

Jaime Fabián Nieto Castellanos

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Results for $\beta = 2.0$ are shown. Everything was obtained with simulations using the parameters of the following table:

Ntime	64
Ntherm	1000
Nmeasure	1000
Trajectory Steps	10
Nsteps	10
β	2

Table 1: All the simulations were performed with this parameters.

The residual pion mass is extrapolated with two different methods. From the plots of m_π^2 vs. m_{pcac} one can fit a parabola

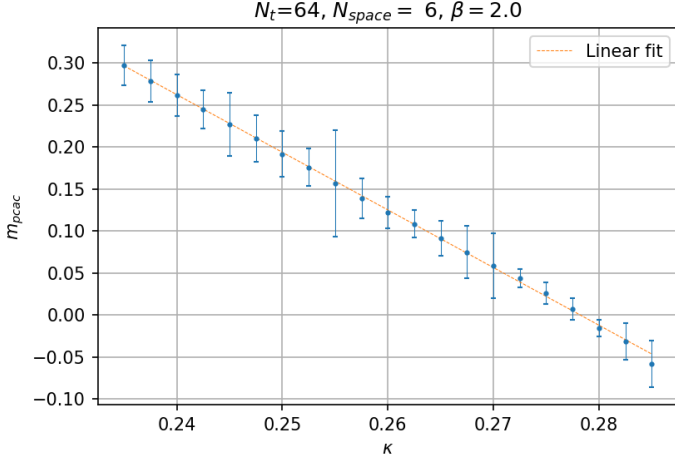
$$y = a + bx^2. \quad (1)$$

Meanwhile, from the plots of m_π vs. $(gm_{pcac}^2)^{1/3}$ one can fit a function of the form

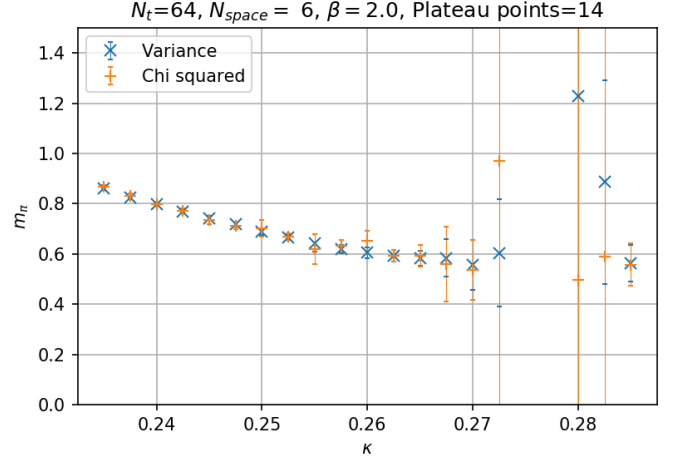
$$y = \sqrt{a + bx^3}. \quad (2)$$

$\beta = 2.0$ results

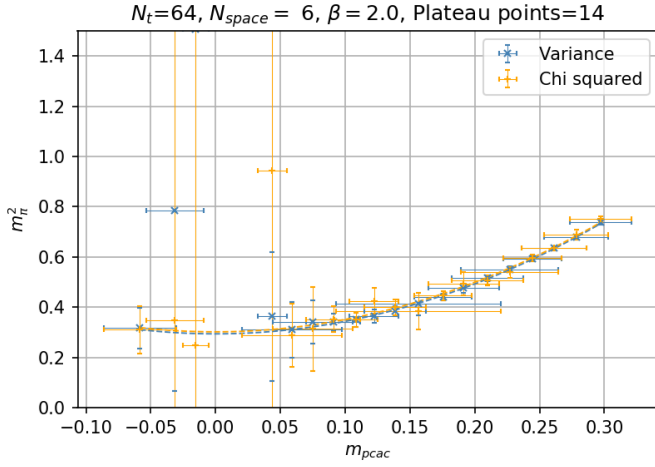
6x64



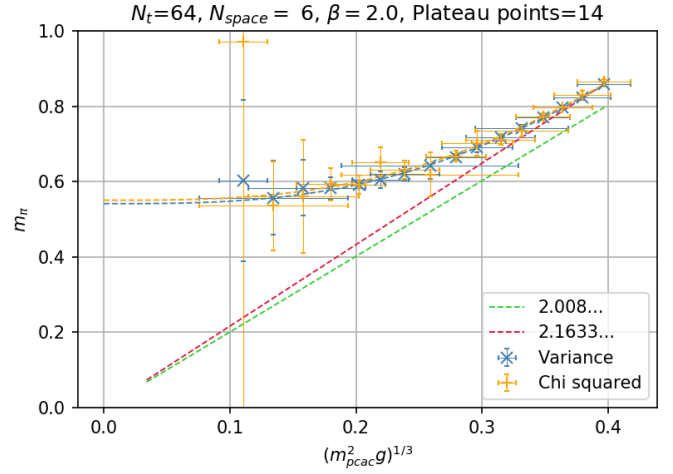
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27824 \pm 0.00307$



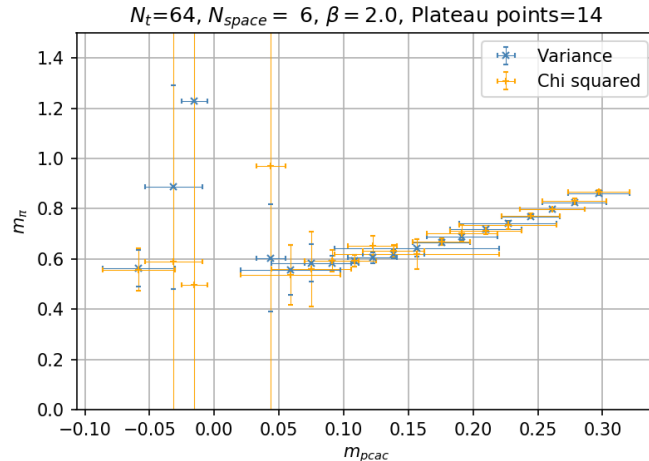
(b) Pion mass as a function of κ

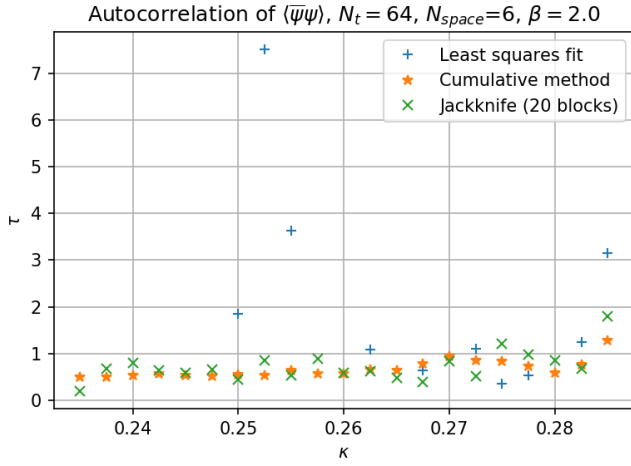


(c) m_π^2 vs. m_{pcac} . m_π^2 vs. m_{pcac} . A function of the form $a + bx^2$ was fitted, the coefficients are $a = 0.29377 \pm 0.00403$, $b = 4.96973 \pm 0.11092$, $m_\pi = 0.54201 \pm 0.00372$ for variance and $a = 0.30172 \pm 0.00672$, $b = 4.93474 \pm 0.16813$, $m_\pi = 0.54929 \pm 0.00612$ for chi squared.

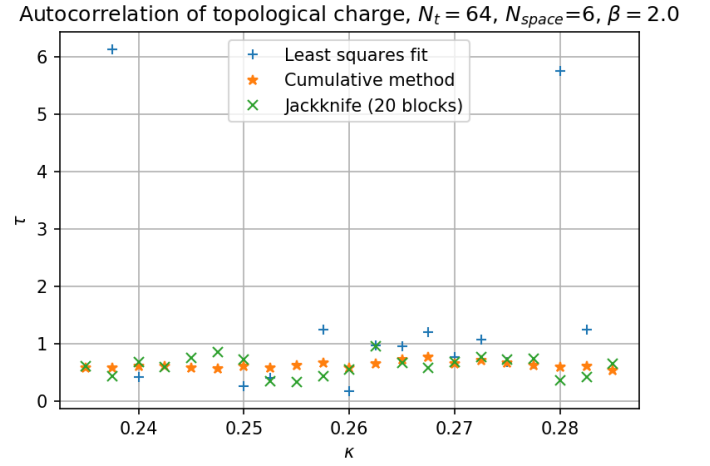


(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. $a = 0.29312 \pm 0.00239$, $b = 7.04747 \pm 0.09194$, $m_\pi = 0.54141 \pm 0.00221$ for variance and $a = 0.303 \pm 0.00769$, $b = 6.94703 \pm 0.26847$, $m_\pi = 0.55045 \pm 0.00698$ for chi squared.





