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March 12, 2021

Low statistics results

The following results were obtained through several simulations on different lattices: 6×64, 7×64, 8×64, 9×64, 10×64, 11×64 and 12×64 with the parameters:

Ntime	64
Ntherm	500
Nmeasure	1000
Trajectory Steps	10
Nsteps	10
β	4

Table 1: All the simulations were performed with this parameters.

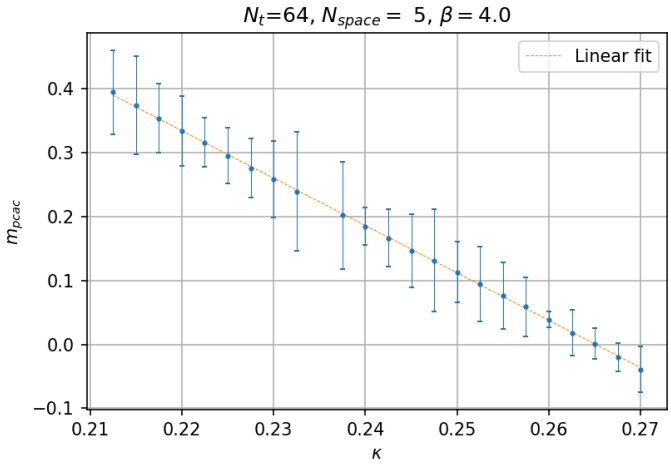
Variance stands for the var option in the masscoll program and Chi squared for the χ^2 option. $g = \frac{1}{\sqrt{\beta}} = \frac{1}{2}$. The residual pion mass is extrapolated with two different methods. From the plots of m_π^2 vs. m_{pcac} ona can fit a parabola

$$y = a + bx^2. \tag{1}$$

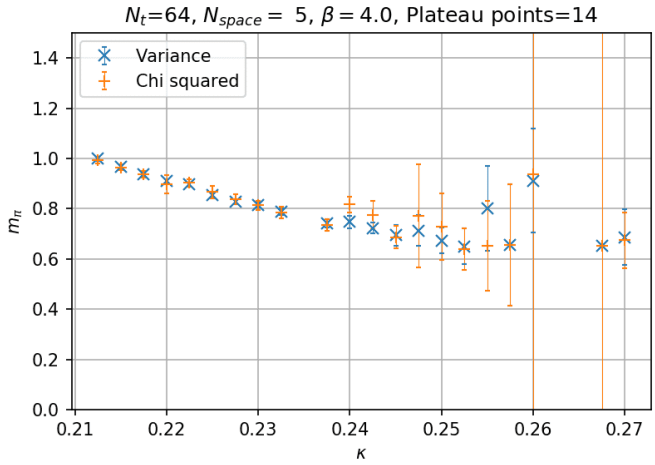
Meanwhile, from the plots of m_π vs. $(gm_{pcac}^2)^{1/3}$ one can fit a function of the form

$$y = \sqrt{a + bx^3}. \tag{2}$$

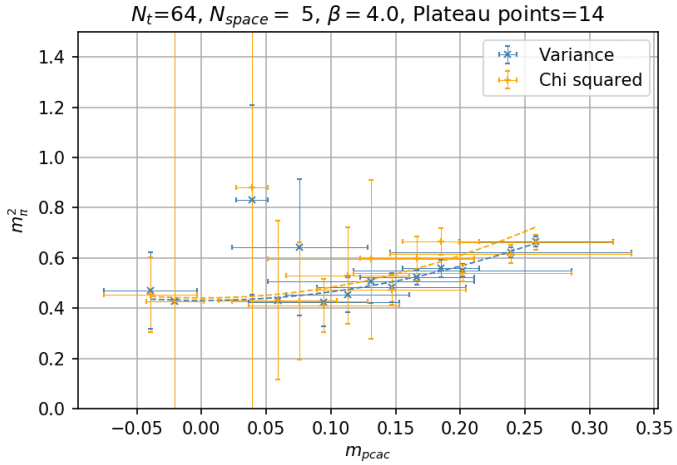
5x64



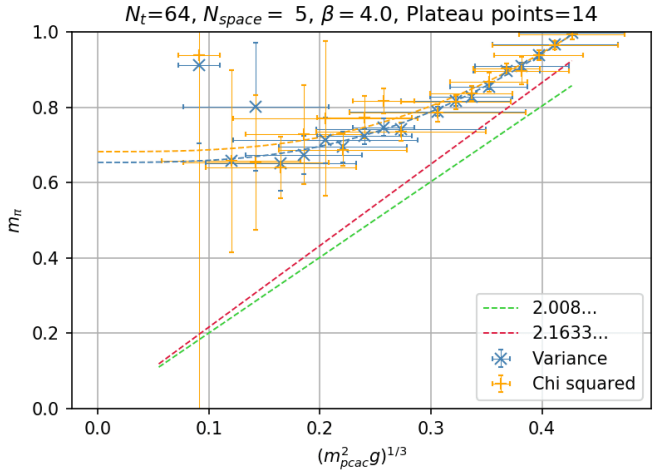
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26508 \pm 0.00119$



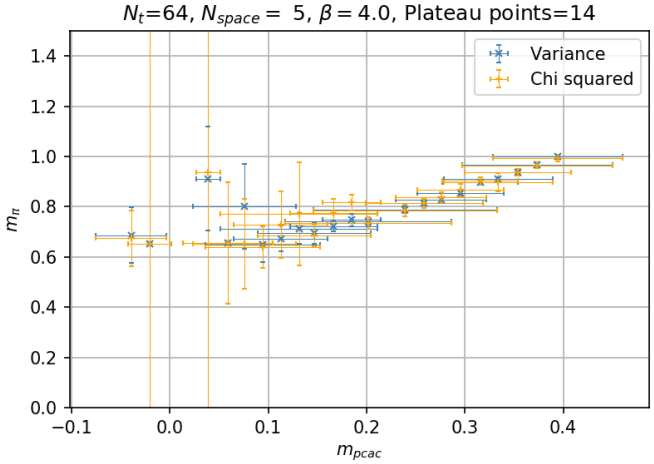
(b) Pion mass as a function of κ.



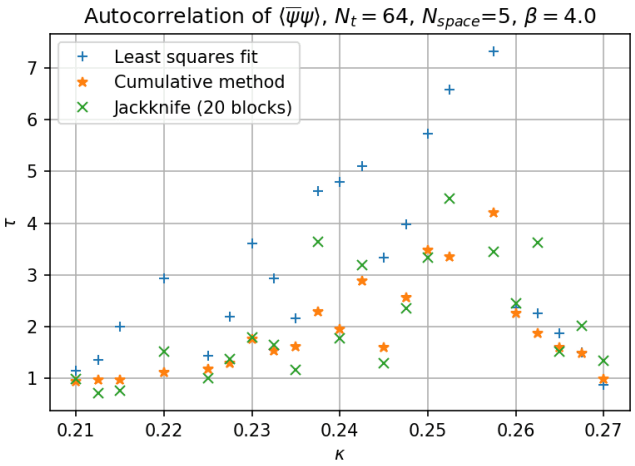
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.43078 ± 0.02268, b =3.41885 ± 0.71614, m_π =0.65634 ± 0.01728 for variance and a =0.44075 ± 0.03599, b =4.23048 ± 1.0871, m_π =0.66389 ± 0.02711 for chi squared.



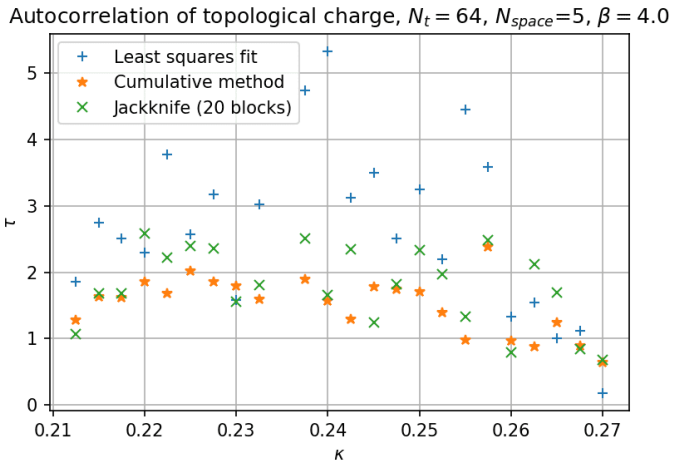
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.42714 ± 0.01502, b =7.17786 ± 0.492, m_π =0.65356 ± 0.01149 for variance and a =0.46526 ± 0.02379, b =6.66985 ± 0.67166, m_π =0.6821 ± 0.01744 for chi squared.



(e) m_π vs. m_{pcac}.



(f) Autocorrelation of $\langle\bar{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

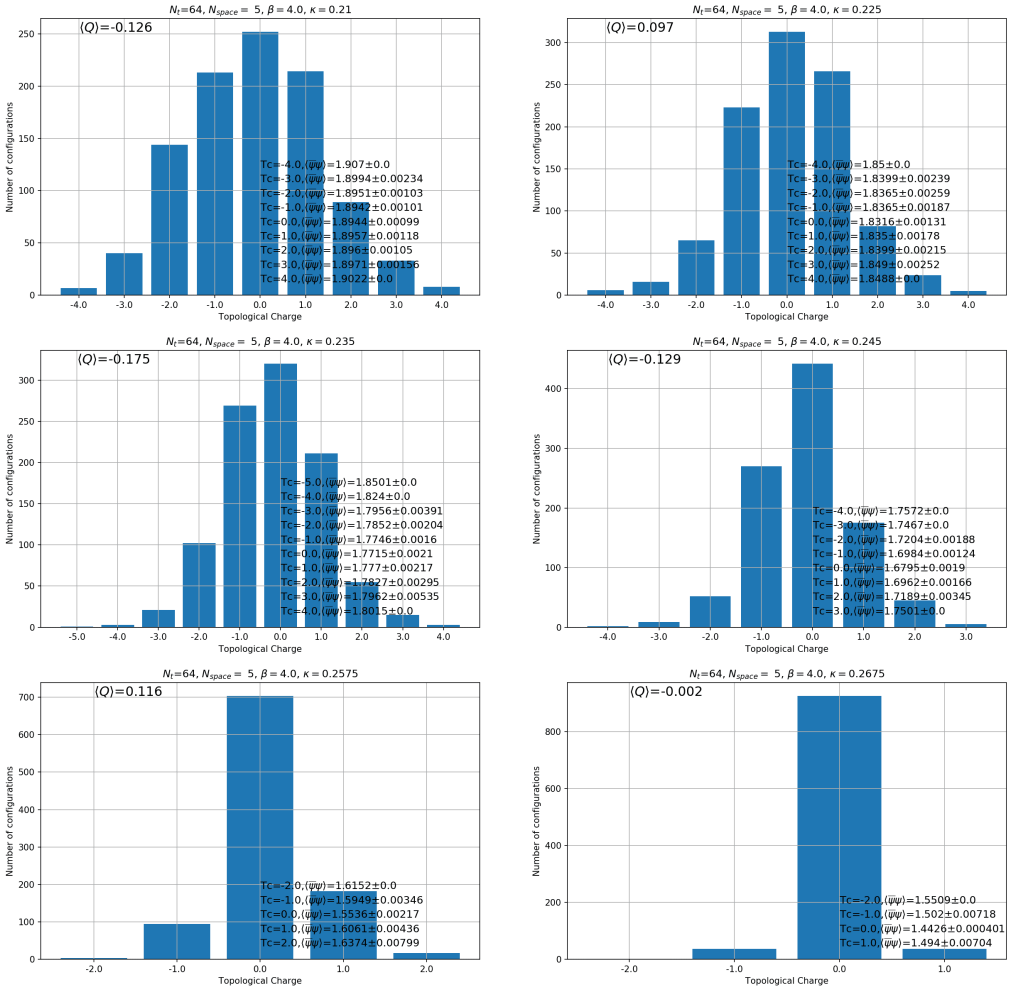
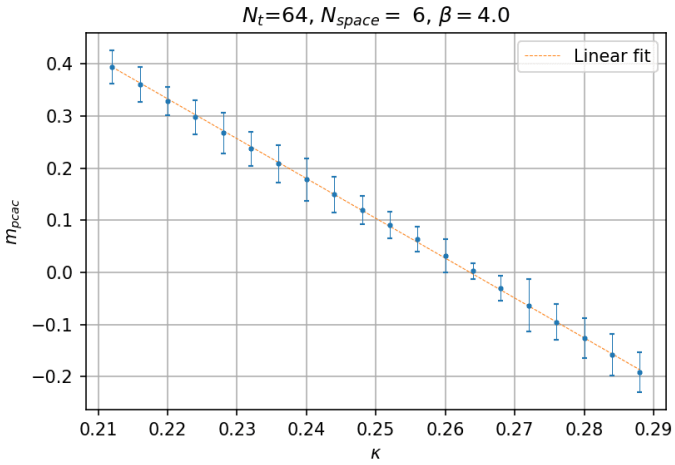
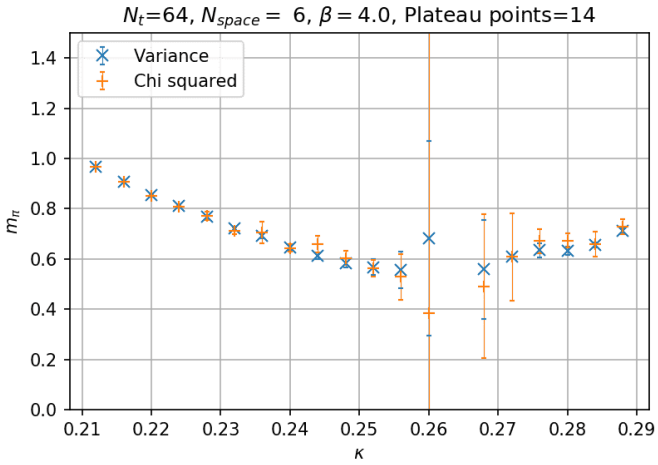


Figure 1: Number of configurations vs. topological charge on a 5×64 lattice.

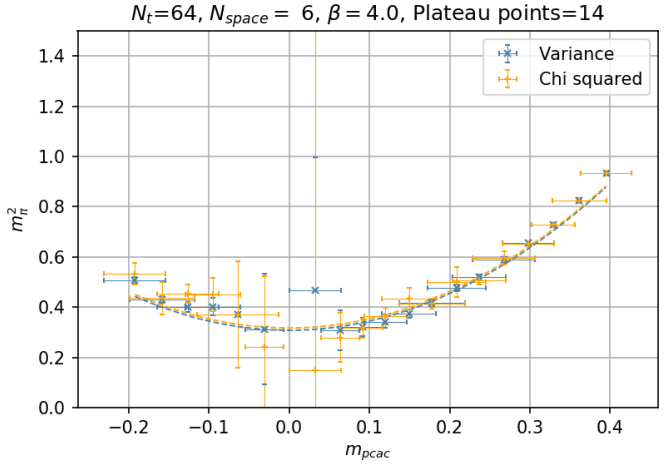
6x64



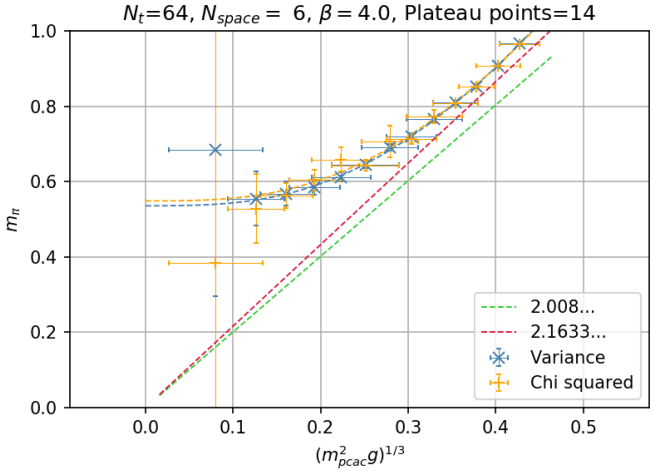
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26356 \pm 0.00318$



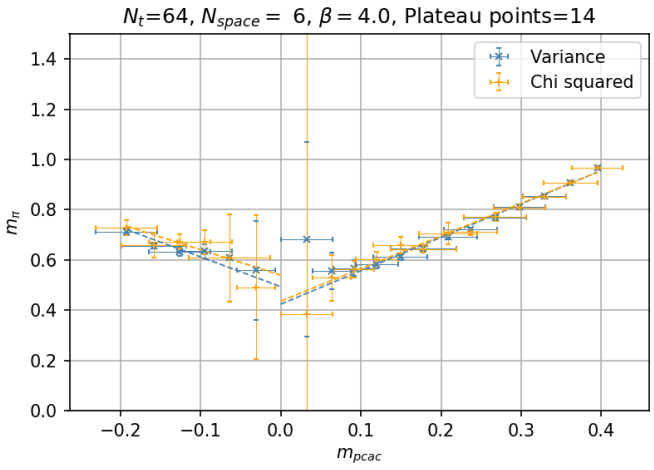
(b) Pion mass as a function of κ.



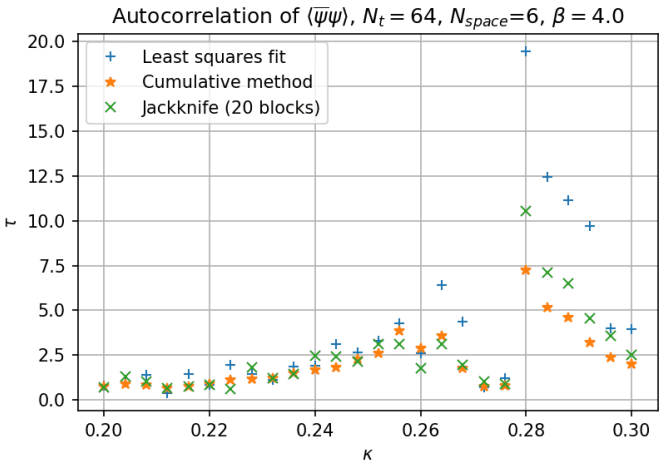
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.3079 ± 0.01585, b =3.65833 ± 0.63195, m_π =0.55489 ± 0.01428 for variance and a =0.31726 ± 0.02114, b =3.62967 ± 0.79829, m_π =0.56326 ± 0.01876 for chi squared.



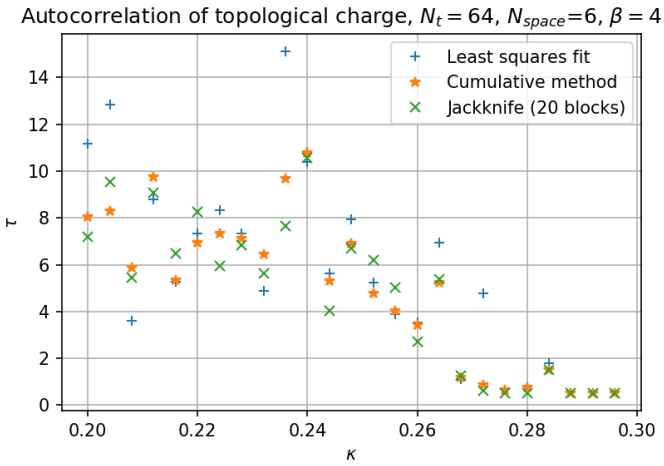
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.28716 ± 0.0094, b =8.27208 ± 0.33396, m_π =0.53588 ± 0.00877 for variance and a =0.30109 ± 0.0128, b =8.04211 ± 0.40997, m_π =0.54871 ± 0.01166 for chi squared.



(e) m_π vs. m_{pcac}. Var fit constants. Left side m_π=0.49366±0.0186. Right side m_π=0.42375±0.01578. Chi fit constants. Left side m_π=0.5394±0.02252. Right side m_π=0.43565±0.01682.



