PROCEEDINGS OF SCIENCE

Finite temperature and δ-regime in the Schwinger model

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The Schwinger model is often used as a testbed for conceptual and numerical approaches in lattice field theory. Nevertheless, some of the rich physical properties of the model in anisotropic volumes have not yet been tested. For the multi-flavor finite temperature Schwinger model there is an approximate solution by Hosotani et al. based on bosonization. We perform thorough comparisons with the lattice results and check the validity and limitations of the Hosotani approach. By inverting the physical interpretation of the coordinates we explore the delta-regime and measure the dependence of the pion mass on the spatial size at zero temperature. Our results confirm universal features of theoretical predictions by Leutwyler, Hasenfratz and Niedermayer and enable the computation of the Schwinger model counterpart of pion decay constant. This is further compared with the 2d version of the Witten-Veneziano formula.

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1. Introduction

- introduced by [Schwinger, 1961][1] *two-dimensional quantum electrodynamics* — fermions coupled to Abelian gauge field
- simple example for chiral anomaly, topology, confinement
- often used as a testbed for conceptual and numerical approaches in lattice field theory
- nevertheless, some of the rich physical properties of the model in anisotropic volumes have not yet been tested:

Finite temperature : Hosotani et al. approximate solution has not been compared with the lattice simulation results

δ**-regime** : conjecture for the residual pion mass

- massless case has analytic solution [Belvedere et al., 1979][2]
- $N - 1$ massless bosons ("pions")
- one massive boson ("eta")

$$m_\eta^2 = N\,\frac{g^2}{\pi}$$

  where g is the gauge coupling
- massive case (fermion mass $m > 0$) has no exact solution
- semiclassical prediction at infinite volume [Hetrick, Hosotani and Iso, 1995][3]

$$m_\pi = \left(4e^{2\gamma}\sqrt{\frac{2}{\pi}}\right)(m^2 g)^{1/3} = 2.1633\ldots(m^2 g)^{1/3}$$

2. Finite temperature

- approximate solution by Hosotani et al.[4] based on bosonization
- finite temperature massive Schwinger model reduced to quantum mechanical system with $N - 1$ degrees of freedom
- set of nonlinear equations valid when

$$m \ll m_\eta$$

- boson masses can be computed by solving this set of equations in a self-consistent way
- we compare the Hosotani predictions for two flavors with HMC simulation results (Wilson fermions — fermion mass m is measured on the lattice by the PCAC relation)

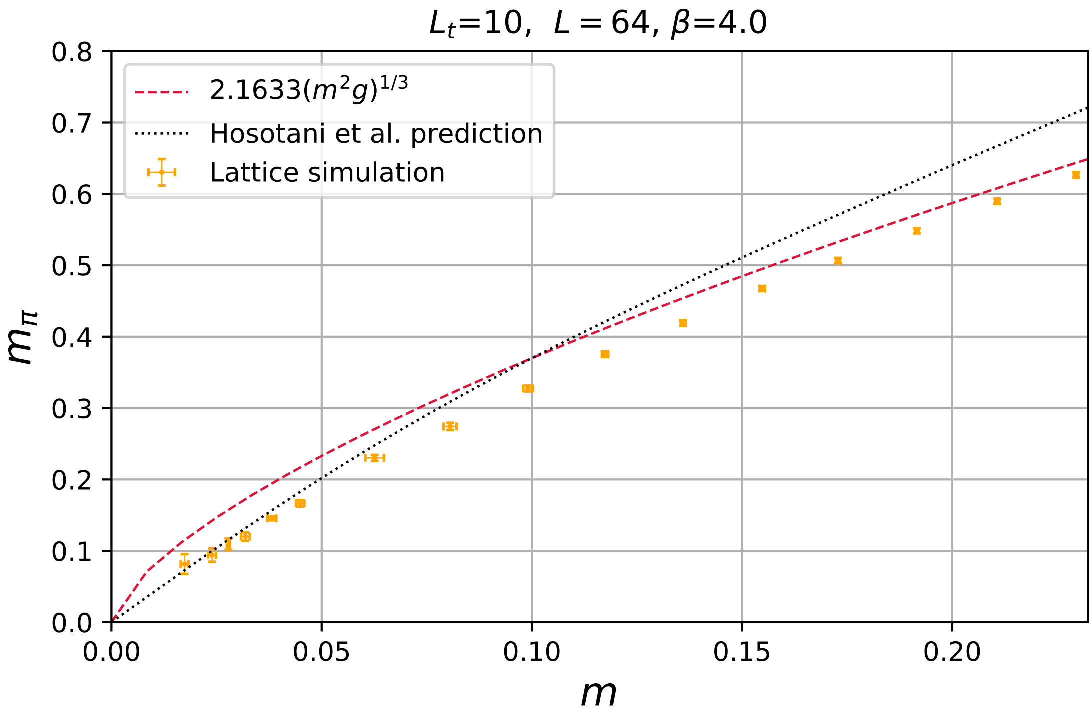

Figure 1: Pion mass as a function of quark mass.