(f) Autocorrelation of $\langle \bar{\psi} \psi \rangle$



(g) Autocorrelation of the topological charge

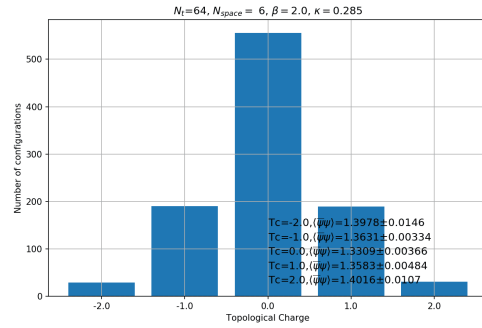
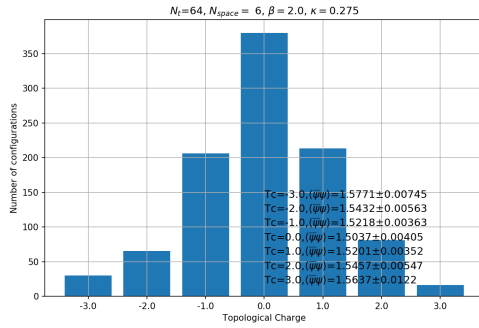
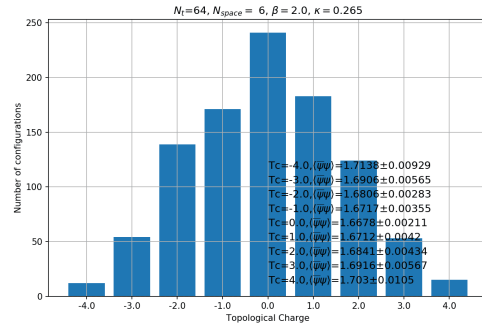
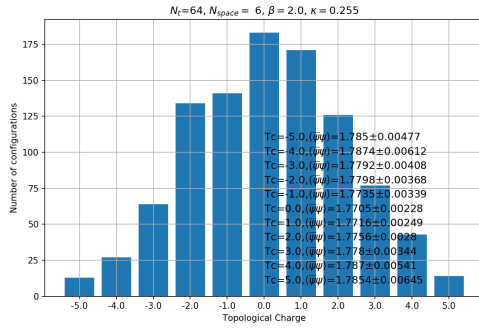
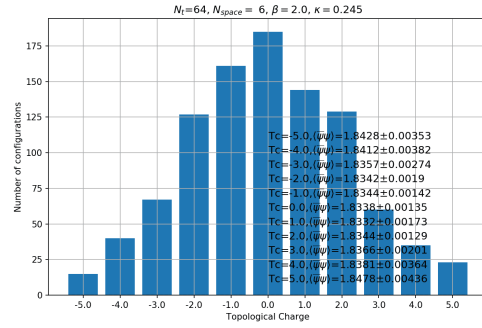
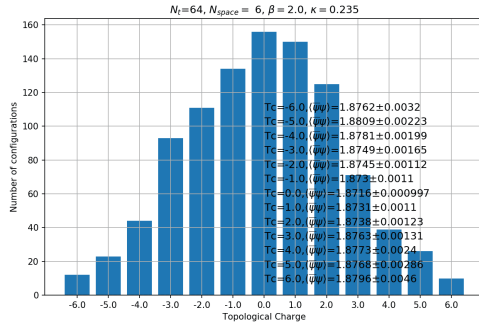
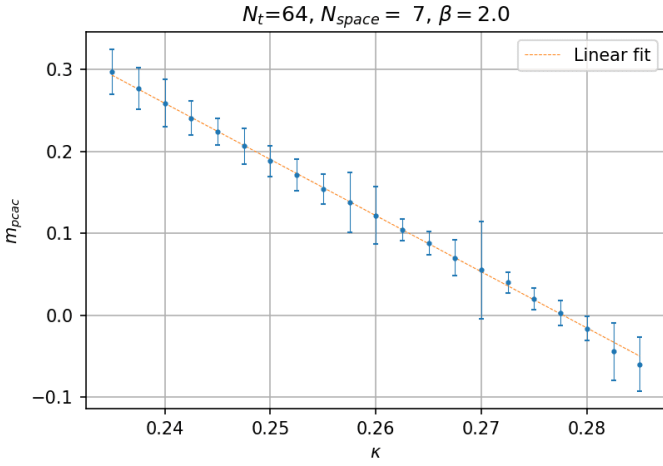
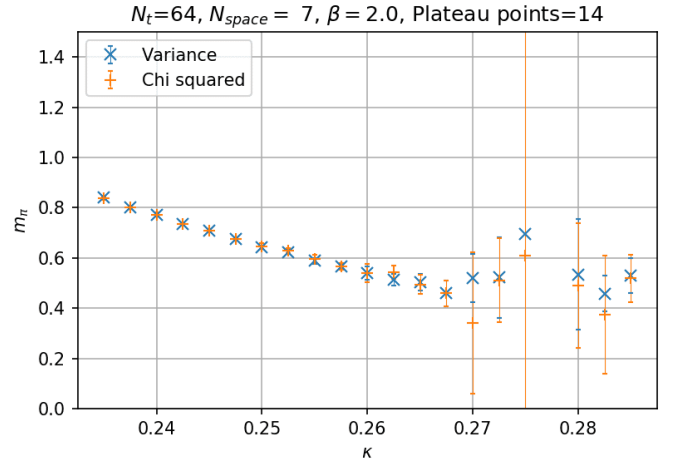
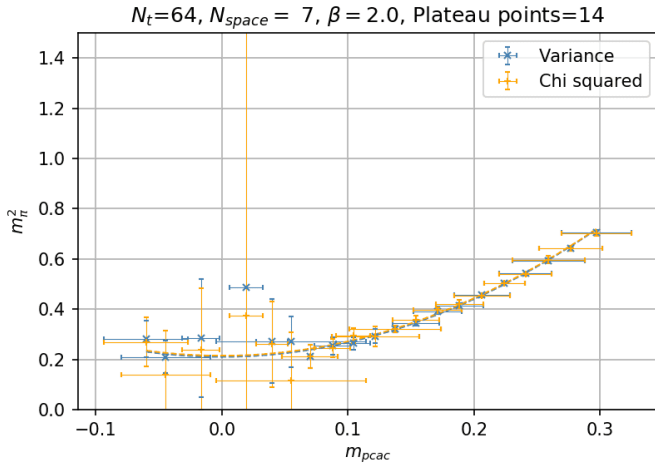
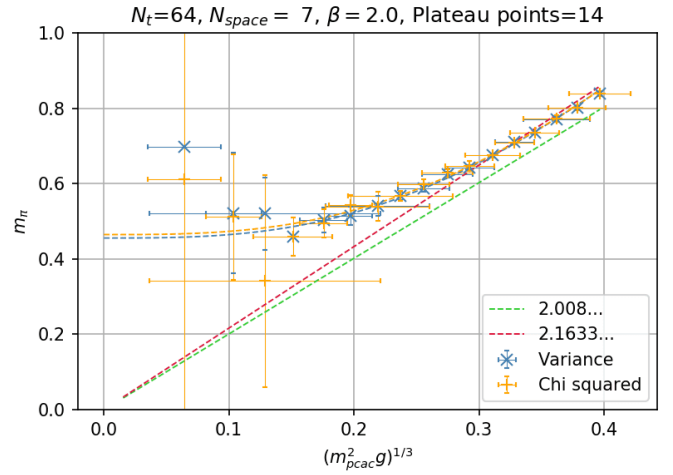


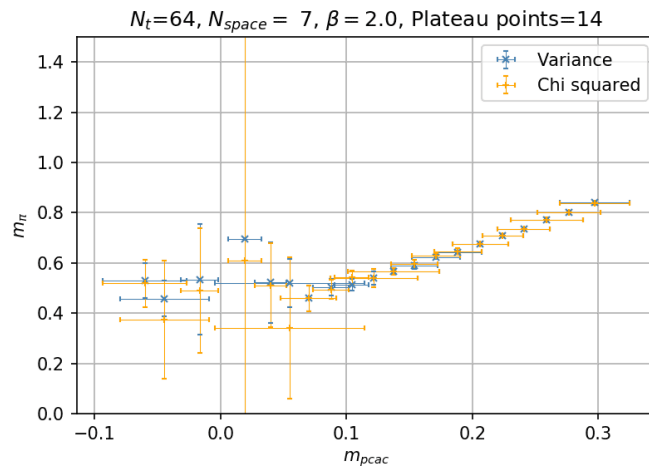
Figure 1: Number of configurations vs. topological charge on a 6×64 lattice.

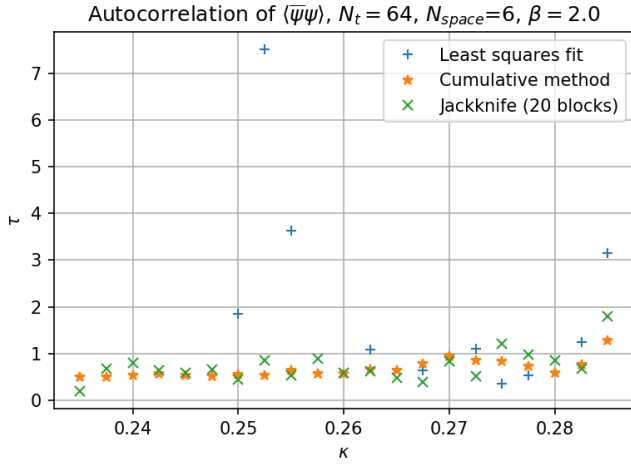
(a) Fermion mass using PCAC relation, $\kappa_c = 0.2777 \pm 0.00236$.(b) Pion mass as a function of κ 

(c) m_π^2 vs. m_{pcac} . m_π^2 vs. m_{pcac} . m_π^2 vs. m_{pcac} . A function of the form $a + bx^2$ was fitted, the coefficients are $a = 0.20986 \pm 0.00523$, $b = 5.7568 \pm 0.15985$, $m_\pi = 0.45811 \pm 0.00571$ for variance and $a = 0.2147 \pm 0.00684$, $b = 5.72429 \pm 0.19724$, $m_\pi = 0.46336 \pm 0.00738$ for chi squared.

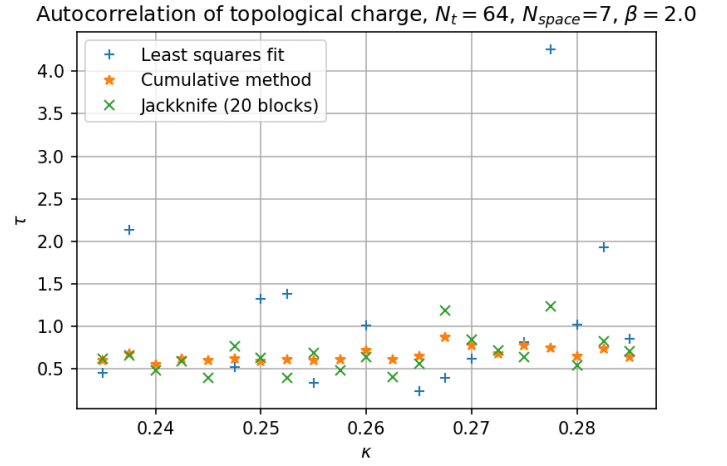


(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. $a = 0.20749 \pm 0.00464$, $b = 8.21704 \pm 0.19521$, $m_\pi = 0.45551 \pm 0.0051$ for variance and $a = 0.21579 \pm 0.00667$, $b = 8.06097 \pm 0.26745$, $m_\pi = 0.46453 \pm 0.00718$ for chi squared.

