

BBE V3 Robot Firmware

Documentation of Main Components



CORBOTICS

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Preface

This document provides information on the key components of the robot firmware of the BBE V3. The corresponding GitHub repository can be found [here](#). The robot firmware of V3 is mostly extended upon its predecessor, and contains work dating back from July, 2020.

*Fabian Ballast, Corbotics
Delft, Thursday 7th April, 2022*

Chapter 1: Motor Speed Calculations

This chapter describes the calculations used to synchronize the motors in the BBE. It assumes second order bang-coast-bang trajectories, which has up to three stages:

1. In the first stage, the motor accelerates with maximum acceleration from its initial velocity v_0 to its cruising velocity v_{cruise} . This is the first bang.
2. In the second stage, the motor maintains a constant velocity (v_{cruise}). This is the coast.
3. In the third and final stage, the motor decelerates to its target velocity v_{target} . This is the second bang.

The goal is to make v_{cruise} as high as possible (as long as it's lower than the maximum velocity v_{max}), to reach the target as quickly as possible. Note that, by changing v_0 , v_{target} or the total distance we need to travel, the total trajectory can be any combination of these three stages, as long as they remain in this order (e.g., not bang-bang-coast, but bang-bang or bang-coast are possible with certain parameters).

Section 1.1: Time Estimation

This section elaborates on the calculation of the time estimation. This time estimation is calculated per motor from which the highest time estimation is used to synchronize all the motors.

To estimate the time needed to reach a target, two situations have to be considered, with v_0 the initial velocity, v_{max} the maximum allowable velocity of that motor and v_{target} the final velocity of that motor:

1. v_{max} is reached;
2. v_{max} is not reached.

Whether or not v_{max} can be reached can be checked with $t_{crit,0}/t_{crit,target}$ and s_{crit} , where $t_{crit,x}$ is the time required to reach v_{max} from v_x and s_{crit} is the distance travelled when reaching v_{max} from v_0 and immediately decelerating to v_{target} (i.e., a bang-bang-trajectory). This type of trajectory is shown in Fig. 1.1, where the area under $v(t)$ is equal to s_{crit} .

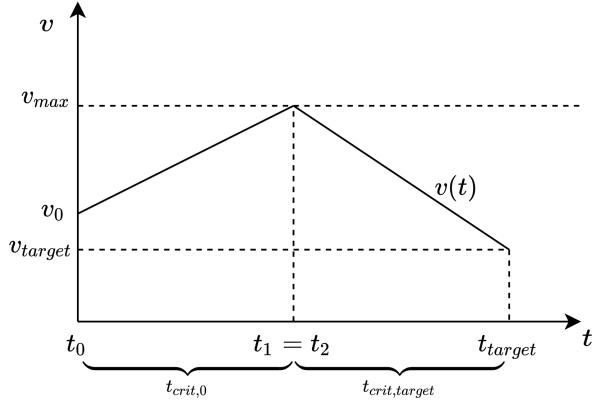


Figure 1.1: Critical trajectory when reaching maximum speed.

Expressed in the given terms, $t_{crit,0}/t_{crit,target}$ can be found as shown in Eq. (1.1), and s_{crit} can be found as shown in Eq. (1.2).

$$\begin{aligned} t_{crit,x} &= \frac{\Delta v}{a_{max}} = \frac{v_{max} - v_x}{a_{max}} \Rightarrow t_{crit,0} = \frac{v_{max} - v_0}{a_{max}}, \quad t_{crit,F} = \frac{v_{max} - v_F}{a_{max}} \quad (1.1) \\ s_{crit} &= \underbrace{\frac{1}{2}(v_{max} + v_0)t_{crit,0}}_{\text{First bang}} + \underbrace{\frac{1}{2}(v_{max} + v_{target})t_{crit,target}}_{\text{Second bang}} \\ &= \frac{v_{max}^2 - v_0^2}{2a_{max}} + \frac{v_{max}^2 - v_{target}^2}{2a_{max}} \\ &= \frac{2v_{max}^2 - v_0^2 - v_{target}^2}{2a_{max}} \quad (1.2) \end{aligned}$$

Then, it is very easy to decide which of the two situations the motor will be in, given a target distance s_{target} :