(f) Autocorrelation of $\langle\bar{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

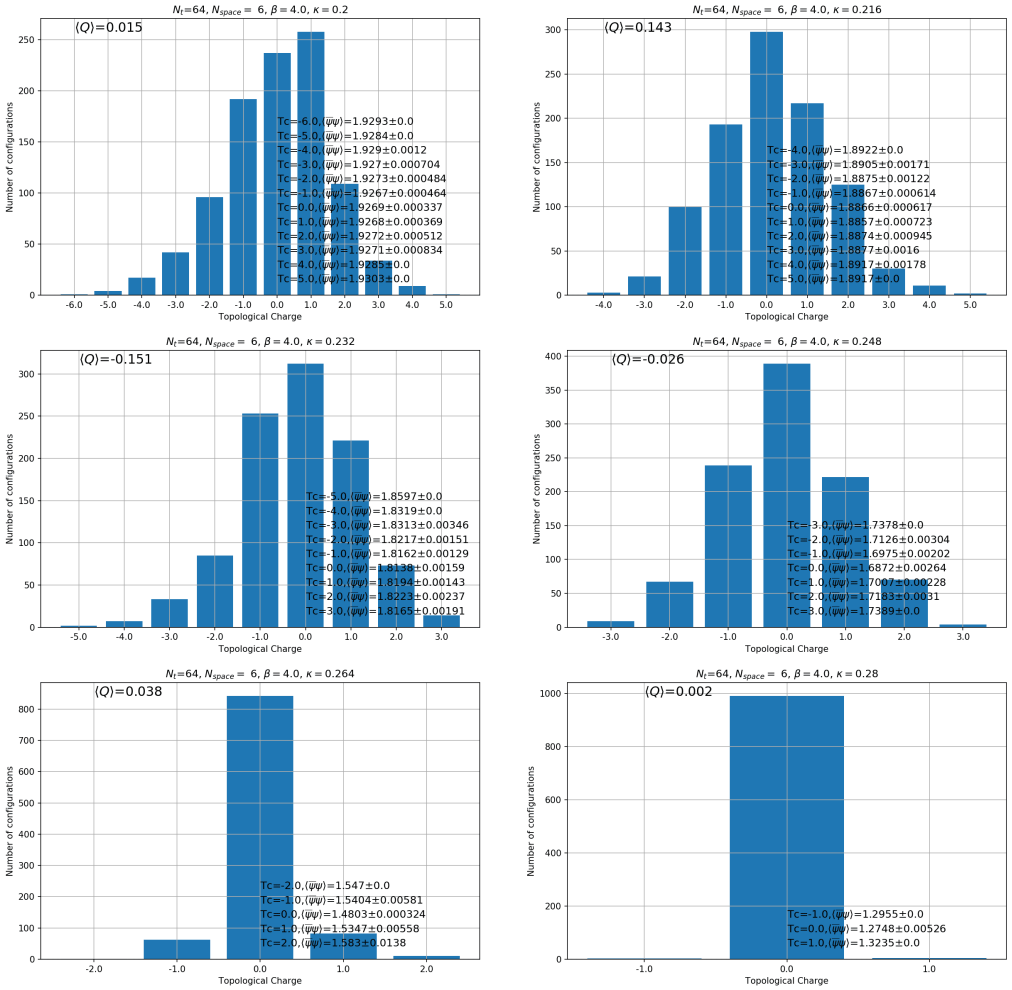
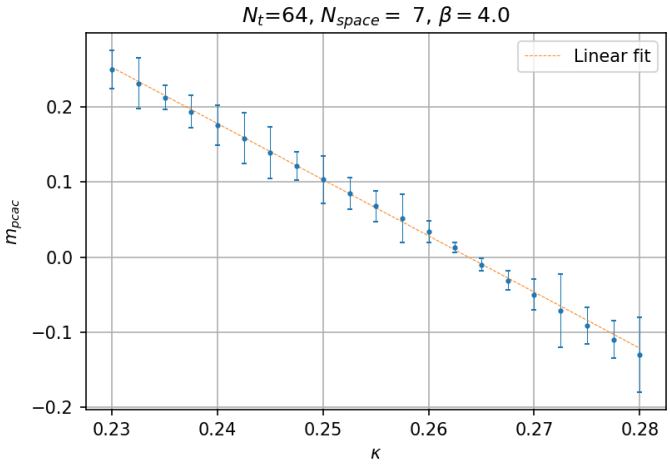
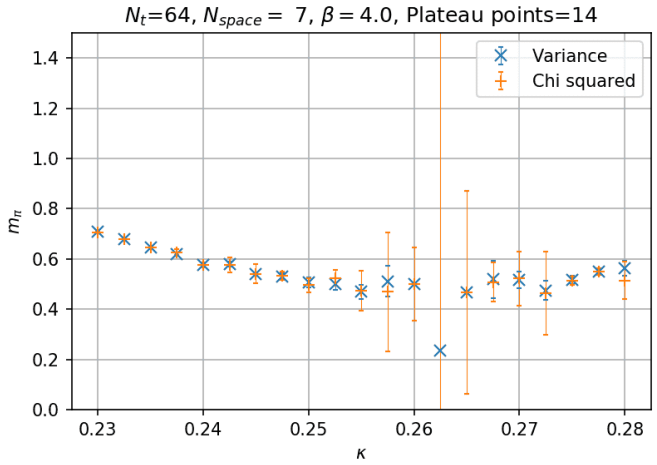


Figure 2: Number of configurations vs. topological charge on a 6×64 lattice.

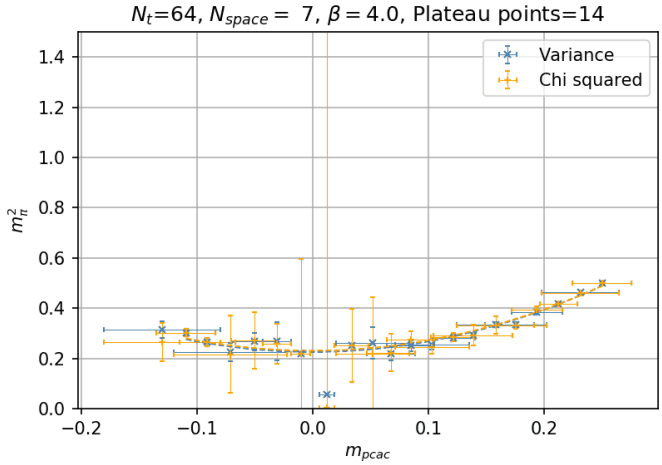
7x64



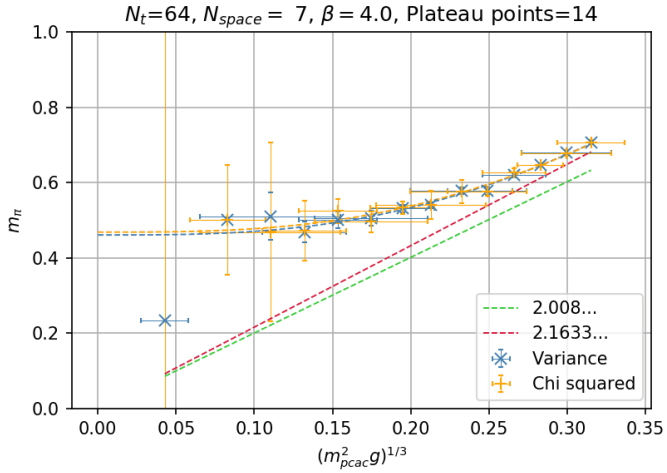
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26378 \pm 0.00323$



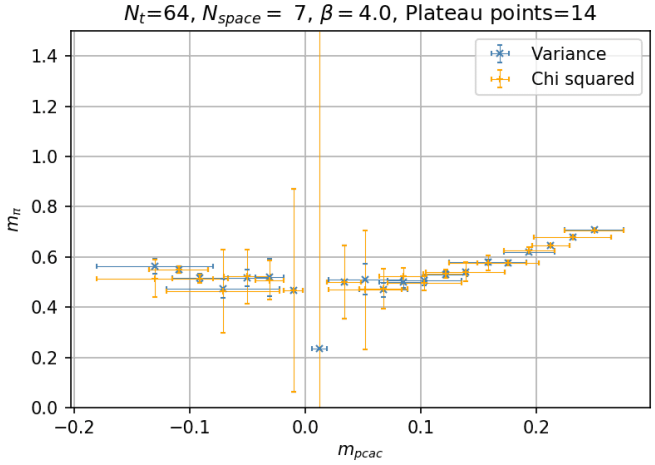
(b) Pion mass as a function of κ



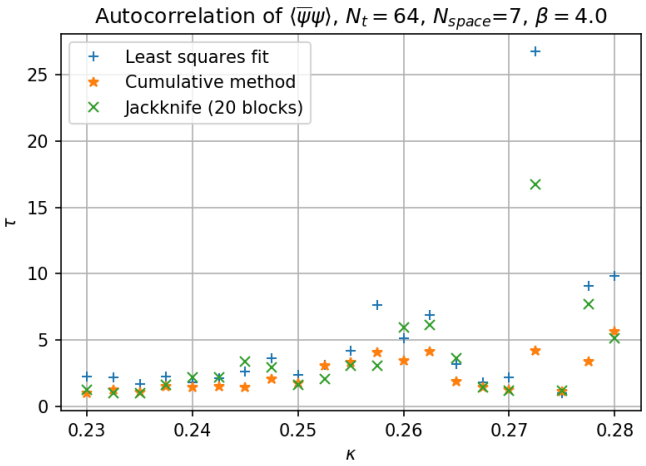
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.22561 ± 0.00592, b =4.1984 ± 0.25992, m_π =0.47498 ± 0.00624 for variance and a =0.23081 ± 0.00603, b =4.11223 ± 0.22776, m_π =0.48042 ± 0.00628 for chi squared.



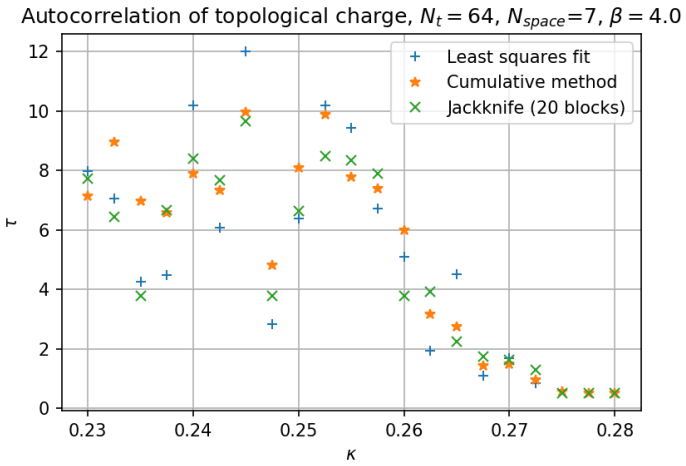
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.21237 ± 0.0048, b =9.0339 ± 0.36531, m_π =0.46083 ± 0.00521 for variance and a =0.21938 ± 0.00726, b =8.73561 ± 0.46984, m_π =0.46838 ± 0.00775 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

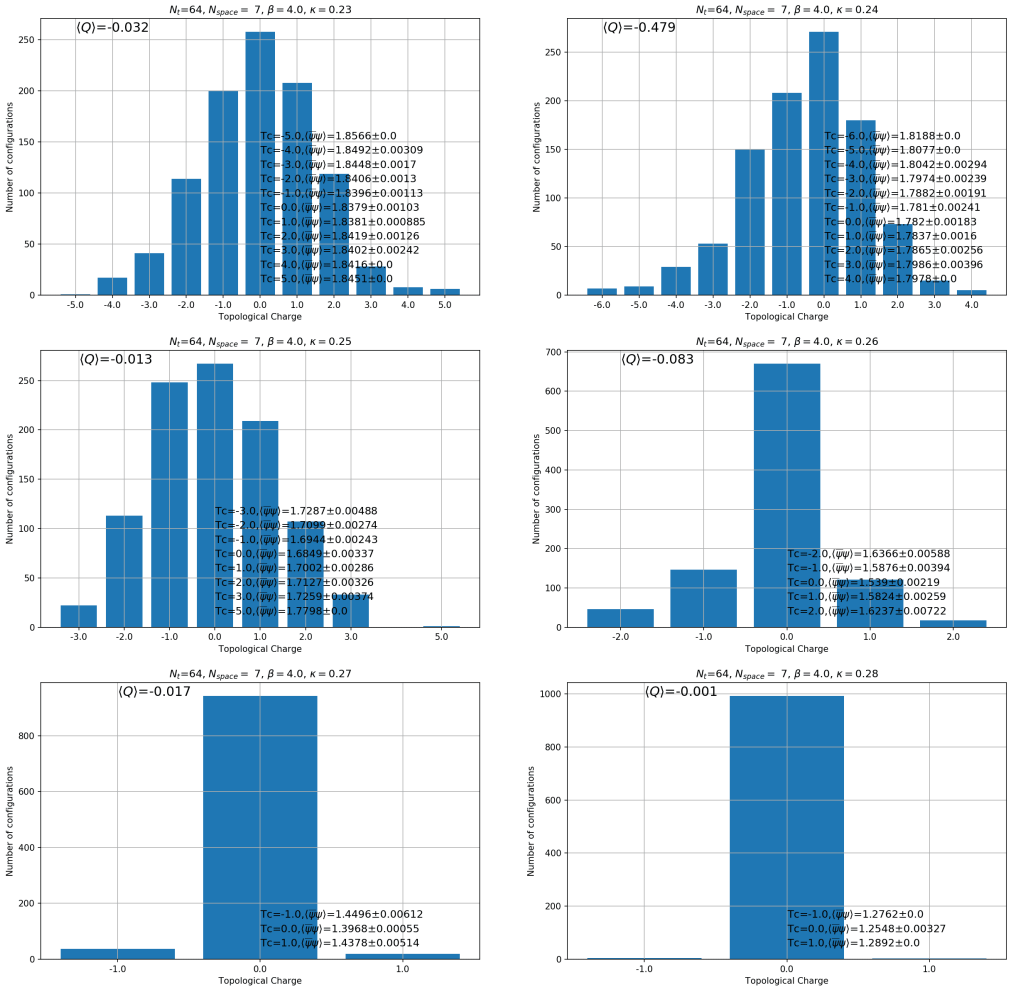
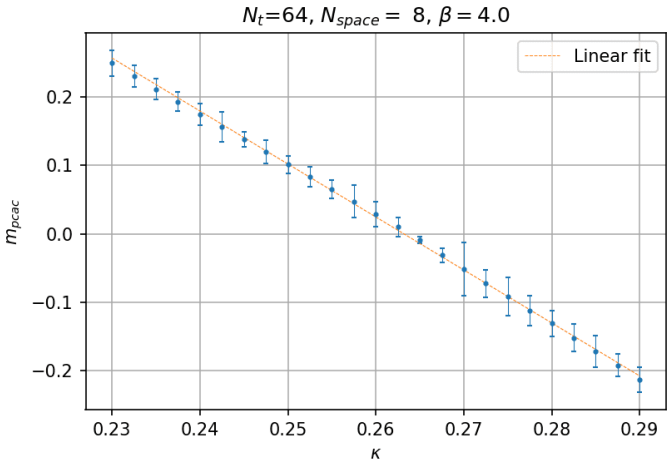
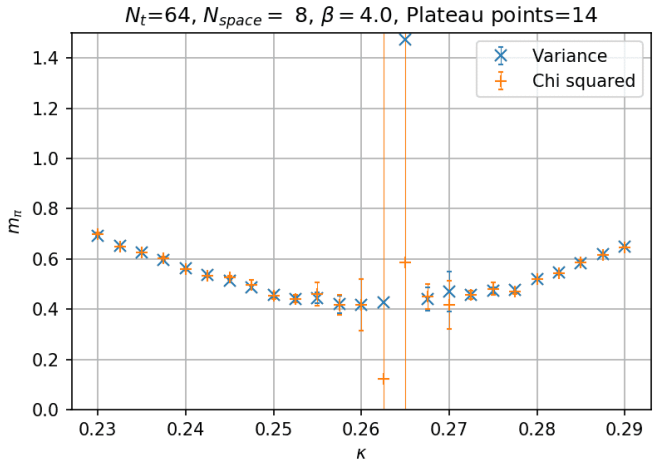


Figure 3: Number of configurations vs. topological charge on a 7×64 lattice.

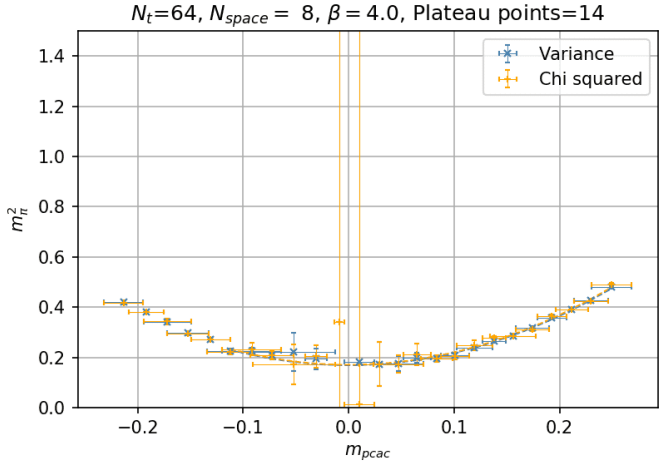
8x64



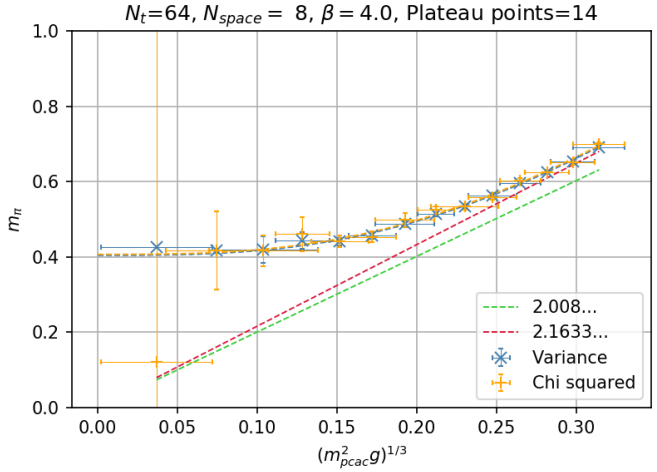
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26316 \pm 0.00263$



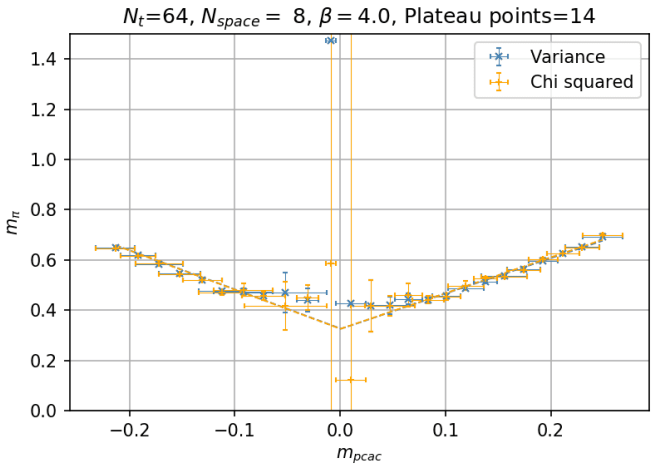
(b) Pion mass as a function of κ



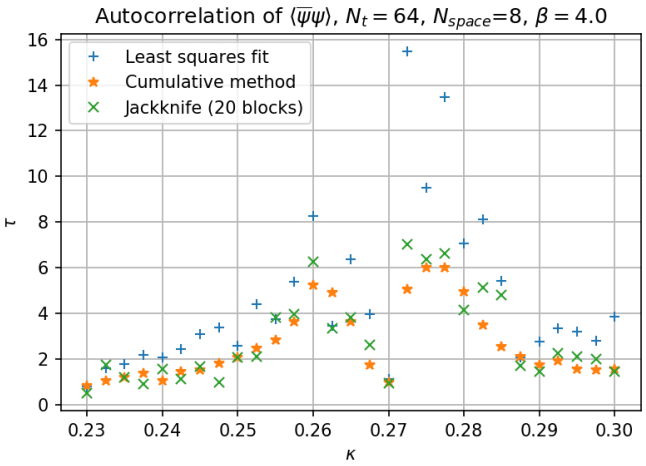
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.16869 ± 0.00315, b =4.91552 ± 0.1548, m_π =0.41072 ± 0.00383 for variance and a =0.17081 ± 0.00449, b =4.96313 ± 0.20058, m_π =0.41329 ± 0.00543 for chi squared.



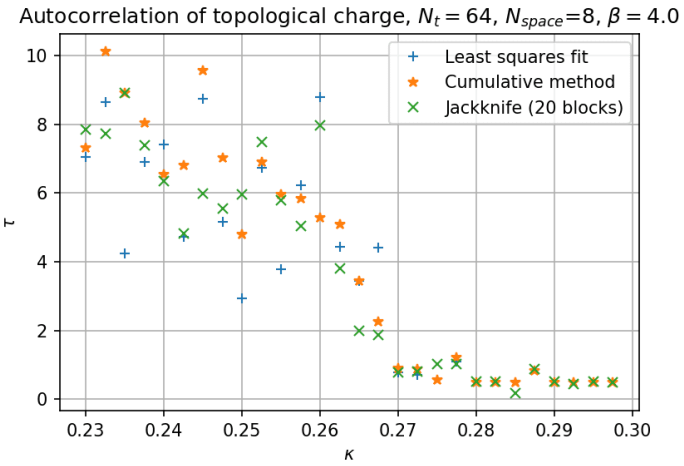
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.16342 ± 0.00256, b =10.15484 ± 0.23169, m_π =0.40426 ± 0.00317 for variance and a =0.16548 ± 0.00512, b =10.25655 ± 0.41141, m_π =0.40679 ± 0.00629 for chi squared.