(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle \bar{\psi}\psi \rangle$



(g) Autocorrelation of the topological charge

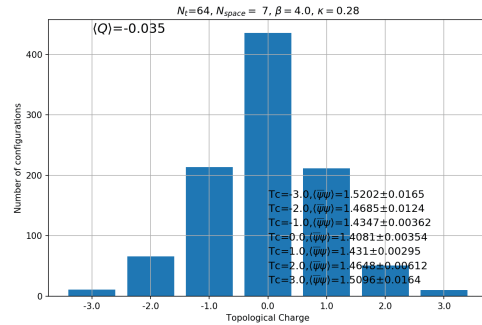
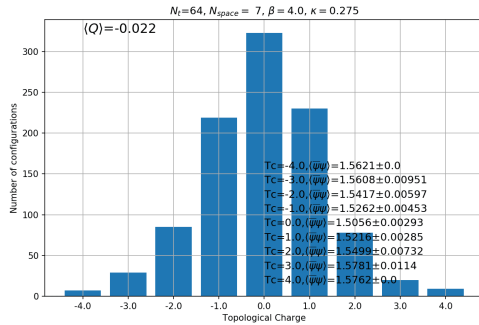
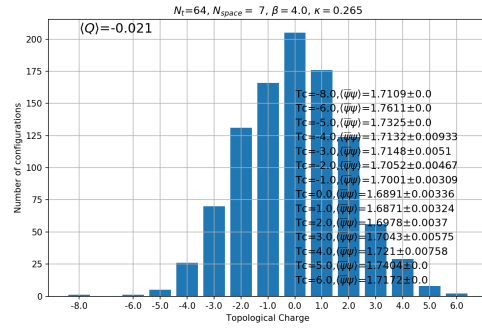
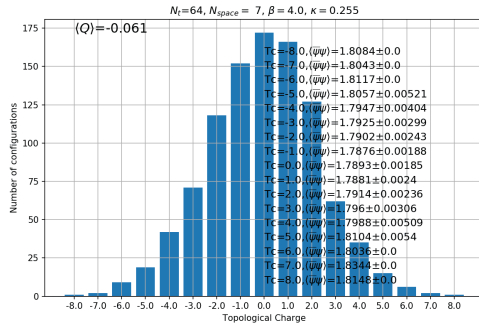
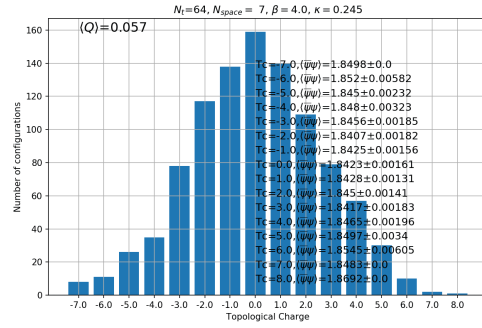
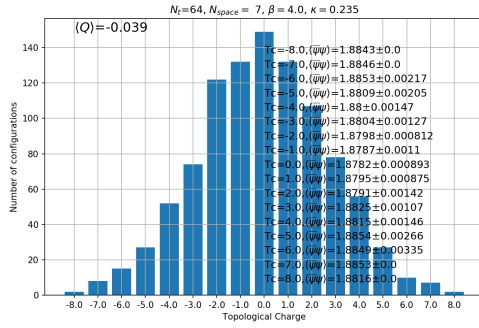
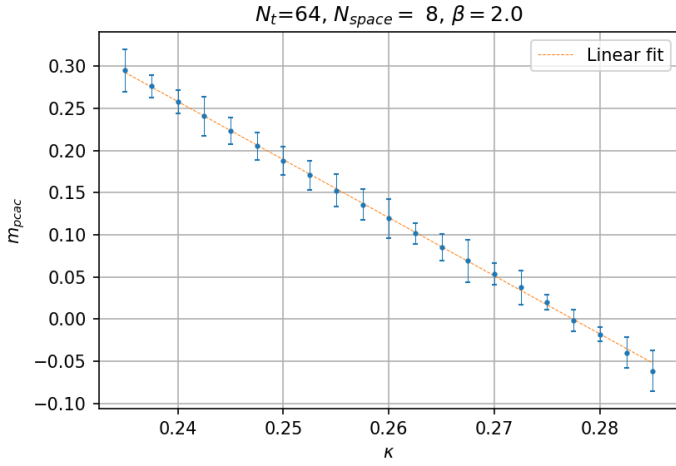
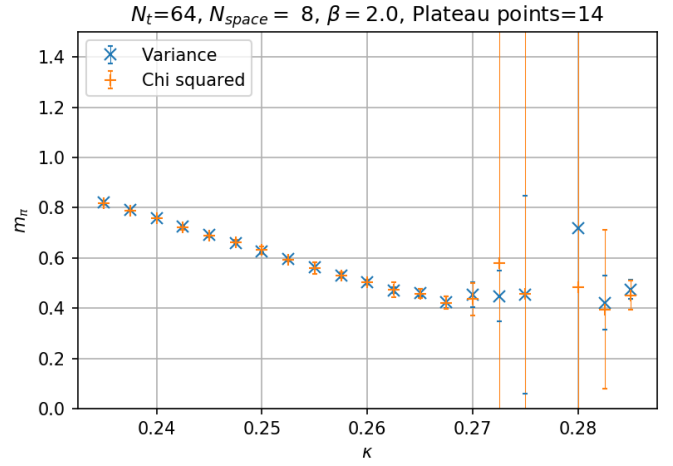


Figure 2: Number of configurations vs. topological charge on a 7×64 lattice.

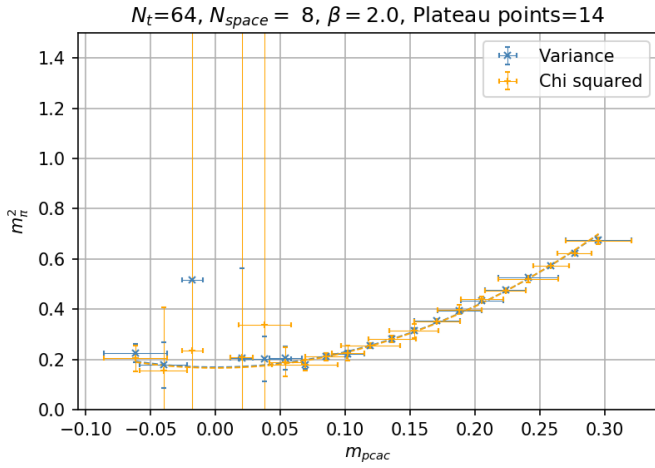
8x64



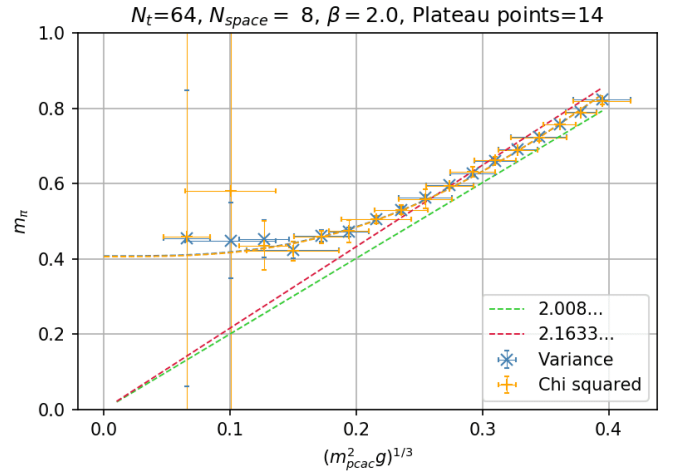
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27744 \pm 0.00196$



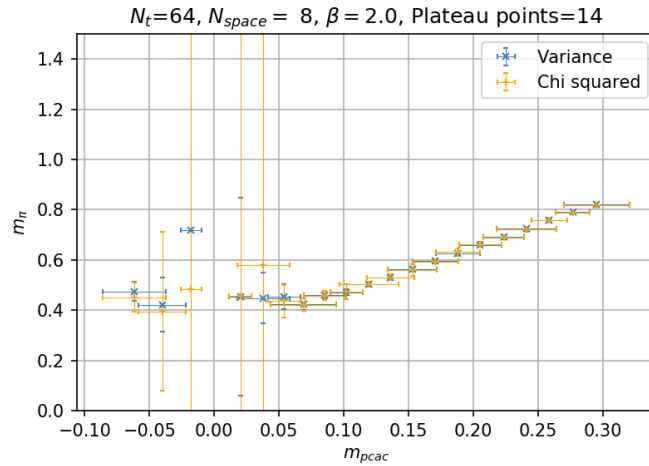
(b) Pion mass as a function of κ



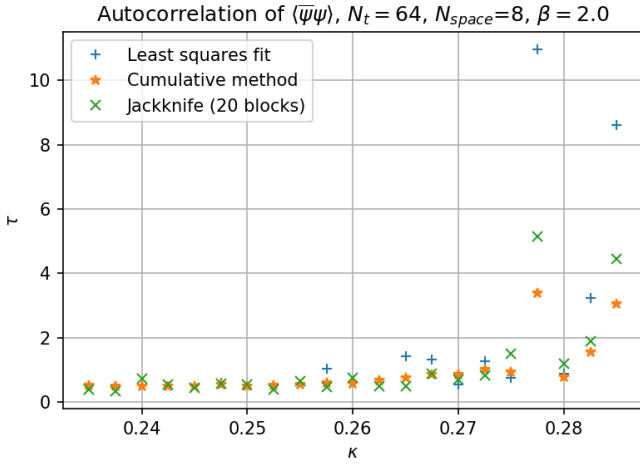
(c) m_π^2 vs. m_{pcac} . m_π^2 vs. m_{pcac} . A function of the form $a + bx^2$ was fitted, the coefficients are $a = 0.16902 \pm 0.00391$, $b = 6.07619 \pm 0.13206$, $m_\pi = 0.41112 \pm 0.00475$ for variance and $a = 0.16587 \pm 0.00337$, $b = 6.13639 \pm 0.10268$, $m_\pi = 0.40727 \pm 0.00413$ for chi squared



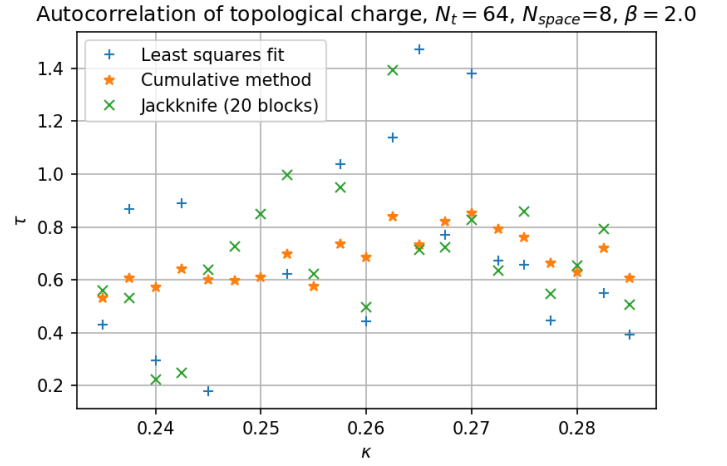
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. $a = 0.16636 \pm 0.00355$, $b = 8.67422 \pm 0.16583$, $m_\pi = 0.40787 \pm 0.00435$ for variance and $a = 0.16505 \pm 0.00366$, $b = 8.70103 \pm 0.15485$, $m_\pi = 0.40627 \pm 0.0045$ for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle \bar{\psi}\psi \rangle$



