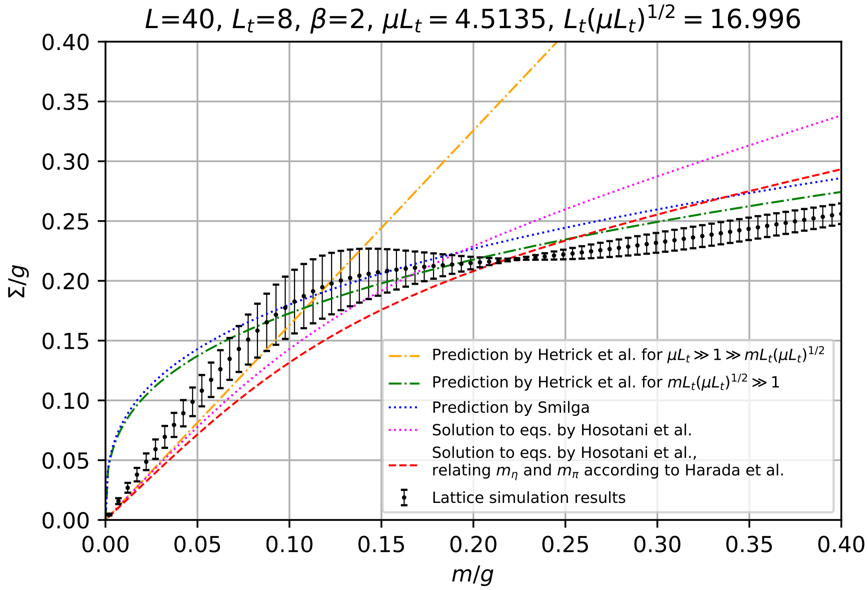
# $\langle\overline{\psi}\psi\rangle$ at finite temperature with the overlap operator.

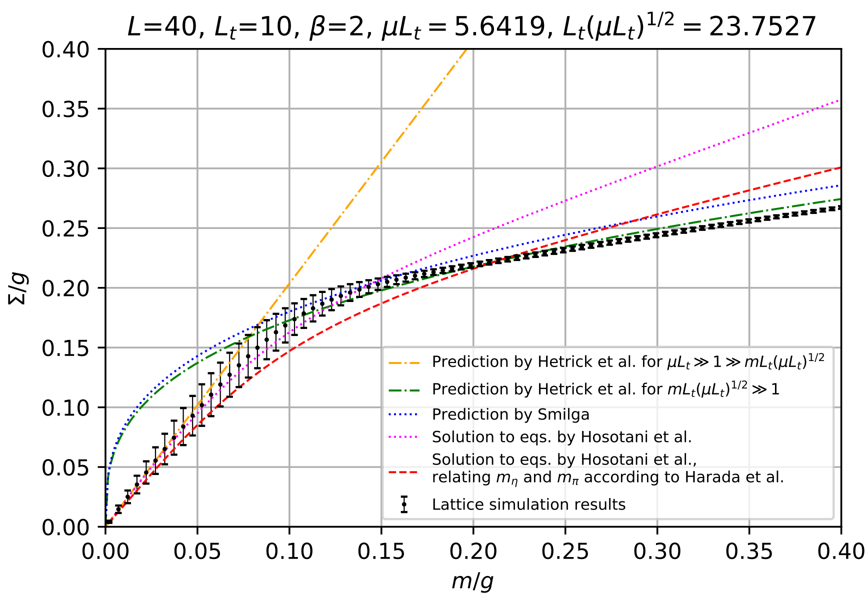
March 31, 2022

# 1 $\beta = 2$

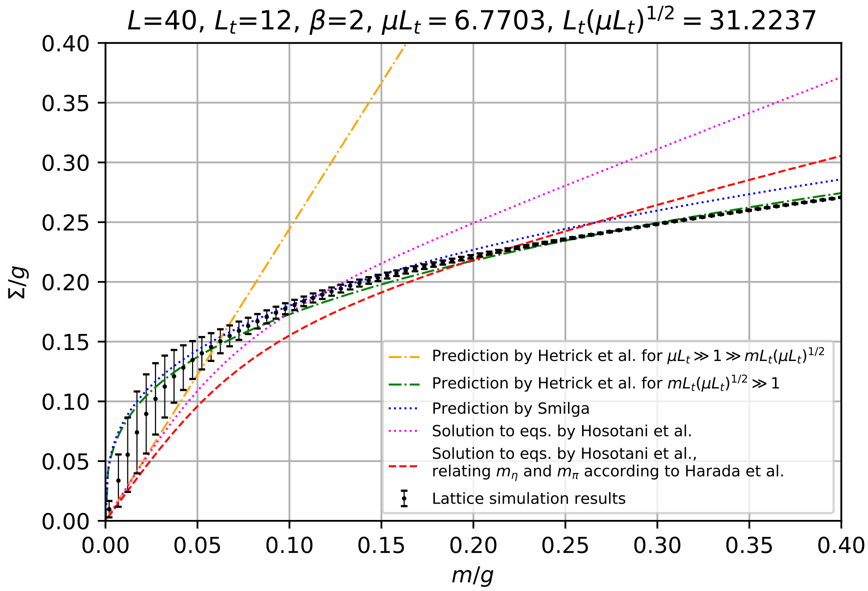
## 1.1 Lattice vs. equations by Hosotani



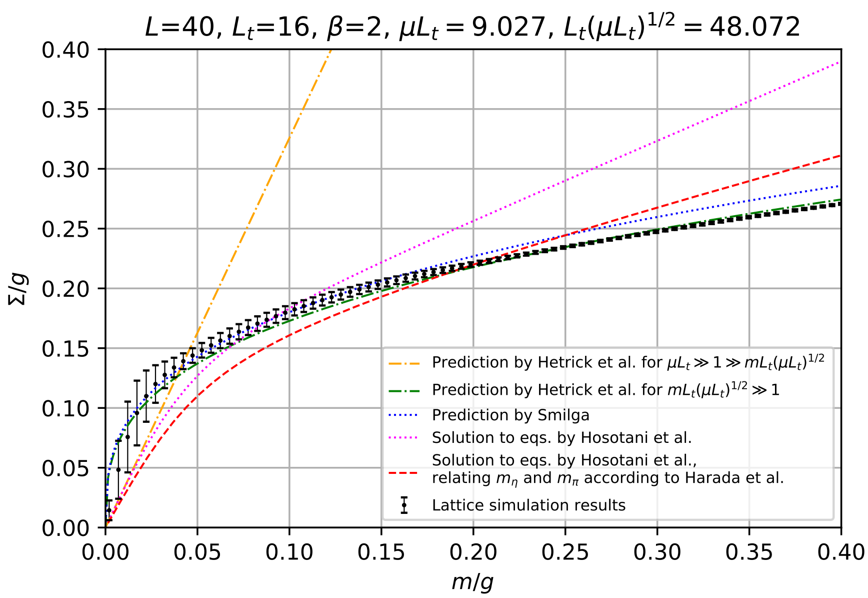

(a) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x8.




(b) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x10.

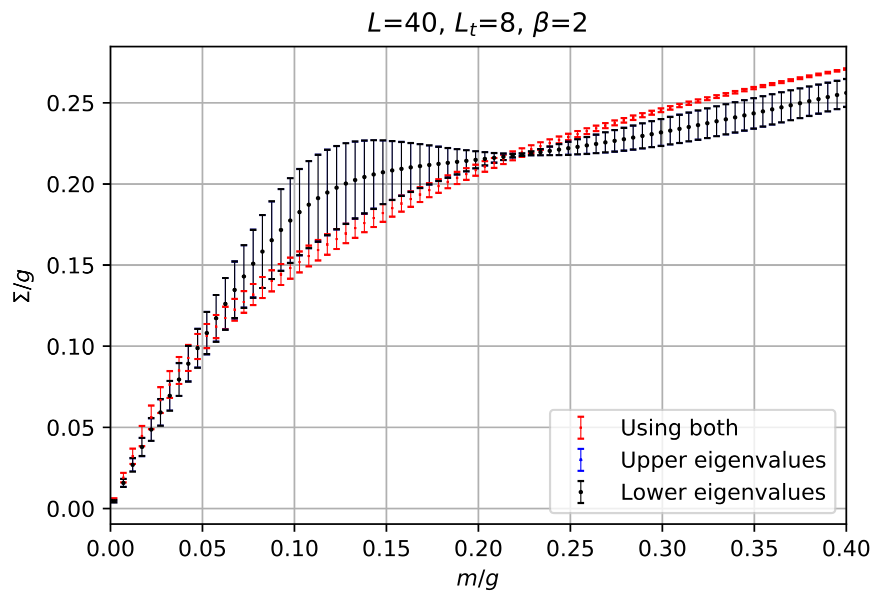



(c) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x12.

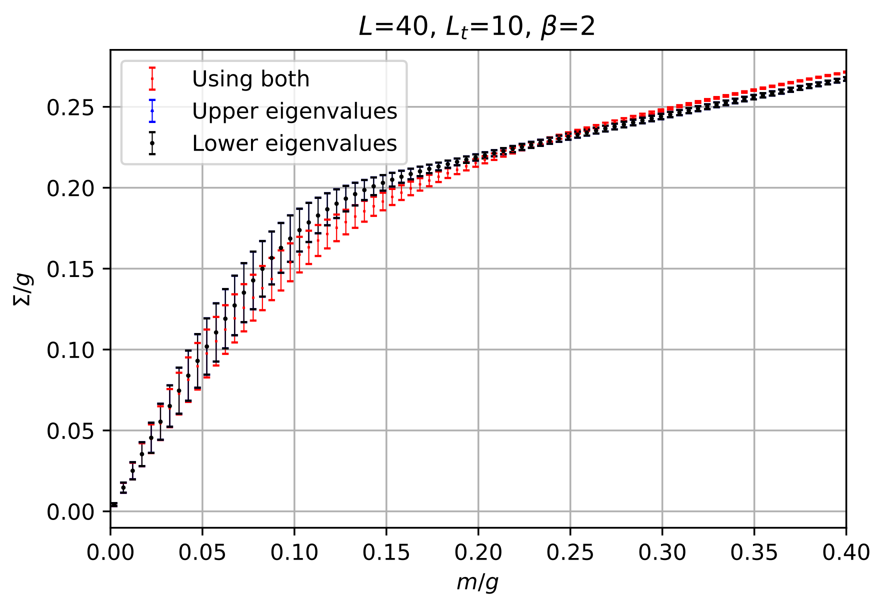



(d) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x16.

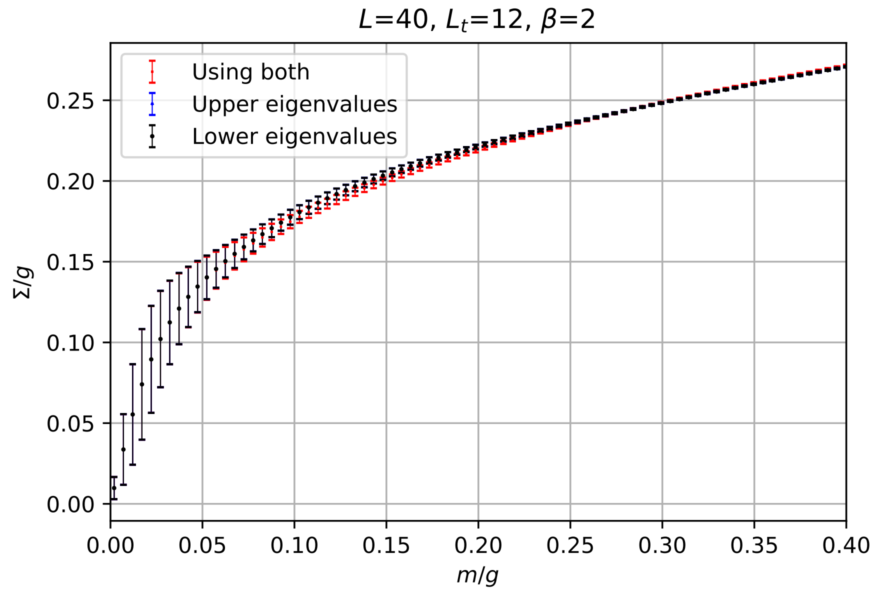
### 1.2 Comparison of $\Sigma$ with the lower and upper half plane eigenvalues.



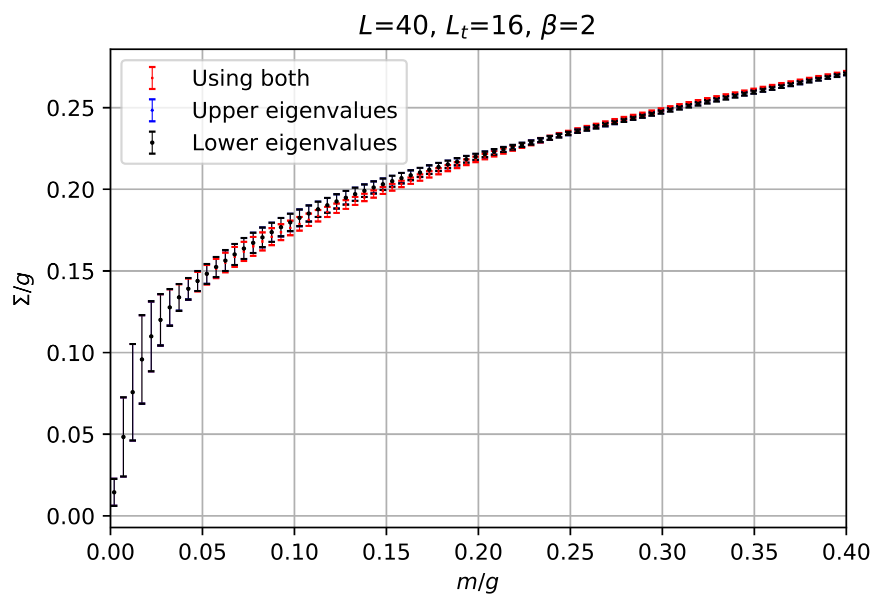

(e) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x8.




(f) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x10.



