.4.1. idealized model 15
4.5. Resolución Espacial vs Ancho de Banda 16

5. Simulaciones 17
5.1. introducción 17
5.1.1. tablas de experimentos 18
5.2. Implementación de modelos 18
5.2.1. Modelo de ruido blanco 18
5.2.2. Modelo Autómata 18
5.3. Modelos de leadfield 18
5.4. Resolución espacial del sistemas de electrodos 18
5.5. Relación ancho de banda 18
5.6. concluciones 18

III Discusión y Conclusiones 19

6. Conclusiones 21

IV Apéndices 23

Bibliografía 27

Índice de figuras

Índice de tablas

Acrónimos y abreviaturas

A lo largo de este documento se mantendrán en su forma original aquellos acrónimos derivados de una expresión inglesa cuyo uso se encuentre extendido en la literatura científica.

De acuerdo con las recomendaciones de la Real Academia Española, en esta Tesis los acrónimos y siglas no se modifican para formar el plural.

DF Frecuencia Dominante.

LF Lead Field.

Capítulo 1

Introducción

Resumen: La presente Proyecto Tesis Doctoral tiene como pilares el estudio de los efectos espectrales en sistemas de electrodos mediante ... procesados digital de señales .....

1.1. Motivación y Estado del Arte

el mapeo de Frecuencia Dominante (DF) ...

1.2. Objetivos

En esta Proyecto Tesis Doctoral se analiza y simula, el efecto espectral del sistema de electrodos,(Lead Field (LF)), en el registro señales eléctricas, ...

1.3. Metodología

Esta Proyecto Tesis Doctoral , se enmarca en el análisis matemático del espectro, junto con los modelos bioeléctricos. así mismo, las simulaciones, se sustentan en la implementación numérica de los modelos bioeléctricos y la estimación espectral de las diversas dinámicas cardiacas.

1.4. Estructura

Esta Proyecto Tesis Doctoral , se estructura en tres partes, que agrupan 5 capítulos. la Parte 1 consta de ... la Parte 2 consta de ... la Parte 3 consta de ...

Parte I

Métodos

Capítulo 2

Modelos Bioeléctricos

Resumen:

.....

2.1. introducción

2.1.1. Anatomía y fisiología de las celular

algo de acá http://www.bem.fi/book/, Malmivuo and Plonsey (1995)

2.2. Modelo Electrofisiología

bibliografia Sachse (2004) para el modelado del corazon, sin embargo falta ve el modelado de la actividad celular ejemplo neurociencia

2.2.1. Modelo Celular

2.2.2. Modelo de Tejido

2.3. Sistemas de medida

algo de acá http://www.bem.fi/book/ Malmivuo and Plonsey (1995)

Cardiac sources are the bioelectric processes generated by the heart during contraction. There exist different, equivalent mathematical paradigms to model the activity of cardiac sources, such as the monopole field and the

dipole field Malmivuo and Plonsey (1995). In this study, we model cardiac sources as a time-varying dipole field, i.e. as a spatial distribution of time-varying dipoles $\mathbf{J}(v,t) = [J_x(v,t), J_y(v,t), J_z(v,t)]^T$ on a volume V, where v denotes a point located inside V and t denotes the time instant.

The time-varying activity of cardiac sources can be measured by lead systems, producing cardiac signals. Taking the dipole field as our reference description for cardiac sources, we follow a lead-field approach to model cardiac signals Malmivuo and Plonsey (1995). According to the lead-field theory, the cardiac signal $c(t)$ that is induced at a lead system by a dipole field $\mathbf{J}(v,t)$ can be expressed as

$$c(t) = \int_V \mathbf{L}^T(v)\mathbf{J}(v,t)dv \tag{2.1}$$

where the vector field $\mathbf{L}(v) = [L_x(v), L_y(v), L_z(v)]^T$ is the measurement sensitivity distribution (MSD) and describes the ability of the l ead system to measure cardiac dipoles located at $v \in V$. In words, cardiac si gnals are a weighted linear combination of the underlying cardiac sources.