(g) Autocorrelation of the topological charge

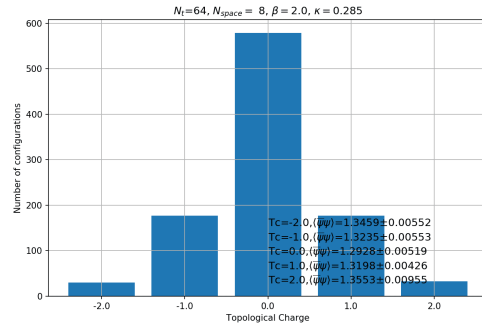
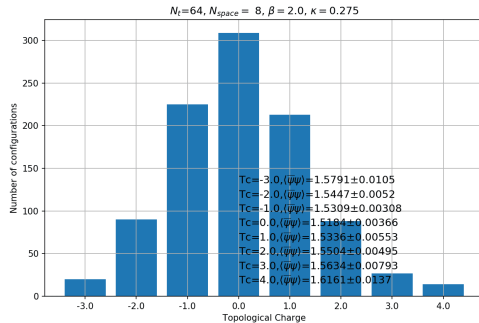
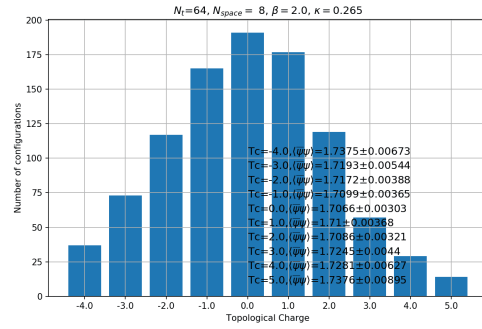
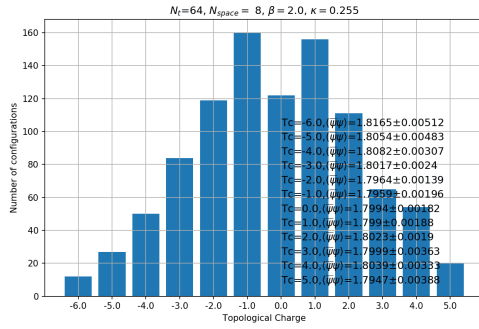
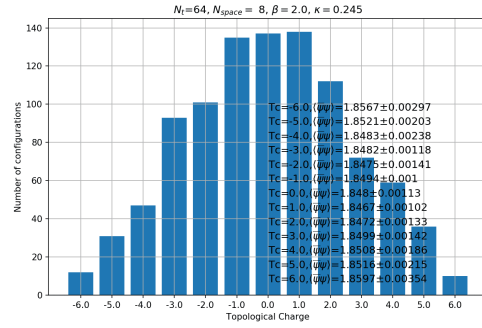
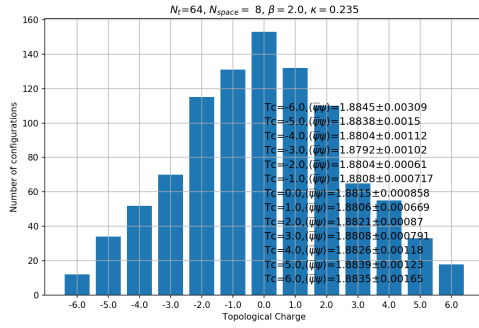
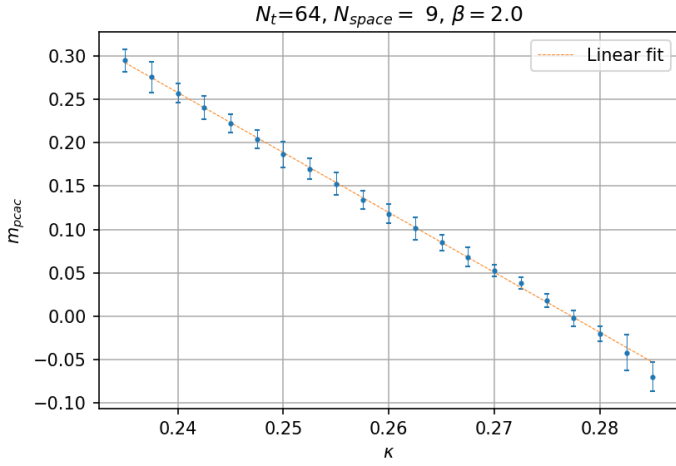
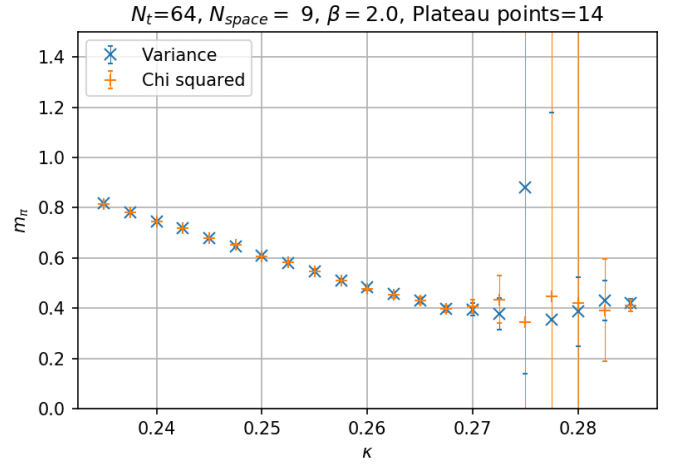
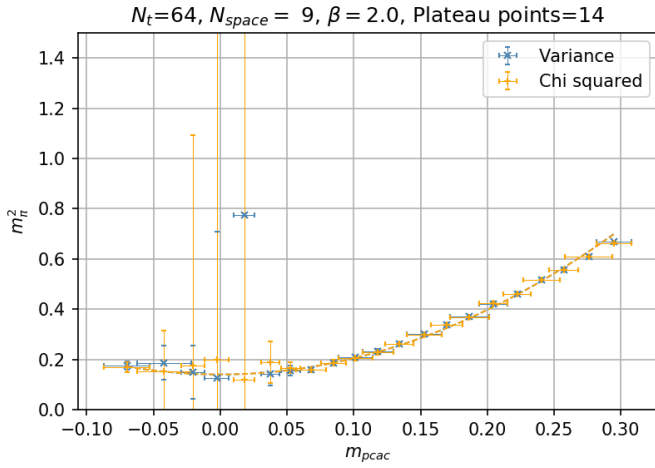
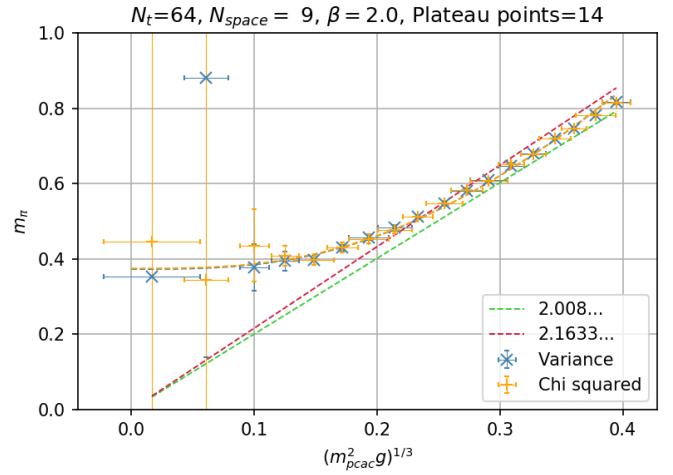
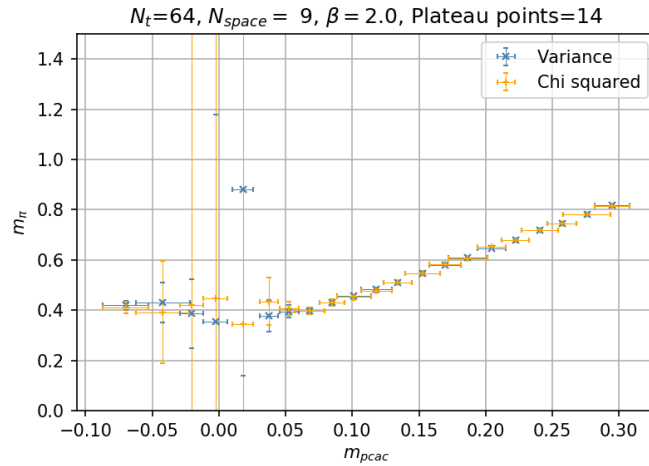
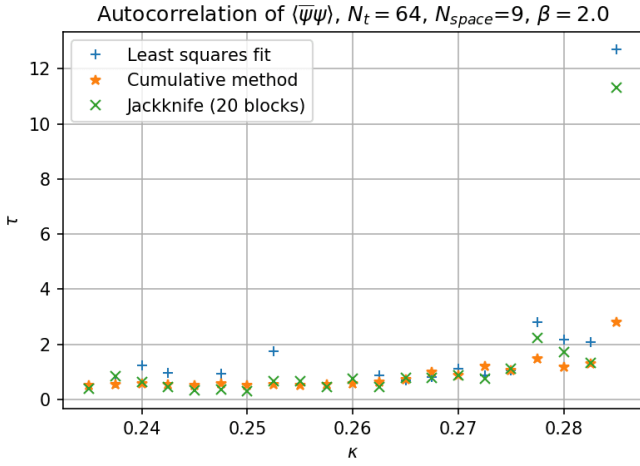
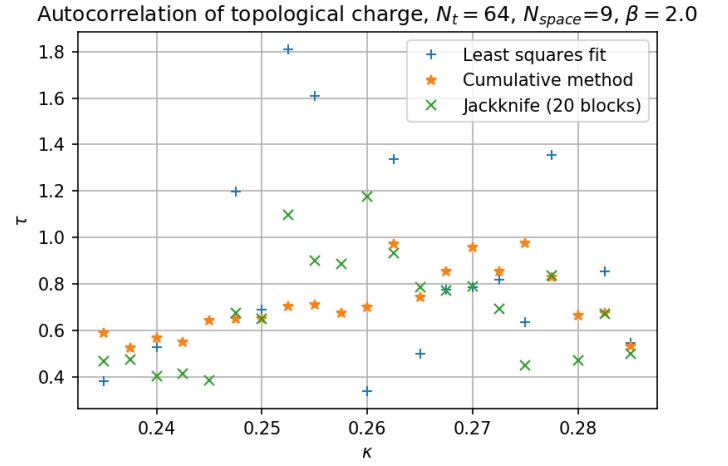


Figure 3: Number of configurations vs. topological charge on a 8×64 lattice.