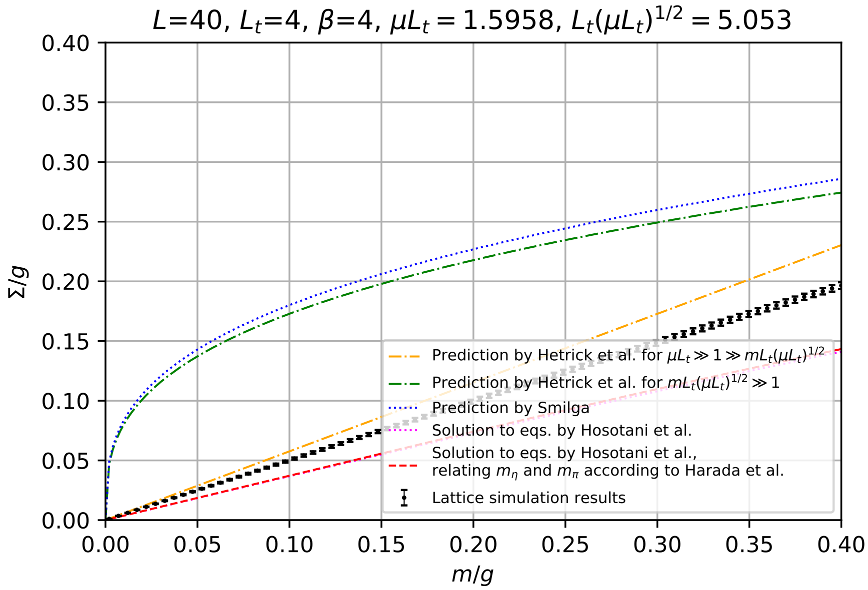

(g) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x12.



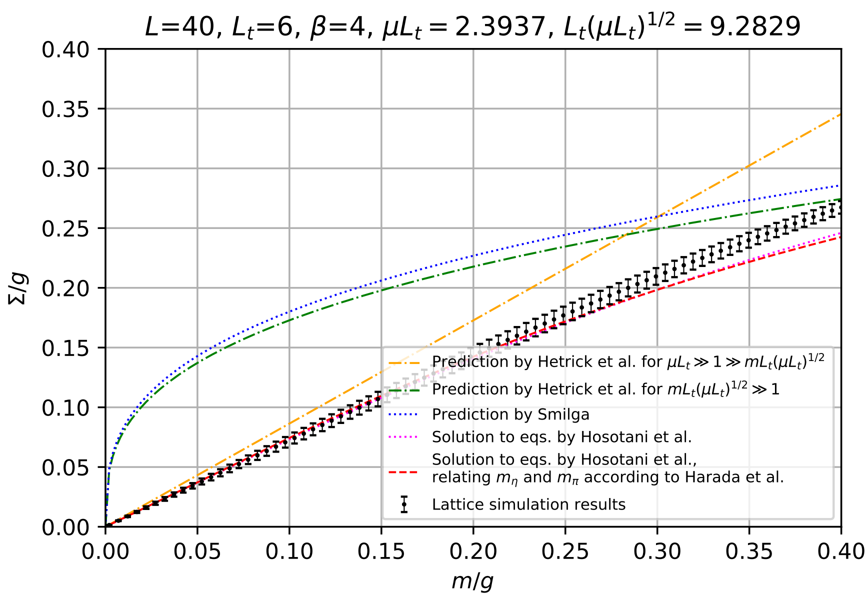

(h) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x16

# 2 $\beta = 4$

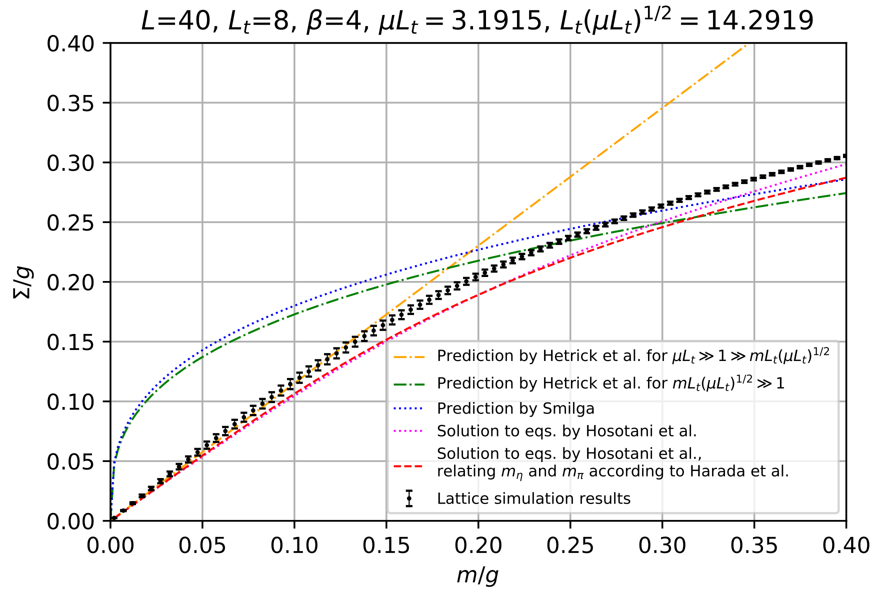
## 2.1 Lattice vs. equations by Hosotani



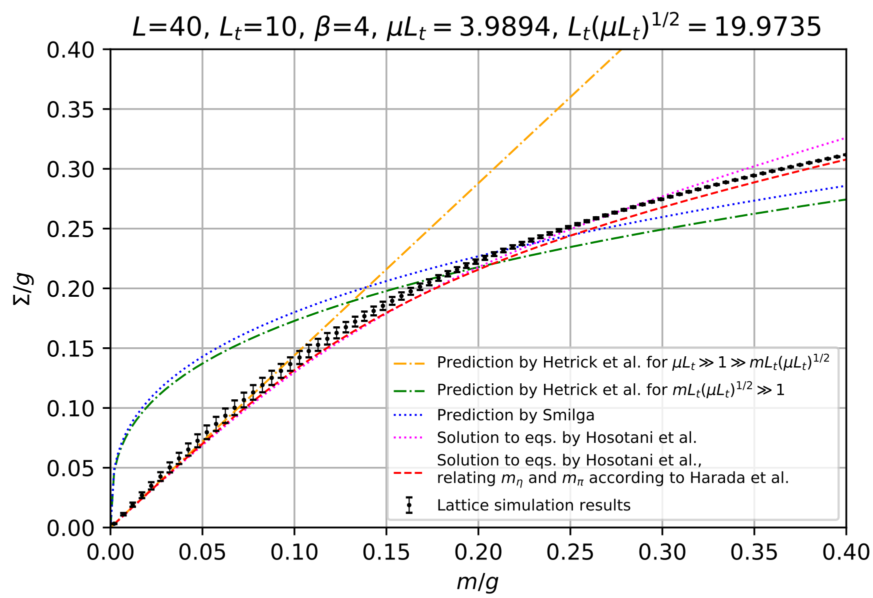

(i) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x4.



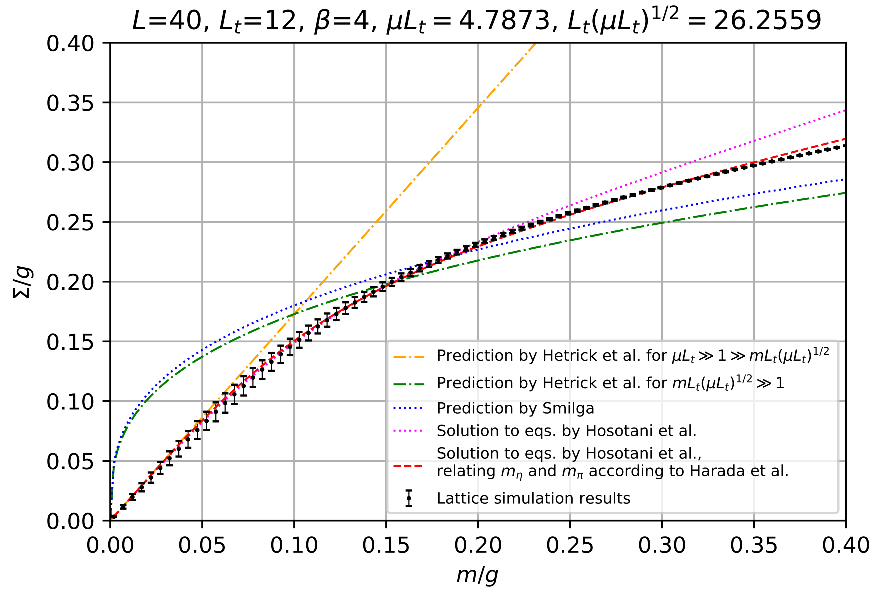

(j) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x6.



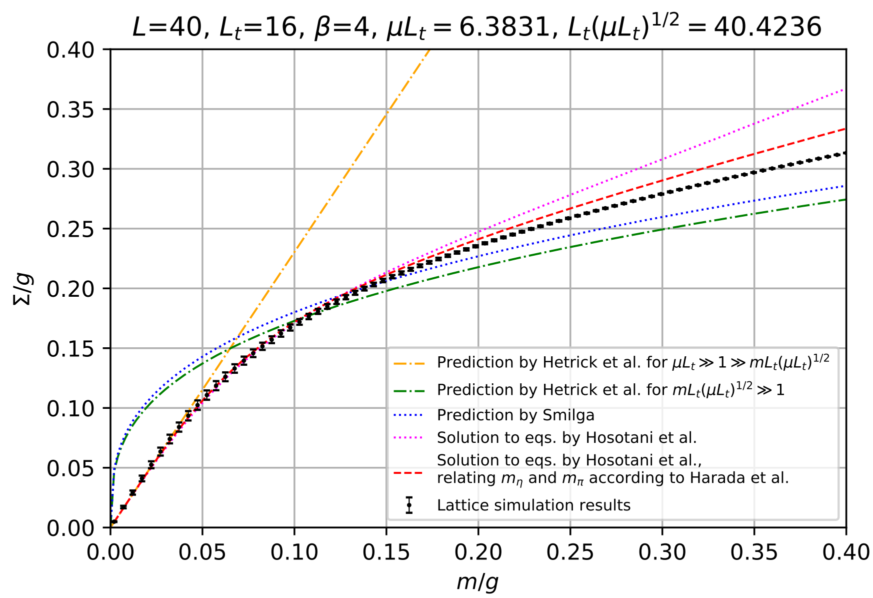

(k) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x8.



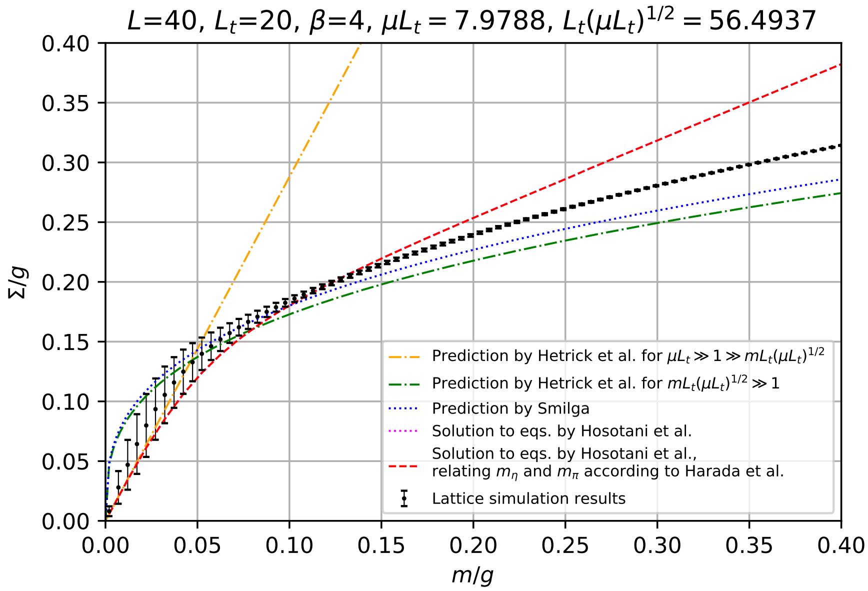

(l) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x10.




(m) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x12.

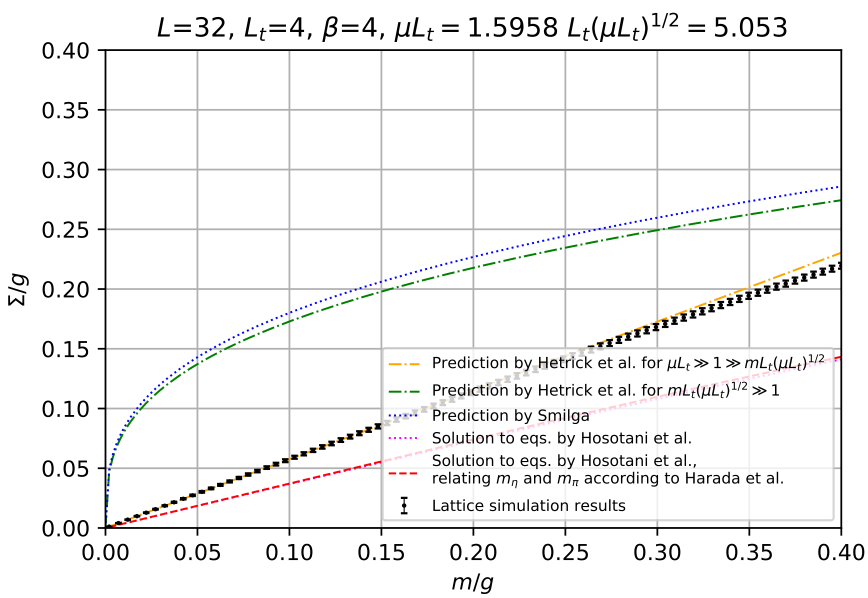



(n) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x16.

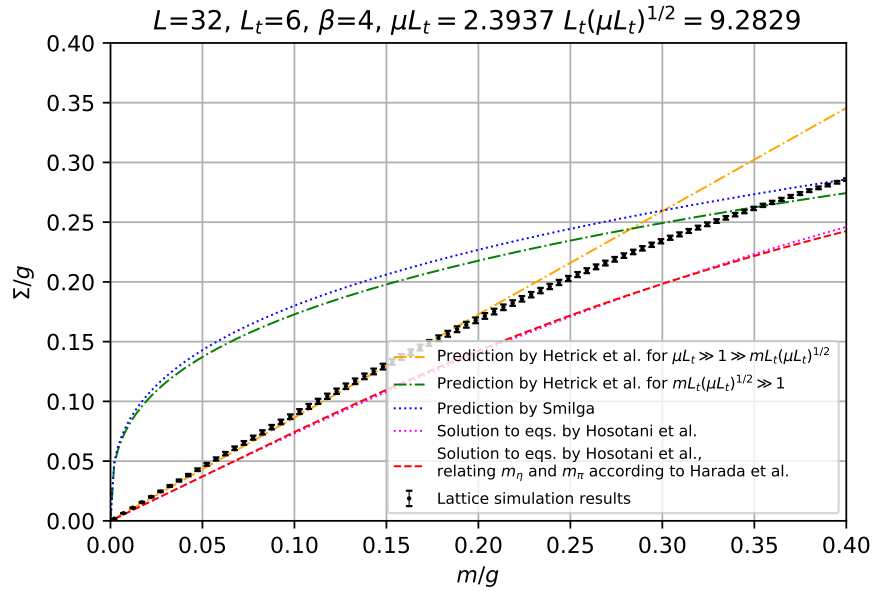



(o) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x20.

