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## High precision results

The following results were obtained through several simulations on lattices of size  $6 \times 64$ ,  $7 \times 64$ ,  $8 \times 64$ ,  $9 \times 64$ ,  $10 \times 64$ ,  $11 \times 64$  and  $12 \times 64$  with the parameters shown in Table 1

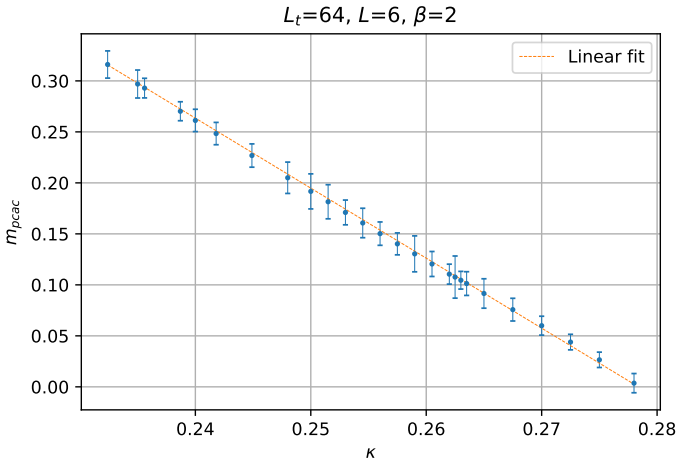
Ntime	64
Ntherm	1000
Nmeasure	10000
Trajectory Steps	10
Nsteps	20
$\beta$	2

Table 1: All the simulations were performed with this parameters.

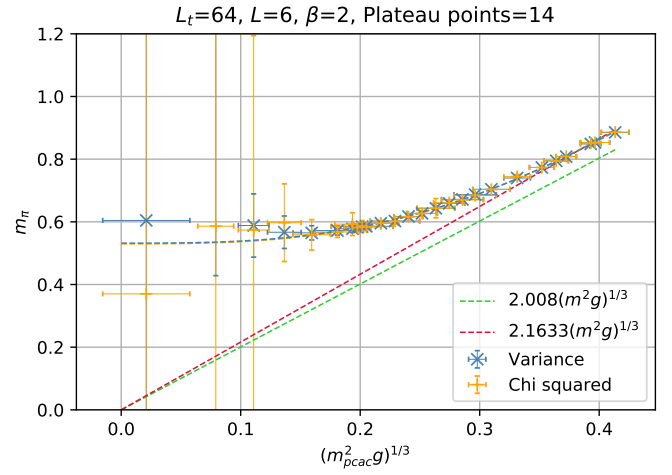
Variance stands for the var option in the masscoll program and Chi squared for the  $\chi^2$  option.  $g = \frac{1}{\sqrt{\beta}}$ . The residual pion mass is extrapolated with two different methods. In the plots of  $m_\pi$  vs.  $(g m_{pcac}^2)^{1/3}$  we fit a function of the form

$$y = \sqrt{a + bx^3}. \quad (1)$$

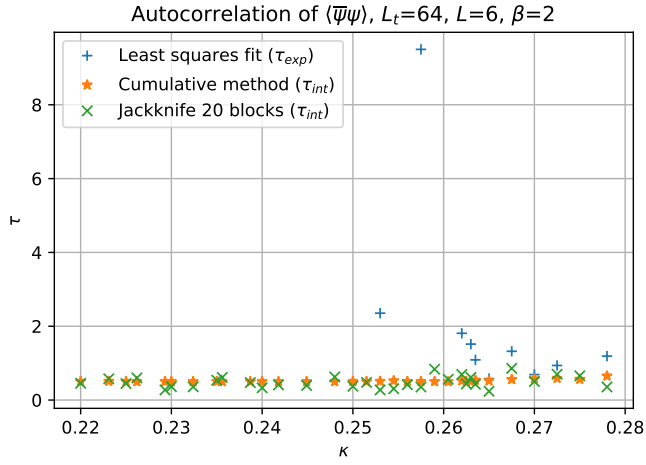
6x64



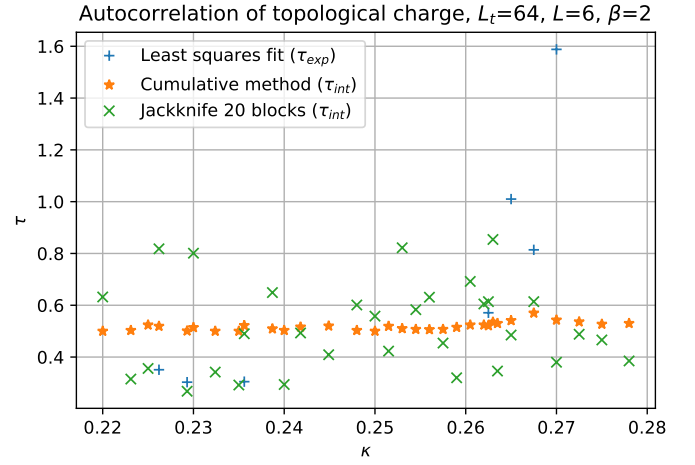
(a) Fermion mass using PCAC relation.



(b) A function of the form  $y = \sqrt{a + bx^3}$  was fitted. Only  $m_{pac} > 0$  is considered.



