Finite temperature and δ-regime in the Schwinger model

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Schwinger model

- introduced by [Schwinger, 1961]: *two-dimensional quantum electrodynamics* — fermions coupled to Abelian gauge field
- simple example for chiral anomaly and confinement
- often used as a testbed for conceptual and numerical approaches in lattice field theory
- nevertheless, some of the rich physical properties of the model in anisotropic volumes have not yet been tested:

Finite temperature : Hosotani solution has not been compared with the lattice simulation results

δ-regime : conjecture for the residual pion mass

N-flavor Schwinger model

- massless case has analytic solution [Belvedere et al., 1979]
- one massive boson ("eta") — g is gauge coupling

$$m_\eta^2 = N\,\frac{g^2}{\pi}$$

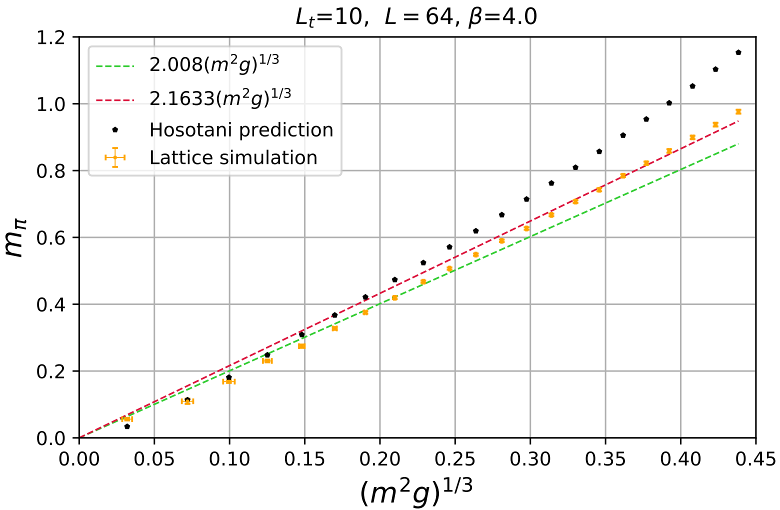
- $N - 1$ massless bosons ("pions")
- massive case (fermion mass $m > 0$) has no exact solution
- semiclassical prediction at infinite volume

$$m_\pi(m) = \left(4e^{2\gamma}\sqrt{\frac{2}{\pi}}\right)(m^2 g)^{1/3} = 2.1633...(m^2 g)^{1/3}$$

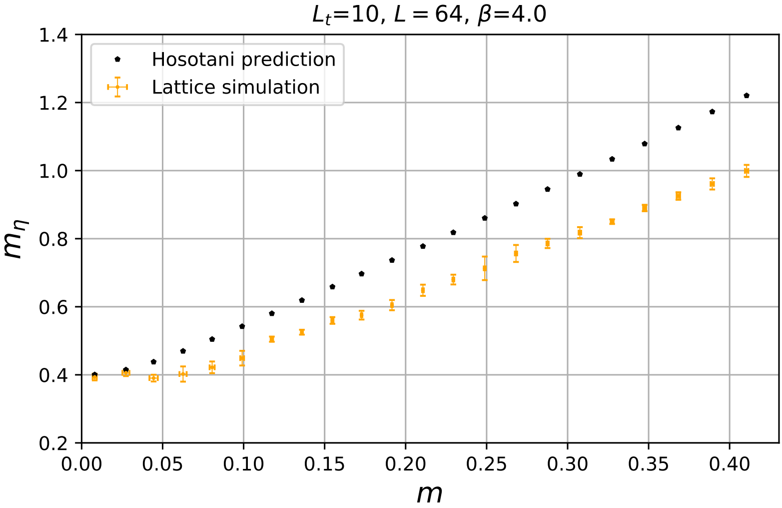
Finite temperature - Hosotani approximate solution

- approximate solution by Hosotani et al. based on bosonization
- finite temperature Schwinger model reduced to quantum mechanical system with $N-1$ degrees of freedom
- set of equations which can be solved numerically to arbitrary precision to get the boson masses μ_1 and μ_2
- for N degenerate flavors:
    - $N-1$ bosons with the mass μ_2
    - one boson with mass $\mu_1 > \mu_2$
- for the fermion mass $m \to 0$:
    - $\mu_2 \to 0$ ("pion")
    - $\mu_1 \to m_\eta$ ("eta")
- in the limit $L_t \to 0$ Hosotani solution converges to semiclassical result

