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Results for $\beta = 2.0$ are shown. Everything was obtained with simulations using the parameters of the following table:

Ntime	64
Ntherm	1000
Nmeasure	1000
Trajectory Steps	10
Nsteps	10
β	2

Table 1: All the simulations were performed with this parameters.

The residual pion mass is extrapolated with two different methods. From the plots of m_π^2 vs. m_{pcac} ona can fit a parabola

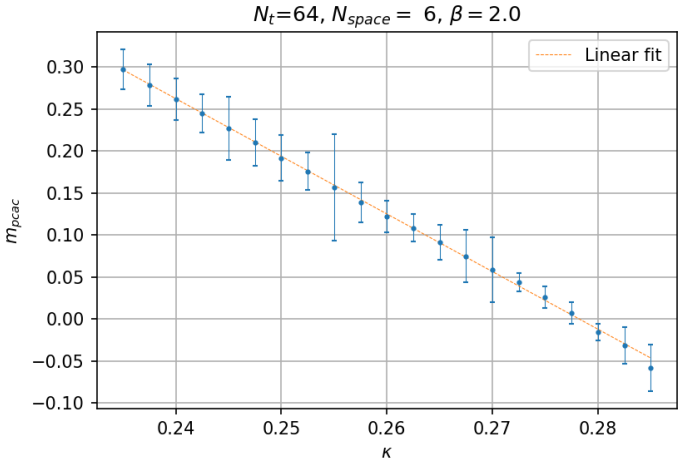
$$y = a + bx^2. \tag{1}$$

Meanwhile, from the plots of m_π vs. $(gm_{pcac}^2)^{1/3}$ one can fit a function of the form

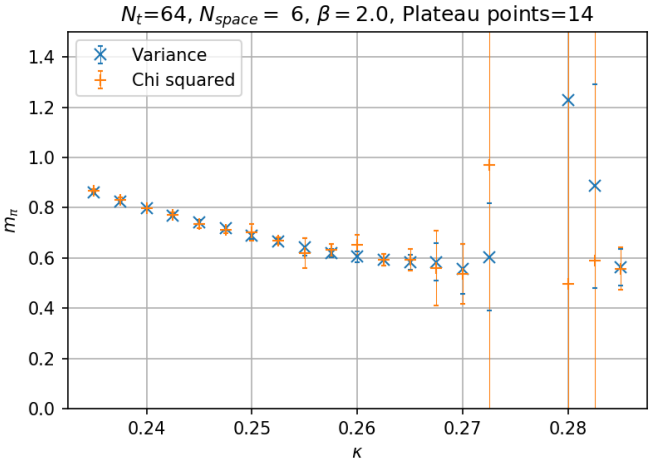
$$y = \sqrt{a + bx^3}. \tag{2}$$

$\beta = 2.0$ results

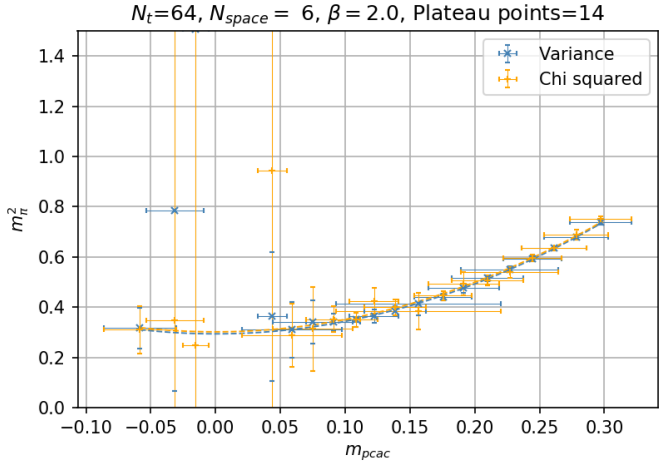
6x64



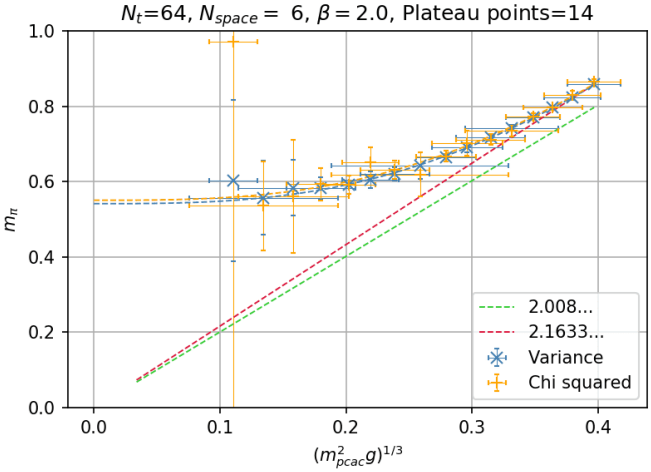
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27824 \pm 0.00307$



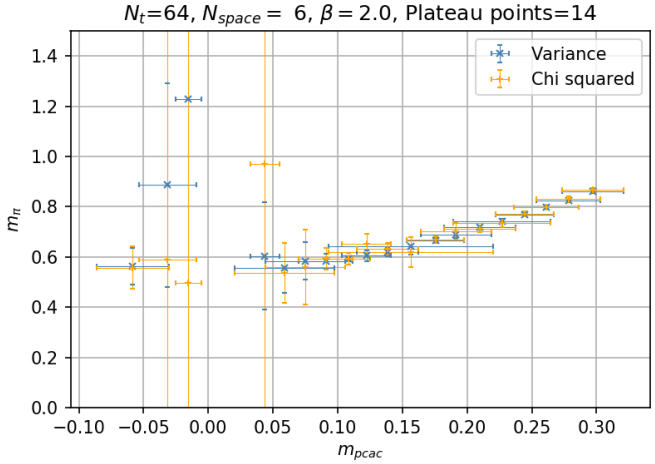
(b) Pion mass as a function of κ

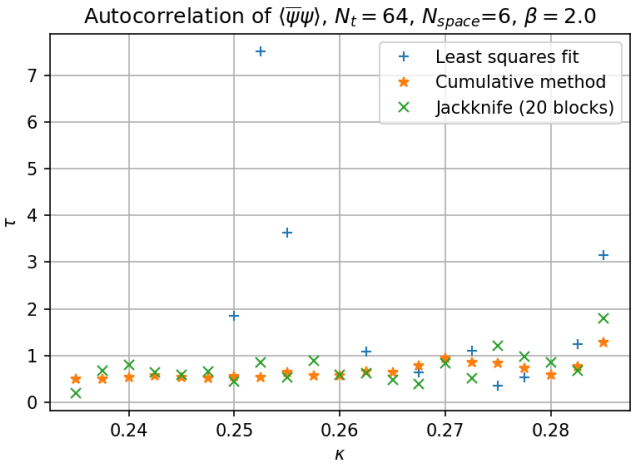


(c) m_π^2 vs. m_{pcac}. m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.29377 ± 0.00403, b =4.96973 ± 0.11092, m_π =0.54201 ± 0.00372 for variance and a =0.30172 ± 0.00672, b =4.93474 ± 0.16813, m_π =0.54929 ± 0.00612 for chi squared.

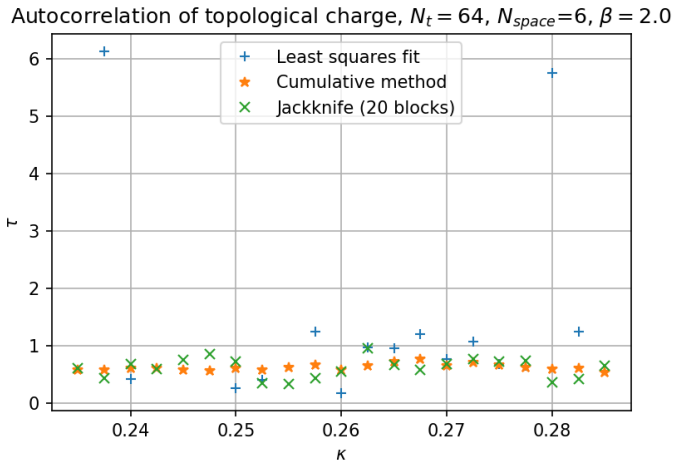


(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.29312 ± 0.00239, b =7.04747 ± 0.09194, m_π =0.54141 ± 0.00221 for variance and a =0.303 ± 0.00769, b =6.94703 ± 0.26847, m_π =0.55045 ± 0.00698 for chi squared.