(c) Autocorrelation of  $\langle \bar{\psi}\psi \rangle$



(d) Autocorrelation of the topological charge

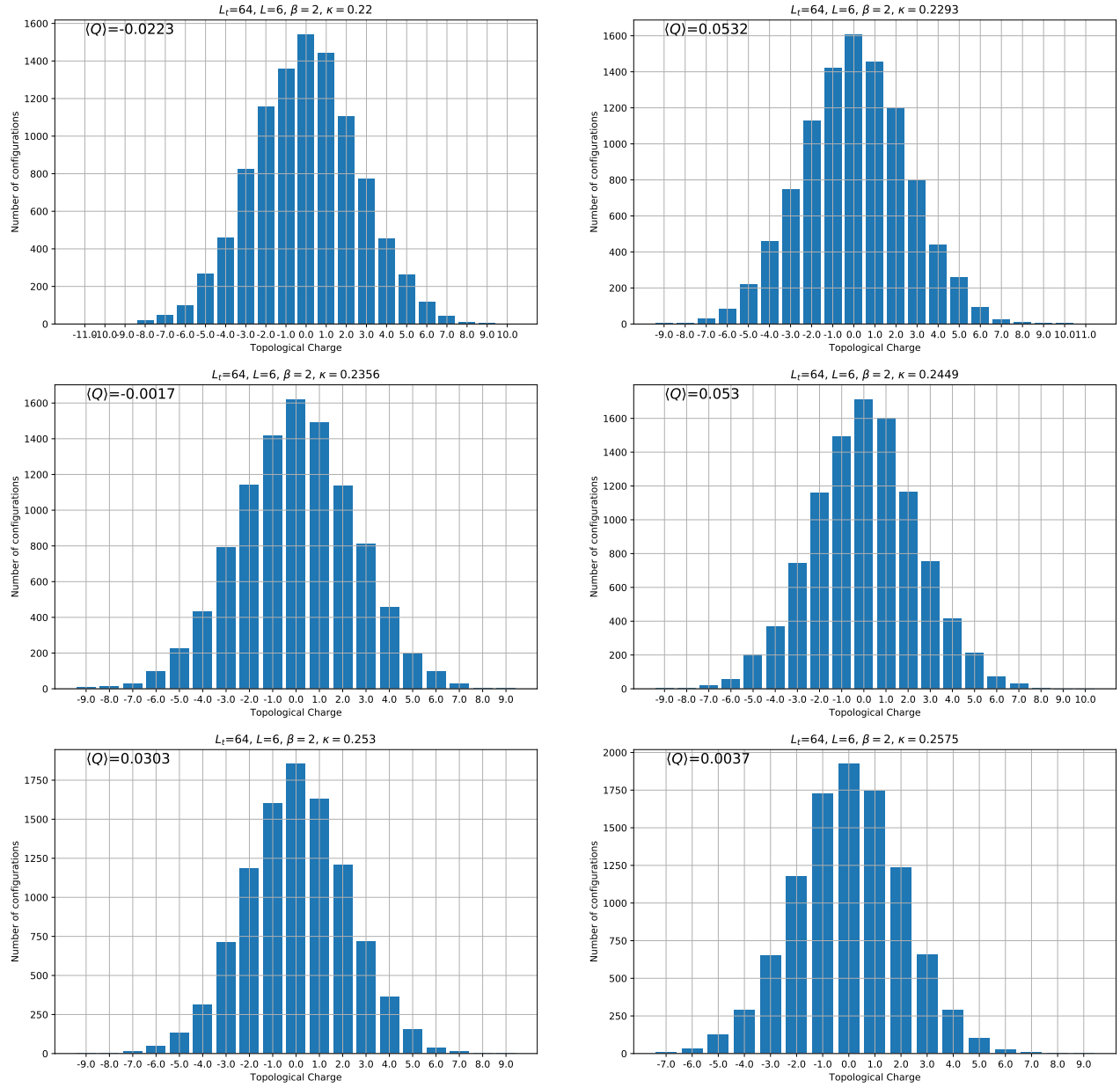
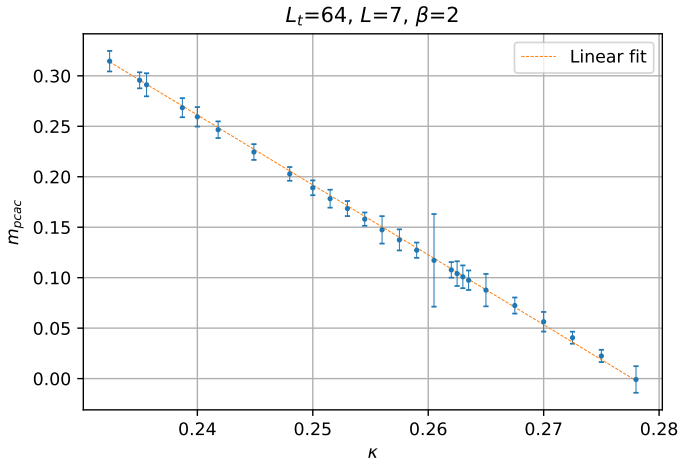
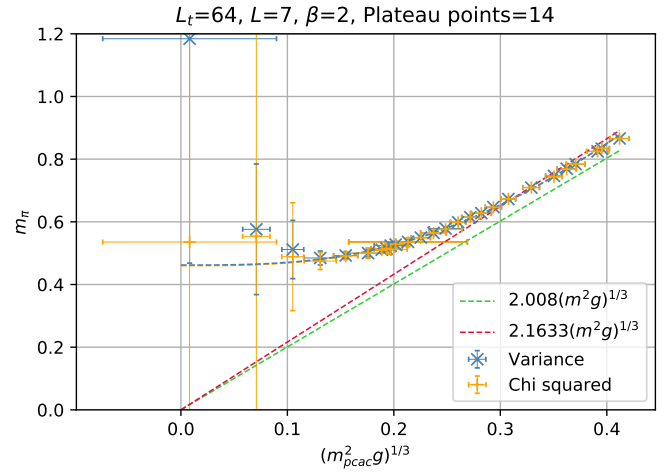
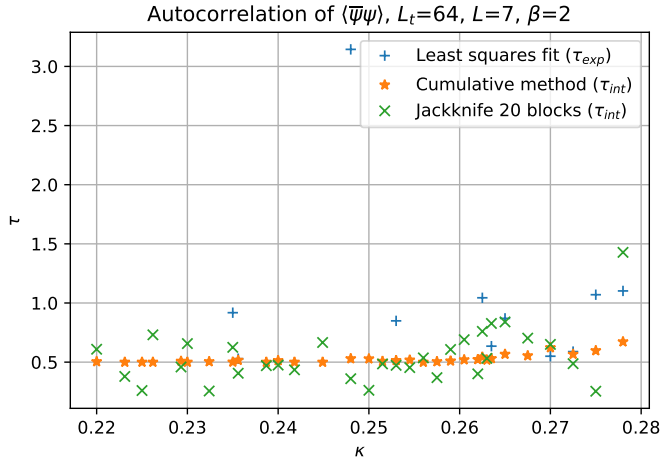
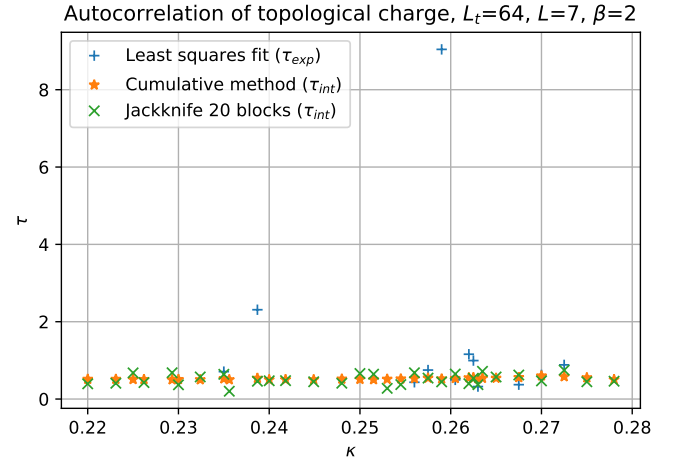


Figure 1: Number of configurations vs. topological charge on a  $6 \times 64$  lattice.



(a) Fermion mass using PCAC relation.

(b) A function of the form  $y = \sqrt{a + bx^3}$  was fitted. Only  $m_{pac} > 0$  is considered.(c) Autocorrelation of  $\langle \bar{\psi}\psi \rangle$ 

(d) Autocorrelation of the topological charge

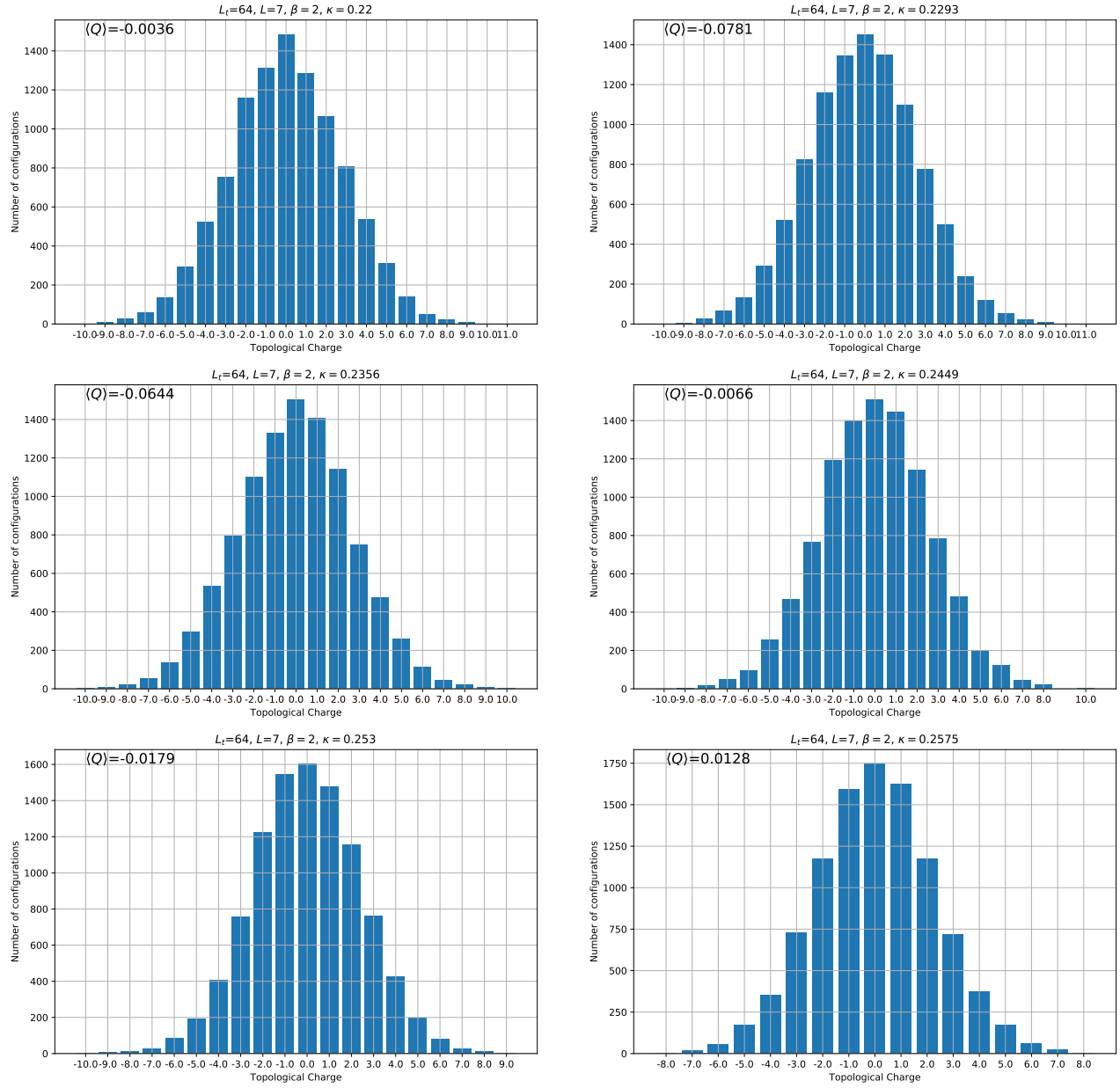
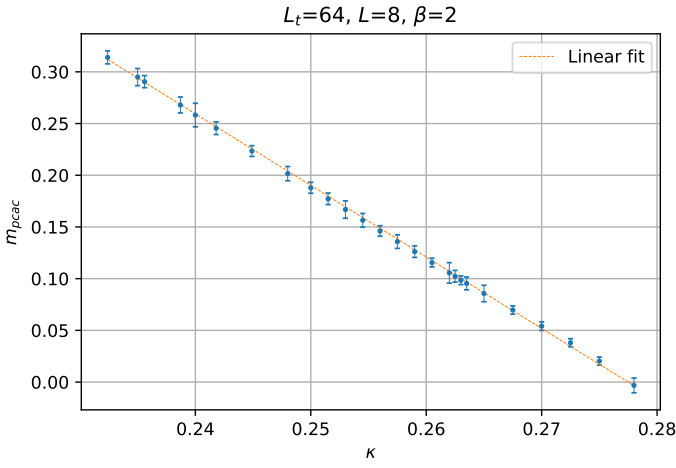
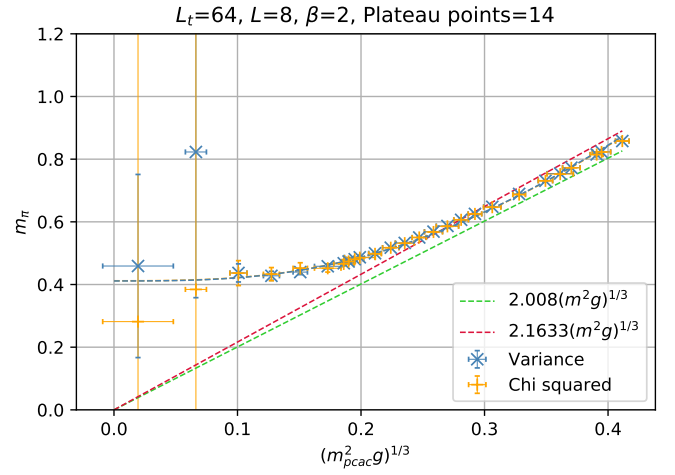