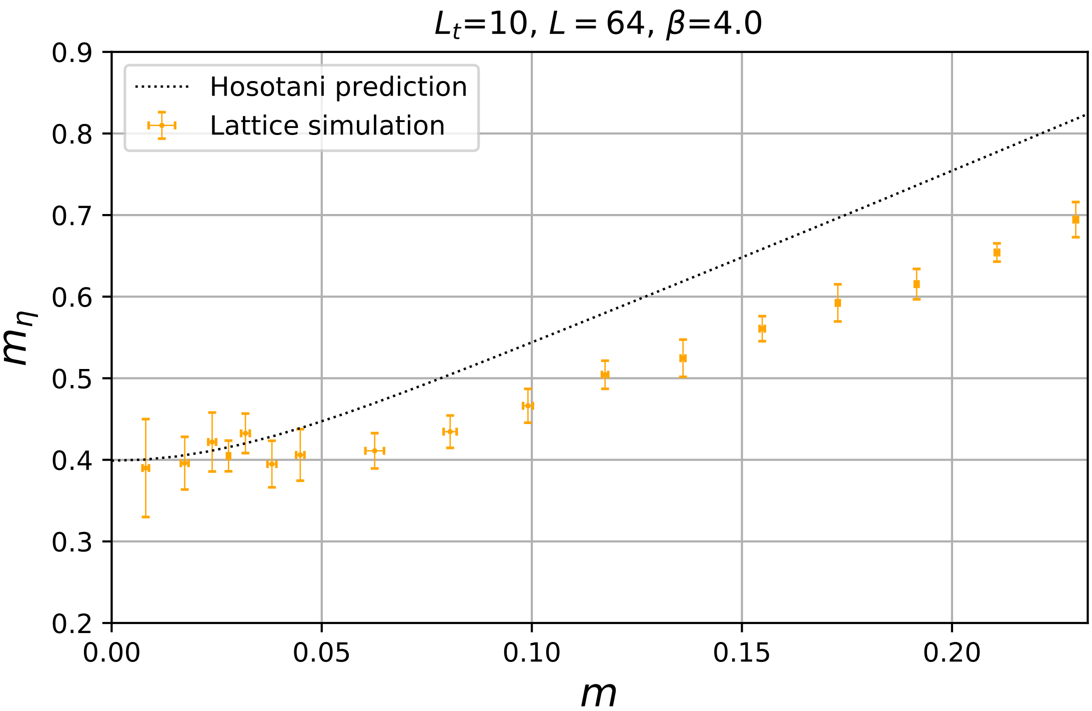

Figure 2: Eta mass as a function of quark mass.

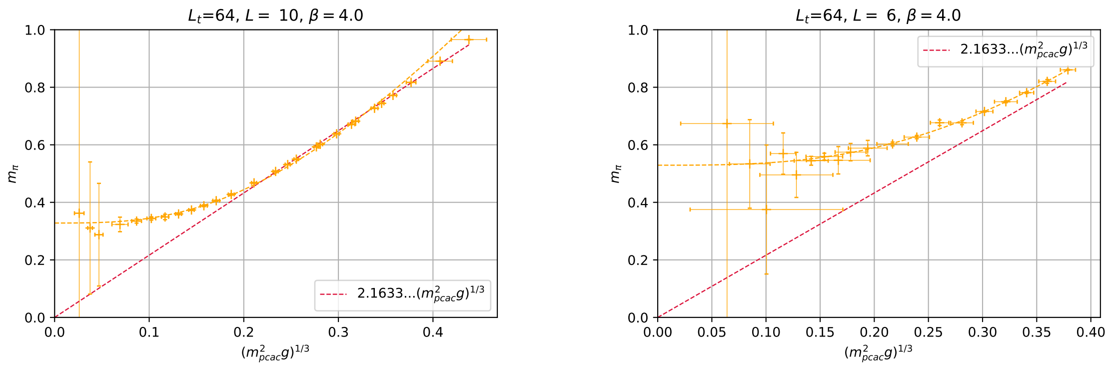

Figure 3: Residual pion mass plateaus for two different spatial volumes.

