

Title of Report

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Preface

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Chapter 1.

Introduction

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This is a citation [1]. Hello, here is some text without a meaning. $d\Omega = \sin\vartheta d\vartheta d\varphi$. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. $\sin^2(\alpha) + \cos^2(\beta) = 1$. This text should contain all letters of the alphabet and it should be written in of the original language $E = mc^2$. There is no need for special content, but the length of words should match the language. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

See Table 1.1 for a floating table and (1.1) for an equation.

$$y = ax + b = cz + d \tag{1.1}$$

a	b	c	d
a	b	c	d
a	b	c	d
a	b	c	d

Table 1.1.: This is a floating table

Chapter 2.

Forward Kinematics

2.1. Task 1

Denavit-Hartenberg (DH) is a convention broadly used in robotics. It was introduced in order to standardize the attachment of coordinate frames to robots. A robot can be seen as a kinematic chain of rigid bodies. These rigid bodies are called links and are connected with joints. A robot arm is represented by a kinematic chain which is a concatenation of several links and joints. If one object is manipulated by one robot the system is called open kinematic chain. If several robots manipulate one object simultaneously the system is called closed kinematic chain. In this task we will focus on open chain robots. Forward kinematics is a way of calculating the end effector pose based on the joint parameters. The relation from the end effector to the base is done by attaching one frame to the end effector and one to the base of the robot. The relation between those two frames is then determined by the homogenous transformation matrix T_E^B . To make things easier a frame can be attached to each link and then the relation between each consecutive frame can be calculated by T_{i+1}^i . Concatenating all these homogenous Transformations and building their dot product determines the homogenous Transformation from the robot base to end effector:

$$T_E^B = T_0^1 * T_1^2 * \dots * T_{i-1}^i \quad (2.1)$$

Denavit-Hartenberg (DH) is a convention which standardizes the attachment of coordinate frames to a robot. It is a convention broadly used in the community which makes the application of the recursive formula in 2.1 more intuitive. The DH convention provides rules how to attach the frames to the link. In general the result should be the same if the frames are attached differently to the links as long as every link is considered. In the DH convention four different parameters are used in order to attach frames to the robot links. The frame of the link $i+1$ is determined based on the frame of the link i . The placement of a frame is visualized in figure 2.1. The following consecutive rules are applied:

- 1 z_i aligns with the rotational axis of joint $i+1$
- 2 O_i is placed at the intersection of z_i and the common normal of z_{i-1} and z_i
- 3 $O_{i'}$ is placed at the intersection of z_{i-1} and the common normal of z_{i-1} and z_i
- 4 x_i aligns with the common normal of z_{i-1} and z_i pointing away from O_i

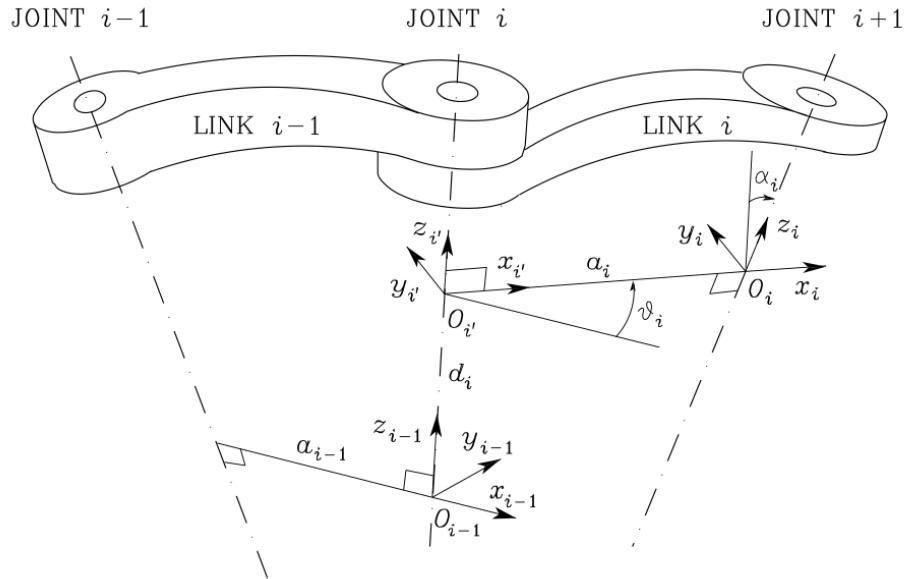


Figure 2.1.: Denavit Hartenberg convention rules [2]

5 y_i is chosen according to the right hand rule for coordinate frames

The DH convention does not define enough rules to make sure that there is just one unique solution.