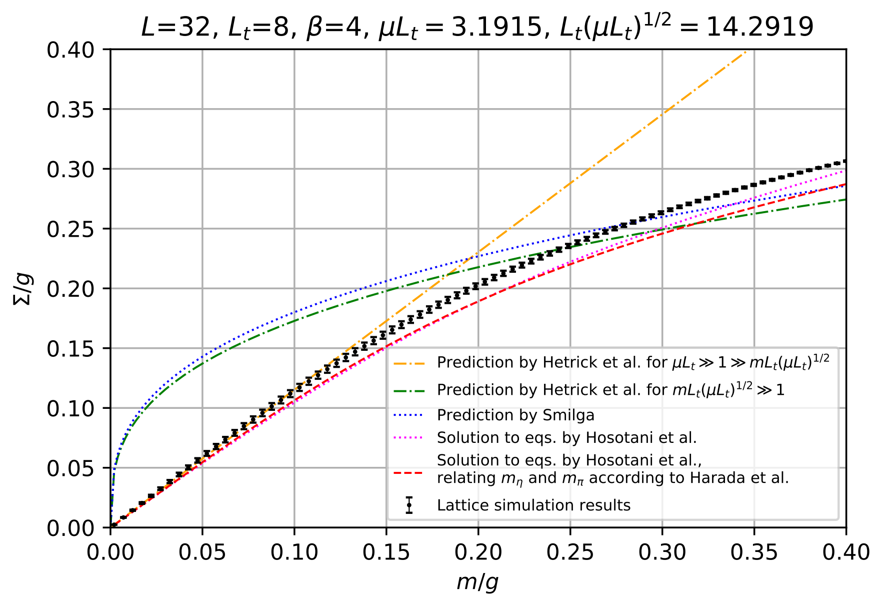
Figure 1: $L = 40$.



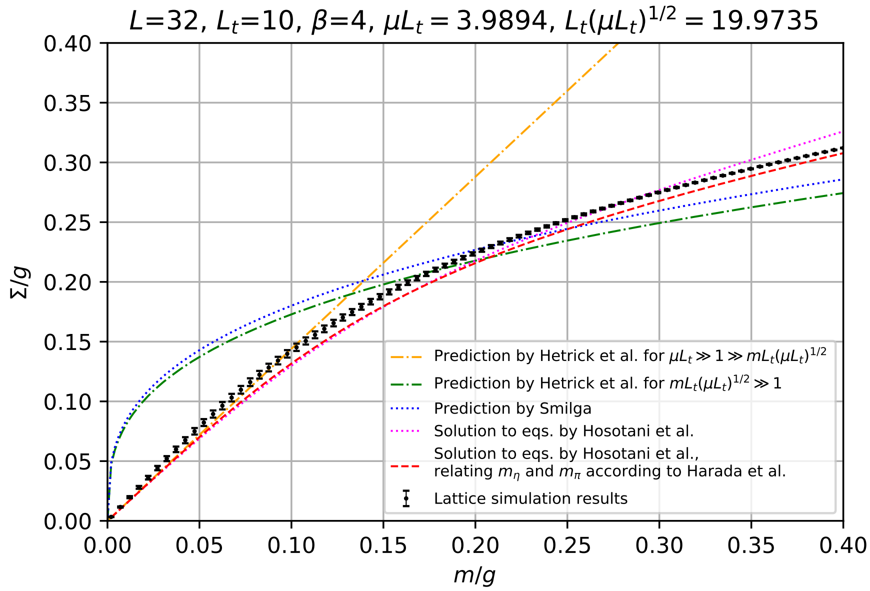

(a) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x4.



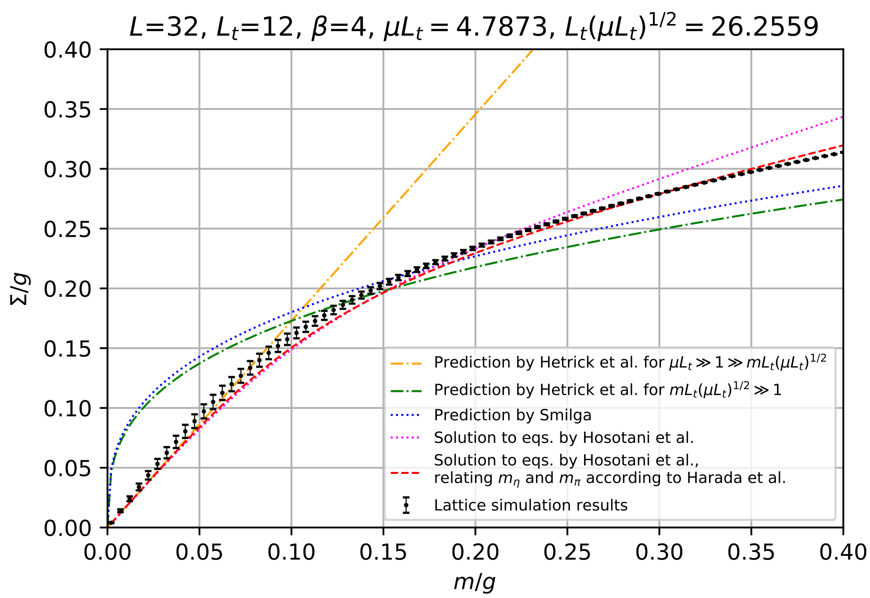

(b) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x6.



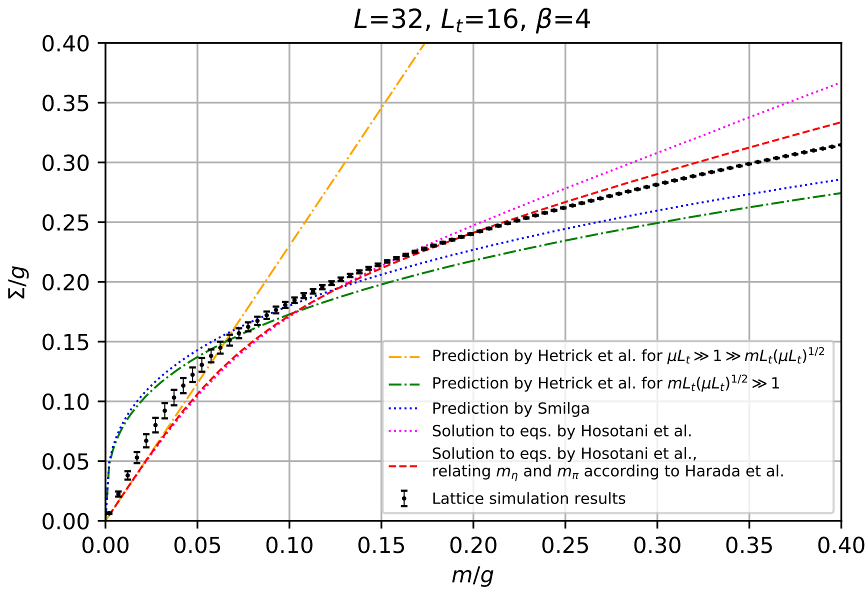

(c) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x8.



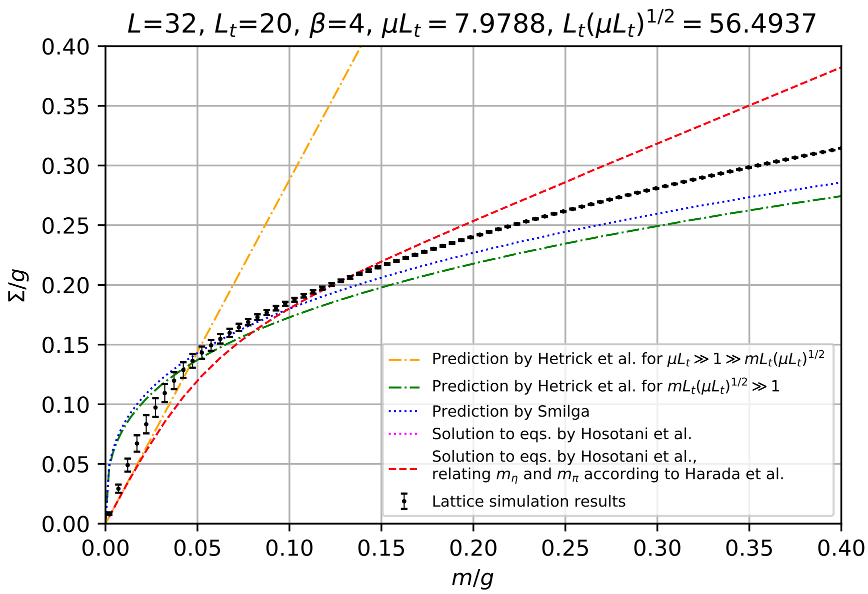

(d) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x10.




(e) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x12.



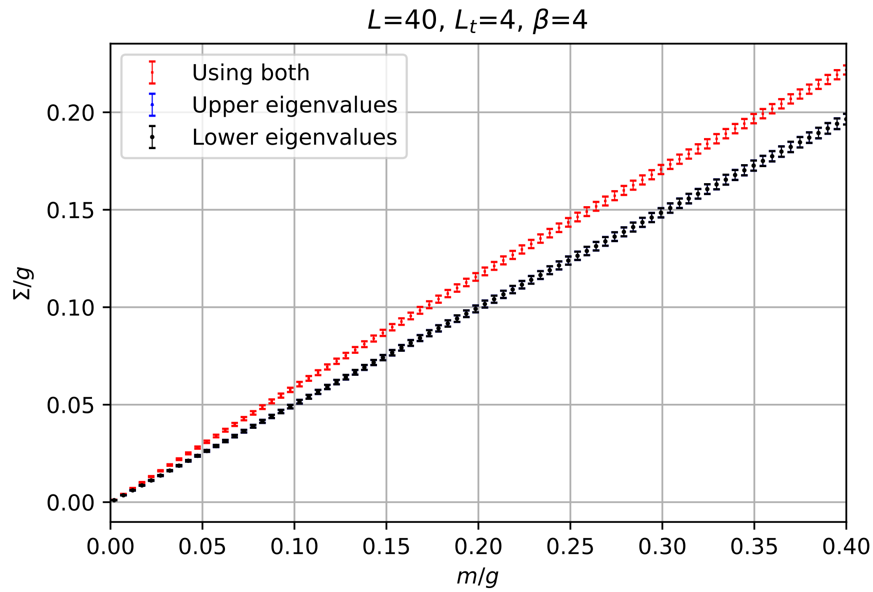

(f) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x16.



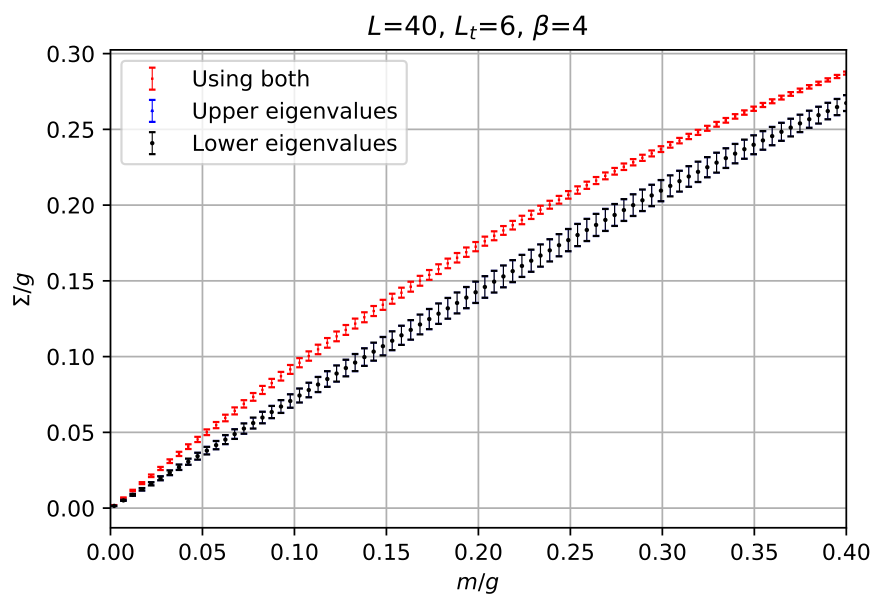

(g) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x20.

Figure 2: $L = 32$.

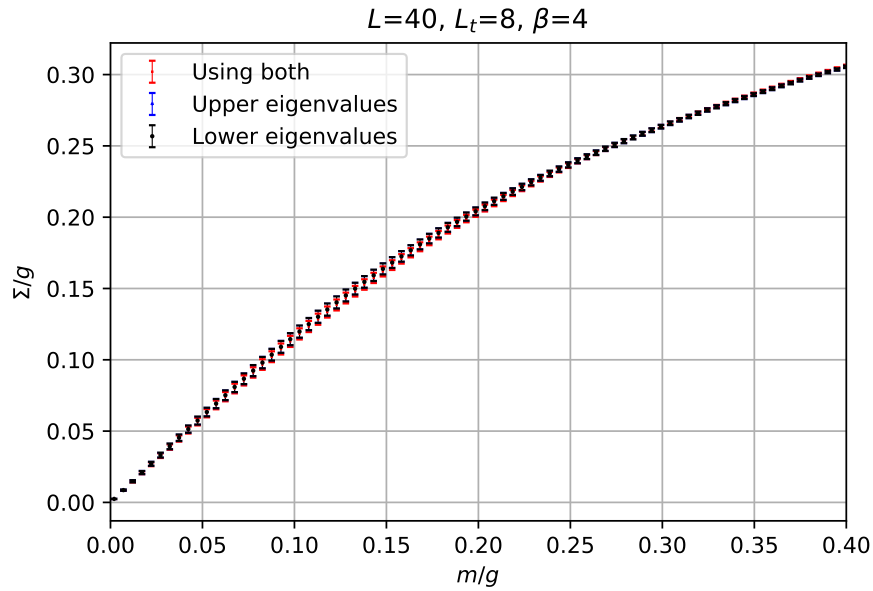
## 2.2 Comparison of $\Sigma$ with the lower and upper half plane eigenvalues.



