Optimal fit of m_π vs. $(m^2_{pcac}g)^{1/3}$.

Jaime Fabián Nieto Castellanos

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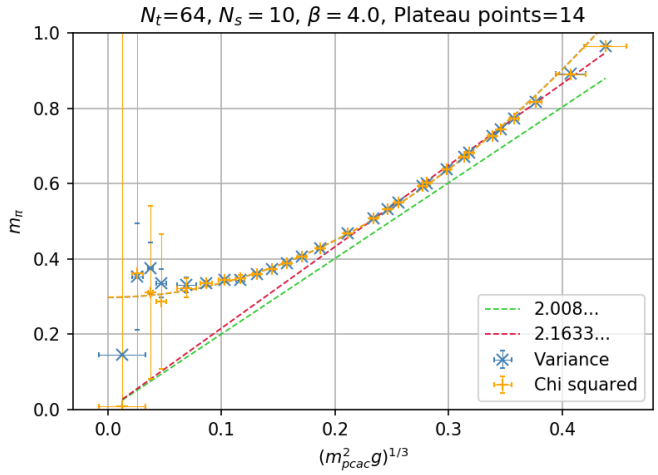
Several experiments were performed in order to determine the most proper way to fit the data of m_π vs. $(m^2_{pcac}g)^{1/3}$. Functions of the form $\sqrt{a+bx^3}$, $\sqrt{a+bx^2}$, $\sqrt{a+bx^4}$, $\sqrt{a+bx^c}$, $a+bx^2$, $a+bx^c$, where $x=(m^2_{pcac}g)^{1/3}$ and a, b and c are fit parameters were used. The best results were obtained with functions of the form

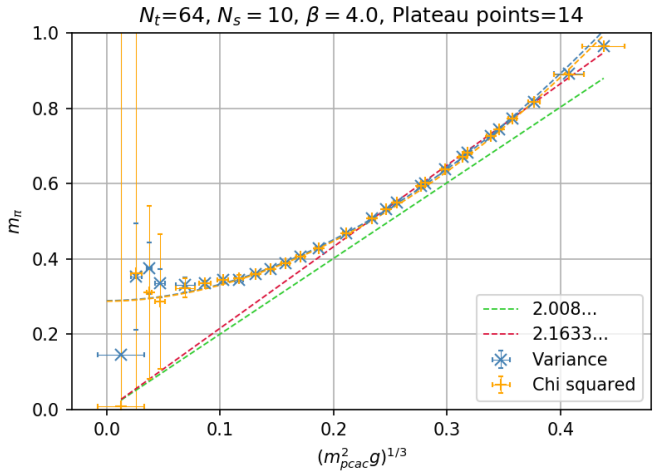
$$\begin{aligned} y &= a+bx^c, \\ y &= \sqrt{a+bx^c}, \\ y &= \sqrt{a+bx^3}. \end{aligned}$$

Results of the pion decay constant F_π are shown as well. Everything was done for $\beta = 4.0$

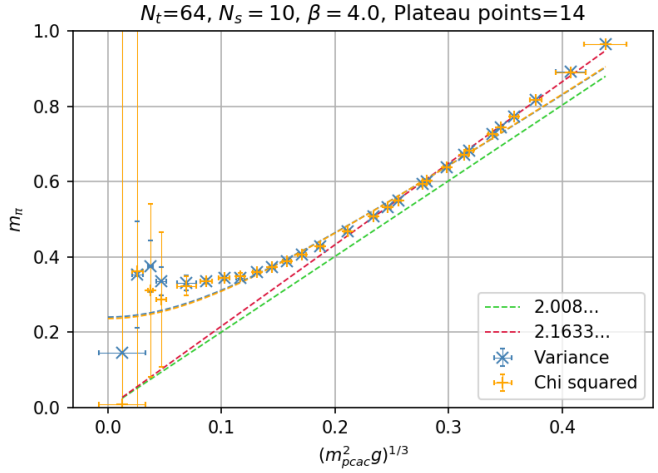
1 Experiments



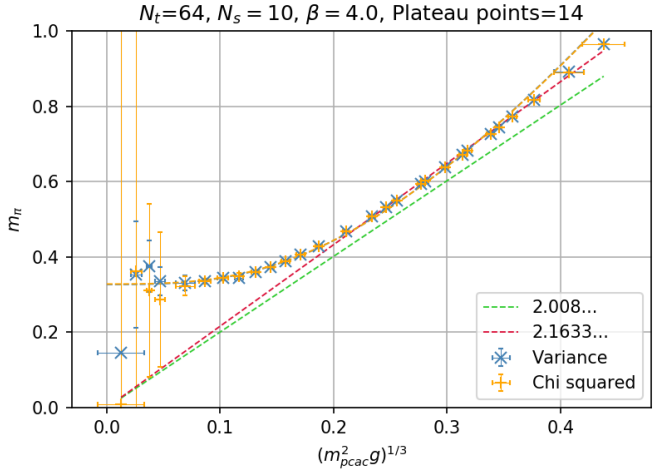
(a) $y = a + bx^c$ was fitted, $m_{pcac} > 0$. $a = 0.298 \pm 0.0018$, $b = 3.7842 \pm 0.0304$, $c = 2.0$, $m_\pi = 0.2980(18)$ for variance and $a = 0.2984 \pm 0.0017$, $b = 3.7777 \pm 0.028$, $c = 2.0$, $m_\pi = 0.2984(17)$ for chi squared.



