From Flow Based Cost Allocation to Locational Market Prices

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Linear Energy Modelling and LMP

We linearly optimize the capacity and dispatch of a simple power system.

$$\max_{d_{n,a,t}, g_{n,s,t}, G_{n,s}} \left(U_{n,a,t}(d_{n,a,t}) - \sum_{n,s} c_{n,s} G_{n,s} - \sum_{n,s,t} o_{n,s} g_{n,s,t} - \sum_{\ell} c_\ell F_\ell \right) \tag{1}$$

where

$U_{n,a,t}$ denotes the utility function per bus n, demand type a time step t

$d_{n,a,t}$ denotes the eletric demand

$c_{n,s}$ denotes the capital expenditure (CAPEX) per node n and generator type s

$G_{n,s}$ denotes the generation capacity

$o_{n,s}$ denotes the operational cost (OPEX)

$g_{n,s,t}$ denotes the net generation in MW

c_ℓ denotes the CAPEX per transmission line

F_ℓ denotes the transmission capacity.

In the following we neglect the utility $U_{n,a,t}$ of the nodal demand while fixing the demand $d_{n,a,t}$ to a predefined time-series. The generation $g_{n,s,t}$ is constraint to its capacity

$$g_{n,s,t} - \bar{g}_{n,s,t} G_{n,s} \leq 0 \quad \perp \quad \bar{\mu}_{n,s,t} \quad \forall\, n, s, t \tag{2}$$

$$-g_{n,s,t} \leq 0 \quad \perp \quad \underline{\mu}_{n,s,t} \quad \forall\, n, s, t \tag{3}$$

where $\bar{g}_{n,s,t} \in [0, 1]$ is the capacity factor for renewable generators. The constraints yield the KKT variables $\bar{\mu}_{n,s,t}$ and $\underline{\mu}_{n,s,t}$ which due to complementary slackness are only non-zero if the corresponding constraint is binding.
The transmission capacity F_ℓ limits the flow $f_{\ell,t}$ in both directions, such that

$$f_{\ell,t} - F_\ell \leq 0 \quad \perp \quad \bar{\mu}_{\ell,t} \quad \forall\, \ell, t \tag{4}$$

$$-f_{\ell,t} - F_\ell \leq 0 \quad \perp \quad \underline{\mu}_{\ell,t} \quad \forall\, \ell, t \tag{5}$$

The yielding KKT variables $\bar{\mu}_{\ell,t}$ and $\underline{\mu}_{\ell,t}$ are only non-zero if $f_{\ell,t}$ is limited by the trasmission capacity in positive or negative direction, i.e. 4 or 5 are binding.

The nodal balance constraint ensures that the amount of power that flows into a bus equals the power that flows out of a bus, thus reflects the Kirchhoff Current Law (KCL)

$$\sum_s g_{n,s,t} - \sum_a d_{n,a,t} - \sum_l K_{n,\ell}\, f_{\ell,t} = 0 \qquad \perp \quad \lambda_{n,t} \qquad \forall\; n, t \tag{6}$$

Its shadow price mirrors the Locational Marginal Prizes (LMP) $\lambda_{n,t}$ per bus and time step. In a power market this is the €/MWh$_{\mathrm{el}}$-price which a consumer has to pay.

As the shown in [1], the OPEX and CAPEX of generators follow the non-profit rule which states that their expenses equals their total revenue

$$c_{n,s}\, G_{n,s} + \sum_t o_{n,s}\, g_{n,s,t} = \sum_t \lambda_{n,t}\, g_{n,s,t} \qquad \forall\; n, s \tag{7}$$

The relation counts for transmission lines where the CAPEX equals their total congestion revenue

$$c_\ell\, F_\ell = \sum_{n,t} \lambda_{n,t}\, K_{n,\ell}\, f_{\ell,t} \qquad \forall\; \ell \tag{8}$$

Note here that we are neglecting the Kirchhoff Voltage Law (KVL) constraints. The total cost $\mathcal{TC}$, consisting of all CAPEX and OPEX finally have to be payed by the consumers

$$\mathcal{TC} = \sum_{n,s} c_{n,s}\, G_{n,s} + \sum_{n,s,t} o_{n,s}\, g_{n,s,t} + \sum_\ell c_\ell\, F_\ell = \sum_{n,a,t} \lambda_{n,t}\, d_{n,a,t} \tag{9}$$