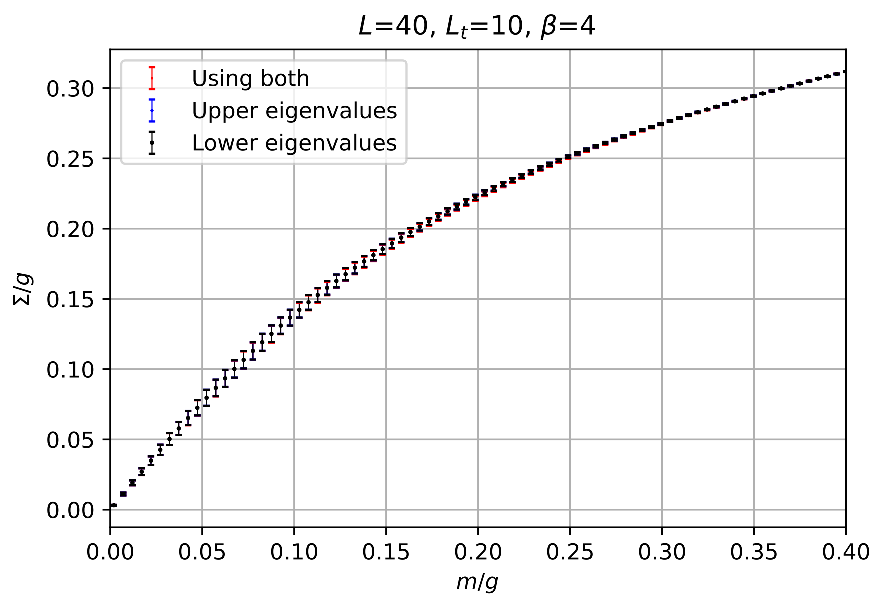

(a) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x8.



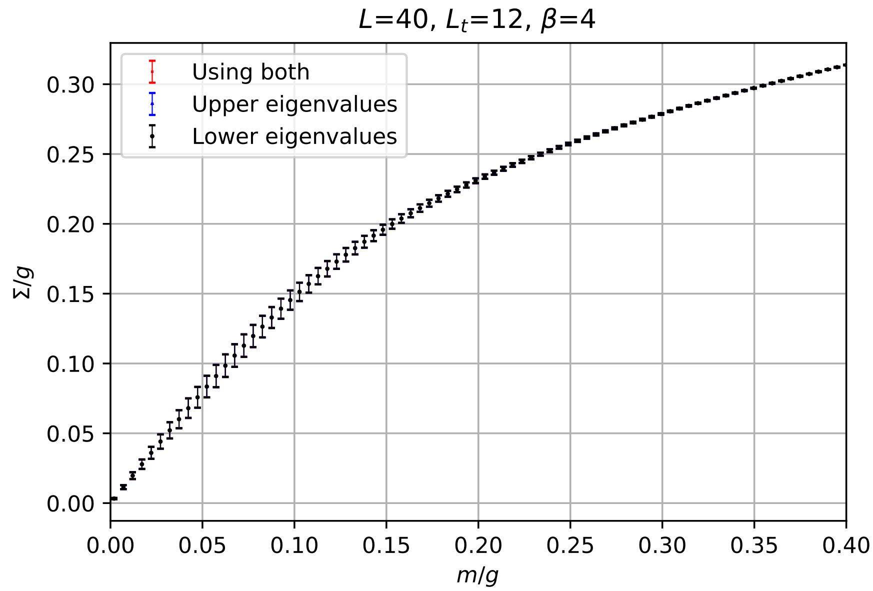

(b) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x10.



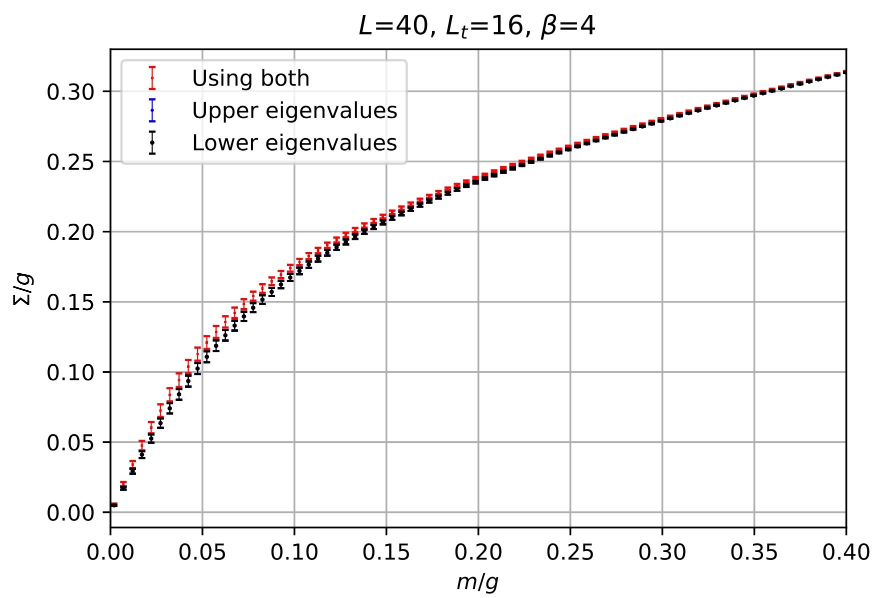

(c) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x8.



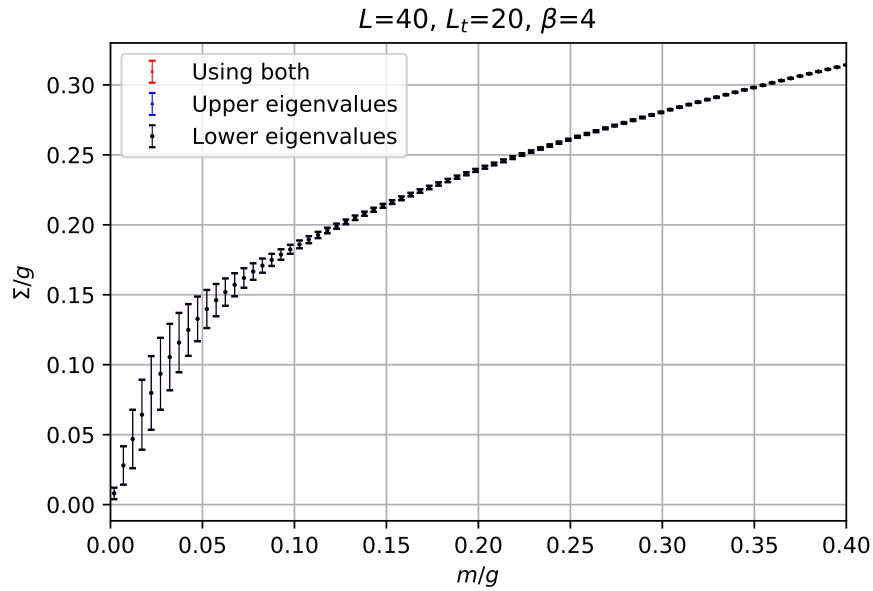

(d) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x10.




(e) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x12.

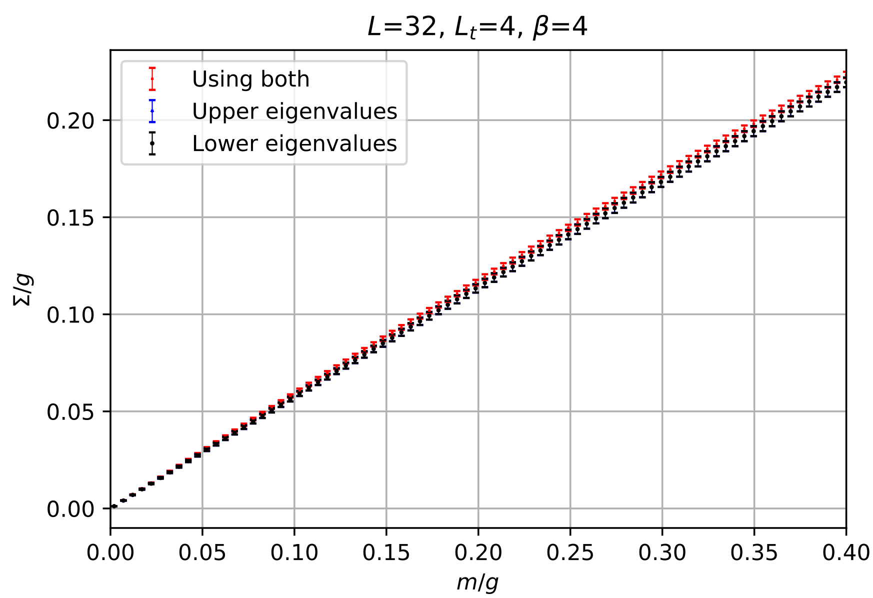



(f) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x16.

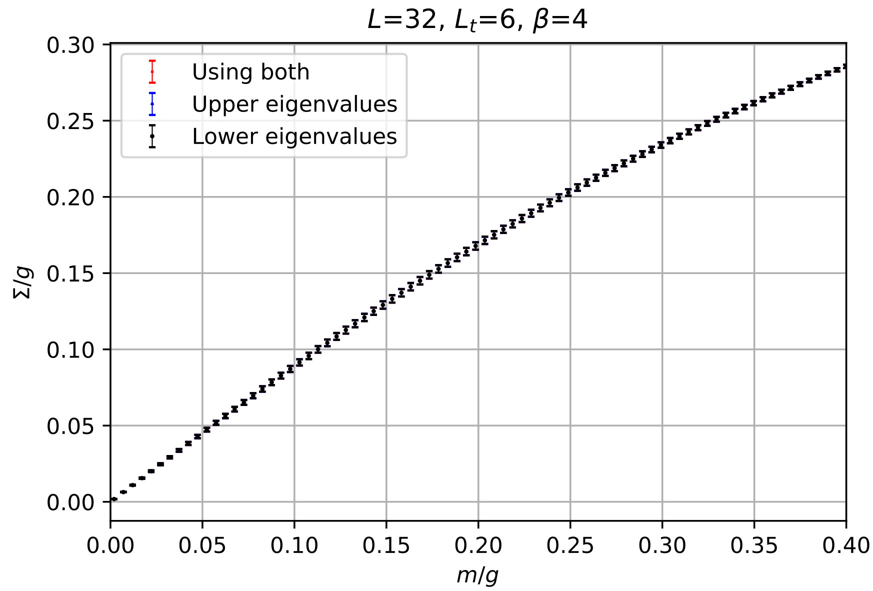



(g) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x20.

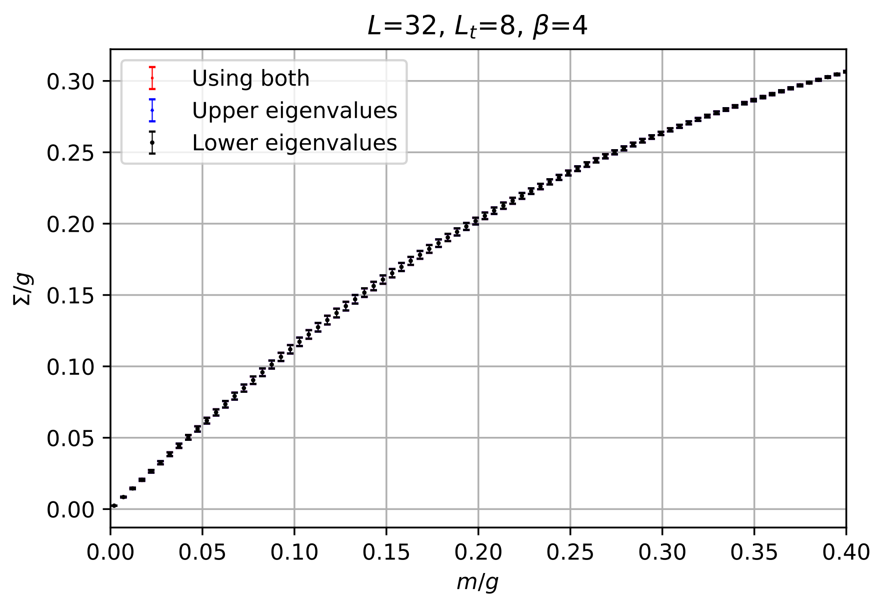
Figure 3: $L = 40$



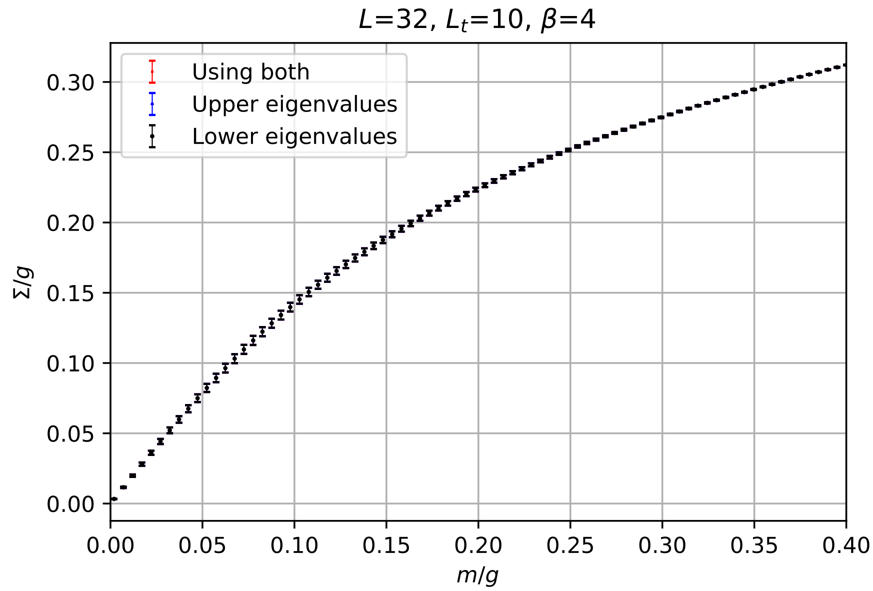

(a) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x4.



