Title of Report

Author(s)

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Preface

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Chapter 1.

Introduction

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See Table 1.1 for a floating table and (1.1) for an equation.

$$y = ax + b = cz + d \tag{1.1}$$

a	b	c	d
a	b	c	d
a	b	c	d
a	b	c	d

Table 1.1.: This is a floating table

Chapter 2.

Forward Kinematics

2.1. Task 1

Denavit-Hartenberg (DH) is a convention broadly used in robotics. It was introduced in order to standardize the attachmemt of cordinate frames to robots. A robot can be seen as a kinematic chain of rigid bodies. These rigid bodies are called links and are connected with joints. A robot arm is represenetd by a kinematic chain which is a concatenation of several links and joints. If one object is manipulated by one robot the system is called open kinematic chain. If several robots manipulate one object simultanously the system is called closed kinematic chain. In this task we will focus on open chain robots. Forward kinematics is a way of of calculating the end effector pose based on the joint parameters. The relation from the end effector to the base is done by attaching one frame to the end effector and one to the base of the robot. The relation between those two frames is then determined by the homogenious transformation matrix T_E^B. To make things easier a frame can be attached to each link and then the relation between each consecutive frame can be calculated by T_{i+1}^i. Concatenating all these homogenious Transformations and building their dot product determines the homogenious Transformation from the robot base to end effector:

$$T_E^B = T_0^1 * T_1^2 * ... * T_{i-1}^i \tag{2.1}$$

Denavit-Hartenberg (DH) is a convention which standardizes the attachmemt of cordinate frames to a robot. It is a convention broadly used in the community which makes the application of the recursive formula in 2.1 more intuitive. The DH convention provides rules how to attach the frames to the link. In general the result should be the same if the frames are attached differently to the links as long as every link is considered. In the DH convention four different parameters afe used in order to attach frames to the robot links. The frame of the link i+1 is determined based on the frame of the link i. The placement of a frame is visualized in figure 2.1. The followinfg consecutive rules are applied:

1 z_i alligns with the rotational axis of joint i+1

2 O_i is placed at the intersetion of z_i and the common normal of z_{i-1} and z_i

3 $O_{i'}$ is placed at the intersetion of z_{i-1} and the common normal of z_{i-1} and z_i

4 x_i alligns with the common normal of z_{i-1} and z_i pointing away from O_i

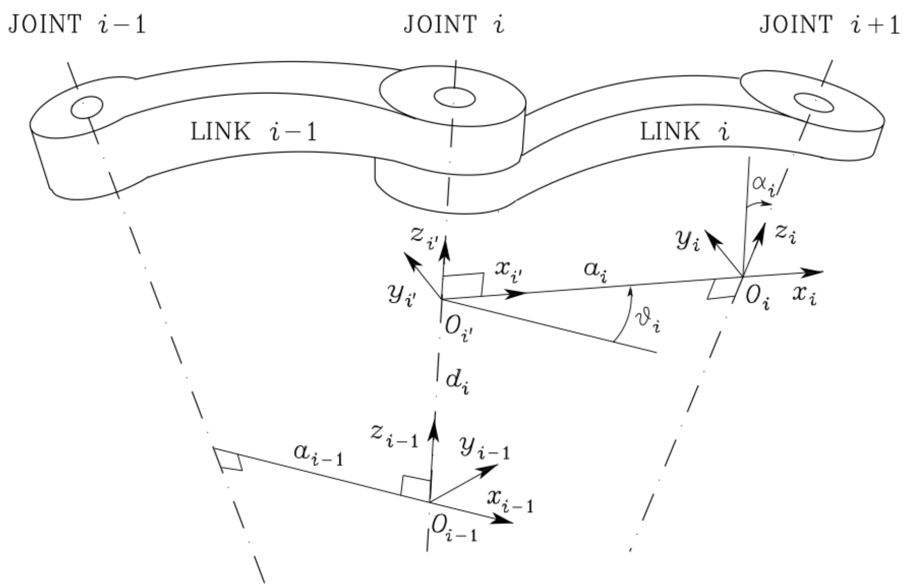

Figure 2.1.: Denavit Hartenberg convention rules [2]

5 y_i is chosen according to the right hand rule for coordinate frames

The DH convention does not define enough rules to make sure that there is just one uniquie solution.