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if  $s_{target} \geq s_{crit}$  then
     $v_{max}$  reached
else
     $v_{max}$  not reached
end if

```

These situations require different calculations to get the required time to reach the target. These calculations when v_{max} is reached are worked out in Section 1.1.1, and Section 1.1.2 shows the calculations for the other scenario.

Section 1.1.1: Situation 1: Maximum Speed Reached

In this situation, the motor reaches v_{max} and thus follows a bang-coast-bang trajectory. The velocity of the motor over time in this situation is shown in Fig. 1.2.

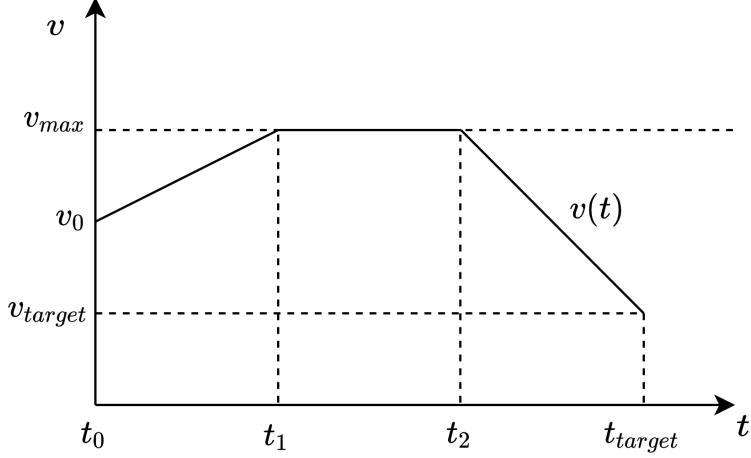


Figure 1.2: Bang-coast-bang trajectory when v_{max} is reached.

Now, filling in v_0 and v_{target} in Eqs. (1.1) and (1.2) to obtain t_{xy} and s_{xy} , the time/distance when moving from point x to y :

$$\begin{aligned} t_{01} = t_{crit,0} &= \frac{v_{max} - v_0}{a_{max}} \\ t_{2target} = t_{crit,target} &= \frac{v_{max} - v_{target}}{a_{max}} \\ s_{01} + s_{2target} = s_{crit} &= \frac{2v_{max}^2 + v_0^2 + v_{target}^2}{2a_{max}} \end{aligned}$$

Then, the total duration from start of finish can be found, where the final result is shown in Eq. (1.3).

$$\begin{aligned} s_{12} &= s_{target} - s_{crit} = s_{target} - \frac{2v_{max}^2 - v_0^2 - v_{target}^2}{2a_{max}} \\ t_{12} &= \frac{s_{12}}{v_{max}} = \frac{s_{target} - \frac{2v_{max}^2 - v_0^2 - v_{target}^2}{2a_{max}}}{v_{max}} = \frac{s_{target}}{v_{max}} - \frac{v_{max}}{a_{max}} + \frac{v_0^2 + v_{target}^2}{2v_{max}a_{max}} \\ t_{0target} &= t_{01} + t_{12} + t_{2target} = \frac{v_{max} - v_0}{a_{max}} + \frac{s_{target}}{v_{max}} - \frac{v_{max}}{a_{max}} + \frac{v_0^2 + v_{target}^2}{2v_{max}a_{max}} + \frac{v_{max} - v_{target}}{a_{max}} \\ &= \frac{v_{max} - v_0 - v_{target}}{a_{max}} + \frac{s_{target}}{v_{max}} + \frac{v_0^2 + v_{target}^2}{2v_{max}a_{max}} \end{aligned} \quad (1.3)$$

Section 1.1.2: Situation 2: Maximum Speed Not Reached

In this situation, the motor does not reach v_{max} and thus follows a bang-bang trajectory. The velocity of the motor over time in this situation is shown in Fig. 1.2.

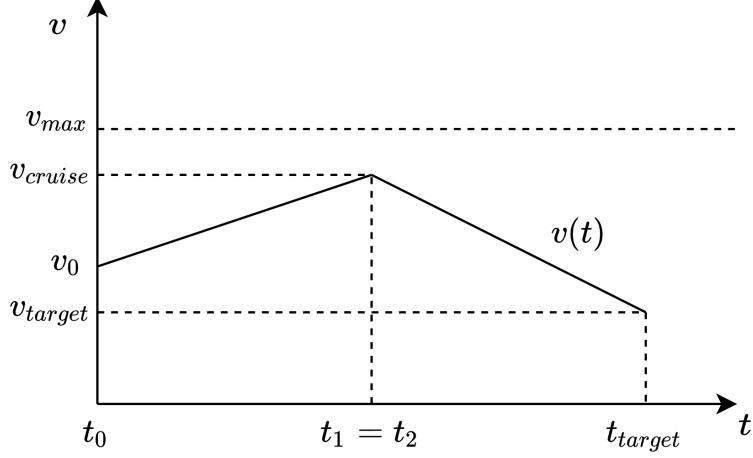


Figure 1.3: Bang-bang trajectory when v_{max} is not reached.

Because v_{max} is not reached, its maximum reached speed is (currently) unknown. This speed, v_{cruise} , has to be chosen such that the desired distance is exactly traversed with the bang-bang trajectory. This can be done by expressing the distance as a function of v_{cruise} , which can be done by replacing all terms of v_{max} in Eqs. (1.1) and (1.2) with v_{cruise} . The end result is shown in Eq. (1.4).