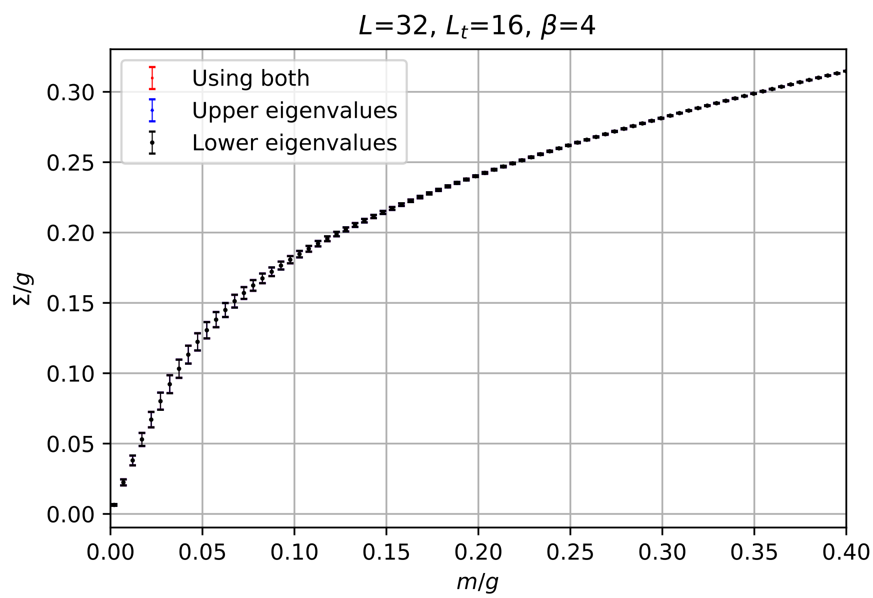

(b) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x6.



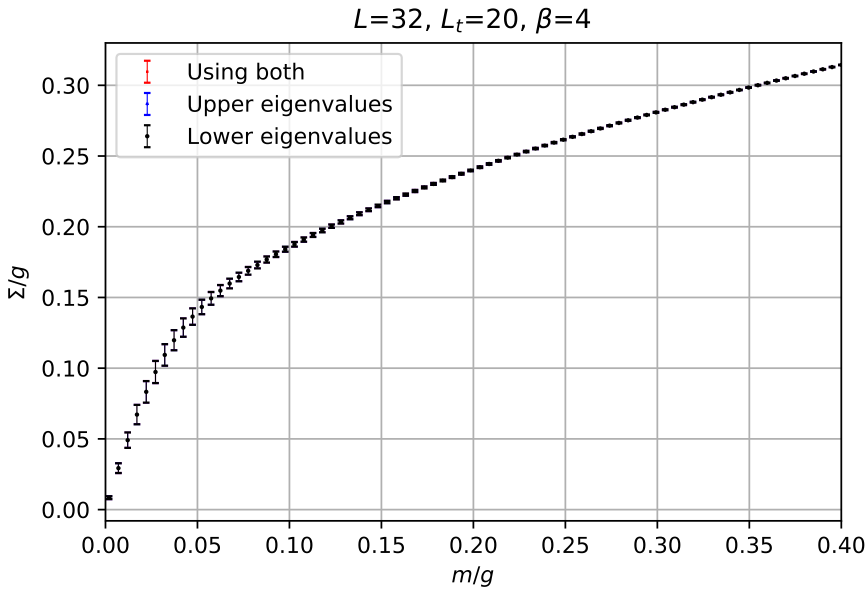

(c) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x8.




(d) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x10.




(e) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x16.

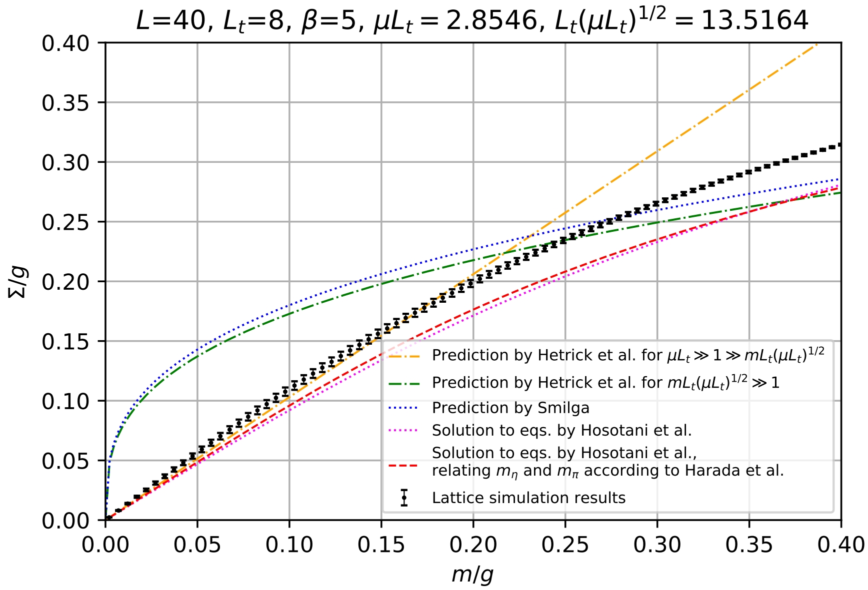



(f) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 32x20.

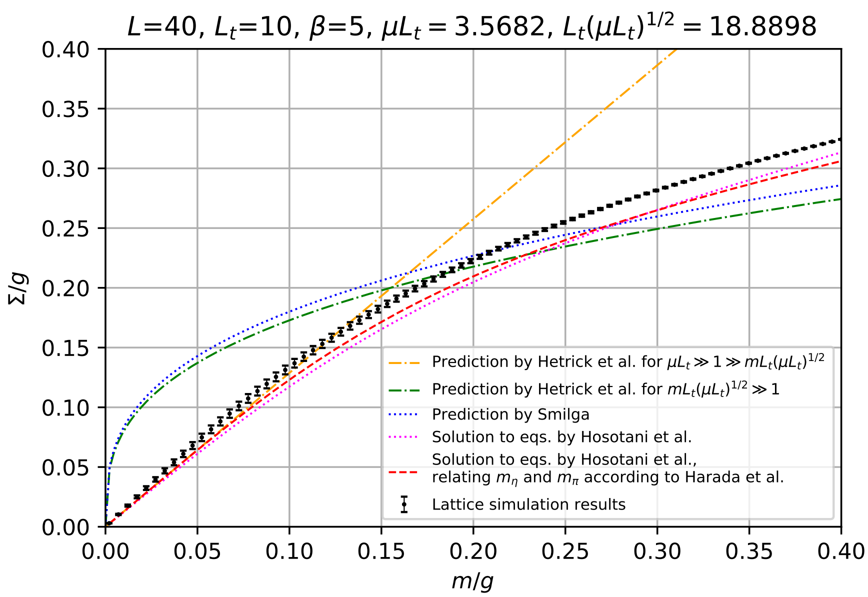
Figure 4: $L = 32$

# 3 $\beta = 5$

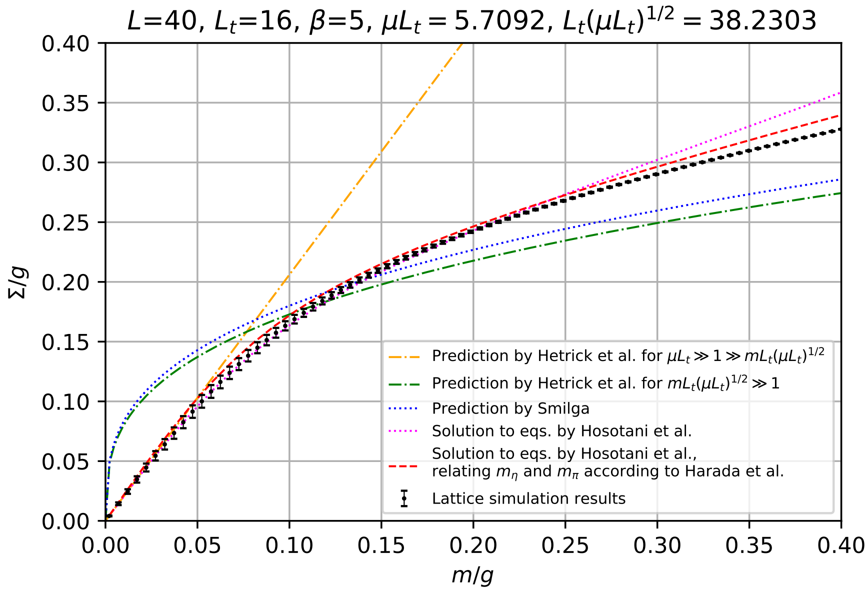
## 3.1 Lattice vs. equations by Hosotani




(a) $\langle \overline{\psi}\psi \rangle$ for a lattice of size 40x8.

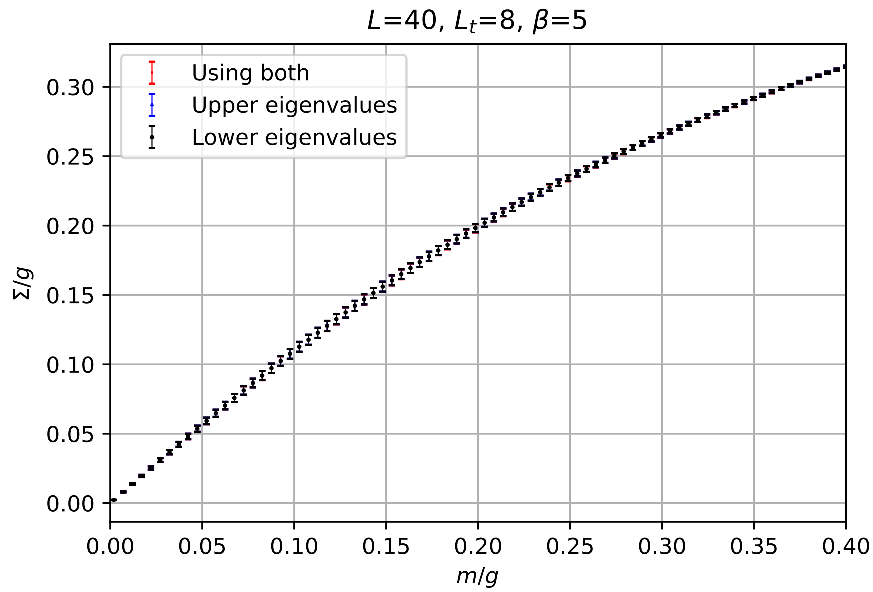



(b) $\langle \overline{\psi}\psi \rangle$ for a lattice of size 40x10.

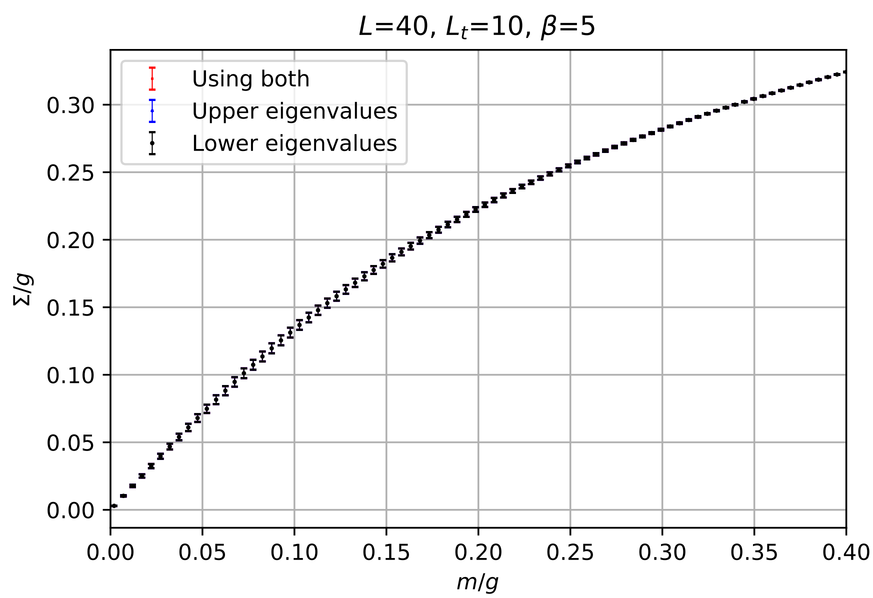



(c) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x16.

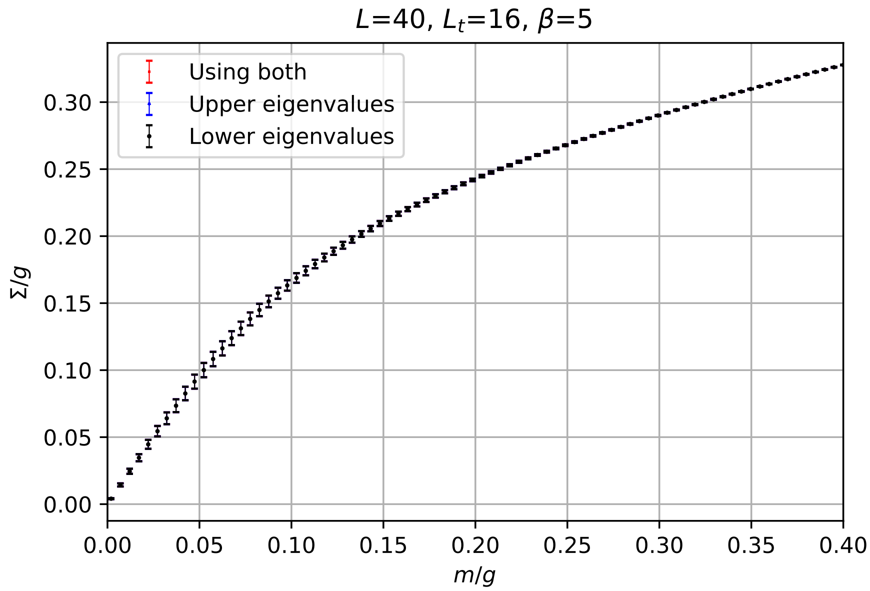
### 3.2 Comparison of $\Sigma$ with the lower and upper half plane eigenvalues.




(d) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x8.




(e) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x10.




(f) $\langle\overline{\psi}\psi\rangle$ for a lattice of size 40x16.