Figure 2: Number of configurations vs. topological charge on a  $7 \times 64$  lattice.

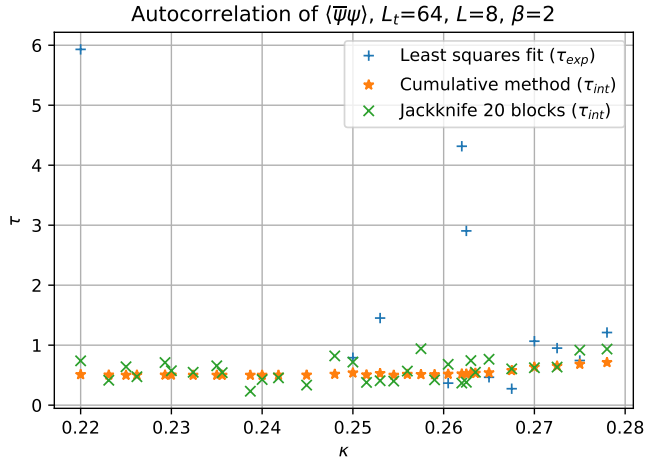
8x64



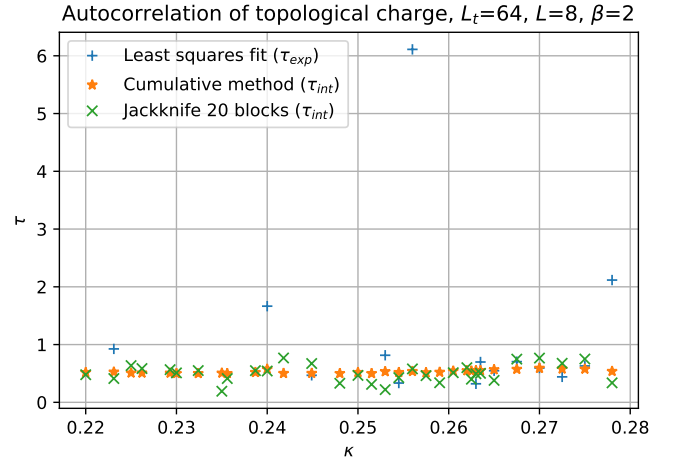
(a) Fermion mass using PCAC relation.



(b) A function of the form  $y = \sqrt{a + bx^3}$  was fitted. Only  $m_{pac} > 0$  is considered.



