Aufgabe 1

$$
\begin{aligned}
T_l^-(P_0) =&(l_2, \underline{\exists x_{post}}.\exists y_{post}.\exists z_{post}.\exists t.t \geq 0 \wedge y_{post} \geq 60 \wedge 20z_{post} > y_{post} \wedge \underline{x_{post} = x + t} \wedge y_{post} = y + t \\
&\wedge z_{post} = z) \\
=&(l_2, \underline{\exists y_{post}}.\exists z_{post}.\exists t.t \geq 0 \wedge \underline{y_{post} \geq 60} \wedge \underline{20z_{post} > y_{post}} \wedge \underline{y_{post} = y + t} \wedge z_{post} = z) \\
=&(l_2, \underline{\exists z_{post}}.\exists t.t \geq 0 \wedge \underline{20z_{post} > 60} \wedge \underline{20z_{post} > y + t} \wedge y + t \geq 60 \wedge \underline{z_{post} = z}) \\
=&(l_2, \underline{\exists t.t \geq 0} \wedge 20z > 60 \wedge \underline{20z > y + t} \wedge \underline{y + t \geq 60}) \\
=&(l_2, 20z > 60 \wedge 20z > y \wedge 20z - y \geq 60 - y) \\
=&(l_2, 20z > 60 \wedge 20z > y)
\end{aligned}
$$

$$
\begin{aligned}
P_1 =&D_e^-(T_l^-(P_0)) \\
=&(l_1, \underline{\exists x_{post}}.\exists y_{post}.\exists z_{post}.20z_{post} > y_{post} \wedge 20z_{post} > 60 \wedge \underline{x_{post} = 0} \wedge y = y_{post} \wedge z = z_{post} \\
&\wedge x \leq 1) \\
=&(l_1, \underline{\exists y_{post}}.\exists z_{post}.\underline{20z_{post} > y_{post}} \wedge 20z_{post} > 60 \wedge \underline{y = y_{post}} \wedge z = z_{post} \wedge x \leq 1) \\
=&(l_1, \underline{\exists z_{post}}.\underline{20z_{post} > y} \wedge \underline{20z_{post} > 60} \wedge \underline{z = z_{post}} \wedge x \leq 1) \\
=&(l_1, 20z > y \wedge 20z > 60 \wedge x \leq 0)
\end{aligned}
$$

$$
\begin{aligned}
T_l^-(P_1) =&(l_1, \underline{\exists x_{post}}.\exists y_{post}.\exists z_{post}.\exists t.t \geq 0 \wedge 20z_{post} > y_{post} \wedge 20z_{post} > 60 \wedge \underline{x_{post} \leq 1} \wedge \underline{x_{post} = x + t} \\
&\wedge y_{post} = y + t \wedge z_{post} = z + t \wedge x \leq 1) \\
=&(l_1, \underline{\exists y_{post}}.\exists z_{post}.\exists t.t \geq 0 \wedge \underline{20z_{post} > y_{post}} \wedge 20z_{post} > 60 \wedge x + t \leq 1 \wedge \underline{y_{post} = y + t} \\
&\wedge z_{post} = z + t \wedge x \leq 1) \\
=&(l_1, \underline{\exists z_{post}}.\exists t.t \geq 0 \wedge \underline{20z_{post} > y + t} \wedge \underline{20z_{post} > 60} \wedge x + t \leq 1 \wedge \underline{z_{post} = z + t} \wedge x \leq 1) \\
=&(l_1, \underline{\exists t.t \geq 0} \wedge \underline{20z + 20t > y + t} \wedge \underline{20z + 20t > 60} \wedge \underline{t \leq 1 - x} \wedge x \leq 1) \\
=&(l_1, 1 - x \geq 0 \wedge 20z + 19 - 19x > y \wedge 20z + 20 - 20x > 60 \wedge x \leq 1) \\
=&(l_1, 1 \geq x \wedge 20z + 19 > y + 19x \wedge z > 2 + x)
\end{aligned}
$$

