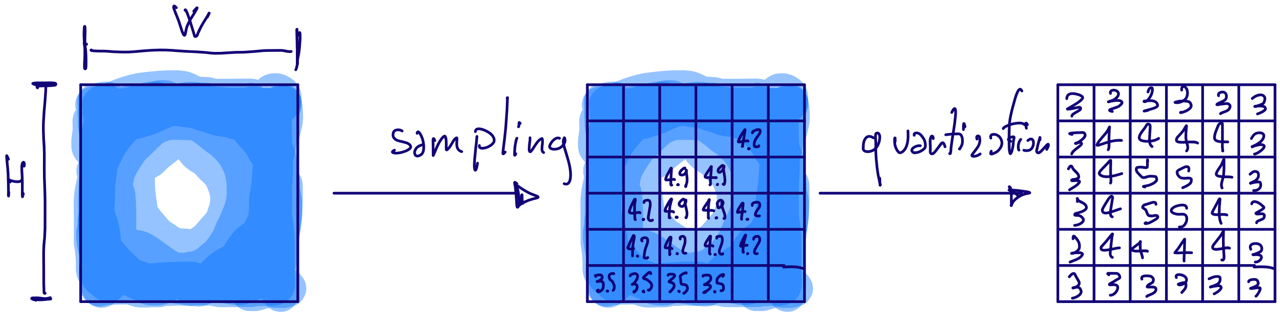
IMAGE PROCESSING BASICS

○ SAMPLING AND QUANTIZATION



$0, L$ intensity interval

$\Delta H = H / M$

$\Delta W = W / N$

for i in $(0, M-1)$

  for j in $(0, N-1)$:

    $x = (i + 0.5) \cdot \Delta H$

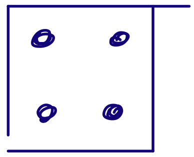
    $y = (j + 0.5) \cdot \Delta W$

    $f_{i,j} = F(x, y)$ // SAMPLING

    $a_{i,j} = \lfloor L \cdot f_{ij} \rfloor$ // QUANTIZATION

⊙ SUPERSAMPLING

- CALCULATE MEAN OF MULTIPLE POINTS IN ONE PIXEL



$$\frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

- SPATIAL INTEGRATION

$$f_{i,j} = \frac{1}{\Delta W \cdot \Delta H} \int_{j \Delta W}^{(j+1)\Delta W} \int_{i \Delta H}^{(i+1)\Delta H} f(x,y)\, dx\, dy$$

⊙ SOLVE QUANTIZATION ISSUES

$a_{ij} = \lfloor L \cdot f_{ij} \rfloor$

IF Range is 0 to $L-1$

if f_{ij} is max, then $f_{ij} = L > L-1$!

Sol 1

round$(L \cdot f_{ij} - 0.5)$ only if $r(0.5) = 0$!

Sol 2

round$((L-1) f_{ij})$ works but extreme bins are small

Sol 3 INCREASE DYNAMIC RANGE

$$a_{i,j} = \left\lfloor L \cdot \frac{f_{ij} - f_*}{f^* - f_*} \right\rfloor \qquad f_* = \min_{x,y} f(x,y) \qquad f^* = \max_{x,y} f(x,y)$$

IMAGE SIZE

Data $M \cdot N \cdot K$ where $L = 2^K$

Spatial $1 \text{ inch} = 2.54 \text{ cm}$ dpi

$X \cdot Y$ image @ n dpi

$\frac{X}{dpi}$ = size in inch width

$\frac{Y}{dpi} \cdot 2.54$ = size in cm height

ZOOM IN (repetition)

in: A_1 $M_1 \times N_1$

out: A_2 $M_2 \times N_2$

scale factor z

so

$M_2 = z M_1$ $N_2 = z N_1$

for i in $(0, M_1 - 1)$
  for j in $(0, N_1 - 1)$
    for k in $(0, z-1)$
      for l in $(0, z-1)$
        $a^2_{z \cdot i + k, zj + l} = a^1_{i,j}$

ZOOM OUT (averaging)

in: A_1 $M_1 \times N_1$

out: A_2 $M_2 \times N_2$

scale factor z

So

$M_1 = z M_2$ $N_1 = z N_2$

For i in $(0, M_2 - 1)$
  for j in $(0, N_2 - 1)$
    $a^2_{i,j} = 0$
    for k in $(0, z-1)$
      for l in $(0, z-1)$
        $a^2_{ij} = a^2_{i,j} + a^1_{zi+k, zj+l}$
    $a^2_{i,j} = a^2_{i,j} / (z^2)$

SCALING BY FACTOR c (GENERAL SAMPLING)

in A_1 $M_1 \times N_1$

out A_2 $M_2 \times N_2$

scaling c

so

$M_2 = cM_1$ $N_2 = cN_1$

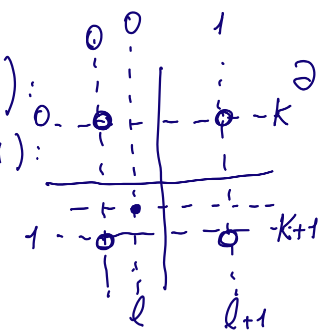
---

for i in $(0, M_2-1)$
  for j in $(0, N_2-1)$
    $x = (i+0.5)/c$ // continuous pixel coords
    $y = (j+0.5)/c$ //
    $k = \lfloor x \rfloor$ // nearest neighbour interpol
    $\ell = \lfloor y \rfloor$ //
    $a^2_{ij} = a^1_{k,\ell}$

~ ALTERNATIVE TO NEAREST NEIGHBOUR INTERPOL ~

BILINEAR

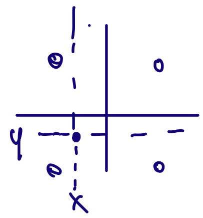
for i in $(0, M_2-1)$:
  for j in $(0, N_2-1)$:
    $x = (i+0.5)/c$
    $y = (j+0.5)/c$
    $k = \text{round}(x) - 1$
    $\ell = \text{round}(y) - 1$
    $u = x - (k+0.5)$
    $v = y - (\ell+0.5)$
    $a^2_{ij} = \text{round}\Big((1-v)(1-u)\,a^1_{k,\ell} + (1-v)(u)\,a^1_{k+1,\ell} + (v)(1-u)\,a^1_{k,\ell+1} + (v)(u)\,a^1_{k+1,\ell+1}\Big)$



$$a^2_{i,j} = a^1_{k,\ell}(1-u)(1-v) + a^1_{k+1,\ell}(u)(1-v) + a^1_{k,\ell+1}(u-1)(v) + a^1_{k+1,\ell+1}(u)(v)$$

BILINEAR WITH INTEGRAL

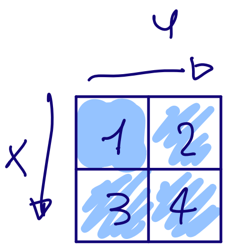
$$\partial_{i,j}^{2} = \int_{y-0.5}^{y+0.5} \int_{x-0.5}^{x+0.5} F_1(s,t)\, ds\, dt$$



