

Exercise 05.04.2021

Consider the one-dimensional Poisson equation

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon}$$

in the interval $[x_1, x_n]$ with boundary conditions $\phi(x_1) = \phi_a$ and $\phi(x_n) = \phi_b$, where $\epsilon = 1$ for simplicity, and pursue an approximate solution by the finite difference method: discretize the equation, remember the three-point approximation for the second derivative and write down the resulting set of linear equations in the matrix form.

Tasks

- Use Thomas method to find the electric field $\phi(x)$ generated in the interval $[x_1, x_n] = [0, 1]$ by the local charge distribution

$$\rho(x) = A \cos(Bx)$$

where $A = 10$ and $B = 2\pi$, with boundary conditions $\phi_a = 1$ and $\phi_b = 0$

- Calculate the exact solution and plot the numerical solution against the exact solution for $n = 10$ and 100
- Plot the average accuracy as a function of n (suggestion: log-log scale)
- Plot the average cpu-time as a function of n (suggestion: log-log scale)

Instructions

During the exercise a short protocol must be made and saved as `PROTOKOLL.txt` in the directory of the respective exercise day. The protocol is a simple ASCII text file that is created with a text editor with which you can also write your programs. The protocol must contain the following

1. Date, exercise number, group number, name(s) of the participating students
2. Time required for the tasks (approximately)
3. Name of the created files, the files must be located in the directory of the respective exercise day
4. The answers to any questions asked above
5. Possible problems or peculiarities, if they have occurred.