- 1 There is no frame $i-1$ for frame 0 we can not define O_0 and x_0 : O_0 and x_0 can be defined arbitrarily.
- 2 There is no joint $i+1$ when defining frame n we can not define z_n : z_n is defined parallel to z_{n-1} .

Usually there are some ways of defining those two parameters which make applying the convention easier.

There are some special cases which also do not allow a unique definition of frames:

- 1 z_{i+1} and z_i are parallel: O_i and x_i can be selected arbitrarily.
- 2 z_{i+1} and z_i intersect: O_i and x_i can be selected arbitrarily.
- 3 joint i is prismatic: x_{i-1} can be selected arbitrarily.

The DH convention makes finding the homogenous transformations especially convenient. After establishing the frames at each joint the four DH parameters can be identified.

- a_i : distance between O_i and $O_{i'}$
- d_i : distance between $O_{i'}$ and O_{i-1} along z_{i-1}
- α_i : angle between z_{i-1} and z_i about x_i
- ϑ_i : angle between x_{i-1} and x_i about z_{i-1}

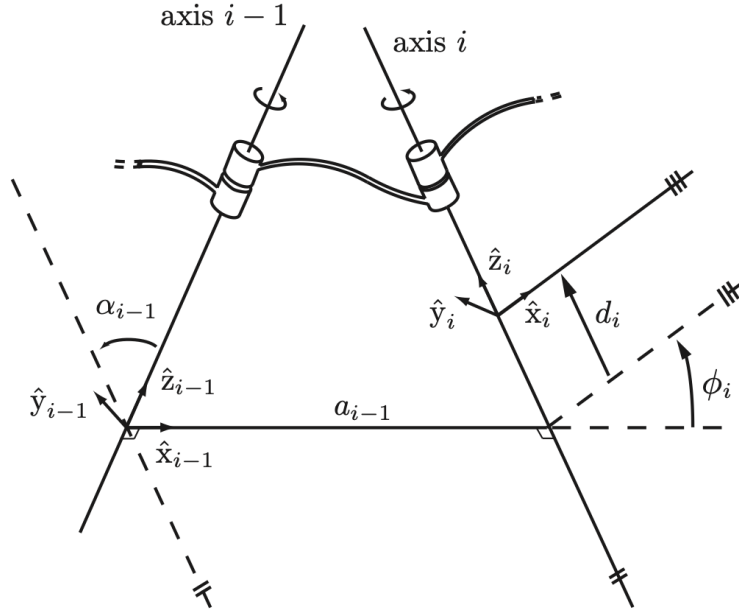


Figure 2.2.: Modified Denavit Hartenberg convention rules [3]

The parameters a_i and α_i are constant and just depend on the geometry of the link i . Non constant is the parameter d_i if joint i is prismatic and the parameter ϑ_i if joint i is rotational.

Having the four parameters a_i , d_i , α_i , ϑ_i one can write down the homogenous transformations T immediately:

$$T_i^{i-1} = \begin{bmatrix} c_{\vartheta_i} & -s_{\vartheta_i}c_{\alpha_i} & s_{\vartheta_i}s_{\alpha_i} & a_i c_{\vartheta_i} \\ s_{\vartheta_i} & c_{\vartheta_i}c_{\alpha_i} & -c_{\vartheta_i}s_{\alpha_i} & a_i s_{\vartheta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.2)$$

[2]

2.2. Task 2

The modified DH convention presented in [3] differs to the original DH presented in [2]. In figure 2.2 The frames are attached to the links differently (see table 2.1). Getting the DH parameters from the attached frames therefore also differs (see table 2.2). Like mentioned before the attachment of the frames does not change the forward dynamics. Anyway, sticking with one convention makes calculations more intuitive and easier to understand for others.