Pion mass - Hosotani vs. lattice simulation



Pion mass - Hosotani vs. lattice simulation



δ-regime

- spatial volume is small compared to the correlation length

$$\xi = m_\pi^{-1}$$

but the Euclidean time extent is large

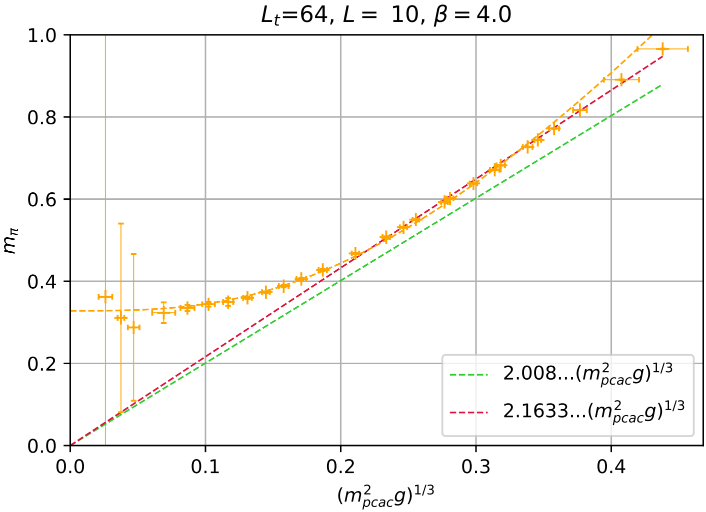
$$L_t \gg \xi \gtrsim L$$

- the system is quasi one dimensional - approximation by quantum rotor [Leutwyler, 1987]
- pion has *residual mass*

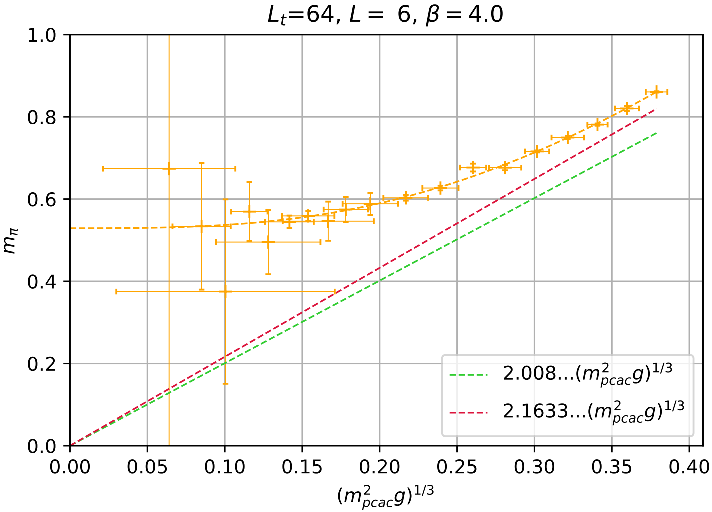
$$m_\pi^R = \frac{3}{2\Theta}$$

where Θ is the moment of inertia

Residual pion mass plateau: $L_t = 64, L = 10$



Residual pion mass plateau: $L_t = 64, L = 6$



Conjecture

- [Hasenfratz and Niedermayer, 1993] computed Θ up to next-to-leading order, for a general dimension $d > 2$

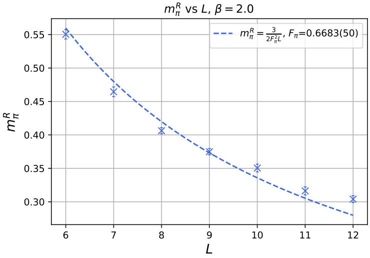
$$\Theta = F_\pi^2 L^{d-1} \left[1 + \frac{\mathcal{N} - \in}{4\pi F_\pi^2 L^{d-2}} \left(2\frac{d-1}{d-2} + \ldots \right) \right]$$