GEOMETRIC TRANSFORMATIONS

- SCALING

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} C_x & 0 \\ 0 & C_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = C \begin{pmatrix} x \\ y \end{pmatrix}$$



scale $C_x = 2$

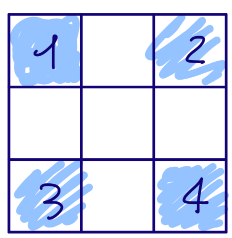
$C_y = 2$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$



- TRANSLATION

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \end{pmatrix}$$

- ROTATION

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix}$$

- VERTICAL SHEAR

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + S_v y \\ y \end{pmatrix} = \begin{pmatrix} 1 & S_v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = S_v \begin{pmatrix} x \\ y \end{pmatrix}$$

- HOROZONTOL SHEAR

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y + S_h x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ S_h & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = S_h \begin{pmatrix} x \\ y \end{pmatrix}$$

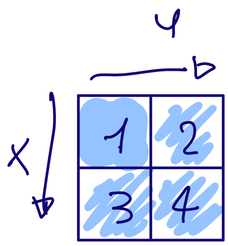
- GEOMETRIC AFFINE TRANFORMATION

From Cartesian $\Psi : \mathbb{R}^2 \to \mathbb{R}^2$

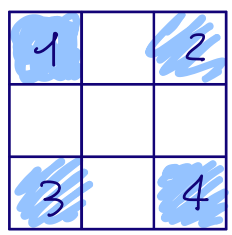
To $\begin{pmatrix} \Psi \\ 1 \end{pmatrix}$

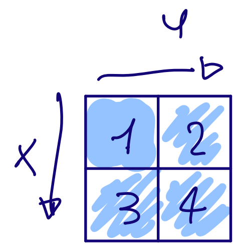
- SCALING

$$C(C_x, C_y) = \begin{pmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad C(C_x, C_y)^{-1} = C\left(\frac{1}{C_x}, \frac{1}{C_y}\right)$$



$C_x = C_y = 2$





$$\begin{pmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

INVERSE

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- TRANSLATION

$$T(t_x, t_y) = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \quad T(t_x, t_y)^{-1} = T(-t_x, -t_y)$$

- ROTATION

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R(\theta)^{-1} = R(-\theta)$$

- VERTICAL SHEAR

$$S_v(\lambda) = \begin{pmatrix} 1 & \lambda & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad S_v(\lambda)^{-1} = S_v(-\lambda)$$

- HORIZONTAL SHEAR

$$S_h(\lambda) = \begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad S_h(\lambda)^{-1} = S_h(-\lambda)$$

FORWARD MAPPING

Transform A_1 into A_2 using A

- For all pixels (x', y') in A_2 use A^{-1} to compute (x, y) in A_1 and get an intensity at (x', y') using an interpolation method

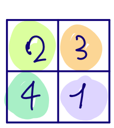
3-SHEARS = ROTATION

$$R(\theta) = S_v(\lambda)\, S_h(\nu)\, S_v(\lambda)$$

$$\lambda = -\tan\frac{\theta}{2} \qquad \nu = \sin\theta$$

for $-\pi < \theta < \pi$

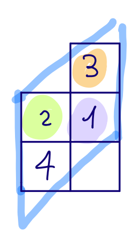
ex: $30° = \theta$ $\quad \lambda = -0.85 \quad \nu = 0.6560$



vert 1

$$\begin{pmatrix} 1 & \lambda & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \lambda & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & \lambda & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda \\ 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \lambda & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+\lambda \\ 1 \\ 1 \end{pmatrix}$$

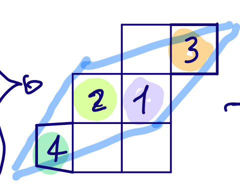


$$\begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$



+ STEP 3

- CHANGE INTENSITY RESOLUTION

map $\{0, \dots, L_1-1\}$ to $\{0, \dots, L_2-1\}$

$$r \rightarrow \text{round}\left(\left(r+0.5\right)\frac{L_2}{L_1} - 0.5\right)$$

ex:

$$in = 23 \rightarrow \text{round}\left(\left(23+0.5\right)\frac{255}{100} - 0.5\right) = 59$$

- REMOVE NOISE (TEMPORAL INTENSITY SUPERSAMPL)

sample the source k times

$$\bar{A}_K = \frac{1}{k}\sum_{i=1}^{K} A_i$$

$k \rightarrow \infty$
NO NOISE!

Recoursive

$$\bar{A}_{K+1} = \frac{K \cdot \bar{A}_K + A_{K+1}}{K+1}$$

if uncorrelated noise with zero μ and STD

then $$\sigma_{\bar{A}_K} = \frac{1}{\sqrt{K}}\sigma_\eta$$

INTENSITY TRANSFORMATIONS

$T: I \to I$ usually bijective

injectivity: all intensity in f is used

surjectivity: all intensity in g is used

- DISCRETE SETTING

$$\hat{T}: \{0, \dots, L-1\} : \{0, \dots, L-1\}$$

$$\hat{T}(r) = \text{round}\left(L \cdot T\left(\frac{r+0.5}{L}\right) - 0.5\right)$$

NEGATIVE

$$T(r) = 1 - r \qquad \hat{T}(r) = L - 1 - r$$

LOG TRANSFORM

enhance contrast in dark regions

$$T(r) = a \log(r/\beta + 1)$$

- $a = 1/(\log(1/\beta + 1))$
- $b = 1/(\exp(1/a) - 1)$

EXP TRANSFORM

enhance contrast in bright region

$$T(r) = \beta\left(\exp(r/a) - 1\right)$$

- $a = 1/(\log(1/\beta + 1))$
- $b = 1/(\exp(1/a) - 1)$

GAMMA CORRECTION

$$T(r) = r^{\gamma} \qquad \gamma > 0$$

if $\gamma < 1$ enhance contrast in dark

if $\gamma > 1$ enhance contrast in light

CONTRAST STRECHING

Medium intensity contrast enhance

$$T(r) = \frac{1}{2}\left(\frac{\arctan(\delta(2r-1))}{\arctan(\delta)} + 1\right) \qquad \delta > 0$$

or

$$T(r) = \frac{1}{2}\left(\lambda\frac{\arctan(\delta(2r-1))}{\arctan(\delta)} + 1\right) \qquad \delta > 0, \quad \lambda \geq 1$$