(e) m_π vs. m_{pcac}. Var fit constants. Left side m_π=0.32734±0.01422. Right side m_π=0.32476±0.00903. Chi fit constants. Left side m_π=0.32631±0.01496. Right side m_π=0.32495±0.01099.



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

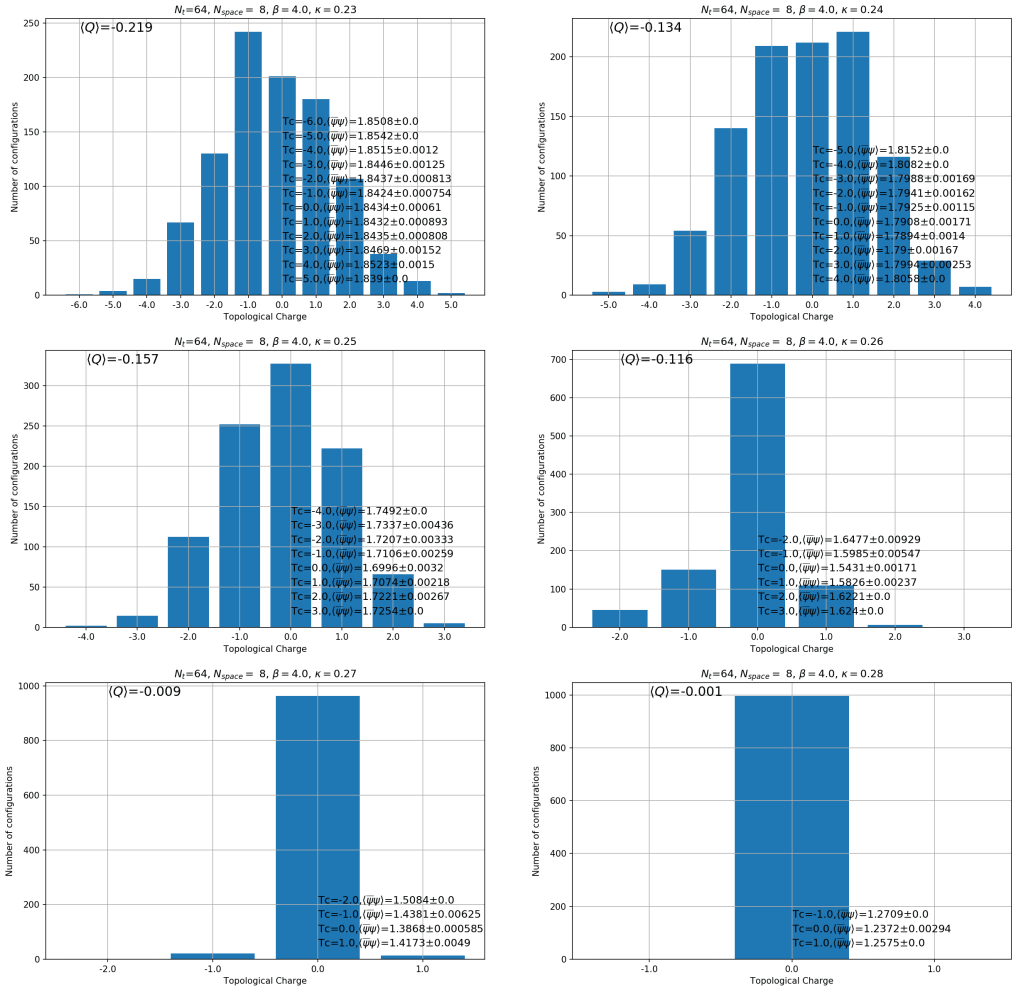
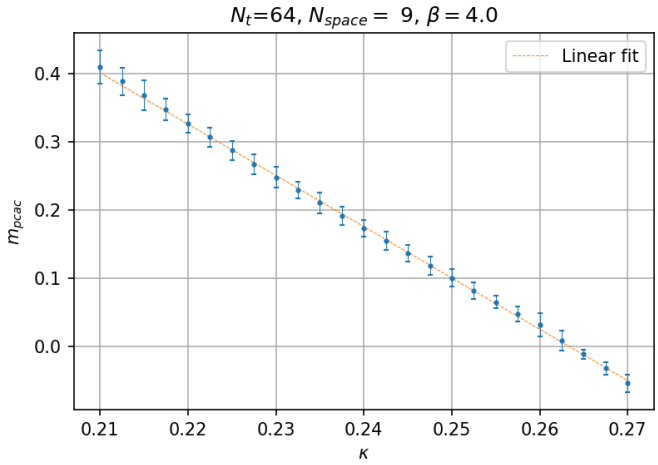
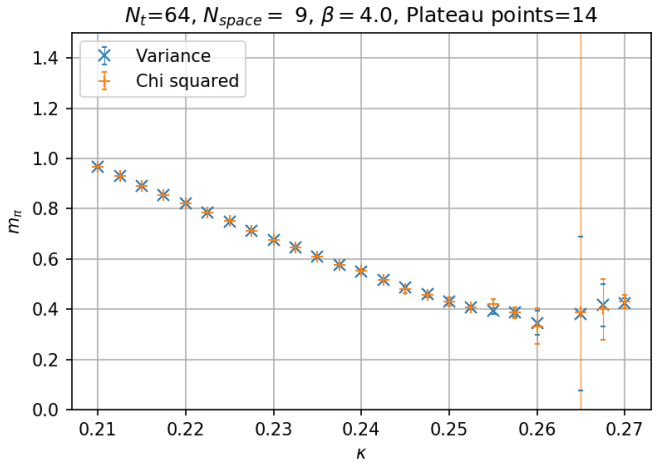


Figure 4: Number of configurations vs. topological charge on a 8×64 lattice.

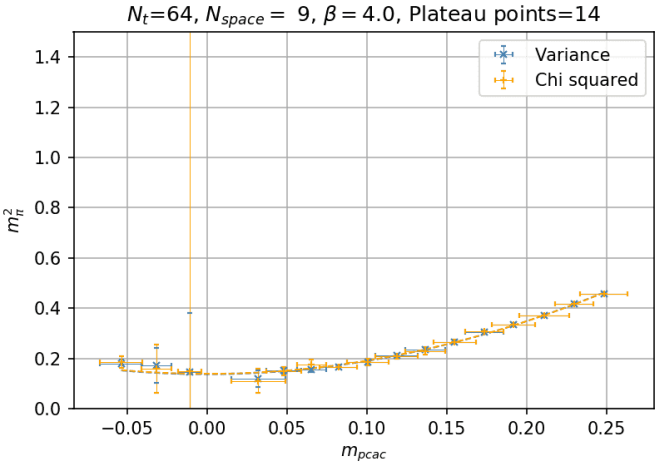
9x64



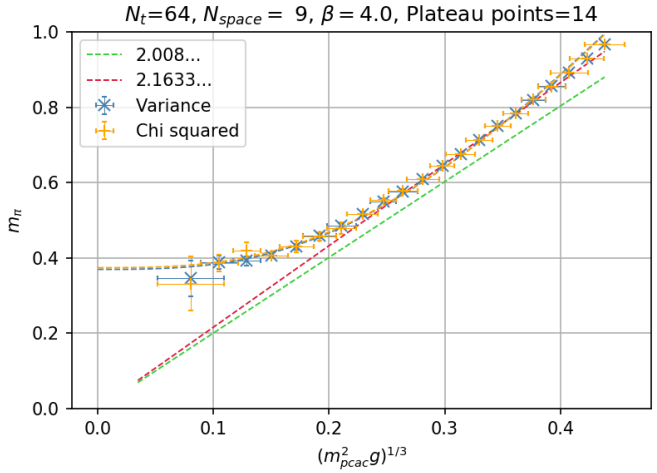
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26343 \pm 0.00153$



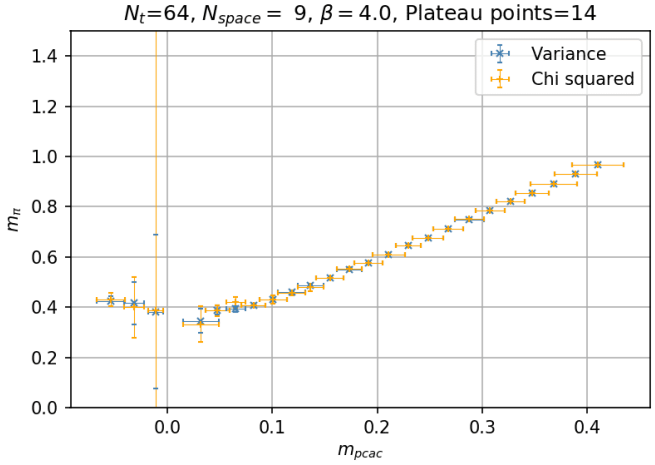
(b) Pion mass as a function of κ



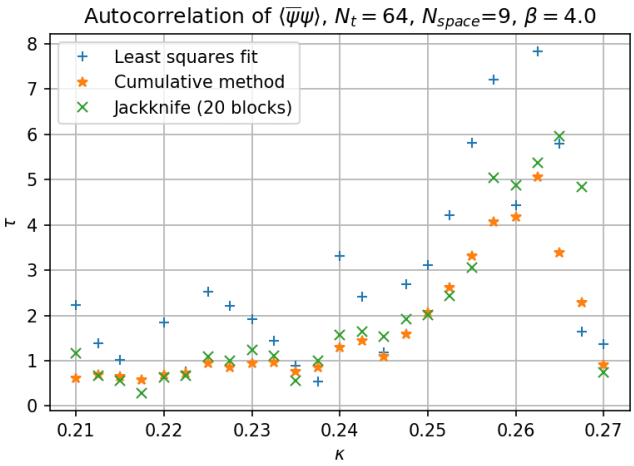
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.13652 ± 0.00356, b =5.29252 ± 0.19094, m_π =0.36948 ± 0.00482 for variance and a =0.13866 ± 0.00427, b =5.23423 ± 0.19744, m_π =0.37237 ± 0.00574 for chi squared.



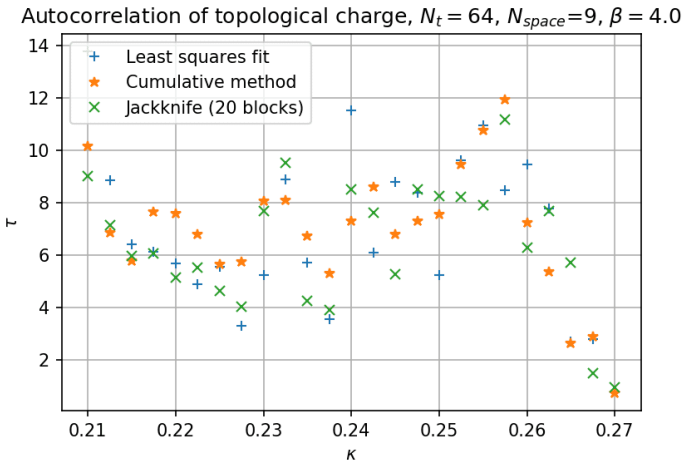
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.13667 ± 0.00228, b =10.15266 ± 0.13843, m_π =0.36969 ± 0.00309 for variance and a =0.13997 ± 0.00301, b =10.04565 ± 0.15739, m_π =0.37412 ± 0.00402 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\bar{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

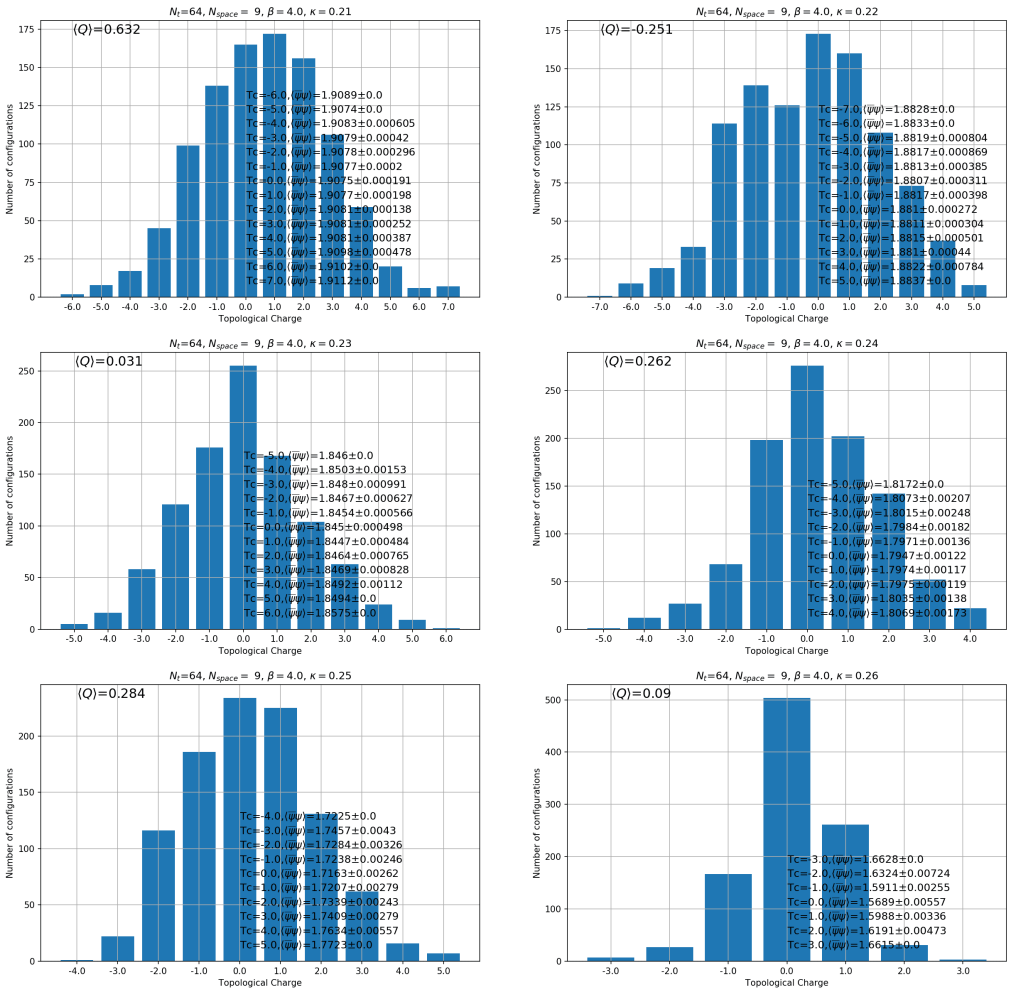
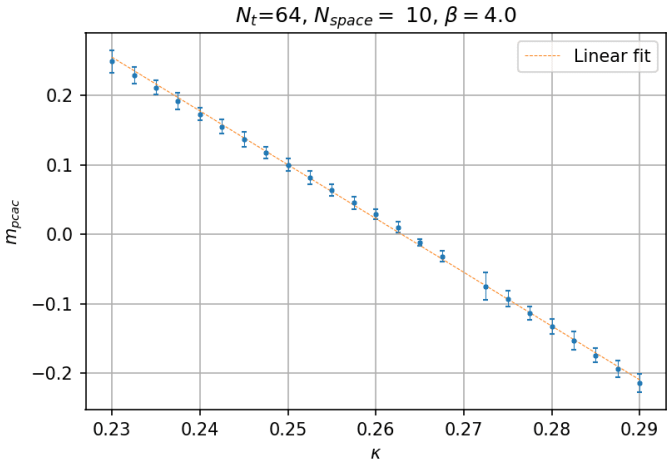
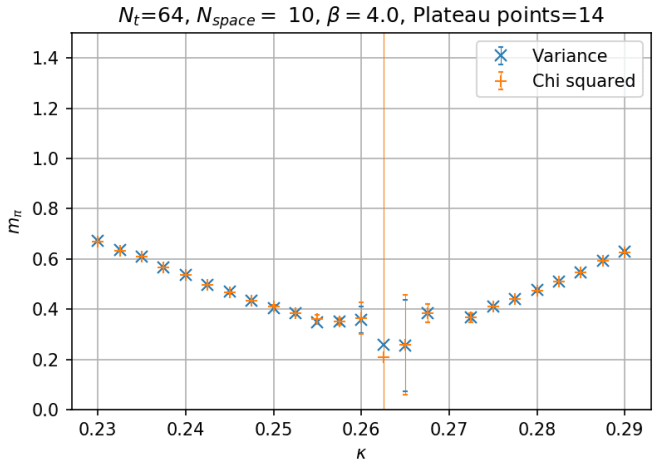


Figure 5: Number of configurations vs. topological charge on a 9×64 lattice.

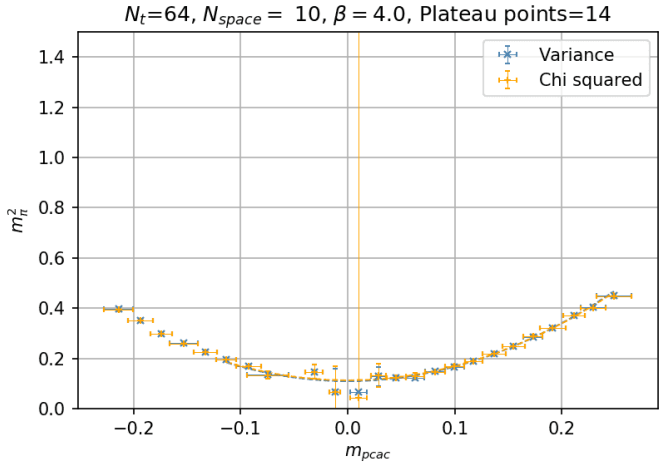
10x64



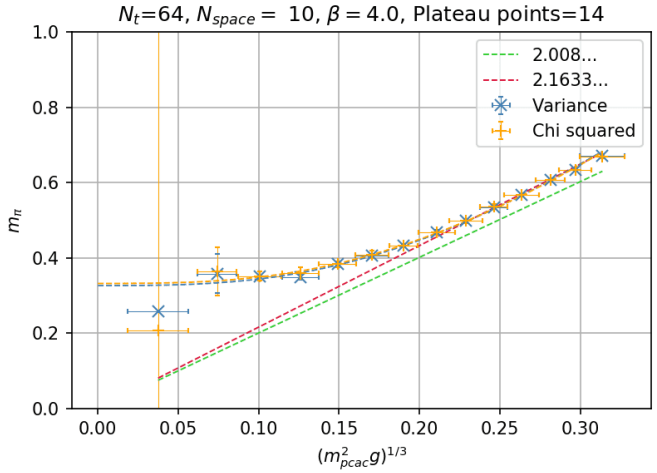
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26293 \pm 0.0023$



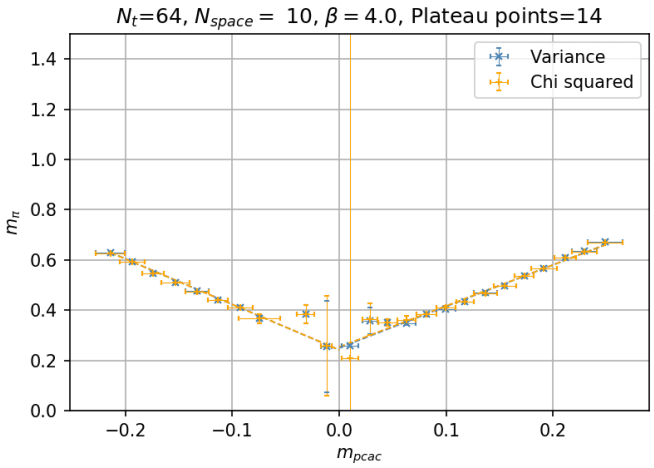
(b) Pion mass as a function of κ



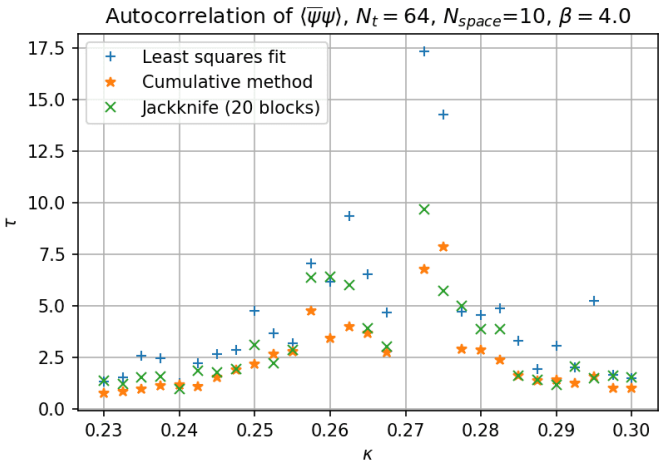
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.10923 ± 0.00272, b =5.79338 ± 0.1752, m_π =0.3305 ± 0.00412 for variance and a =0.11293 ± 0.00235, b =5.66633 ± 0.14032, m_π =0.33606 ± 0.0035 for chi squared.