Figure 5: $\beta = 5$

# 4 Histograms

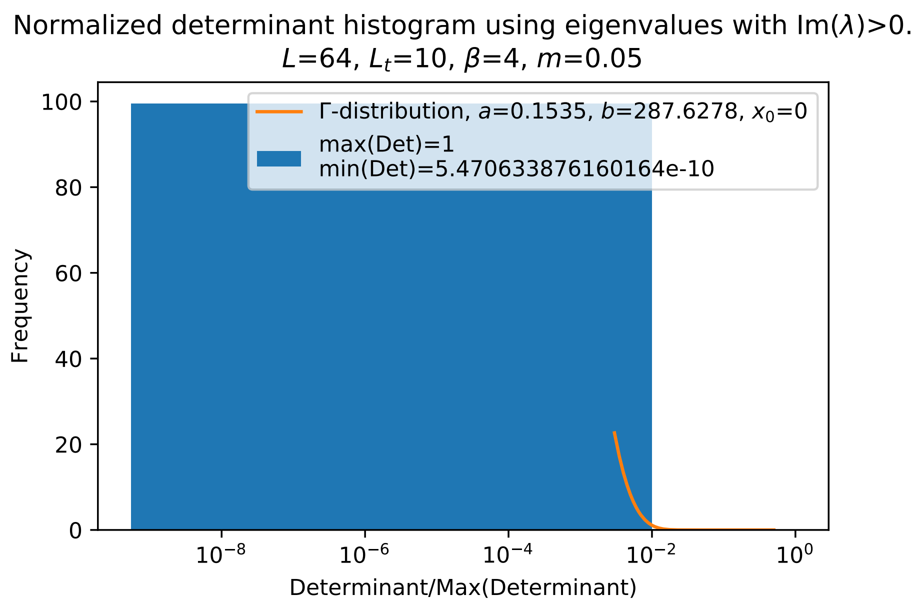
We show histograms of the determinant

$$\det(D_m^{\text{ov}}) = \prod_{\lambda} \left[ \left(1 - \frac{m}{2}\right) \lambda + m \right]. \tag{1}$$

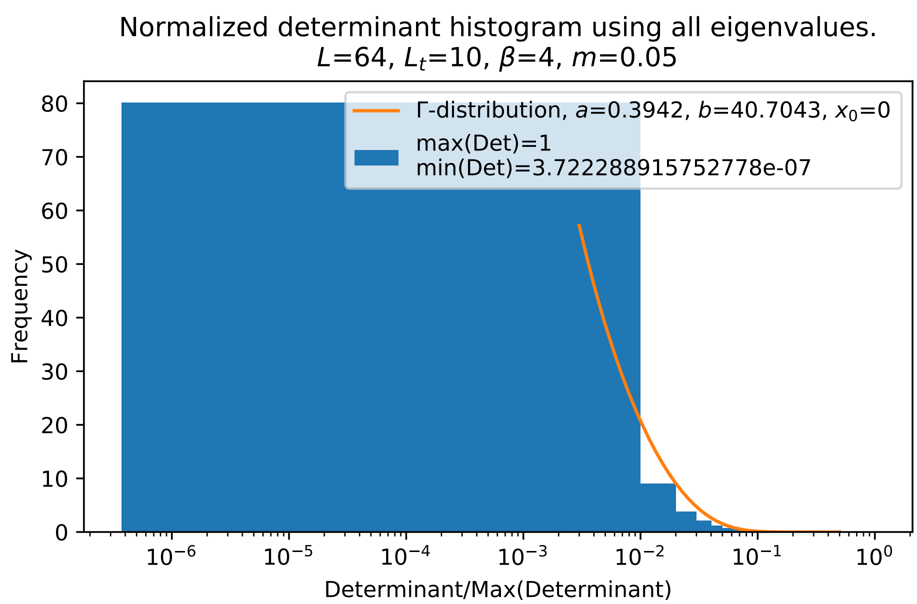
for $m = 0.05$ and different lattices. We fitted a Gamma distribution,

$$f(x) = \frac{b^{a-1}(x - x_0)^{a-1} e^{-b(x-x_0)}}{\Gamma(a)}, \quad a,\ b \text{ and } x_0 \text{ fitting paremeters,} \tag{2}$$

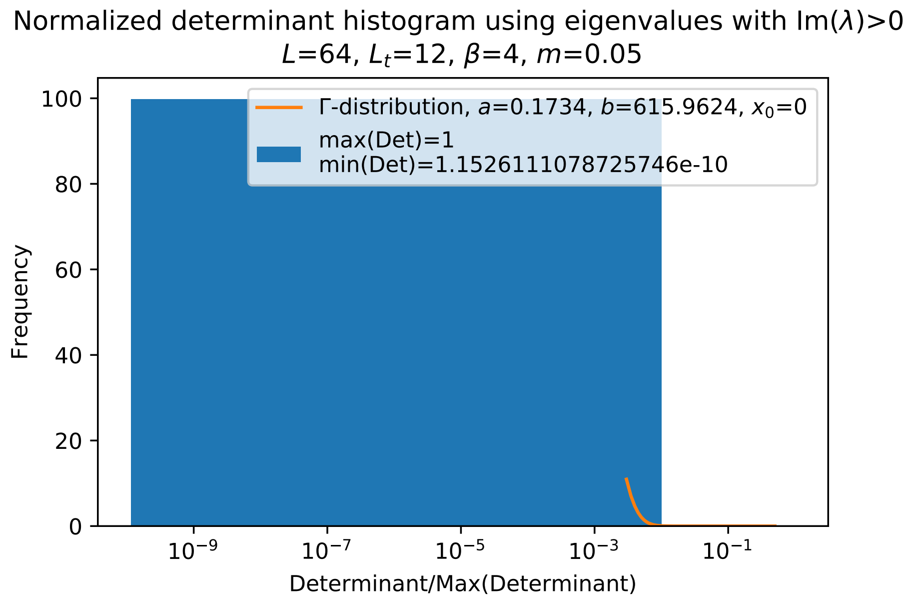
to the region of the histograms where they decay. By fixing $x_0 = 0$ we obtained the best fits. The histograms are normalized on the vertical axis by the number of measurements and on the horizontal axis by the maximum value that the determinant takes.




(a)

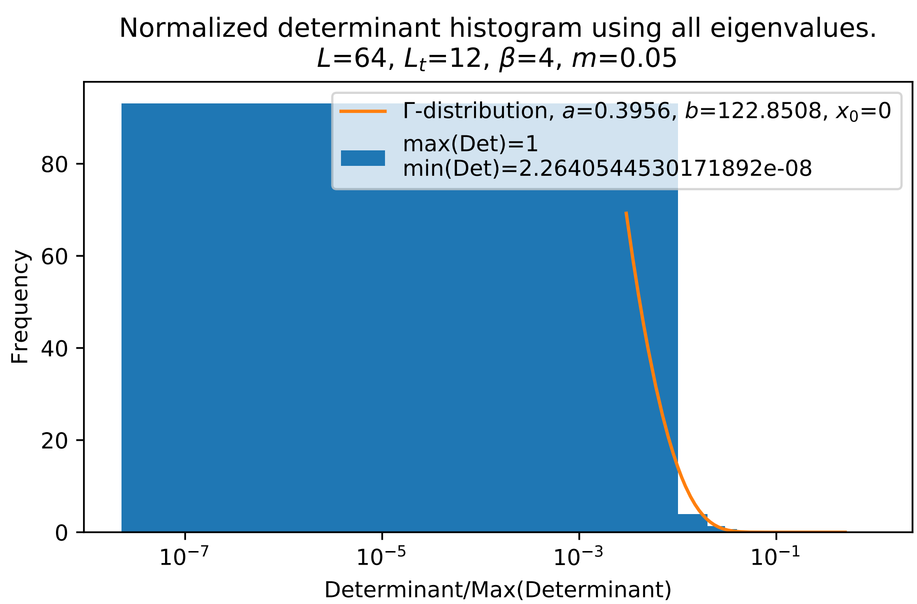



(b)



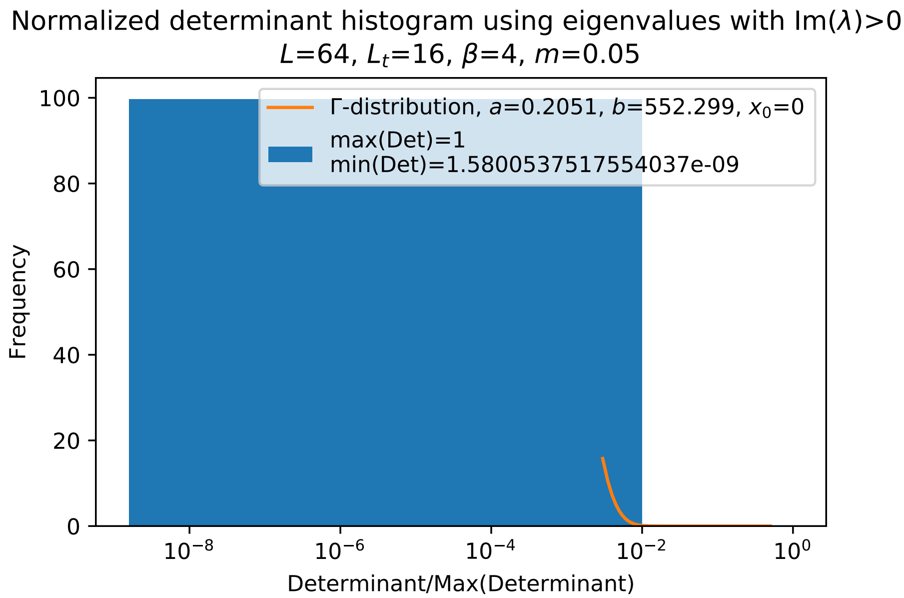
Normalized determinant histogram using eigenvalues with Im(λ)>0.
L=64, Lt=12, β=4, m=0.05
Γ-distribution, a=0.1734, b=615.9624, x0=0
max(Det)=1
min(Det)=1.1526111078725746e-10
Frequency
Determinant/Max(Determinant)


(c)

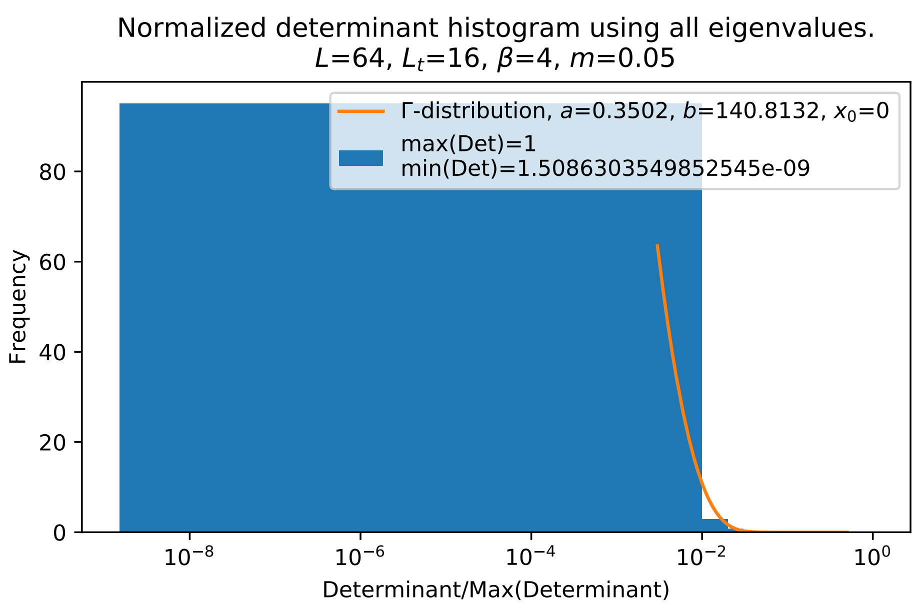


Normalized determinant histogram using all eigenvalues.
L=64, Lt=12, β=4, m=0.05
Γ-distribution, a=0.3956, b=122.8508, x0=0
max(Det)=1
min(Det)=2.2640544530171892e-08
Frequency
Determinant/Max(Determinant)


(d)

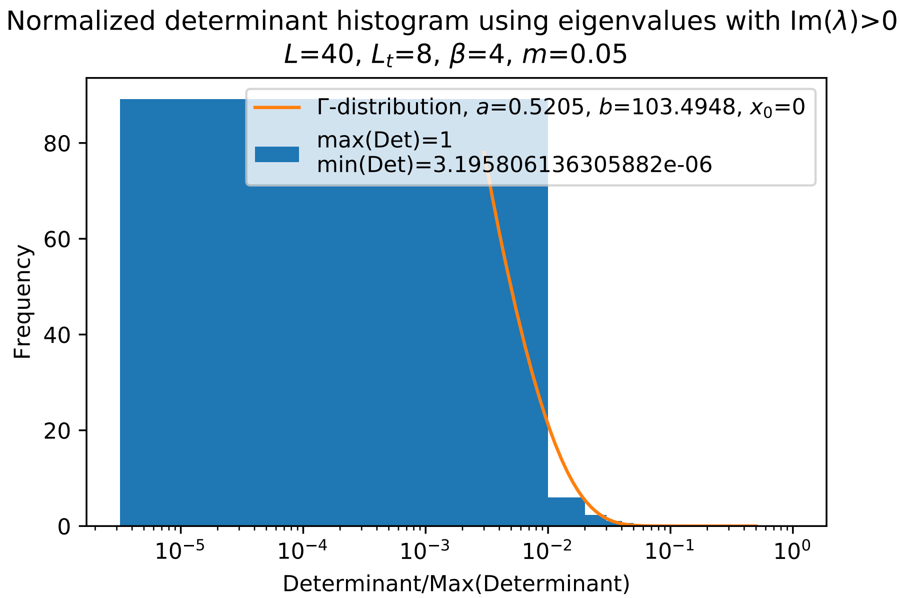



(e)

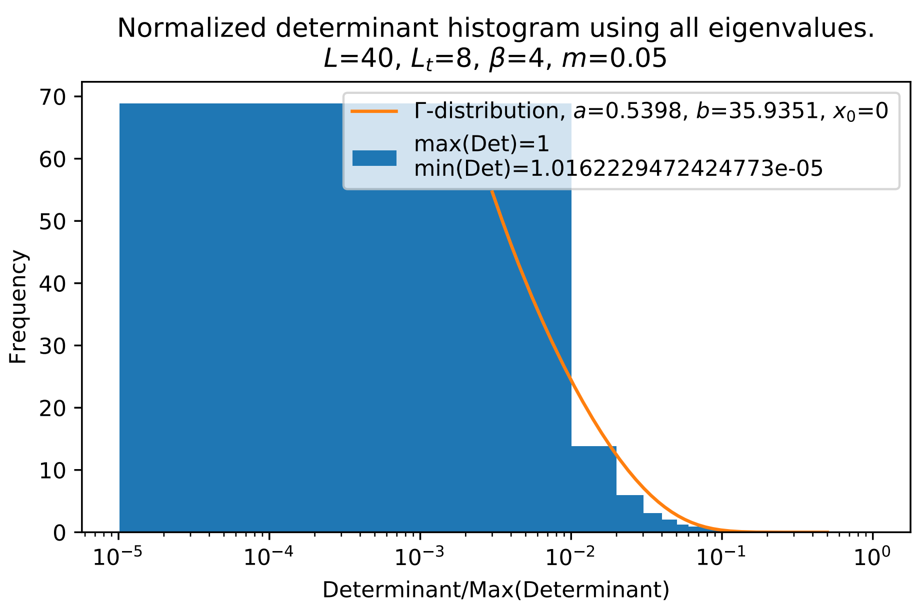



(f)

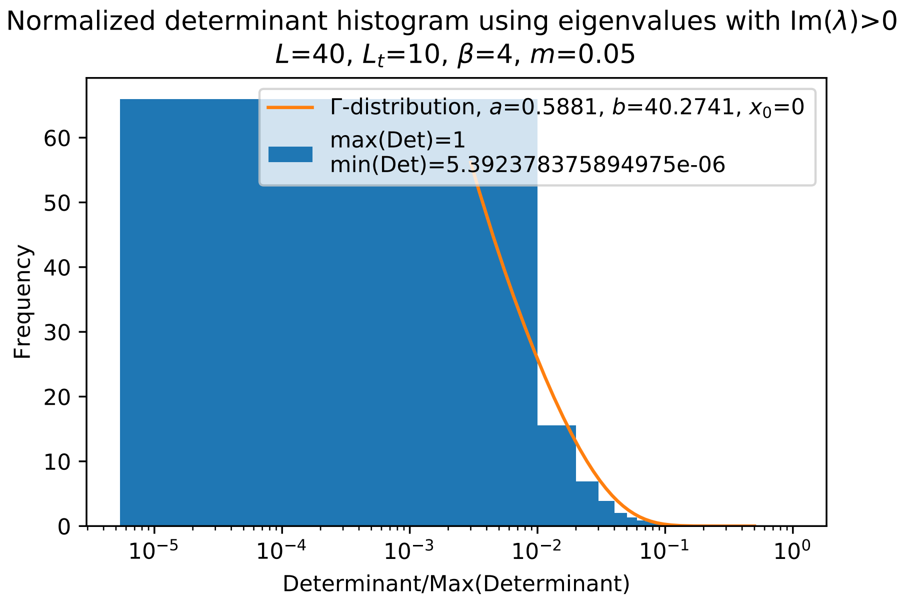
Figure 6: $\beta = 4$, $L = 64$. The horizontal axis of the histogram is in logarithmic scale.