3. δ-regime

{HIP: Wolfgang would probably like to cite [5] ;-)}

- spatial volume is small compared to the correlation length

$$\xi = m_\pi^{-1}$$

  but the Euclidean time extent is large

$$L_t \gg \xi \gtrsim L$$

- the system is quasi one dimensional - approximation by quantum mechanical rotor [Leutwyler, 1987[6]
- pion has ***residual mass***

$$m_\pi^R = \frac{\mathcal{N}-1}{2\Theta}$$

- $\mathcal{N}-1$ is the number of pions and Θ is the moment of inertia

- [Hasenfratz and Niedermayer, 1993][7] computed Θ up to next-to-leading order, for a general dimension $d > 2$

$$\Theta = F_\pi^2 L^{d-1}\left[1 + \frac{\mathcal{N}-2}{4\pi F_\pi^2 L^{d-2}}\left(2\frac{d-1}{d-2} + \ldots\right)\right]$$

- for $\mathcal{N} = 2$ we ignore the next to leading term despite the division by $d-2$, so we just consider the leading term

$$m_\pi^R \simeq \frac{\mathcal{N}-1}{2F_\pi^2 L}$$

- we verify the relation $m_\pi^R \propto 1/L$ with simulation data and extract the value of pion decay constant F_π

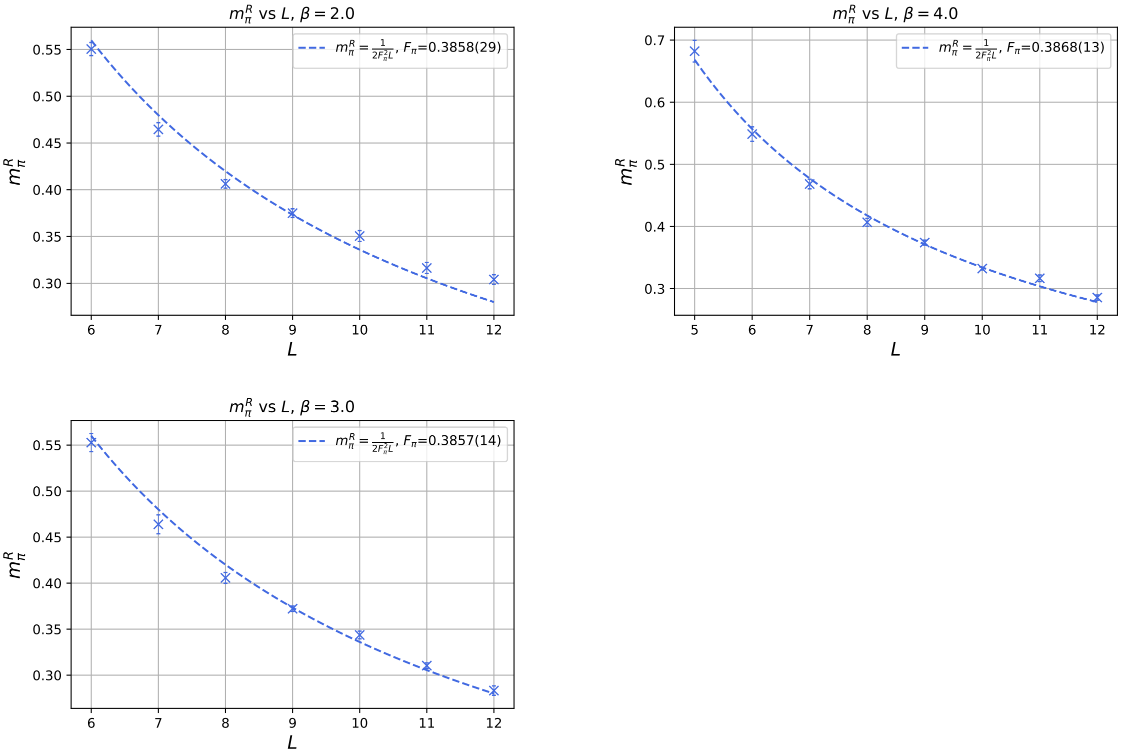

Figure 4: 1 / L confirmed by lattice simulation (β = 5.0 still missing).

