Exercise 20.04.2021

Given the matrix A

$$A = \begin{pmatrix} 6 & -2 & 3 \\ 1 & 10 & -4 \\ -2 & 1 & 0 \end{pmatrix}$$

1. find the geometric estimate of its eigenvalues by the Gershgorin circles theorem and plot the circles in the complex plane. On this basis, provide an estimate of the condition number of the problem; is the problem well-conditioned?

2. find its largest eigenvalue by applying the power method, with initial vector $z_0 = (1, 1, 1)^T$ and convergence criteria $\epsilon = 10^{-6}$. Tabulate and plot the sequence of λ_{max}-values as a function of the iteration step i, until convergence is reached within the chosen criterium; also tabulate and plot the difference $|\lambda_i - \lambda_{i-1}|$ at each iteration step and check that it goes to zero within the chosen criterium. How many steps do you need to reach convergence within the chosen tolerance criterium?

3. find its complete set of eigenvalues with a self-developed QR-algorithm and with a publicly available routine. How many steps do you need to reach convergence within a tolerance of $\epsilon = 10^{-6}$? Plot the sequence of $(\lambda_1, \lambda_2, \lambda_3)$-values as a function of the iteration step i, until convergence is reached within the chosen criterium. Add $(\lambda_1, \lambda_2, \lambda_3)$ to the plot(s) of the Gershgorin circles.

Instructions

During the exercise a short protocol must be made and saved as `PROTOKOLL.txt` in the directory of the respective exercise day. The protocol is a simple ASCII text file that is created with a text editor with which you can also write your programs. The protocol must contain the following

1. Date, exercise number, group number, name(s) of the participating students

2. Time required for the tasks (approximately)

3. Name of the created files, the files must be located in the directory of the respective exercise day

4. The answers to any questions asked above

5. Possible problems or peculiarities, if they have occurred.