(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

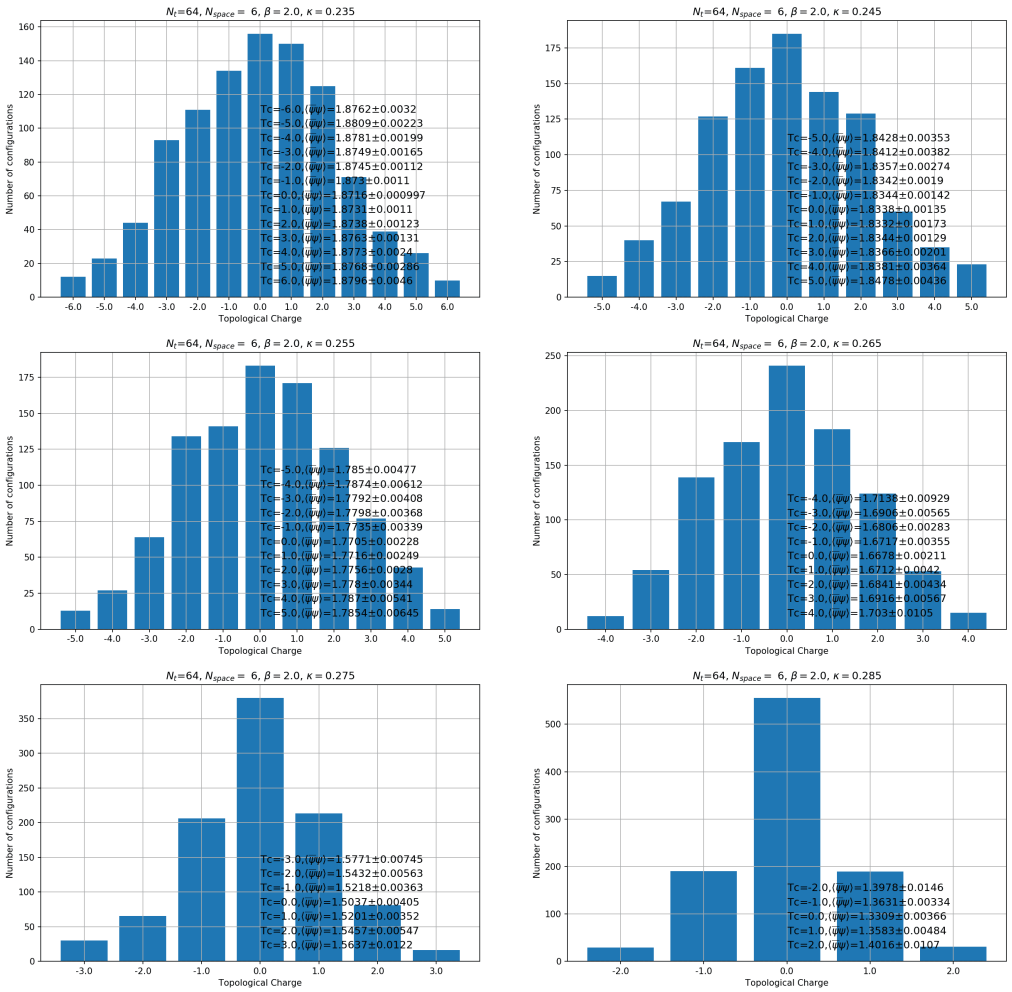
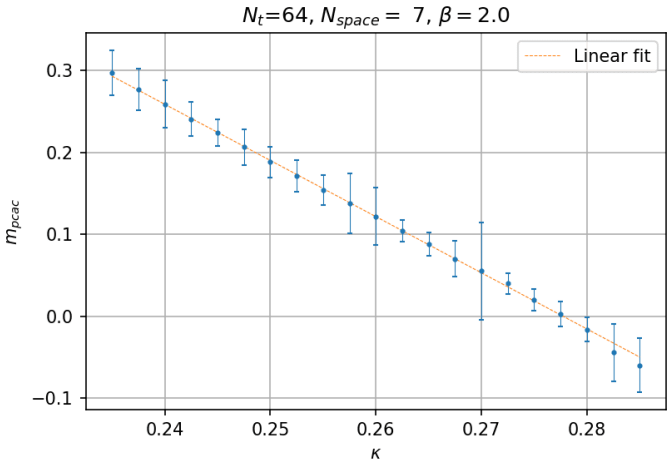
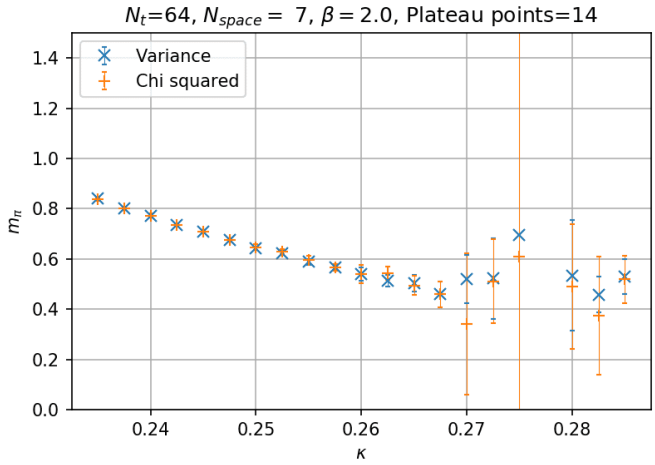


Figure 1: Number of configurations vs. topological charge on a 6×64 lattice.

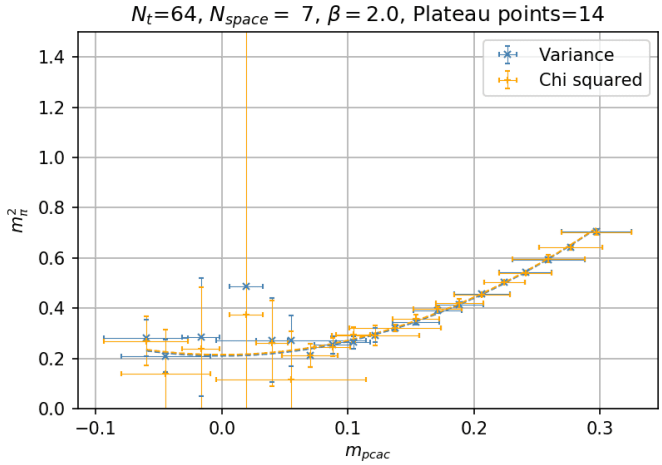
7x64



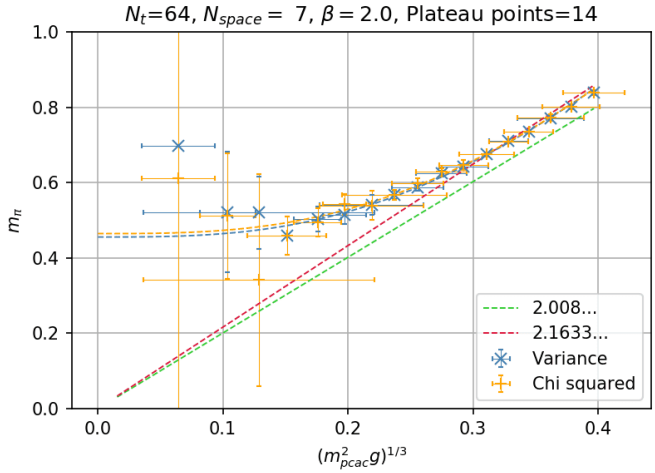
(a) Fermion mass using PCAC relation, $\kappa_c = 0.2777 \pm 0.00236$.



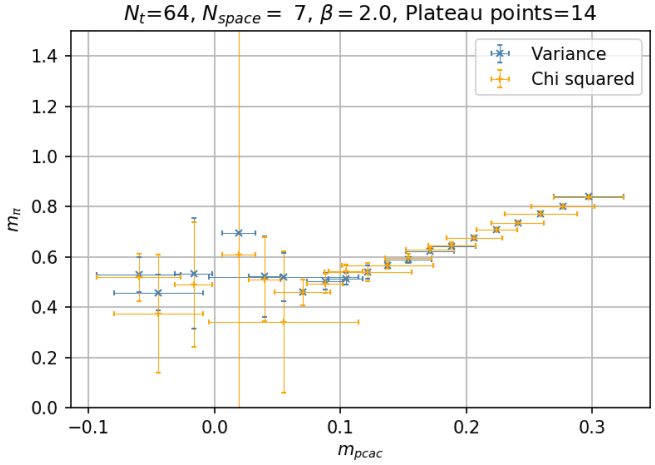
(b) Pion mass as a function of κ



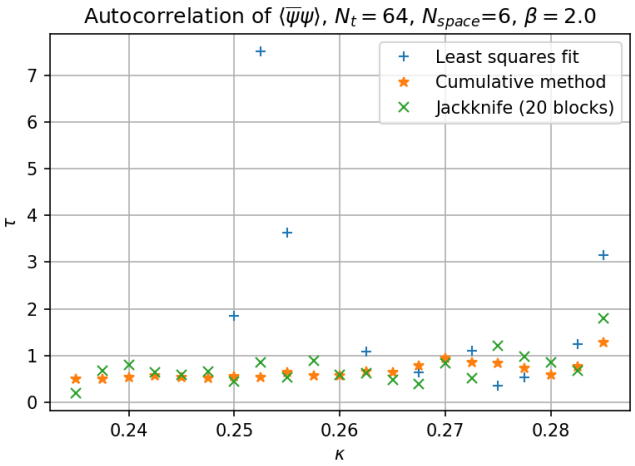
(c) m_π^2 vs. m_{pcac}. m_π^2 vs. m_{pcac}. m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.20986 ± 0.00523, b =5.7568 ± 0.15985, m_π =0.45811 ± 0.00571 for variance and a =0.2147 ± 0.00684, b =5.72429 ± 0.19724, m_π =0.46336 ± 0.00738 for chi squared.



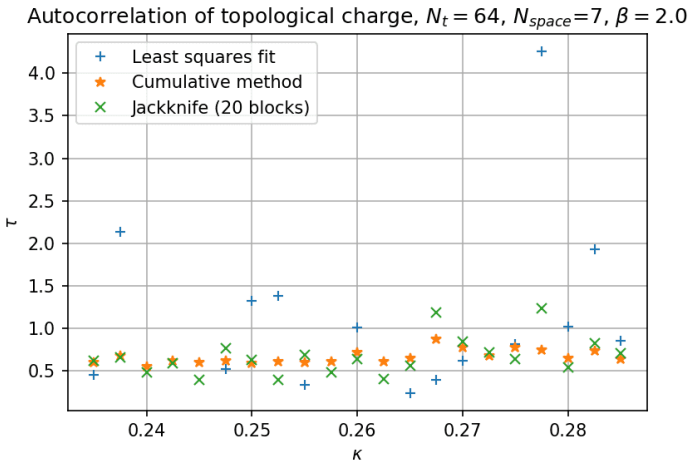
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.20749 ± 0.00464, b =8.21704 ± 0.19521, m_π =0.45551 ± 0.0051 for variance and a =0.21579 ± 0.00667, b =8.06097 ± 0.26745, m_π =0.46453 ± 0.00718 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\bar{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