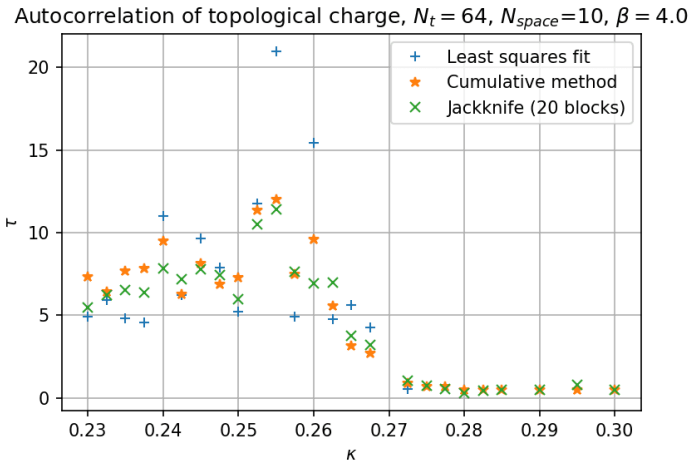
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.1067 ± 0.00218, b =11.65238 ± 0.26154, m_π =0.32665 ± 0.00334 for variance and a =0.11045 ± 0.00145, b =11.41887 ± 0.15804, m_π =0.33234 ± 0.00218 for chi squared.



(e) m_π vs. m_{pcac}. Var fit constants. Left side m_π=0.24139±0.01272. Right side m_π=0.24982±0.00837. Chi fit constants. Left side m_π=0.24181±0.01281. Right side m_π=0.25364±0.00839.



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

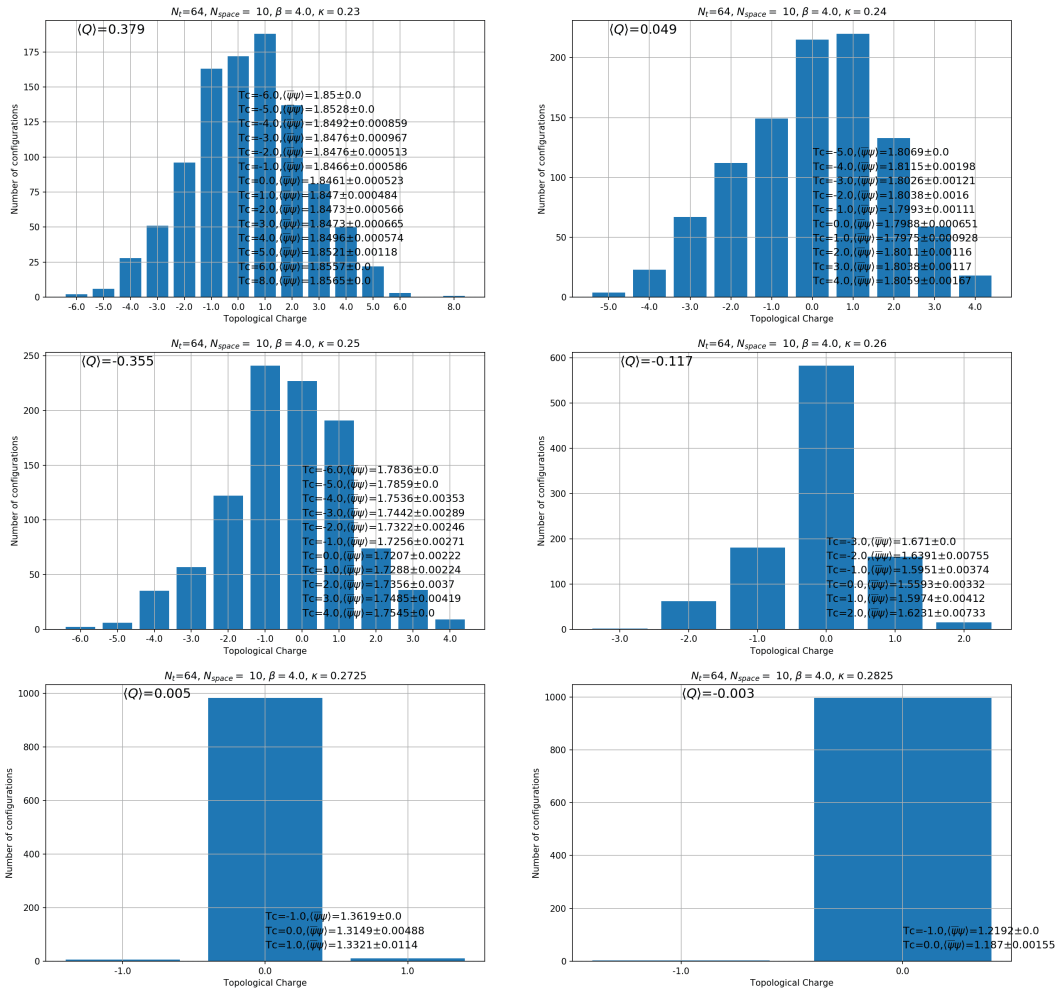
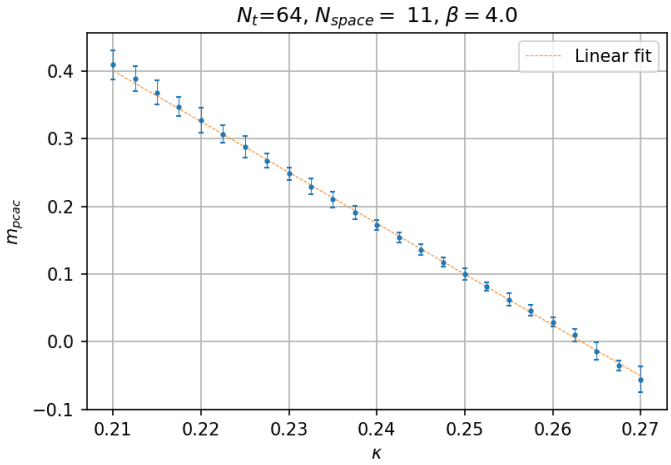
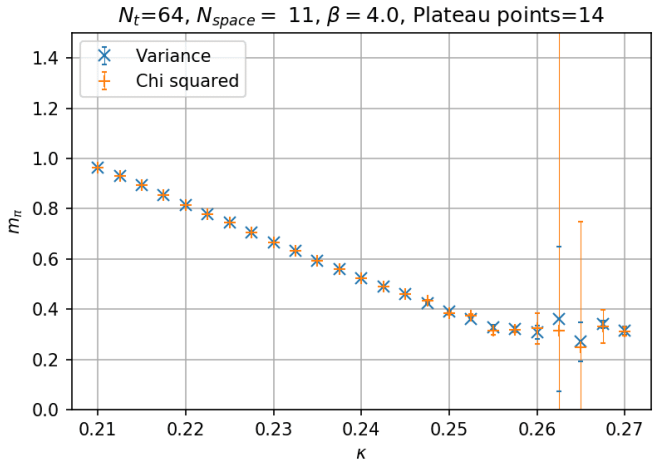


Figure 6: Number of configurations vs. topological charge on a 10×64 lattice.

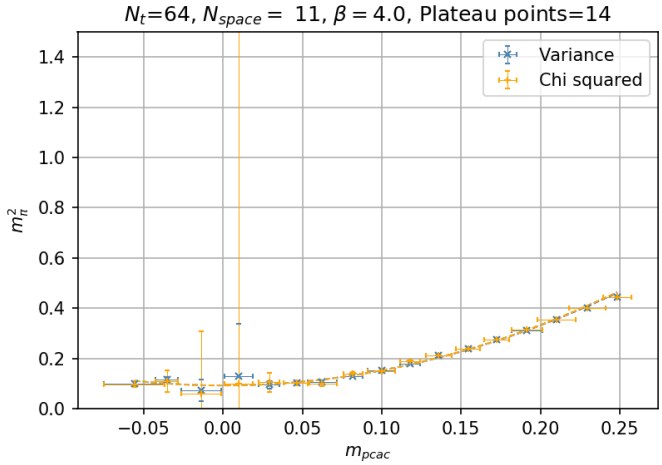
11x64



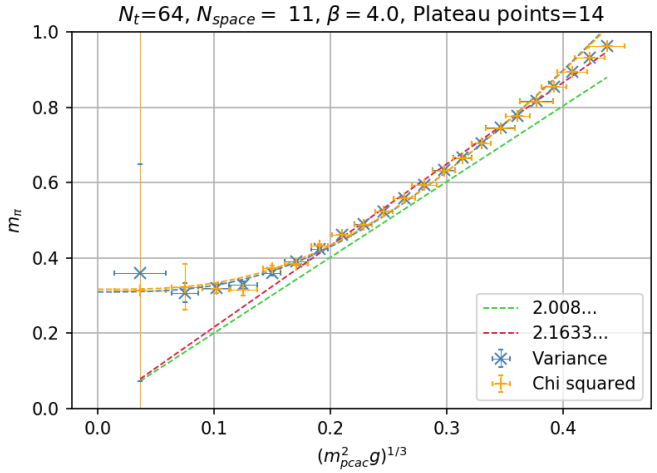
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26327 \pm 0.00189$



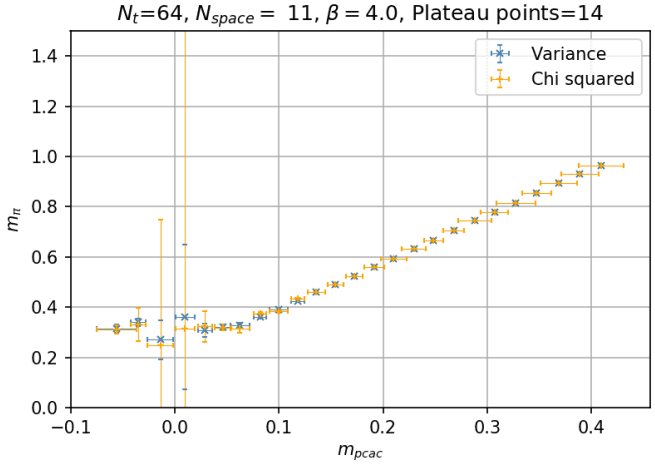
(b) Pion mass as a function of κ



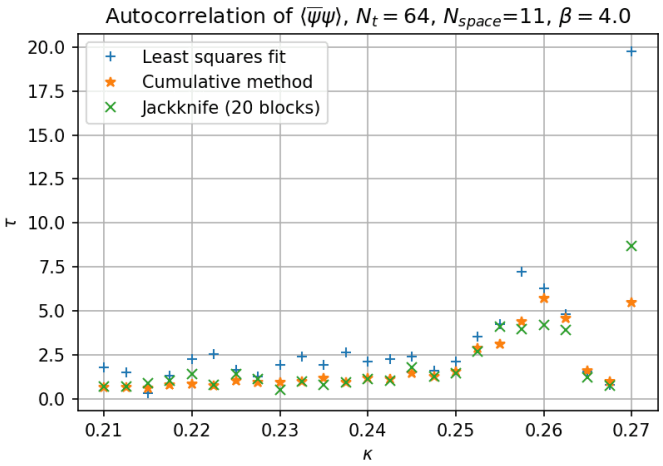
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.09215 ± 0.00255, b =5.97476 ± 0.16241, m_π =0.30356 ± 0.0042 for variance and a =0.09236 ± 0.00327, b =6.02194 ± 0.18321, m_π =0.3039 ± 0.00537 for chi squared.



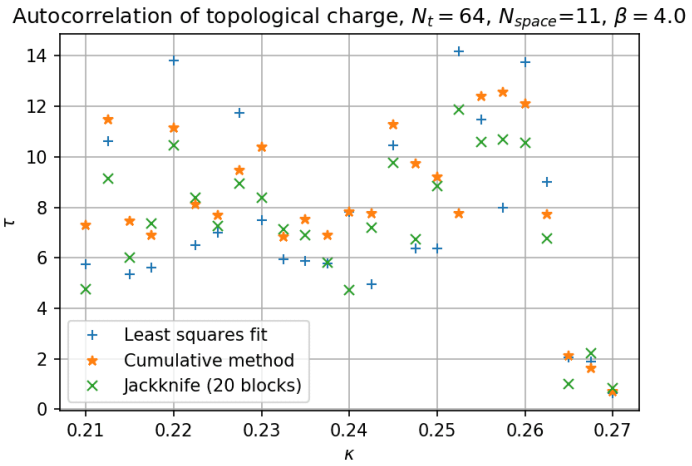
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.09598 ± 0.00272, b =11.0958 ± 0.22224, m_π =0.3098 ± 0.00439 for variance and a =0.10028 ± 0.0034, b =10.95341 ± 0.251, m_π =0.31667 ± 0.00538 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

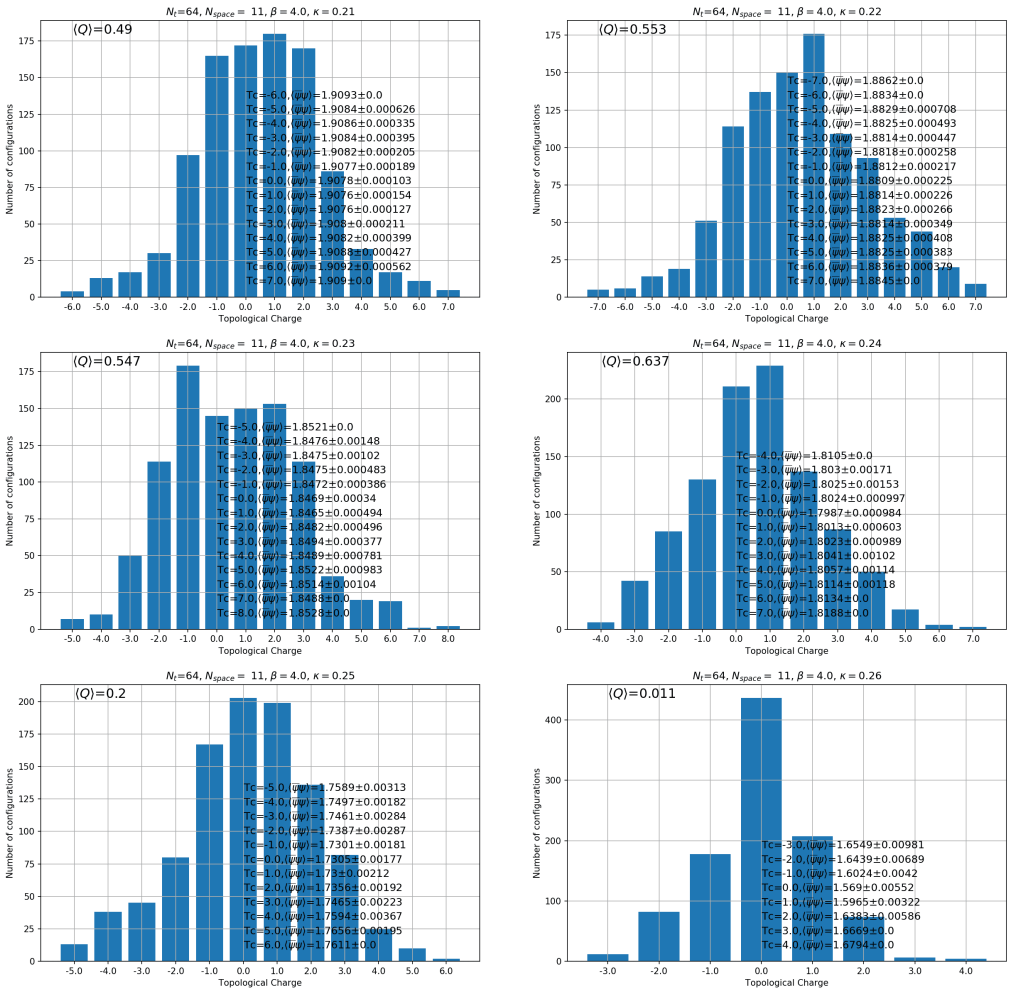
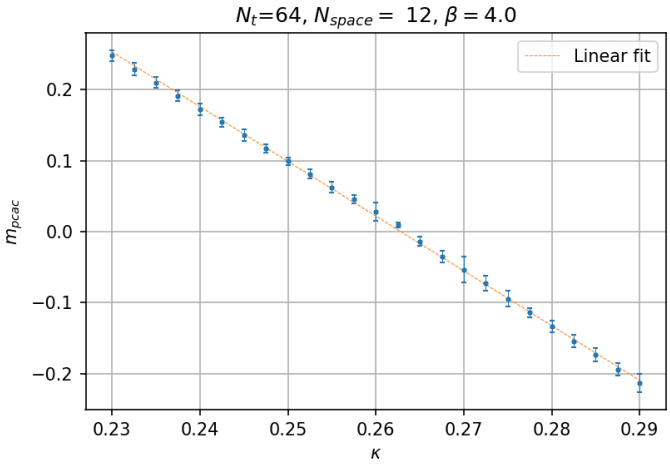
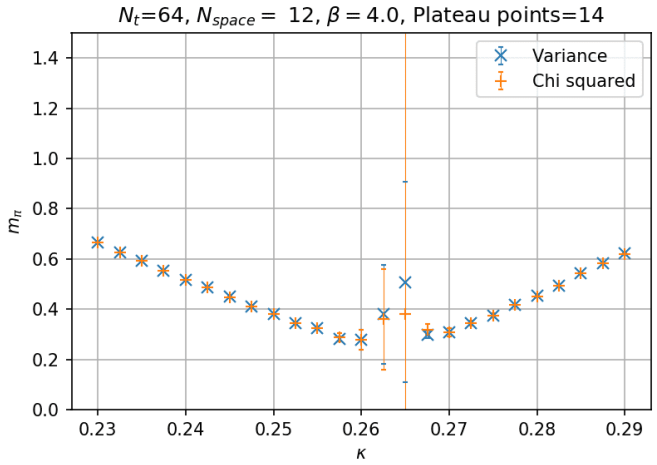


Figure 7: Number of configurations vs. topological charge on a 11×64 lattice.

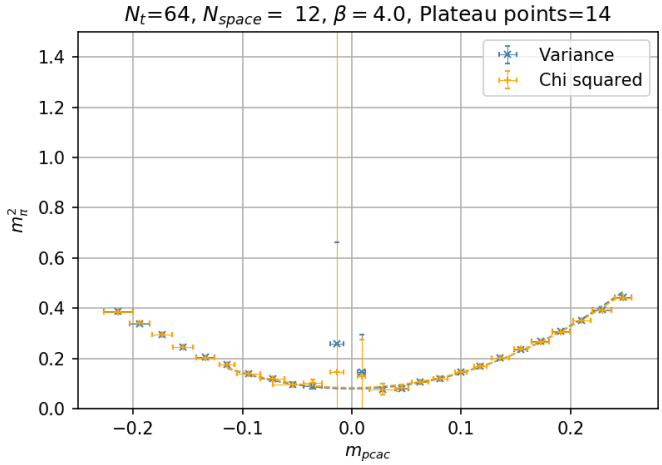
12x64



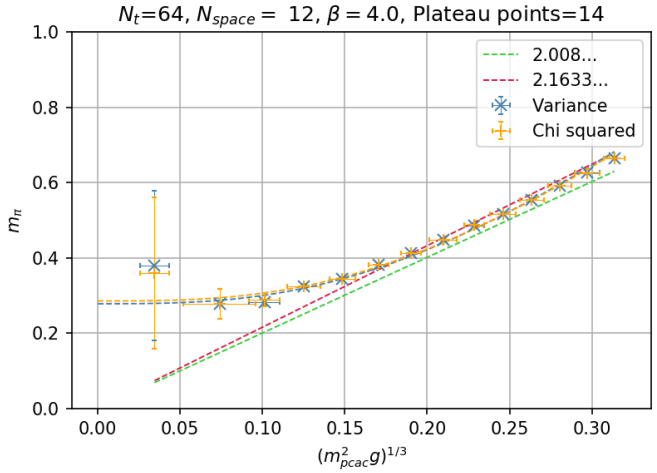
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26283 \pm 0.00211$



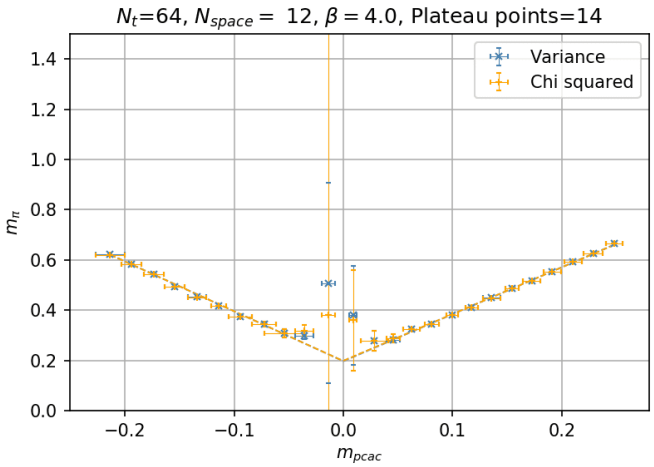
(b) Pion mass as a function of κ



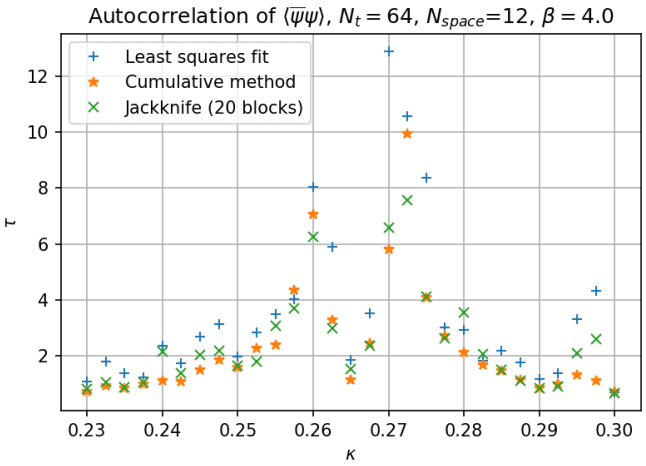
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.07922 ± 0.00249, b =6.2819 ± 0.16946, m_π =0.28146 ± 0.00443 for variance and a =0.08344 ± 0.00217, b =6.09893 ± 0.13452, m_π =0.28886 ± 0.00375 for chi squared.