2.3.1. sistema de electrodos

definición de las ecuaciones lapalaciano

2.3.2. Volumen equivalente de sistema de electrodos

LF

2.4. Conclusiones

Capítulo 3

Definición del Espectro

Resumen: In this section we present the mathematical formalism for investigating the spectrum of cardiac signals. Firstly, we introduce the lead-field bioelectric model of cardiac sources and signals. Then, based on multivariate signal analysis, we define the autocorrelation and the spectrum of cardiac sources.

Throughout this paper the following notation is used: $\langle\cdot\rangle_t$ denotes time-average, $\mathcal{F}[\cdot]$ is the Fourier Transform operator, $(*)$ denotes convolution and $\delta(\cdot)$ is the Dirac's delta. We use the following vector definitions: $\mathbf{1} = [1,1,1]^T$ and $\mathbf{0} = [0,0,0]^T$.

3.1. Introducción

3.1.1. Señal

3.1.2. Vector

3.1.3. Campo Vectorial

3.2. Autocorrelación

3.2.1. Autocorrelation and spectrum of cardiac sources

The autocorrelation of a cardiac source, $\boldsymbol{\rho}(v,w,\tau)$, $\forall v,w \in V$, is defined as the collection of the cross-correlations between all pairs of dipoles in V. Since cardiac dipoles are vectorial entities, the cross-correlation between two dipoles consists of the cross-correlations between all three components of each

dipole Papoulis (1991). In order to define the autocorrelation of a cardiac source, the average dipole field $\bar{\mathbf{J}}(v)$ needs to be introduced:

$$\begin{aligned} \bar{\mathbf{J}}(v) &= \langle \mathbf{J}(v,t) \rangle_t \\ &= [\langle J_x(v,t) \rangle_t, \langle J_y(v,t) \rangle_t, \langle J_z(v,t) \rangle_t]^T. \end{aligned} \tag{3.1}$$

Based on $\bar{\mathbf{J}}(v)$, we define the zero-average dipole field $\mathbf{J}'(v,t) = \mathbf{J}(v,t) - \bar{\mathbf{J}}(v)$, so that $\langle \mathbf{J}'(v,t) \rangle_t = \mathbf{0}$. The cross-correlation matrix between two cardiac dipoles $\mathbf{J}(v,t)$ and $\mathbf{J}(w,t)$, where $v, w \in V$, is then defined as

$$\begin{aligned} \boldsymbol{\rho}(v,w,\tau) &= \langle \mathbf{J}'(v,t+\tau) \mathbf{J}'^T(w,t) \rangle_t \\ &= \begin{pmatrix} \rho_{xx}(v,w,\tau) & \rho_{xy}(v,w,\tau) & \rho_{xz}(v,w,\tau) \\ \rho_{yx}(v,w,\tau) & \rho_{yy}(v,w,\tau) & \rho_{yz}(v,w,\tau) \\ \rho_{zx}(v,w,\tau) & \rho_{zy}(v,w,\tau) & \rho_{zz}(v,w,\tau) \end{pmatrix}. \end{aligned} \tag{3.2}$$

Therefore, each entry of $\boldsymbol{\rho}(v,w,\tau)$ contains the cross-correlation between one component of $\mathbf{J}(v,t)$ and one component of $\mathbf{J}(w,t)$. For instance, matrix entry $\rho_{zy}(v,w,\tau)$ is $\langle J'_z(v,t+\tau) J'_y(w,t) \rangle_t$. Also, the average power of dipole component $J'_x(v,t)$ is by definition $P_x(v) = \rho_{xx}(v,v,0)$, and analogous expressions can be obtained for the average power of dipole components $J'_y(v,t)$ and $J'_z(v,t)$.