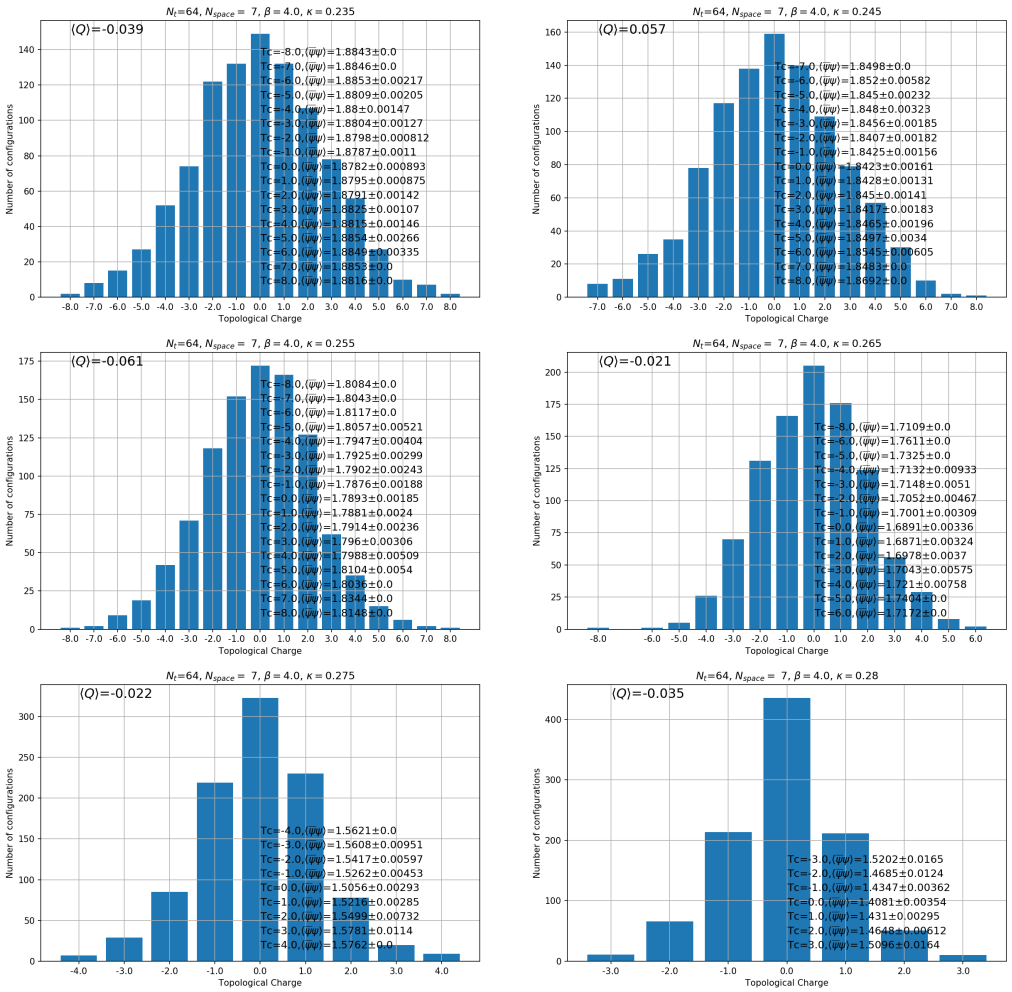
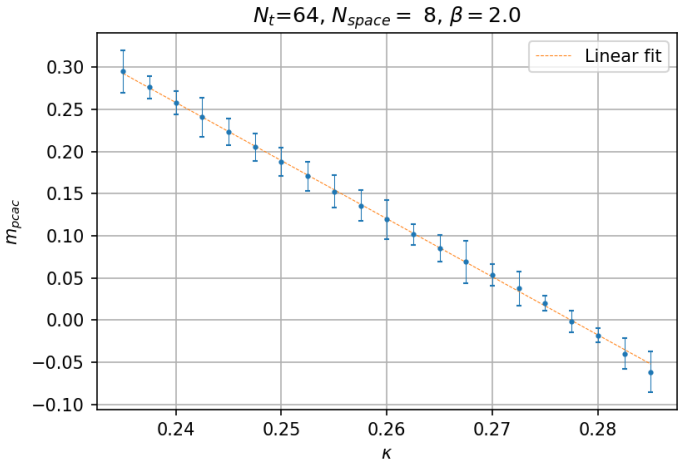
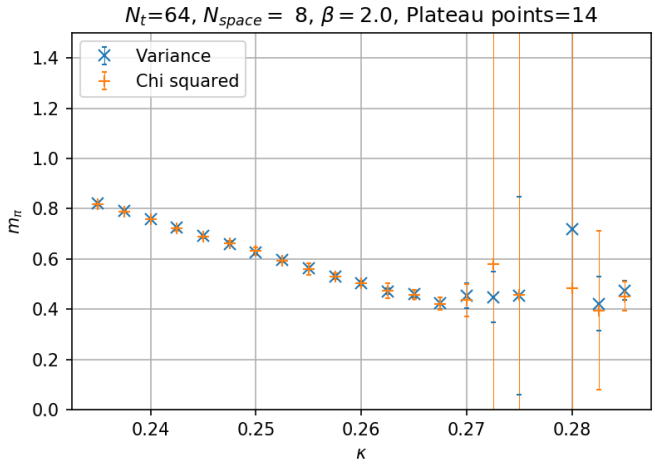


Figure 2: Number of configurations vs. topological charge on a 7×64 lattice.

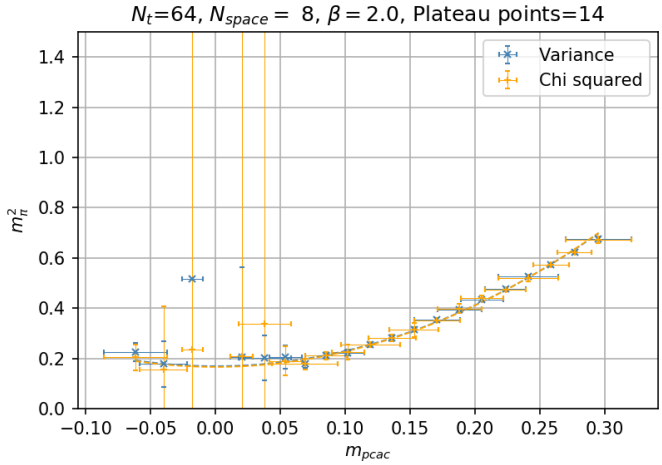
8x64



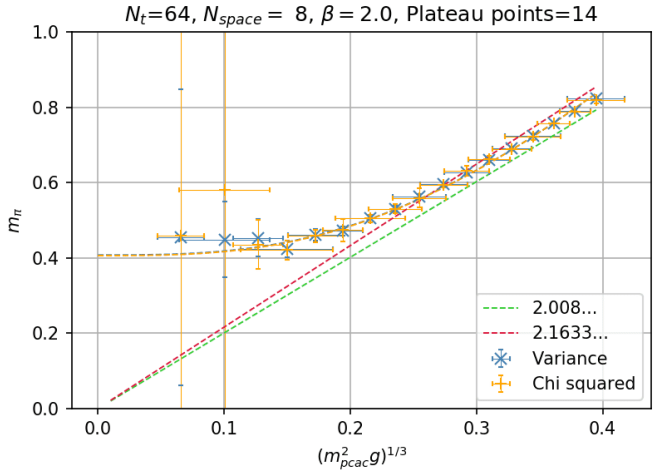
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27744 \pm 0.00196$



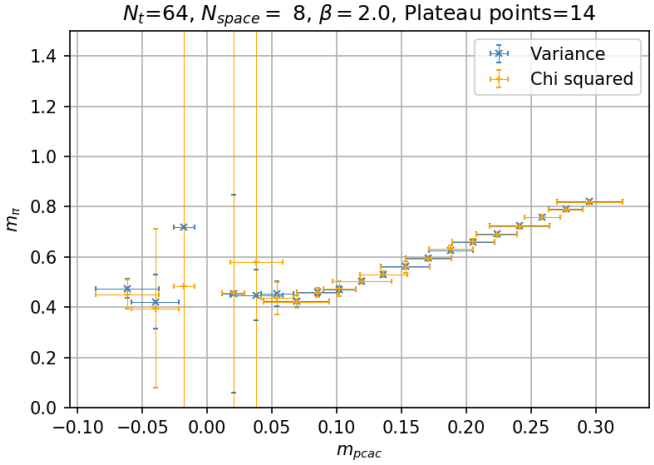
(b) Pion mass as a function of κ



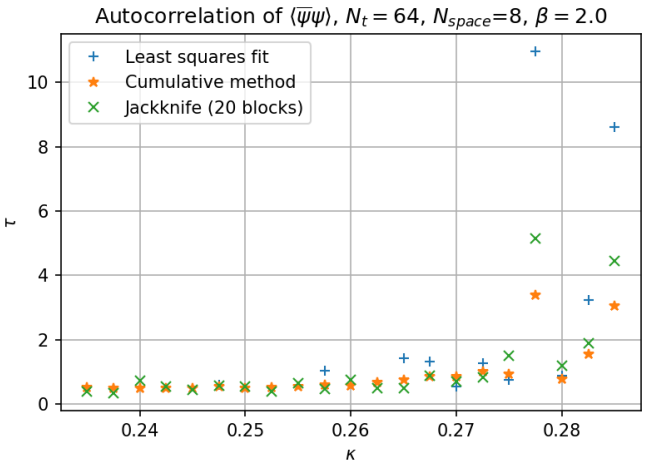
(c) m_π^2 vs. m_{pcac}. m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.16902 ± 0.00391, b =6.07619 ± 0.13206, m_π =0.41112 ± 0.00475 for variance and a =0.16587 ± 0.00337, b =6.13639 ± 0.10268, m_π =0.40727 ± 0.00413 for chi squared



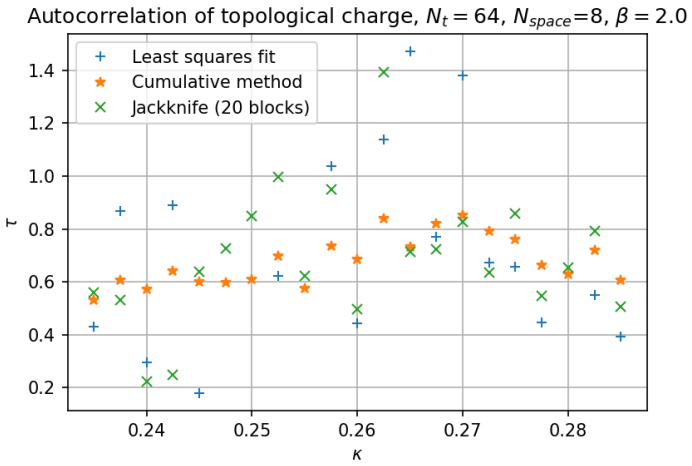
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.16636 ± 0.00355, b =8.67422 ± 0.16583, m_π =0.40787 ± 0.00435 for variance and a =0.16505 ± 0.00366, b =8.70103 ± 0.15485, m_π =0.40627 ± 0.0045 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\bar{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