(c) Autocorrelation of  $\langle \bar{\psi}\psi \rangle$



(d) Autocorrelation of the topological charge

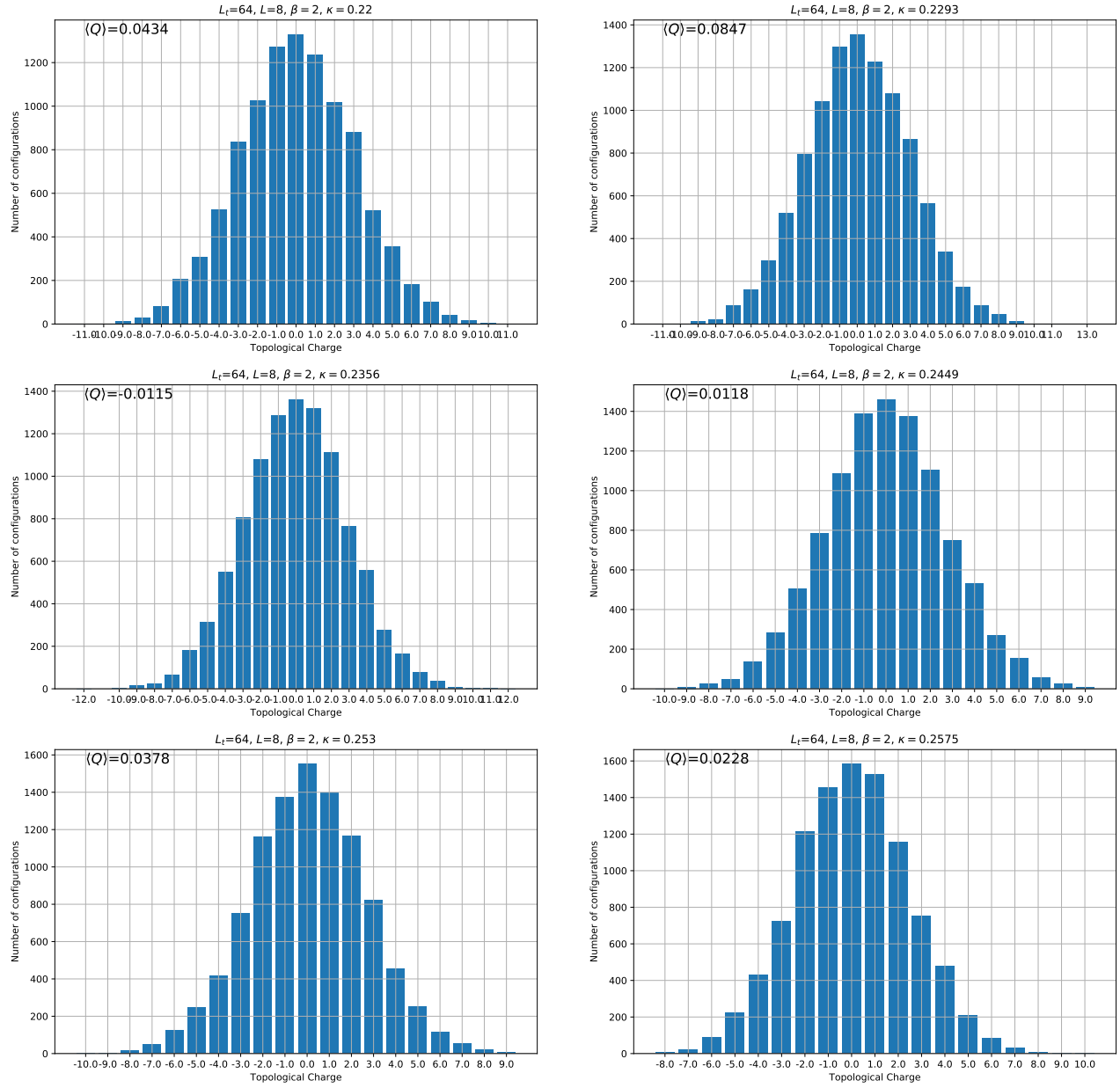
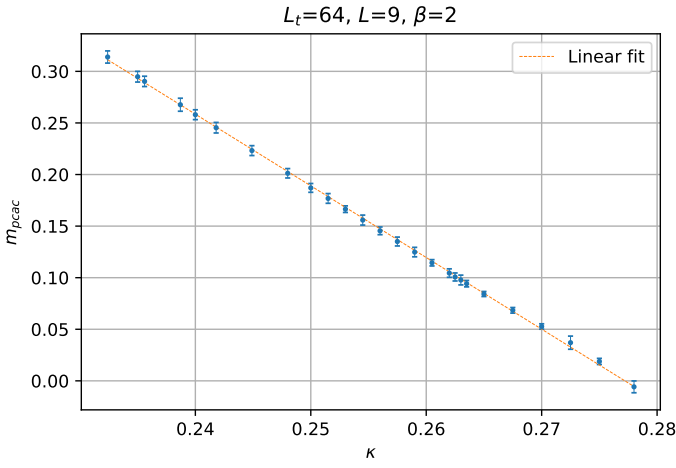
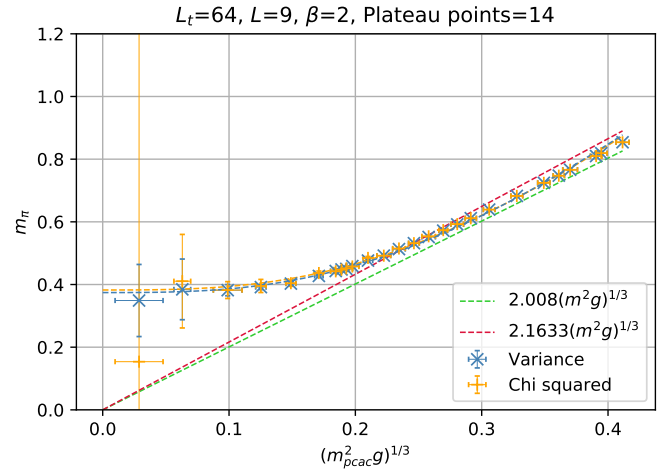
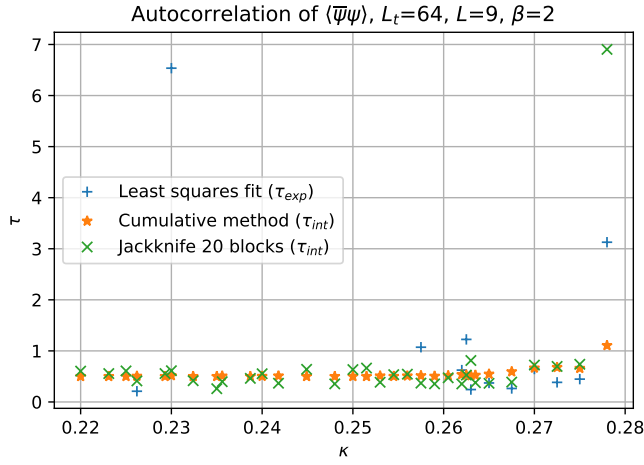
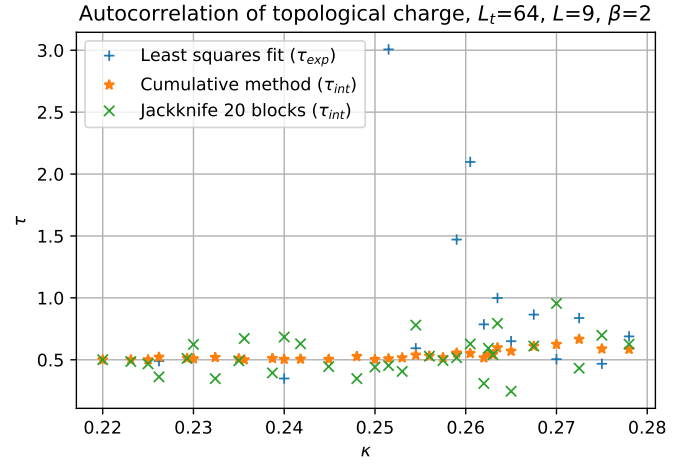


Figure 3: Number of configurations vs. topological charge on a  $8 \times 64$  lattice.



(a) Fermion mass using PCAC relation.

(b) A function of the form  $y = \sqrt{a + bx^3}$  was fitted. Only  $m_{pac} > 0$  is considered.(c) Autocorrelation of  $\langle \bar{\psi}\psi \rangle$ 

