Finite temperature Schwinger model

/preliminary results/

September 5, 2020

1 Notation

Schwinger model at finite temperature T in equilibrium is equivalent to an Euclidean field theory satisfying boundary conditions [Hos95a]

$$\psi(x, t + \frac{1}{T}) = -\psi(x, t), \quad A(x, t + \frac{1}{T}) = A(x, t) \tag{1}$$

In the literature it is usual to use β for inverse temperature. However, we are not going to use β because we use it for inverse gauge coupling, i.e.

$$\beta \equiv \frac{1}{g^2} \tag{2}$$

If we denote number of lattice points in temporal direction with n_t and lattice spacing with a, inverse temperature is

$$\frac{1}{T} = a\, n_t \tag{3}$$

2 Predictions

Massless Schwinger model with N_f flavors is exactly solvable [Aff86][1] and there exists $N_f - 1$ massless ("pions") and one massive ("eta") boson with mass

$$\mu = g\sqrt{\frac{N_f}{\pi}} \tag{4}$$

or, using definition (2)

$$\mu = \sqrt{\frac{N_f}{\pi\beta}} \tag{5}$$

[1] Cited after [HHI95] — I have no access to full text of [Aff86] to check it!

In [HHI96] the Schwinger model with N_f flavors of massive fermions at finite temperature T is reduced to quantum mechanical system of $N_f - 1$ degrees of freedom. For degenerate fermion masses m this system can be numerically solved for general values of T/μ and m/μ to get predictions for chiral condensate $\frac{1}{\mu}\langle\bar{\psi}\psi\rangle$. Fig. 3 in [HHI96] shows that for $N_f = 2$ ($\theta = 0$) the condensate which is non-vanishing at $T = 0$ smoothly goes to zero at finite temperature, i.e. *there is no phase transition at finite temperature!*

More detailed presentation of the results from [HHI96] is given in the two workshop proceedings [Hos95a, Hos95b]. Especially interesting is Fig. 1 in [Hos95a] where the dependence of chiral condensate $\frac{1}{\mu}\langle\bar{\psi}\psi\rangle$ on T/μ for several fermion masses in 3-flavor Schwinger model is given. It is compared to $N_f = 1$ massless case [SW92] where the crossover takes place at $\mu = 1$.

Although there is no chance to get the right values of chiral condensate using Wilson fermions, one can hope to see some kind of crossover near $T/\mu = 1$ similar to those shown on fig. 1 in [Hos95a]. I started several runs on lattices 16×16, 32×8, 64×4 and 128×2 to see what happens.

Fig. 1 shows the results for $\beta = 2$ (1000 measurements on 16×16 and 500 measurements on other lattices). Qualitatively, picture looks promising: there seems to be crossover near $T/\mu = 1$ and the crossover is smoother for smaller masses (larger κ is closer to κ critical where the fermion mass m vanishes, i.e. larger κ means smaller fermion mass m). Values of κ and corresponding values of effective fermion mass m which was measured with the help of PCAC relation [HLT98, GHL99] (see Fig. 3) are given in Tab. 1.

$\beta = 2.0$		$\beta = 6.0$	
κ	m	κ	m
0.25	0.186(2)	0.23	0.220(3)
0.26	0.117(2)	0.24	0.142(3)
0.27	0.052(5)	0.25	0.070(3)

Table 1: Effective fermion masses at $\beta = 2.0$ and $\beta = 6.0$ from PCAC relation.