(a) Fermion mass using PCAC relation, $\kappa_c = 0.27728 \pm 0.00299$.(b) Pion mass as a function of κ (c) m_π^2 vs. m_{pcac} . A function of the form $a + bx^2$ was fitted, the coefficients are $a = 0.1399 \pm 0.00274$, $b = 6.44178 \pm 0.11343$, $m_\pi = 0.37403 \pm 0.00366$ for variance and $a = 0.14015 \pm 0.00305$, $b = 6.42378 \pm 0.11876$, $m_\pi = 0.37436 \pm 0.00408$ for chi squared.(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. $a = 0.13913 \pm 0.00312$, $b = 9.1345 \pm 0.17585$, $m_\pi = 0.373 \pm 0.00418$ for variance and $a = 0.14058 \pm 0.0035$, $b = 9.06774 \pm 0.18662$, $m_\pi = 0.37494 \pm 0.00466$ for chi squared.(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle \bar{\psi} \psi \rangle$



(g) Autocorrelation of the topological charge

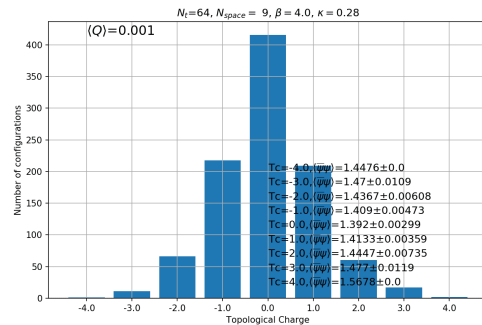
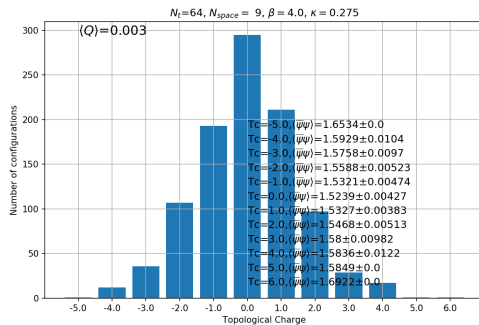
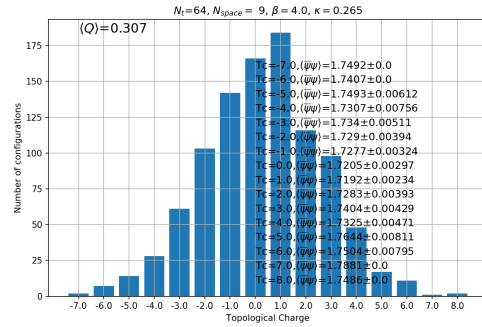
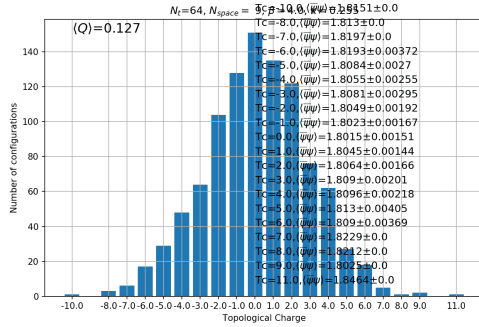
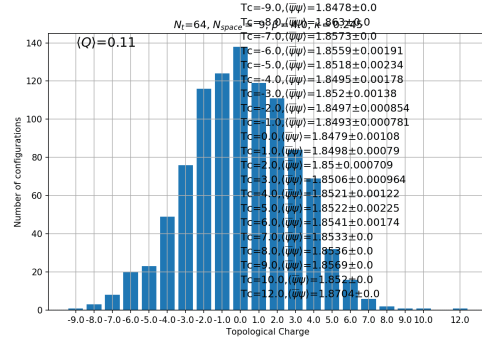
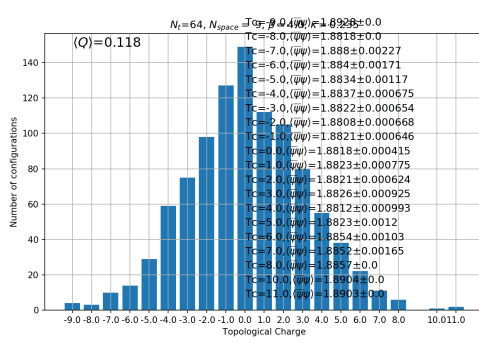
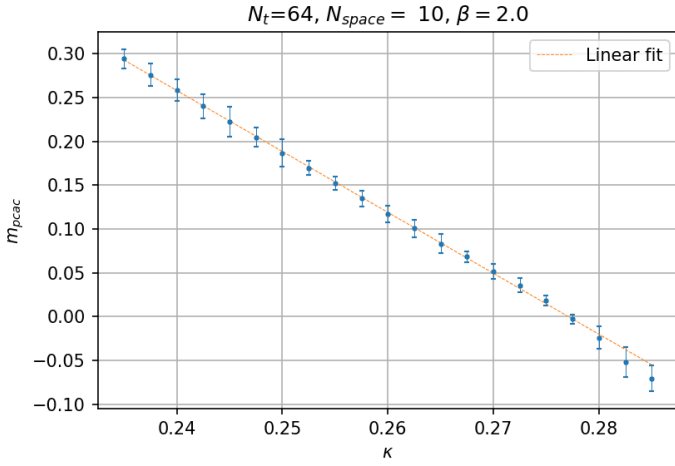
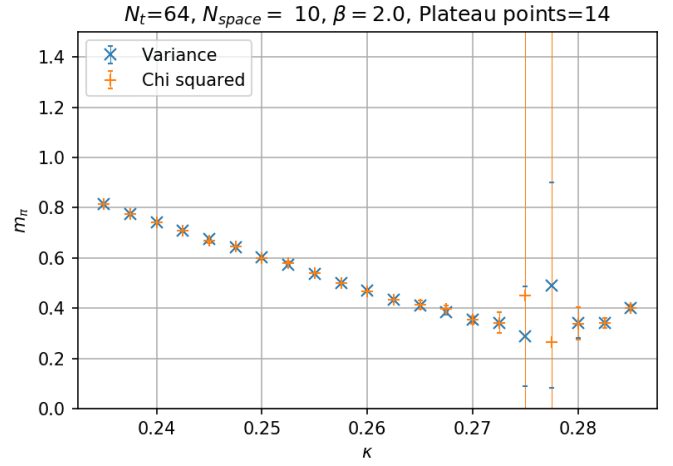
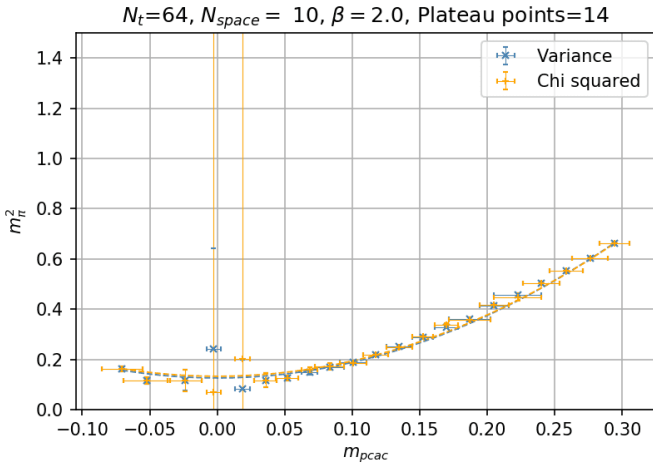
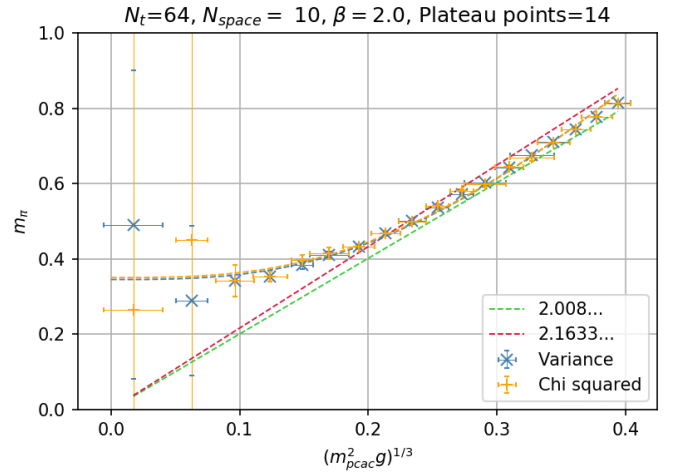
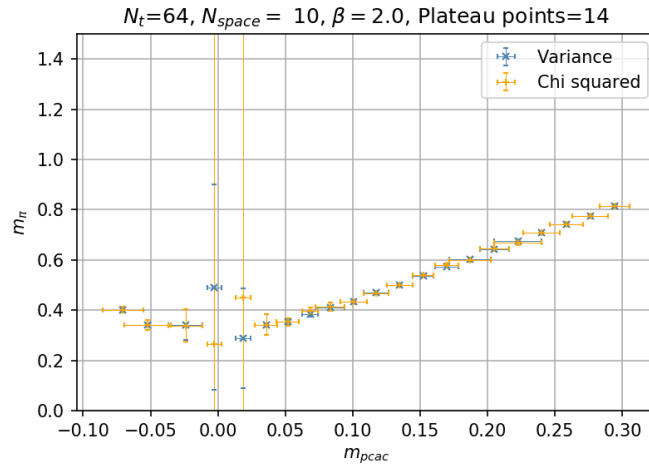
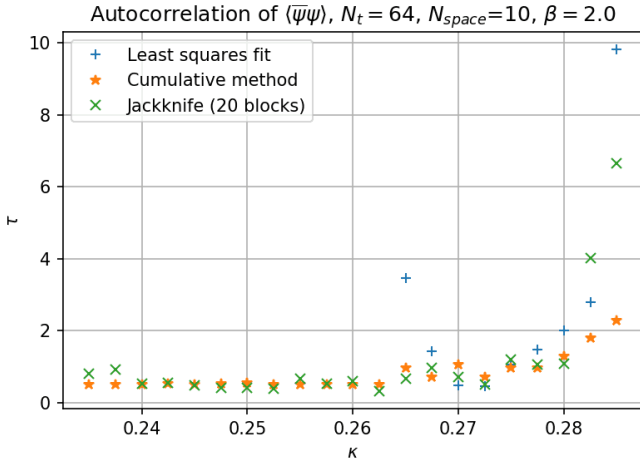
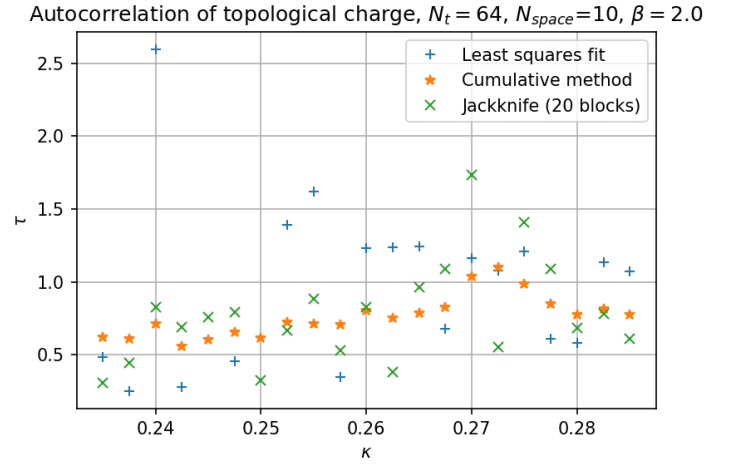


Figure 4: Number of configurations vs. topological charge on a 9×64 lattice.