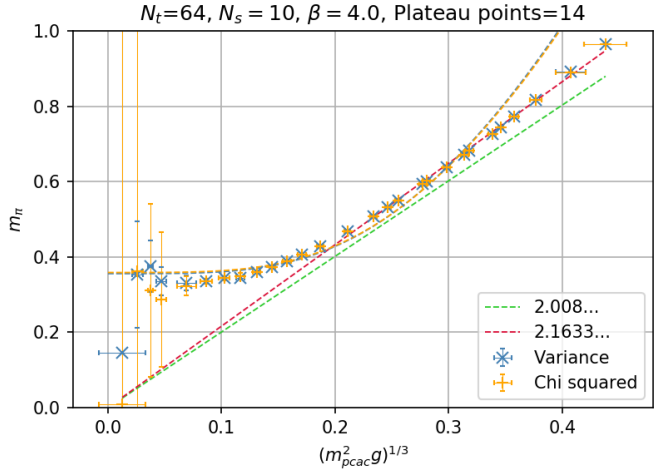
(b) $y = a + bx^c$ was fitted, $m_{pcac} > 0$. $a = 0.2892 \pm 0.0047$, $b = 3.4319 \pm 0.1535$, $c = 1.8969 \pm 0.0475$, $m_\pi = 0.2892(47)$ for variance and $a = 0.2878 \pm 0.0044$, $b = 3.3815 \pm 0.1315$, $c = 1.8969 \pm 0.0475$, $m_\pi = 0.2878(44)$ for chi squared.



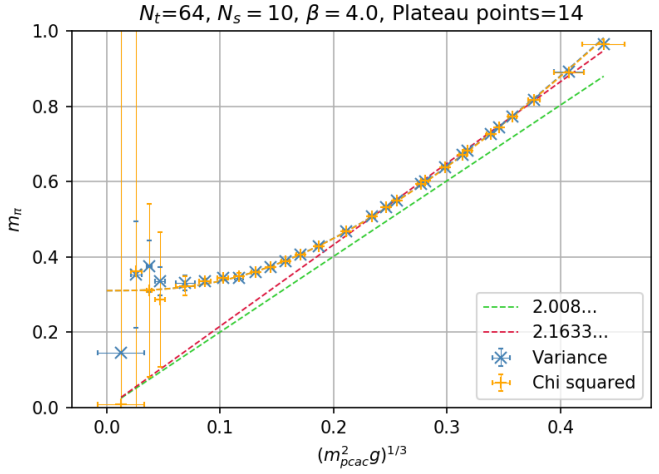
(c) $y = \sqrt{a + bx^c}$ was fitted, $m_{pcac} > 0$. $a = 0.0578 \pm 0.0045$, $b = 3.9498 \pm 0.082$, $c = 2.0$ $m_\pi = 0.2404(94)$ for variance and $a = 0.0559 \pm 0.0046$, $b = 3.9763 \pm 0.0799$, $c = 2.0$, $m_\pi = 0.2364(97)$ for chi squared



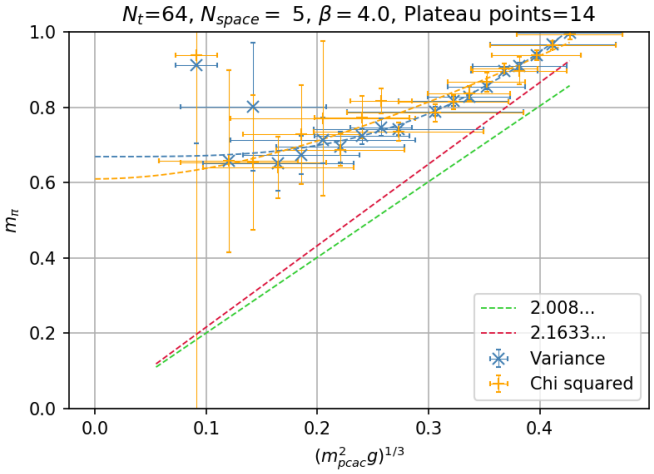
(d) $y = \sqrt{a + bx^c}$ was fitted, $m_{pcac} > 0$. $a = 0.1068 \pm 0.0013$, $b = 11.238 \pm 0.1228$, $c = 3.0$ $m_\pi = 0.3269(21)$ for variance and $a = 0.1077 \pm 0.0014$, $b = 11.1881 \pm 0.1152$, $c = 3.0$, $m_\pi = 0.3281(21)$ for chi squared