Normalized determinant histogram using eigenvalues with Im(λ)>0.
L=40, Lt=8, β=4, m=0.05
Γ-distribution, a=0.5205, b=103.4948, x0=0
max(Det)=1
min(Det)=3.195806136305882e-06
Frequency
0
20
40
60
80
10−5
10−4
10−3
10−2
10−1
10^0
Determinant/Max(Determinant)

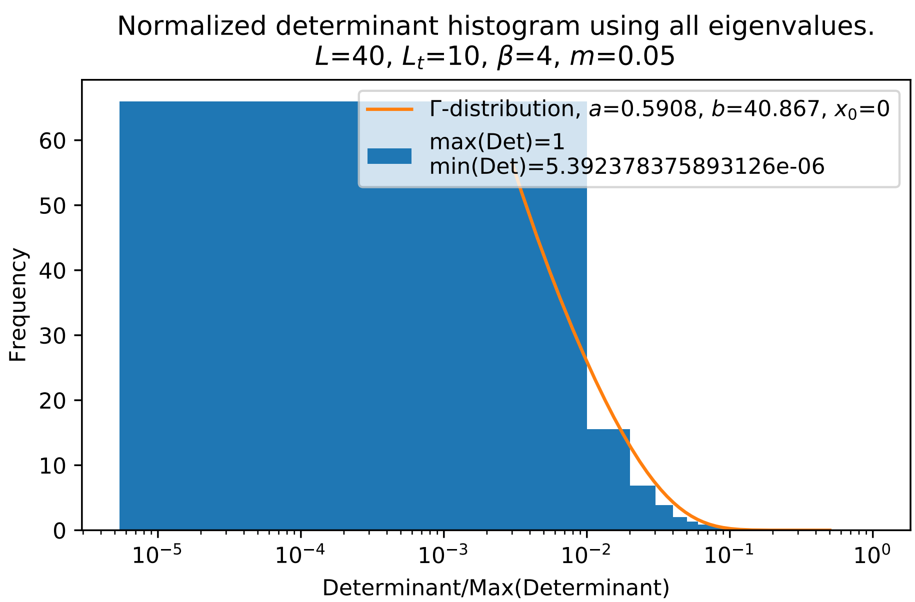
(a)


Normalized determinant histogram using all eigenvalues.
L=40, Lt=8, β=4, m=0.05
Γ-distribution, a=0.5398, b=35.9351, x0=0
max(Det)=1
min(Det)=1.0162229472424773e-05
Frequency
0
10
20
30
40
50
60
70
10−5
10−4
10−3
10−2
10−1
10^0
Determinant/Max(Determinant)

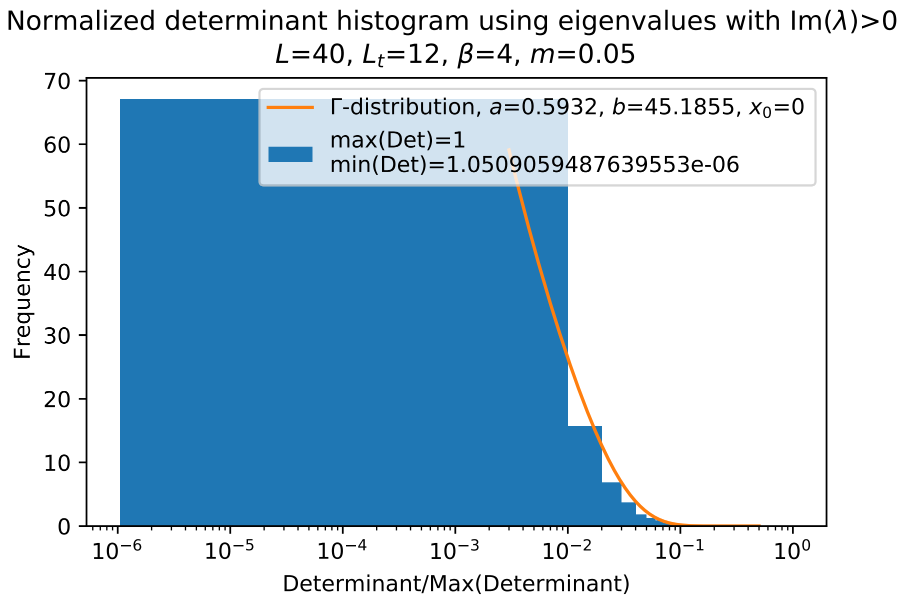
(b)


Normalized determinant histogram using eigenvalues with Im(λ)>0.
L=40, L_t=10, β=4, m=0.05
Γ-distribution, a=0.5881, b=40.2741, x_0=0
max(Det)=1
min(Det)=5.392378375894975e-06
Frequency
Determinant/Max(Determinant)

(c)

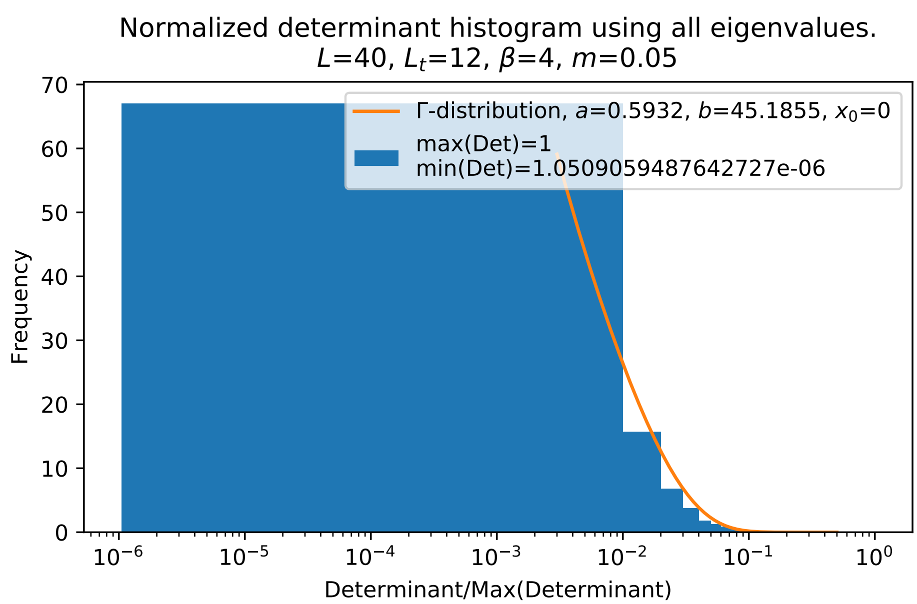

Normalized determinant histogram using all eigenvalues.
L=40, L_t=10, β=4, m=0.05
Γ-distribution, a=0.5908, b=40.867, x_0=0
max(Det)=1
min(Det)=5.392378375893126e-06
Frequency
Determinant/Max(Determinant)

(d)



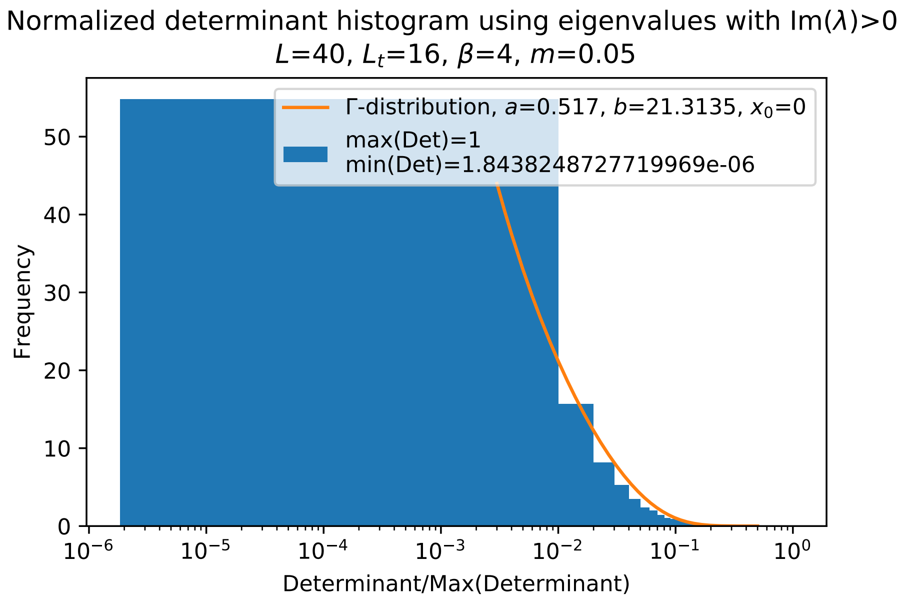
Normalized determinant histogram using eigenvalues with Im(λ)>0.
L=40, Lt=12, β=4, m=0.05
Γ-distribution, a=0.5932, b=45.1855, x0=0
max(Det)=1
min(Det)=1.0509059487639553e-06
Frequency
Determinant/Max(Determinant)


(e)

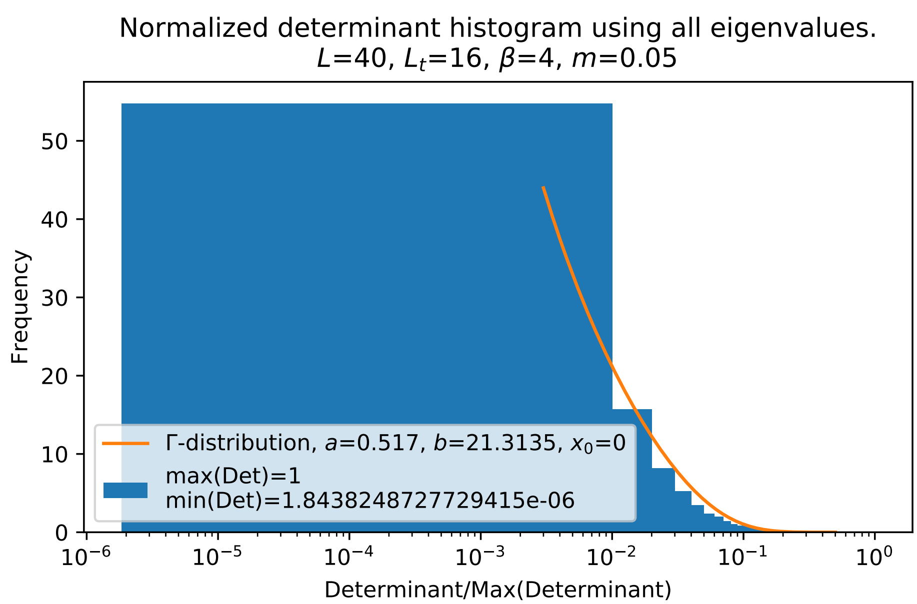


Normalized determinant histogram using all eigenvalues.
L=40, Lt=12, β=4, m=0.05
Γ-distribution, a=0.5932, b=45.1855, x0=0
max(Det)=1
min(Det)=1.0509059487642727e-06
Frequency
Determinant/Max(Determinant)


(f)

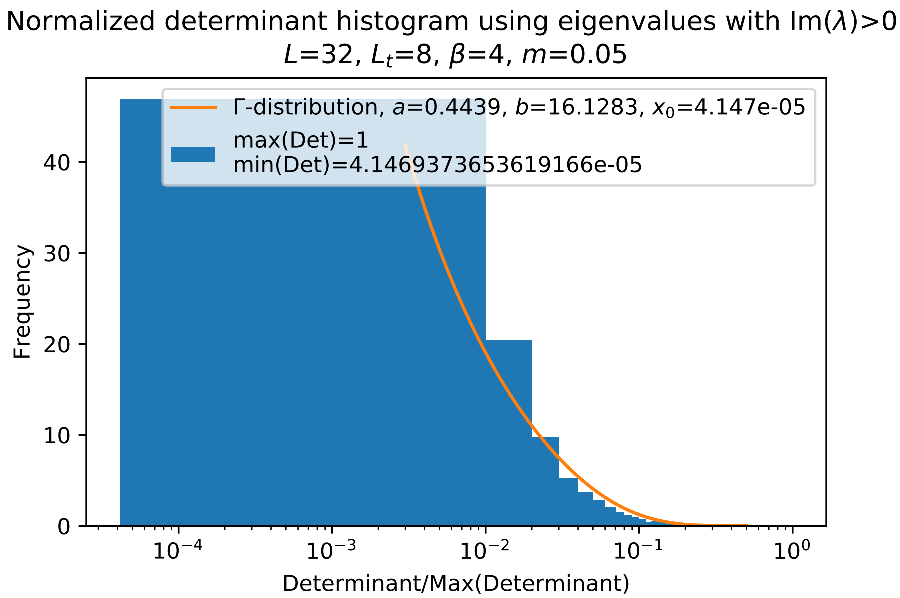



(g)

