Finite temperature and δ-regime in the Schwinger model

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Schwinger model

- introduced by [Schwinger, 1961]: *two-dimensional quantum electrodynamics* — fermions coupled to Abelian gauge field
- simple example for chiral anomaly, topology, confinement
- often used as a testbed for conceptual and numerical approaches in lattice field theory
- nevertheless, some of the rich physical properties of the model in anisotropic volumes have not yet been tested:

Finite temperature : Hosotani approximate solution has not been compared with the lattice simulation results

δ-regime : conjecture for the residual pion mass

N-flavor Schwinger model

- massless case has analytic solution [Belvedere et al., 1979]
- $N-1$ massless bosons ("pions")
- one massive boson ("eta")

$$m_\eta^2 = N\,\frac{g^2}{\pi}$$

where g is the gauge coupling

- massive case (fermion mass $m>0$) has no exact solution
- semiclassical prediction at infinite volume

$$m_\pi = \left(4e^{2\gamma}\sqrt{\frac{2}{\pi}}\right)(m^2 g)^{1/3} = 2.1633\ldots(m^2 g)^{1/3}$$

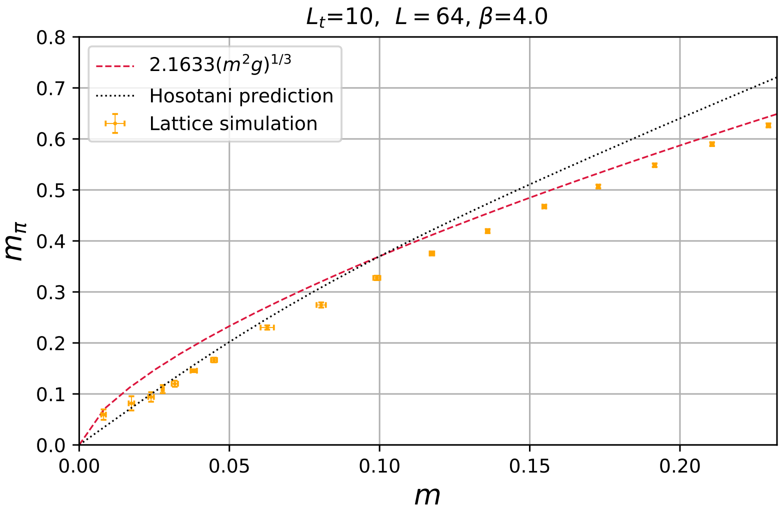
Finite temperature - Hosotani approximate solution

- approximate solution by Hosotani et al. based on bosonization
- finite temperature massive Schwinger model reduced to quantum mechanical system with $N - 1$ degrees of freedom
- set of nonlinear equations valid when

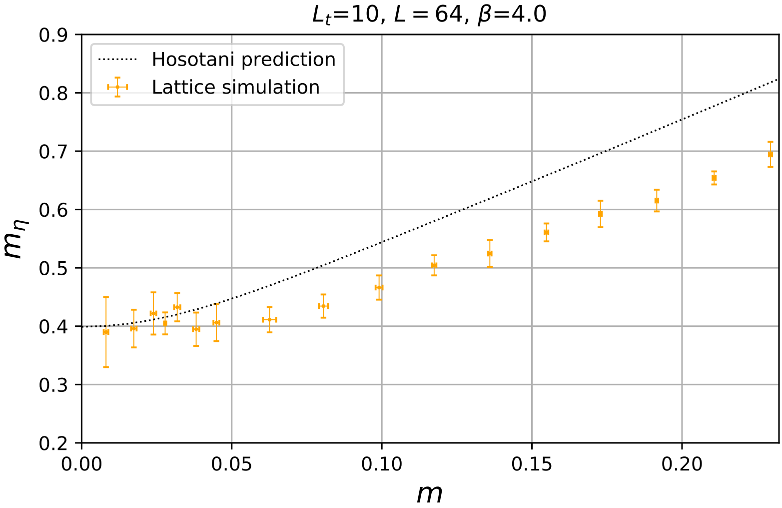
$$m \ll m_\eta$$

- boson masses can be computed by solving this set of equations in a self-consistent way
- we compare the Hosotani predictions for two flavors with HMC simulation results (Wilson fermions — fermion mass m is measured on the lattice by the PCAC relation)