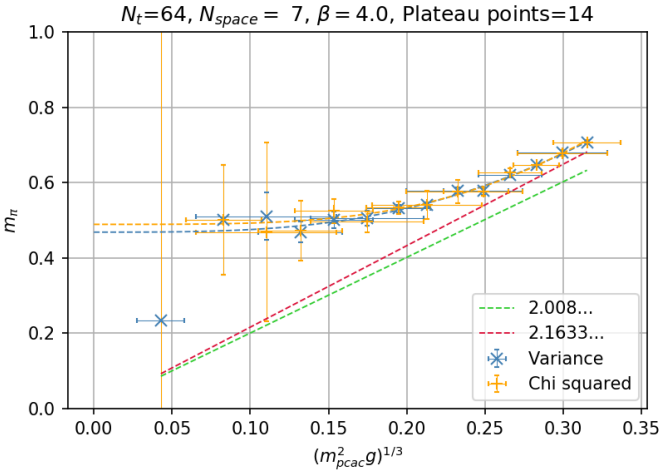
(e) $y = \sqrt{a + bx^c}$ was fitted, $m_{pcac} > 0$. $a = 0.1265 \pm 0.0032$, $b = 35.3606 \pm 1.4906$, $c = 4.0$ $m_\pi = 0.3557(46)$ for variance and $a = 0.1286 \pm 0.0035$, $b = 34.8539 \pm 1.4156$, $c = 4.0$, $m_\pi = 0.3586(49)$ for chi squared



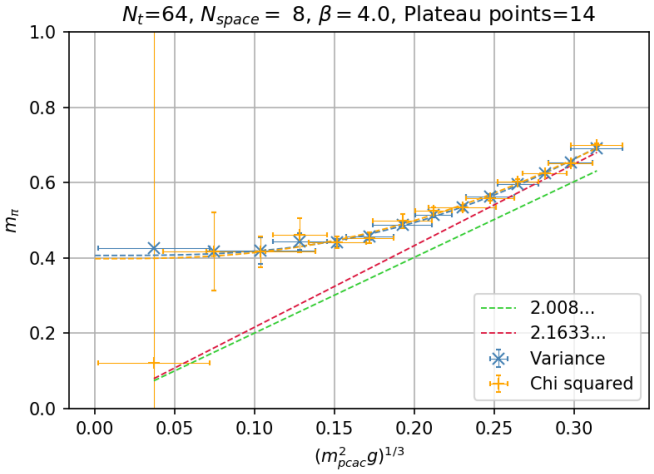
(f) $y = \sqrt{a + bx^c}$ was fitted, $m_{pcac} > 0$. $a = 0.0964 \pm 0.0014$, $b = 7.9435 \pm 0.275$, $c = 2.6832 \pm 0.0319$ $m_\pi = 0.3105(23)$ for variance and $a = 0.0965 \pm 0.0012$, $b = 7.9065 \pm 0.2137$, $c = 2.6832 \pm 0.0319$ $m_\pi = 0.3106(19)$ for chi squared



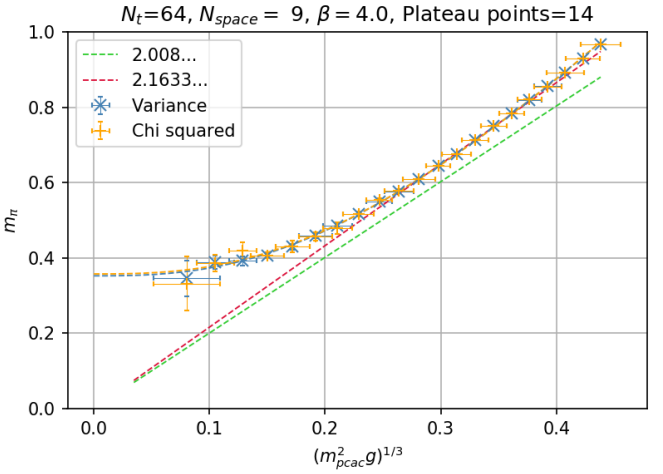
(g) A function of the form $y = \sqrt{a + bx^c}$ was fitted. $m_{pcac} > 0$. $c = 3.4651 \pm 0.7174$, $m_\pi = 0.6689 \pm 0.0232$ for variance and $c = 1.9228 \pm 0.7116$, $m_\pi = 0.6097 \pm 0.0854$ for chi squared.



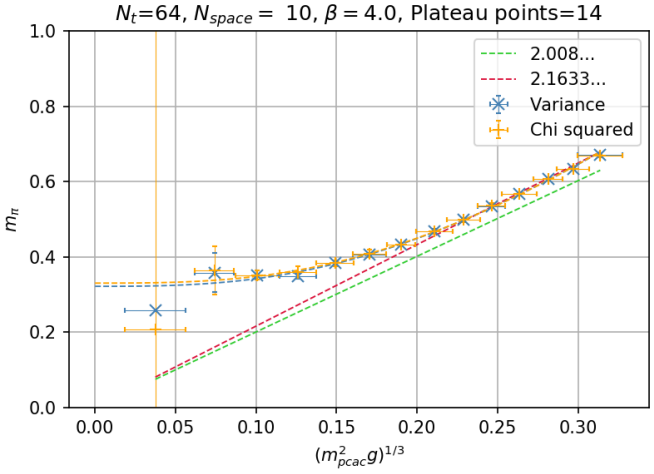
(h) A function of the form $y = \sqrt{a + bx^c}$ was fitted. $m_{pcac} > 0$. $c = 3.2848 \pm 0.4176$, $m_\pi = 0.4684 \pm 0.0111$ for variance and $c = 3.8378 \pm 0.6821$, $m_\pi = 0.4892 \pm 0.0143$ for chi squared