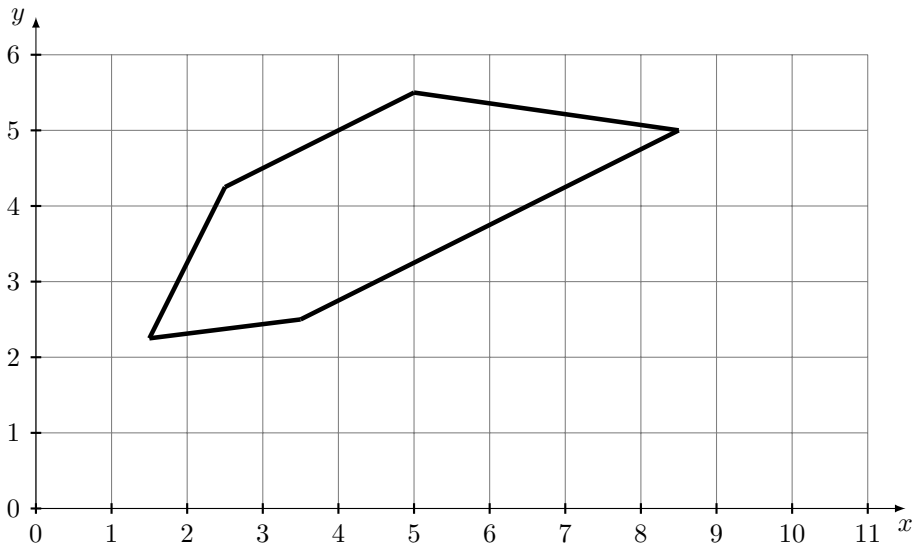
$$
\begin{aligned}
P_2 =&D_e^-(T_l^-(P_1)) \\
=&(l_2, \exists x_{post}.\underline{\exists y_{post}}.\exists z_{post}.1 \geq x_{post} \wedge \underline{20z_{post} + 19 > y_{post} + 19x_{post}} \wedge z_{post} > 2 + x_{post} \wedge x \geq 30 \\
&\wedge x_{post} = 0 \wedge \underline{y_{post} = y} \wedge z_{post} = z) \\
=&(l_2, \exists x_{post}.\underline{\exists z_{post}}.1 \geq x_{post} \wedge \underline{20z_{post} + 19 > y + 19x_{post}} \wedge \underline{z_{post} > 2 + x_{post}} \wedge x \geq 30 \wedge x_{post} = 0 \\
&\wedge \underline{z_{post} = z}) \\
=&(l_2, \underline{\exists x_{post}}.\underline{1 \geq x_{post}} \wedge \underline{20z + 19 > y + 19x_{post}} \wedge \underline{z > 2 + x_{post}} \wedge x \geq 30 \wedge \underline{x_{post} = 0}) \\
=&(l_2, 1 \geq 0 \wedge 20z + 19 > y \wedge z > 2 \wedge x \geq 30) \\
=&(l_2, 20z + 19 > y \wedge z > 2 \wedge x \geq 30)
\end{aligned}
$$

$$
\begin{aligned}
T_l^-(P_2) =&(l_2, \underline{\exists x_{post}}.\exists y_{post}.\exists z_{post}.\exists t.t \geq 0 \wedge 20z_{post} + 19 > y_{post} \wedge z_{post} > 2 \wedge \underline{x_{post} \geq 30} \wedge \underline{x_{post} = x + t} \\
&\wedge y_{post} = y + t \wedge z_{post} = z) \\
=&(l_2, \underline{\exists y_{post}}.\exists z_{post}.\exists t.t \geq 0 \wedge \underline{20z_{post} + 19 > y_{post}} \wedge z_{post} > 2 \wedge x + t \geq 30 \wedge \underline{y_{post} = y + t} \\
&\wedge z_{post} = z) \\
=&(l_2, \underline{\exists z_{post}}.\exists t.t \geq 0 \wedge \underline{20z_{post} + 19 > y + t} \wedge \underline{z_{post} > 2} \wedge x + t \geq 30 \wedge \underline{z_{post} = z}) \\
=&(l_2, \underline{\exists t}.\underline{t \geq 0} \wedge \underline{20z + 19 > y + t} \wedge z > 2 \wedge \underline{x + t \geq 30}) \\
=&(l_2, 20z + 19 > y \wedge z > 2 \wedge 19 + 20z + x \geq y + 30) \\
=&(l_2, 20z + 19 > y \wedge z > 2 \wedge 20z + x \geq y + 11)
\end{aligned}
$$