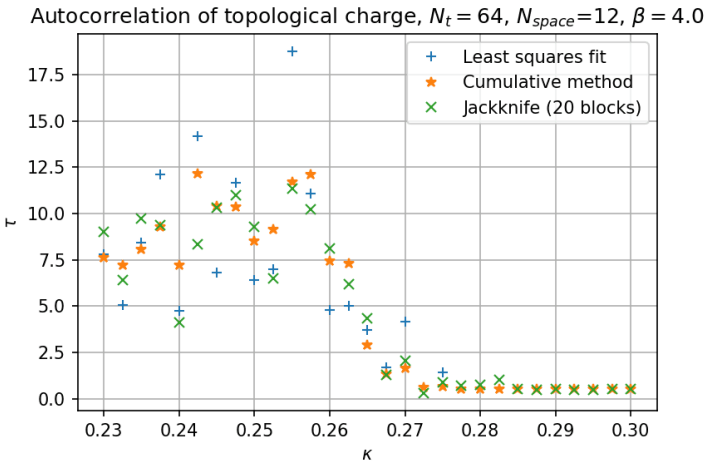
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.07753 ± 0.003, b =12.57244 ± 0.36746, m_π =0.27844 ± 0.00539 for variance and a =0.08164 ± 0.00237, b =12.24683 ± 0.26713, m_π =0.28572 ± 0.00414 for chi squared.



(e) m_π vs. m_{pcac}. Var fit constants. Left side m_π=0.19691±0.00741. Right side m_π=0.19678±0.00398. Chi fit constants. Left side m_π=0.19743±0.00917. Right side m_π=0.19857±0.00417.



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

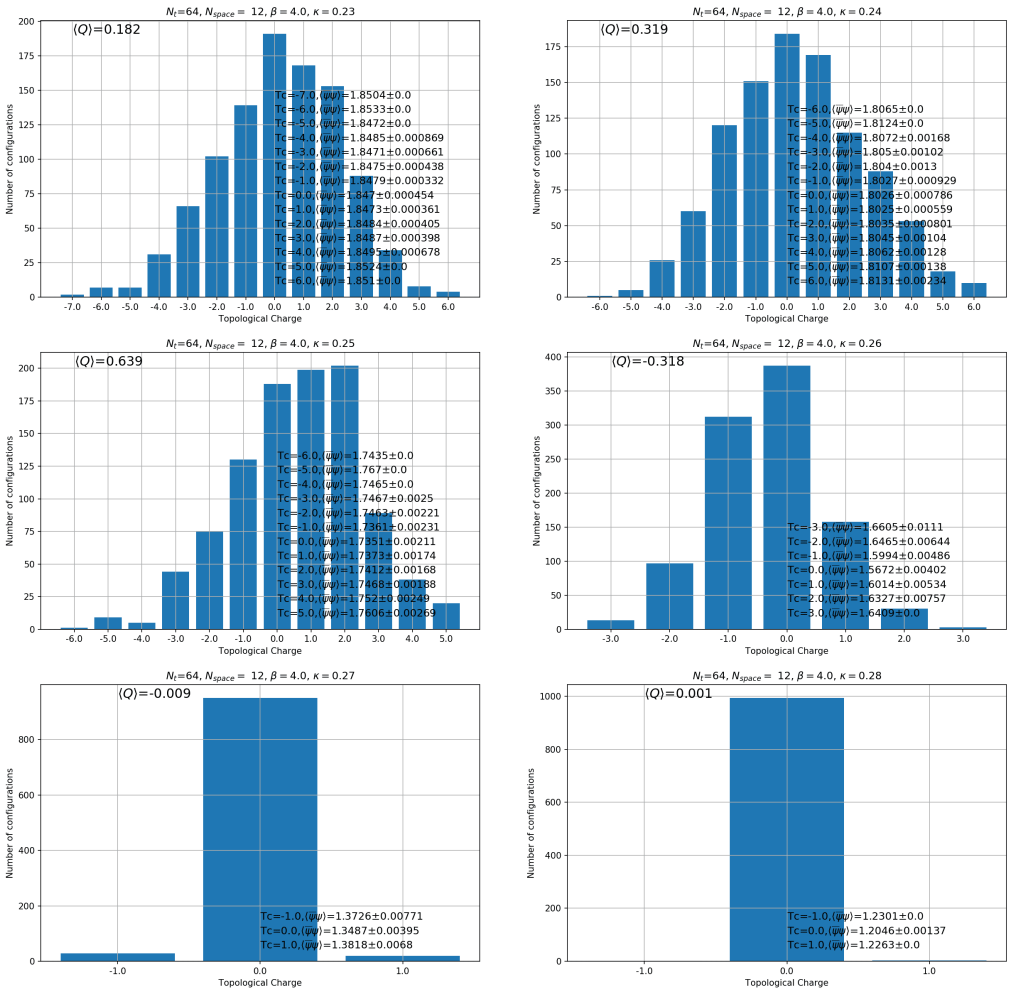


Figure 8: Number of configurations vs. topological charge on a 12×64 lattice.

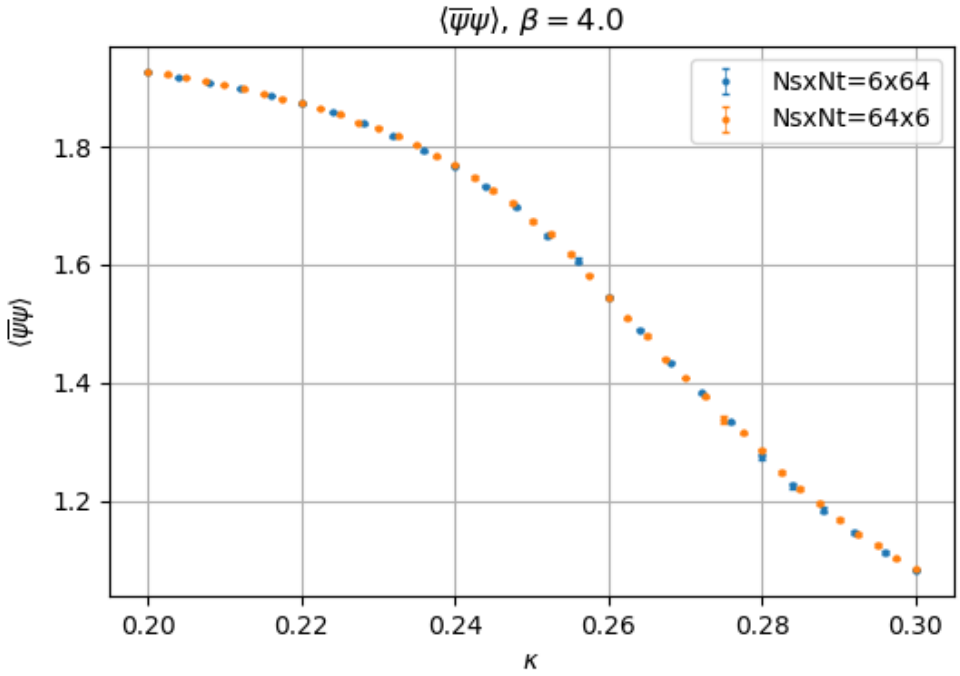


Figure 9: $\langle\overline{\psi}\psi\rangle$ vs. κ

Two plots of the Smilga prediction for all N_{space} are shown.

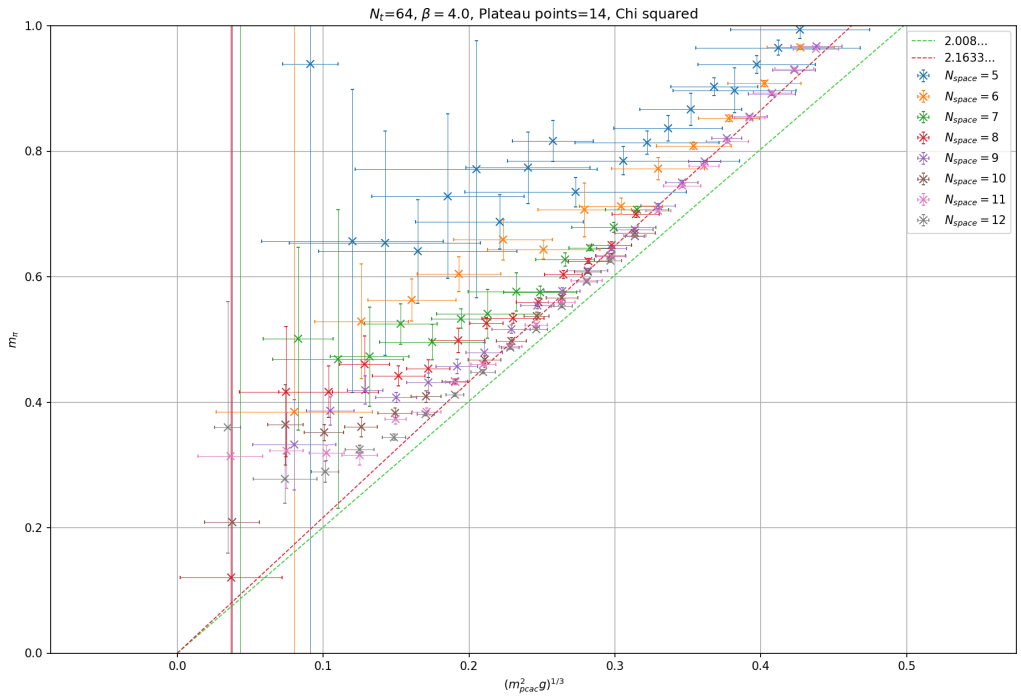


Figure 10: m_π vs. $(m^2_{pcac}g)^{1/3}$ for Chi squared

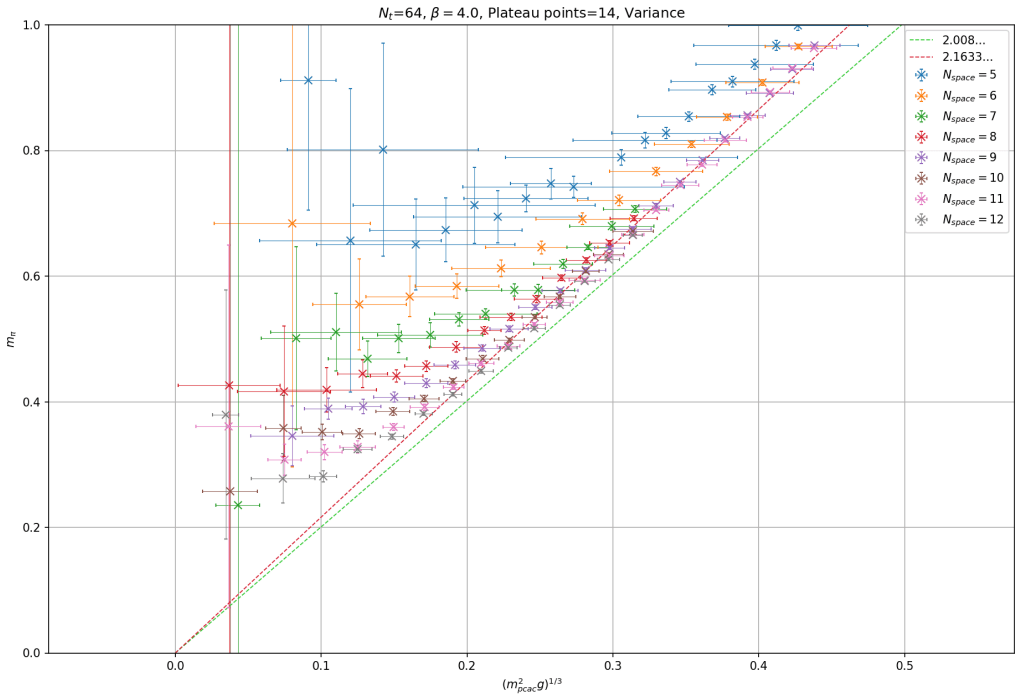


Figure 11: m_π vs. $(m^2_{pcac}g)^{1/3}$ for Variance

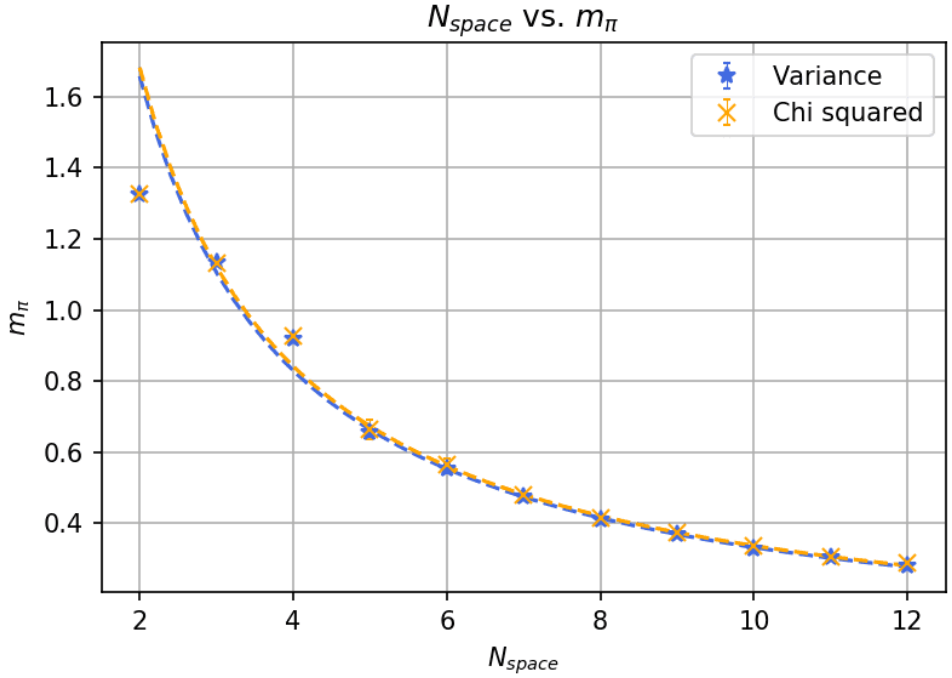


Figure 12: A function of the form $m_\pi = a/N_{space}$ was fitted. For variance the fit parameter is a =3.36547±0.01812, while for Chi squared a =3.31621±0.01044. To perform the fit only the points where $N_{space} > 4$ were considered, the other points in the plot correspond to high statistics results, however they are shown together with this low statistics results.

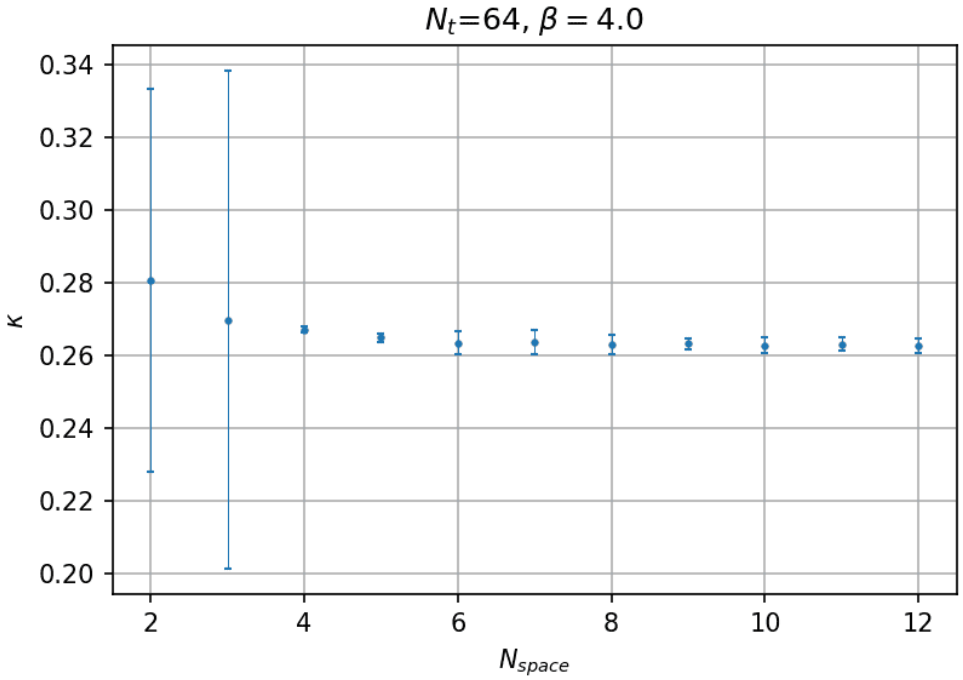


Figure 13: Kappa critical as a function of the volume size.

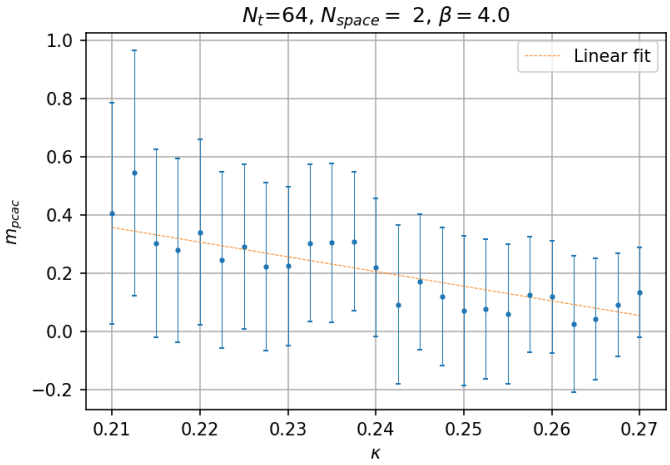
High statistics results

The following results were obtained through several simulations on different lattices: 2×64, 3×64, 4×64 and 10×64 with the parameters:

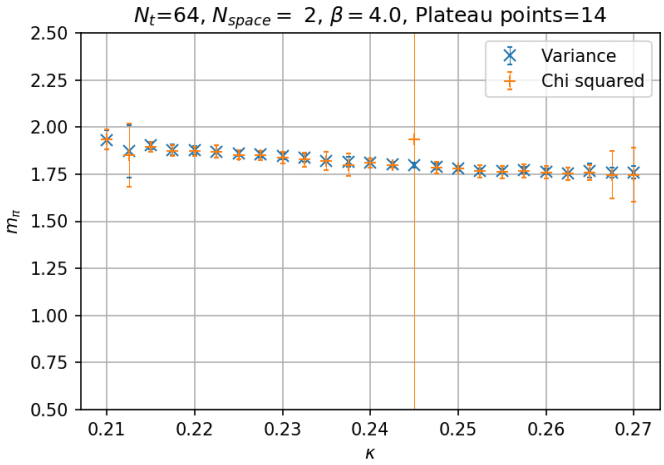
Ntime	64
Ntherm	1000
Nmeasure	10000
Trajectory Steps	10
Nsteps	100
β	4

Table 2: All the simulations were performed with this parameters.

