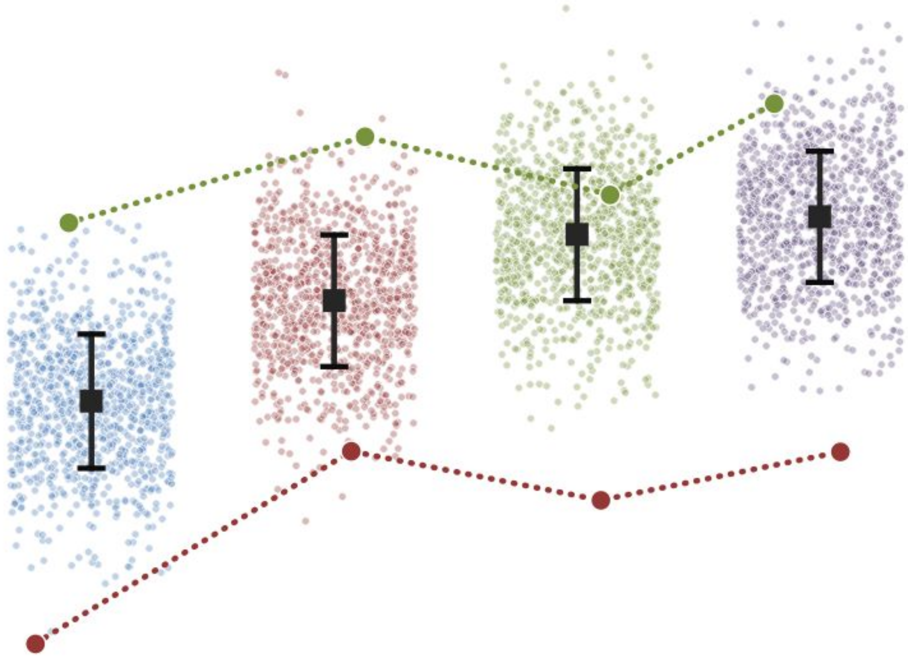
Sphericity Assumption

Sphericity is a less strict version of **Compound Symmetry**; both are patterns we assume about (or impose onto) the *covariance matrix* (a sort of table with the variances and covariances). By adding the **Person** effect, we could assume a **Compound Symmetry** pattern for instance. In that case we'd assume that the variances of each time-point are the same, likewise we can assume that the covariances, or correlations, between all time-points are the same. It doesn't matter if you compare T1 with T2 or T1 with T4, a high score on one means a high score on the other (or a low score if the correlation is negative). There's no magic or math to it, we assume, we decide, that this is the pattern and we have.

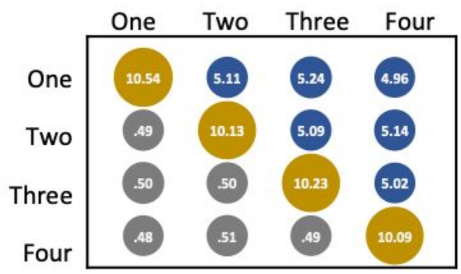
Such a dataset would look like the image below. Over time we see an increase in the score (the average goes up, which technically isn't necessary but makes the example easier). This increase is pretty much the same for each person, the variances within each time-point don't change at all. The **highest scoring** and **lowest scoring** individuals differ by more or less the same, they both have the same time effect.

We can see the *Covariance Matrix* below, with the **variances** on the diagonal, **covariances** on the off diagonal (the top), and I transformed the bottom covariances into **correlations** (correlations are just standardized covariances).