THREE PIECIES

$s_1 = 0$

$s_2 = 1$

r_1 = tresh 1

r_2 = tresh 2

$T(r)$ = piecewise, bijective, linear function

INTENSITY LEVEL SLICING

highlight one area

NORMALIZE HISTOGRAM

$$p_K = \frac{h_K}{M \cdot N} \qquad \sum_{K=0}^{L-1} p_K = 1$$

HISTOGRAM EQUALIZATION

Make histogram as uniform as possible

continuous setting $p(r) \geq 0$ probability density funct

with $\int_0^1 p(r)\,dr = 1$

at a position (x,y), $f(x,y)$

has to be between r_1 and r_2

ad is $\int_{r_1}^{r_2} p(r)\,dr$

$$q(s) = \frac{p(r)}{T'(r)},$$

$$q(s) = 1 \qquad T'(r) = p(r)$$

digital setting

$$p(r/L) = L \cdot p_K \qquad \text{for } r \in [K, K+1]$$

then $$T(K/L) = \sum_{j=0}^{K-1} p_j$$

for $K = 0, .., L$

FORMULA : $$\hat{T}(K) = \text{round}(\tilde{S}_K)$$

$$\tilde{S}_K = L \sum_{j=0}^{K-1} p_j + \frac{L}{2} p_K - \frac{1}{2}$$

$K = 0, \dots, L-1$

HISTOGRAM MATCHING

$$T = T_q^{-1} \circ T_p \qquad T_p(k/L) = \sum_{j=0}^{k-1} p_j$$

$$T_q^{-1} = \frac{K}{L}$$

LINEAR SPATIAL FILTERING

- box $a = b$, $m = n = 2a+1$, $W_{s,t} = \frac{1}{m^2}$

- gaussian $a = b$

$$W_{s,t} = \frac{\exp\left(-\frac{s^2+t^2}{2\sigma^2}\right)}{-a\left(\exp\left(-\frac{s^2+t^2}{2\sigma^2}\right)\right)}$$

PADDING

- zero
- mirror
- replicate

PERFORMANCE

$$O(MNmn)$$

can be improved with separable kernel

- MEDIAN FILTER (KERNEL)

  Median value of Kernel window

- LAPLACIAN (BIVARIATE)

$$L = l \star A \qquad l = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$l' = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- PREWITT (KERNEL) (Box smoothing

$$g_x = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad g_y = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Gradient magnitude

$$\sqrt{(g_x \star A)^2 + (g_y \star A)^2}$$

- SOBEL (Gaussian smoothing)

$$g_x = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad g_y = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

gradient magnitude

$$\sqrt{(g_x * A)^2 + (g_y * A)^2}$$

- BASIC BINARIZE

$$\begin{cases} 0 & \text{if } A_{ij} \leq t \\ 1 & \text{else} \end{cases}$$

Treshold selection

- Fit poly or Fit gauss. dists

- IMAGE SEGMENTATION

$\bigcup_{i=1}^{n} R_i = R$

R_i connected set $i = 0, \dots, n$

$R_i \cap R_j = \emptyset$ for all i, j

If same value but different regions $\Rightarrow$ different sets

- SIGNAL SMOOTHING (square Kernel)

h = Kernel size

$$g_i = \frac{1}{h}\int_{i-h/2}^{i+h/2} f(x)\,dx$$

- AVG SMOOTHING ($n \cdot m$ Kernel)

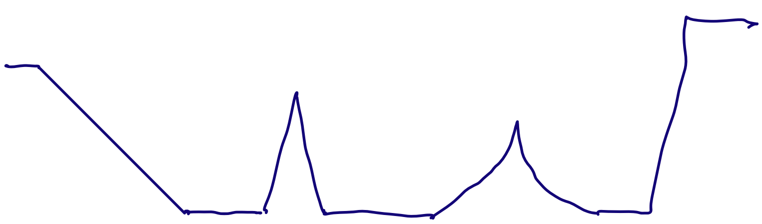
$$B_{ij} = \frac{1}{n\cdot m}\int_{i-n/2}^{i+n/2}\int_{j-m/2}^{j+m/2} A(s,t)\,dt\,ds$$

$$B_{ij} = \sum_{s=-n/2}^{n/2}\ \sum_{t=-m/2}^{m/2} A_{ij}\, w(i+s,\ j+t)$$

where

$$w = \begin{bmatrix} 1/n\cdot m & \cdots & 1/n\cdot m \\ \vdots & \ddots & \vdots \\ 1/n\cdot m & \cdots & 1/n\cdot m \end{bmatrix}$$

- USE 1st DERIVATIVE TO DETECT regions



$$\text{Der: } 0\ \underbrace{-1\,-1\,-1}_{\text{Ramp}}\ 0\ 0\ \underbrace{4\,-4}_{\text{Isolated point}}\ 0\ 0\ 0\ \underbrace{-1\,-2\,2\,1}_{\text{Line}}\ 0\ 0\ \underbrace{7}_{\text{Step}}\ 0\ 0$$

CANNY EDGE DETECTION

1) Integration to smooth img

2) Differentiation to compute

$$G = \sqrt{G_x^2 + G_y^2} \qquad \Theta = \operatorname{atan} \frac{G_y}{G_x}$$

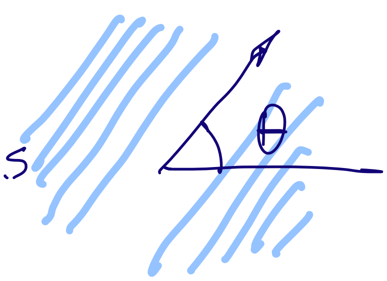
$$G_x = W_x \star A \qquad W_x = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$G_y = W_y \star A \qquad W_y = \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

3) Non maxima suppression

- Group angles

$$\Theta'_{ij} = \begin{cases} 0 & -22.5 \le \; < -22.5 \\ 45 & 22.5 \le \; < 67.5 \\ 90 & 67.5 \le \; < 112.5 \\ 135 & 112.5 \le \; < 157.5 \end{cases}$$



4) Magnitude comparison

if $\theta'_{ij} = 0$

  if $G_{ij} < G_{(i-1)j}$ or $G_{ij} < G_{(i+1)j}$:

    $G'_{ij} = 0$

if $\theta'_{ij} = 45$

  if $G_{ij} < G_{(i-1)(j+1)}$ or $G_{(i+1)(i-j)}$

    $G'_{ij} = 0$

if $\theta'_{ij} = 90$

  if $G_{ij} < G_{(i-1)j}$ or $G_{ij} < G_{(i+1)j}$

    $G'_{ij} = 0$

if $\theta'_{ij} = 135$

  if $G_{ij} < G_{(i-1)j}$ or $G_{ij} < G_{(i+1)j}$

    $G'_{ij} = 0$

else

  $G'_{ij} = G_{ij}$

5) Hysteresis tresholding

choose 2 tresholds

and form an image with

points of intensity $> t_h$

and 1 other image of points

of intensity $< t_l$

Label points as 2 if $G'_{ij} \geq t_h$

1 if $t_l \leq G'_{ij} \leq t_h$

0 if $G'_{ij} \leq t_l$

- Connectivity analysis

Using result of Hysteresis

1) Choose a point in class 2

2) and recursively grow seed to include neighbours with magnitude ≥ 1

3) Repeat 1 and 2 until all values in class 2 are included

HARRIS-STEPHENS CORNER DETECTOR

- Sum of square differences

$$C(x,y) = \sum_{s=-n/2}^{n/2} \sum_{t=-m/2}^{m/2} \left[A(s+x,t+y) - A(s,t)\right]^2$$