(a) Fermion mass using PCAC relation, $\kappa_c = 0.27712 \pm 0.00335$ (b) Pion mass as a function of κ (c) m_π^2 vs. m_{pcac} . A function of the form $a + bx^2$ was fitted, the coefficients are $a = 0.11777 \pm 0.0032$, $b = 6.78654 \pm 0.16208$, $m_\pi = 0.34317 \pm 0.00466$ for variance and $a = 0.12042 \pm 0.0038$, $b = 6.75557 \pm 0.18446$, $m_\pi = 0.34702 \pm 0.00547$ for chi squared.(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. $a = 0.11927 \pm 0.00336$, $b = 9.52723 \pm 0.22213$, $m_\pi = 0.34536 \pm 0.00486$ for variance and $a = 0.12283 \pm 0.00406$, $b = 9.45064 \pm 0.25607$, $m_\pi = 0.35048 \pm 0.00579$ for chi squared(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle \bar{\psi}\psi \rangle$



(g) Autocorrelation of the topological charge

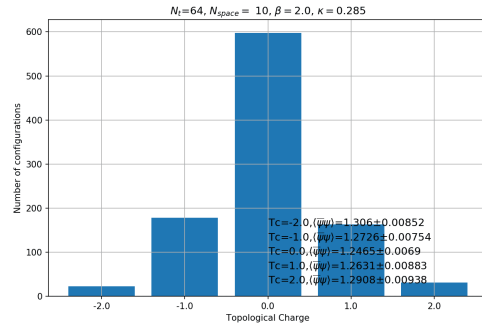
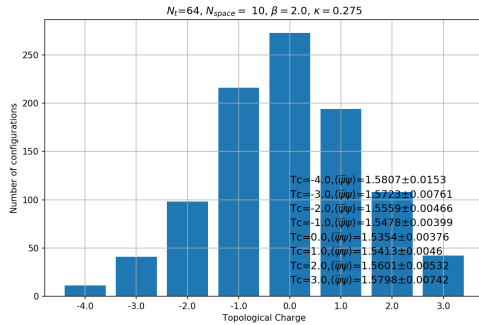
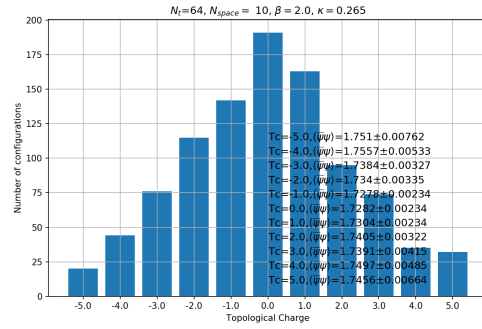
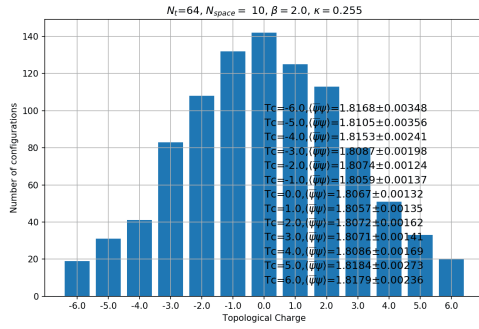
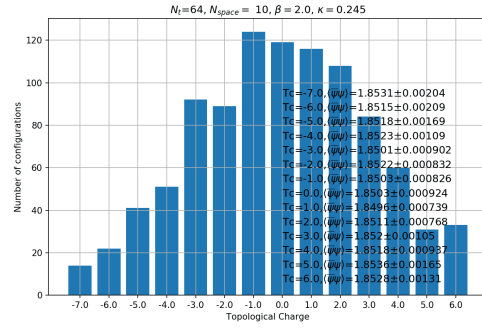
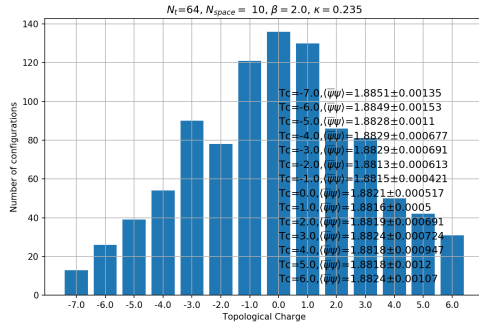
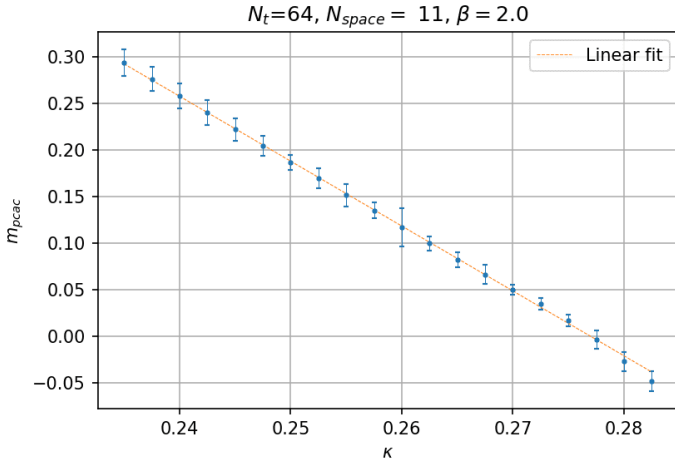
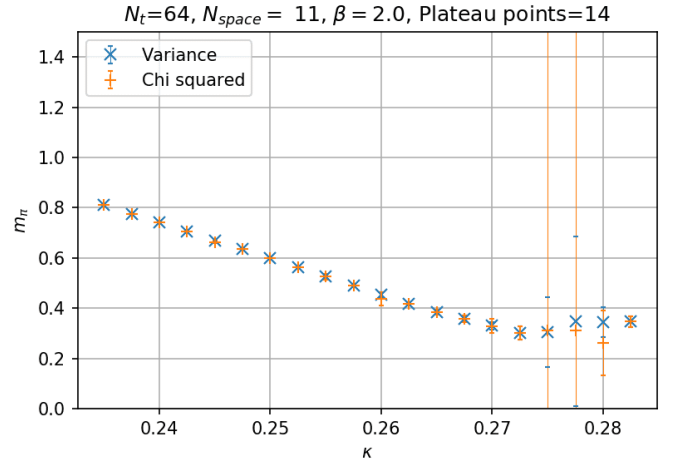
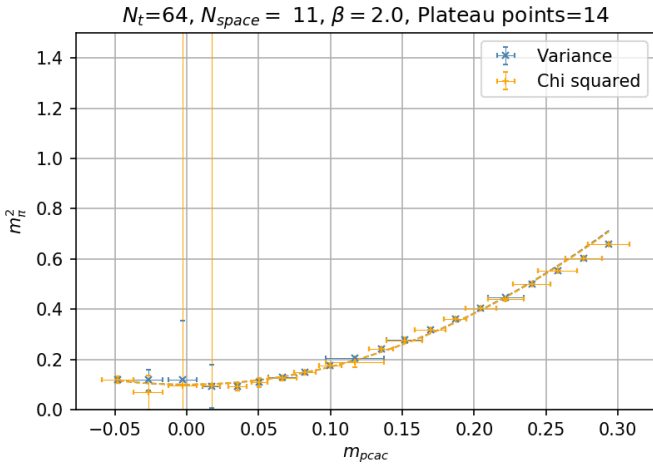
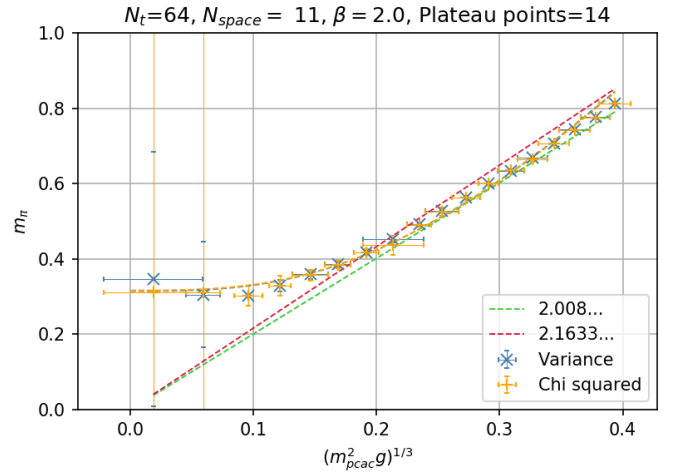


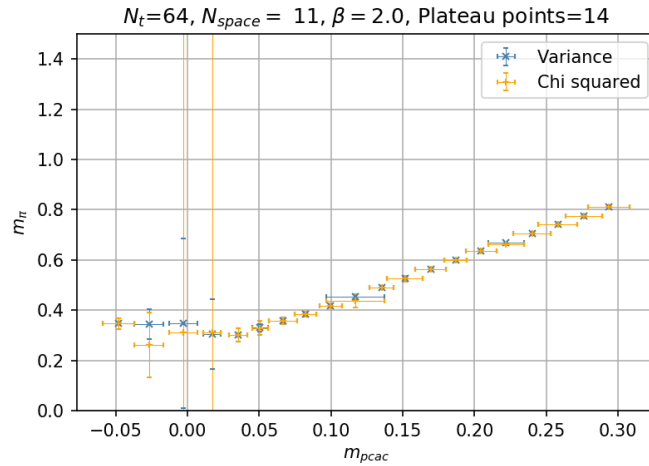
Figure 5: Number of configurations vs. topological charge on a 10×64 lattice.

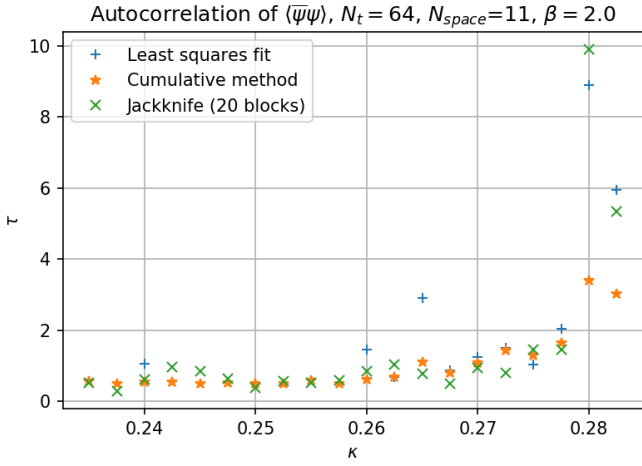
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27703 \pm 0.00297$ (b) Pion mass as a function of κ 

(c) m_π^2 vs. m_{pcac} . A function of the form $a + bx^2$ was fitted, the coefficients are $a = 0.09948 \pm 0.003$, $b = 7.1086 \pm 0.16062$, $m_\pi = 0.31541 \pm 0.00476$ for variance and $a = 0.09989 \pm 0.00334$, $b = 7.06439 \pm 0.16118$, $m_\pi = 0.31605 \pm 0.00528$ for chi squared.