The spectrum of a cardiac source, $\boldsymbol{\sigma}(v,w,f)$, $\forall v, w \in V$, corresponds to the collection of the cross-spectra between all pairs of dipoles in V, and is defined as

$$\begin{aligned} \boldsymbol{\sigma}(v,w,f) &= \mathcal{F}[\boldsymbol{\rho}(v,w,\tau)] \\ &= \begin{pmatrix} \sigma_{xx}(v,w,f) & \sigma_{xy}(v,w,f) & \sigma_{xz}(v,w,f) \\ \sigma_{yx}(v,w,f) & \sigma_{yy}(v,w,f) & \sigma_{yz}(v,w,f) \\ \sigma_{zx}(v,w,f) & \sigma_{zy}(v,w,f) & \sigma_{zz}(v,w,f) \end{pmatrix} \end{aligned} \tag{3.3}$$

where the operator $\mathcal{F}[\cdot]$ is applied to $\boldsymbol{\rho}(v,w,\tau)$ on a component-by-component basis. For instance, $\sigma_{zy}(v,w,f)$ is $\mathcal{F}[\rho_{zy}(v,w,\tau)]$.

We also define the *total cross-correlation* $R_J(v,w,\tau)$ between two cardiac dipoles $\mathbf{J}(v,t)$ and $\mathbf{J}(w,t)$ as the sum of the entries of $\boldsymbol{\rho}(v,w,\tau)$ and the *total cross-spectrum* $S_J(v,w,f)$ as the Fourier Transform of $R_J(v,w,\tau)$. Mathematically, they can be expressed as

$$R_J(v,w,\tau) = \mathbf{1}^T \boldsymbol{\rho}(v,w,\tau) \mathbf{1}, \tag{3.4}$$

$$S_J(v,w,f) = \mathbf{1}^T \boldsymbol{\sigma}(v,w,f) \mathbf{1}. \tag{3.5}$$

Finally, we define the *normalized* cross-correlation $\hat{\boldsymbol{\rho}}(v, w, \tau)$ as the matrix of entries

$$\hat{\rho}_{ab}(v, w, \tau) = \frac{\rho_{ab}(v, w, \tau)}{\sqrt{\rho_{aa}(v, v, 0)\rho_{bb}(w, w, 0)}} \tag{3.6}$$

where $a, b \in \{x, y, z\}$, and the *normalized* total cross-correlation $\hat{R}_J(v, w, \tau)$ as

$$\hat{R}_J(v, w, \tau) = \frac{R_J(v, w, \tau)}{\underset{\tau}{\text{máx}}\{R_J(v, w, \tau)\}}. \tag{3.7}$$

3.2.2. Autocorrelation and spectrum of cardiac signals

Let $c(t)$ be a cardiac signal measured by applying $\mathbf{L}(v)$ to a cardiac source of autocorrelation $\boldsymbol{\rho}(v, w, \tau)$ and spectrum $\boldsymbol{\sigma}(v, w, f)$ in V. Define $c'(t)$ as the cardiac signal $c(t)$ minus its time-average value $\bar{c} = \langle c(t) \rangle_t$,

$$c'(t) = c(t) - \bar{c}. \tag{3.8}$$

The autocorrelation function $R_c(\tau)$ of the cardiac signal $c(t)$ is defined as the following average Papoulis (1991):

$$R_c(\tau) = \langle c'(t + \tau)c'(t) \rangle_t, \tag{3.9}$$

and its power spectrum $S_c(f)$ is defined as the Fourier Transform of its autocorrelation function,

$$S_c(f) = \mathcal{F}[R_c(\tau)]. \tag{3.10}$$

Based on (2.1), the following relationships can be derived between $R_c(\tau)$ and $\boldsymbol{\rho}(v, w, \tau)$, and betwen $S_c(f)$ and $\boldsymbol{\sigma}(v, w, f)$ (see Appendix A):

$$R_c(\tau) = \int_{V \times V} \mathbf{L}^T(v)\boldsymbol{\rho}(v, w, \tau)\mathbf{L}(w)dvdw, \tag{3.11}$$

$$S_c(f) = \int_{V \times V} \mathbf{L}^T(v)\boldsymbol{\sigma}(v, w, f)\mathbf{L}(w)dvdw. \tag{3.12}$$