$$
\begin{aligned}
P_3 =&D_e^-(T_l^-(P_2)) \\
=&(l_1, \underline{\exists x_{post}}.\exists y_{post}.\exists z_{post}.20z_{post} + 19 > y_{post} \wedge z_{post} > 2 \wedge \underline{20z_{post} + x_{post} \geq y_{post} + 11} \\
&\wedge \underline{x_{post} = 0} \wedge y = y_{post} \wedge z = z_{post} \wedge x \leq 1) \\
=&(l_1, \underline{\exists y_{post}}.\exists z_{post}.\underline{20z_{post} + 19 > y_{post}} \wedge z_{post} > 2 \wedge \underline{20z_{post} \geq y_{post} + 11} \wedge \underline{y = y_{post}} \\
&\wedge z = z_{post} \wedge x \leq 1) \\
=&(l_1, \underline{\exists z_{post}}.\underline{20z_{post} + 19 > y} \wedge \underline{z_{post} > 2} \wedge \underline{20z_{post} \geq y + 11} \wedge \underline{z = z_{post}} \wedge x \leq 1) \\
=&(l_1, 20z + 19 > y \wedge z > 2 \wedge 20z \geq y + 11 \wedge x \leq 1) \\
=&(l_1, z > 2 \wedge 20z \geq y + 11 \wedge x \leq 1)
\end{aligned}
$$

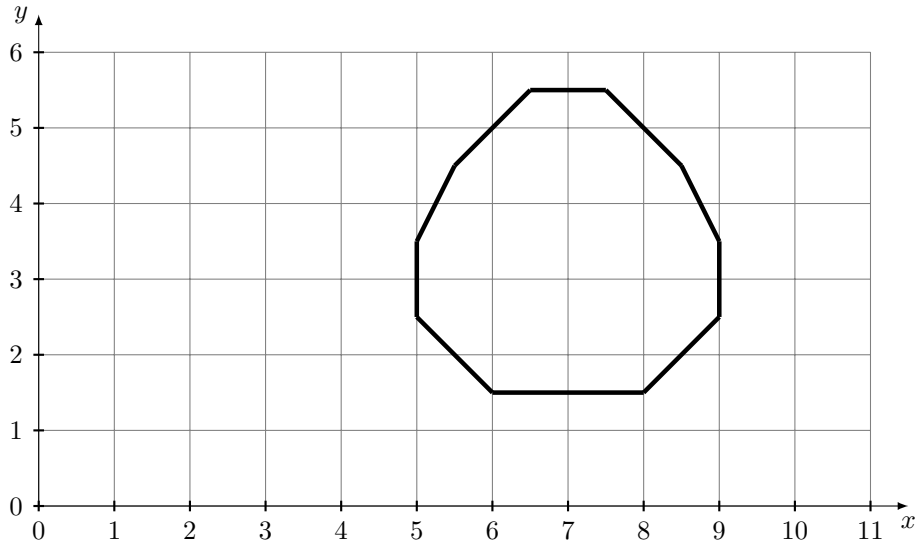
Aufgabe 2

Aufgabenteil a:



$$P' = \text{cHull}(\{\begin{pmatrix}1.5\\2.25\end{pmatrix}, \begin{pmatrix}2.5\\4.25\end{pmatrix}, \begin{pmatrix}5\\5.5\end{pmatrix}, \begin{pmatrix}8.5\\5\end{pmatrix}, \begin{pmatrix}3.5\\2.5\end{pmatrix}\})$$