Adding CO_2 Constraints

Imposing an additional CO_2 constraint limiting the total emission to K,

$$\sum_{n,s,t} e_{n,s}\, g_{n,s,t} \leq \mathrm{K} \qquad \perp \quad \mu_{\mathrm{CO2}} \tag{10}$$

with $e_{n,s}$ being the emission factor in tonne-CO_2 per MWh$_{\mathrm{el}}$, returns an effective CO_2 price μ_{CO2} in €/tonne-CO_2. The CO_2 price shifts the right hand side of the non-profit relation for generators Eq. (7) to

$$c_{n,s}\, G_{n,s} + \sum_t o_{n,s}\, g_{n,s,t} = \sum_t \left(\lambda_{n,t} - e_{n,s}\, \mu_{\mathrm{CO2}}\right)\, g_{n,s,t} \qquad \forall\; n, s \tag{11}$$

This shows nicely the duality for exchanging the CO_2 Constr. (10) for a shifted OPEX which includes the CO_2 costs

$$o_{n,s} \rightarrow o_{n,s} + e_{n,s}\, \mu_{\mathrm{CO2}} \tag{12}$$

Flow Based Cost Allocation

Besides using LMP, there are others methods for distributing costs in a power system. A common approach is based on Flow Allocation (FA) methods which determine peer-to-peer (P2P) relations between producers (sources) and consumers (sinks) and thus enable to directly assign OPEX to the consumers. Let $A_{n\to m,t}$ denotes the power that is produced at bus n at time t and consumed at bus m. We will refer to it as a P2P relation between n and m. Further let $A_{n\to m,\ell,t}$ denote partial flow on line ℓ at time t caused by bus n.
In the following we show how the LMP can be reproduced on the basis of FA.

Allocating Operational Expenditures

Consumers at bus m compensate the OPEX proportionally to the assigned production, thus

$$\mathcal{O}_{m,t} = \sum_{n,s} o_{n,s}\, \tilde{g}_{n,s,t}\, A_{n\to m,t} \qquad \forall\; m,t \tag{13}$$

where $\tilde{g}_{n,s,t} = g_{n,s,t}/\sum_s g_{n,s,t}$ denotes the share of generator s of the nodel production.

Allocating CO_2 Cost

Similar to the OPEX (compare Rel. (12)) consumers at bus m compensate the total CO_2 cost, thus

$$\mathcal{E}_{m,t} = \sum_{n,s} \mu_{\mathrm{CO2}}\, e_{n,s}\, \tilde{g}_{n,s,t}\, A_{n\to m,t} \qquad \forall\; m,t \tag{14}$$

Allocating Capital Expenditures for Generators

The generator CAPEX allocation, on the other hand, requires more systematic approach. According to the polluter pays principle, it should differentiate between consumers who are 'responsible' for investments and those who are not. Thus, it is crucial to know in the first place whether a time step t makes investments in $G_{n,s}$ necessary or if it would be feasible without (further) investments. The KKT variable of the upper generator capacity constraint $\bar{\mu}_{n,s,t}$ gives us the needed estimator: If $\bar{\mu}_{n,s,t} > 0$, the upper capacity constraint Constr. (2) is binding and it can be fairly assumed that these times steps cause investments in $G_{n,s}$. If in contrast $\bar{\mu}_{n,s,t} = 0$, the generation $g_{n,s,t}$ is not bound and does not push further investments in $G_{n,s}$. Thus, on the basis of the KKT variable $\bar{\mu}_{n,s,t}$ we are able to define a measure $\Phi_{n,s,t}$ for the impact on the investments in $G_{n,s}$

$$\Phi_{n,s,t} = \frac{\bar{\mu}_{n,s,t}}{\sum_t \bar{\mu}_{n,s,t}\, g_{n,s,t}} \tag{15}$$

which weights the contributions of consumers at bus m at time t to the CAPEX

$$\mathcal{C}^G_{m,t} = \sum_{n,s} c_{n,s}\, G_{n,s}\, \Phi_{n,s,t}\, A_{n\to m,t} \qquad \forall\; m,t \tag{16}$$

A way more simple approach, would neglect these thoughts and let consumers at m compensate CAPEX for generator s at bus n proportional to their retrieved power. This would lead us to $\Phi_{n,s,t} = (g_{n,s,t})^{-1}$.

Allocating Capital Expenditures for Lines

The allocation of CAPEX per the transmission line $c_\ell\, F_\ell$ follows a similar procedure as for the generators. Again, we buid up on KKT variables $\bar{\mu}_{\ell,t}$ and $\underline{\mu}_{\ell,t}$ which direct to terms when transmission expansion become necessary. The measure $\Phi_{\ell,t}$ respects the need of CAPEX of both flow directions when set to

$$\Phi_{\ell,t} = \frac{\bar{\mu}_{\ell,t} + \underline{\mu}_{\ell,t}}{\sum_t \left(\bar{\mu}_{\ell,t} + \underline{\mu}_{\ell,t}\right)\, f_{\ell,t}} \tag{17}$$

which weights the contributions of consumers at bus m at time t to the CAPEX in line ℓ,

$$\mathcal{C}^F_{m,t} = \sum_{n,\ell} c_\ell\, F_\ell\, \Phi_{\ell,t}, A_{n\to m,\ell,t} \qquad \forall\; m,t \tag{18}$$