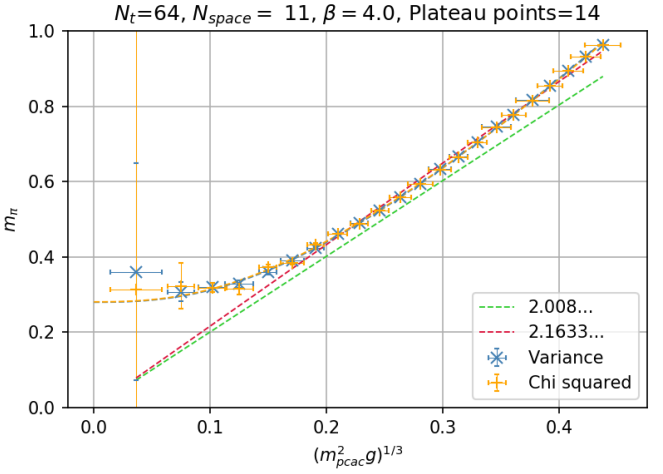
(i) A function of the form $y = \sqrt{a + bx^c}$ was fitted. $m_{pcac} > 0$. $c = 3.0663 \pm 0.2013$, $m_\pi = 0.4065 \pm 0.0075$ for variance and $c = 2.783 \pm 0.3346$, $m_\pi = 0.3979 \pm 0.017$ for chi squared.



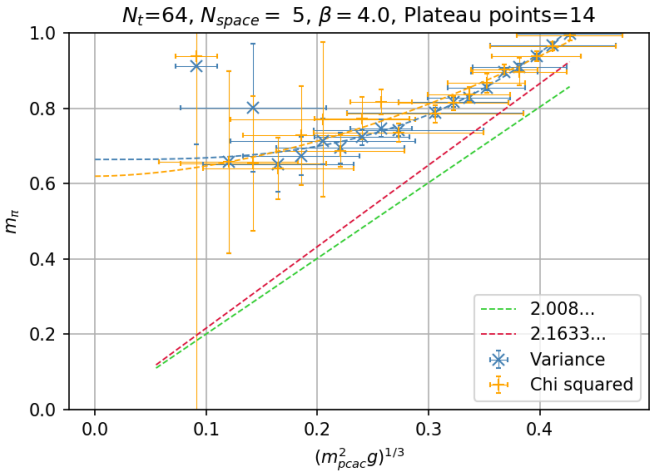
(j) A function of the form $y = \sqrt{a + bx^c}$ was fitted. $m_{pcac} > 0$. $c = 2.7049 \pm 0.0456$, $m_\pi = 0.3525 \pm 0.0038$ for variance and $c = 2.7325 \pm 0.0753$, $m_\pi = 0.3575 \pm 0.0067$ for chi squared.



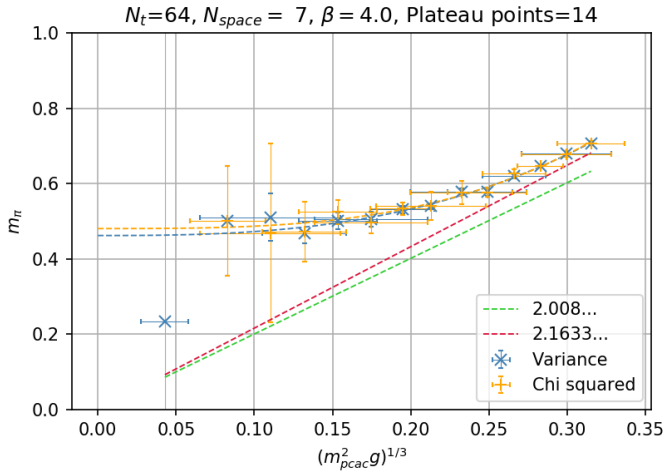
(k) A function of the form $y = \sqrt{a + bx^c}$ was fitted. $m_{pcac} > 0$. $c = 2.8691 \pm 0.1585$, $m_\pi = 0.322 \pm 0.0074$ for variance and $c = 2.9509 \pm 0.1014$, $m_\pi = 0.3305 \pm 0.0045$ for chi squared.



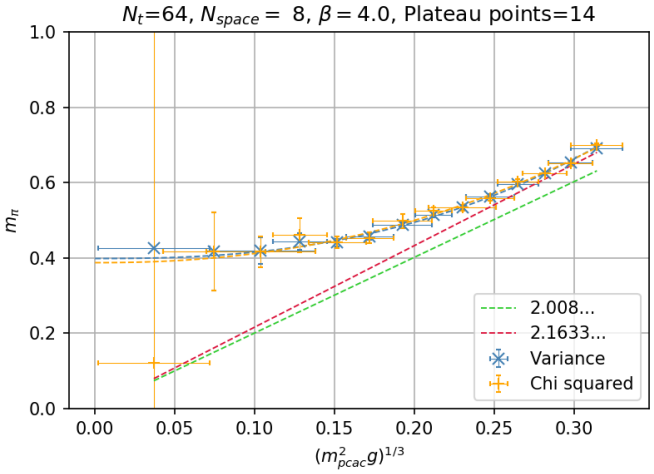
(l) A function of the form $y = \sqrt{a + bx^c}$ was fitted. $m_{pcac} > 0$. $c = 2.5818 \pm 0.0323$, $m_\pi = 0.2798 \pm 0.0036$ for variance and $c = 2.5528 \pm 0.0539$, $m_\pi = 0.2807 \pm 0.0066$ for chi squared.



