

Mass Reweighting

May 12, 2022

We show the pion mass as a function of $(m^2g)^{1/3}$. In order to extrapolate the pion mass we fitted a function of the form $m_\pi = \sqrt{a + bx^c}$, with $x = (m^2g)^{1/3}$ and a, b, c fitting parameters. We only fit the region $x > 0.076$.

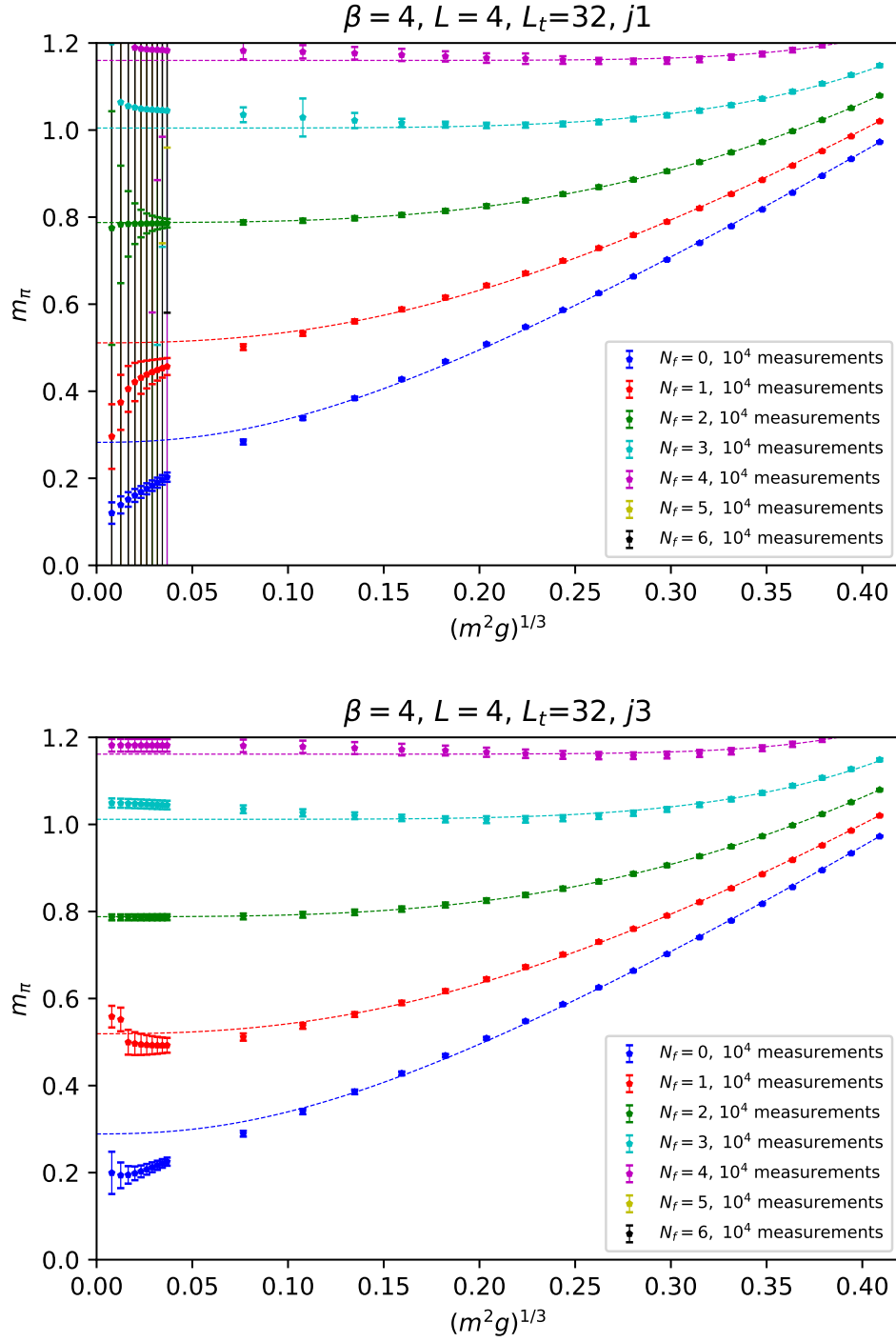


Figure 1: Pion mass as a function of the degenerate $(m^2 g)^{1/3}$ for different flavors. $L = 4$, $L_t = 32$.

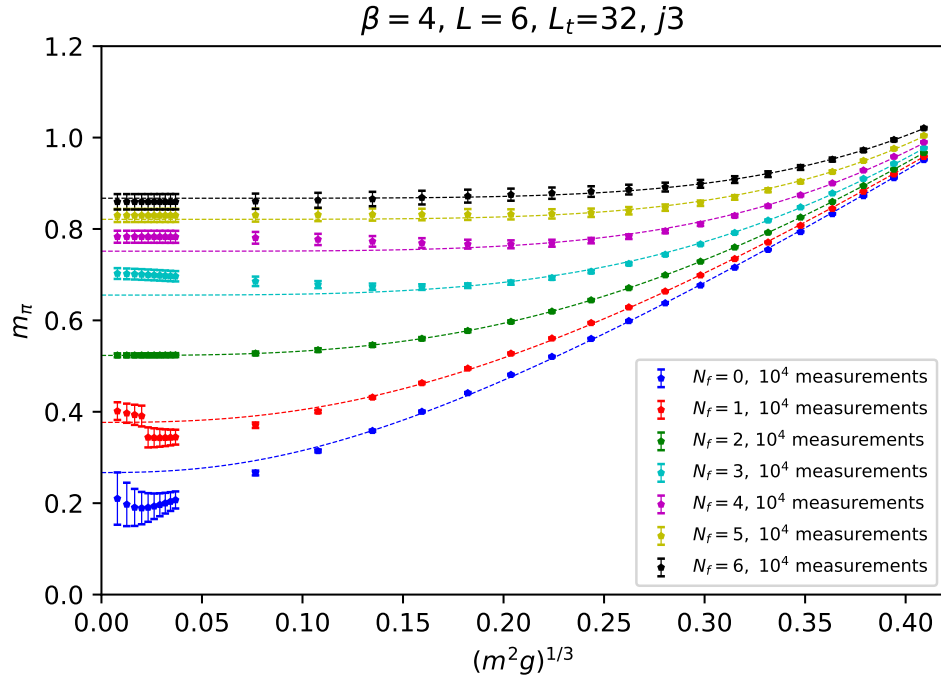
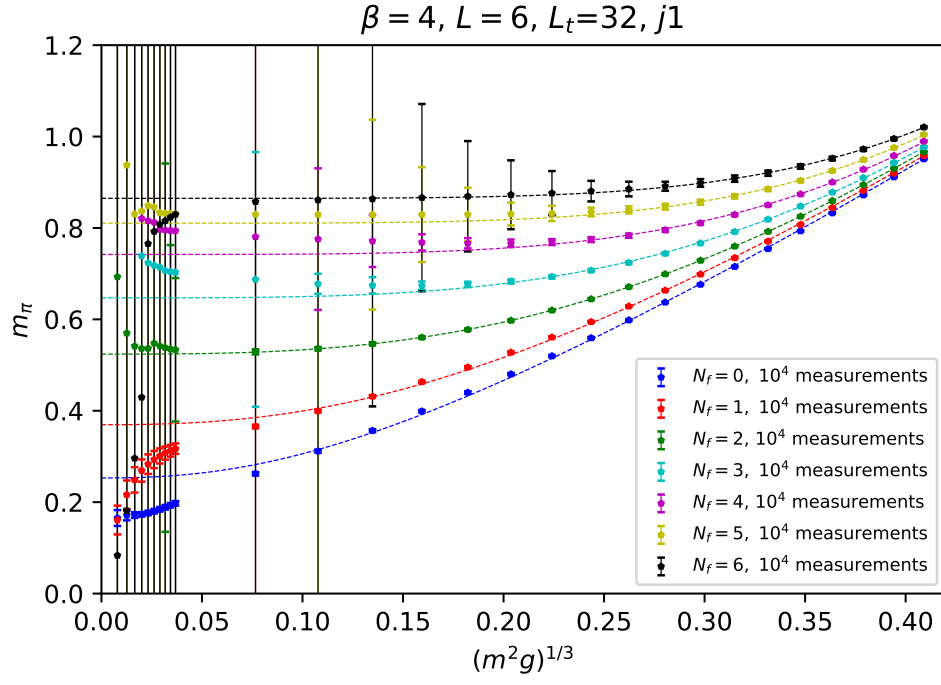


Figure 2: Pion mass as a function of the degenerate $(m^2 g)^{1/3}$ for different flavors. $L = 6$, $L_t = 32$.