1 0

or

0 1

- weighted sum of square differences

$$C(x,y) = \sum_{s=-n/2}^{n/2} \sum_{t=-m/2}^{m/2} W(s,t)\left[A(s+x,t+y) - A(s,t)\right]^2$$

Smoothing kernel

- bilinear approximation

$$\boxed{A(s+x,t+y)} \approx A(s,t) + x\,G_x(s,t) + y\,G_y(s,t)$$

$$G_x = A \star \begin{bmatrix} -0.5 \\ 0 \\ 0.5 \end{bmatrix} \qquad G_y = A \star \begin{bmatrix} -0.5 & 0 & 0.5 \end{bmatrix}$$

$$C(x,y) = \sum_{s=-n/2}^{n/2} \sum_{t=-m/2}^{m/2} W(s,t)\left[\boxed{A(s+x,t+y)} - A(s,t)\right]^2$$

Smoothing kernel

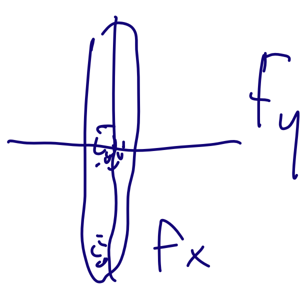
[illegible]

$$C(x,y) = \begin{bmatrix} x & y \end{bmatrix} M \begin{bmatrix} x \\ y \end{bmatrix}$$

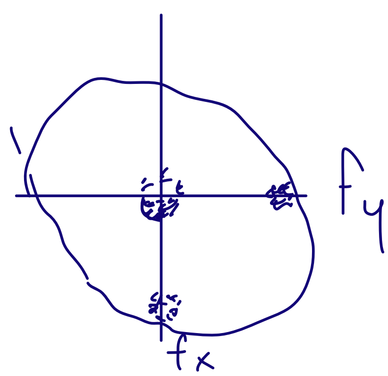
$$M = \sum_s \sum_t w(s,t) \begin{bmatrix} G_x(s,t)^2 & G_x(s,t)\,G_y(s,t) \\ G_x(s,t)\,G_y(s,t) & G_y(s,t)^2 \end{bmatrix}$$

covariance matrix representing the form $(Ax^2 + Bxy + Cy^2)$

So if we compute eigenvectors of Harris matrix, we get:



straight edge



corner

We can summarize eigenvalue computation by:

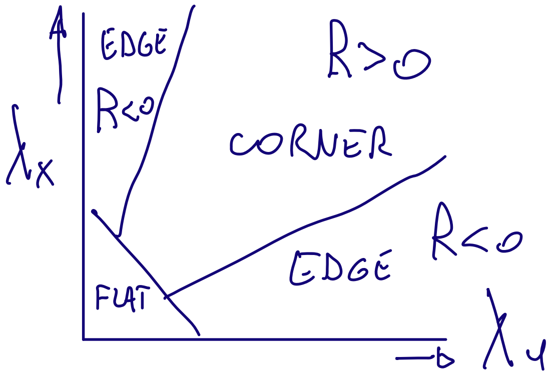
$$R = \lambda_x \lambda_y - K(\lambda_x + \lambda_y)^2 = \det(M) - K\,\mathrm{trace}^2(M) \qquad K \in [0.04, 0.06]$$

$$\det(M) = G_x(s,t)^2\, G_y(s,t)^2 - [G_x(s,t)\, G_y(s,t)]^2 = \lambda_x \lambda_y$$

$$\mathrm{trace}(M) = G_x(s,t)^2 + G_y(s,t)^2 = \lambda_x + \lambda_y$$

$$G_x = A * \begin{bmatrix} -0.5 \\ 0 \\ 0.5 \end{bmatrix} \qquad G_y = A * \begin{bmatrix} -0.5 & 0 & 0.5 \end{bmatrix}$$

- READ RESULTS:



- LIGHT TYPES:

monochrome : 1 wavelength

achromatic : all wavelength $\approx$ equal

chromatic : all spectrum with dominant length

- XYZ COLOR SPACE

$$\mathbf{X} = \int \underbrace{\bar{x}(\lambda)\, I(\lambda)}\, d\lambda \qquad \text{L cone} = x$$

$$\mathbf{Y} = \int \underbrace{\bar{y}(\lambda)\, I(\lambda)}\, d\lambda \qquad \text{M cone} = y$$

$$\mathbf{Z} = \int \bar{z}(\lambda)\, I(\lambda)\, d\lambda$$

SPD of light

S cone = z

- xyY COLOR SPACE

$$x = \frac{\mathbf{X}}{\mathbf{X}+\mathbf{Y}+\mathbf{Z}} \qquad y = \frac{\mathbf{Y}}{\mathbf{X}+\mathbf{Y}+\mathbf{Z}} \qquad z = \frac{\mathbf{Z}}{\mathbf{X}+\mathbf{Y}+\mathbf{Z}} = 1-x-y$$

So we use

x, y and $Y \rightarrow$ luminance

- RGB

$R: \{0, \ldots, 255\}$

$G: \{0, \ldots, 255\}$

$B: \{0, \ldots, 255\}$

CONVERTION TO XYZ

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.06 & -1.39 & -0.48 \\ -0.97 & 1.88 & 0.04 \\ 0.07 & -0.23 & 1.07 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

- RGB $\leftrightarrow$ CMY

$$\begin{pmatrix} C \\ M \\ Y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} R \\ G \\ B \end{pmatrix} \qquad \begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} C \\ M \\ Y \end{pmatrix}$$

- CMY TO CMYK

$$(C\,M\,Y) = \frac{(C-K,\ M-K,\ Y-K)}{(1-K)}$$

H hue 0, 360
S saturat 0,1
I intens 0,1

RGB to HSI

$$H = \begin{cases} \theta & \text{if } B \le G \\ 360 - \theta & \text{if } B > G \end{cases}$$

$$\theta = \arccos\left(\frac{\frac{1}{2}\left[(R-G)+(R-B)\right]}{\sqrt{(R-G)^2+(R-G)(G-B)}} \right)$$

$$S = 1 - \frac{3\min(R,G,B)}{R+G+B} = 1 - \frac{\min(R,G,B)}{I}$$

$$I = \frac{R+G+B}{3}$$

HSI to RGB

if $H \in [0, 120)$

$B = I(1-S)$

$R = I\left(1 - \frac{S\cos H}{\cos(60-H)}\right)$

$G = 3I - (R+B)$

elif $H \in [120, 240]$

$H = H - 120$

$R = I(1-S)$

$G = I\left(1 - \frac{S\cos H}{\cos(60-H)}\right)$

$B = 3I - (R+G)$

elif $H \in [240, 360)$

$H = H - 240$

$G = I(1-S)$

$B = I\left(1 - \frac{S\cos 60}{\cos(60-H)}\right)$

$R = 3I - (G+B)$

- PSEUDO COLORS

input

$f : X \times Y \to I$

$T_R, T_G, T_B : I \to I$

output

$g_R, g_G, g_B : X \times Y \to I$ where $g_*(x,y) = T_*(F(x,y))$

- CARTESIAN

(x, y)