$$\begin{aligned}
 t_{01} &= \frac{v_{cruise} - v_0}{a_{max}} \quad (\text{N.B.: } t_{12} = 0) \\
 t_{2target} &= \frac{v_{cruise} - v_{target}}{a_{max}} \\
 s_{01} + s_{2target} &= s_{target} = \frac{2 v_{cruise}^2 - v_0^2 - v_{target}^2}{2 a_{max}} \Rightarrow v_{cruise} = \sqrt{s_{target} a_{max} + \frac{1}{2}(v_0^2 + v_{target}^2)} \\
 t_{0target} &= t_{01} + t_{2target} = \frac{v_{cruise} - v_0}{a_{max}} + \frac{v_{cruise} - v_{target}}{a_{max}} = \frac{2 v_{cruise} - v_0 - v_{target}}{a_{max}} \\
 &= \frac{2 \sqrt{s_{target} a_{max} + \frac{1}{2}(v_0^2 + v_{target}^2)} - v_0 - v_{target}}{a_{max}}
 \end{aligned} \tag{1.4}$$

Section 1.2: Scaling Factor

This section describes the calculations for the scaling factor based on a time requirement. The time requirement is calculated from a time estimation, described in Section 1.1. In order to synchronize different motors, the following procedure is followed:

- Calculate the duration of the movement of each motor to its target position.
- For all motors that arrive before the slowest motor, decrease their v_{max} such that they arrive at the same time as the slowest motor.

The ratio by which v_{max} is reduced is denoted with α . In Section 1.1, essentially two types of trajectories are discussed: bang-coast-bang trajectories (v_{max} was reached) and bang-bang trajectories (v_{max} was not reached). A bang-coast-bang trajectory with a lower v_{max} always results in another bang-coast-bang trajectory, whereas a bang-bang trajectory now also must result in a bang-coast-bang trajectory (otherwise nothing was altered). Thus, all new trajectories follow the trajectory and calculations from Section 1.1.1 (except for the slowest motor, which may follow a bang-bang trajectory).

All motors reach their (scaled) v_{max} and the goal is to reach the target at t_{req} , where it must be that $t_{req} > t_{fastest}$ ¹ with $t_{fastest}$ being the fastest trajectory as derived in Eq. (1.3). Then, if v_{max} is scaled with a factor α , the time to the target is found as:

$$\begin{aligned} t_{req} = t_{0target} &= \frac{\alpha v_{max} - v_0 - v_{target}}{a_{max}} + \frac{s_{target}}{\alpha v_{max}} + \frac{v_0^2 + v_{target}^2}{2 \alpha v_{max} a_{max}} \\ \alpha t_{req} &= \frac{\alpha^2 v_{max}}{a_{max}} - \alpha \frac{v_0 + v_{target}}{a_{max}} + \frac{s_{target}}{v_{max}} + \frac{v_0^2 + v_{target}^2}{2 v_{max} a_{max}} \\ 0 &= \underbrace{\frac{v_{max}}{a_{max}}}_{a} \alpha^2 + \underbrace{\left(-\frac{v_0 + v_{target}}{a_{max}} - t_{req} \right)}_{b} \alpha + \underbrace{\frac{s_{target}}{v_{max}} + \frac{v_0^2 + v_{target}^2}{2 v_{max} a_{max}}}_{c} \end{aligned}$$