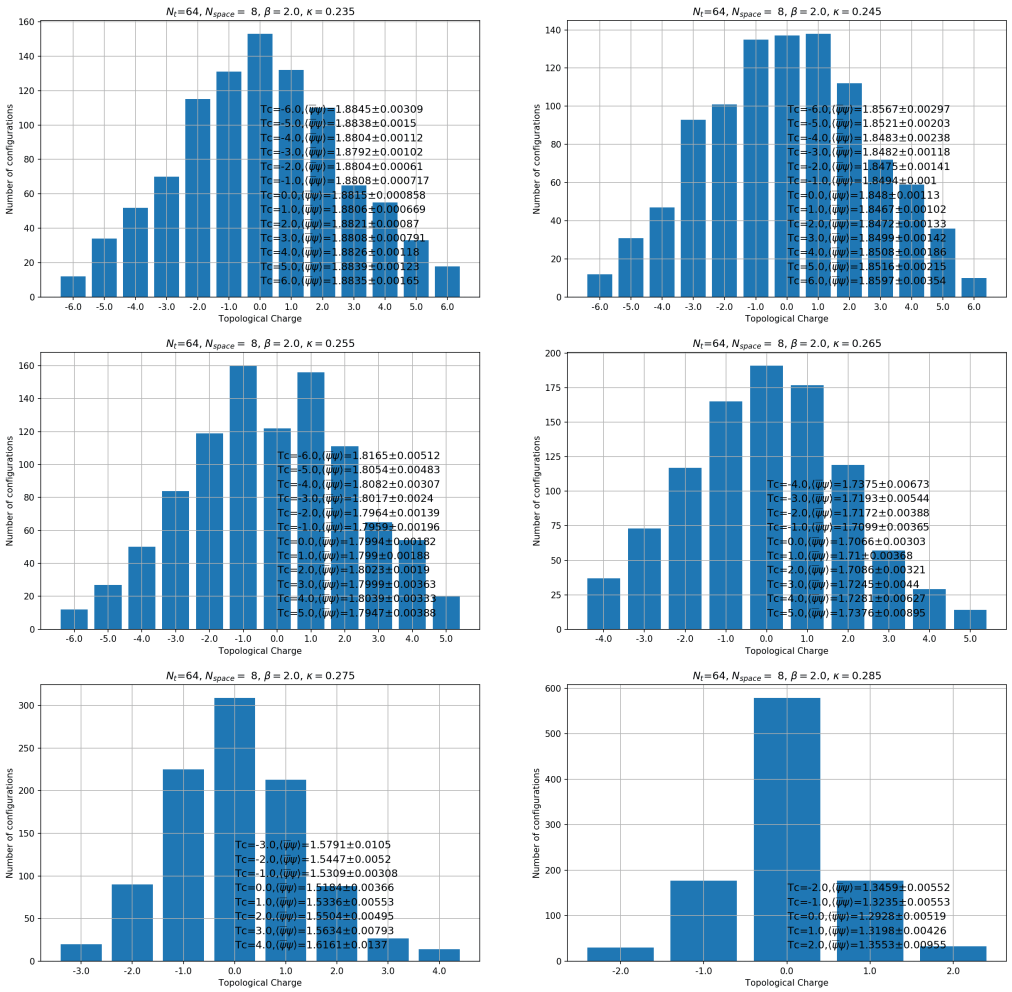
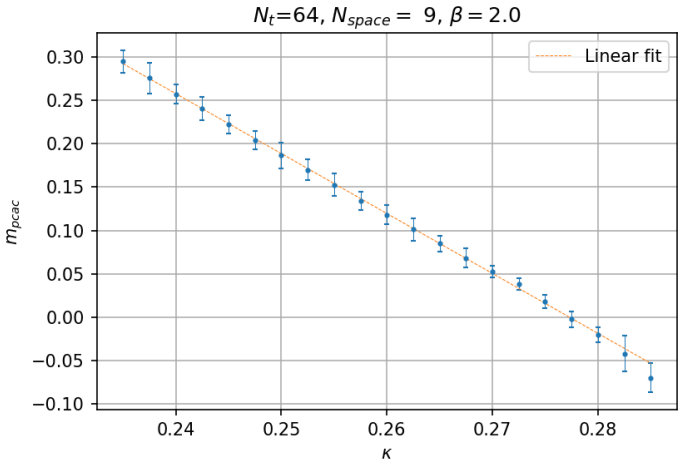
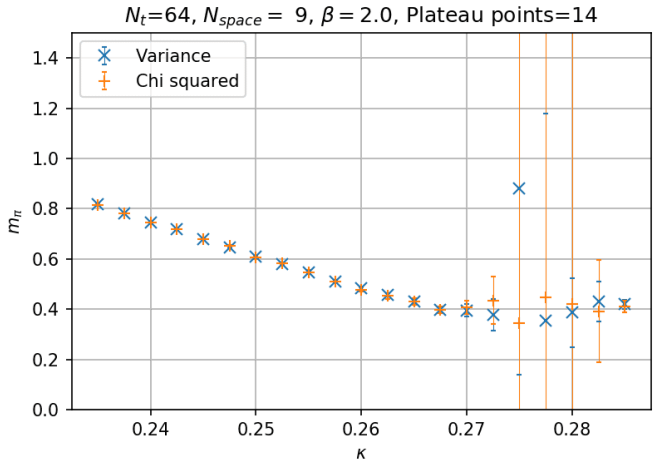


Figure 3: Number of configurations vs. topological charge on a 8×64 lattice.

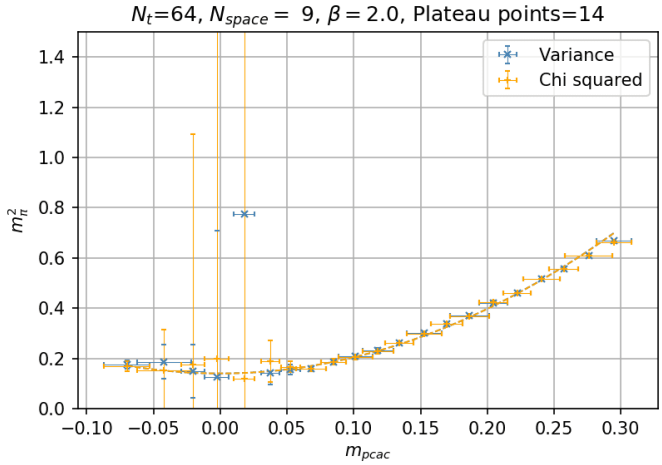
9x64



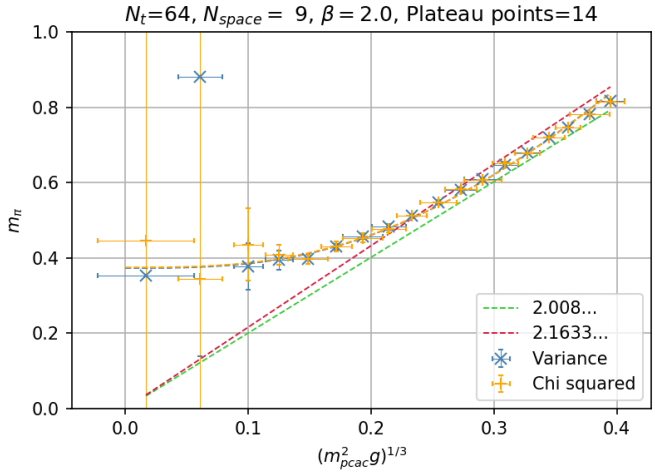
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27728 \pm 0.00299$.



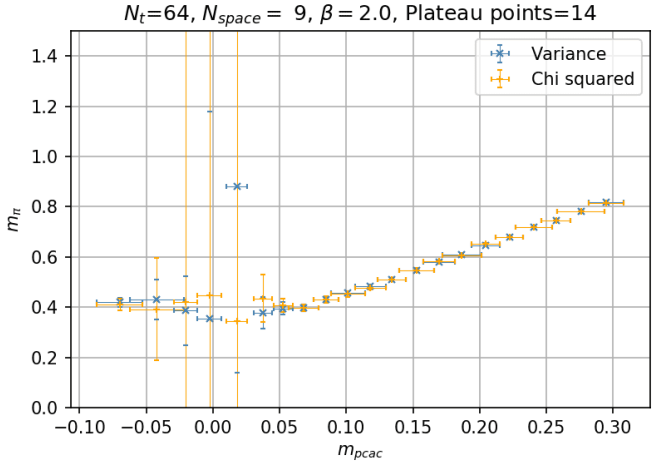
(b) Pion mass as a function of κ



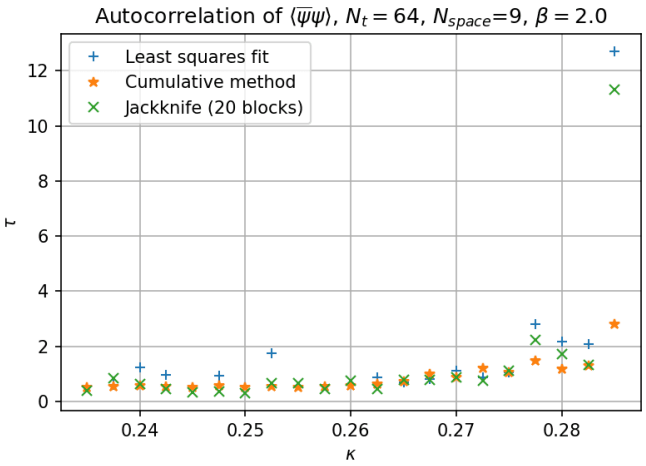
(c) m_π^2 vs. m_{pcac}. A function of the form $a+bx^2$ was fitted, the coefficients are a =0.1399 $\pm$ 0.00274, b =6.44178 $\pm$ 0.11343, m_π =0.37403 $\pm$ 0.00366 for variance and a =0.14015 $\pm$ 0.00305, b =6.42378 $\pm$ 0.11876, m_π =0.37436 $\pm$ 0.00408 for chi squared.