In the following passage the differences in the frame attachment are presented.

In the special case where two consecutive z_i and z_{i-1} intersect [3] recommends to pick a x_{i-1} which is perpendicular to the plane spanned by z_i and z_{i-1} . [2] just says that an

frame parameters	original DH	modified DH
z_i	aligns with the rotational axis of joint i+1	aligns with the rotational axis of joint i
O_i	is placed at the intersetion of z_i and the common normal of z_{i-1} and z_i	is placed at the intersetion of z_i and the common normal of z_i and z_{i+1}
x_i	aligns with the common normal of z_{i-1} and z_i pointing away from O_i	aligns with the common normal of z_{i+1} and z_i and pointing away from O_i

Table 2.1.: frame attachment original versus modified DH

DH paramters	original DH	modified DH
a_i	length of common normal of z_{i-1} and z_i	length of common normal of z_i and z_{i+1}
d_i	distance between O_{i-1} and intersection of common normal of z_{i-1} and z_i with z_{i-1}	distance between O_i and intersection of common normal of z_{i-1} and z_i with z_i
α_i	angle between z_{i-1} and z_i about x_i	angle between z_{i-1} and z_i about x_{i-1}
ϑ_i	angle between x_{i-1} and x_i about z_{i-1}	angle between x_{i-1} and x_i about z_i

Table 2.2.: DH parameter original versus modified DH

arbitrary x_i should be picked.

2.3. Task 3

2.3.1. Twist

Rigid body motions can be expressed by consecutively applying a rotation and translation on a body. So called twists which use rotations around a screw axis and translation along the screw axis can also be used to represent rigid body motions. The screw axis S can be represented by three parameters q, \hat{s}, h , where q is any point on the screw axis, \hat{s} is the unit vector representing the screw axis and h is the screw pitch, which deifnes the ratio of linear and angular velocity. Figure 2.3 shows:

- angular velocity $\dot{\theta}$ which rotates the coordinate frame around the screw axis
- the angular speed due to the angular velocity: $\dot{\theta}$: $-\hat{s}\dot{\theta} \times q$
- the linear speed due to the screw pitch h : $h\hat{s}\dot{\theta}$

The twist can than be written as:

$$V = \begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} \hat{s}\dot{\theta} \\ -\hat{s}\dot{\theta} \times q + h\hat{s}\dot{\theta} \end{bmatrix}$$

Instead of representing the screw axis with q, \hat{s}, h we can also represent it as a normalized version of any twist. Still the previous example gives a good intuition for twists and screw axis. This will help to understand the PoE formula later in this section.

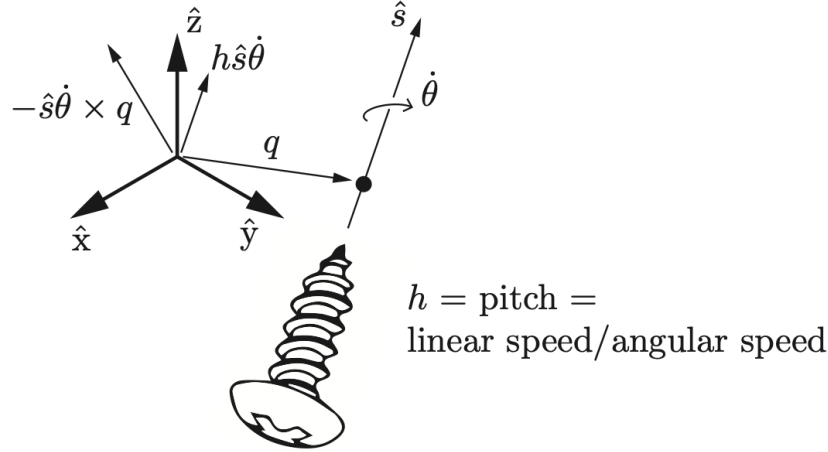


Figure 2.3.: Screw axis Visualization [3]