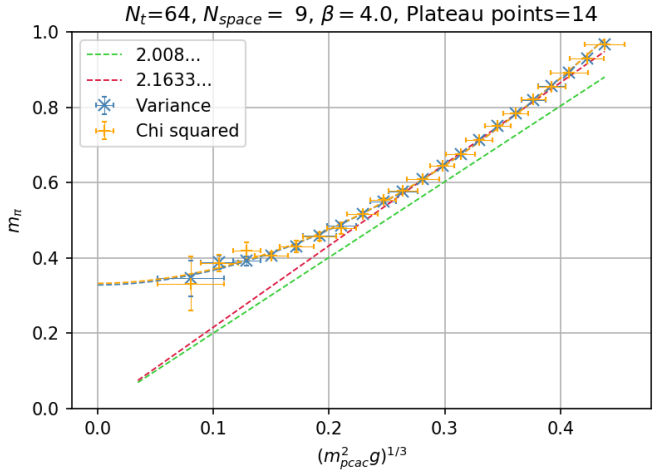
(m) A function of the form $y = a + bx^c$ was fitted. $m_{pcac} > 0$. $c = 3.0182 \pm 0.7033$, $m_\pi = 0.664 \pm 0.0259$ for variance and $c = 1.7735 \pm 0.7512$, $m_\pi = 0.6196 \pm 0.0806$ for chi squared.



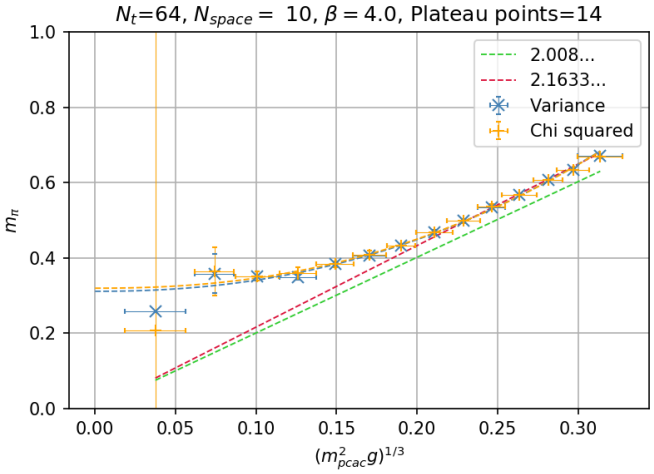
(n) A function of the form $y = a + bx^c$ was fitted. $m_{pcac} > 0$. $c = 2.7568 \pm 0.4221$, $m_\pi = 0.4622 \pm 0.0136$ for variance and $c = 3.132 \pm 0.6695$, $m_\pi = 0.4806 \pm 0.0183$ for chi squared.



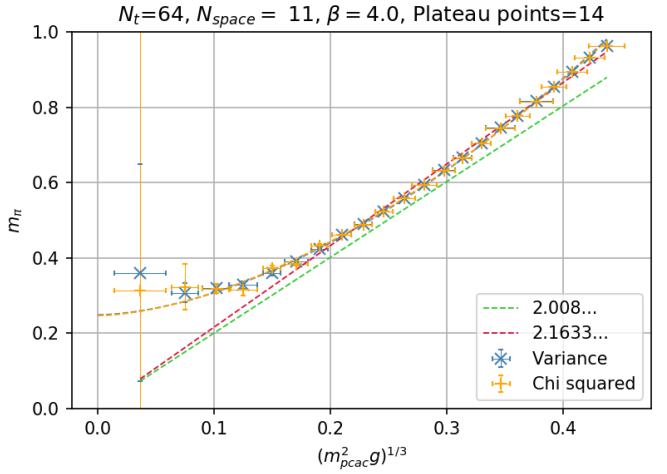
(o) A function of the form $y = a + bx^c$ was fitted. $m_{pcac} > 0$. $c = 2.4945 \pm 0.2091$, $m_\pi = 0.3984 \pm 0.0095$ for variance and $c = 2.2133 \pm 0.3326$, $m_\pi = 0.3874 \pm 0.0206$ for chi squared.