- EUCLEDIAN

using cartesian

2-points $\Rightarrow$ line

2 lines are parallel = never touch

- BARYCENTRIC

$\cdot (1,0,0)$

$\cdot (0,25, -0,25, 1)$

$(0,1,0)$

$\cdot (0,0,1)$

- PROJECTIVE

homo coords

2 points $\to$ line

2 lines are parallel = touch

- ADD ELEMENTS IN P^n

$$\begin{pmatrix} x \\ v \end{pmatrix} \oplus \begin{pmatrix} y \\ w \end{pmatrix} = \begin{pmatrix} v & x \\ 0 & v \end{pmatrix} \begin{pmatrix} y \\ w \end{pmatrix} = \begin{pmatrix} vy + wx \\ vw \end{pmatrix}$$

$\odot$ MULTIPLY ELEMENT IN P^n $\begin{pmatrix} x \\ v \end{pmatrix} \odot \begin{pmatrix} y \\ w \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & v \end{pmatrix} \begin{pmatrix} y \\ w \end{pmatrix}$
$= \begin{pmatrix} xy \\ vw \end{pmatrix}$

CONICS

IMPLICIT FORM : $(x,y)^T \in \mathbb{R}^2 : ax^2 + bxy + cy^2 + dx + ey + f = 0$

DISCRIMINANT : $\delta = -4\Delta$ where $\Delta = \det M$
where $M = \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix}$

non-degenerate: $(a,b,c) \neq (0,0,0)$

Parabola $\delta = 0$

Ellipse $\delta < 0$

Circle $(a = c, b = 0)$ & $\delta < 0$

Hyperbola $\delta > 0$

Rectangular hyperb $\delta > 0$ & $a + c = 0$

- Point on conic with homo

if point $=(x,y,z)^T$ on conic homo

then

$$ax^2+bxy+cy^2+dxz+eyz+fz^2=0$$

other form

$$x^T C x=0 \qquad C=\begin{pmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{pmatrix}$$

- CONIC THEORY

Conic is uniquely defined by 5 points $X_i=(x_i,y_i,z_i)$ for $i=1,\dots,5$

each point places a constraint on the coefficient

$$(x_i^2, x_iy_i, y_i^2, x_iz_i, y_iz_i, z_i^2)\,c=0 \qquad c=(a,b,c,d,e)^T$$

IF 3 OF THESE X_i are collinear then not unique $\longrightarrow$ DEGENERATE

- DEGENERATE (Not unique conic)
  - $\delta = 0 \rightarrow$ two parallel lines (degenerate parabola)
  - $\delta = 0$ & $(d^2 + e^2 = 4(a+c)f) \rightarrow$ one (double line)
  - $\delta < 0 \rightarrow$ Single point (degenerate ellipse)
  - $\delta > 0 \rightarrow$ 2 intersecting lines
- if $\det \neq 0$, $\delta < 0$ and $(a+c)\det C > 0$ then imaginary ellipse $\Rightarrow$ empty in $\mathbb{R}^2$

- TANGENT LINE

  $l = Cx$ is line l tangent to C at x

  If all l is in C:

  $l \in C$, C must be degenerate

- DUAL CONIC

All lines tangent to C

So adjoint of conic C...

$M \cdot adj(M) = det(M) I$

where $M = (M^*)^T$ where M^* is the cofactor matrix

- $adj(M) = det(M) I \cdot M^{-1}$ if M invertible
- $\underline{adj(C) = C^* = C^{-1}}$

IF FULL RANK

l is tangent of C and element of C^*

$\Leftrightarrow l^T C^* l = 0$

PROJECTIVE TRANSFORMATION

$\mathbb{P}^2 \Rightarrow \mathbb{P}^2$

$h(x) = Hx$ where H is a non-singular matrix

H is homognous: so it can be scaled by any $\lambda \neq 0$

DEGREES OF FREEDOM

$H \in \mathbb{R}^{3 \times 3}$ has 8 degrees freedom

use 4 points in original and 4 in projected

~ NO 3 COLLINEAR ~

- TRANSFORMATION EXAMPLES

$$x \rightarrow x' = Hx$$

$$l \rightarrow l' = H^{-T} l$$

$$C \rightarrow C' = H^{-T} C H^{-1}$$

$$C^* \rightarrow C^{*\prime} = H C^* H^T$$

- ISOMETRY

$$H_1 = \begin{pmatrix} \epsilon \cos\theta & -\sin\theta & t_x \\ \epsilon \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \quad \epsilon = \pm 1$$

- $\epsilon = 1 \rightarrow$ orientation-preserving

$\epsilon = -1 \rightarrow$ orientation-reversing (reflection)

3 degrees of freedom $+ \epsilon$

INVARIANT AREA, DISTANCES!

• EUCLIDEAN TRANSFORMATION

$$H_E = \begin{pmatrix} R_\theta & t \\ 0^T & 1 \end{pmatrix} \qquad R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

• SIMILARITIES

$$H_S = \begin{pmatrix} \epsilon s\cos\theta & -s\sin\theta & t_x \\ \epsilon s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \quad \epsilon = \pm 1$$

4 degrees $+ \epsilon$

invariant angles

ratios of lengths

- AFFINE TRANSFORMATIONS

$$H_A = \begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} A & t \\ 0^T & 1 \end{pmatrix}$$

where $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is non-regular

6 degrees of freedom

Preserve lines & parallelism

- PROJECTIVE TRANSFORMATION

$$H_P = \begin{pmatrix} A & t \\ v^T & w \end{pmatrix}$$

$v^T = (v_1, v_2)$ and $\det H_P \neq 0$

8 deg of freedom

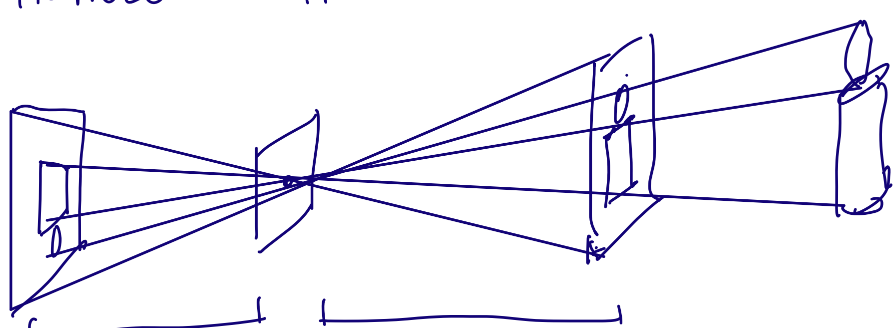
CAMERA GEOMETRY

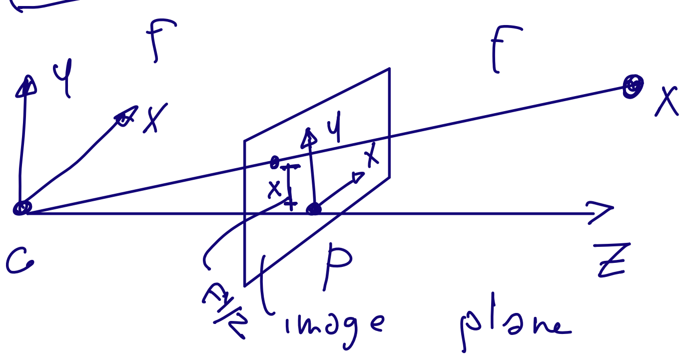
Map a world point $\mathbb{R}^3$ to an image point $\mathbb{R}^2$