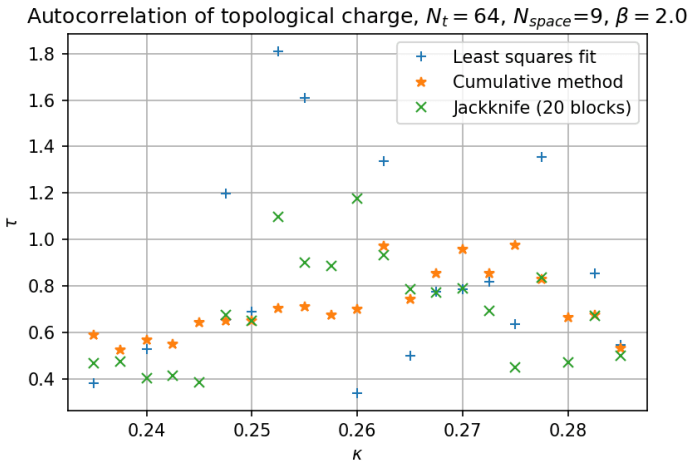
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.13913 $\pm$ 0.00312, b =9.1345 $\pm$ 0.17585, m_π =0.373 $\pm$ 0.00418 for variance and a =0.14058 $\pm$ 0.0035, b =9.06774 $\pm$ 0.18662, m_π =0.37494 $\pm$ 0.00466 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

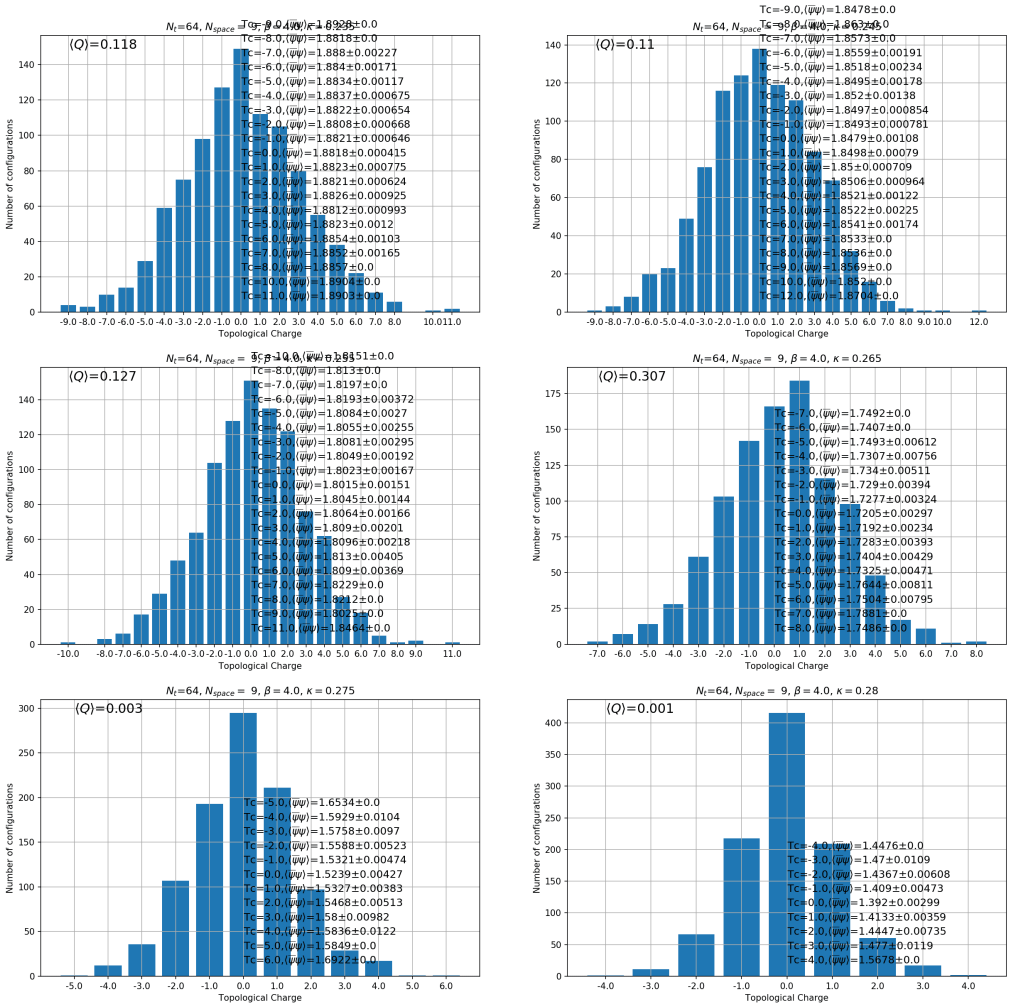
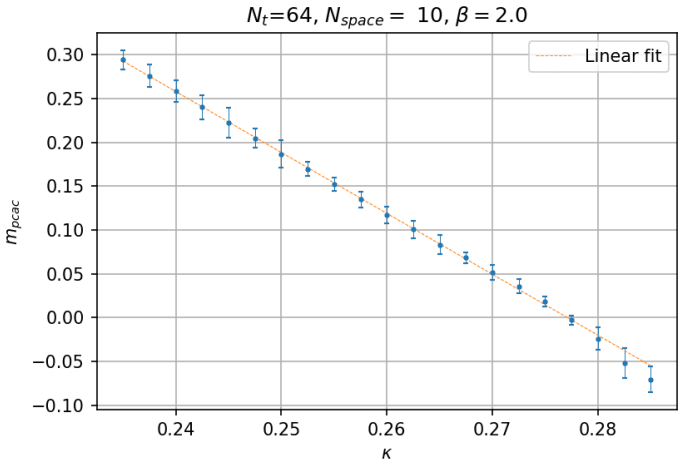
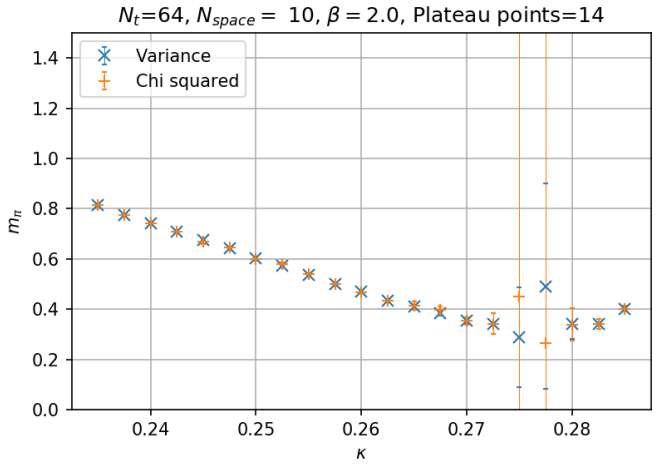


Figure 4: Number of configurations vs. topological charge on a 9×64 lattice.

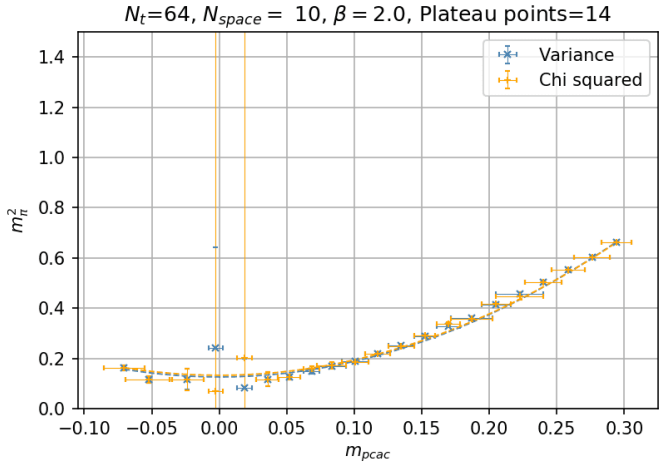
10x64



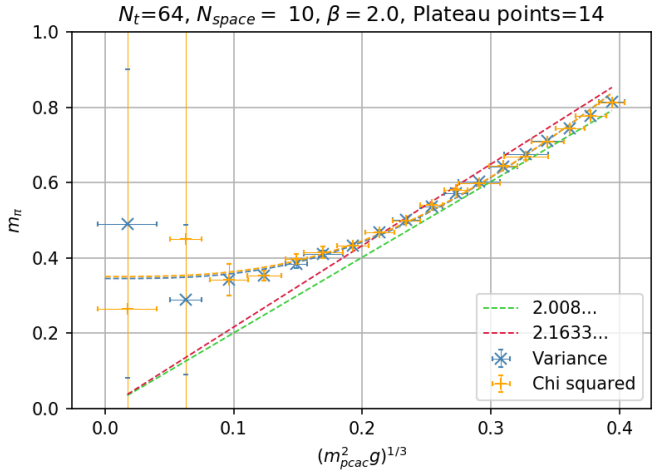
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27712 \pm 0.00335$



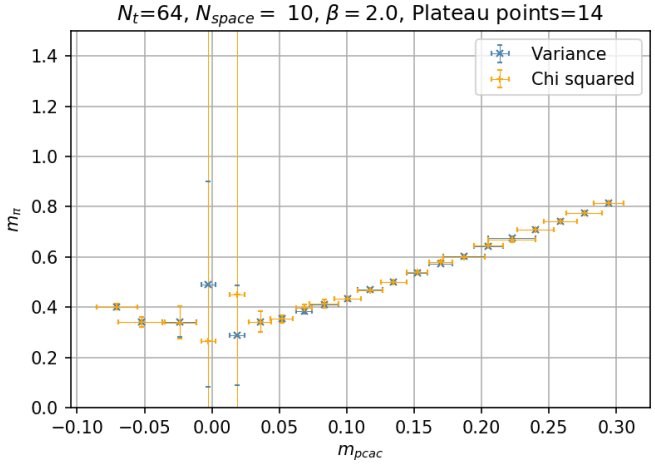
(b) Pion mass as a function of κ



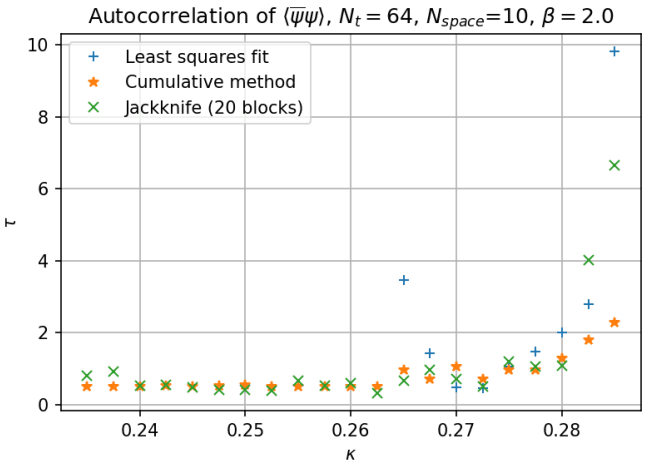
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.11777 ± 0.0032, b =6.78654 ± 0.16208, m_π =0.34317 ± 0.00466 for variance and a =0.12042 ± 0.0038, b =6.75557 ± 0.18446, m_π =0.34702 ± 0.00547 for chi squared.