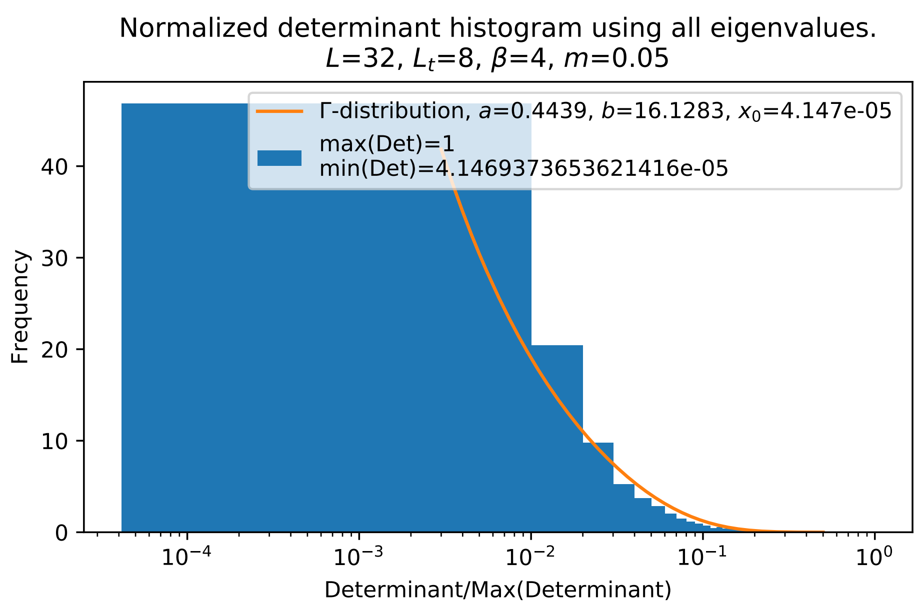



(h)

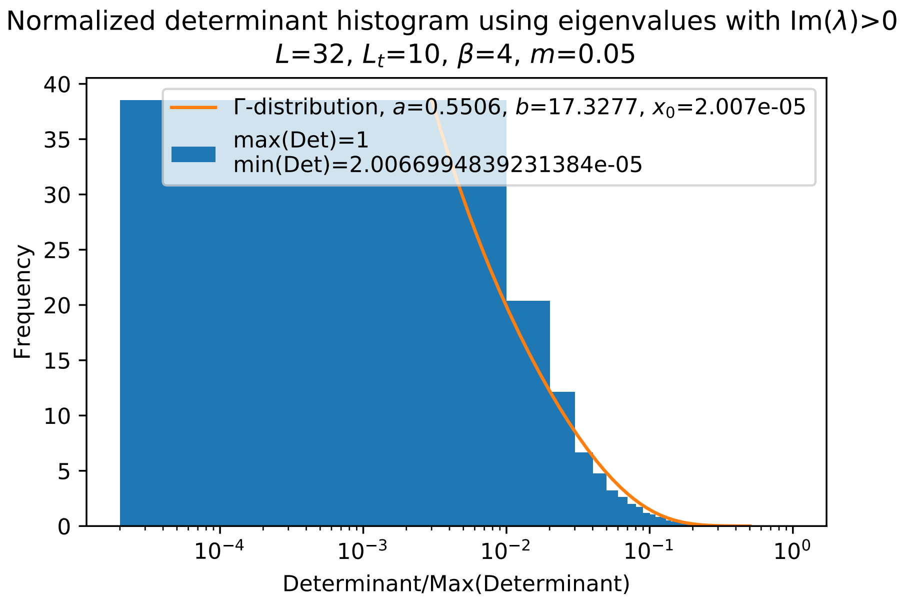
Figure 7: $\beta = 4$, $L = 40$. The horizontal axis of the histogram is in logarithmic scale.


Normalized determinant histogram using eigenvalues with Im(λ)>0.
L=32, L_t=8, β=4, m=0.05
Γ-distribution, a=0.4439, b=16.1283, x_0=4.147e-05
max(Det)=1
min(Det)=4.1469373653619166e-05
Frequency
Determinant/Max(Determinant)

(a)

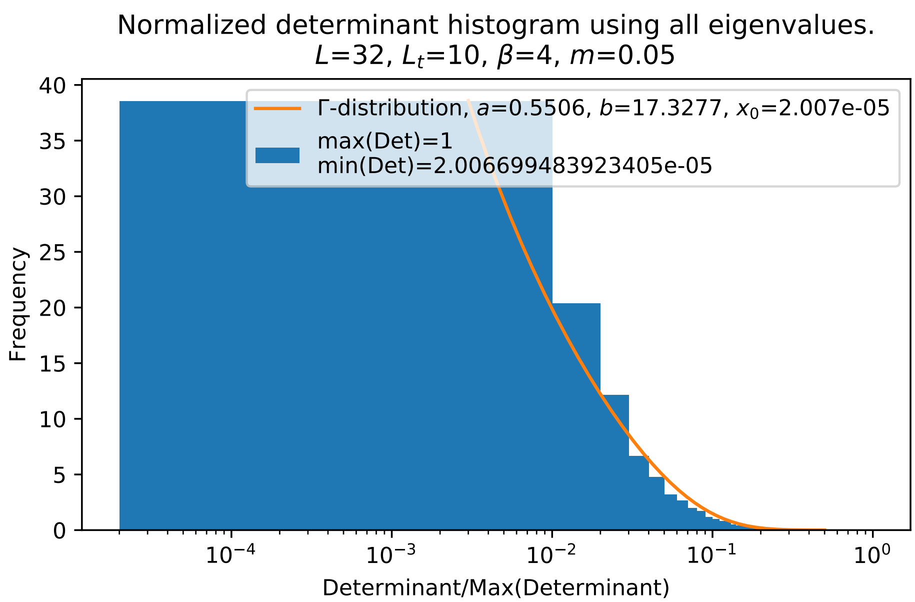

Normalized determinant histogram using all eigenvalues.
L=32, L_t=8, β=4, m=0.05
Γ-distribution, a=0.4439, b=16.1283, x_0=4.147e-05
max(Det)=1
min(Det)=4.1469373653621416e-05
Frequency
Determinant/Max(Determinant)

(b)



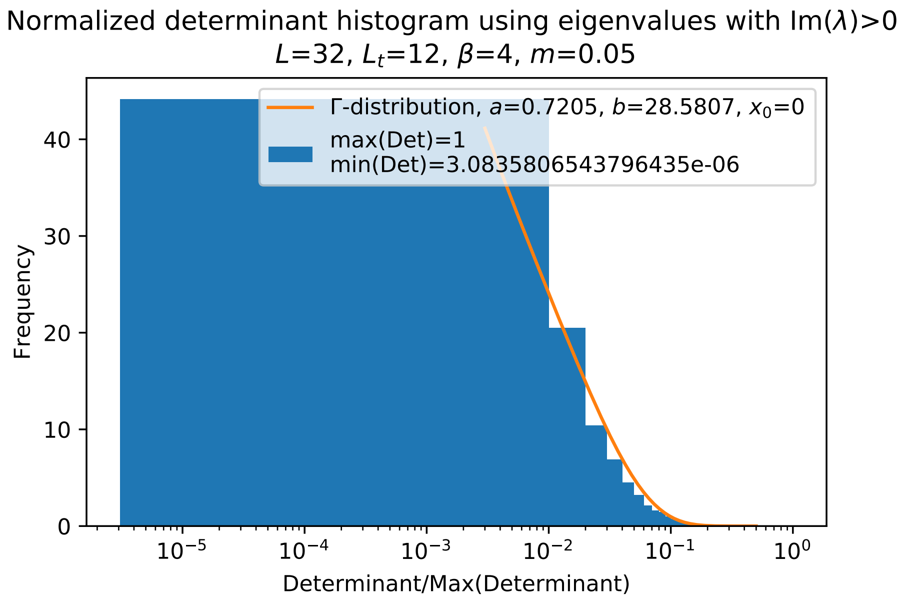
Normalized determinant histogram using eigenvalues with Im($\lambda$)>0.
$L$=32, $L_t$=10, $\beta$=4, $m$=0.05
Γ-distribution, $a$=0.5506, $b$=17.3277, $x_0$=2.007e-05
max(Det)=1
min(Det)=2.0066994839231384e-05
Frequency
Determinant/Max(Determinant)


(c)



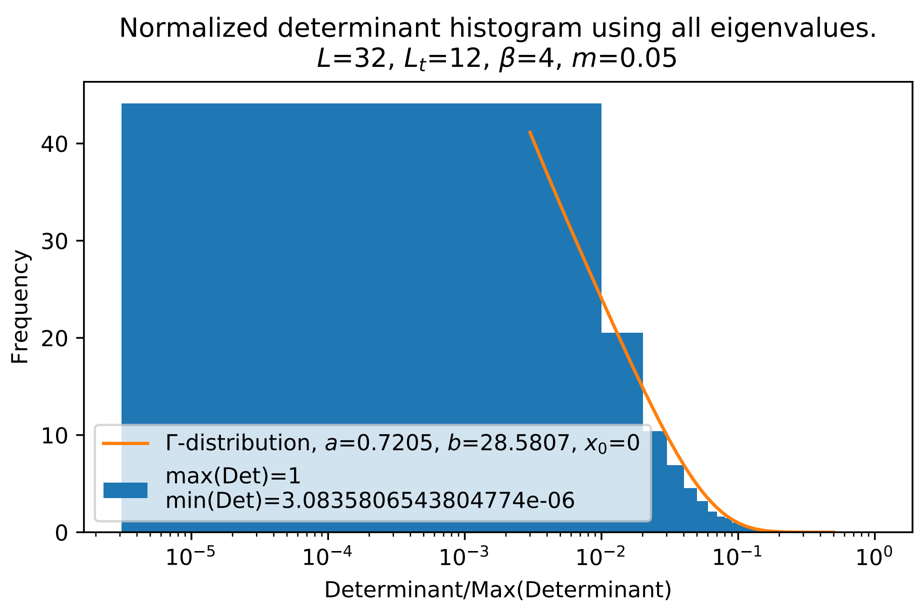
Normalized determinant histogram using all eigenvalues.
$L$=32, $L_t$=10, $\beta$=4, $m$=0.05
Γ-distribution, $a$=0.5506, $b$=17.3277, $x_0$=2.007e-05
max(Det)=1
min(Det)=2.006699483923405e-05
Frequency
Determinant/Max(Determinant)


(d)



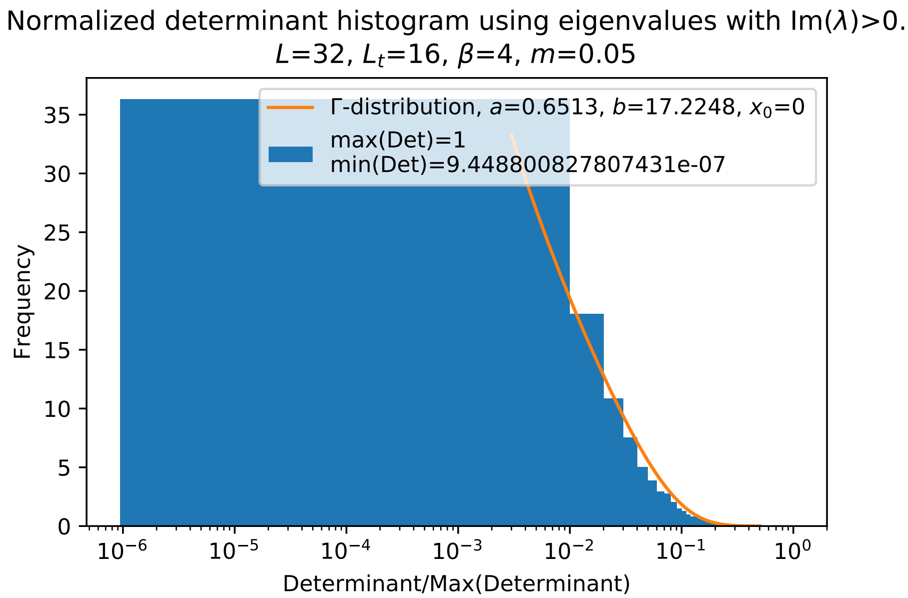
Normalized determinant histogram using eigenvalues with Im($\lambda$)>0.
$L$=32, $L_t$=12, $\beta$=4, $m$=0.05
Γ-distribution, $a$=0.7205, $b$=28.5807, $x_0$=0
max(Det)=1
min(Det)=3.0835806543796435e-06
Frequency
Determinant/Max(Determinant)


(e)

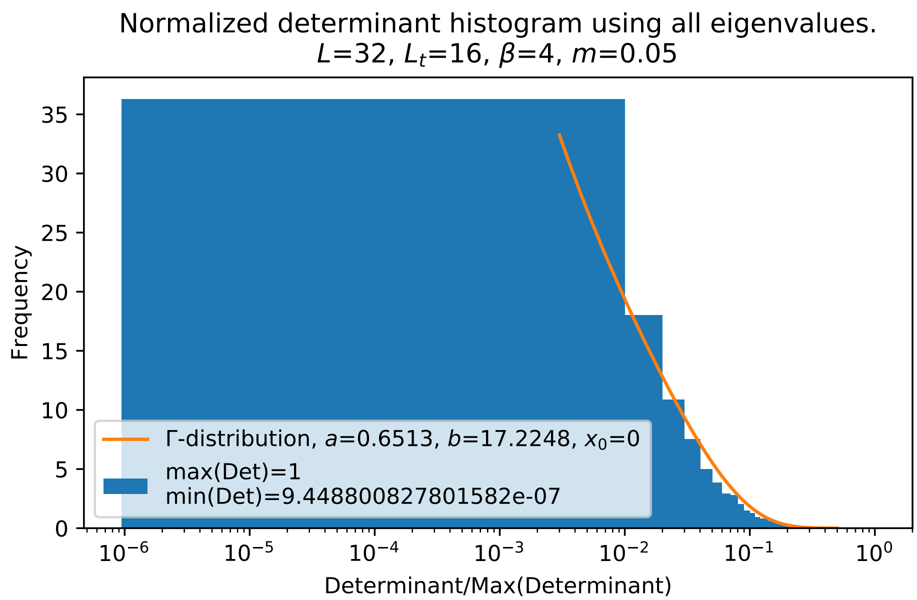


Normalized determinant histogram using all eigenvalues.
$L$=32, $L_t$=12, $\beta$=4, $m$=0.05
Γ-distribution, $a$=0.7205, $b$=28.5807, $x_0$=0
max(Det)=1
min(Det)=3.0835806543804774e-06
Frequency
Determinant/Max(Determinant)


(f)




(g)




(h)

Figure 8: $\beta = 4$, $L = 32$. The horizontal axis of the histogram is in logarithmic scale.