Finite temperature Schwinger model

/preliminary results/

August 20, 2020

1 Notation

Schwinger model at finite temperature T in equilibrium is equivalent to an Euclidean field theory satisfying boundary conditions [Hos95]

$$\psi(x, t + \frac{1}{T}) = -\psi(x, t), \quad A(x, t + \frac{1}{T}) = A(x, t) \tag{1}$$

In the literature it is usual to use β for inverse temperature. However, we are not going to use β because we use it for inverse gauge coupling, i.e.

$$\beta \equiv \frac{1}{g^2} \tag{2}$$

If we denote number of lattice points in temporal direction with n_t and lattice spacing with a, inverse temperature is

$$\frac{1}{T} = a\, n_t \tag{3}$$

2 Predictions

Massless Schwinger model with N_f flavors is exactly solvable [Aff86][1] and there exists $N_f - 1$ massless ("pions") and one massive ("eta") boson with mass

$$\mu = g\sqrt{\frac{N_f}{\pi}} \tag{4}$$

or, using definition (2)

$$\mu = \sqrt{\frac{N_f}{\pi\beta}} \tag{5}$$

[1] Cited after [HHI95] — I have no access to full text of [Aff86] to check it!

In [HHI96] the Schwinger model with N_f flavors of massive fermions at finite temperature T is reduced to quantum mechanical system of $N_f - 1$ degrees of freedom. For degenerate fermion masses m this system can be numerically solved for general values of T/μ and m/μ to get predictions for chiral condensate $\frac{1}{\mu}\langle\bar{\psi}\psi\rangle$. Fig. 3 in [HHI96] shows that for $N_f = 2$ ($\theta = 0$) the condensate which is non-vanishing at $T = 0$ smoothly goes to zero at finite temperature, i.e. *there is no phase transition at finite temperature!*

Somewhat more detailed presentation of the results from [HHI96] is given in the proceedings [Hos95]. Especially interesting is fig. 1 where the dependence of chiral condensate $\frac{1}{\mu}\langle\bar{\psi}\psi\rangle$ on T/μ for several fermion masses in 3-flavor Schwinger model is given. It is compared to $N_f = 1$ massless case [SW92] where the crossover takes place at $\mu = 1$.

Although there is no chance to get the right values of chiral condensate using Wilson fermions, one can hope to see some kind of crossover near $T/\mu = 1$ similar to those shown on fig. 1 in [Hos95]. I started several runs on lattices 16×16, 32×8, 64×4 and 128×2 to see what happens.

Fig. 1 shows the results for $\beta = 2$ (1000 measurements on 16×16 and 500 measurements on other lattices). Qualitatively, picture looks promising: there seems to be crossover near $T/\mu = 1$ and the crossover is smoother for smaller masses (larger κ is closer to κ critical where the fermion mass m vanishes, i.e. larger κ means smaller fermion mass m).

Fig. 2 shows the results for $\beta = 6$ for the same number of measurements. Smaller values for κ are chosen because κ_c for $\beta = 6$ is lower then κ_c for $\beta = 2$.[2] Although the statistical fluctuations are small, it seems that there are some problems with ergodicity (for large β topological transitions are rare, so simulation could be stuck in the same topological sector). However, the picture is similar as for $\beta = 2$ but it is shifted to the right. That doesn't really makes sense and it seems to me that it comes because there is T divided by μ on horizontal axis: from (3) and (5) follows

$$\frac{T}{\mu} = \frac{1}{an_t}\sqrt{\frac{\pi\beta}{N_f}} \tag{6}$$

so there is explicit β dependence. Therefore, I decided to plot both β-s on the same graph (fig. 3), but with

$$T = \frac{1}{n_t} \tag{7}$$

[2] I guess that the fermion masses in fig. 1 and fig. 2 are similar but that should be checked, e.g. with the help of PCAC relation [HLT98, GHL99]

instead of T/μ on the horizontal axis. The crossovers at both β seems to take place at the same temperature, but it is against pyhsical intuition: boson mass μ sets the scale and it is to be expected that the position of crossover depends on μ, exactly as it is claimed in [HHI96, Hos95]?!

References

[Aff86] Ian Affleck. On the Realization of Chiral Symmetry in (1+1)-dimensions. *Nucl. Phys. B*, 265:448–468, 1986.

[GHL99] Christof Gattringer, Ivan Hip, and C.B. Lang. The Chiral limit of the two flavor lattice Schwinger model with Wilson fermions. *Phys. Lett. B*, 466:287–292, 1999.

[HHI95] J.E. Hetrick, Y. Hosotani, and S. Iso. The Massive multi - flavor Schwinger model. *Phys. Lett. B*, 350:92–102, 1995.

[HHI96] J.E. Hetrick, Y. Hosotani, and S. Iso. The Interplay between mass, volume, vacuum angle and chiral condensate in N flavor QED in two-dimensions. *Phys. Rev. D*, 53:7255–7259, 1996.

[HLT98] I. Hip, C.B. Lang, and R. Teppner. Chiral symmetry in the two flavor lattice Schwinger model. *Nucl. Phys. B Proc. Suppl.*, 63:682–684, 1998.

