Homework #4

Train set

x_1	1	2	3	4
y	2	5	6	7.5

Compute a linear regression model with no intercept and determine its MAE on the training set.

Fabian Gobet
MAI
13 March 2024

$Y = X_1\beta + \varepsilon$

$y_i = X_{1i}\beta + \varepsilon_i$

$\varepsilon_i = y_i - X_{1i}\beta$

$$\beta^* = (X_1^T X_1)^{-1} X_1^T y =$$

$$= \left(\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \\ 7.5 \end{bmatrix} =$$

$$= \frac{1}{25}(2+10+18+30) = \frac{60}{25} = \frac{12}{5}$$

$Y^* = \frac{12}{5} X$

i	Y	X_1	Y^*	$\lvert Y - Y^* \rvert$
1	2	1	$\frac{12}{5}$	$2/5$
2	5	2	$\frac{24}{5}$	$1/5$
3	6	3	$\frac{36}{5}$	$6/5$
4	7.5	4	$\frac{48}{5}$	$21/10$

$$MAE = \frac{1}{|X_1|} \sum_{i=1}^{4} |y_i - y_i^*| = \frac{1}{4}\left(\frac{21+4+2+12}{10}\right) = \frac{39}{40}$$