Pion mass - Hosotani vs. lattice simulation



Eta mass - Hosotani vs. lattice simulation



δ-regime

- spatial volume is small compared to the correlation length

$$\xi = m_\pi^{-1}$$

but the Euclidean time extent is large

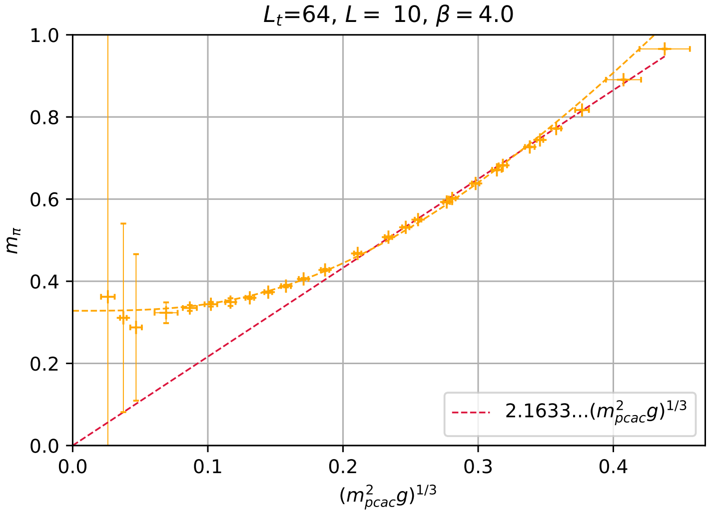
$$L_t \gg \xi \gtrsim L$$

- the system is quasi one dimensional - approximation by quantum rotor [Leutwyler, 1987]
- pion has *residual mass*

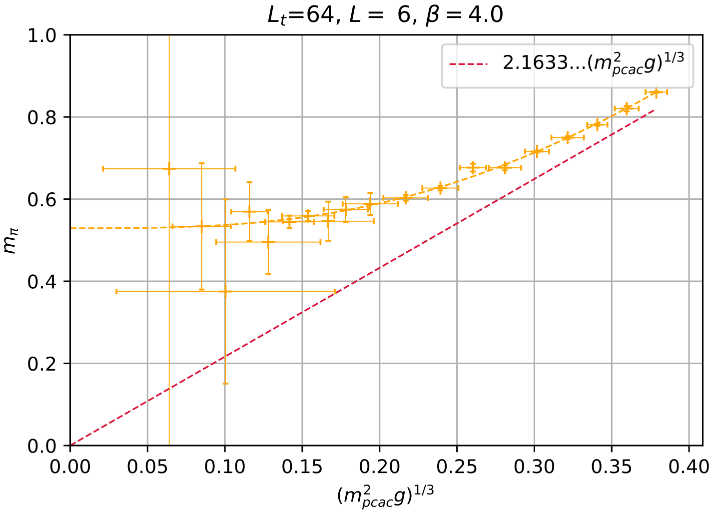
$$m_\pi^R \propto \frac{1}{\Theta}$$

where Θ is the moment of inertia

Residual pion mass plateau: $L_t = 64, L = 10$



Residual pion mass plateau: $L_t = 64, L = 6$



Conjecture

- [Hasenfratz and Niedermayer, 1993] computed Θ up to next-to-leading order, for a general dimension $d > 2$

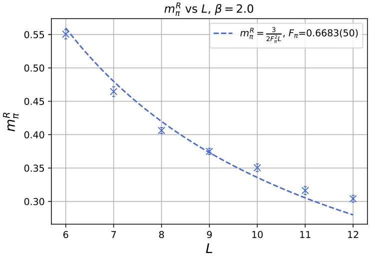
$$\Theta = F_\pi^2 L^{d-1} \left[1 + \frac{\mathcal{N} - 2}{4\pi F_\pi^2 L^{d-2}} \left(2 \frac{d-1}{d-2} + \dots \right) \right]$$