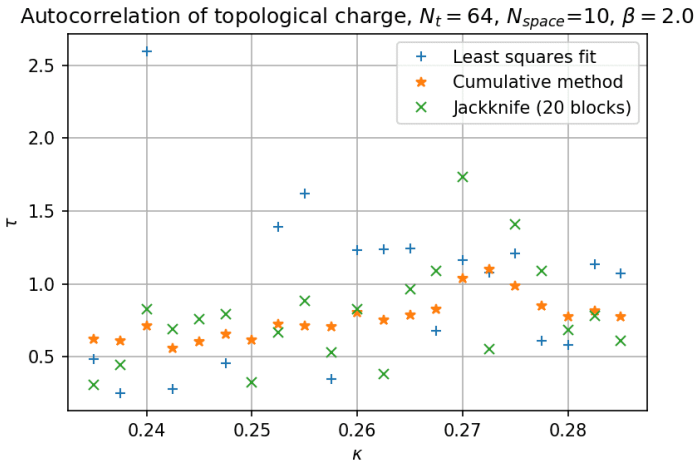
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.11927 ± 0.00336, b =9.52723 ± 0.22213, m_π =0.34536 ± 0.00486 for variance and a =0.12283 ± 0.00406, b =9.45064 ± 0.25607, m_π =0.35048 ± 0.00579 for chi squared



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

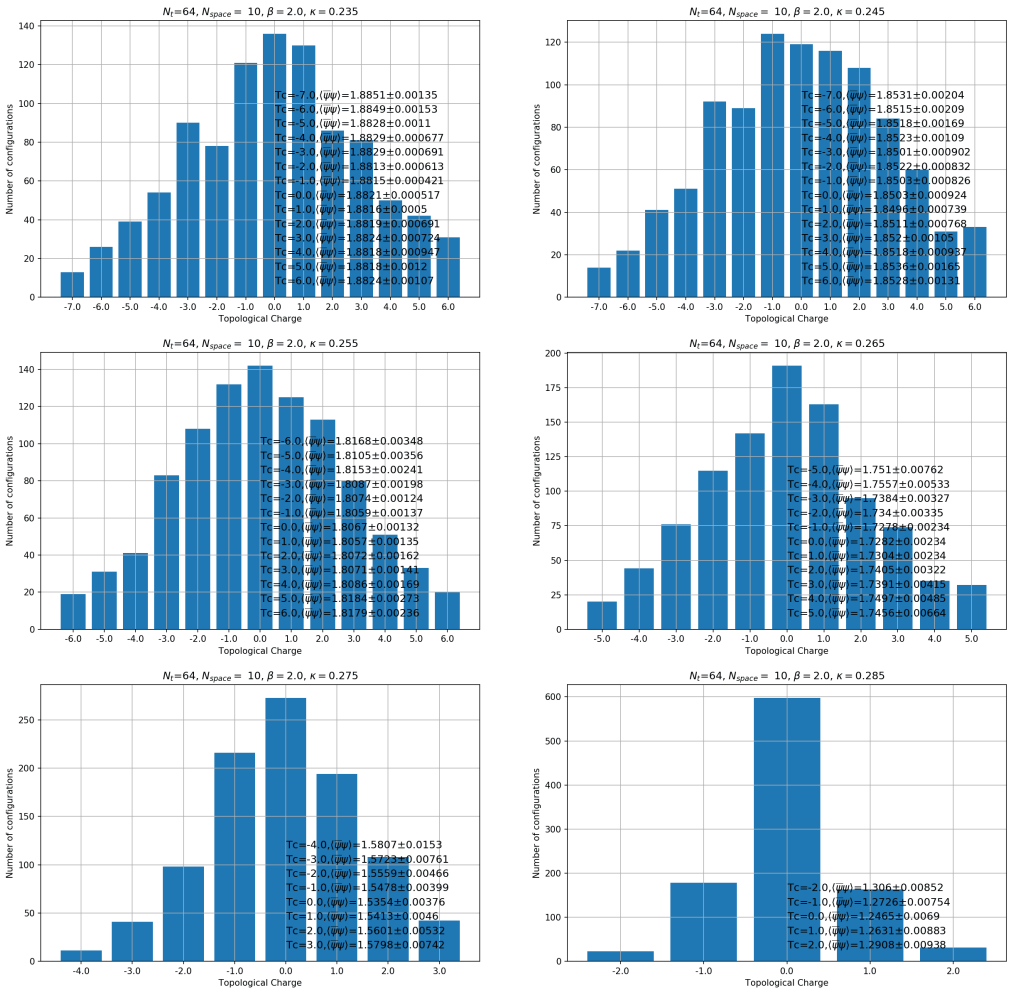
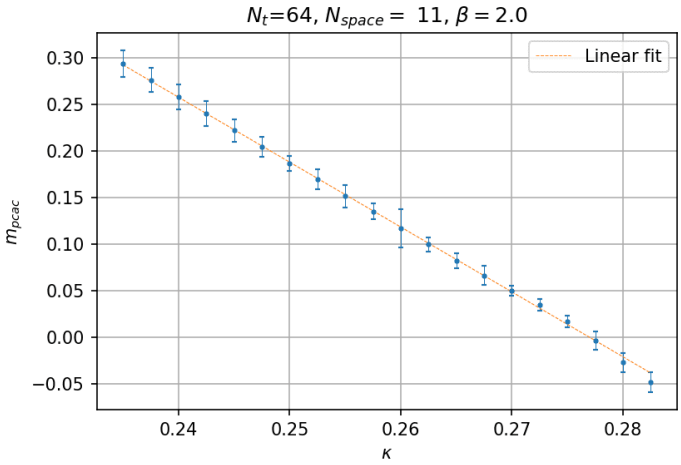
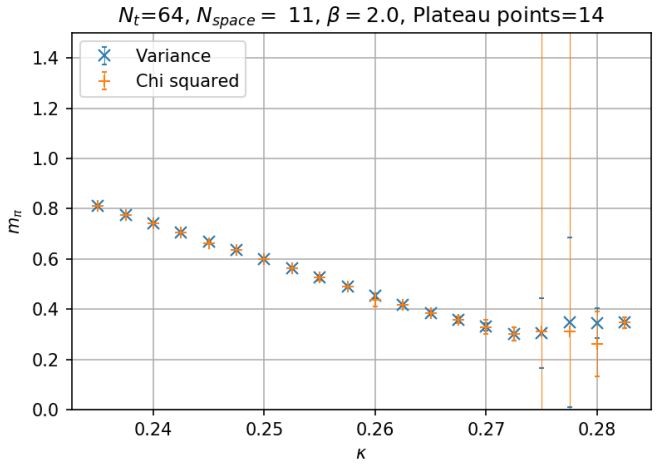


Figure 5: Number of configurations vs. topological charge on a 10×64 lattice.

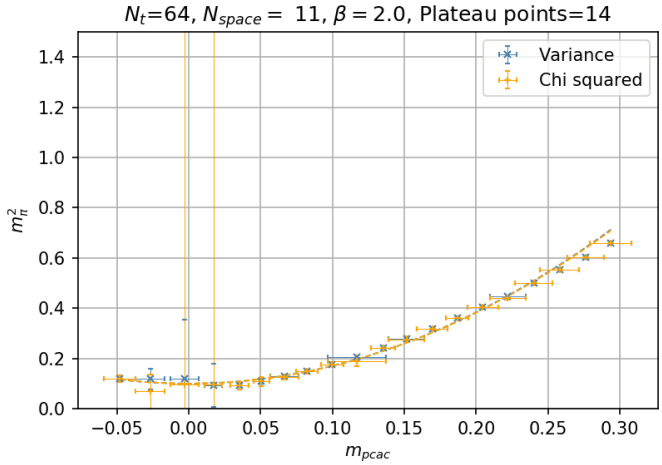
11x64



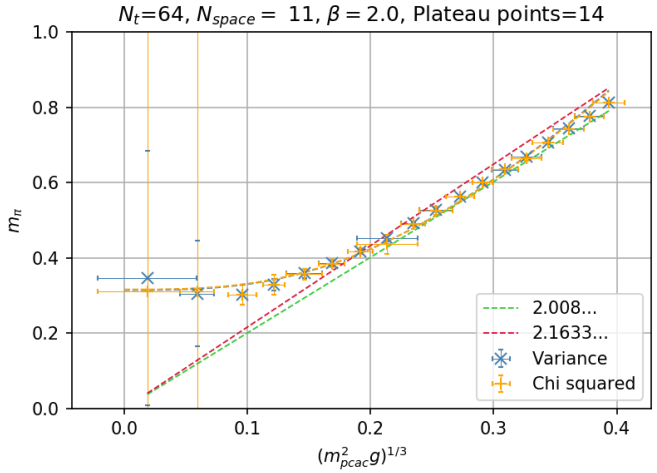
(a) Fermion mass using PCAC relation, κ_c =0.27703 ± 0.00297



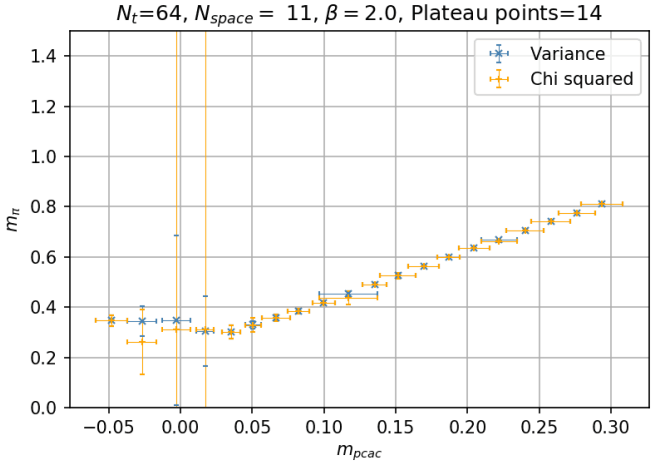
(b) Pion mass as a function of κ



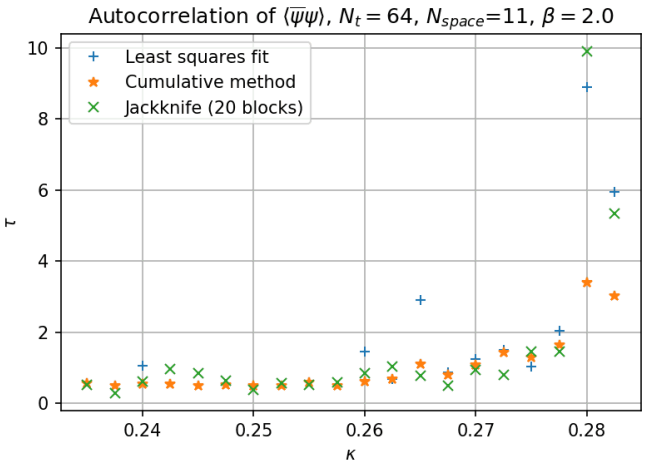
(c) m_π^2 vs. m_{pcac}. A function of the form $a + bx^2$ was fitted, the coefficients are a =0.09948 ± 0.003, b =7.1086 ± 0.16062, m_π =0.31541 ± 0.00476 for variance and a =0.09989 ± 0.00334, b =7.06439 ± 0.16118, m_π =0.31605 ± 0.00528 for chi squared.