Fig. 2 shows the results for $\beta = 6$ for the same number of measurements. Smaller values for κ are chosen because κ_c for $\beta = 6$ is lower then κ_c for $\beta = 2$. Although the statistical fluctuations are small, it seems that there are some problems with ergodicity (for large β topological transitions are rare, so simulation could be stuck in the same topological sector). However, the picture is similar as for $\beta = 2$ but it is shifted to the right. That doesn't really makes sense and it seems to me that it comes because there is T divided by μ on horizontal axis: from (3) and (5) follows

$$\frac{T}{\mu} = \frac{1}{an_t}\sqrt{\frac{\pi\beta}{N_f}} \tag{6}$$

so there is explicit β dependence. Therefore, I decided to plot both β-s on the same graph (fig. 4), but with

$$T = \frac{1}{n_t} \tag{7}$$

instead of T/μ on the horizontal axis. The crossovers at both β seems to take place at the same temperature, but it is against pyhsical intuition: boson mass μ sets the scale and it is to be expected that the position of crossover depends on μ, exactly as it is claimed in [HHI96, Hos95a]?!

3 To-do list

- to analyze Polyakov loops
- to compute predictions for $\langle\bar{\psi}\psi\rangle$ dependence for two flavor Schwinger model and arbitrary fermion masses m as explained in [HHI96, Hos95a, Hos95b] (numerical solution of Shrödinger equation is necessary! — in [Hos95b], pg. 4, is claimed that it is the equation which describes quantum pendulum)
- to start runs for given m on different lattice sizes and inverse gauge couplings β

References

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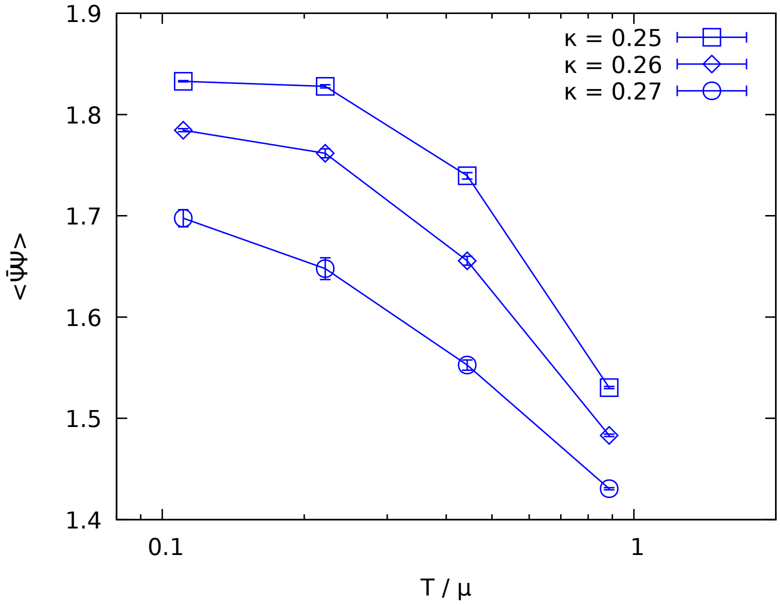