Equations (3.11) and (3.12) reflect the linear relationship between cardiac signals and sources [c.f. (2.1)], and can be used to gain insight into the nature of the autocorrelation and spectrum of cardiac signals. According to (3.12), two factors determine the spectrum of cardiac signals. The first factor is the spectrum $\boldsymbol{\sigma}(v, w, f)$ of cardiac sources. It is worth noting that this is the solely feature of the spatiotemporal dynamics of cardiac sources that manifests on the spectrum of cardiac signals. The second factor is the MSD of the lead system, $\mathbf{L}(v)$. Since $\mathbf{L}(v)$ is specific for each lead system, (3.12) reveals that cardiac signals measured by different lead systems will in general have different spectra for the same underlying spatiotemporal dynamics.

3.3. Métodos estimación del espectro

3.4. Frecuencia Dominante

3.5. Concluciones

Parte II

Análisis y simulaciones

Capítulo 4

Análisis

Resumen: In this section we study analytically the spectral manifestation of two second-order models of cardiac sources, namely the fully correlated (FC) source and the fully uncorrelated (FU) source. The FC and FU models are physiologically meaningful and can be used to describe the dynamics of, respectively, highly organized and highly disorganized cardiac rhythms.

We firstly define the autocorrelation and the spectrum of the following models of spatiotemporal dynamics: identically distributed (ID), FC and FU. The ID model is introduced for facilitating the comparison of the spectrum of cardiac signals measured during FC and FU dynamics. Secondly, we define a simple, idealized model of MSD, namely the pulse model. Because of its simplicity, the pulse model is used in the analytical derivations and in the simulation experiments throughout this study. Finally, we derive analytically the spectrum of cardiac signals measured by pulse MSD during FU and FC dynamics.

4.1. Introducción

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4.2. Modelos de fuentes de 2 orden

4.2.0.1. Identically distributed spatiotemporal dynamics

In this model of spatiotemporal dynamics all cardiac dipoles have the same autocorrelation and spectrum,

$$\boldsymbol{\rho}(v, v, \tau) = \boldsymbol{\rho}(\tau), \tag{4.1}$$
$$\boldsymbol{\sigma}(v, v, f) = \boldsymbol{\sigma}(f). \tag{4.2}$$

By substituting (4.1) and (4.2) respectively into (3.4) and (3.5), it can be proved that the total autocorrelation and total spectrum of all the dipoles are also identical,

$$R_J(v, v, \tau) = R_J(\tau) = \mathbf{1}^T \boldsymbol{\rho}(\tau)\mathbf{1}, \tag{4.3}$$
$$S_J(v, v, f) = S_J(f) = \mathbf{1}^T \boldsymbol{\sigma}(f)\mathbf{1}. \tag{4.4}$$

Note that this model only describes the activity of cardiac dipoles individually, and does not specify $\boldsymbol{\rho}(v, w, \tau)$ nor $\boldsymbol{\sigma}(v, w, f)$ for $v \neq w$.

4.2.0.2. Fully correlated spatiotemporal dynamics

This model of spatiotemporal dynamics corresponds to highly regular rhythms, such as sinus rhythm, in which the activity of one dipole $\mathbf{J}(w, t)$ can be expressed as a delayed version of the activity of another dipole $\mathbf{J}(v, t)$,