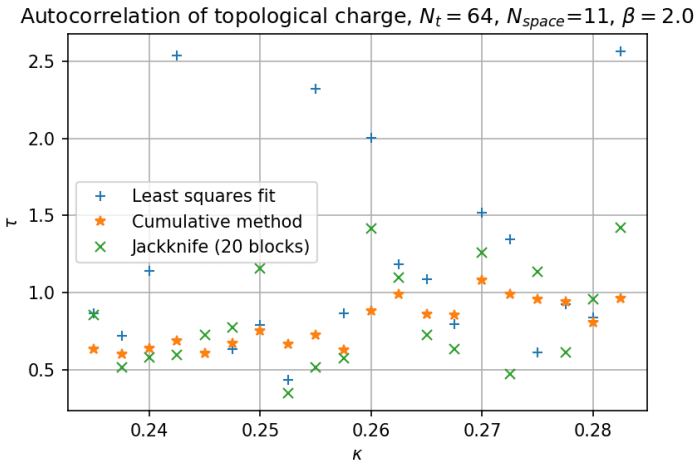
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.09902 ± 0.00337, b =10.06628 ± 0.24085, m_π =0.31467 ± 0.00535 for variance and a =0.10005 ± 0.00374, b =9.97696 ± 0.23947, m_π =0.3163 ± 0.00591 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

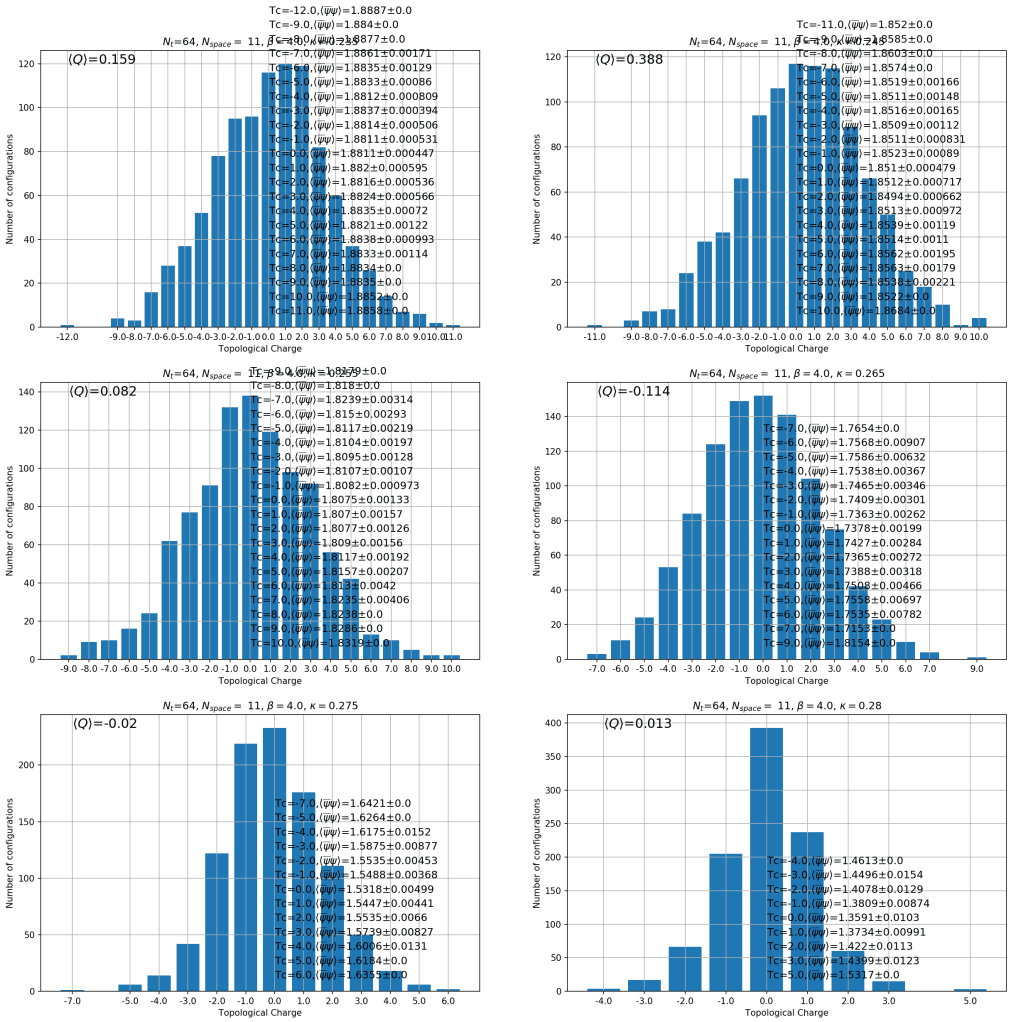
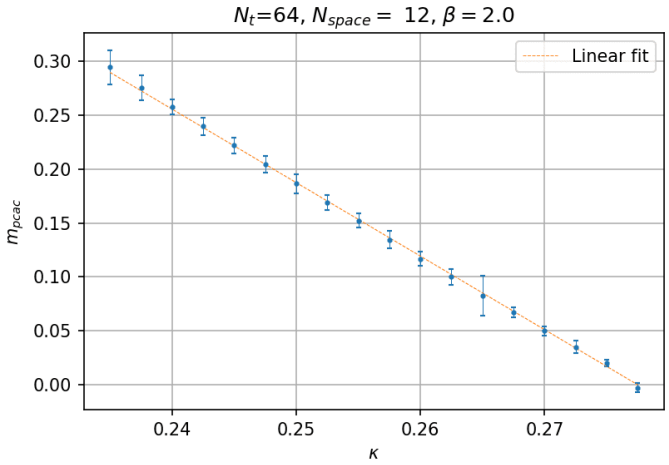
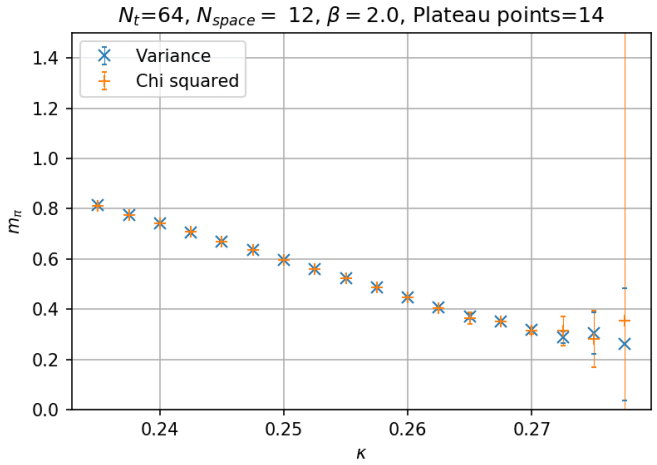


Figure 6: Number of configurations vs. topological charge on a 11×64 lattice.

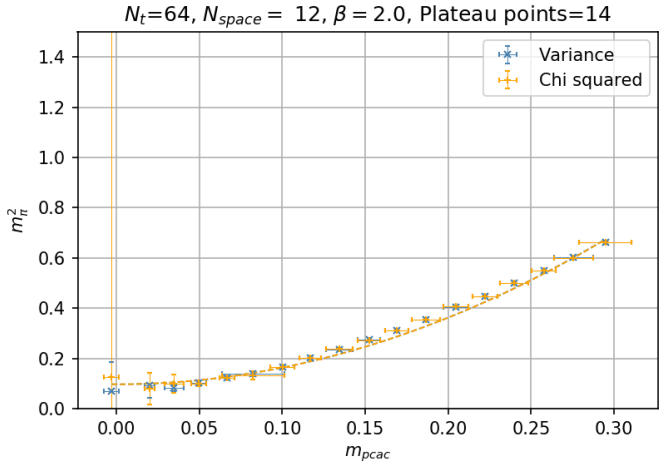
12x64



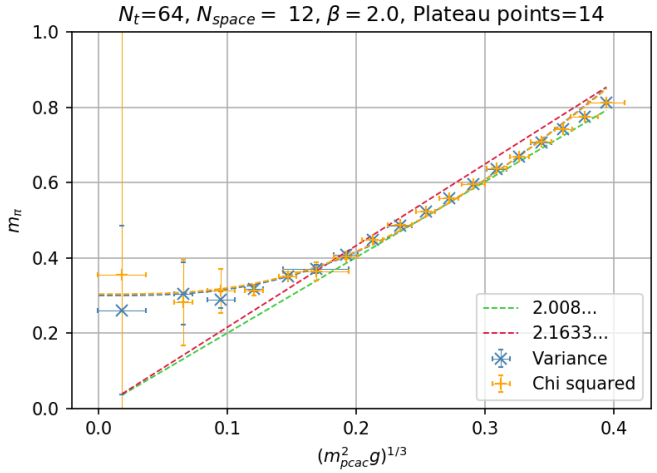
(a) Fermion mass using PCAC relation, $\kappa_c = 0.27747 \pm 0.00246$