- Projective transformation, represented by $P \in \mathbb{R}^{3\times4}$ matrix
- 11 DOF!

INTERNAL CAMERA PARAMS, $K \in \mathbb{R}^{3\times3}$
EXTERNAL: position & orientation

PINHOLE CAM





map world point $(x, y, z)^T$ into image plane

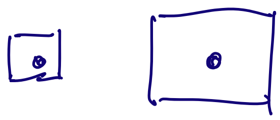
MAPPING: $X \mapsto x = PX$

$(fx, fy, f)^T$ in $\mathbb{R}^2_{\text{img}}$

$(X, Y, Z, 1)^T$ in $\mathbb{R}^3_{\text{world}}$

- CENTRAL PROJECTION (PINHOLE)

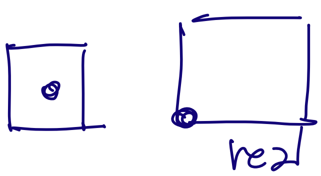
$$P = \operatorname{diag}(f, f, 1)[I \mid \mathbf{0}] = \begin{pmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{pmatrix}$$



central projection with focal length f

- OFFSET PROJECTION (PINHOLE)

$$P = K[I \mid \mathbf{0}], \quad K = \begin{pmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{pmatrix}$$



⊚ OFFSET, DIFFERENT ORIENTATION PINHOLE

R: Rotation matrix (orientation)

$\tilde{C}$: World coordinate of camera center

$$X = \begin{pmatrix} R & -R\tilde{C} \\ 0^T & 1 \end{pmatrix} \tilde{X}$$

world point

camera point

$$R = (\tilde{x}, \tilde{y}, \tilde{z})^T$$

$$X = P\tilde{X} \qquad P = K \cdot R [I \mid -\tilde{C}] = K [R \mid t]$$

$t = -R\tilde{C}$

skew

$$K = \begin{pmatrix} a_x & s & x_0 \\ & a_y & y_0 \\ & & 1 \end{pmatrix}$$

$a_x = f m_x \qquad x_0 = m_x \cdot p_x$

$a_y = f m_y \qquad y_0 = m_y \cdot p_y$

m_x, m_y pixel size

$$P = [M \mid p_4]$$

$M = KR$ RQ decomp $\Rightarrow$ QR of $M^T \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = QR$

once we have Q & R

$$K = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} R^T \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} Q^T$$

$$p_4 = -M\tilde{C}$$

○ CALIBRATION

P has 11 deg freedom so we need 5.5 points $(x_1, y_2) \cdot 5.5 = 11$

use X_i world, x_i image $\Big\}$ Find P such that $x_i = PX_i$

DLT

$$
\begin{array}{cc}
 & 12 \\
2n & \begin{bmatrix} 0, 0, 0, 0, -X_1, -Y_1, -Z_1, -1, y_1 X_1, y_1 Y_1, y_1 Z_1, y_1 \\ X_1, Y_1, Z_1, 1, 0, 0, 0, 0, -x_1 X_1, -x_1 Y_1, -x_1 Z_1, -x_1 \\ \vdots \end{bmatrix}
\end{array}
$$

For each point

PATTERN RECOGNITION

- MINIMUM-DISTANCE CLASSIFIER

$$m_j = \frac{1}{n_j} \sum_{x \in C_j} x \qquad j = 1, 2, \ldots, N_c$$

class j

n° classes

n° pattern vector for class j

prototype vector

Measue euclidian distance between a pattern vector and the prototype of each class, the pattern will be in the lowest euclidian distance class

$$\underset{j \in 1 \ldots N_c}{\operatorname{argmin}} \| x - m_j \|$$

TEMPLATE MATCHING

$$(w \star f)(x,y) = \sum_{s}\sum_{t} w(s,t)\, f(x+s, y+t)$$

examples of CROSS CORRELATION

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{f} \star \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}}_{w} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 8 & 7 & 0 \\ 0 & 6 & 5 & 4 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{f} \star \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{w} = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 6 & 8 & 6 & 2 \\ 3 & 8 & 12 & 8 & 3 \\ 2 & 6 & 8 & 6 & 2 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix}$$

Example of convolution

$$f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \star \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & 6 & 0 \\ 0 & 7 & 8 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

w

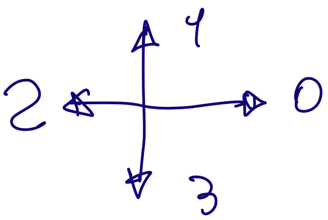
$$\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad \text{same thing}$$

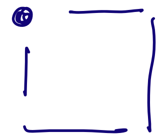
- Correlation coefficient

  Get mean of window and subtract it out of each value!

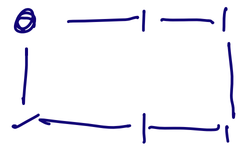
⓪ MATCHING STRUCTURAL PATTERNS

- Shape numbers





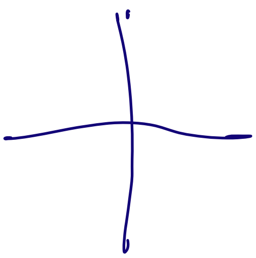
0 3 2 1

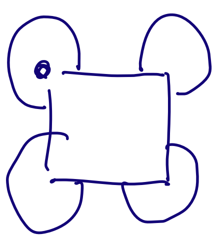


$\underbrace{0\,0\,3\,2\,2\,1}$

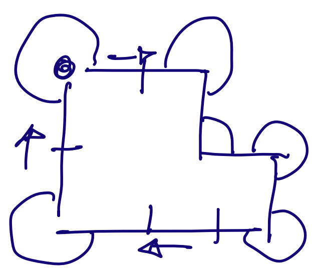
If number is even we have cycle

- Shape numbers (rotation invariant)