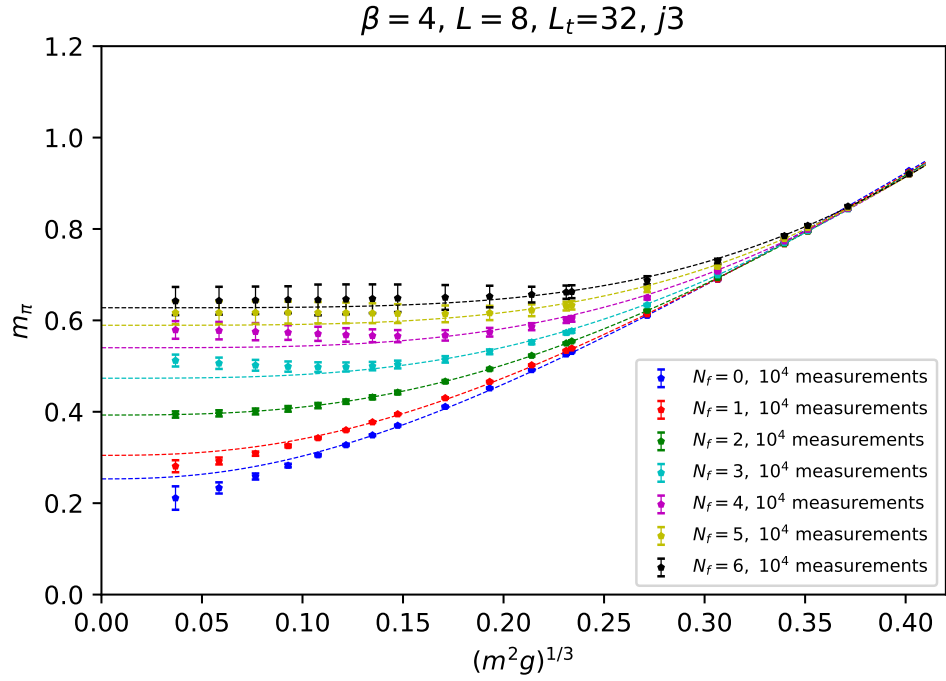
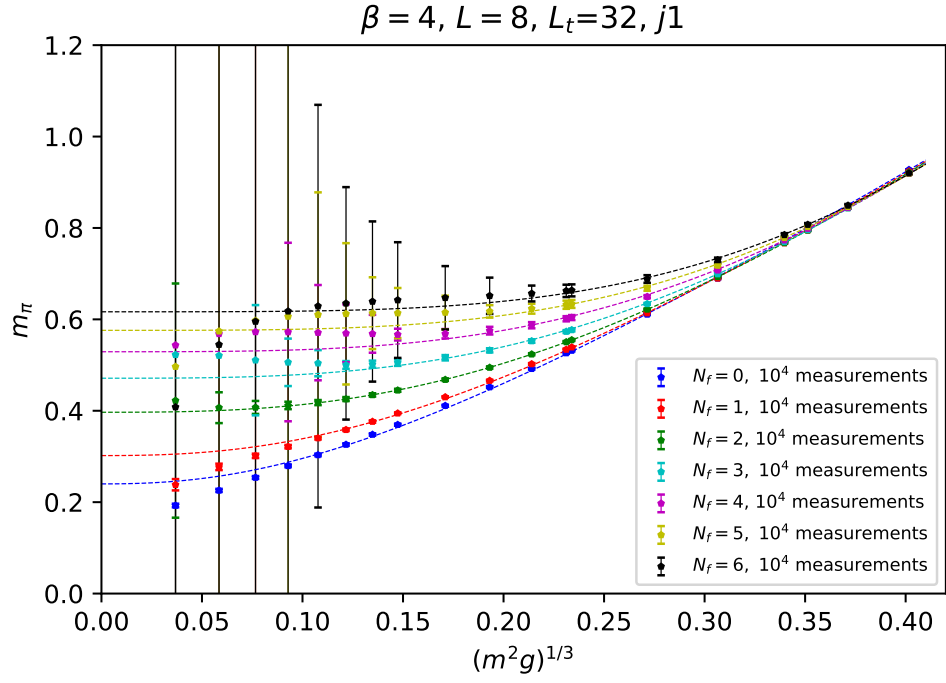


Figure 3: Pion mass as a function of the degenerate $(m^2 g)^{1/3}$ for different flavors. $L = 8, L_t = 32$.

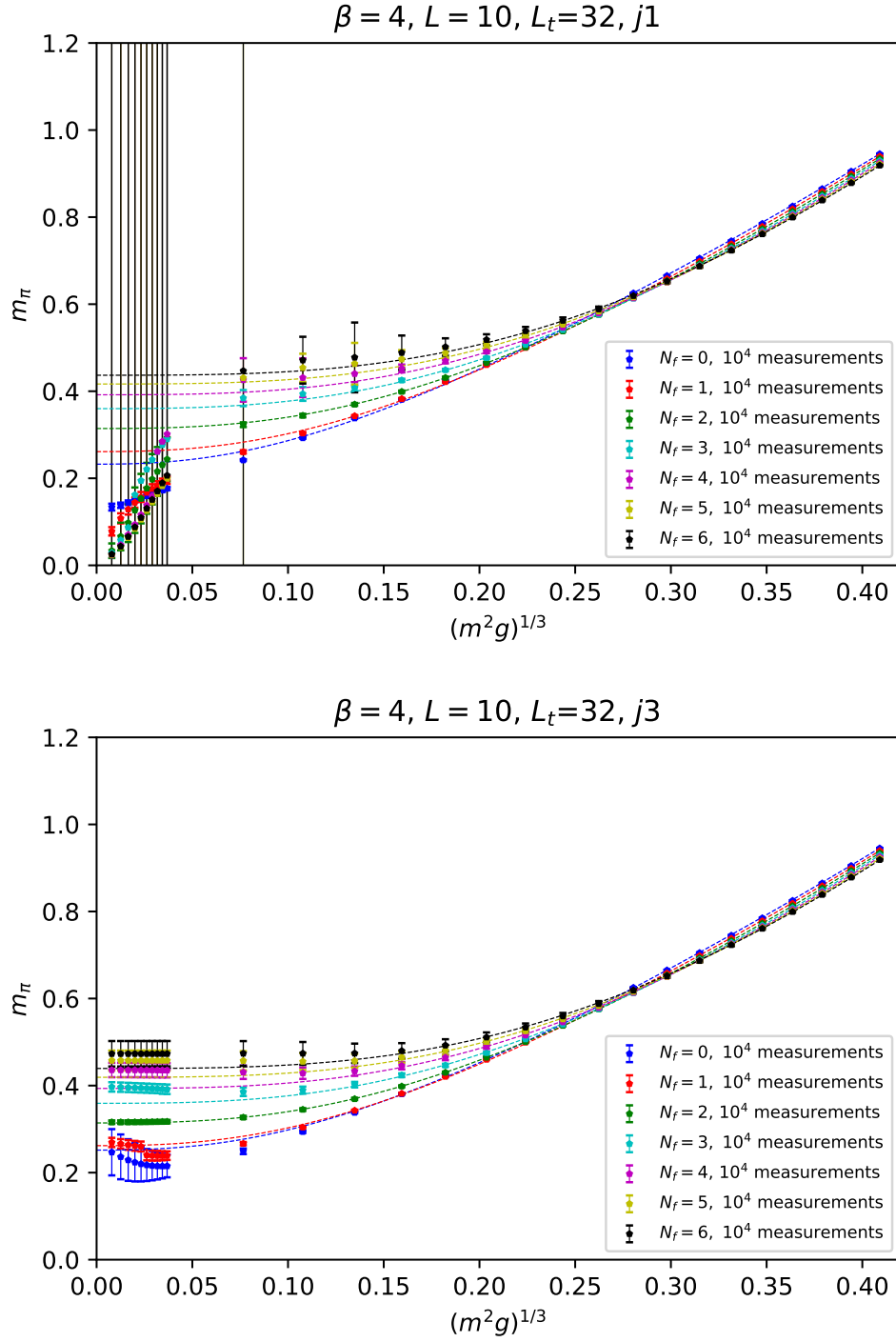


Figure 4: Pion mass as a function of the degenerate $(m^2 g)^{1/3}$ for different flavors. $L = 10, L_t = 32$.