[Hos95] Yutaka Hosotani. Massive multi - flavor Schwinger model at finite temperature and on compact space. In *4th Workshop on Thermal Field Theories and Their Applications*, pages 355–364, 10 1995.

[SW92] Ivo Sachs and Andreas Wipf. Finite temperature Schwinger model. *Helv. Phys. Acta*, 65:652–678, 1992.

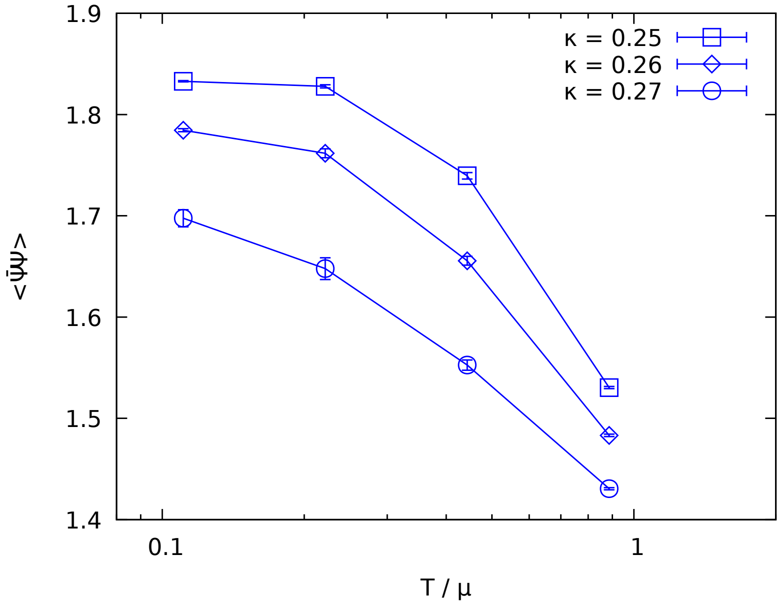

Figure 1: Wilson fermions, $\beta = 2$

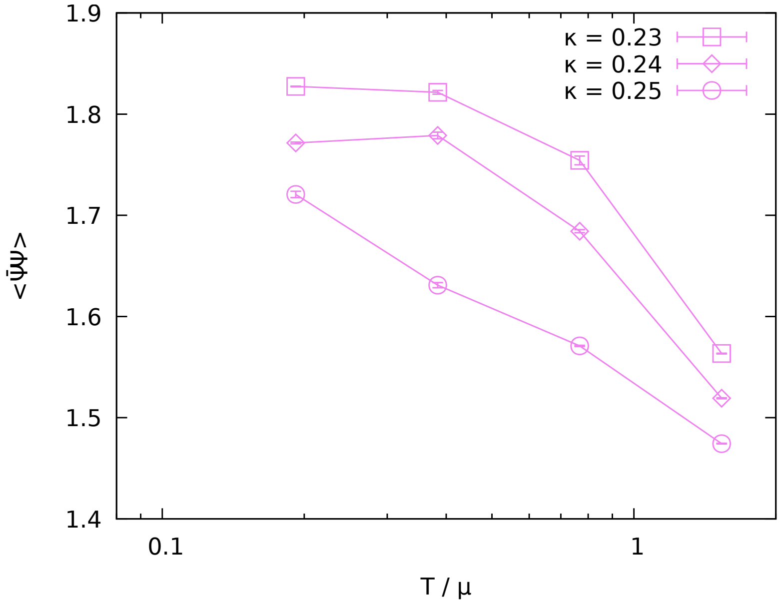

Figure 2: Wilson fermions, $\beta = 6$

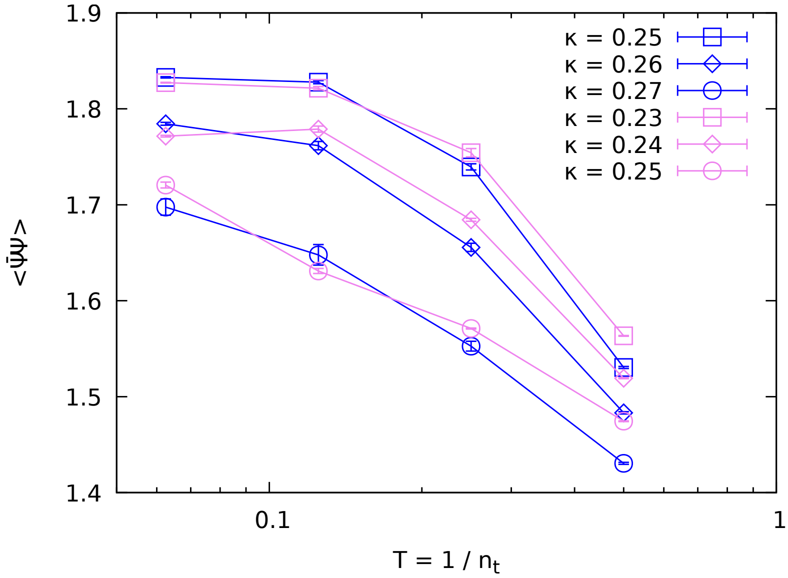

Figure 3: Wilson fermions, $\beta = 2$ (blue) and $\beta = 6$ (violet)