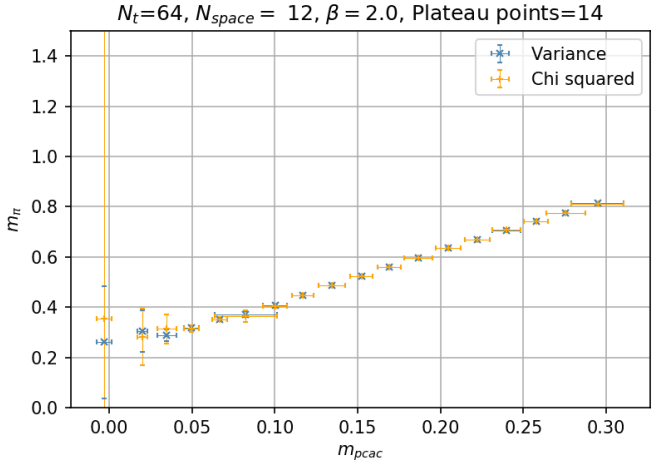
(b) Pion mass as a function of κ



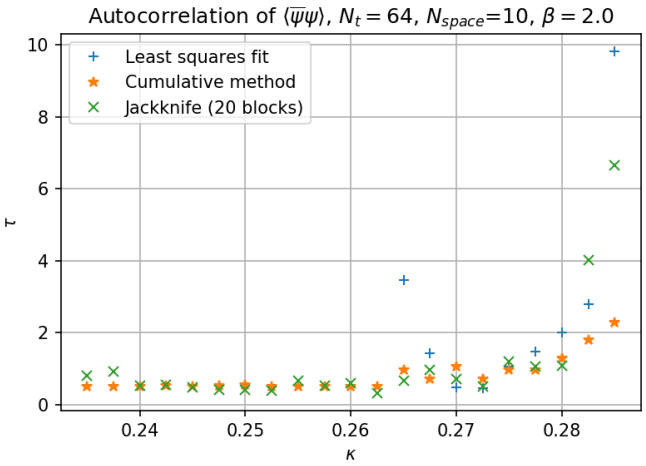
(c) m_π^2 vs. m_{pcac}. A function of the form $a+bx^2$ was fitted, the coefficients are a =0.08988 ± 0.00293, b =7.32002 ± 0.16239, m_π =0.29981 ± 0.00489 for variance and a =0.09219 ± 0.00308, b =7.24626 ± 0.15345, m_π =0.30362 ± 0.00507 for chi squared.



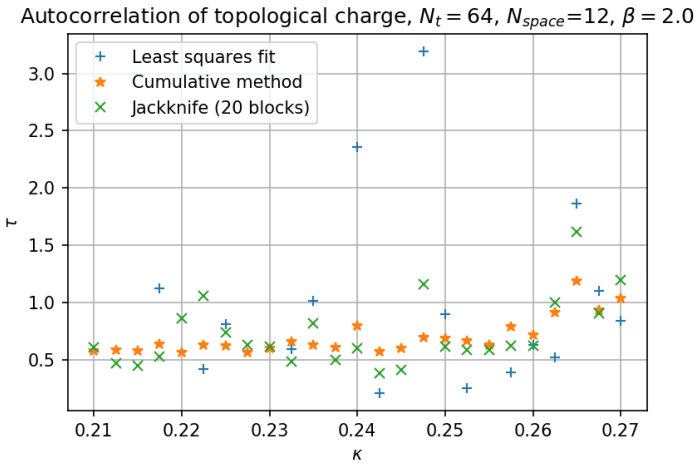
(d) A function of the form $y = \sqrt{a + bx^3}$ was fitted. Only $m_{pcac} > 0$ is considered. a =0.09014 ± 0.00296, b =10.335 ± 0.22912, m_π =0.30023 ± 0.00492 for variance and a =0.09241 ± 0.00311, b =10.23365 ± 0.21701, m_π =0.30398 ± 0.00511 for chi squared.



(e) m_π vs. m_{pcac}



(f) Autocorrelation of $\langle\overline{\psi}\psi\rangle$



(g) Autocorrelation of the topological charge

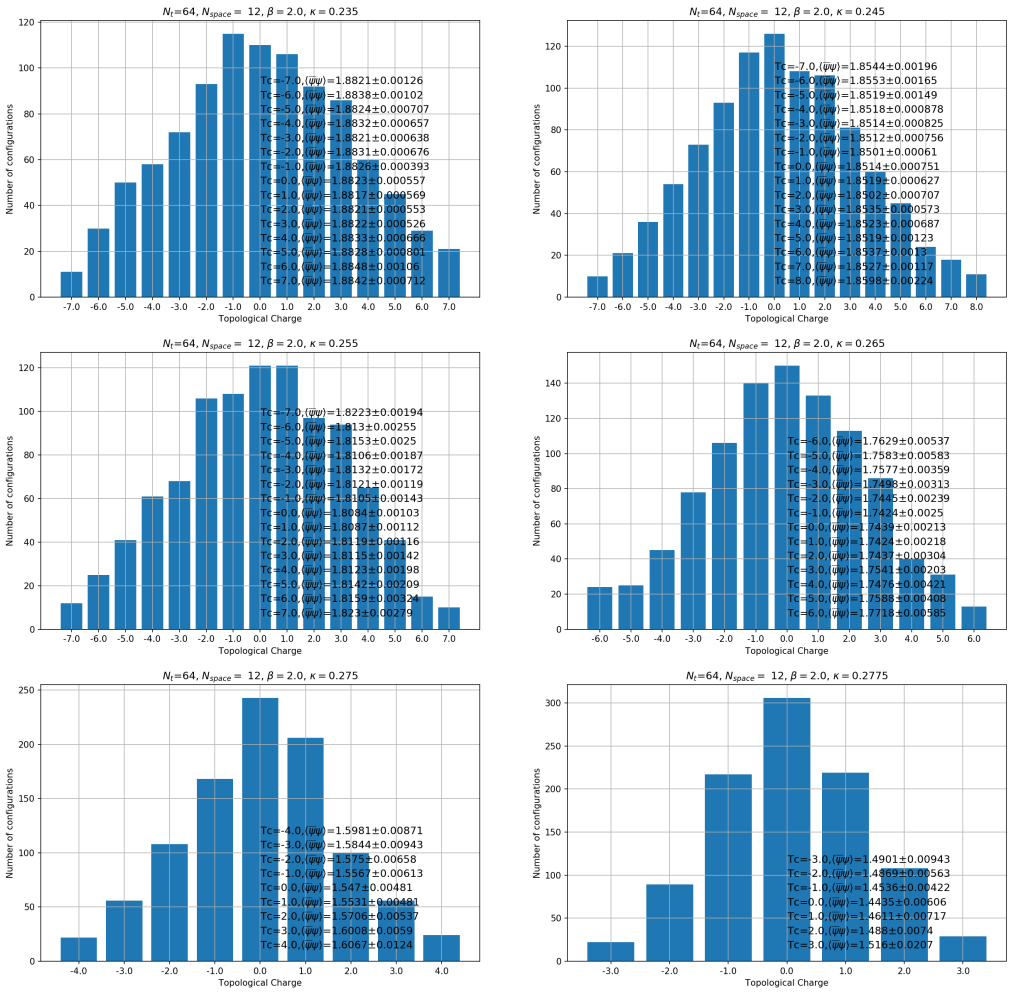


Figure 7: Number of configurations vs. topological charge on a 12×64 lattice.

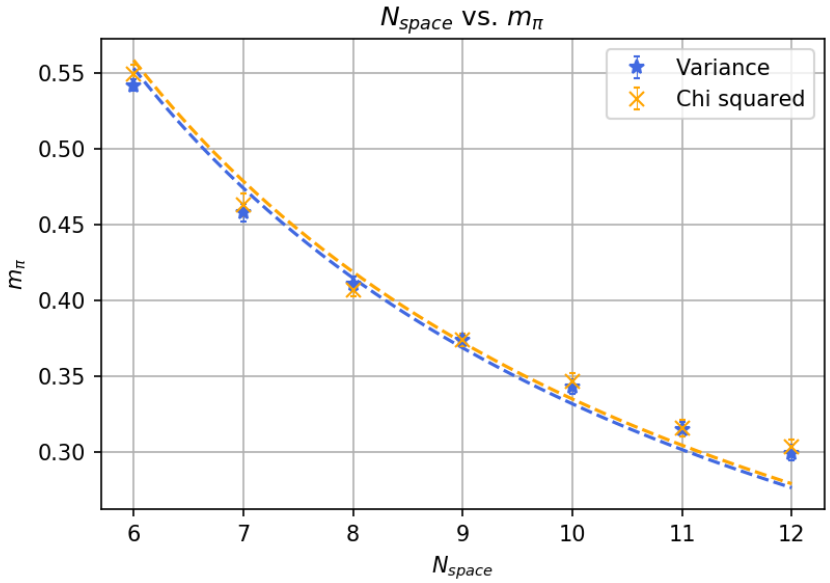


Figure 8: M_π^R vs. L. A curve of the form a/L was fitted. For variance the fit parameter is a =3.3190±0.0418, while for Chi squared a =3.3519±0.0463.

For $\beta = 4.0$, the fit constant is $a = 3.36547 \pm 0.01812$ for variance, while for Chi squared $a = 3.31621 \pm 0.01044$.

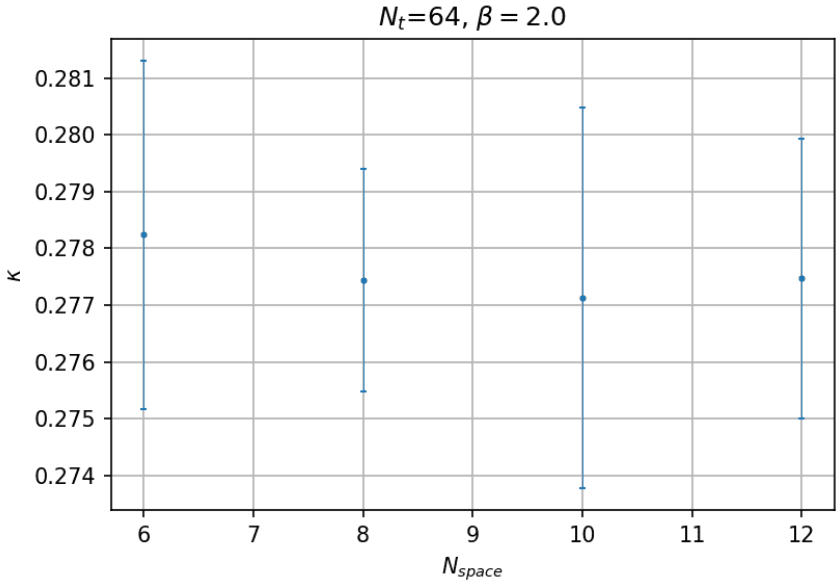


Figure 9: Kappa critical as a function of the volume size