This quadratic equation can easily be solved, yielding two solutions:

$$\begin{aligned} \alpha &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{\frac{v_0 + v_{target}}{a_{max}} + t_{req} \pm \sqrt{\left(\frac{v_0 + v_{target}}{a_{max}} + t_{req} \right)^2 - 4 \left(\frac{s_{target}}{a_{max}} + \frac{v_0^2 + v_{target}^2}{2 a_{max}^2} \right)}}{2 \frac{v_{max}}{a_{max}}} \\ &= \frac{v_0 + v_{target} + t_{req} a_{max} \pm \sqrt{(v_0 + v_{target} + t_{req} a_{max})^2 - 2(2 s_{target} a_{max} + v_0^2 + v_{target}^2)}}{2 v_{max}} \end{aligned}$$

For a given t_{req} , an increasing s_{target} should result in an increasing α . As the square root gets smaller, this must mean the only correct solution is where we subtract the square root². Thus, the correct equation for α is shown in Eq. (1.5).

$$\alpha = \frac{v_0 + v_{target} + t_{req} a_{max} - \sqrt{(v_0 + v_{target} + t_{req} a_{max})^2 - 2(2 s_{target} a_{max} + v_0^2 + v_{target}^2)}}{2 v_{max}} \quad (1.5)$$

¹Note that the slowest motor is excluded from these calculations, as this motor might follow a trajectory. Its scaling factor is already known to be zero.

²Note that the extra solution was added through the square in the beginning of this section.

Chapter 2: Steady Head Calculations

This chapter describes the calculations used to keep the tip of the transducer in the same location in several dimensions. That is, while the transducer might change its orientation (i.e., yaw, pitch and roll) and its x -coordinate, the y -coordinate can be kept constant. These calculations make use of homogeneous transformations, which allow for both translations and rotations between frames using matrix multiplications. For a world frame \mathcal{N} and body frame \mathcal{B} , one can express a vector ${}^{\mathcal{B}}\mathbf{x}$ in the world frame with:

$$\underbrace{\begin{bmatrix} {}^{\mathcal{N}}\mathbf{x} \\ 1 \end{bmatrix}}_{{}^{\mathcal{N}}\mathbf{x}_e} = \underbrace{\begin{bmatrix} {}^{\mathcal{N}}R_{\mathcal{B}} & {}^{\mathcal{N}}\mathbf{r}_{\mathcal{B}} \\ \mathbf{0}^{1 \times 3} & 1 \end{bmatrix}}_{{}^{\mathcal{N}}T_{\mathcal{B}}} \underbrace{\begin{bmatrix} {}^{\mathcal{B}}\mathbf{x} \\ 1 \end{bmatrix}}_{{}^{\mathcal{B}}\mathbf{x}_e}$$
$$= \begin{bmatrix} {}^{\mathcal{N}}R_{\mathcal{B}} {}^{\mathcal{B}}\mathbf{x} + {}^{\mathcal{N}}\mathbf{r}_{\mathcal{B}} \\ 1 \end{bmatrix} \quad (2.1)$$

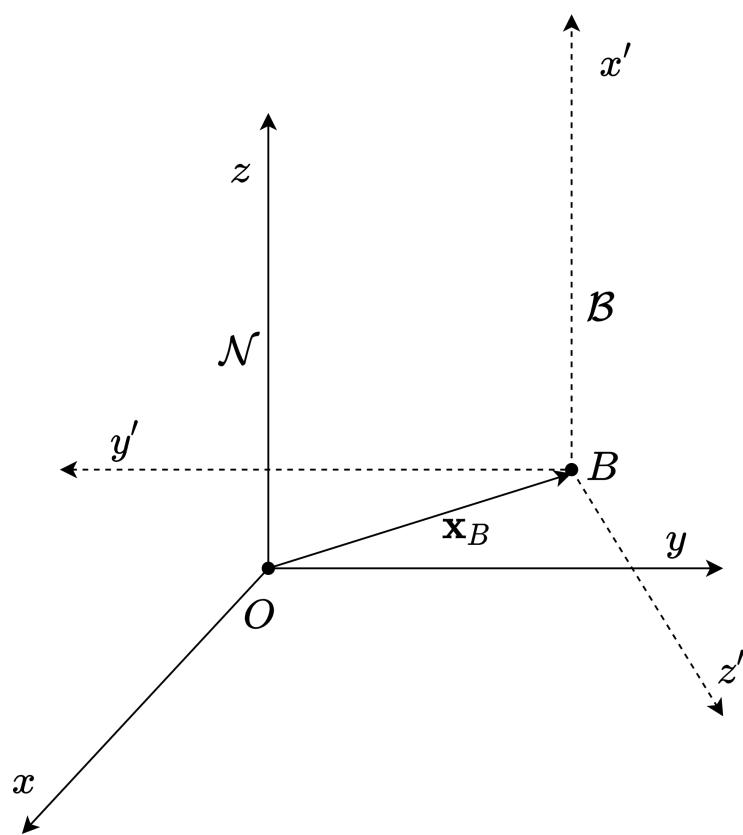


Figure 2.1: Different world frames.