2.3.2. Exponential Coordinates of Twists

Applying the matrix exponential to $w\dot{\theta}$ the corresponding rotation matrix is generated. Equivalently this works for Twists. Applying the matrix exponential to $S\theta \in \mathbb{R}^6$ we receive the homogenous Transformation T

$$\exp : [S]\theta \in se(3) \Rightarrow T \in SE(3)$$

$$\log : T \in SE(3) \Rightarrow [S]\theta \in se(3)$$

In the next section the PoE formula is presented for solving the forward kinematics for complex open chain robots. The homogenous transformations between links is calculated with the matrix exponential of screw motions. In this sense, it is important to the concept of matrix logarithm and exponential of twists.

2.3.3. Change of reference frame - Twists

The same twists can be represented in different reference frames. The relation between those two representations is the following:

$$[S_b] = T_a^b [S_a] T_b^a \quad (2.3)$$

Also the notation of the adjoint mapping is common:

$$[S_b] = Ad_{T_a^b} [S_a] \quad (2.4)$$

2.3.4. Power of Exponentials formulation

In comparison to DH convention in the Product of Exponential Formula (POE Formula) there is no convention for attaching a frame to each link. It is just necessary to attach a frame at a stationary point and the end effector. The rotation of each joint i is represented by a screw motion which influences all links between joint i and end effector. When the

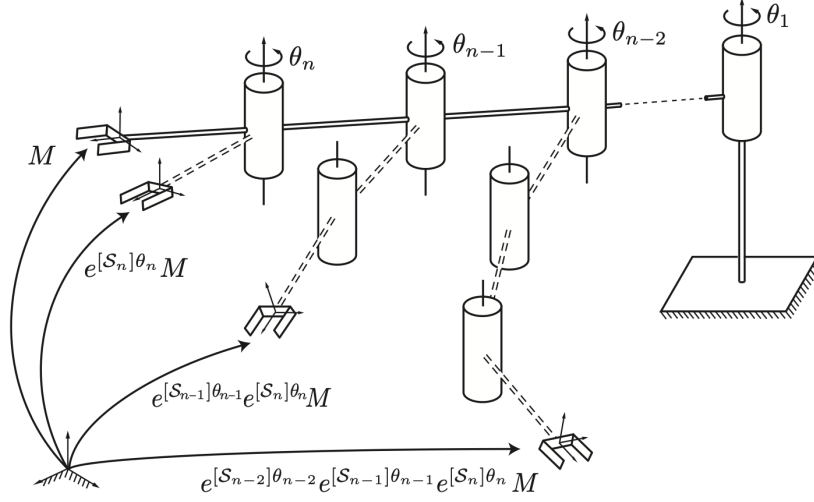


Figure 2.4.: Visualization of PoE Formula

robot is in its zero position and one does just move the last joint with θ_n , the end effector pose is represented by $T = e^{[S_n]\theta_n} M$. In contrast when moving the last two joints θ_n and θ_{n-1} , the end effector pose is represented by $T = e^{[S_{n-1}]\theta_{n-1}} e^{[S_n]\theta_n} M$. This is what figure 2.4 shows. M is a homogenous transformation representing the end effector frame relative to the base frame when the robot is in its home position. S_i is the screw axis of each joint i when the robot is in its home position. This screw axis can be represented in the fixed space frame or end effector frame. The first PoE formula is called spatial form of the Power-of-Exponentials formulations. The second PoE formula is called body form of the Power-of-Exponentials formulations. The homogenous transformation representing the end effector relative to the base frame in the fixed space frame:

$$T = e^{[S_1]\theta_1} \dots e^{[S_{n-1}]\theta_{n-1}} e^{[S_n]\theta_n} M \quad (2.5)$$