β	F_π
2.0	0.3858(29)
3.0	0.3857(14)
4.0	0.3868(13)

$$F_\pi = 0.386(2)$$

4. Witten-Veneziano

{HIP: here we probably should cite [8]}

- in the chiral N-flavor Schwinger model the Witten-Veneziano formula is simplified to [Seiler and Stamatescu, 1987][9] *{HIP: this Seiler and Stamatescu paper from 1987 is unpublished and exists only as a KEK scan — it has just few lines about 1-flavor Schwinger model, so for N-flavor SM makes probably more sense to cite [10]}*

$$m_\eta^2 = \frac{2N}{F_\eta^2}\chi_T^{que}$$

- m_η in the chiral limit is known analytically [Belvedere et al., 1979][2]

$$m_\eta^2 = \frac{N}{\pi\beta}$$

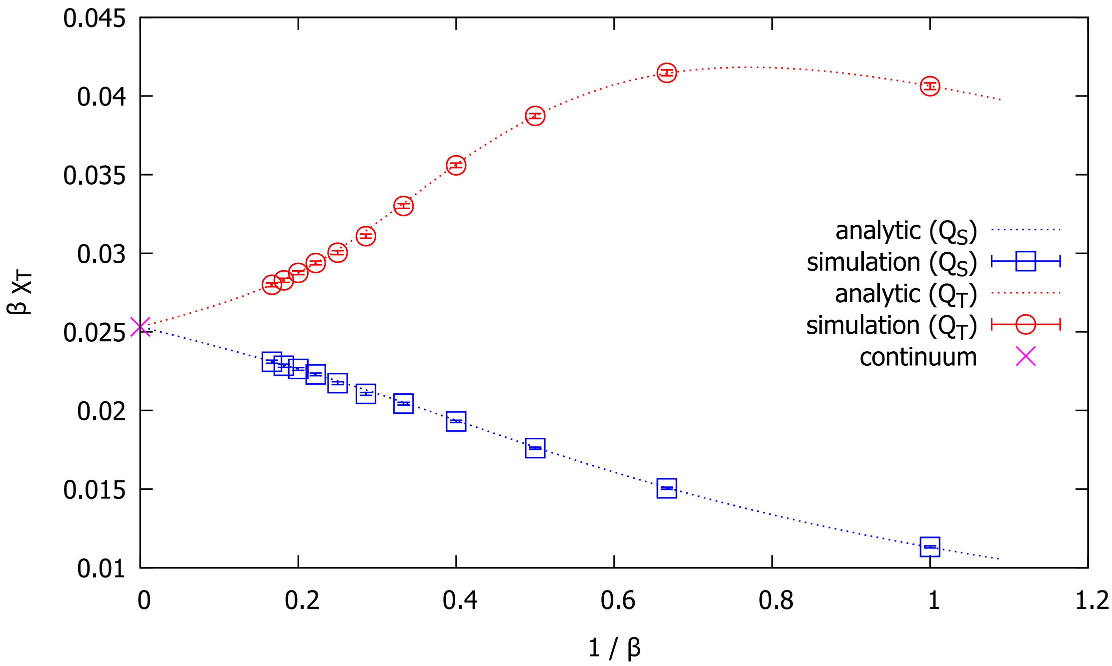

Figure 5: Quenched topological susceptibility.

- continuum prediction for χ_T^{que} [Seiler and Stamatescu, 1987]

$$\beta\chi_T^{que} = \frac{1}{4\pi^2}$$

- [Bardeen et al., 1998][11] were able to analytically compute χ_T^{que} on the lattice

$$\beta\chi_T^{que} = \frac{I_1(\beta)}{4\pi^2 I_0(\beta)}$$

  by using an alternative definition of topological charge

$$Q_S = \frac{1}{2\pi}\sum_P \sin(\theta_P)$$

- for the usual definition of topological charge

$$Q_T = \frac{1}{2\pi}\sum_P \theta_P$$

  it is not possible to find an analytic expression, but using the same line of reasoning it is possible to numerically compute χ_T^{que} to arbitrary precision

- inserting the confirmed values for m_η^2 and χ_T^{que}

$$F_\eta^2 = \frac{2N}{m_\eta^2}\chi_T^{que} = 2N\left(\frac{\pi\beta}{N}\right)\left(\frac{1}{4\pi^2\beta}\right) = \frac{1}{2\pi}$$

- we can compare the two decay constants

$$F_\pi \simeq 0.386 \quad \stackrel{?}{=} \quad F_\eta = 0.399$$

- in large N_c QCD, to the order $1/N_c$

$$F_{\eta'} = F_\pi$$

- in the Schwinger model nothing assures that this relation holds

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