1 There is no frame i-1 for frame 0 we can not define O_0 and x_0: O_0 and x_0 can be defined arbitrarily.

2 There is no joint i+1 when defining frame n we can not define z_n: z_n is defined parallel to z_{n-1} .

Usually there are some ways of defining those two parameters which make applying the convention easier.

There are some special cases which also do not allow a unique definition of frames:

1 z_{i+1} and z_i are parallel: O_i and x_i can be selected arbitrarily.

2 z_{i+1} and z_i intersect: O_i and x_i can be selected arbitrarily.

3 joint i is prismatic: x_{i-1} can be selected arbitrarily.

The DH convention makes finding the homogenious transformations especially convenient. After establishing the frames at each joint the four DH parameters can be identified.

a_i : distance between O_i and $O_{i'}$

d_i : distance between $O_{i'}$ and O_{i-1} along z_{i-1}

α_i : angle between z_{i-1} and z_i about x_i

ϑi : angle between x_{i-1} and x_i about z_{i-1}

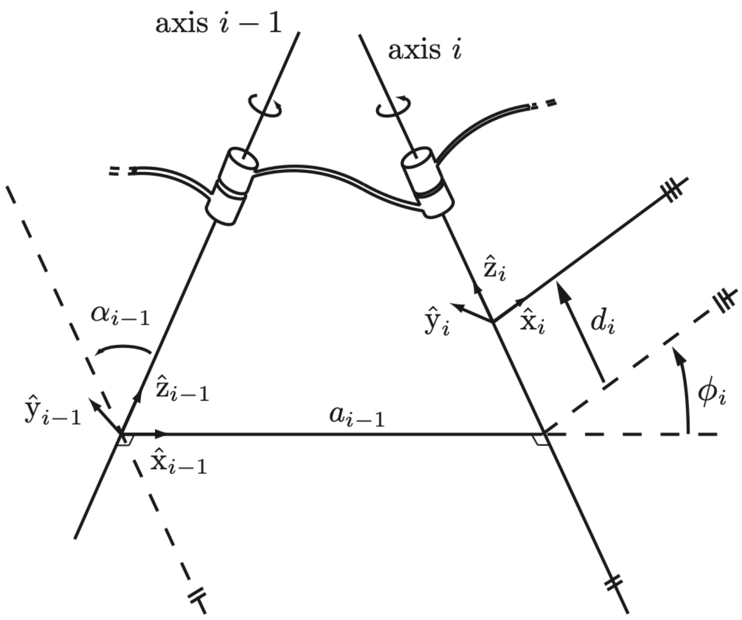

Figure 2.2.: Modified Denavit Hartenberg convention rules [3]

The paramters a_i and α_i are constantg and just depend on the geometry of the link i. Non constant is the parameter d_i if joint i is prismatic and the parameter ϑi if joint i is rotational.

Having the four parameters a_i, d_i, α_i, $\vartheta_i$one can write down the homogenious transformations T immediatelly:

$$T_i^{i-1} = \begin{bmatrix} c_{\vartheta_i} & -s_{\vartheta_i}c_{\alpha_i} & s_{\vartheta_i}s_{\alpha i} & a_i c_{\vartheta_i} \\ s_{\vartheta_i} & c_{\vartheta_i}c_{\alpha i} & -c_{\vartheta_i}s_{\alpha i} & a_i s_{\vartheta_i} \\ 0 & s_{\alpha i} & c_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.2}$$

[2]

2.2. Task 2

The modified DH convention presented in [3] differs to the original DH presented in [2]. In figure 2.2 The frames are attached to the links differntly (see table 2.1). Getting the DH parameters from the attached frames therefore also differs (see table 2.2). Like mentioned before the attachment of the frames does not change the forward dynamics. Anyway, sticking with one convention makes calculatations more intuitive and easier to understand for others.

In the following passage the differences in the frame attachement are presented.