$$\mathbf{J}(w, t) = \mathbf{J}(v, t - \zeta(v, w)), \tag{4.5}$$

where $\zeta(v, w)$ is defined as the time delay between the activities of dipoles $\mathbf{J}(v, t)$ and $\mathbf{J}(w, t)$. Based on (4.5), it can be proved (see Appendix B) that FC dynamics are also ID, $\boldsymbol{\rho}(v, v, \tau) = \boldsymbol{\rho}(\tau)$ and $\boldsymbol{\sigma}(v, v, f) = \boldsymbol{\sigma}(f)$ [cf. (4.1) and (4.2)], and that the autocorrelation and the spectrum of FC sources can be expressed as:

$$\boldsymbol{\rho}(v, w, \tau) = \boldsymbol{\rho}(\tau - \zeta(v, w)), \tag{4.6}$$
$$\boldsymbol{\sigma}(v, w, f) = \boldsymbol{\sigma}(f) \exp[-j2\pi f \zeta(v, w)]. \tag{4.7}$$

Consequently, FC sources are completely characterized by $\boldsymbol{\rho}(\tau)$, $\boldsymbol{\sigma}(f)$ and $\zeta(v, w)$.

4.2.0.3. Fully uncorrelated spatiotemporal dynamics

This model of spatiotemporal dynamics constitutes an idealization of highly irregular and disorganized rhythms, such as fibrillation, in which there is no second-order relationship between the temporal activity of any pair of cardiac dipoles. The autocorrelation and the spectrum of a FU source are defined as

$$\boldsymbol{\rho}(v, w, \tau) = \boldsymbol{\rho}(v, v, \tau)\delta(v - w), \tag{4.8}$$
$$\boldsymbol{\sigma}(v, w, \tau) = \boldsymbol{\sigma}(v, v, \tau)\delta(v - w). \tag{4.9}$$

In words, the cross-correlation between two cardiac dipoles $\mathbf{J}(w, t)$ and $\mathbf{J}(v, t)$, where $v \neq w$, is null.

4.3. Función de distribución de tiempos

4.4. Lead Fields

4.4.1. idealized model

in this section we define one simple, idealized model of MSD, namely the pulse model. The pulse model describes a lead system that measures with the same sensitivity every dipole within a region V_0 of the volume source V, while rejecting the rest. Mathematically, this model is defined as

$$\mathbf{L}_{V_0}(v) = \begin{cases} \mathbf{1} & \text{if } v \in V_0 \\ \mathbf{0} & \text{otherwise} \end{cases} . \tag{4.10}$$

The pulse model can be treated as an approximation of physical MSD that effectively concentrate their measurement in a region V_0.

For the subsequent analysis it is also convenient to quantify the spatial resolution (SR) of pulse leads. The SR can be defined as the region of the cardiac source that contributes the most to the measured signal. In this study we quantify the SR of pulse leads by introducing the notion of the lead equivalent volume (LEV), which is defined as the relative size of V_0 to the size of V,

$$LEV = \frac{\int_{V_0} dv}{\int_V dv} = \frac{M_{V_0}}{M_V}, \tag{4.11}$$

where M_{V_0} and M_V are the sizes of V_0 and V respectively. Thus, for local measurements the LEV is close to zero, whereas for global measurements where $V_0 \simeq V$, the LEV is close to one.

4.5. Resolución Espacial vs Ancho de Banda

we identify the relationship between: the spectrum of cardiac signals, the spatiotemporal dynamics of cardiac sources and the measurement characteristics of lead systems

Capítulo 5

Simulaciones

Resumen: .....

5.1. introducción

el objetivo es generar dinamicas controladas, con respecto al nivel de correlación, ver mapas de correlación

5.1.1. tablas de experimentos

5.2. Implementación de modelos

5.2.1. Modelo de ruido blanco

5.2.2. Modelo Autómata

5.2.2.1. protocolos de estimulacón

5.3. Modelos de leadfield

5.4. Resolución espacial del sistemas de electrodos

5.5. Relación ancho de banda

5.6. concluciones

Parte III

Discusión y Conclusiones

Capítulo 6

Conclusiones

Parte IV

Apéndices

SSSSS

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