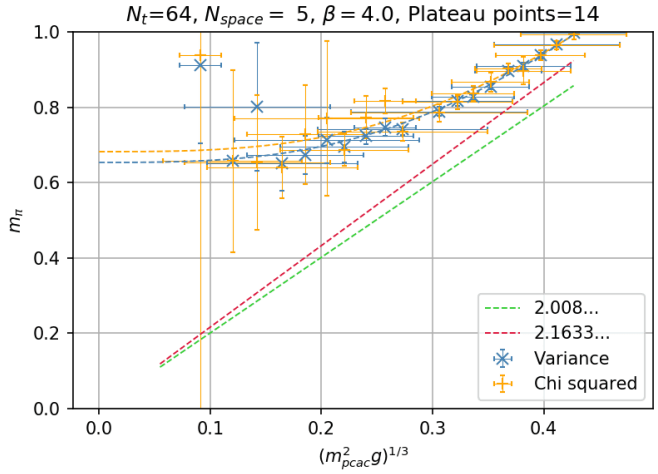
(p) A function of the form $y = a + bx^c$ was fitted. $m_{pcac} > 0$. $c = 1.9056 \pm 0.0561$, $m_\pi = 0.3284 \pm 0.0065$ for variance and $c = 1.9209 \pm 0.0809$, $m_\pi = 0.3325 \pm 0.0098$ for chi squared.



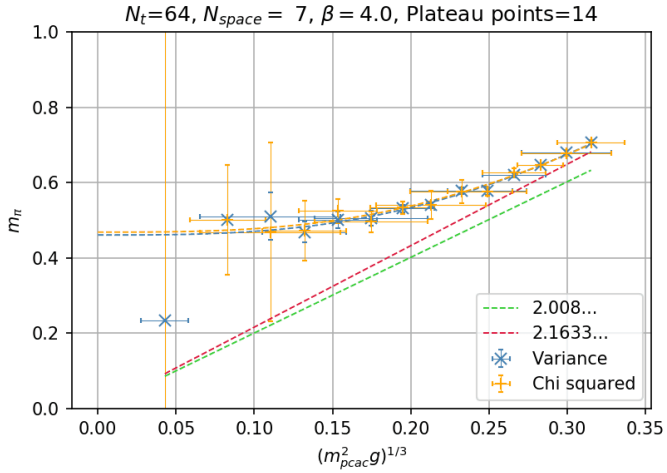
(q) A function of the form $y = a + bx^c$ was fitted. $m_{pcac} > 0$. $c = 2.2171 \pm 0.169$, $m_\pi = 0.3114 \pm 0.01$ for variance and $c = 2.2766 \pm 0.1222$, $m_\pi = 0.3197 \pm 0.007$ for chi squared.



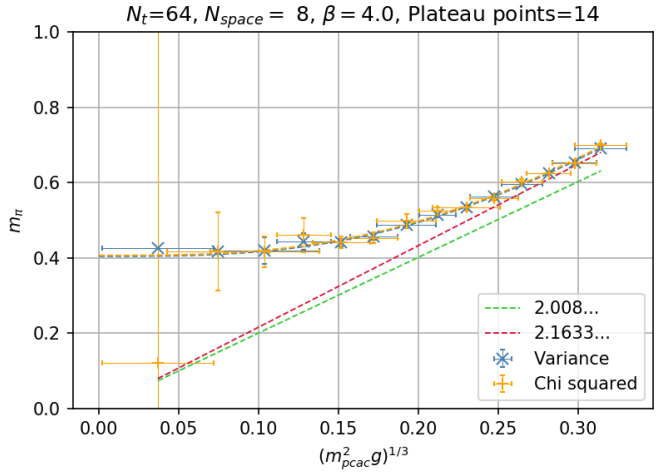
(r) A function of the form $y = a + bx^c$ was fitted. $m_{pcac} > 0$. $c = 1.7062 \pm 0.047$, $m_\pi = 0.2489 \pm 0.0073$ for variance and $c = 1.6732 \pm 0.0557$, $m_\pi = 0.2469 \pm 0.0093$ for chi squared.



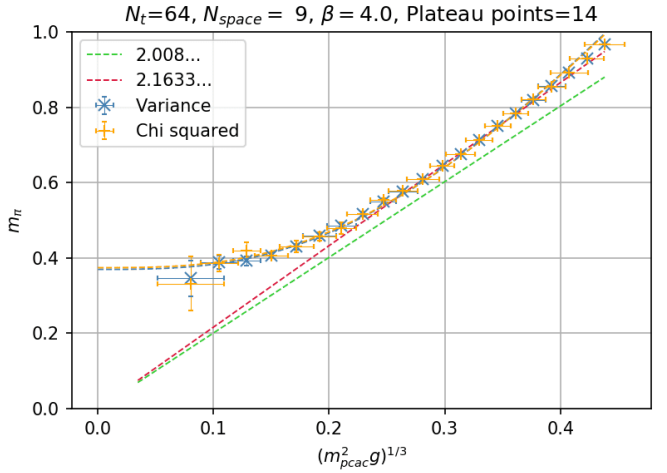
(s) A function of the form $y = \sqrt{a+bx^3}$ was fitted. $m_{pcac} > 0$. m_π =0.65356 ± 0.01149 for variance and m_π =0.6821 ± 0.01744 for chi squared