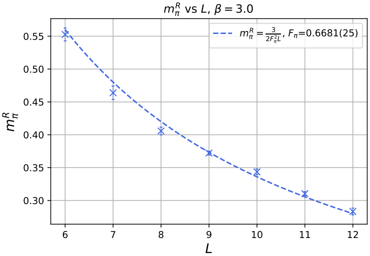
- in two dimensions ($d = 2$) there is division by zero in the next to leading term, so we just consider the leading term

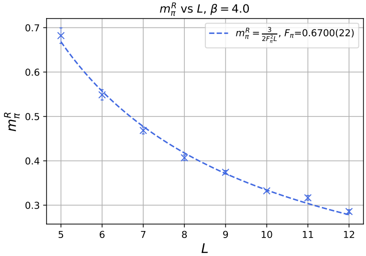
$$m_\pi^R \simeq \frac{1}{2 F_\pi^2 L}$$

- we verify the relation $m_\pi^R \propto 1/L$ with simulation data and extract the value of pion decay constant F_π

1 / L confirmed by lattice simulation







β	F_π
2.0	0.3858(29)
3.0	0.3857(14)
4.0	0.3868(13)

$$F_\pi = 0.386(2)$$

Witten-Veneziano formula

- in the chiral N-flavor Schwinger model the Witten-Veneziano formula is simplified to [Seiler and Stamatescu, 1987]

$$m_\eta^2 = \frac{2N}{F_\eta^2}\chi_T^{que}$$

- mass of the η particle is known analytically [Belvedere et al., 1979]

$$m_\eta^2 = \frac{N}{\pi\beta}$$

- continuum prediction for χ_T^{que} [Seiler and Stamatescu, 1987]

$$\beta\chi_T^{que} = \frac{1}{4\pi^2}$$

Quenched topological susceptibility

- [Bardeen et al., 1998] were able to analytically compute χ_T^{que} on the lattice

$$\beta\chi_T^{que} = \frac{I_1(\beta)}{4\pi^2 I_0(\beta)}$$

by using an alternative definition of topological charge

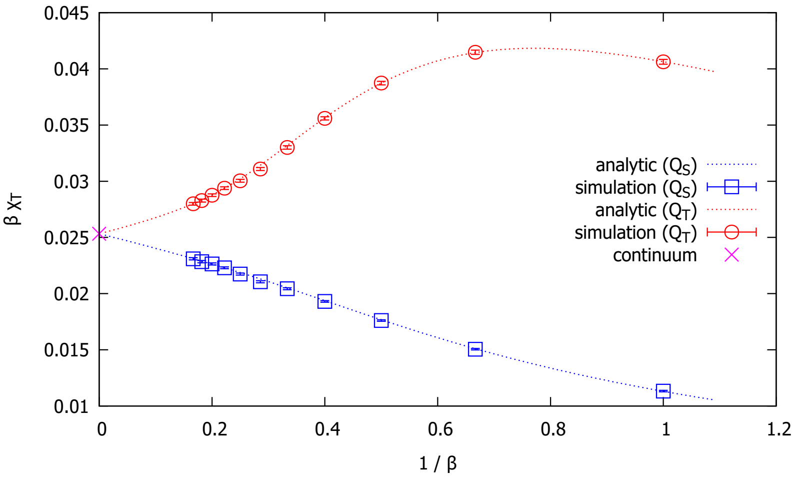
$$Q_S = \frac{1}{2\pi}\sum_P \sin(\theta_P)$$

- for the usual definition of topological charge

$$Q_T = \frac{1}{2\pi}\sum_P \theta_P$$

it is not possible to find an analytic expression, but using the same line of reasoning it is possible to numerically compute χ_T^{que} to arbitrary precision

Quenched topological susceptibility



F_η versus F_π

- inserting the confirmed values for m_η^2 and χ_T^{que}

$$F_\eta^2 = \frac{2N}{m_\eta^2}\chi_T^{que} = 2N\left(\frac{\pi\beta}{N}\right)\left(\frac{1}{4\pi^2\beta}\right) = \frac{1}{2\pi}$$

- we can compare the two decay constants

$$F_\pi = 0.386(2) \quad \stackrel{?}{=} \quad F_\eta = 0.399$$

- in large N_c QCD, to the order $1/N_c$

$$F_{\eta'} = F_\pi$$

- in the Schwinger model nothing assures that this relation holds