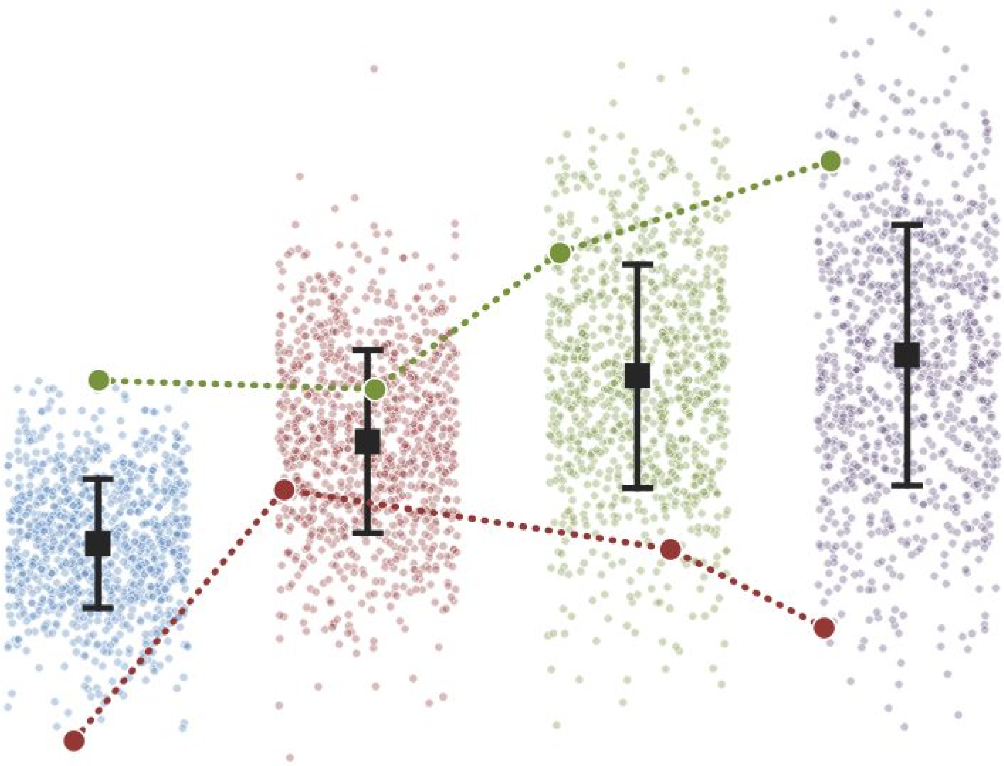
Compound Symmetry

	One	Two	Three	Four
One	10.54	5.11	5.24	4.96
Two	.49	10.13	5.09	5.14
Three	.50	.50	10.23	5.02
Four	.48	.51	.49	10.09



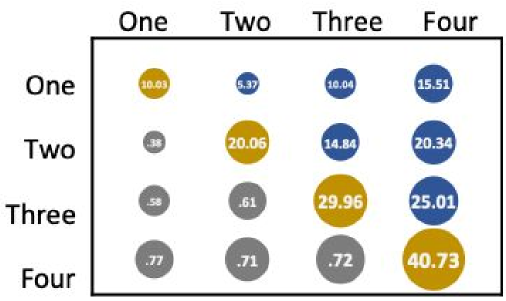
Looks nice, but it's a tad bit unrealistic for real-world data. That's why we instead have **Sphericity**. This assumption says that the **variances of the difference scores are equal**; if they are, we can pool the Standard Error without introducing any kind of bias. It is less strict than **Compound Symmetry** because it doesn't assume the correlations and variances are all the same. Instead it assumes **groups with larger variances will have larger correlations between them**.

The next image has the same set-up but now shows one version of Sphericity (there are many ways Sphericity can be met). Again, we have increasing means with variances on the diagonal and covariances/correlations off the diagonal. At the first measurement all participants score very similar, not a lot of variation there, at the second measurement they start to diverge, and in the final measurement we have a lot of variation. In the beginning the correlation isn't so big yet, but as the time-effects starts setting in, the correlation increases.



Sphericity

	One	Two	Three	Four
One	10.03	5.37	10.03	15.51
Two	.38	20.06	14.84	20.34
Three	.58	.61	29.96	25.01
Four	.77	.71	.72	40.73



Imagine this being an experiment with a treatment. It works better for some people, but not so much for others. Over time the individual differences will become larger, as some people change more than others. At the start of the treatment our participants will score more or less the same, no treatment has been given. After a few treatments some of them start to improve, but others do not. The effect is small at first, so the variation is still a bit low and the correlation as well. Give it few treatments and differences increase, having a high score on the third treatment means you'll be very likely to score high on the fourth as well (since you are one of the people who responds well to the treatment).

How does this relate to difference scores? If we take the difference between each time-point (Q1-Q2, Q1-Q3, Q1-Q4, Q2-Q3, Q2-Q4, Q3-Q4), we'll see that some people have a larger change score than others (we've established that treatment for example works better for some that others). This change is very consistent though, if you are responsive to treatment you will have a larger difference than someone who isn't responsive, no matter which two you compare. **The variance of those change scores? That needs to be the same!** You can do the same for the compound symmetry scores, by definition those too will have equal variances on the change scores (since the change is the same for everyone).