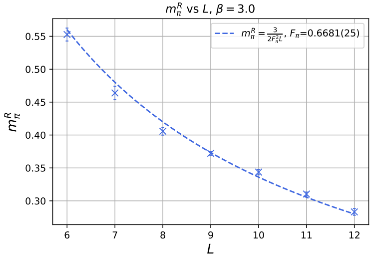
- in two dimensions ($d = 2$) there is a divergence of the next to leading term, so we just consider the leading term

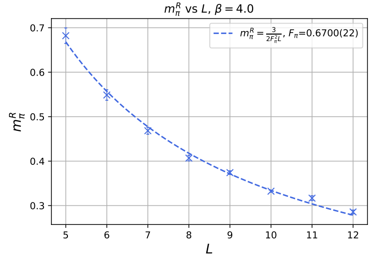
$$m_\pi^R \simeq \frac{3}{2F_\pi^2 L}$$

- we verify the relation $m_\pi^R \propto 1/L$ with simulation data and extract the value of pion decay constant F_π

1 / L confirmed by lattice simulation







β	F_π
2.0	0.6683(50)
3.0	0.6681(25)
4.0	0.6700(22)

$$F_\pi = 0.6688(5)$$

Witten-Veneziano formula

- in the chiral N-flavor Schwinger model the Witten-Veneziano formula is simplified to [Seiler and Stamatescu, 1987]

$$m_\eta^2 = \frac{2N}{F_\eta^2}\chi_T^{que}$$

- mass of the η particle is known analytically [Belvedere et al., 1979]

$$m_\eta^2 = \frac{N}{\pi\beta}$$

- continuum prediction for χ_T^{que} [Seiler and Stamatescu, 1987]

$$\beta\chi_T^{que} = \frac{1}{4\pi^2}$$

Quenched topological susceptibility

- [Bardeen et al., 1998] were able to analytically compute χ_T^{que} on the lattice

$$\beta\chi_T^{que} = \frac{I_1(\beta)}{4\pi^2 I_0(\beta)}$$

by using an alternative definition of topological charge

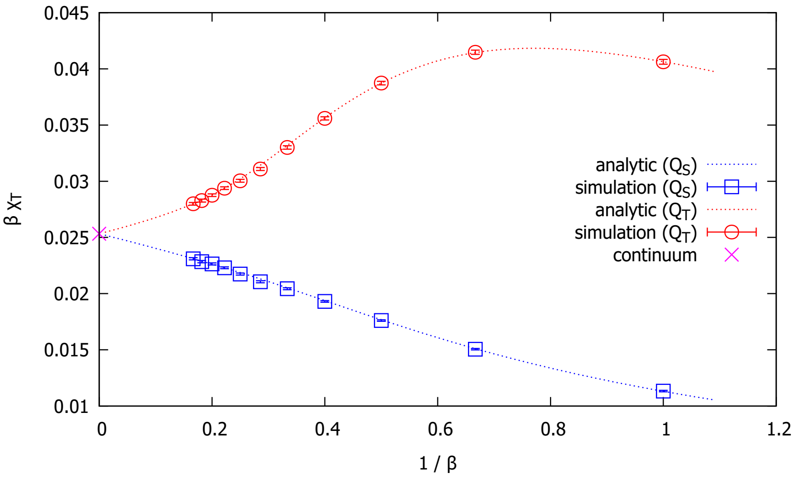
$$Q_S = \frac{1}{2\pi}\sum_P \sin(\theta_P)$$

- for the usual definition of topological charge

$$Q_T = \frac{1}{2\pi}\sum_P \theta_P$$

it is not possible to find analytic expression, but using the same line of reasoning it is possible to numerically compute χ_T^{que} to arbitrary precision

Quenched topological susceptibility



F_η versus F_π

- in large N_c QCD, to the order $1/N_c$

$$F_{\eta'} = F_\pi$$

- in the Schwinger model nothing assures that this relation holds
- inserting the confirmed values for m_η^2 and χ_T^{que}

$$F_\eta^2 = \frac{2N}{m_\eta^2}\chi_T^{que} = 2N\left(\frac{\pi\beta}{N}\right)\left(\frac{1}{4\pi^2\beta}\right) = \frac{1}{2\pi}$$

- our results suggest that in the Schwinger model these two decay constants differ significantly

$$F_\eta = 0.3989 \qquad F_\pi = 0.6688(5)$$