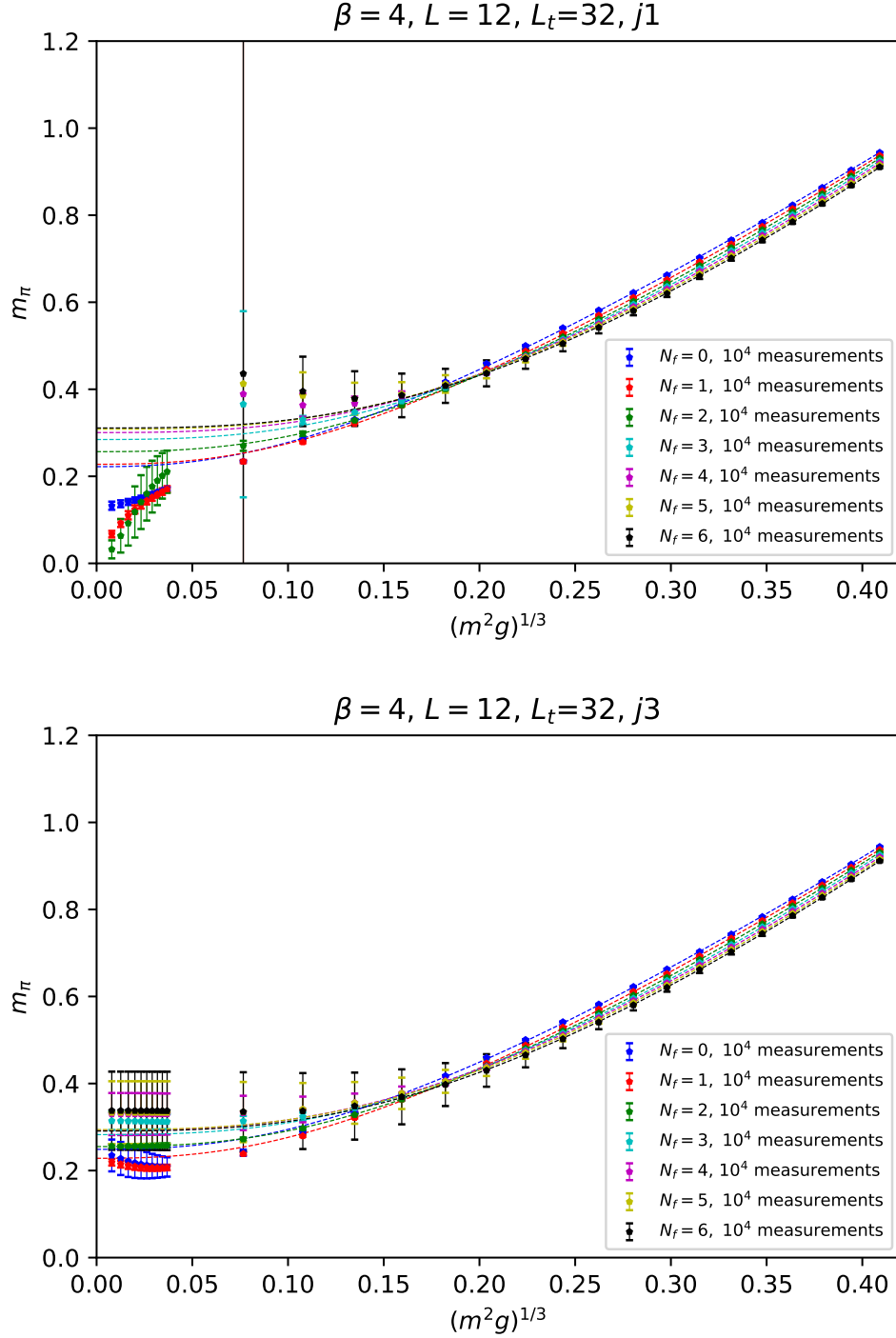


Figure 5: Pion mass as a function of the degenerate $(m^2 g)^{1/3}$ for different flavors. $L = 12, L_t = 32$.

1 Pion decay constant in the delta-regime

We show the residual pion mass m_π^R as a function of the spatial size L and fit a function proportional to $1/L$ to obtain F_π .

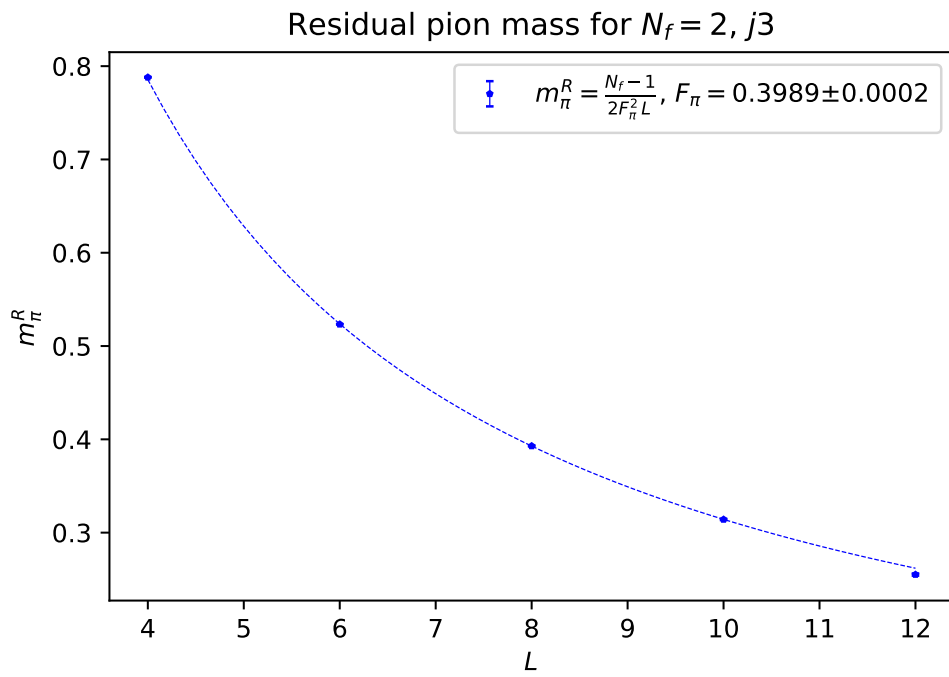
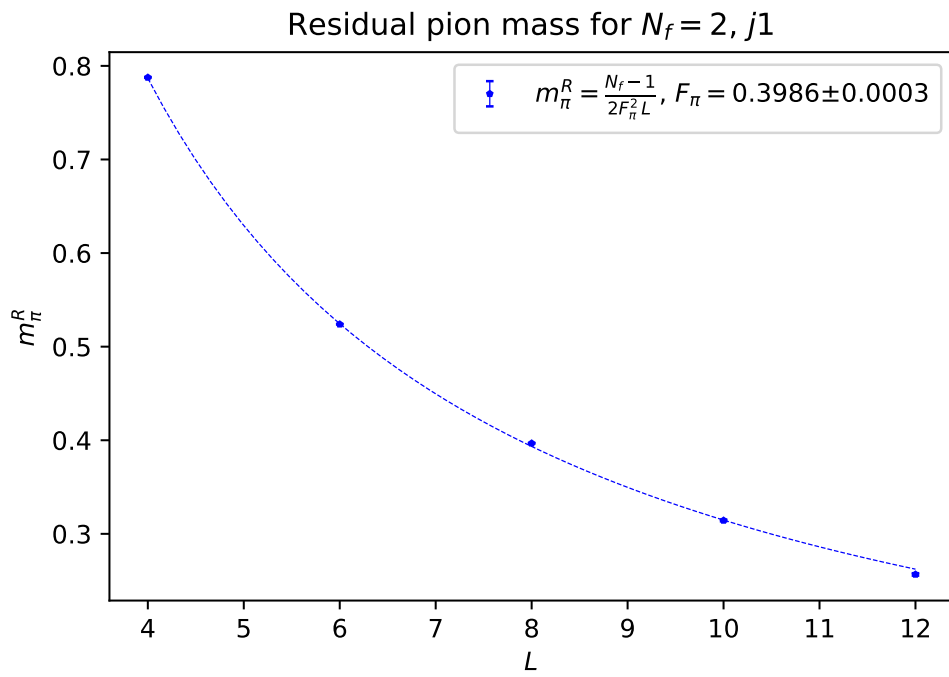