In the special case where two consecutive z_i and z_{i-1} intersect [3] recommends to pick a x_{i-1} which is perpendicular to the plane spanned by z_i andz_{i-1}. [2] just says that an

frame parameters	original DH	modified DH
z_i	alligns with the rotational axis of joint i+1	alligns with the rotational axis of joint i
O_i	is placed at the intersetion of z_i and the common normal of z_{i-1} and z_i	is placed at the intersetion of z_i and the common normal of z_i and z_{i+1}
x_i	alligns with the common normal of z_{i-1} and z_i pointing away from O_i	alligns with the common normal of z_{i+1} and z_i and pointing away from O_i

Table 2.1.: frame attachment original versus modified DH

DH paramters	original DH	modified DH
a_i	length of common normal of z_{i-1} and z_i	length of common normal of z_i and z_{i+1}
d_i	distance between O_{i-1} and intersection of common normal of z_{i-1} and z_i with z_{i-1}	distance between O_i and intersection of common normal of z_{i-1} and z_i with z_i
α_i	angle between z_{i-1} and z_i about x_i	angle between z_{i-1} and z_i about x_{i-1}
ϑi	angle between x_{i-1} and x_i about z_{i-1}	angle between x_{i-1} and x_i about z_i

Table 2.2.: DH parameter original versus modified DH

arbitrary x_i should be picked.

2.3. Task 3

2.3.1. Twist

Rigid body motions can be expressed by consecutively applying a rotation and translation on a body. So called twists which use rotations around a srew axis and translation along the screw axis can also be used to represent rigid body motions. The screw axis S can be represented by three parameters $q, \hat{s}, h$, where q is any point on the screw axis, $\hat{s}$ is the unit vector representing the screw axis and h is the screw pitch, which deifines the ratio of linear and angular velocity. Figure 2.3 shows:

- angular velocity $\dot{\theta}$ which rotates the coordinate frame around the screw axis
- the angular speed due to the angualar velocity: $\dot{\theta}$: -$\hat{s}\dot{\theta} \times q$
- the linear speed due to the screw pitch h: $h\hat{s}\dot{\theta}$

The twist can than be written as:

$$V = \begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} \hat{s}\dot{\theta} \\ -\hat{s}\dot{\theta} \times q + h\hat{s}\dot{\theta} \end{bmatrix}$$

Instead of representing the screw axis with $q, \hat{s}, h$ we can also represent it as a normalized version of any twist. Still the previous example gives a good intuition for twists and screw axis. This will help to understand the PoE formula later in this section.

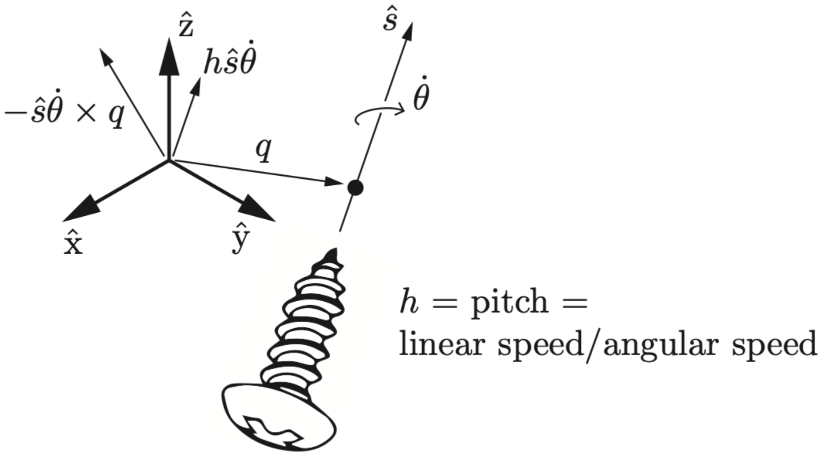

Figure 2.3.: Screw axis Visualization [3]

2.3.2. Exponential Coordinates of Twists

Applying the matrix exponential to $w\dot{\theta}$ the corresponding rotation matrix is generated. Equivalently this works for Twists. Applying the matrix exponential to $S\theta \in \mathbb{R}^6$ we receive the homogenious Transformation T

$$\exp\ : [S]\theta \in se(3) \Rightarrow T \in SE(3)$$

$$\log\ : T \in SE(3) \Rightarrow [S]\theta \in se(3)$$

In the next section the PoE formula is presented for solving the forward kinematics for compelex open chain robots. The homogenious transformations between links is caclulated with the matrix exponential of screw motions. In this sense, it is important to the concept of matrix logarithm and exponential of twists.