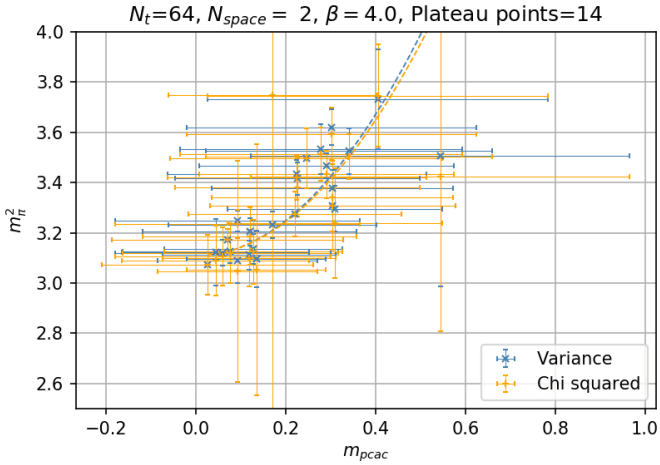
2x64



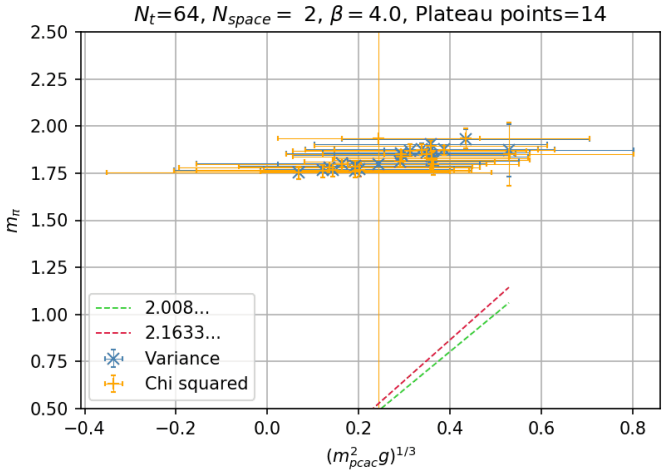
(a) Fermion mass using PCAC relation, $\kappa_c = 0.28085 \pm 0.05262$



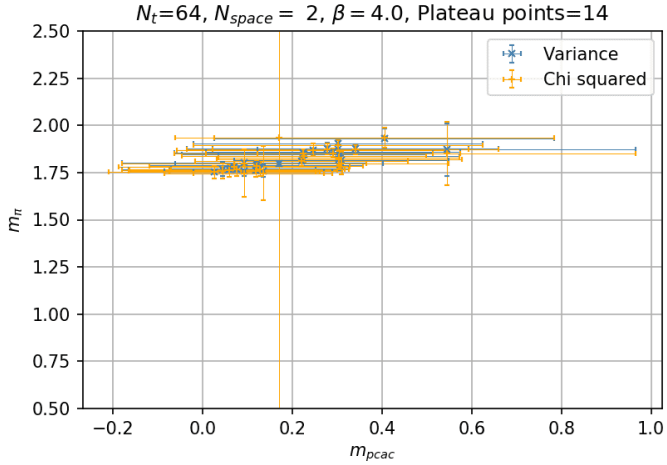
(b) Pion mass as a function of κ



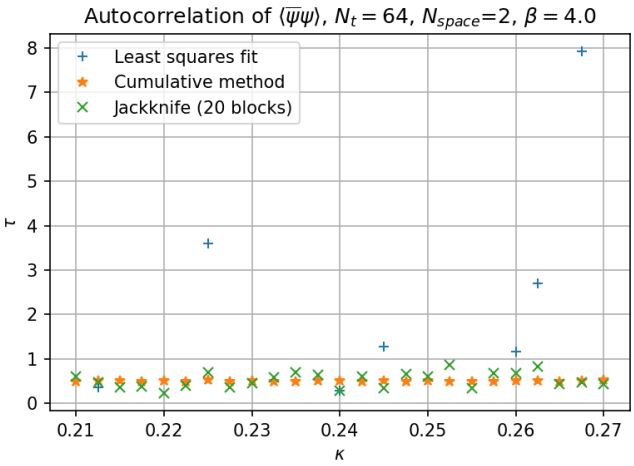
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =3.10771 ± 0.01387, b =3.52539 ± 0.57509, m_π =1.76287 ± 0.00393 for variance and a =3.10897 ± 0.01499, b =3.37729 ± 0.54389, m_π =1.76323 ± 0.00425 for chi squared.



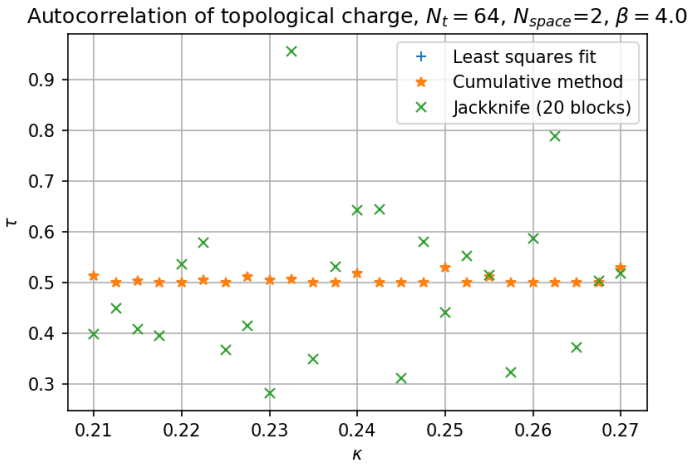
(d) Smilga prediction. Only $m_{pcac} > 0$ is considered.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

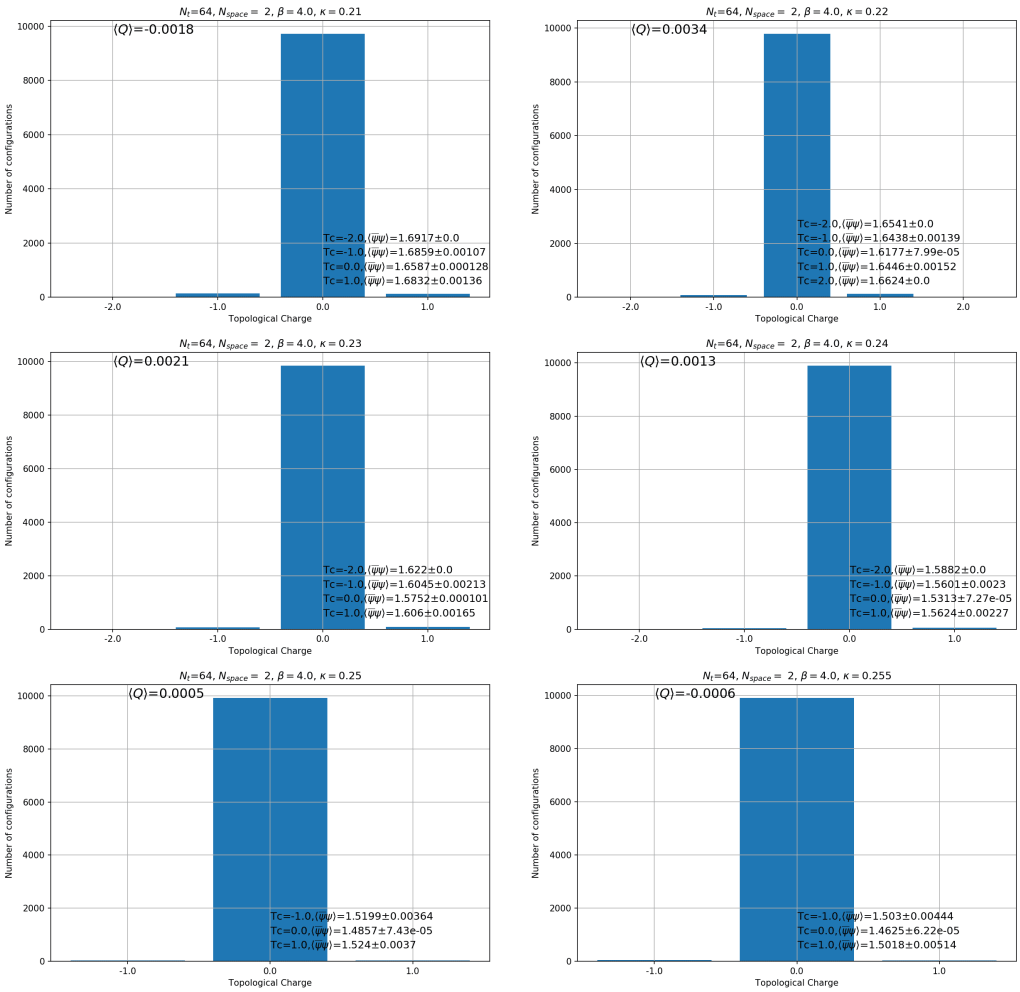
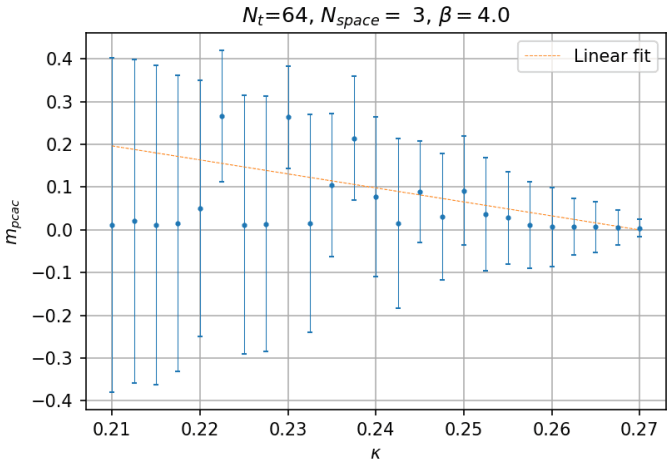
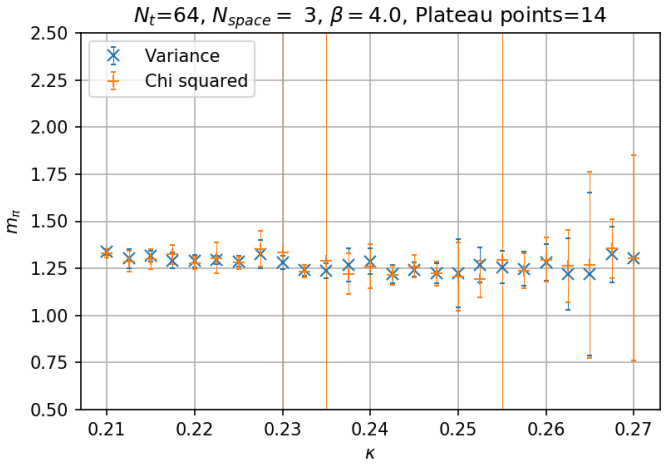


Figure 14: Number of configurations vs. topological charge on a 2×64 lattice.

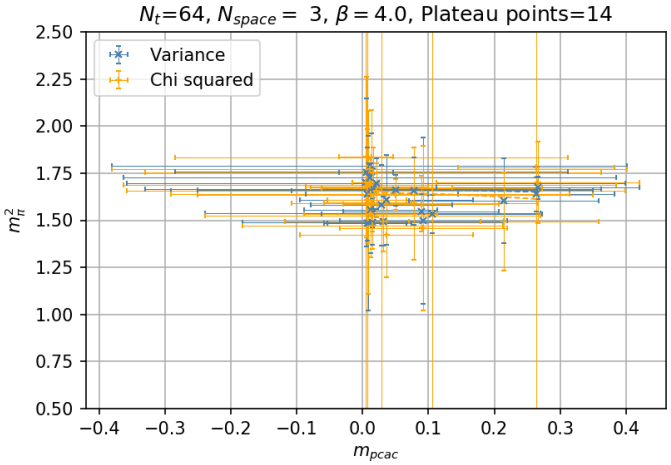
3x64



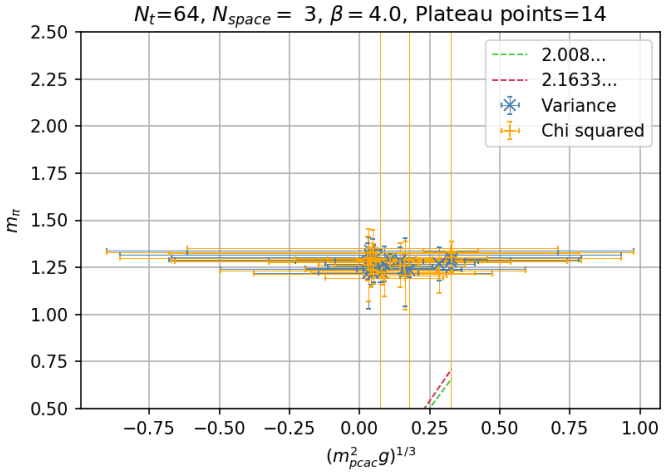
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26996 \pm 0.06863$



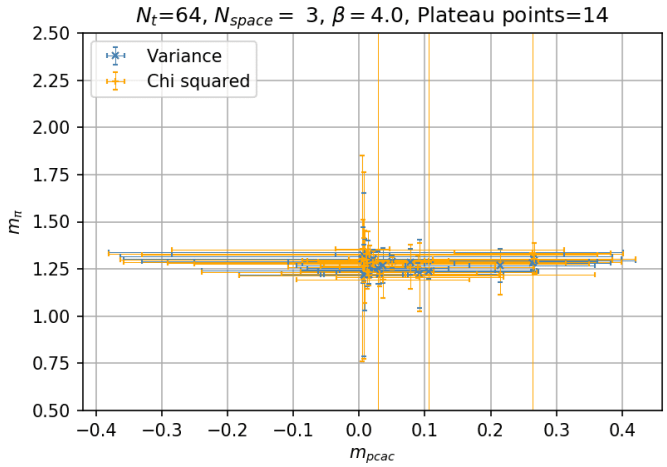
(b) Pion mass as a function of κ



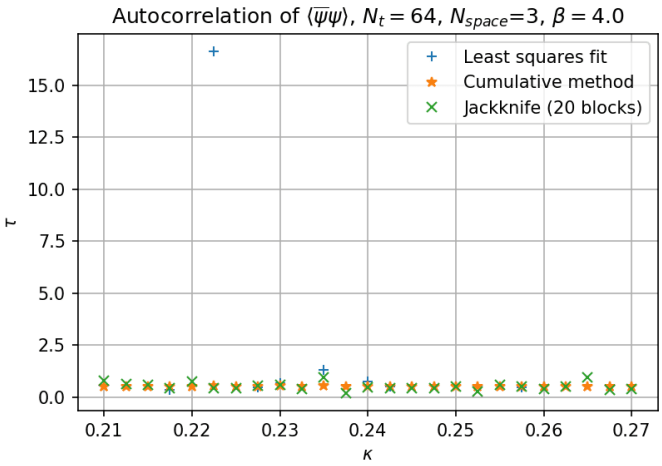
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =1.66304 ± 0.02205, b =-0.17669 ± 0.74098, m_π =1.28959 ± 0.00855 for variance and a =1.64521 ± 0.02288, b =-0.46975 ± 2.04727, m_π =1.28266 ± 0.00892 for chi squared



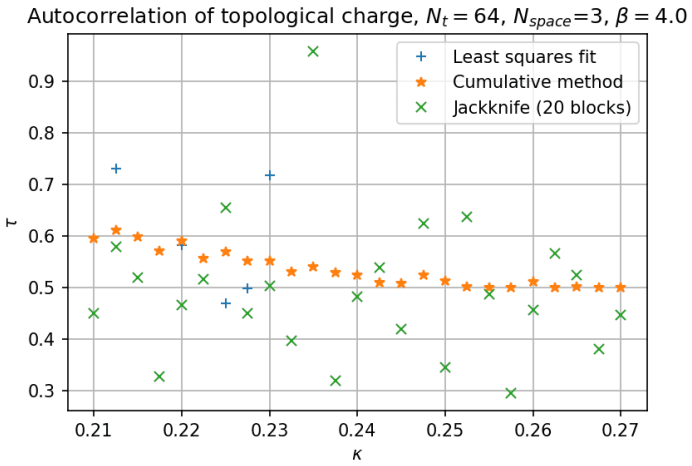
(d) Smilga prediction. Only $m_{pcac} > 0$ is considered.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

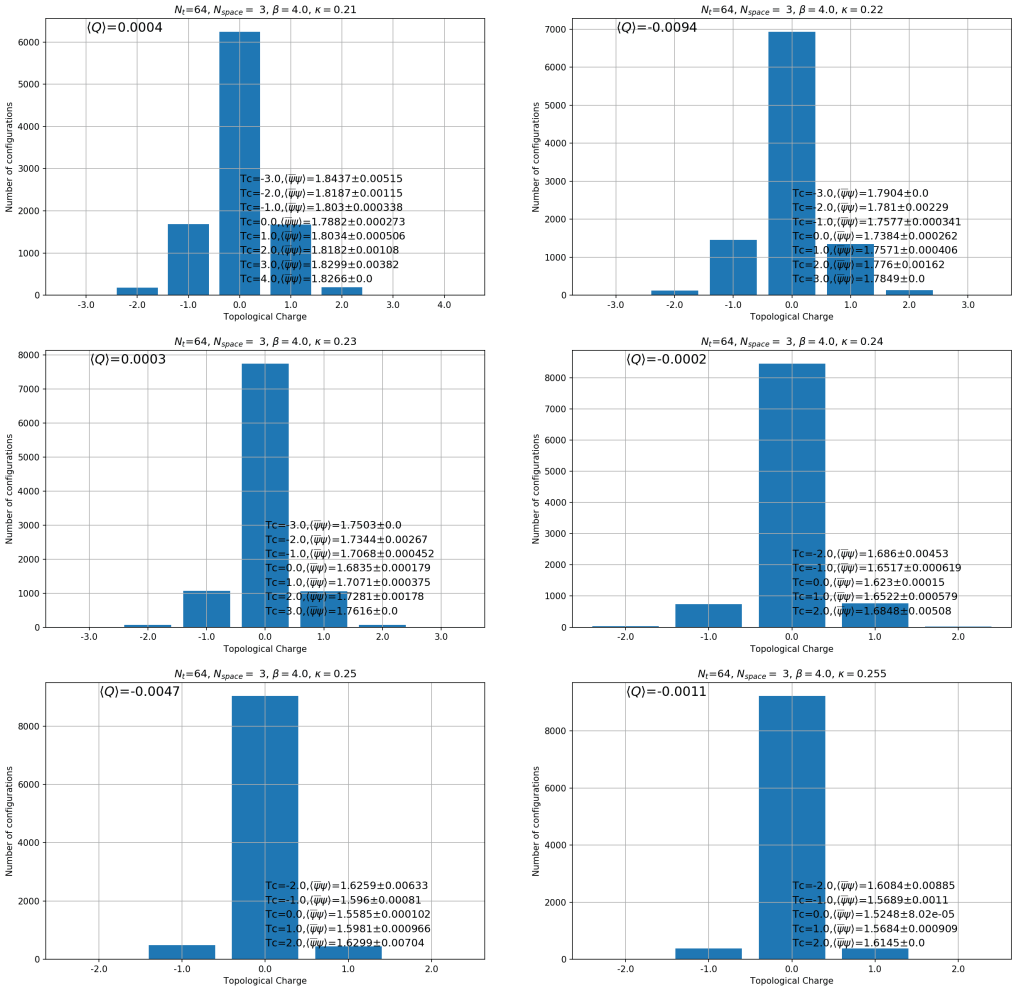
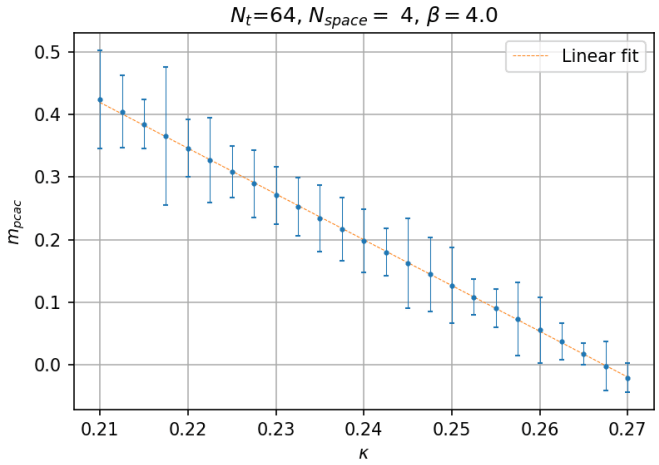
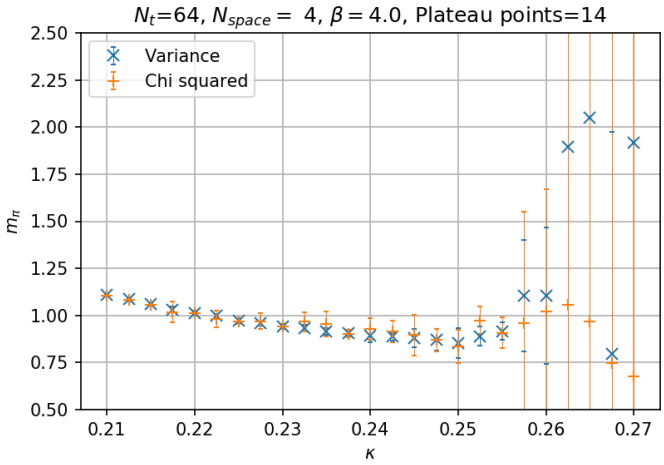


Figure 15: Number of configurations vs. topological charge on a 3×64 lattice.