Figure 6: $N_f = 2; 1/\sqrt{2\pi} = 0.39894 \dots$

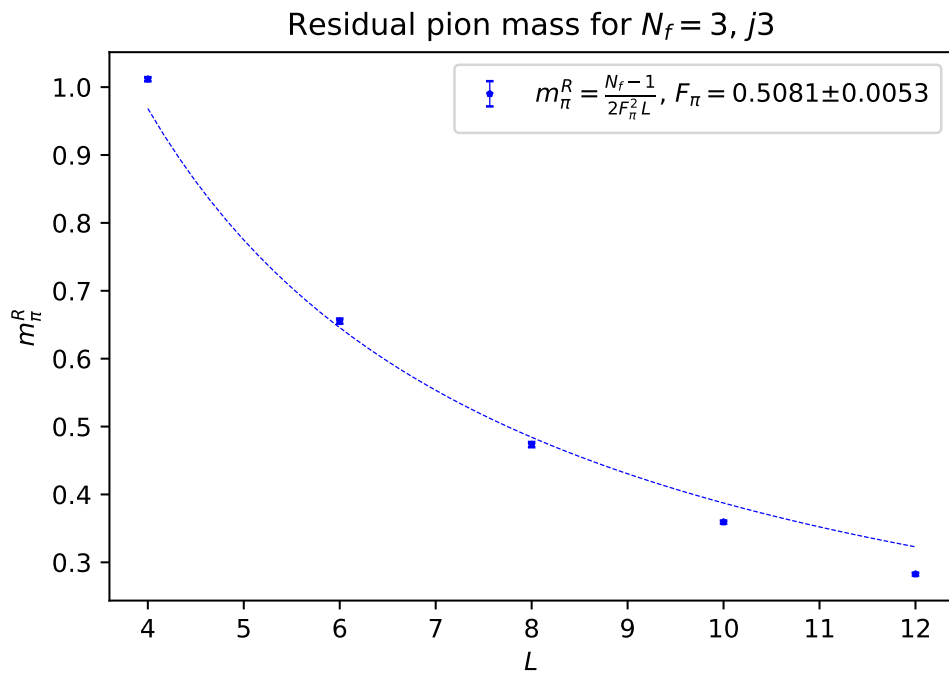
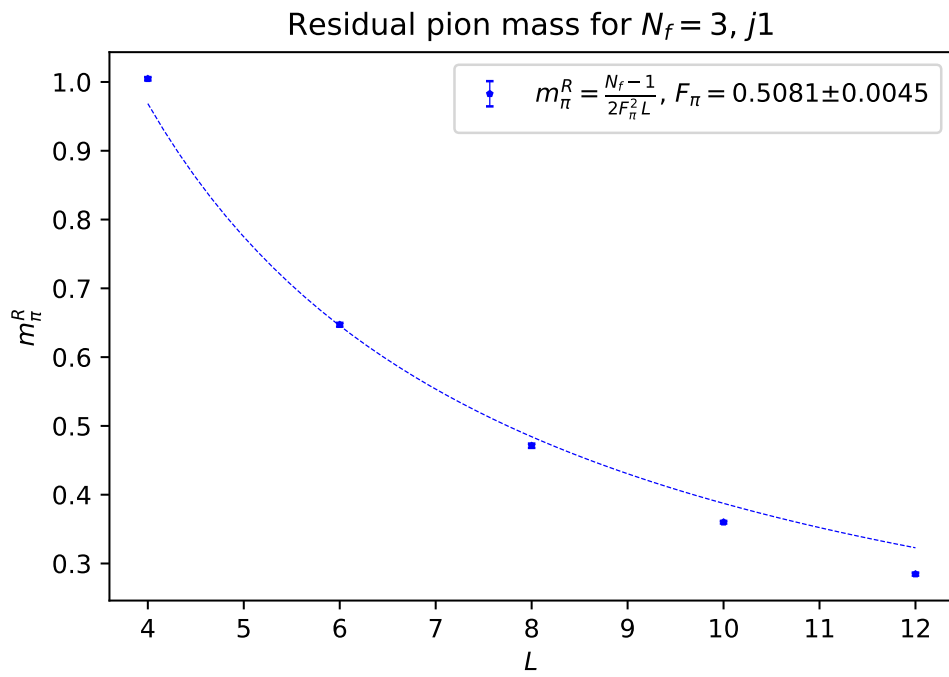


Figure 7: $N_f = 3$

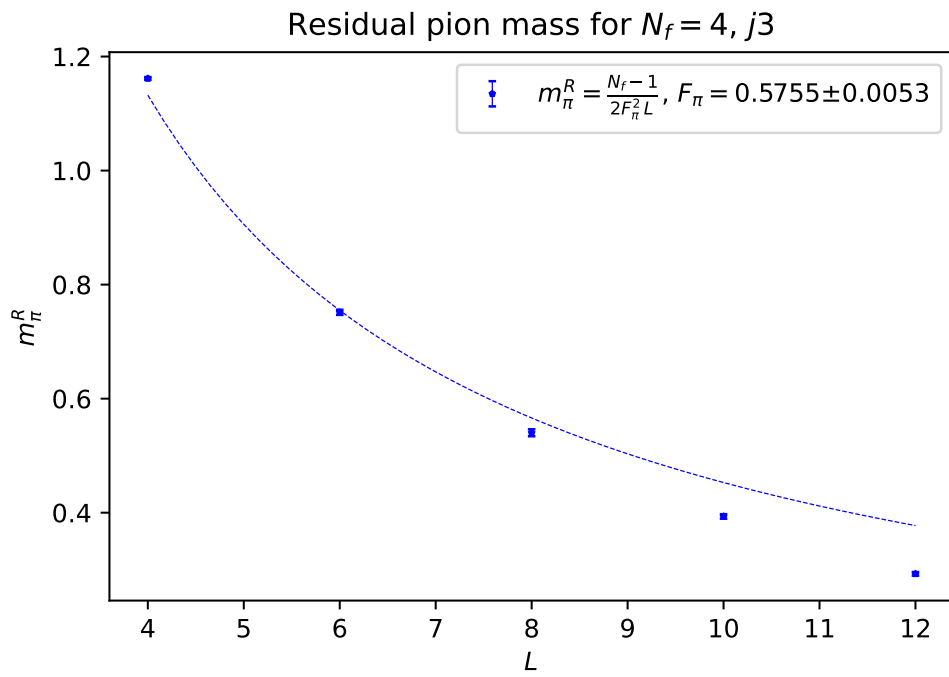
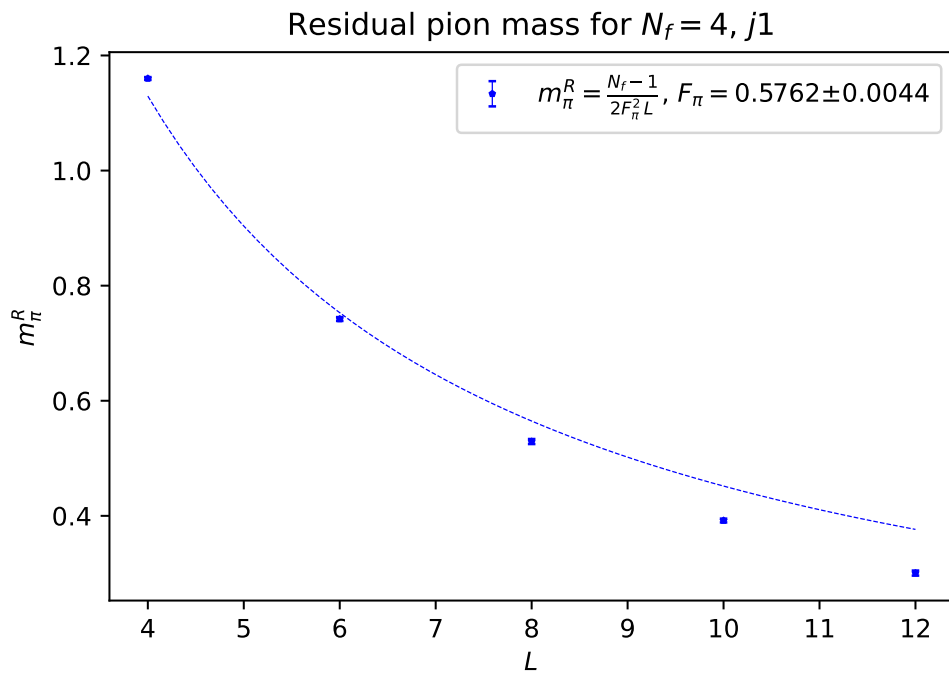