(d) Autocorrelation of the topological charge



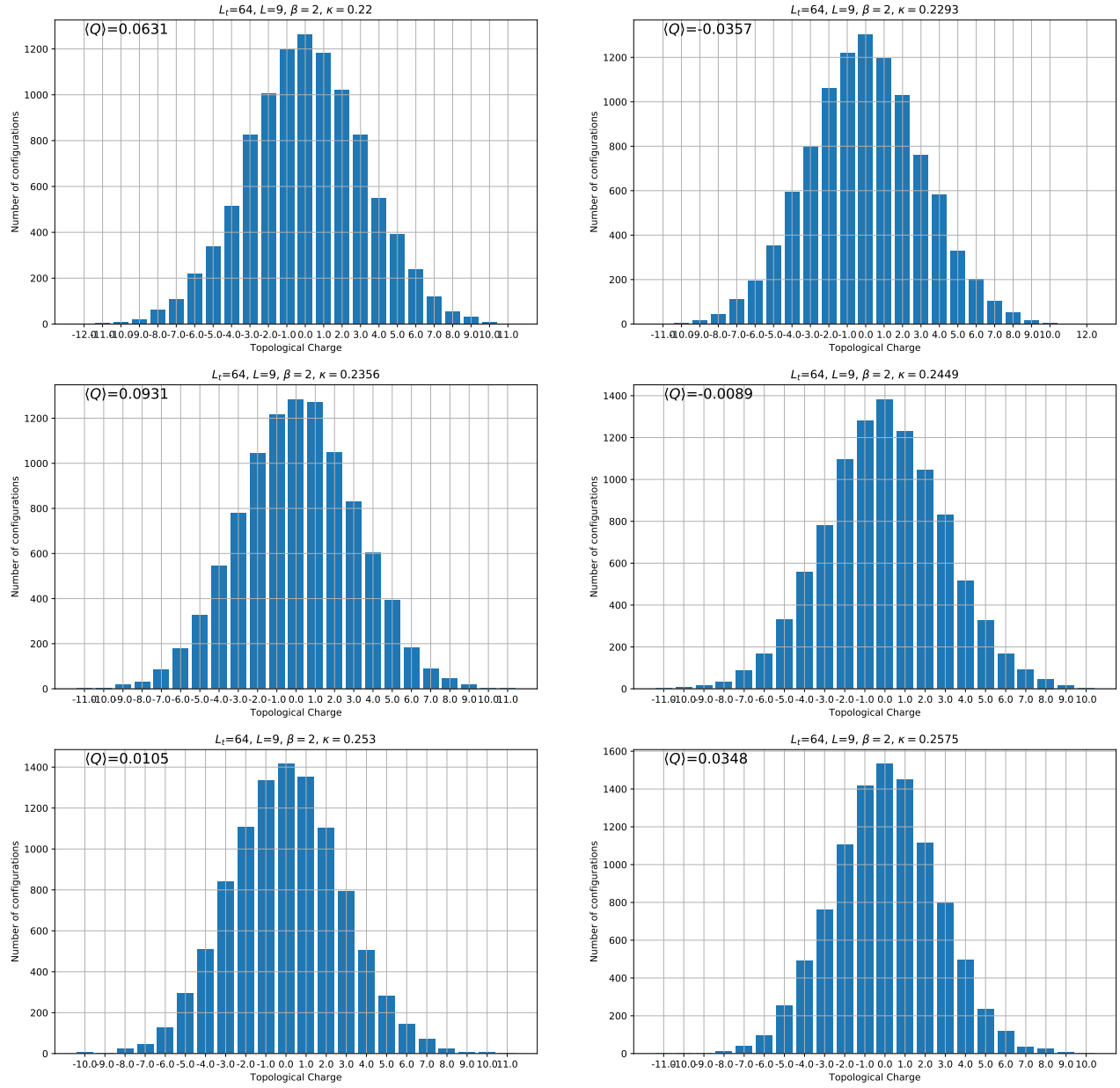
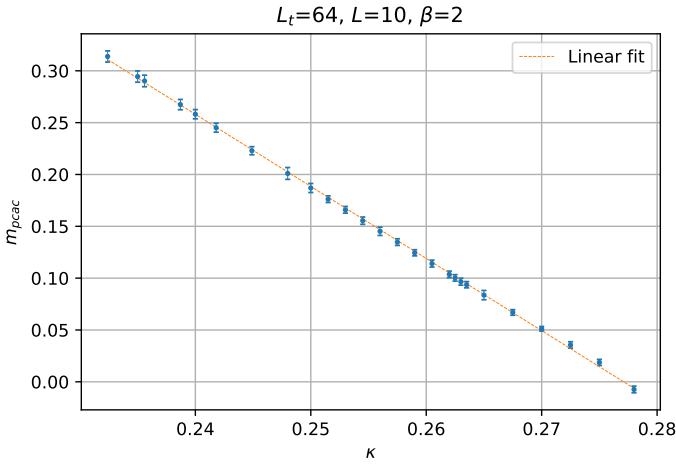
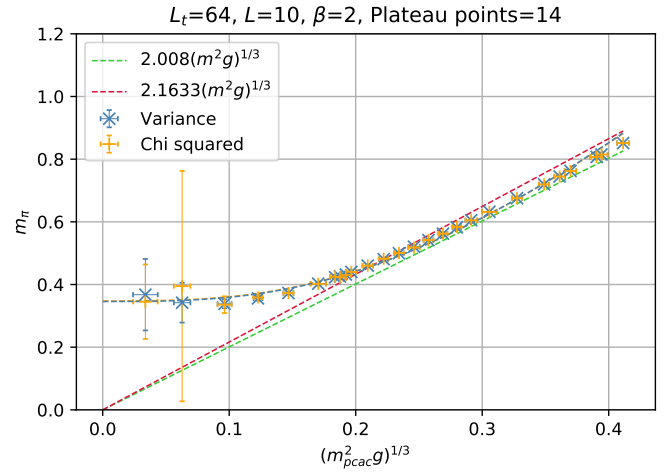
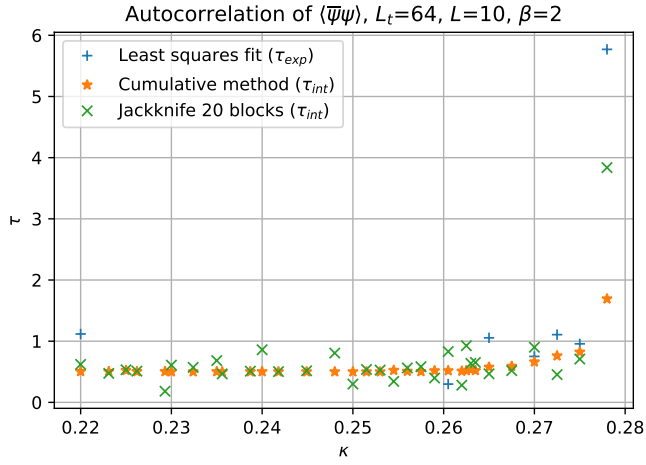
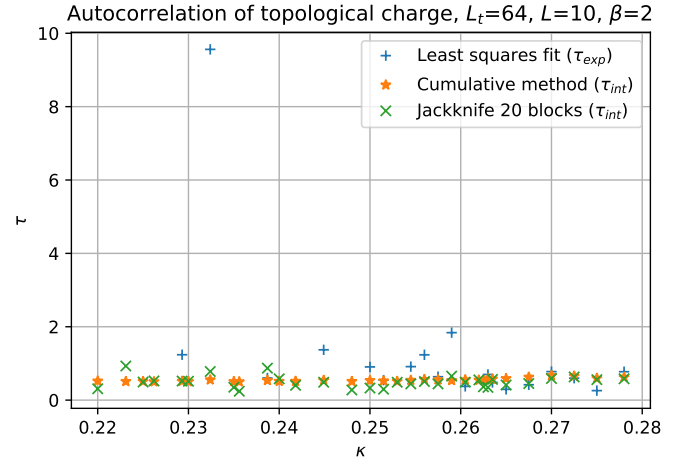


Figure 4: Number of configurations vs. topological charge on a  $9 \times 64$  lattice.



(a) Fermion mass using PCAC relation.

(b) A function of the form  $y = \sqrt{a + bx^3}$  was fitted. Only  $m_{pac} > 0$  is considered.(c) Autocorrelation of  $\langle \bar{\psi}\psi \rangle$ 

(d) Autocorrelation of the topological charge

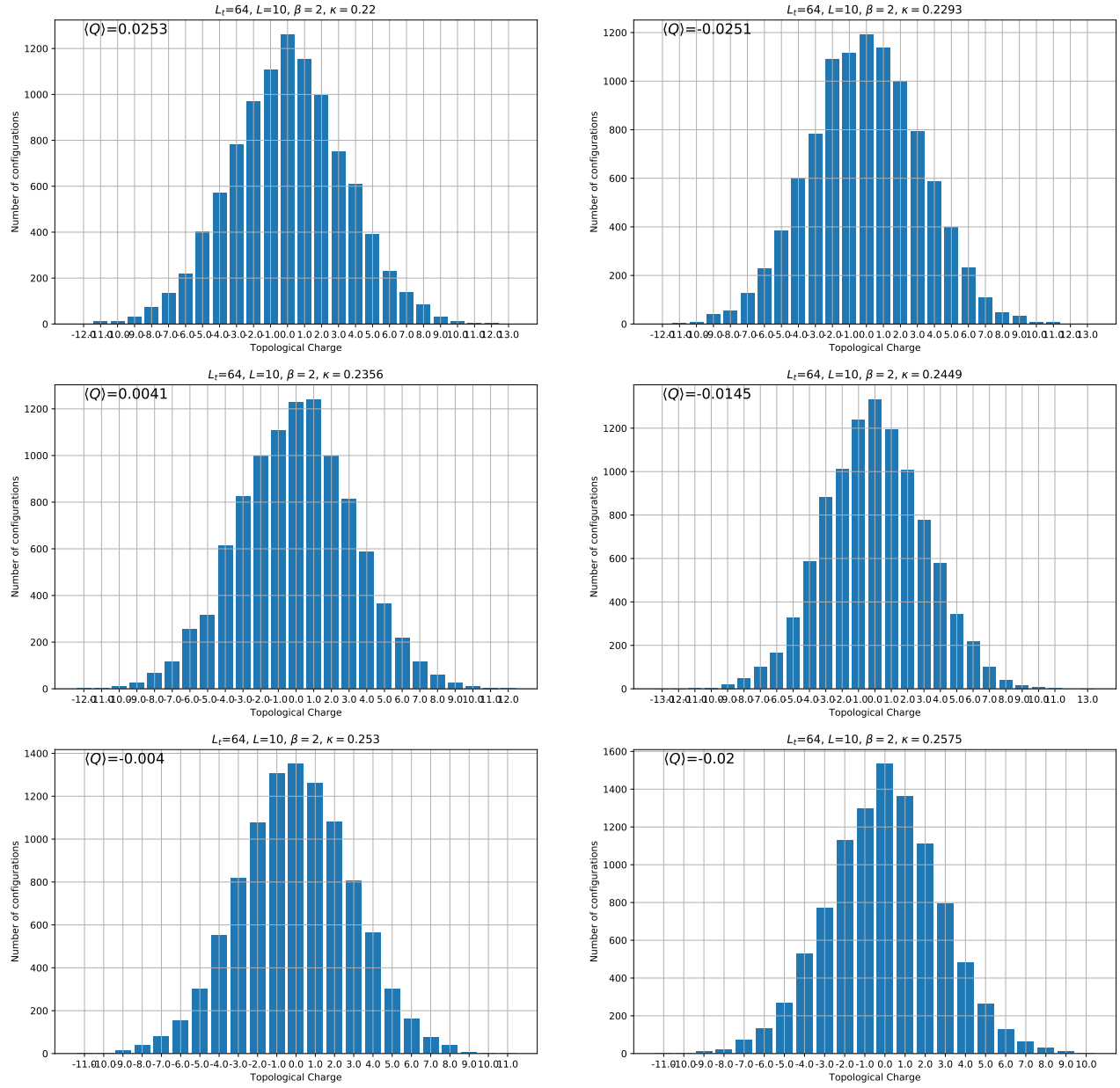
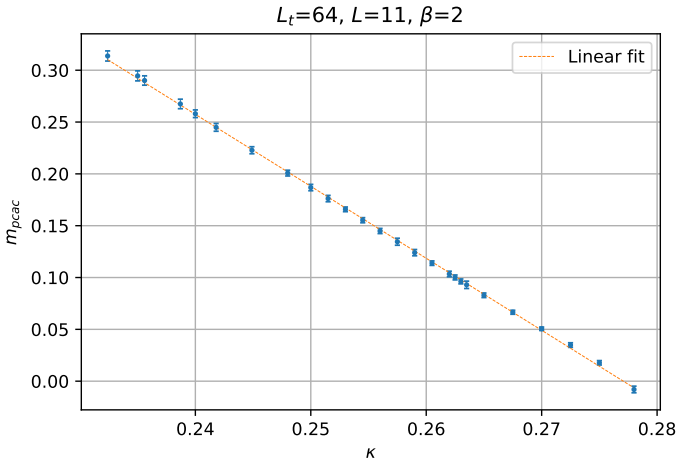
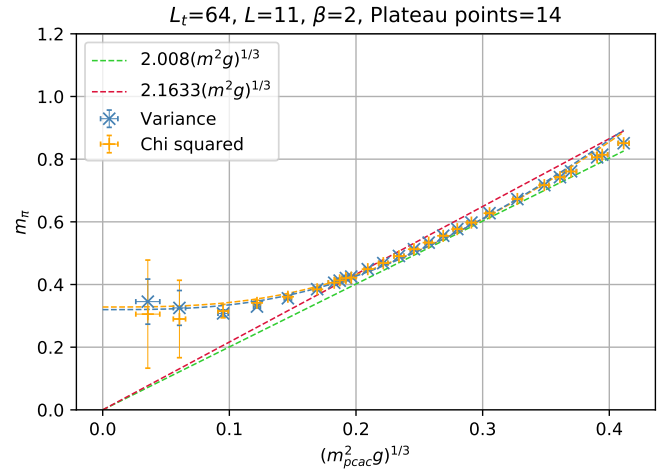
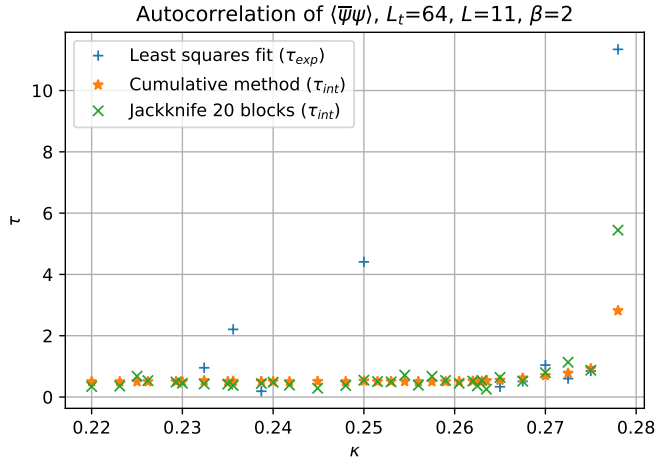
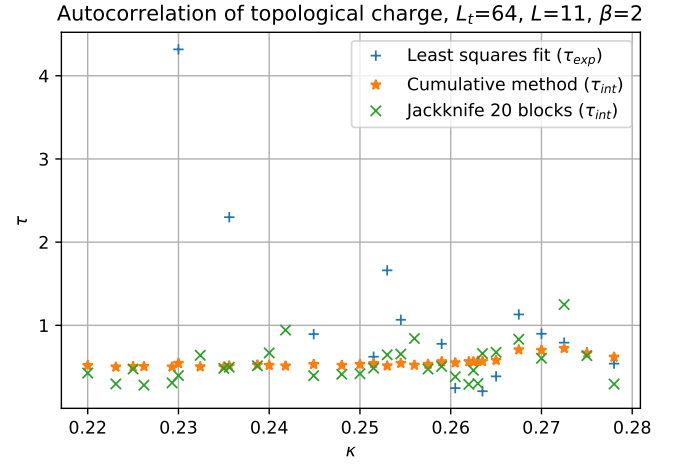