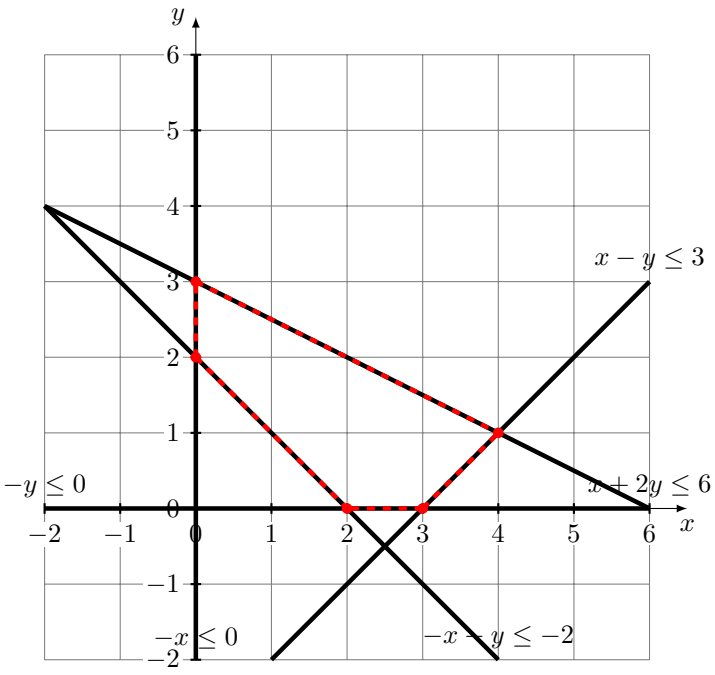
Aufgabenteil b:



$$P' = \text{cHull}(\{\begin{pmatrix}6\\1.5\end{pmatrix}, \begin{pmatrix}8\\1.5\end{pmatrix}, \begin{pmatrix}9\\2.5\end{pmatrix}, \begin{pmatrix}9\\3.5\end{pmatrix}, \begin{pmatrix}8.5\\4.5\end{pmatrix}, \begin{pmatrix}7.5\\5.5\end{pmatrix} \begin{pmatrix}6.5\\5.5\end{pmatrix}, \begin{pmatrix}5.5\\4.5\end{pmatrix}, \begin{pmatrix}5\\3.5\end{pmatrix}, \begin{pmatrix}5\\2.5\end{pmatrix}\})$$

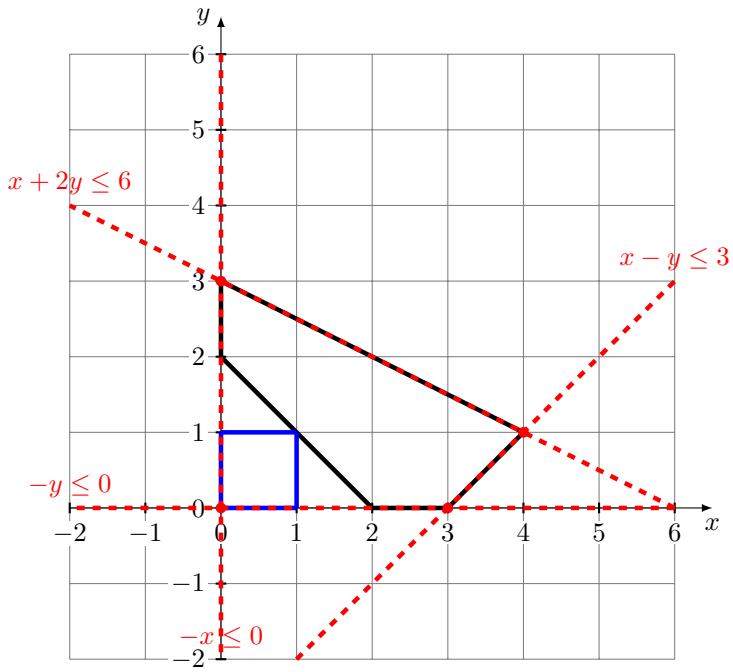
Aufgabe 3

Aufgabenteil a:



$$P' = \text{cHull}(\{\begin{pmatrix}0\\3\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}2\\0\end{pmatrix}, \begin{pmatrix}3\\0\end{pmatrix}, \begin{pmatrix}4\\1\end{pmatrix}\})$$

Aufgabenteil b:



$$P' = \text{cHull}(\{\begin{pmatrix}0\\3\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}2\\0\end{pmatrix}, \begin{pmatrix}3\\0\end{pmatrix}, \begin{pmatrix}4\\1\end{pmatrix}\})$$

$$\begin{aligned} -x &\leq 0 \\ -y &\leq 0 \\ x - y &\leq 3 \\ x + 2y &\leq -2 \end{aligned}$$