2.3.3. Power of Exponentials formulation

In comparison to DH convention in the Product of Exponential Formula (POE Formula) there is no convention for attaching a frame to each link. It is just necessary to attach a frame at a stationary point and the end effector. The rotation of each joint i is represented by a screw motion which influences all links between joint i and end effector. When the robot is in its zero position and one does just move the last joint with θ_n, the end effector pose is represented by $T = e^{[S_n]\theta_n}M$. In contrast when moving the last two joints θ_n and θ_{n-1}, the end effector pose is represented by $T = e^{[S_{n-1}]\theta_{n-1}}e^{[S_n]\theta_n}M$. This is what figure 2.4 shows. M is a homogenious transformation representing the end effector frame relative to the base frame when the robot is in it's home position. S_i is the screw axis of each joint i when the robot is in its home position This screw axis can be represented in the fixed space frame or end effector frame. The first PoE formula is called spatial form of the Power-of-Exponentials formulations. The second PoE formula is called body form of the Power-of-Exponentials formulations. The homogenious transformation representing the end effector relative to the base frame in the fixed space frame:

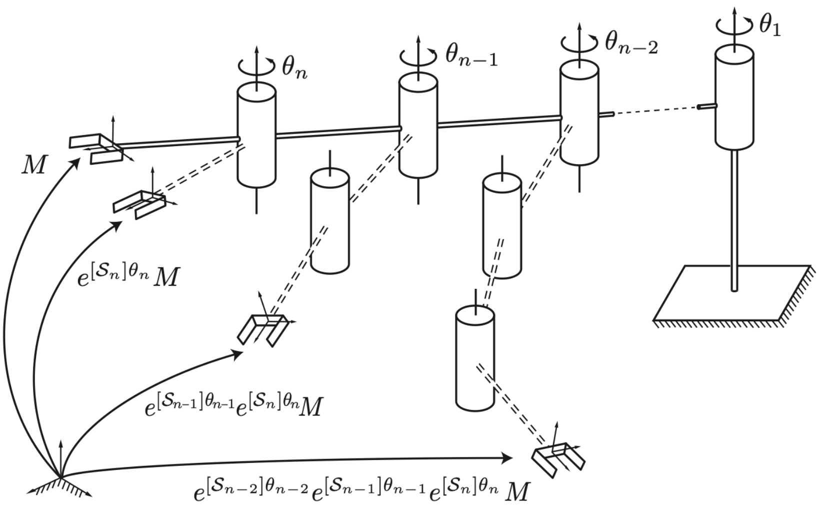

Figure 2.4.: Visualization of PoE Formula

$$T = e^{[S_1]\theta_1}...e^{[S_{n-1}]\theta_{n-1}}e^{[S_n]\theta_n}M \tag{2.3}$$

The screw motion of a joint i just impacts the pose of the the joints i+1 to end effector frame but not any joint between the base and joint i-1. Therefore, it makes sense that in the formula 2.3 the M matrix is first transformed by the the screw motion of the joint n. Using the matrix identity $e^{M^{-1}PM} = M^{-1}e^{P}M$ we can start to move the M matrix on the right side from formula 2.3 to the left side:

$$T = Me^{M^{-1}[S_1]M\theta_1}...e^{[M^{-1}[S_{n-1}]M\theta_{n-1}}e^{M^{-1}[S_n]M\theta_n} \tag{2.4}$$

$M^{-1}[S_i]M$ is representing the screw axis of joint i in the end effector frame. The screw motion of joint i impacts all joints between base and joint i-1 but not any joint between joint i+1 and the end effector. Therefore it makes sense, that in formula 2.4 M is first transformed by the screw motion of the joint 1.

Chapter 3.

Here Comes Chapter 2

Hello, here is some text without a meaning. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. This text should show what a printed text will look like at this place. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$. If you read this text, you will get no information. $\mathrm{d}\Omega = \sin\vartheta \mathrm{d}\vartheta \mathrm{d}\varphi$. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. $\sin^2(\alpha) + \cos^2(\beta) = 1$.

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Figure 3.1.: NTNU logo

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3.1. Title of section 2

Chapter 4.

Here Comes Chapter 3

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Figure 4.1.: NTNU logo

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Chapter 5.

Conclusion

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Appendix A.

Name of Appendix

A.1. This is a section