Figure 5: Number of configurations vs. topological charge on a  $10 \times 64$  lattice.



(a) Fermion mass using PCAC relation.

(b) A function of the form  $y = \sqrt{a + bx^3}$  was fitted. Only  $m_{pac} > 0$  is considered.(c) Autocorrelation of  $\langle \bar{\psi}\psi \rangle$ 

(d) Autocorrelation of the topological charge

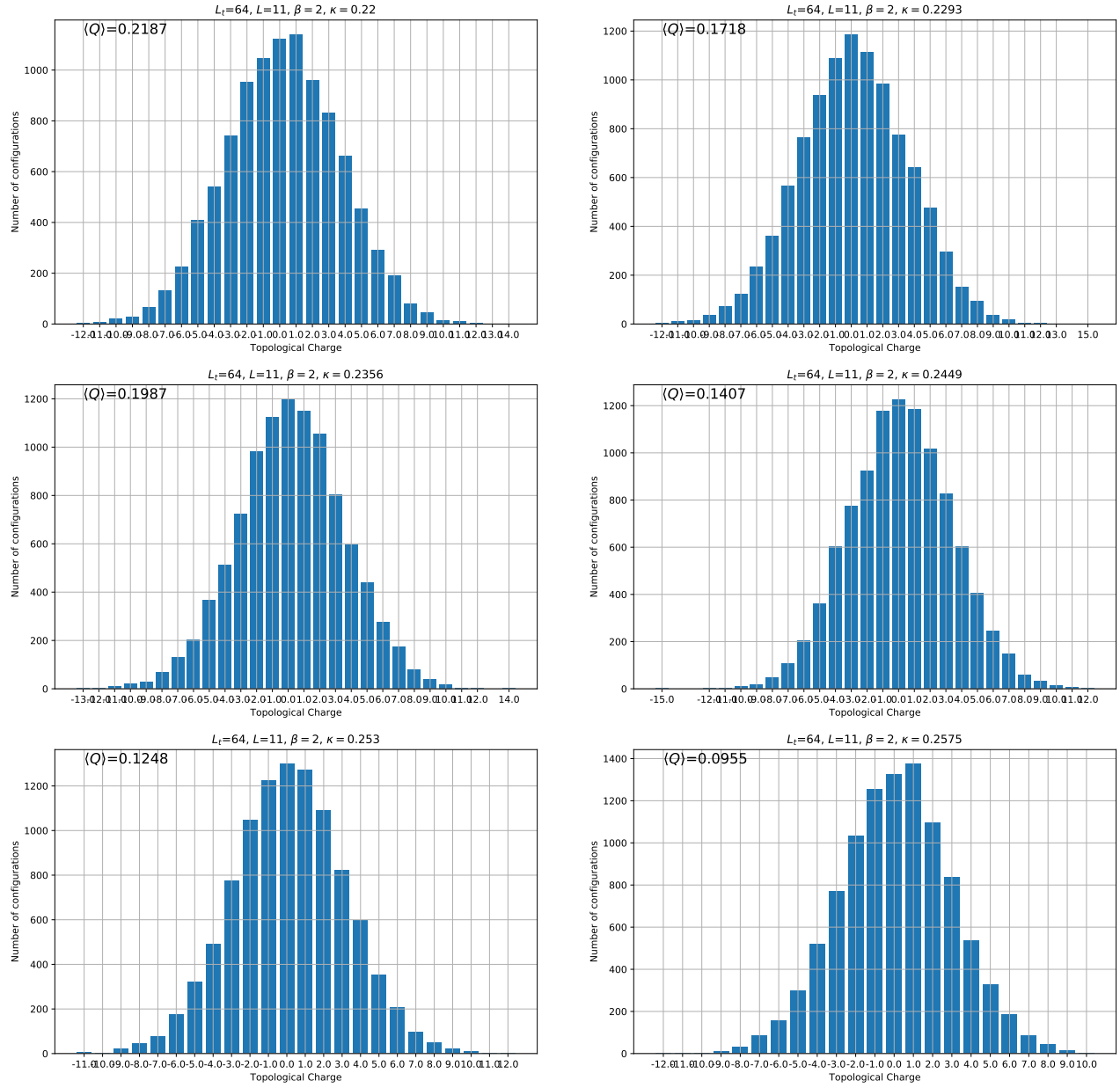
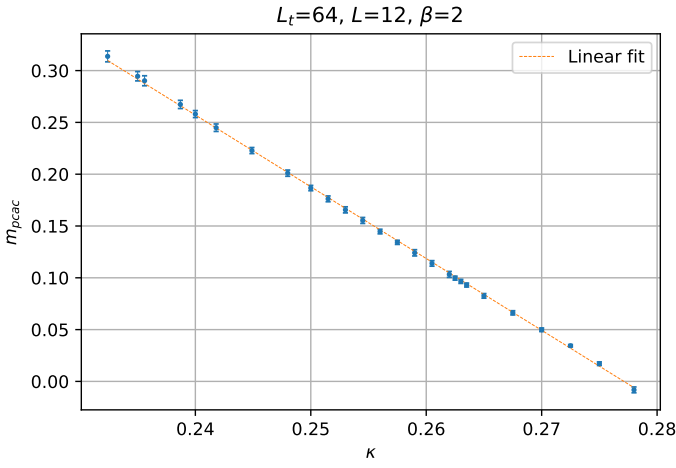
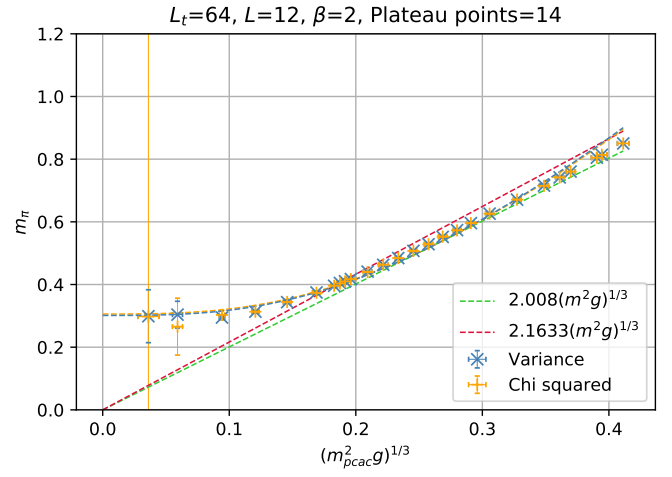
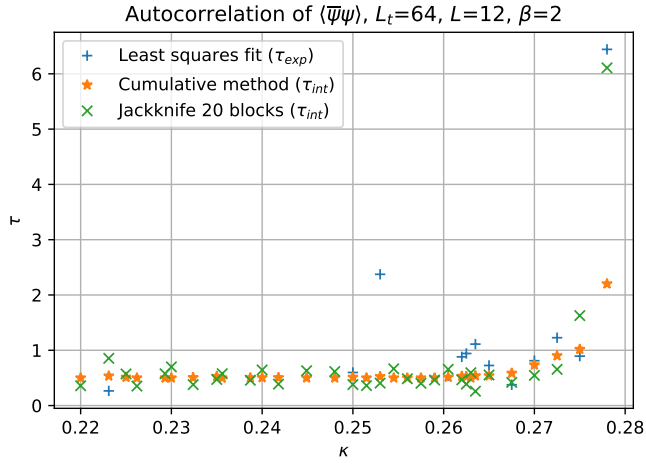
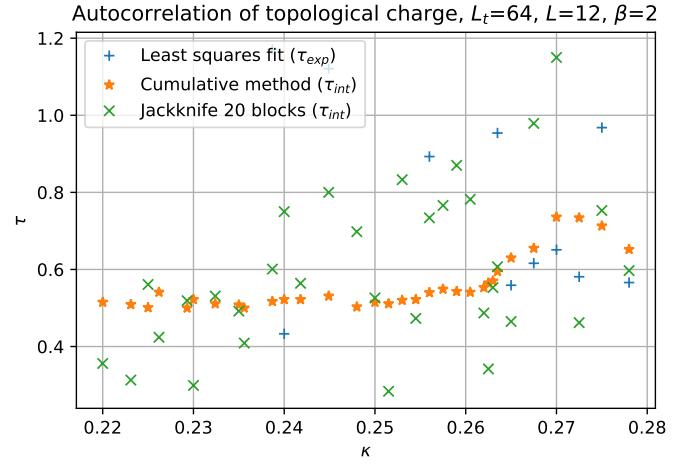