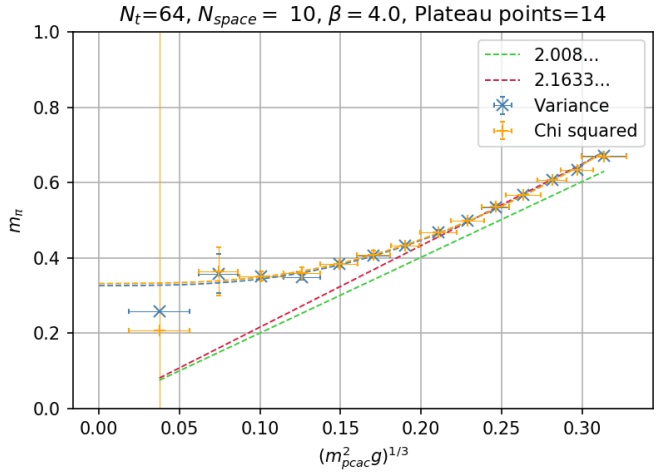
(t) A function of the form $y = \sqrt{a+bx^3}$ was fitted. $m_{pcac} > 0$. m_π =0.46083 ± 0.00521 for variance and m_π =0.46838 ± 0.00775 for chi squared



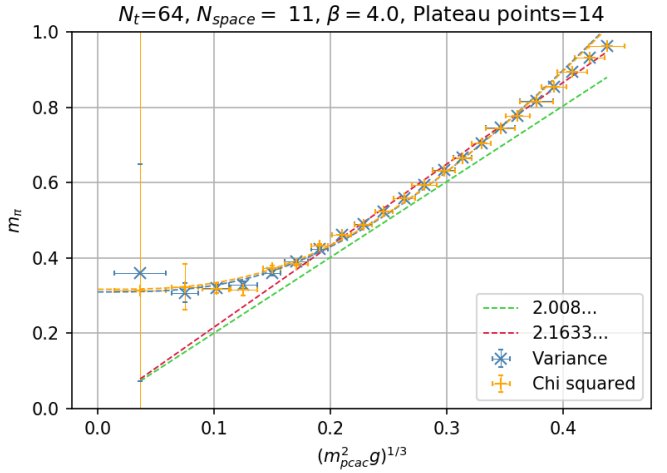
(u) A function of the form $y = \sqrt{a+bx^3}$ was fitted. $m_{pcac} > 0$. m_π =0.40426 ± 0.00317 for variance and m_π =0.40679 ± 0.00629 for chi squared.



(v) A function of the form $y = \sqrt{a+bx^3}$ was fitted. $m_{pcac} > 0$. m_π =0.36969 ± 0.00309 for variance and m_π =0.37412 ± 0.00402 for chi squared.



(w) A function of the form $y = \sqrt{a+bx^3}$ was fitted. $m_{pcac} > 0$. m_π =0.32665 ± 0.00334 for variance and m_π =0.33234 ± 0.00218 for chi squared.



(x) A function of the form $y = \sqrt{a+bx^3}$ was fitted. $m_{pcac} > 0$. m_π =0.3098 ± 0.00439 for variance and m_π =0.31667 ± 0.00538 for chi squared

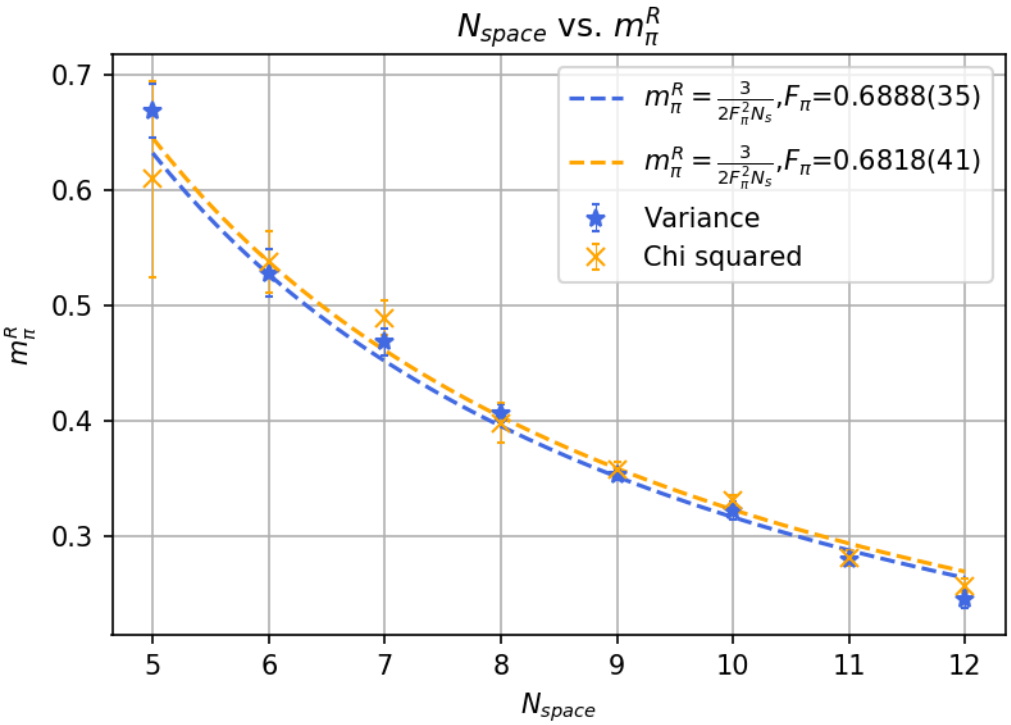


Figure 1: Result obtained by fitting $y = \sqrt{a + bx^c}$. For variance $F_\pi = 0.6888(35)$, while for chi squared $F_\pi = 0.6818(41)$