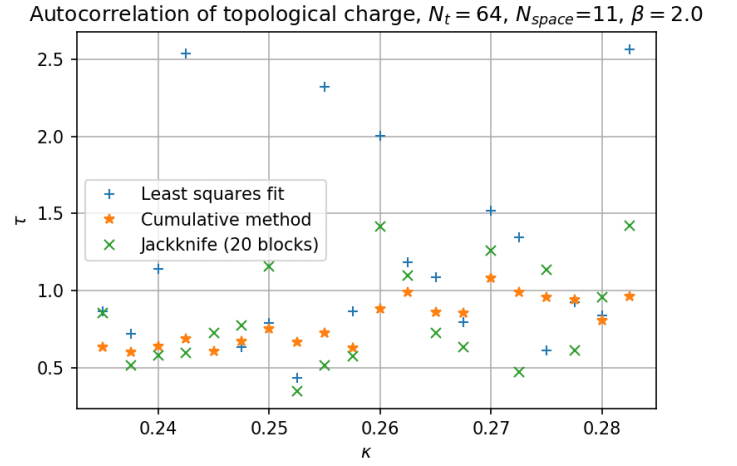


(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. $a = 0.09902 \pm 0.00337$, $b = 10.06628 \pm 0.24085$, $m_\pi = 0.31467 \pm 0.00535$ for variance and $a = 0.10005 \pm 0.00374$, $b = 9.97696 \pm 0.23947$, $m_\pi = 0.3163 \pm 0.00591$ for chi squared.

(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle \bar{\psi}\psi \rangle$



(g) Autocorrelation of the topological charge

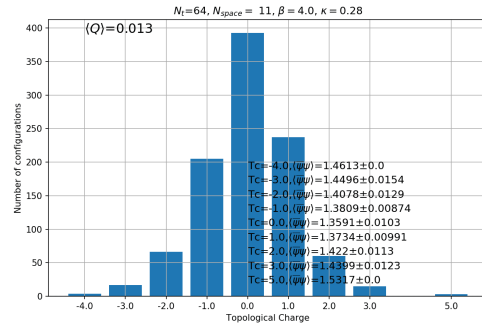
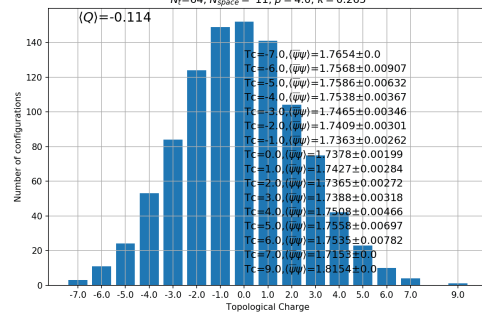
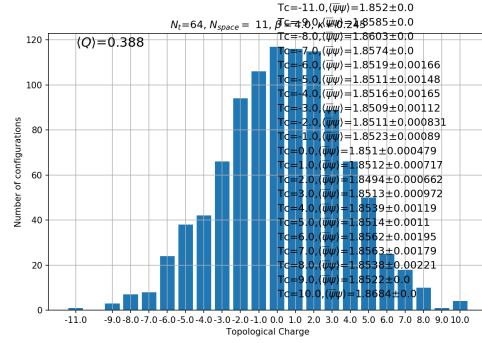
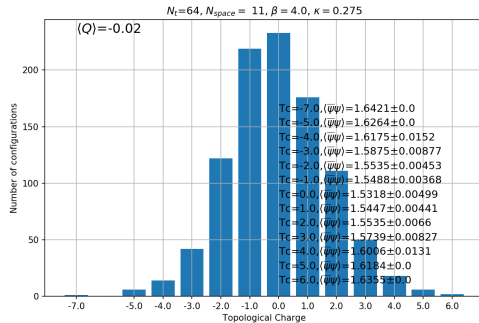
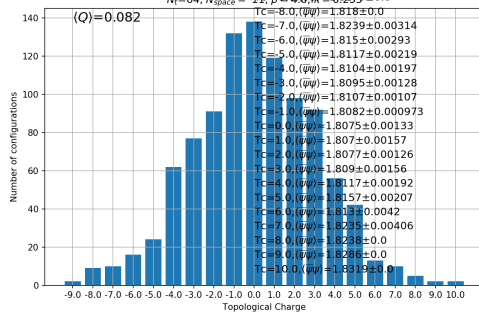
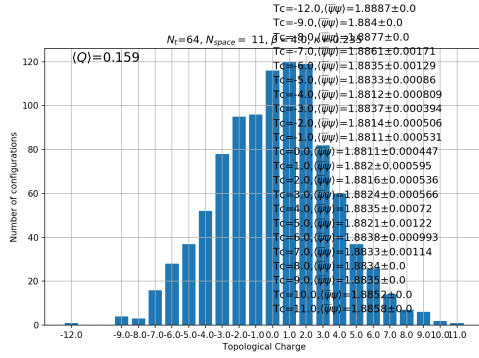
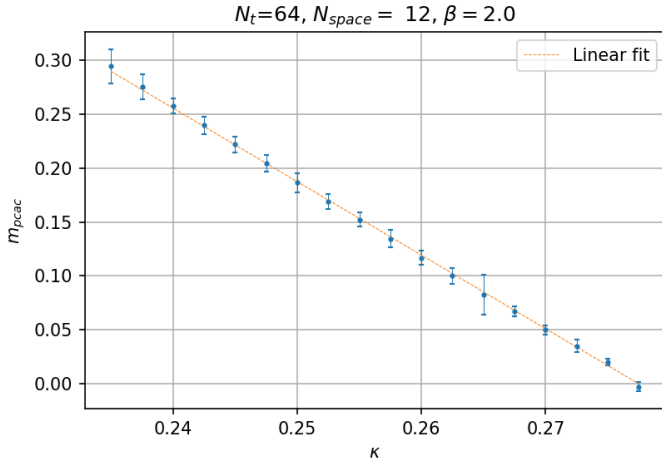
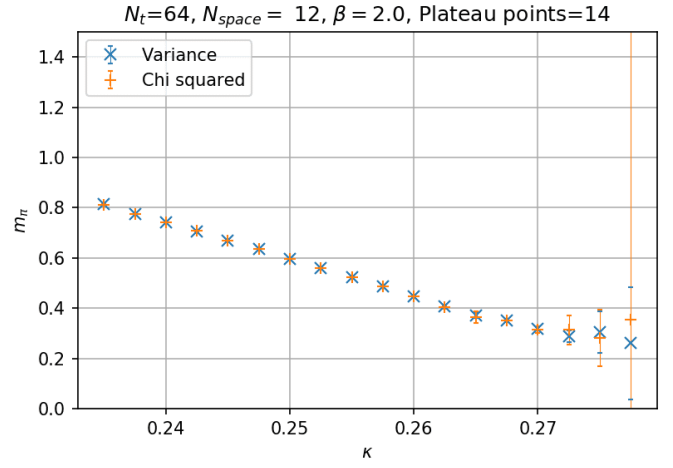
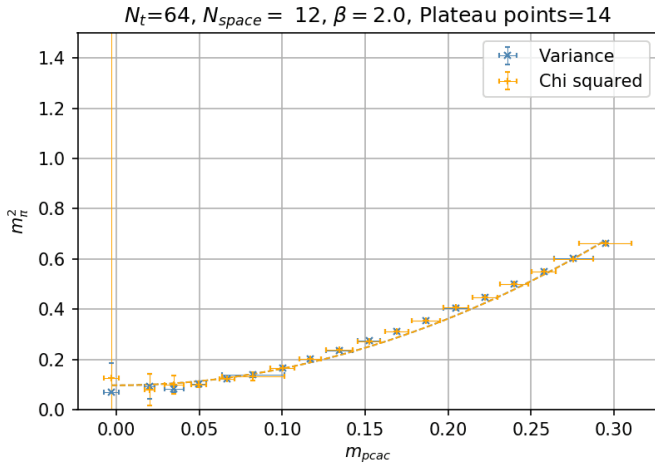
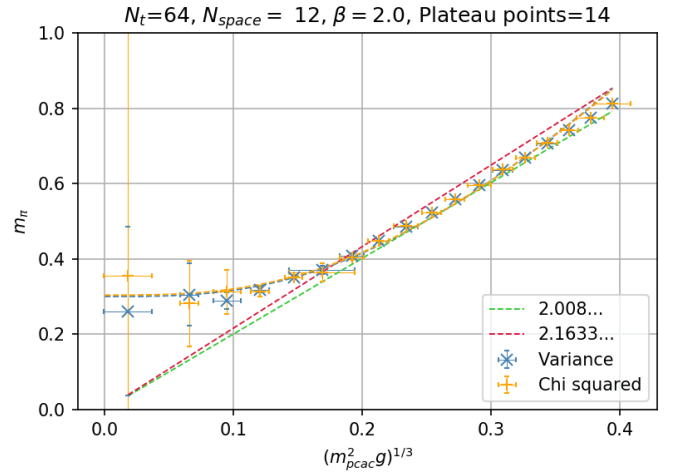
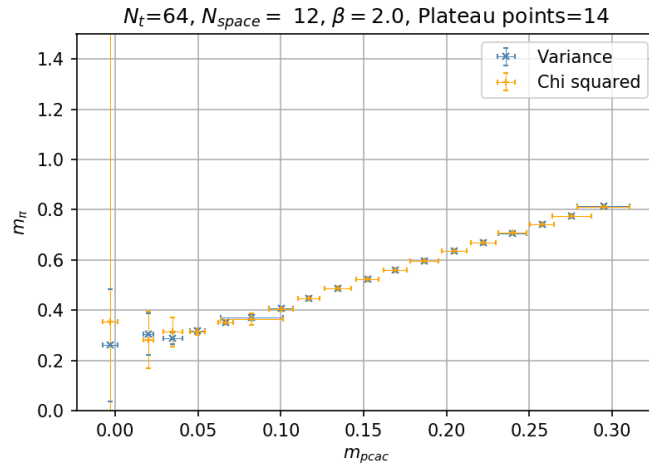
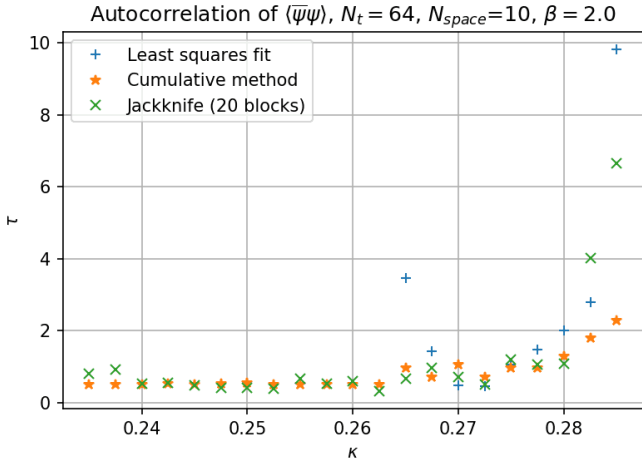
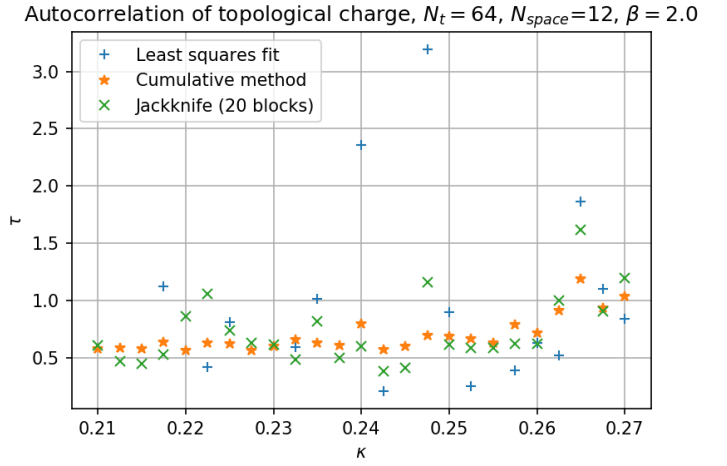


Figure 6: Number of configurations vs. topological charge on a 11×64 lattice.