Figure 8: $N_f = 4$

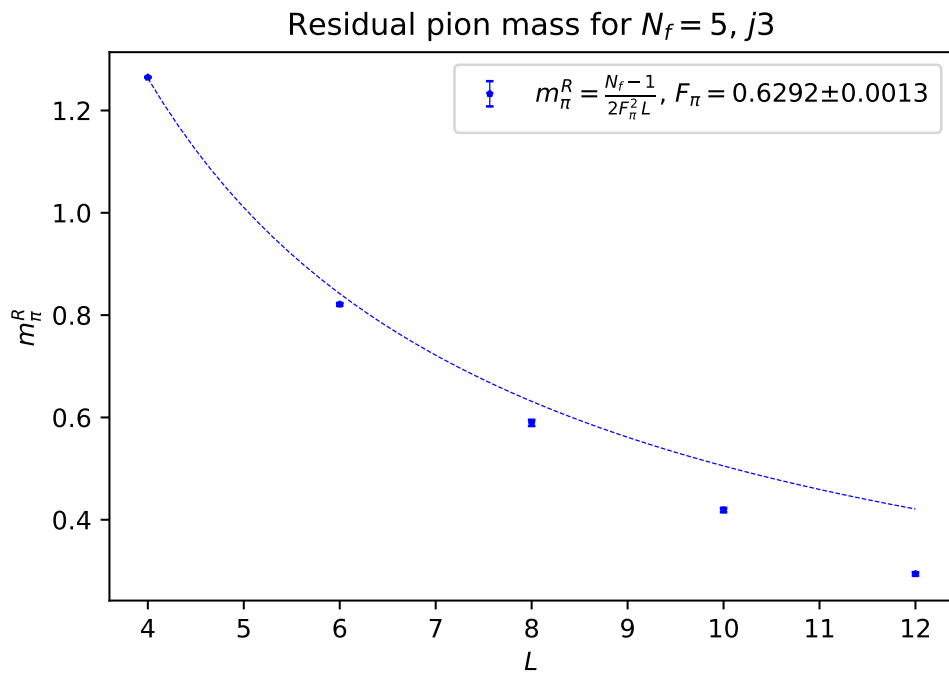
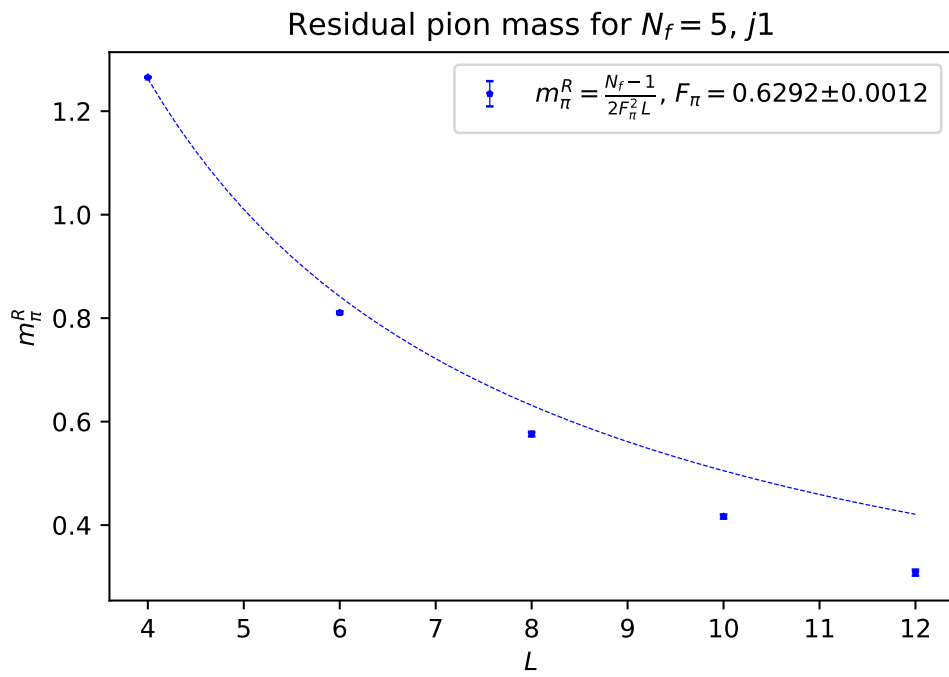


Figure 9: $N_f = 5$

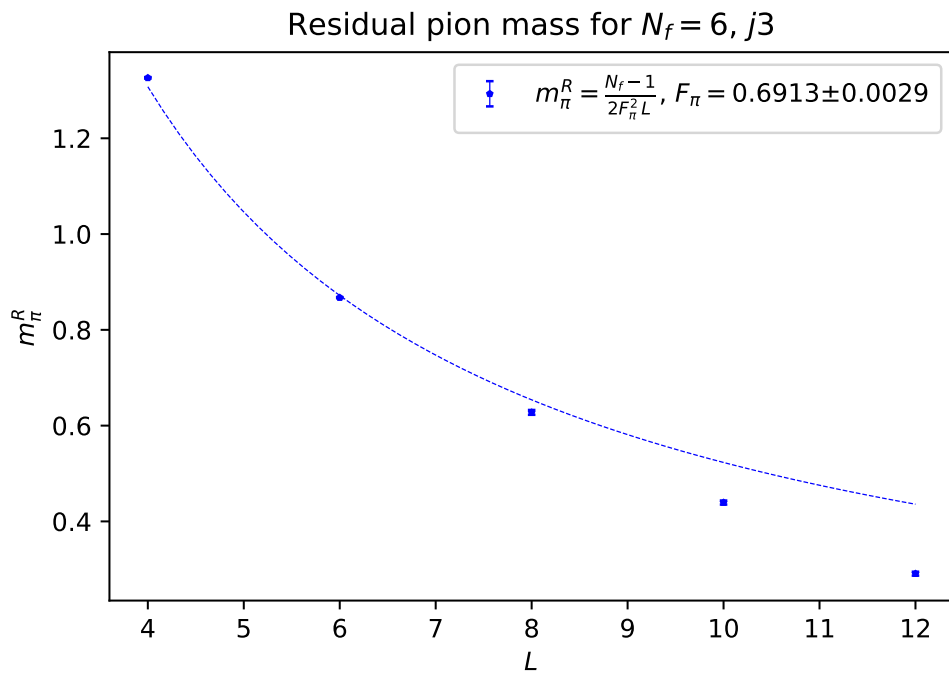
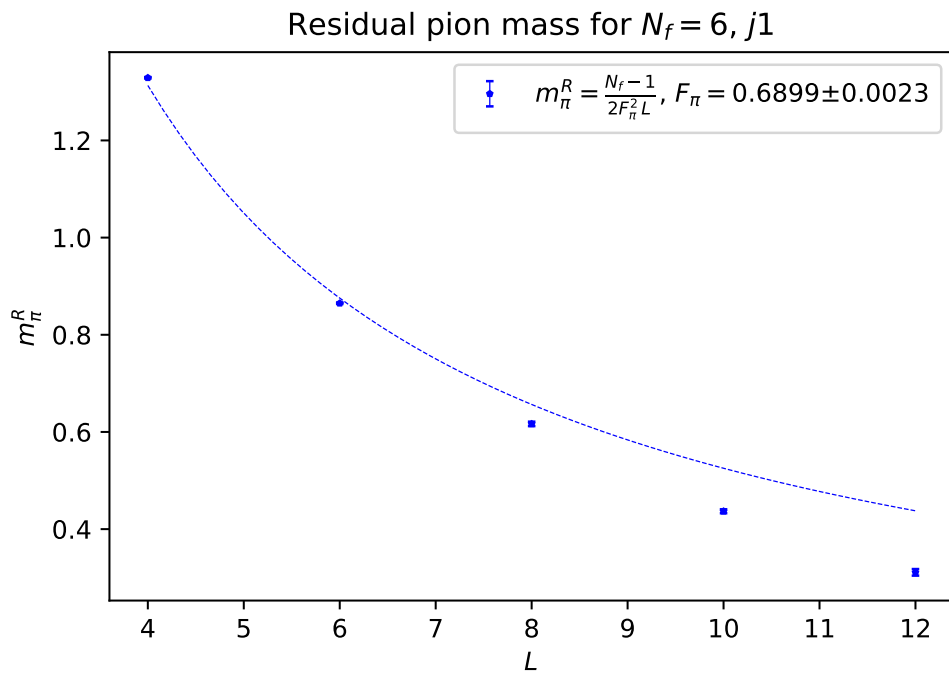
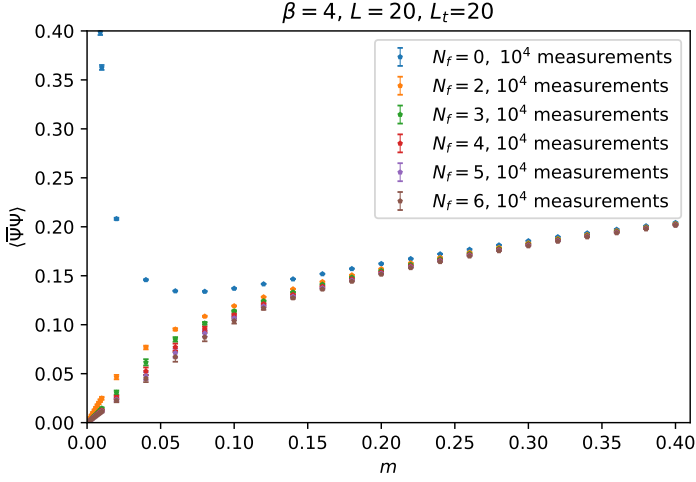