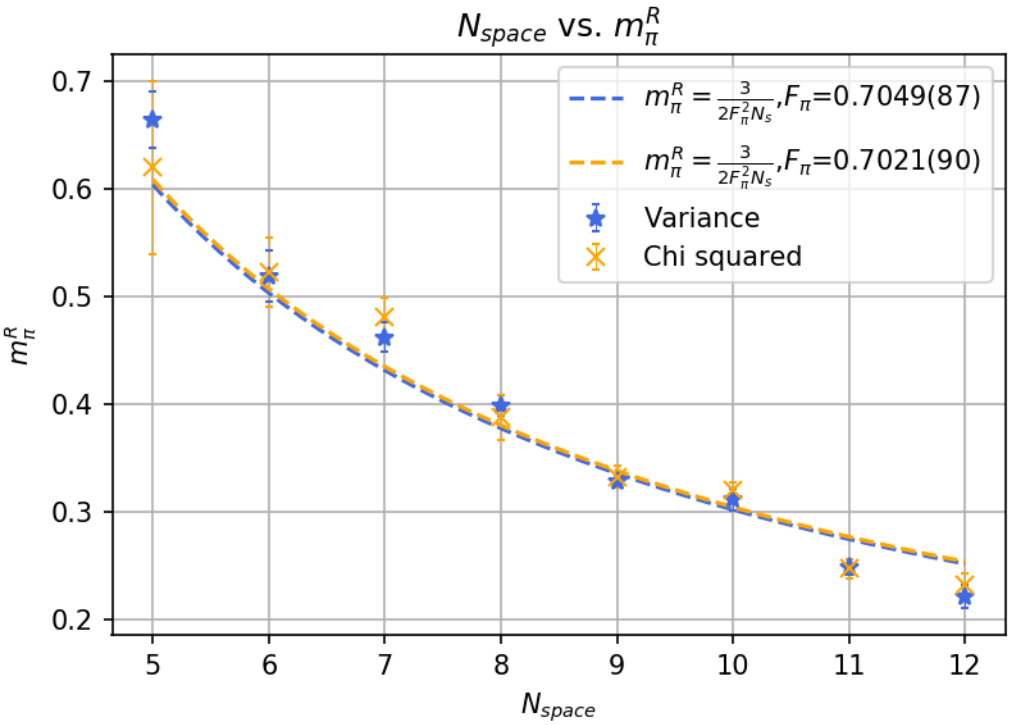


Figure 2: Result obtained by fitting $y = a + bx^c$. For variance $F_\pi = 0.7049(87)$, while for chi squared $F_\pi = 0.7021(90)$

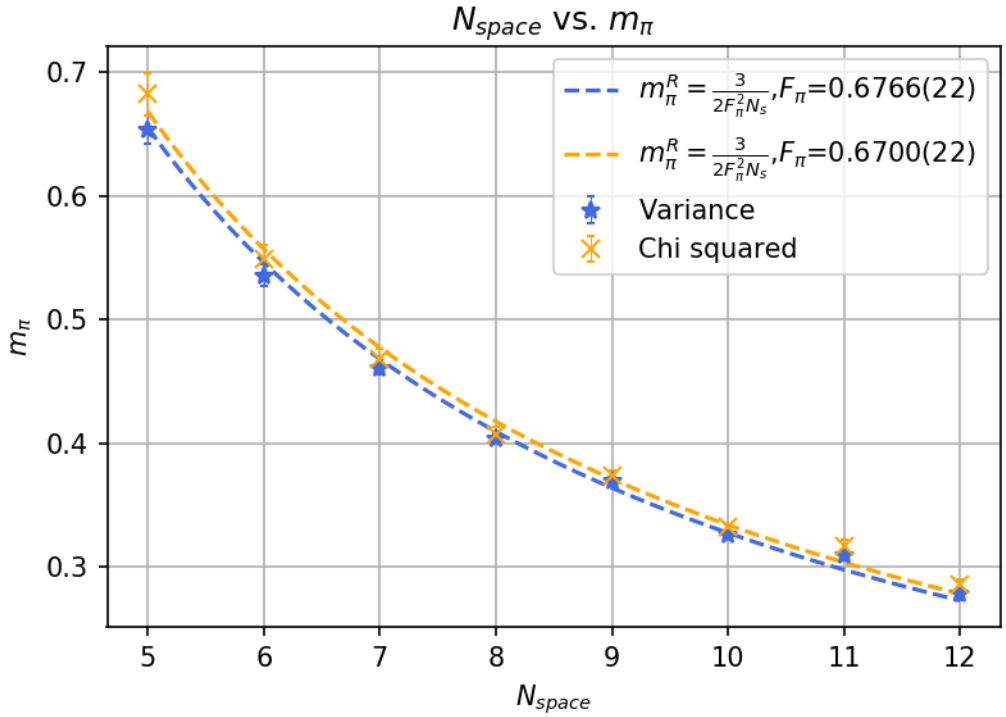


Figure 3: Result obtained by fitting $y = a + bx^3$. For variance $F_\pi = 0.6766(22)$, while for chi squared $F_\pi = 0.6700(22)$.