Figure 1: Wilson fermions, $\beta = 2$

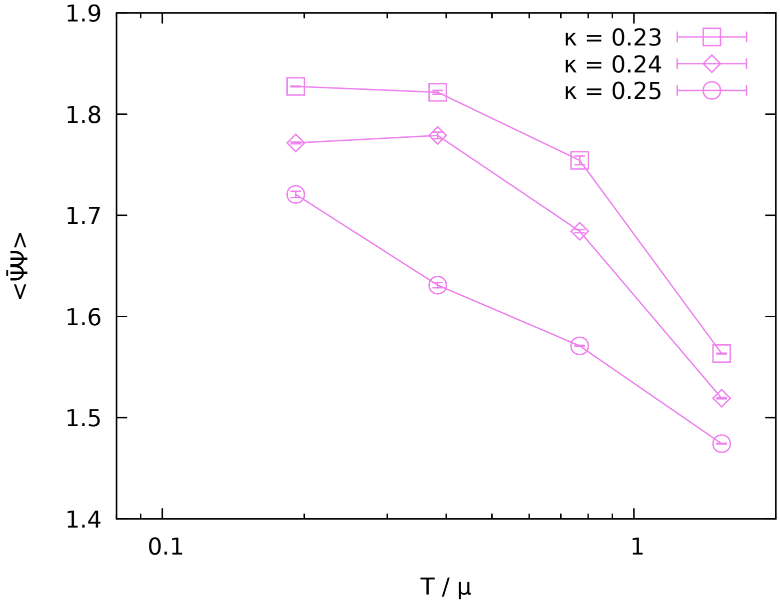

Figure 2: Wilson fermions, $\beta = 6$

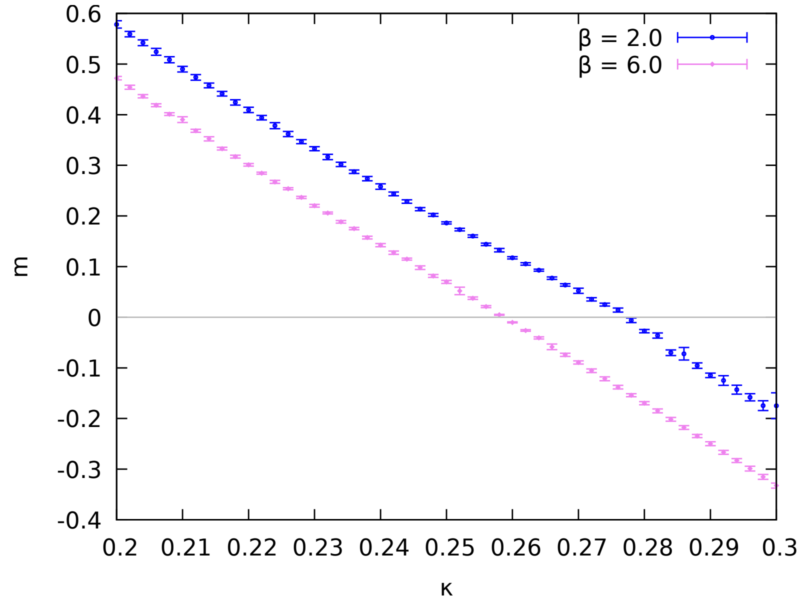

Figure 3: Effective fermion mass m as a function of κ measured via PCAC relation for $\beta = 2$ (blue) and $\beta = 6$ (violet).

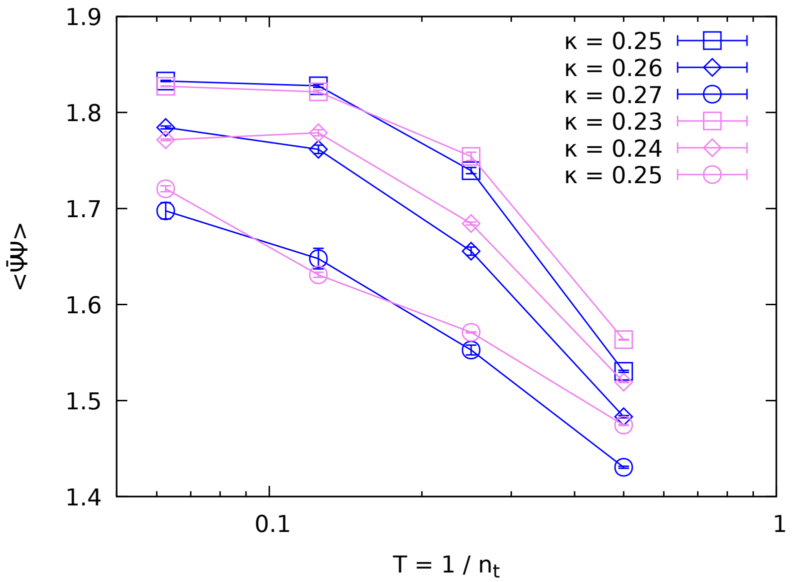

Figure 4: Wilson fermions, $\beta = 2$ (blue) and $\beta = 6$ (violet)