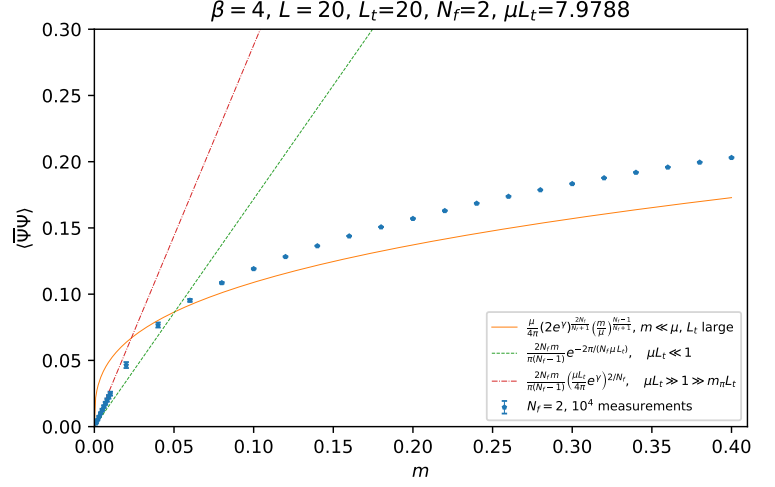
Figure 10: $N_f = 6$

2 Chiral condensate at finite temperature

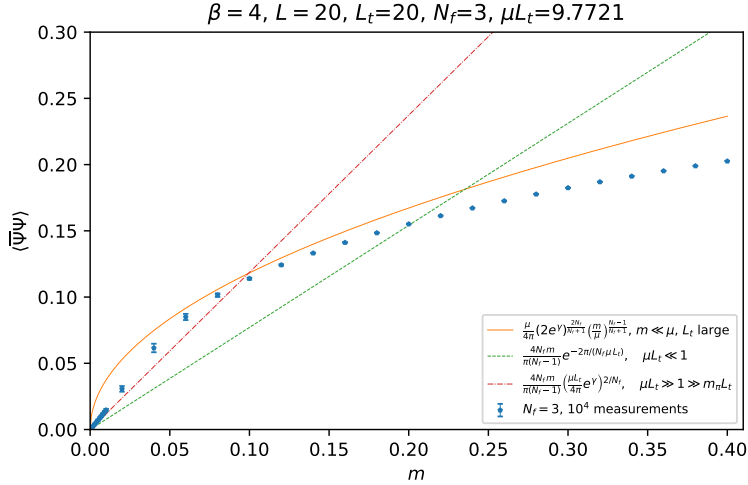
We show the chiral condensate for several lattices together with some predictions, valid in different regimes, for N_f flavors that are written in eqs. (13), (15) and (16) of ref. [1].



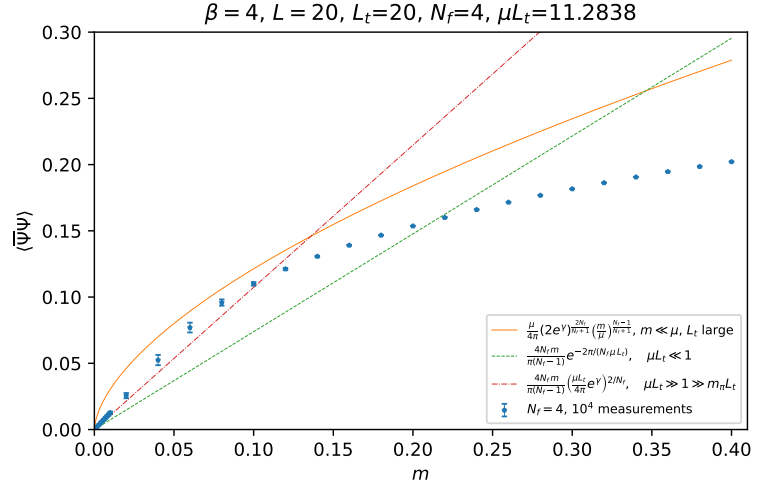
(a)



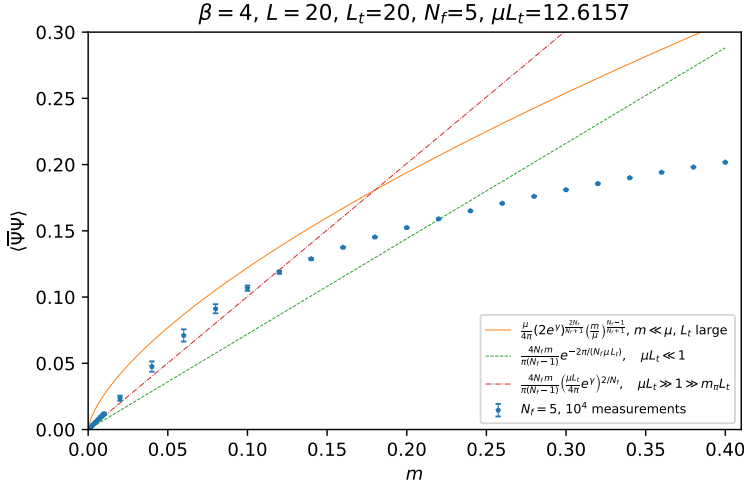
(b)



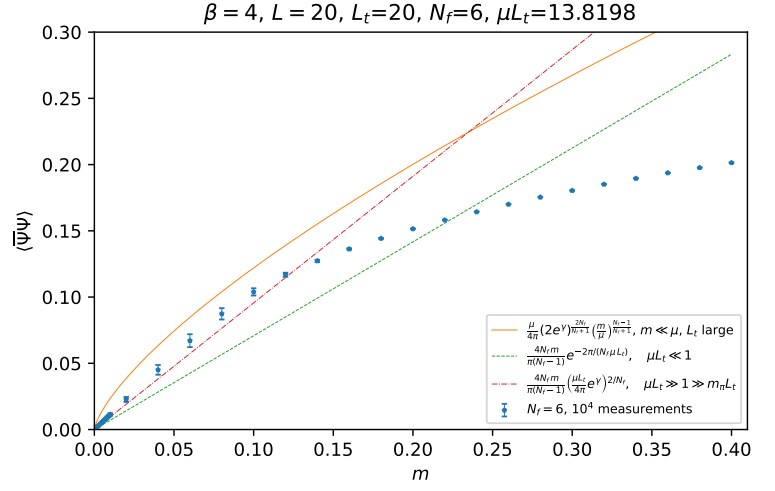
(c)



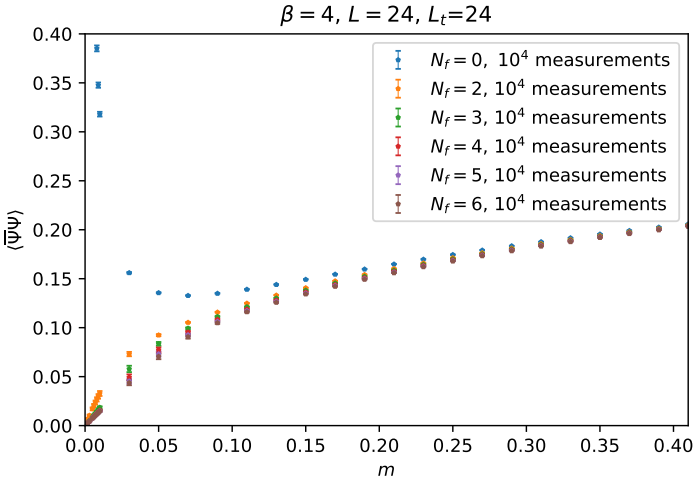
(d)