Figure 6: Number of configurations vs. topological charge on a  $11 \times 64$  lattice.



(a) Fermion mass using PCAC relation.

(b) A function of the form  $y = \sqrt{a + bx^3}$  was fitted. Only  $m_{pac} > 0$  is considered.(c) Autocorrelation of  $\langle \bar{\psi}\psi \rangle$ 

(d) Autocorrelation of the topological charge

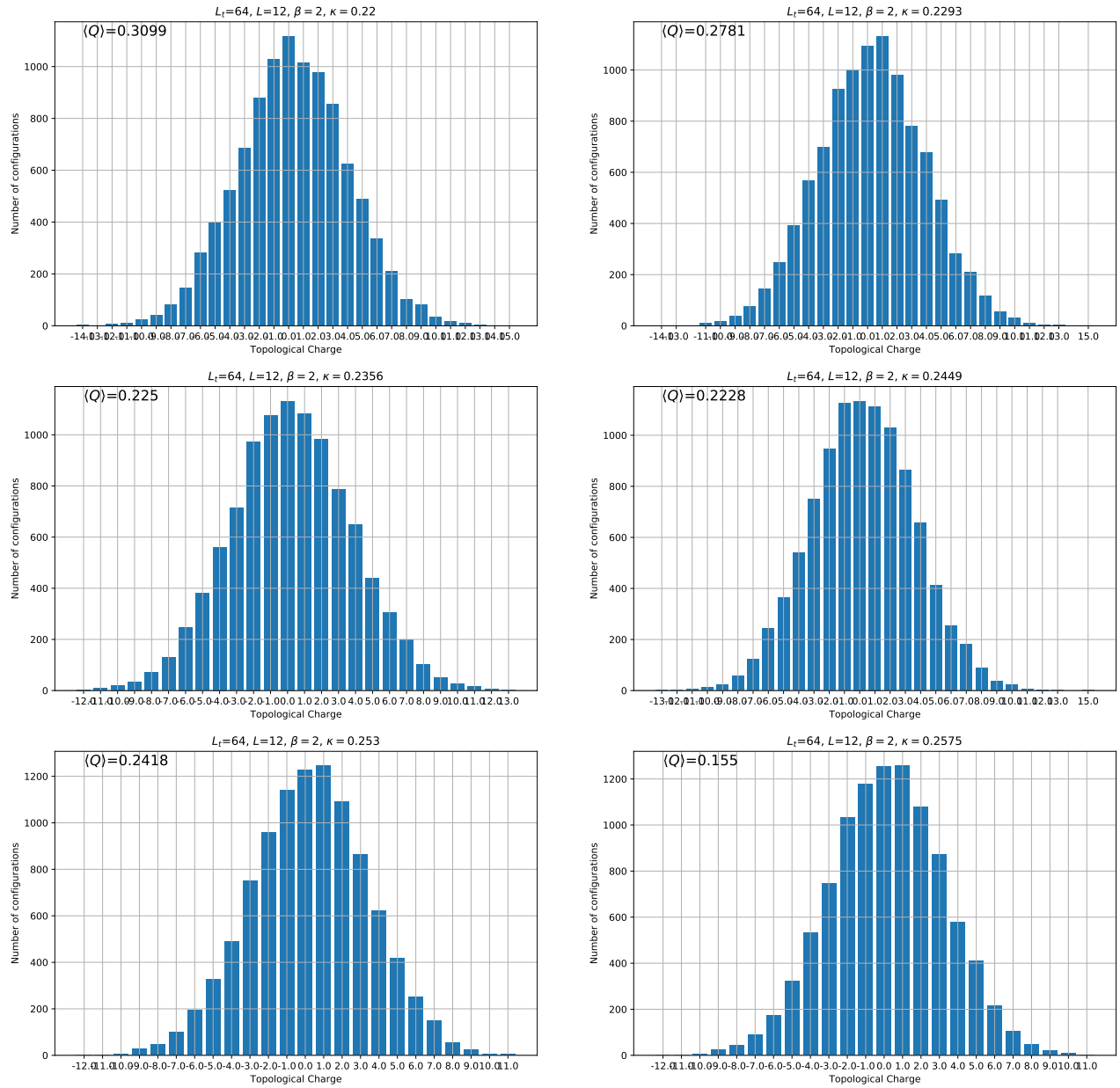


Figure 7: Number of configurations vs. topological charge on a  $12 \times 64$  lattice.