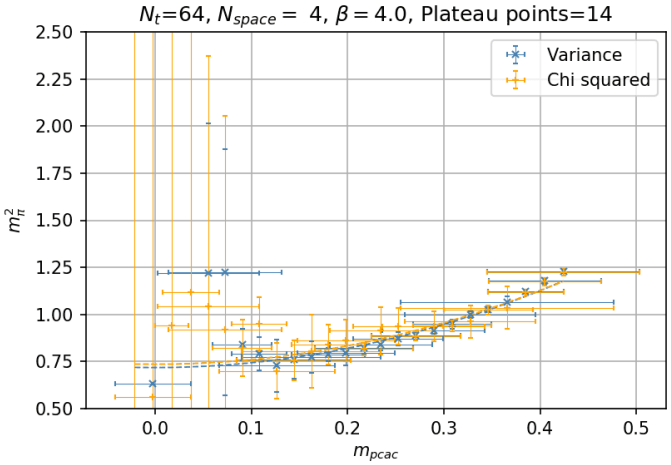
4x64



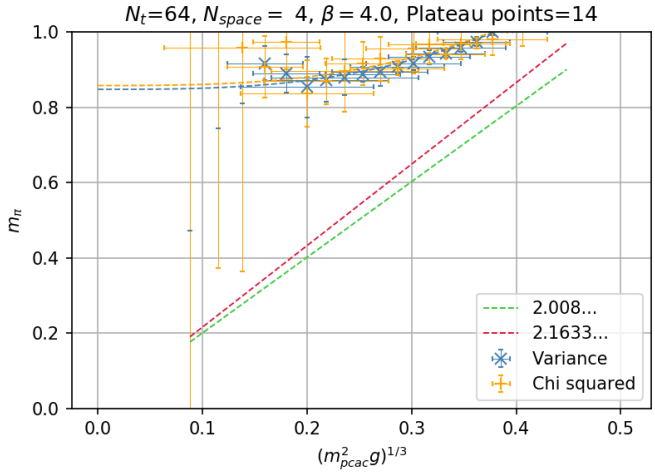
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26727 \pm 0.00083$



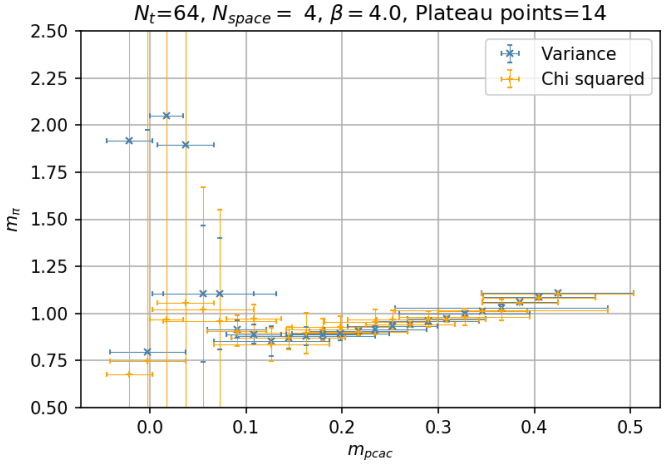
(b) Pion mass as a function of κ



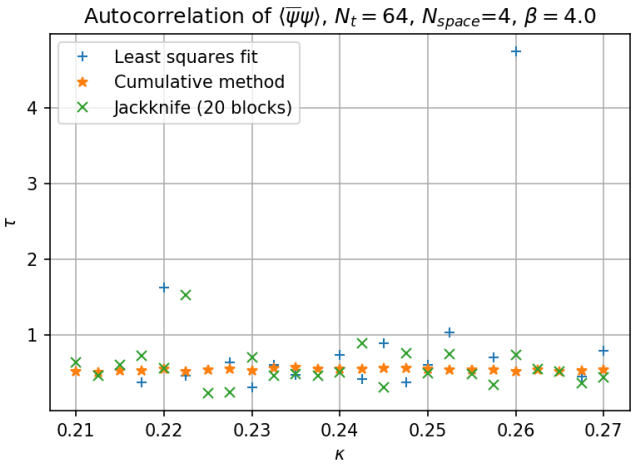
(c) m_π^2 vs. m_{pcac}. m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.71803 ± 0.01385, b =2.55405 ± 0.17842, m_π =0.84736 ± 0.00817 for variance and a =0.73566 ± 0.01899, b =2.44527 ± 0.20696, m_π =0.85771 ± 0.01107 for chi squared



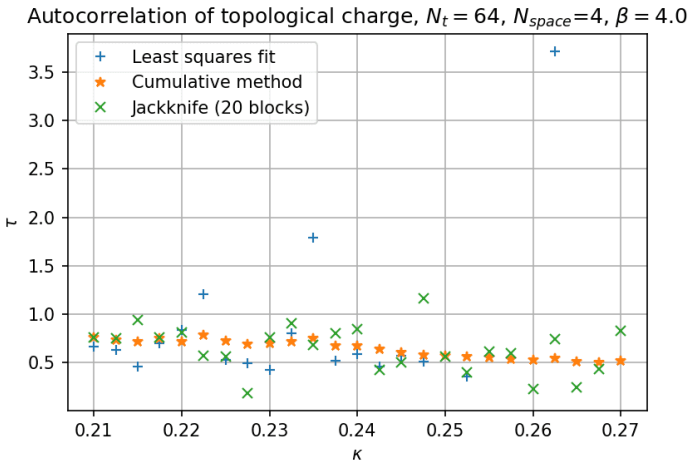
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.71823 ± 0.01589, b =5.13387 ± 0.41096, m_π =0.84748 ± 0.00937 for variance and a =0.73564 ± 0.0207, b =4.92092 ± 0.4536, m_π =0.85769 ± 0.01207 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\bar{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

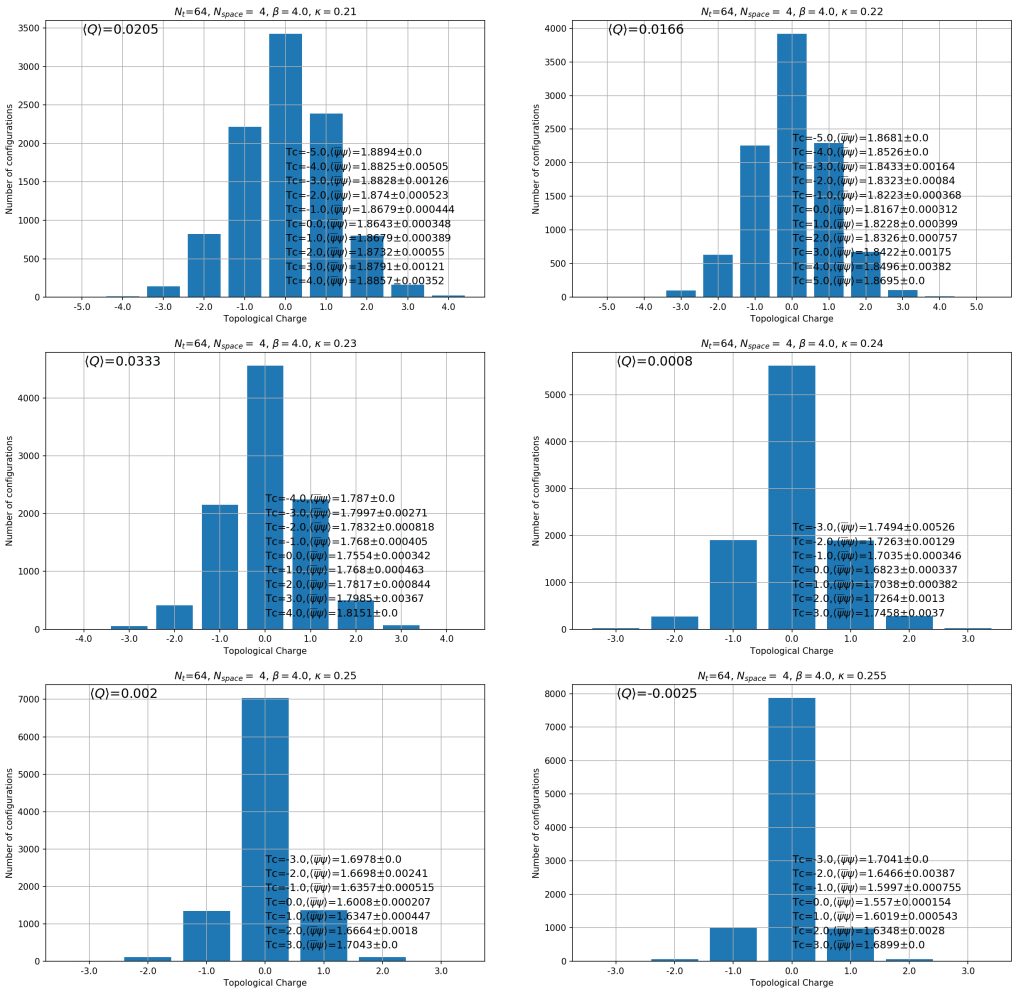
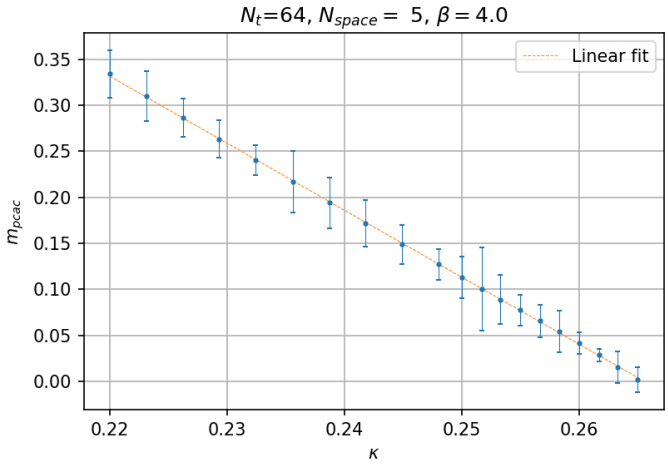
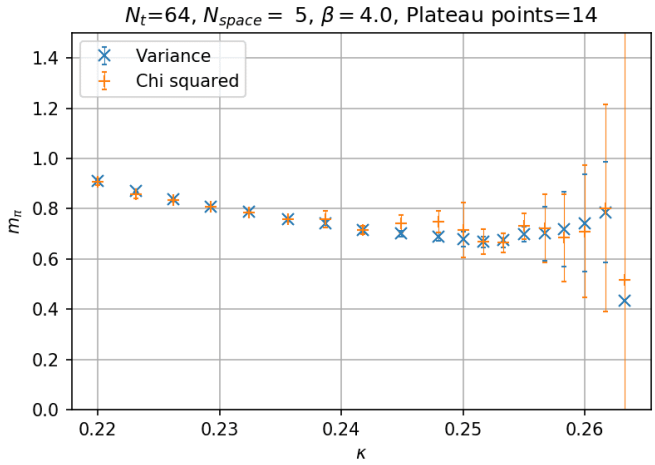


Figure 16: Number of configurations vs. topological charge on a 4×64 lattice.

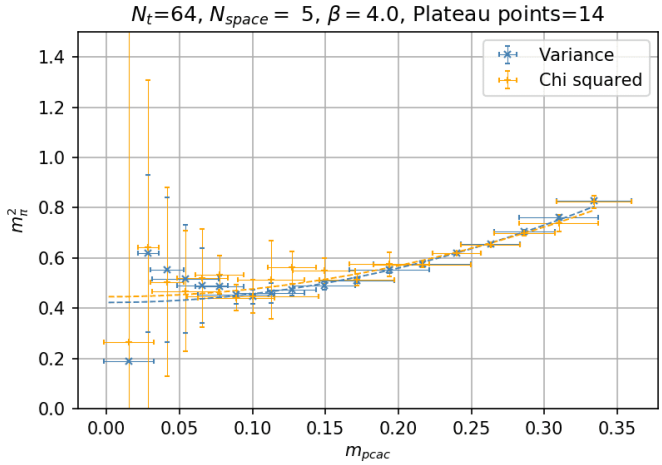
5x64



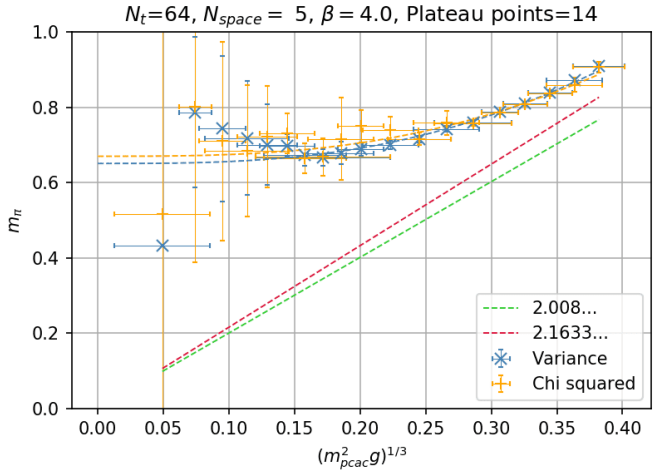
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26556 \pm 0.00109$



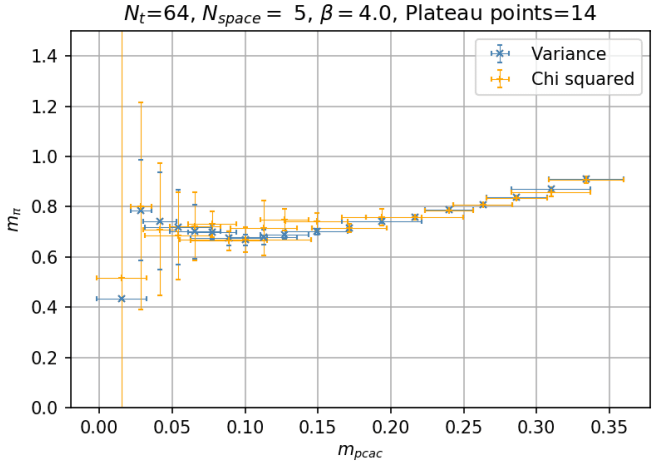
(b) Pion mass as a function of κ



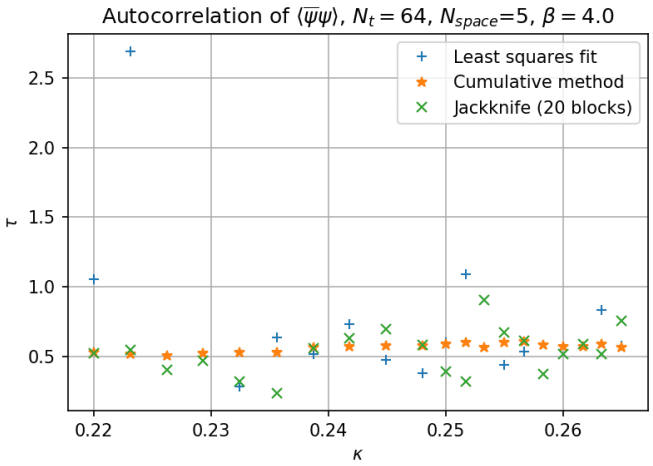
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.42259 ± 0.0063, b =3.42492 ± 0.13595, m_π =0.65007 ± 0.00484 for variance and a =0.44553 ± 0.01245, b =3.08719 ± 0.21368, m_π =0.66748 ± 0.00933 for chi squared



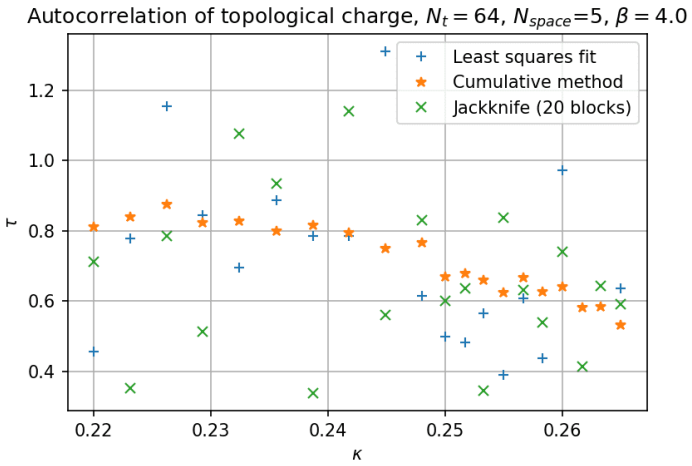
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.42334 ± 0.00646, b =6.82472 ± 0.27928, m_π =0.65064 ± 0.00497 for variance and a =0.44881 ± 0.01253, b =6.06369 ± 0.42939, m_π =0.66993 ± 0.00935 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

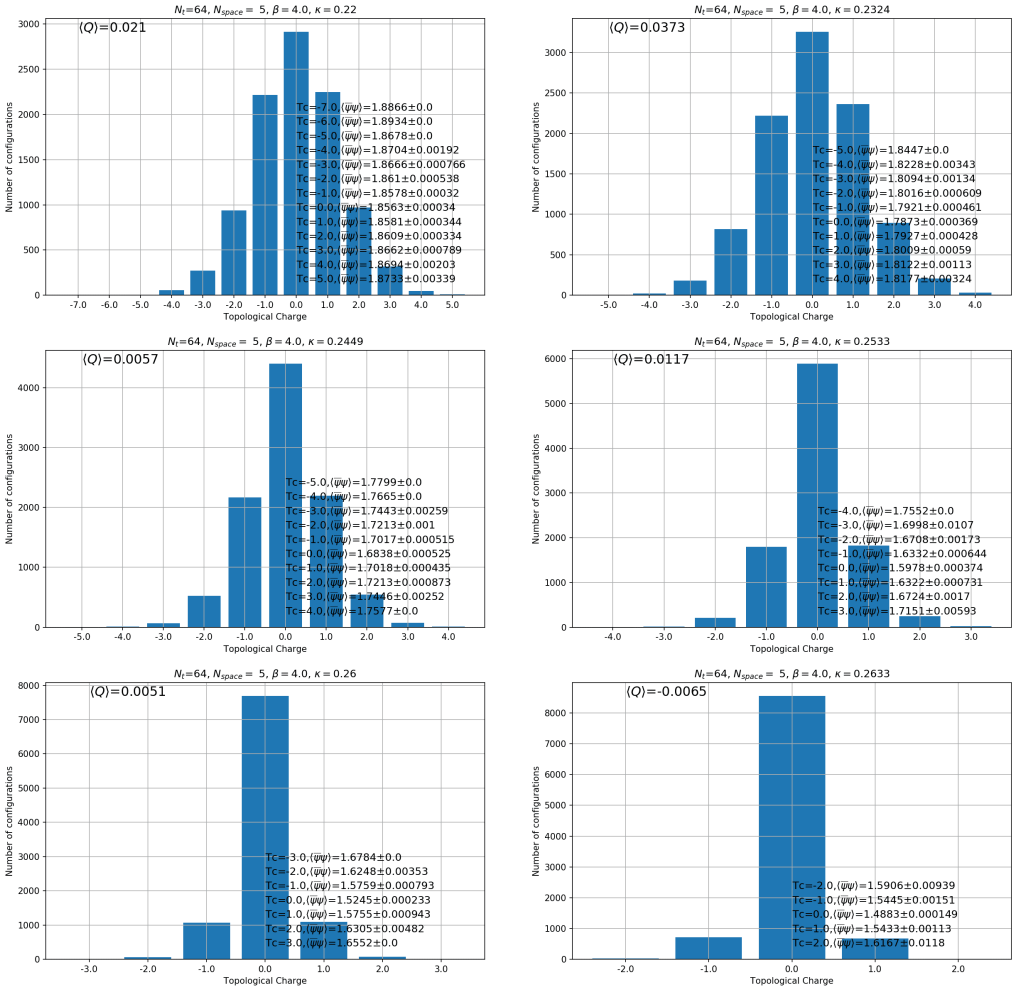
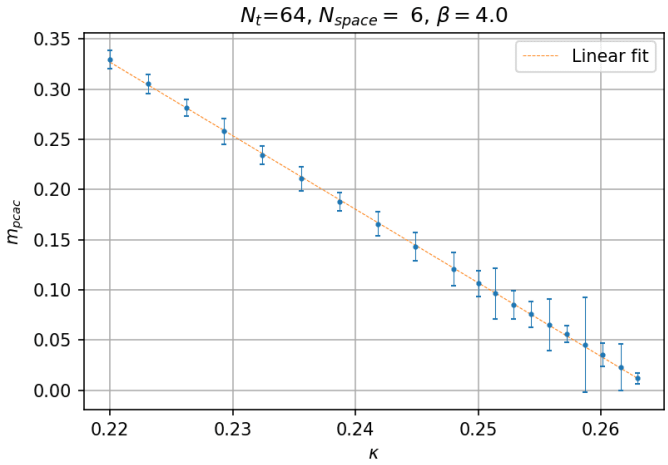
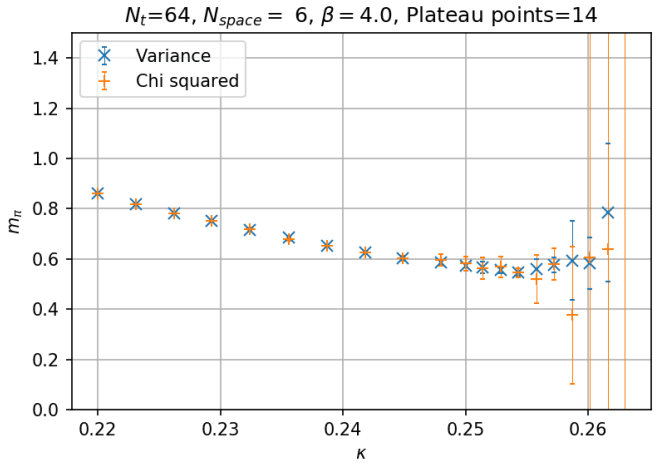


Figure 17: Number of configurations vs. topological charge on a 5×64 lattice.