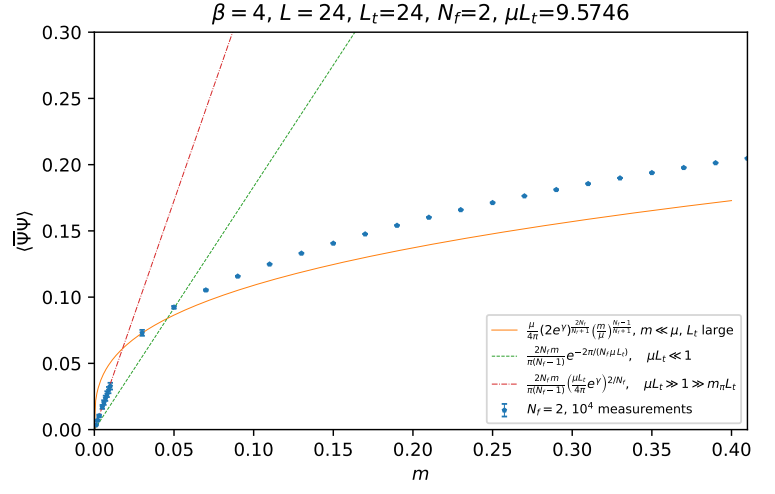
(e)



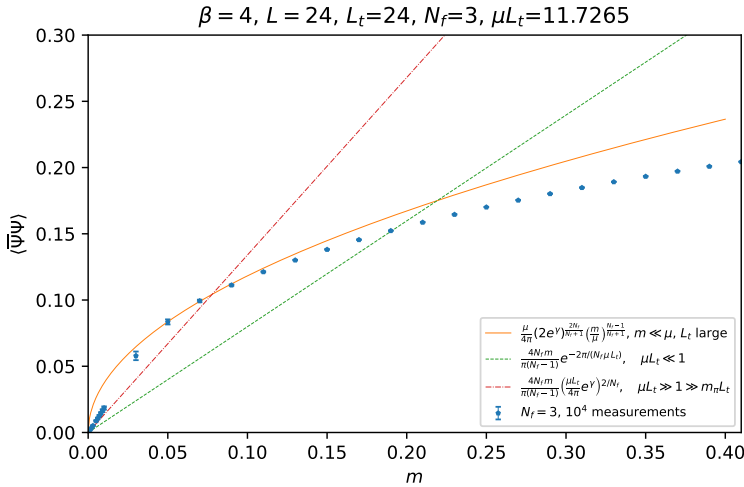
(f)

Figure 11: Chiral condensate as a function of the degenerate fermion mass for different flavors. $L = 20, L_t = 20$ 

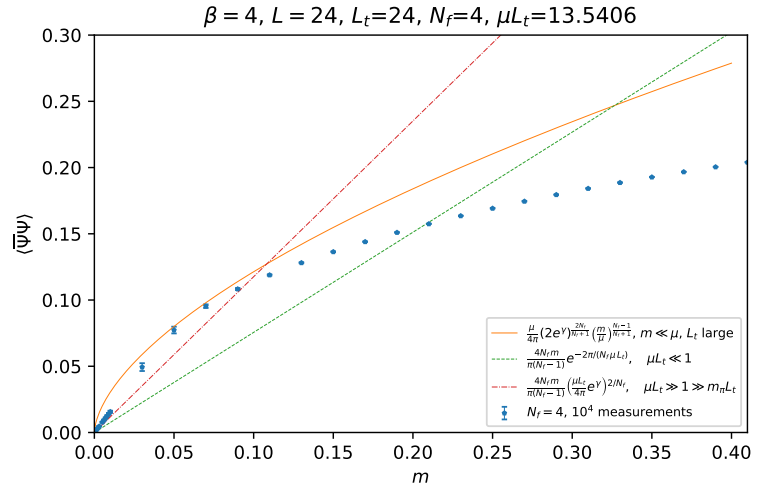
(a)



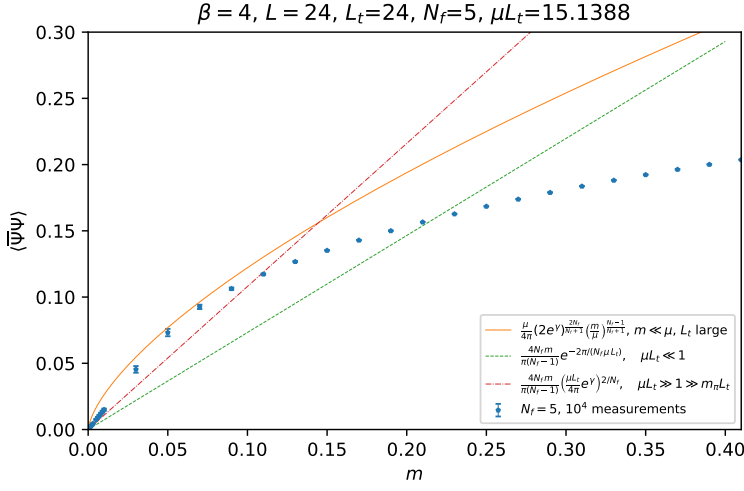
(b)



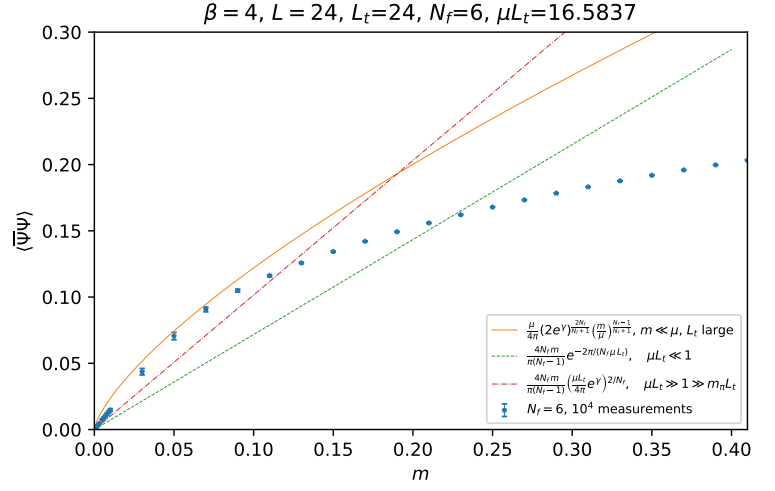
(c)



(d)



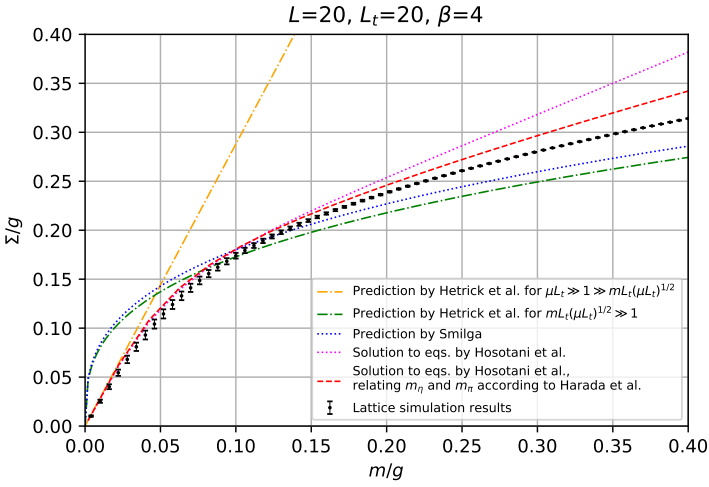
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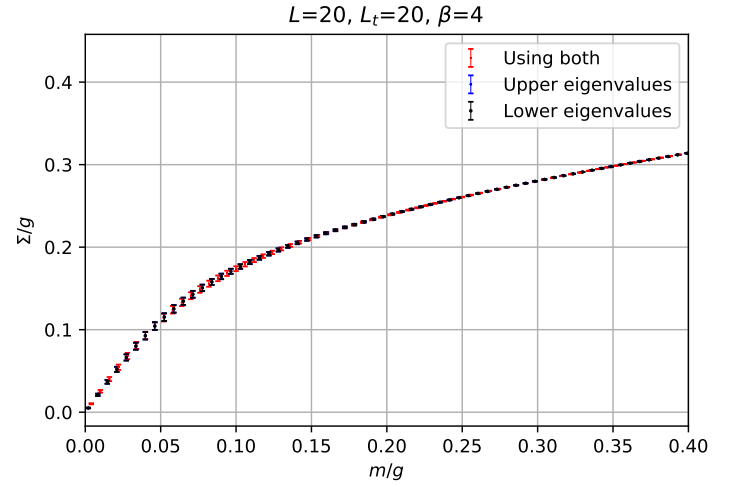
(f)