Reproducing the LMP Based Consumption Costs

The above presented quantities complete an allocation of the total cost $\mathcal{TC}$ which mirrors the cost allocation based on LMP, thus

$$\mathcal{O}_{m,t} + \mathcal{E}_{m,t} + \mathcal{C}^G_{m,t} + \mathcal{C}^F_{m,t} \simeq \lambda_{m,t}\, d_{m,a,t} \qquad \forall\ m, t \tag{19}$$

Showcase

Network with CO_2 constraint

We illustrate the flow based cost allocation under use of the fictive network shown in Fig. 2a. It consists of nine buses and ten time steps. The solver optimizes the capacity of two generators, wind and gas, per bus. ...

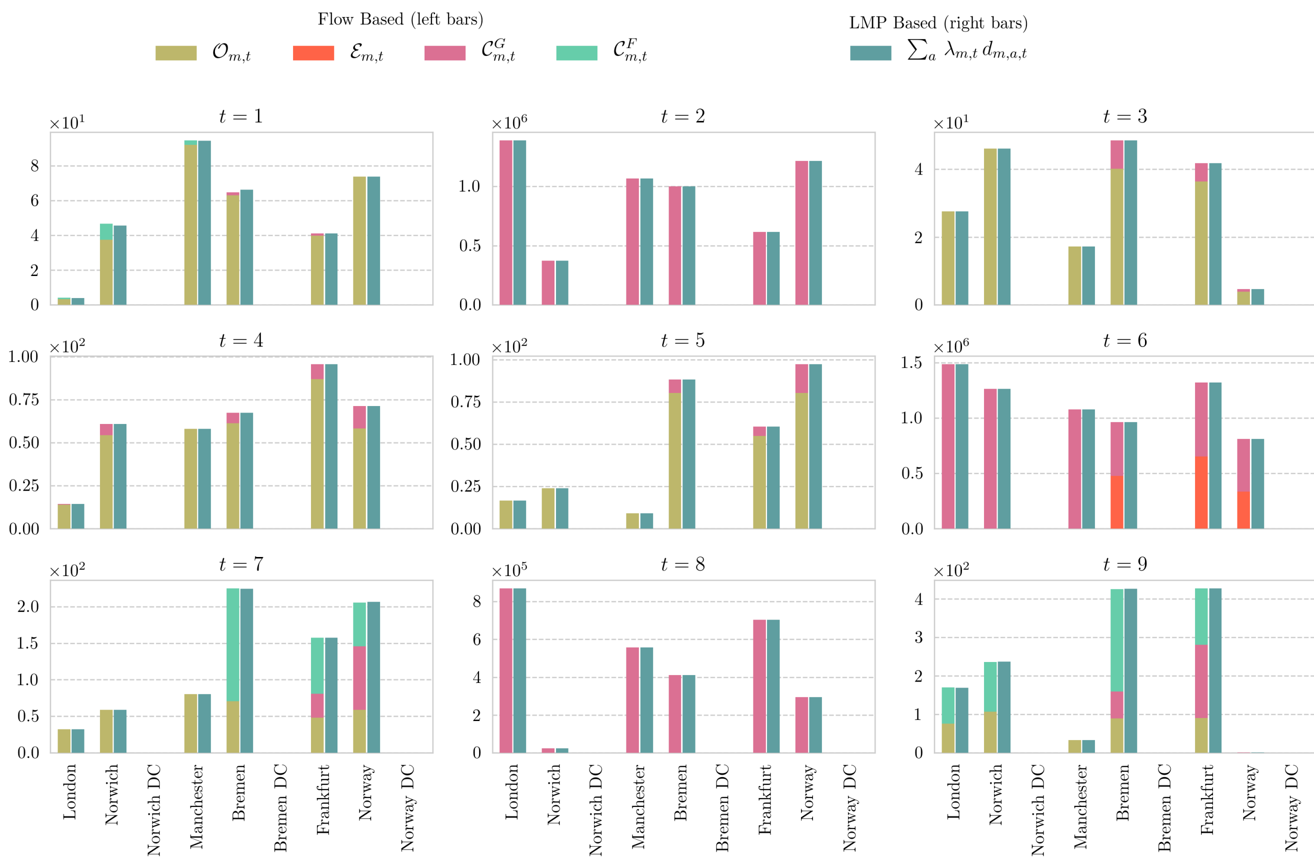
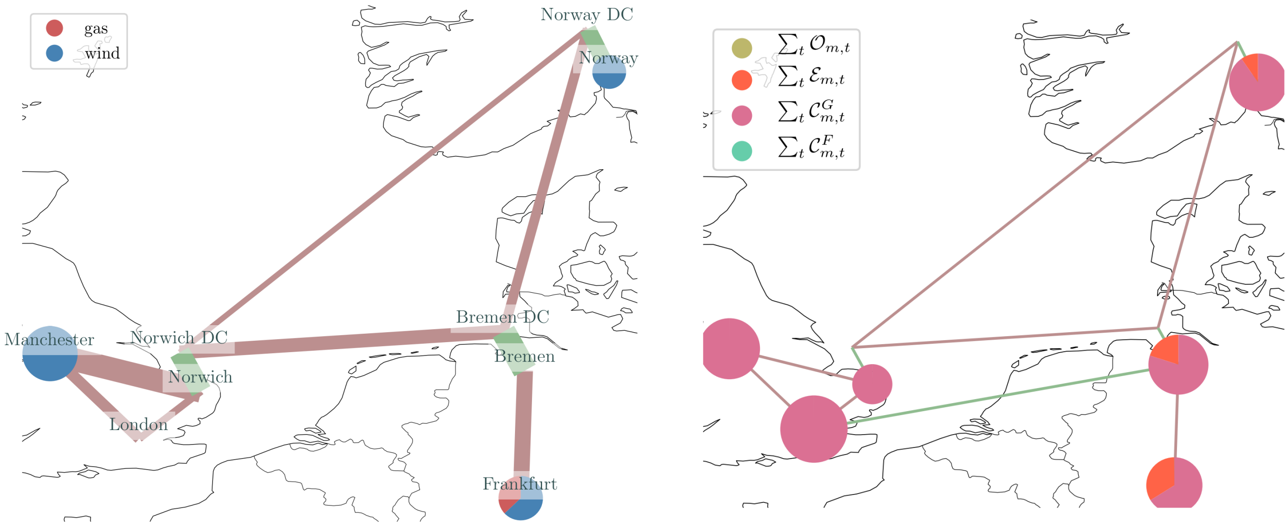