(a) Fermion mass using PCAC relation, $\kappa_c = 0.27747 \pm 0.00246$ (b) Pion mass as a function of κ (c) m_π^2 vs. m_{pcac} . A function of the form $a + bx^2$ was fitted, the coefficients are $a = 0.08988 \pm 0.00293$, $b = 7.32002 \pm 0.16239$, $m_\pi = 0.29981 \pm 0.00489$ for variance and $a = 0.09219 \pm 0.00308$, $b = 7.24626 \pm 0.15345$, $m_\pi = 0.30362 \pm 0.00507$ for chi squared.(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. $a = 0.09014 \pm 0.00296$, $b = 10.335 \pm 0.22912$, $m_\pi = 0.30023 \pm 0.00492$ for variance and $a = 0.09241 \pm 0.00311$, $b = 10.23365 \pm 0.21701$, $m_\pi = 0.30398 \pm 0.00511$ for chi squared.(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle \bar{\psi}\psi \rangle$



(g) Autocorrelation of the topological charge

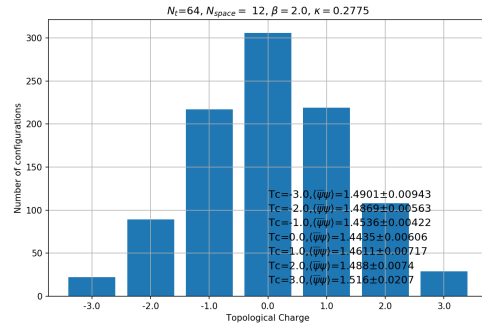
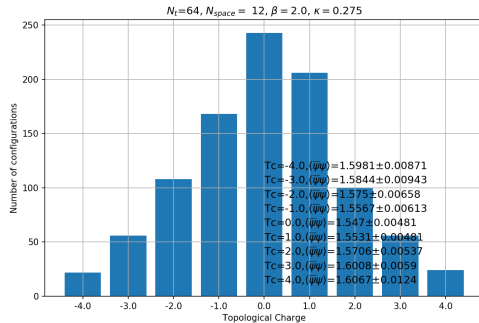
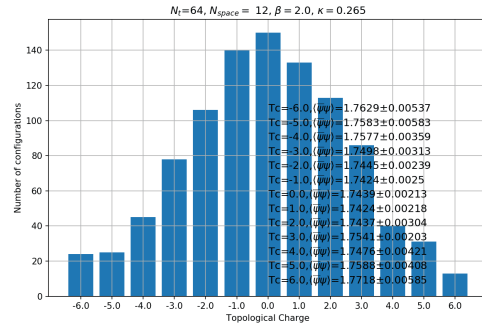
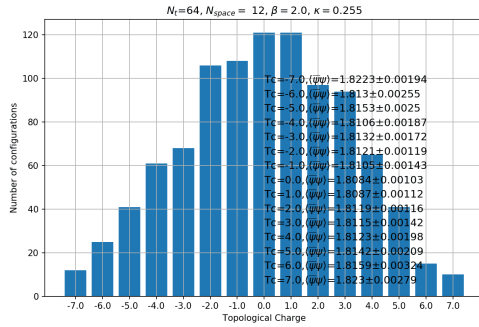
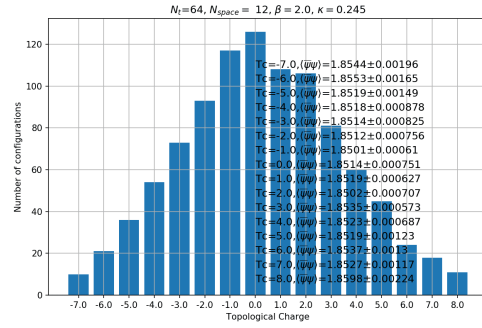
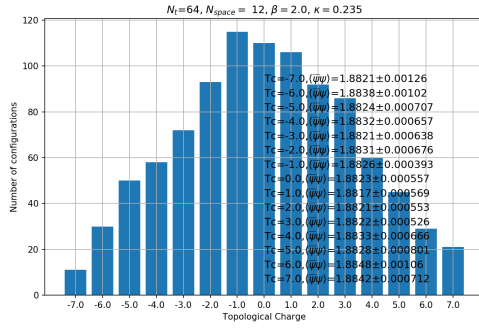


Figure 7: Number of configurations vs. topological charge on a 12×64 lattice.

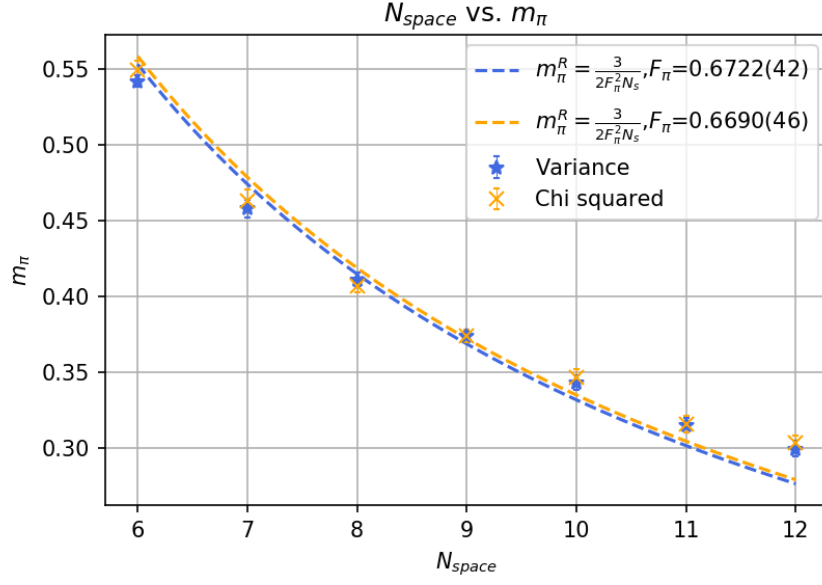


Figure 8: m_π^R vs. L . The results of the pion mass were obtained through the parabolic fit. For variance $F_\pi = 0.6722(42)$, while for chi squared $F_\pi = 0.6690(46)$

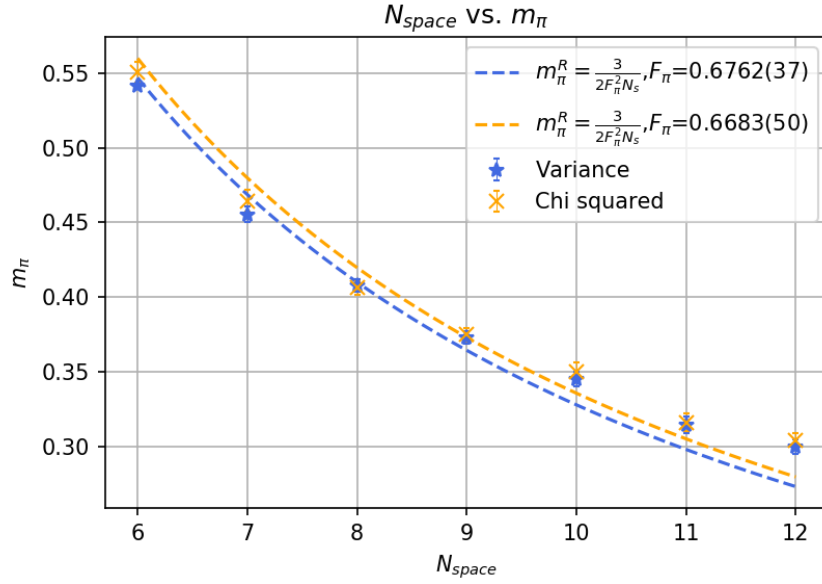


Figure 9: m_π^R vs. L . The results of the pion mass were obtained through the fit of the form $\sqrt{a + bx^3}$ to the data m_π vs. $(m_{pcac}^2 g)^{1/3}$. For variance $F_\pi = 0.6762(37)$, while for chi squared $F_\pi = 0.6683(50)$

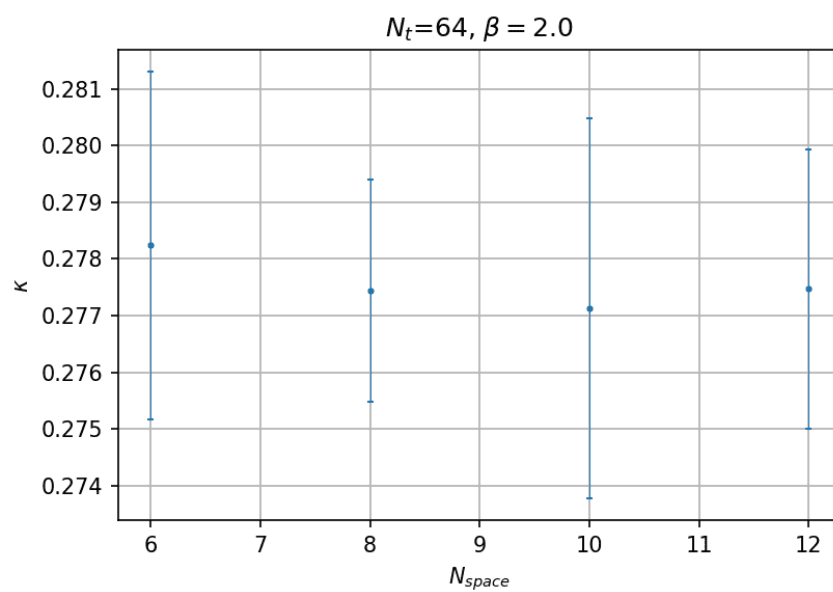


Figure 10: Kappa critical as a function of the volume size