Figure 12: Chiral condensate as a function of the degenerate fermion mass for different flavors. $L = 24, L_t = 24$

2.1 Numerical stability check

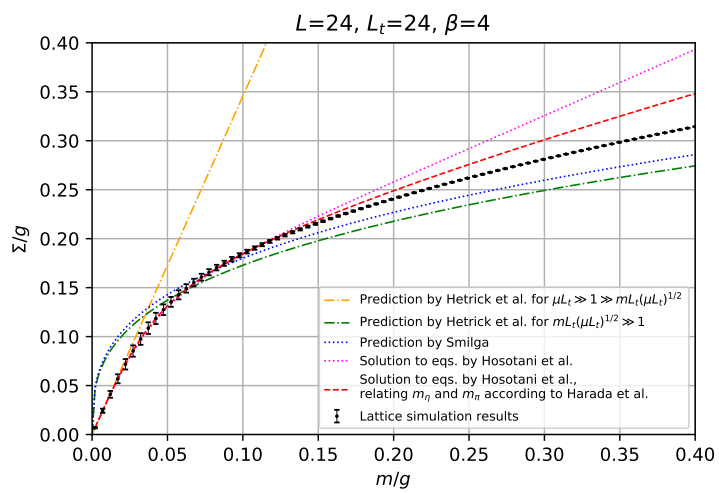


(a)

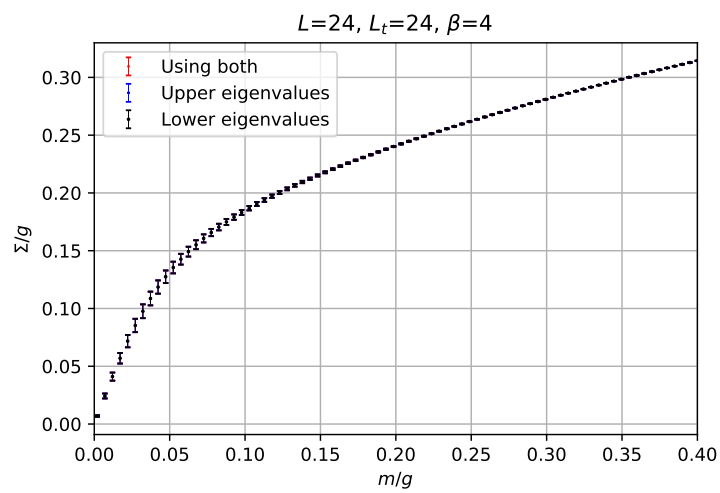


(b)

Figure 13: $L = 20, L_t = 20$



(a)



(b)

Figure 14: $L = 24, L_t = 24$

3 Pion decay constant at finite temperature

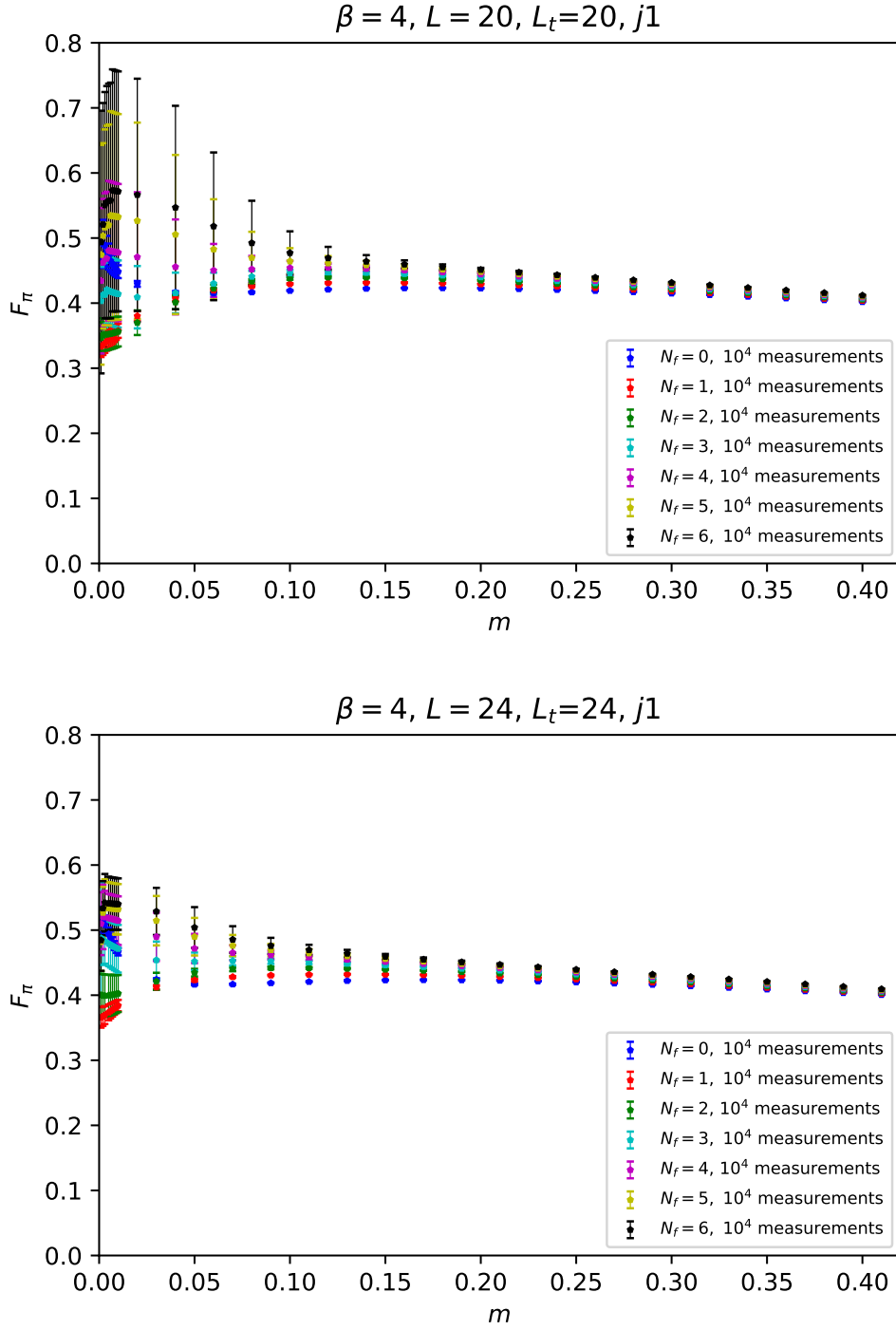


Figure 15: F_π for different flavors assuming the GMOR relation.

| N_f | F_π Leut. form. j_1 | F_π Leut. form. j_3 | F_π magic form. j_1 | F_π magic form. j_3 | F_π finite T |
|-------|---------------------------|---------------------------|---------------------------|---------------------------|--------------------|
| 2 | 0.3986(3) | 0.3989(2) | 0.3986(3) | 0.3989(2) | 0.4004(361) |
| 3 | 0.5081(45) | 0.5081(53) | 0.4148 | 0.4148 | 0.4877(856) |
| 4 | 0.5762(44) | 0.5755(53) | 0.4074 | 0.4069 | 0.5099(483) |
| 5 | 0.6292(12) | 0.6292(13) | 0.3979 | 0.3979 | 0.4819(447) |
| 6 | 0.6899(23) | 0.6913(29) | 0.3983 | 0.3991 | 0.4845(471) |

Table 1: F_π measured with different methods and for different flavors. Leutwyler’s formula stands for $m_\pi^R = (N_f - 1)/2F_\pi^2 L$, while magic formula refers to $m_\pi^R = (N_f - 1)/N_f F_\pi^2 L$. The finite temperature results correspond to the value of F_π at $m = 0.001$ in figure 15, for the 24×24 lattice.

References

- [1] J. E. Hetrick, Y. Hosotani, and S. Iso. Interplay between mass, volume, vacuum angle and chiral condensate in N avor QED in two-dimensions. *Phys. Rev. D*, **53**, 1996. arXiv:hep-th/9510090.