The screw motion of a joint i just impacts the pose of the joints $i+1$ to end effector frame but not any joint between the base and joint $i-1$. Therefore, it makes sense that in the formula 2.5 the M matrix is first transformed by the screw motion of the joint n . Using the matrix identity $e^{M^{-1}PM} = M^{-1}e^PM$ we can start to move the M matrix on the right side from formula 2.5 to the left side:

$$T = M e^{M^{-1}[S_1]M\theta_1} \dots e^{M^{-1}[S_{n-1}]M\theta_{n-1}} e^{M^{-1}[S_n]M\theta_n} \quad (2.6)$$

$M^{-1}[S_i]M$ is representing the screw axis of joint i in the end effector frame. The screw motion of joint i impacts all joints between base and joint $i-1$ but not any joint between joint $i+1$ and the end effector. Therefore it makes sense, that in formula 2.6 M is first transformed by the screw motion of the joint 1.

2.4. Task 4

2.4.1. Relation between DH-convention and PoE formula

When using the DH-convention, the relative motion between two links is represented by four parameters $a_i, d_i, \alpha_i, \vartheta_i$. The homogenous transformation between two links can be written as following:

$$T_i^{i-1} = Rot(\hat{x}, \alpha_{i-1}) Trans(\hat{x}, a_{i-1}) Trans(\hat{z}, d_i) Rot(\hat{z}, \phi_i) \quad (2.7)$$

In the case of rotational joints the rotation around x and the translation along x and z is constant and can be written as following:

$$M_i = Rot(\hat{x}, \alpha_{i-1}) Trans(\hat{x}, a_{i-1}) Trans(\hat{z}, d_i) \quad (2.8)$$

The screw axis of the joint is the following:

$$[A_i] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.9)$$

Applying the matrix exponential of $[A_i]\theta_i$ returns the homogenous transformation for the rotation around z:

$$Rot(\hat{z}, \phi_i) = e^{[A_i]\theta_i} \quad (2.10)$$

The equations from equation 2.8, 2.9 and 2.10 lead to the following homogenous transformation:

$$T_i^{i-1} = M_i e^{[A_i]\theta_i} \quad (2.11)$$

Concatenating equation ?? for all links in the open chain solves the forward kinematics:

$$T_n^0 = M_1 e^{[A_1]\theta_1} M_2 e^{[A_2]\theta_2} \dots M_{n-1} e^{[A_{n-1}]\theta_{n-1}} M_n e^{[A_n]\theta_n} \quad (2.12)$$

By rearranging the following matrix identity $Me^P M^{-1} = e^{MPM^{-1}}$ to $Me^P = e^{MPM^{-1}} M$ we can rewrite equation 2.11 to:

$$T_i^{i-1} = e^{M_i [A_i] \theta_i M_i^{-1}} M_i \quad (2.13)$$

Applying equation 2.13 to 2.12:

$$\begin{aligned} T_0^n &= e^{M_1 [A_1] M_1^{-1} \theta_1} (M_1 M_2) e^{[A_2] \theta_2} \dots M_{n-1} e^{[A_{n-1}] \theta_{n-1}} M_n e^{[A_n] \theta_n} \\ &= e^{M_1 [A_1] M_1^{-1} \theta_1} e^{(M_1 M_2) [A_2] (M_1 M_2)^{-1} \theta_2} (M_1 M_2 M_3) e^{[A_3] \theta_3} \dots M_n e^{[A_n] \theta_n} \\ &= e^{[S_1] \theta_1} e^{[S_2] \theta_2} \dots e^{[S_n] \theta_n} M \end{aligned} \quad (2.14)$$

where

$$[S_i] = (M_1 \dots M_{i-1})[A_n](M_1 \dots M_{i-1})^{-1} \quad (2.15)$$

$$M = M_1 M_2 \dots M_n \quad (2.16)$$

From equation 2.3 and 2.4 we know that the same Twist can be represented in different reference frames. A_i is the screw axis of joint i in the i frame. It is obvious that S_i is the same screw axis as A_i transformed into another frame. The screw axis of joint 1 is transformed by M_1 , whereas the screw axis of joint i is transformed by $M_1 \dots M_{i-1}$. S_i . This means that the screw axis of all joints are transformed from the local joint frames A_i into the base frame S_i . After using the DH-convention to attach frames to each link, applying the matrix exponential to generate the homogenous transformation between consecutive frames in equation 2.10, changing the reference frame of each joint screw axis into the base frame and multiplying the homogenous transformations between consecutive joints we end up with the PoE formula.