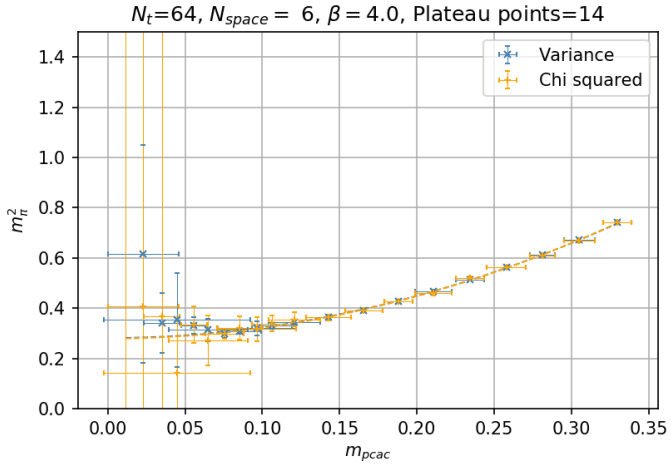
6x64



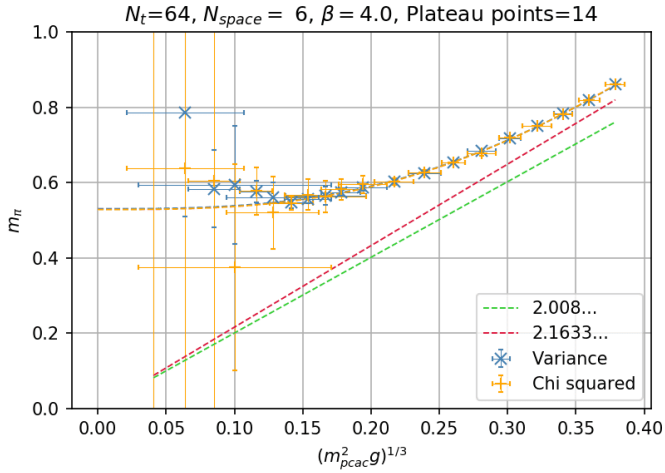
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26459 \pm 0.00106$



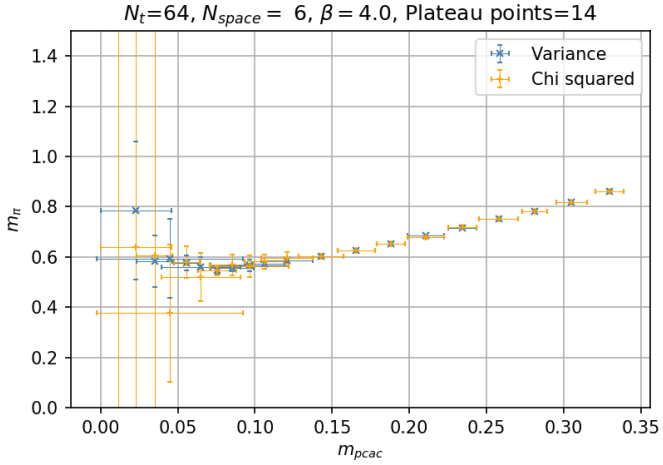
(b) Pion mass as a function of κ



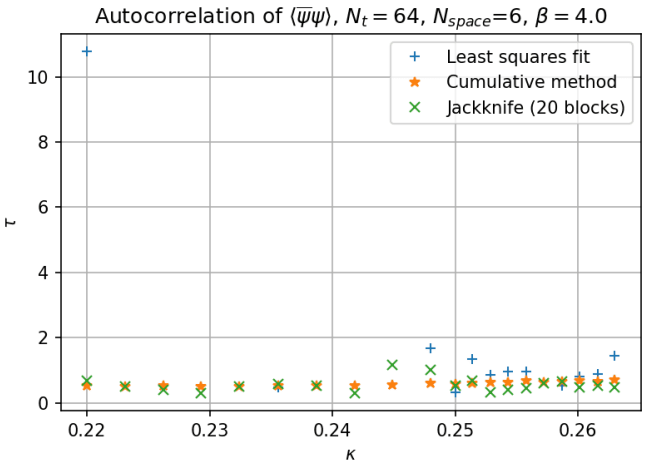
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.28125 ± 0.00295, b =4.1968 ± 0.06567, m_π =0.53033 ± 0.00278 for variance and a =0.27897 ± 0.00335, b =4.22684 ± 0.06326, m_π =0.52818 ± 0.00317 for chi squared



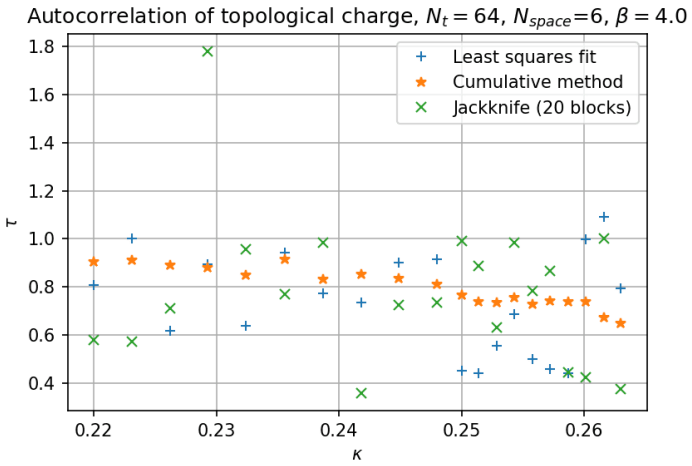
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.28157 ± 0.00313, b =8.3831 ± 0.13964, m_π =0.53063 ± 0.00295 for variance and a =0.27934 ± 0.00321, b =8.44264 ± 0.12115, m_π =0.52852 ± 0.00303 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

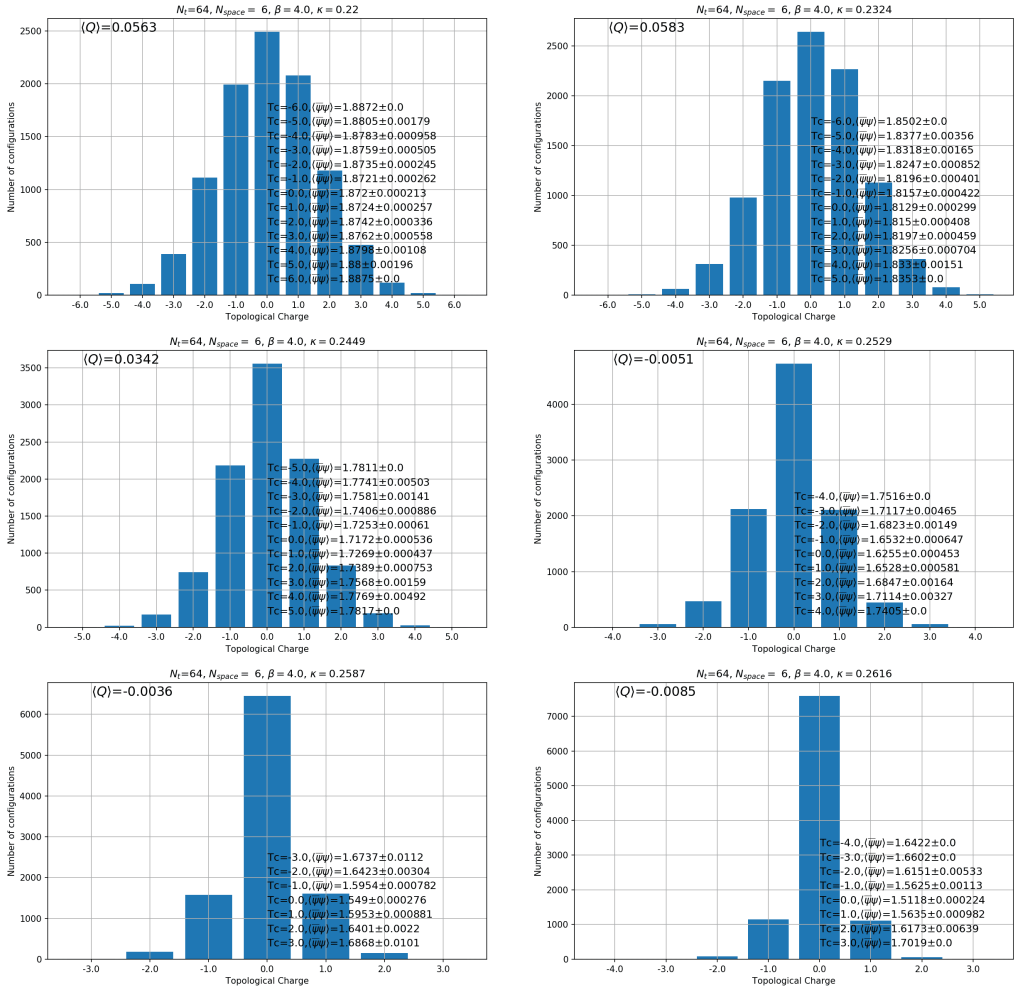
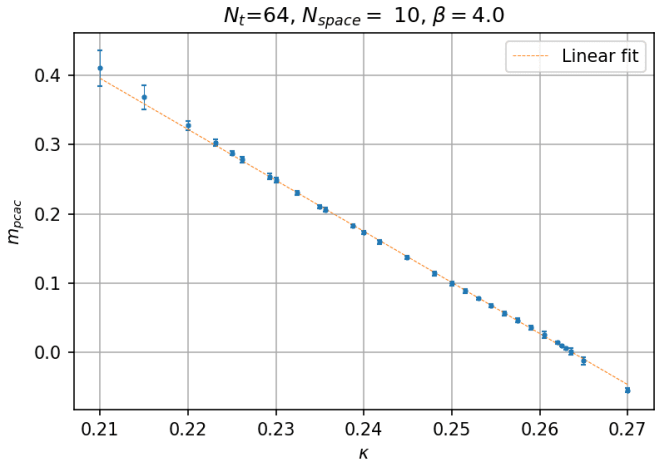
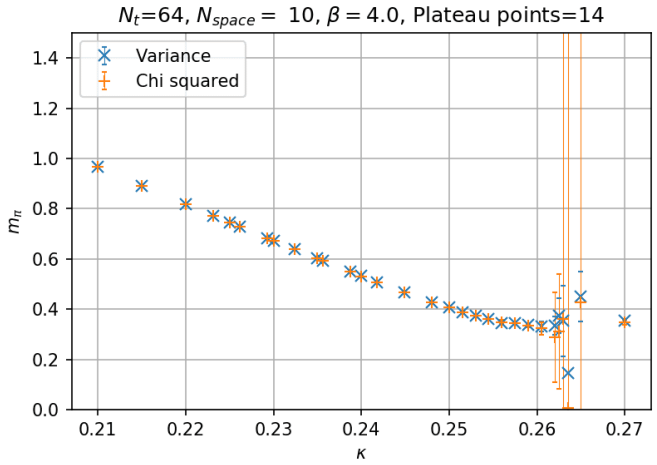


Figure 18: Number of configurations vs. topological charge on a 6×64 lattice.

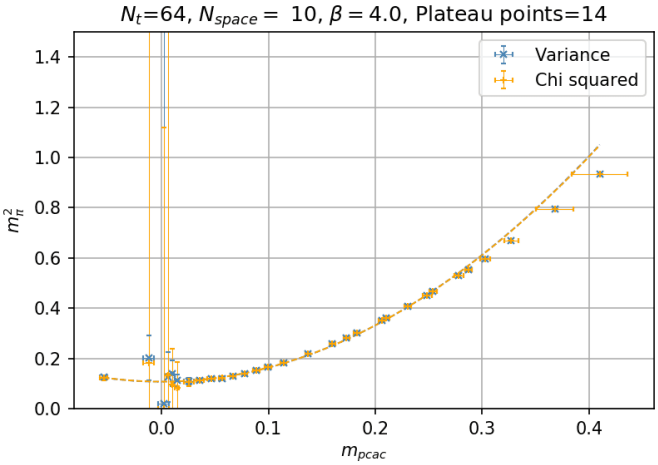
10x64



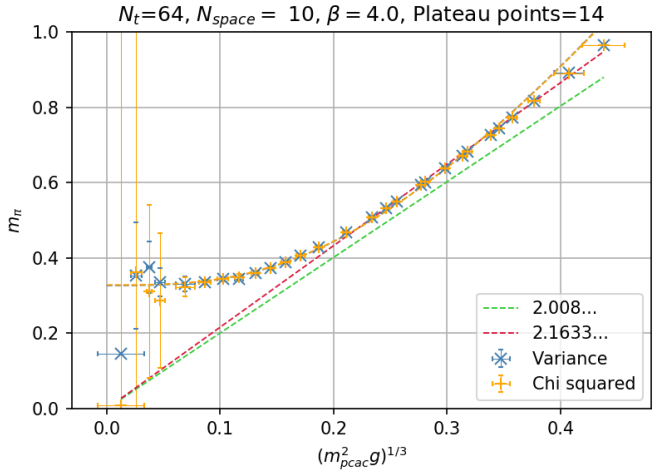
(a) Fermion mass using PCAC relation, $\kappa_c = 0.26373 \pm 0.00137$



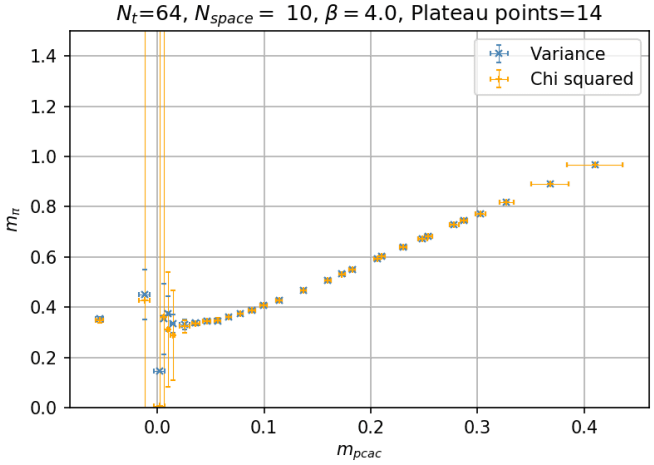
(b) Pion mass as a function of κ



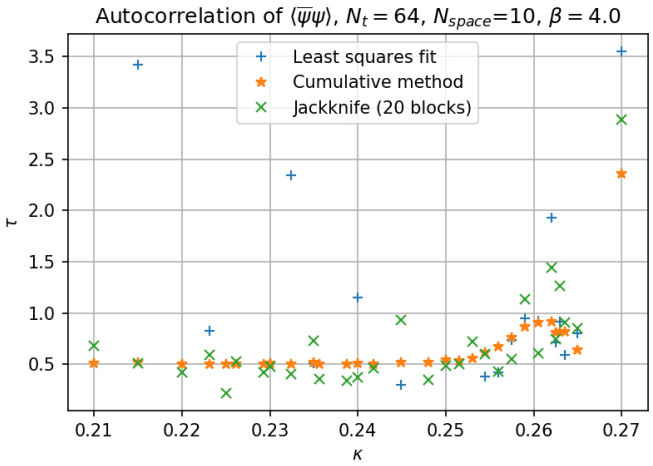
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.10697 ± 0.00127, b =5.61713 ± 0.05967, m_π =0.32706 ± 0.00194 for variance and a =0.10754 ± 0.0013, b =5.59865 ± 0.05522, m_π =0.32793 ± 0.00198 for chi squared.



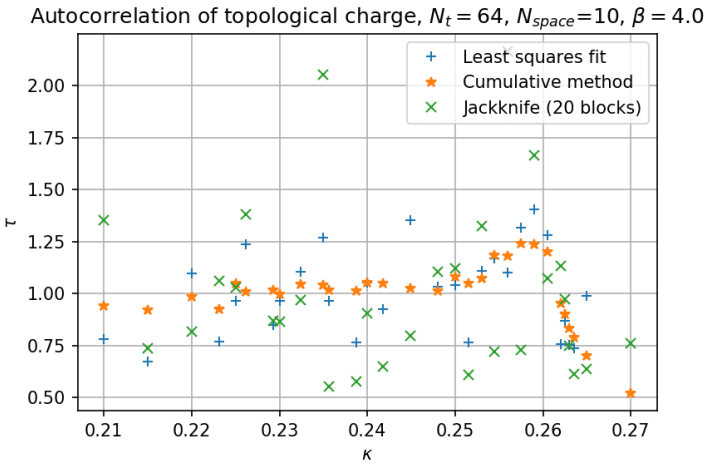
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.10684 ± 0.00134, b =11.23804 ± 0.12283, m_π =0.32686 ± 0.00205 for variance and a =0.10766 ± 0.00137, b =11.18809 ± 0.11521, m_π =0.32812 ± 0.00209 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

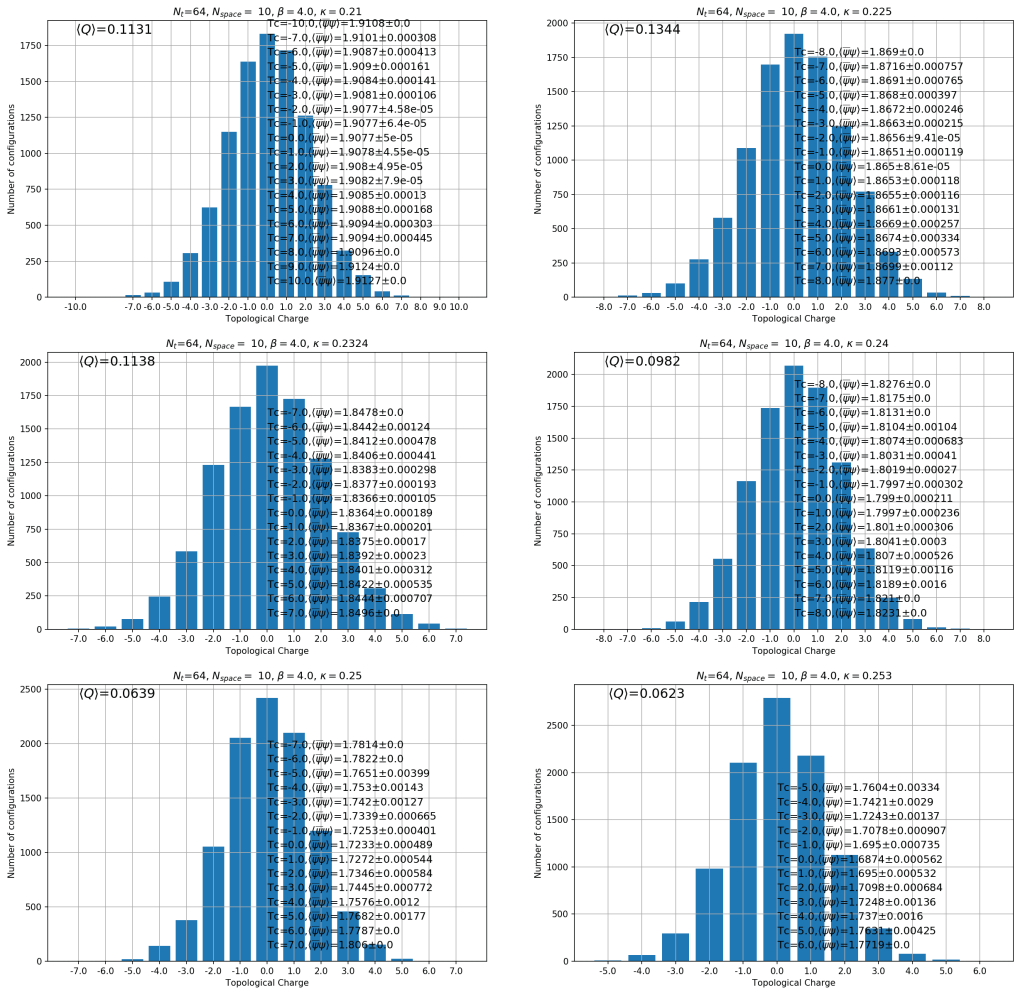


Figure 19: Number of configurations vs. topological charge on a 10×64 lattice.