## Determining $F_\pi$

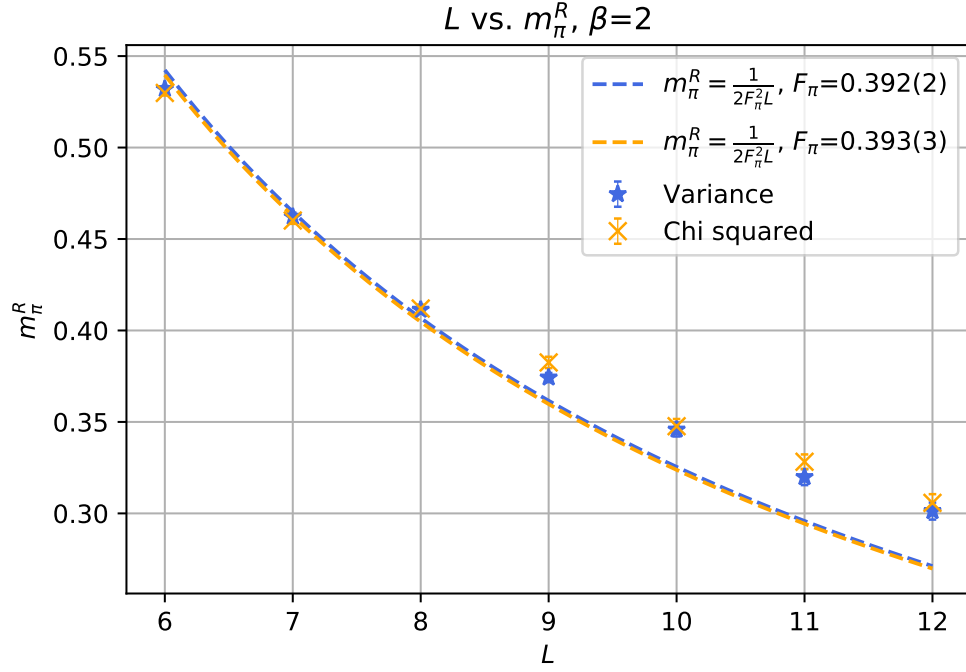


Figure 8:  $m_\pi^R$  vs.  $L$ . We fitted a function of the form  $y = a/x$ . For variance  $F_\pi = 0.392(2)$ , while for chi squared  $F_\pi = 0.393(3)$ .

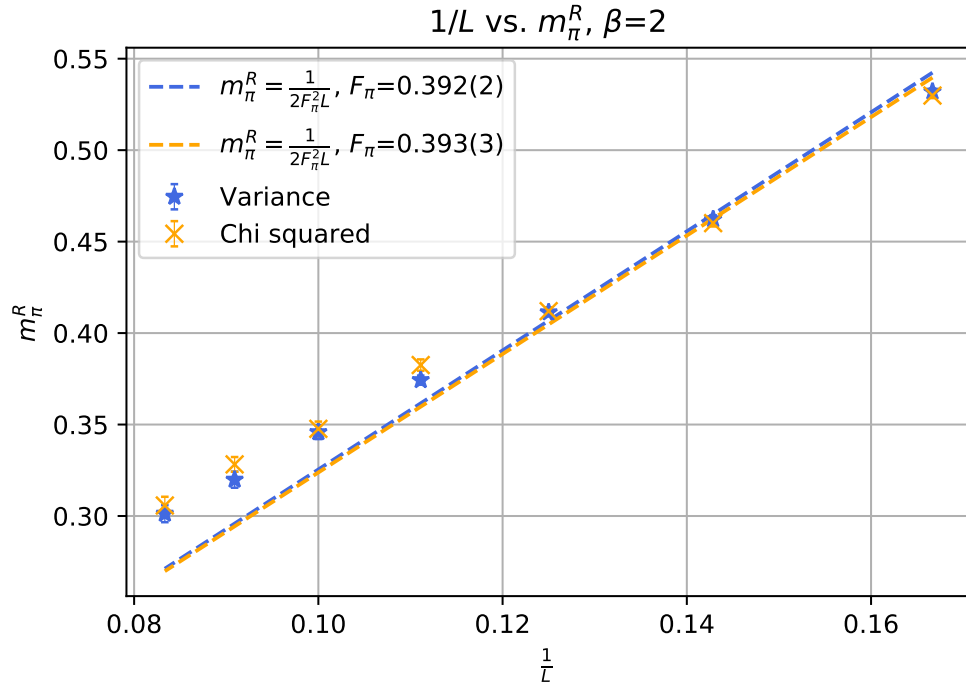
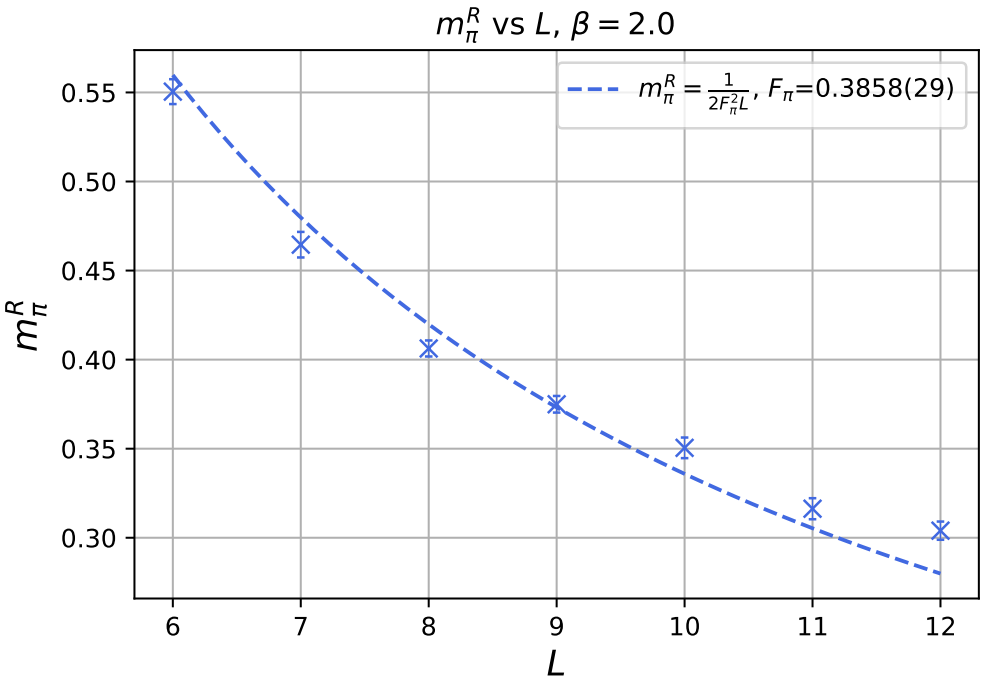


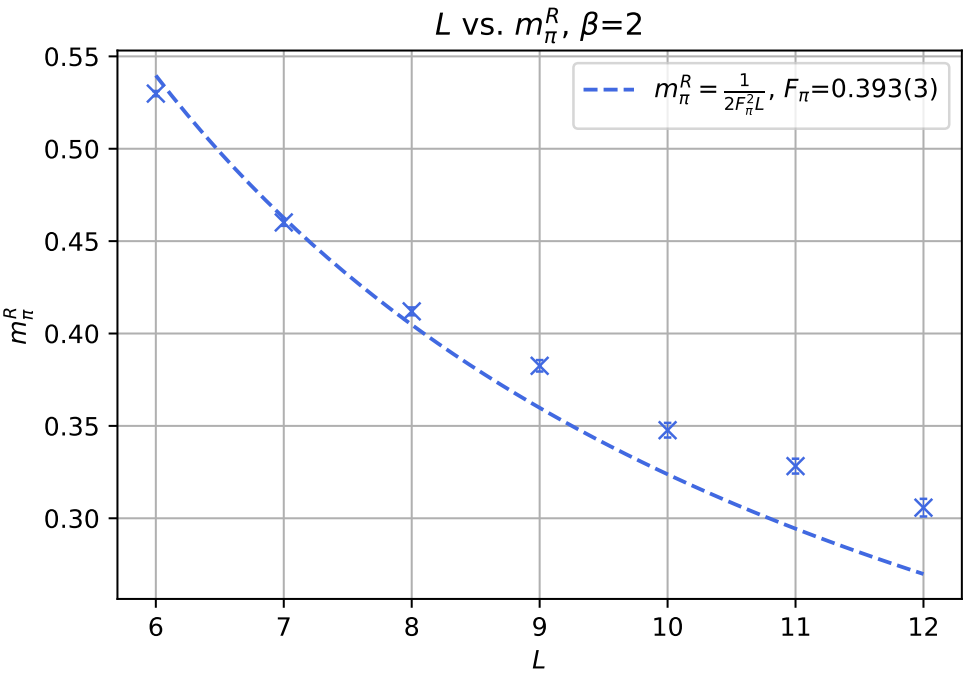
Figure 9:  $m_\pi^R$  vs.  $1/L$ . We fitted a function of the form  $y = a/x$ . For variance  $F_\pi = 0.392(2)$ , while for chi squared  $F_\pi = 0.393(3)$ .



# Comparison with the low statistics results



(a) Results with low statistics  $\sim 10^3$  measurements.



(b) Results with high statistics  $\sim 10^4$  measurements.