2.4.2. DH-convention vs PoE formula - advantages and disadvantages

In this section we try to elaborate the differences between the DH convention and the PoE formula. Both methods differ in several points and have advantages and disadvantages. The DH convention on the one hand has fixed rules to attach frames to joints. This might sometimes seem to be exhaustig. Especially for complex robots with a lot of links it can become a tedious work. If all frames are attached properly one can easily find the homogenous transformations between all consecutive frames T_1^0 to T_n^{n-1} . Using the DH convention the homogenous transformations is established by using the minimal number of parameters to describe joint movements. The minimal number of parameters in the DH convention are $a_i, d_i, \alpha_i, \vartheta_i$. These DH paramters can be used immediatly to build the transformation matrix T . Multipling all these matrices returns the Transformation from base to end effector and the forward kinematics is solved. The easy establishment of the transformation matrix with the minimal number of parameters is the biggest advantage of the DH convention. Still there exist different DH conventions for attaching the frames to the links. This might sometimes be confusing. Another disadvantage is that the DH-parameters can become ill-conditioned. Especially in the special cases where consecutive joint axes are parallel or do intersect little changes in the robot geometry can make huge changes in the DH-parameters. Errors in manufacturing, CAD models or in other areas can therefore lead to bigger problems.

In contrast to that the PoE does not expect to attach frames at each joint. After deciding for a base and end effector frame and establishing the M matrix, which defines the homegenious transformation from base to end effector in the robot's home position, one can use the PoE formula to solve the forward kinematics. There is a smaller effort when attaching frames to the robot and not a such a problem of ill-conditioned parameters. The interpretation of the joints as screw motion might be more intuitive. Besides that the PoE formula does not differ when using it for prismatic and rotational joints. In the DH convention the parameter d_i for prismatic joints and the parameter ϑ_i for rotational joints are variable. In these cases those DH-paramters are not solely determined by the

geometry of the link. This is a disadvantage of the DH-convention. This problem does not appear in the PoE formula where prismatic and rotational joints are treated equally. Still, in the PoE-formula it might be more effort to establish the screw axis and applying the matrix exponential. When using a computer this disadvantage might be irrelevant. Besides that the PoE formula does not work with a minimal number of parameters.

The PoE formula on the other hand does not expect frames to be attached to each link.

Chapter 3.

Here Comes Chapter 2

Hello, here is some text without a meaning. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. This text should show what a printed text will look like at this place. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$. If you read this text, you will get no information. $d\Omega = \sin\vartheta d\vartheta d\varphi$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. $\sin^2(\alpha) + \cos^2(\beta) = 1$.

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Figure 3.1.: NTNU logo

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3.1. Title of section 2

Chapter 4.

Here Comes Chapter 3

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Figure 4.1.: NTNU logo

like this gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain all letters of the alphabet and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain all letters of the alphabet and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

Chapter 5.

Conclusion

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain all letters of the alphabet and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$. Hello, here is some text without a meaning. $d\Omega = \sin\vartheta d\vartheta d\varphi$. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. $\sin^2(\alpha) + \cos^2(\beta) = 1$. This text should contain all letters of the alphabet and it should be written in of the original language $E = mc^2$. There is no need for special content, but the length of words should match the language. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. Hello, here is some text without a meaning. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. This text should show what a printed text will look like at this place. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$. If you read this text, you will get no information. $d\Omega = \sin\vartheta d\vartheta d\varphi$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. $\sin^2(\alpha) + \cos^2(\beta) = 1$. Hello, here is some text without a meaning $E = mc^2$. This text should show what a printed text will look like at this place. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. If you read this text, you will get no information. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$. This text should contain all letters of the alphabet and it should be written in of the original language. $d\Omega = \sin\vartheta d\vartheta d\varphi$. There is no need for special content, but the length of words should match the language.

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Appendix A.

Name of Appendix

A.1. This is a section