3 3 3 3



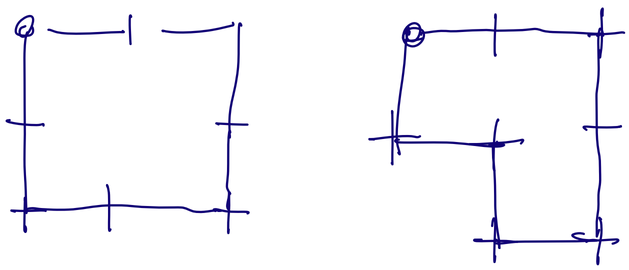
<u>30</u> <u>31</u> <u>33</u> <u>00</u> <u>30</u>

even

- shift by n° of possible directions to get smallest number

We compare 2 images.
2 vals at a time.

ex:



30303030 30303313
shift by 4 dirs
GET THE BEST:

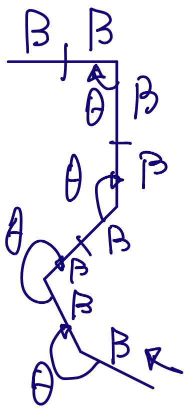
03030303 03033133
check 2 at time

here 4 are same so K=4

$$D(a,b) = \frac{1}{K}$$ Distance

shapes

AS STRINGS...



$a = \beta\theta\beta\theta\beta\beta\theta\beta\beta\theta\beta\beta$

count number of matches between a & b

$$\beta = \left(\text{max length between the 2 strings} - n^{\circ}\text{ matches}\right)$$

$$r = \frac{n^{\circ}\text{ matches}}{\beta}$$

Matching can be done with edit distances

⓪ HOG   Histogram oriented gradients

1 - 64 x 128 DETECTION WINDOW

2 - DEVIDE THE WINDOW INTO 6x6 PIXELS CELLS

3 - COMPUTE QUANTIZED GRADIENT ORIENTATION

$$\theta = \operatorname{atan}\left(\frac{G_y}{G_x}\right)$$

4 - GROUP 3x3 CELLS INTO BLOCKS

5 - NORMALIZE GRADIENT MAGNITUDES BY CELL BLOCKS

6 - TRUNCATE NORMALIZED CELL RESPONSES

7 - TRAIN CLASSIFIER

---

1) n x m window

2) Devide in 6x6 cells

3) Group cells in 3 x 3 blocks

4) COMPUTE

$$G_x = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \quad G_y = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

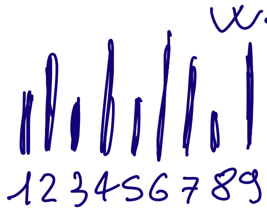
Gradient magnitude $G = \sqrt{G_x^2 + G_y^2}$

orientation: $\theta = \operatorname{atan} \dfrac{G_y}{G_x}$

BIN ORIENTATIONS AS

$$\left\{\begin{array}{ll} 1 & 0 < 20 \\ 2 & 20 < 40 \\ 3 & 40 < 60 \\ 4 & 60 < 80 \\ 5 & 80 < 100 \\ 6 & 100 < 120 \\ 7 & 120 < 140 \\ 8 & 140 < 160 \\ 9 & 160 < 180 \end{array}\right.$$

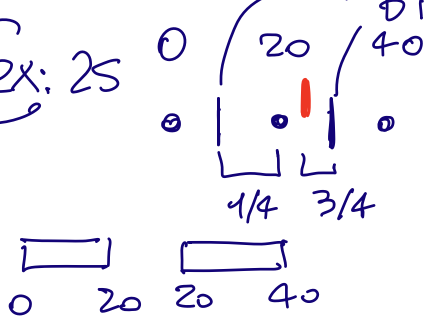
USE THIS TO CREATE AN HISTOGRAM FOR EACH CELL → each pixel is weighted by G: gradient magnitude



ALTERNATIVE TO BINNING:

ex: 25

bin centers



Put 3/4th on the 20-40 bin and 1/4th in the other

4) THEN . GROUP EACH CELL INTO BLOCK

5) Normalize gradient magnitudes by cell blocks

For a block:

$$V = \frac{v}{\sqrt{\|v\|_2^2 + \varepsilon^2}}$$

un-normalized vector of all 9 histograms in a block

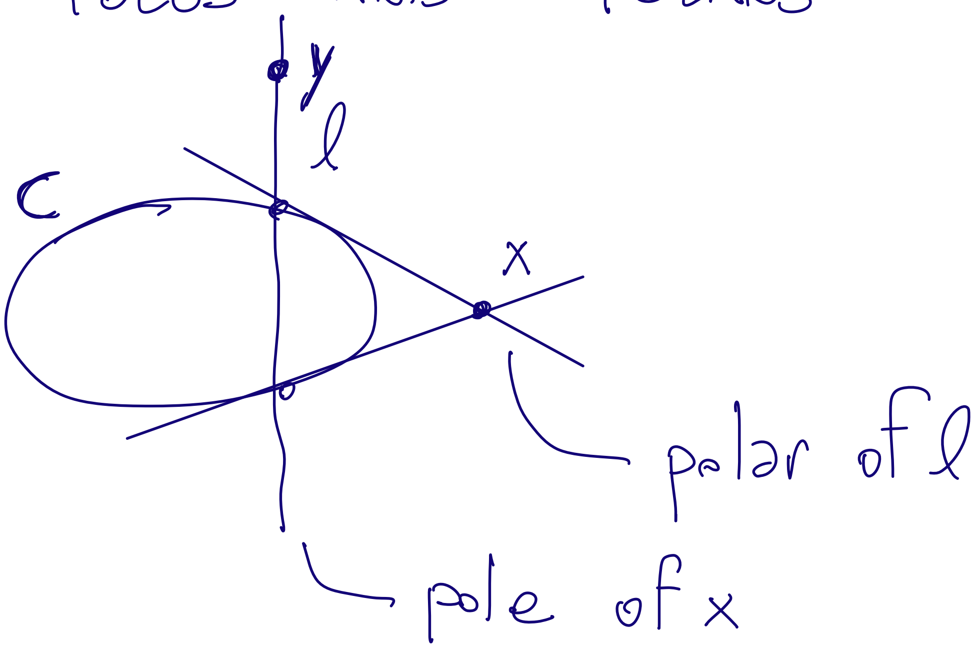
ε constant to avoid $\frac{x}{0}$

6) Truncate normalized vector

~ FLATTEN ~

7) TRAIN WITH SOFT MARGIN SVM

POLES AND POLARS



x, y conjugate if $x^T C y = y^T C x = 0$

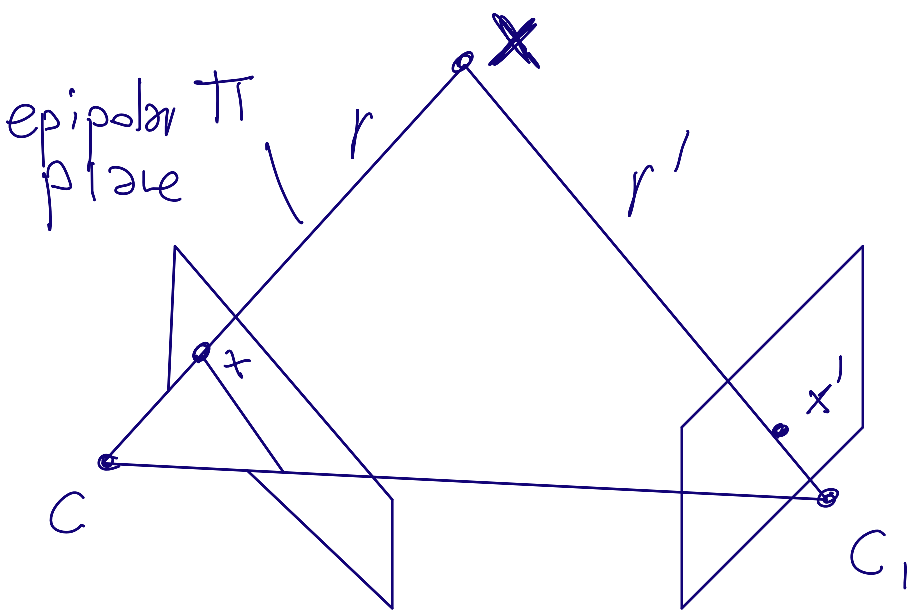
- x and any y on the polar of x are conjugate
- If y is on the polar of x then x is on the polar of y
- two lines l, m are conjugate, if

$$l^T C^* m = m^T C^* l$$

STEREO VISION

2 Projection matrices

given X in first image
where is x' in second image?



camera centers are on a common plane π

$\mathbf{X}$ projects to x and x'

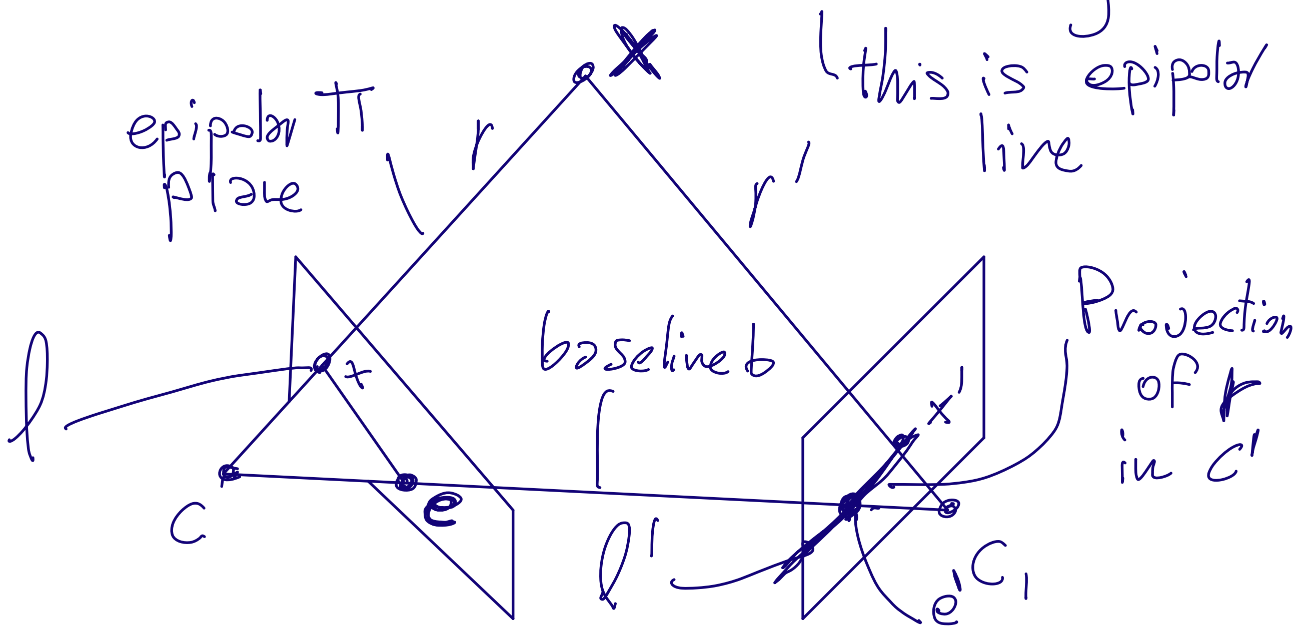
$\mathbf{X}, x, x', C, C'$ are coplanar

- Given x we know $\mathbf{X}$ must lie on $r = \overrightarrow{Cx}$

<u>and</u> x' must lie on $r' = C'\mathbf{X}$

- x' must lie on the projection of r into the image of C'

this is epipolar line



- Mapping from $x \to l'$ from a point in the first image to the corresponding epipolar line in C' picture: l'

THIS MAPPING IS $l' = Fx$

(Fundam. matrix

where $F = [e']_\times P' P^+$

(C in C' image

So we get $e' = \begin{pmatrix} e'_1 \\ e'_2 \\ e'_3 \end{pmatrix}$

$$F = \begin{bmatrix} 0 & -e'_3 & e'_2 \\ e'_3 & 0 & -e'_1 \\ -e'_2 & e'_1 & 0 \end{bmatrix} \cdot P' \left(P^T (P P^T)^{-1} \right)$$

General case to get e:

$$P = [M \mid p_4] \qquad P' = [M' \mid p'_4]$$

then

$$e = -M M'^{-1} p'_4 + p_4 \qquad e' = -M' M^{-1} p_4 + p'_4$$

and

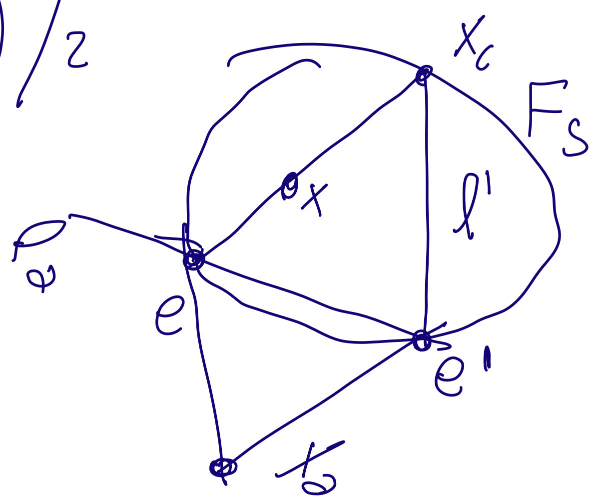
$$F = \begin{bmatrix} 0 & -e'_3 & e'_2 \\ e'_3 & 0 & -e'_1 \\ -e'_2 & e'_1 & 0 \end{bmatrix} M' M^{-1}$$

DECOMPOSE

$F = F_s + F_a$

$F_a = (F - F^T)/2$

$F_s = (F + F^T)/2$



$l' = Fx$

Find intersection x_c of the epipolarline through e and x with F_s