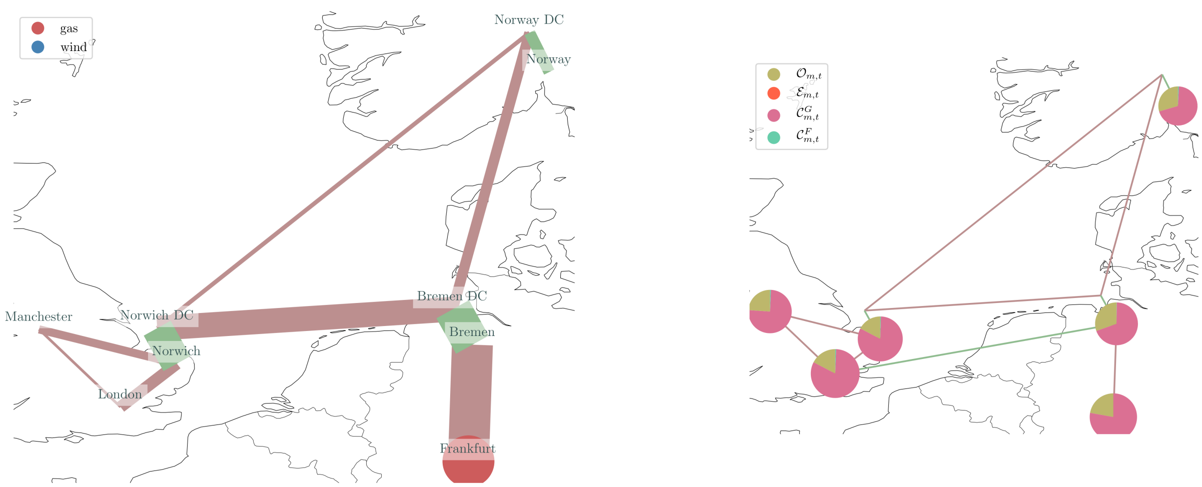
Figure 1: Comparison of the stacked flow based cost allocation with the LMP based cost per consumer for each time step t without CO_2 constaint. The left bars consist of the allocated OPEX $\mathcal{O}_{m,t}$, the allocated CO_2 cost $\mathcal{E}_{m,t}$, the allocated generator CAPEX $\mathcal{C}^G_{m,t}$ and transmission CAPEX $\mathcal{C}^F_{m,t}$, while the right bars show the of the nodal consumption times the LMP.



(a) (b)

Figure 2: Network used for showcasing. (a) shows the distributing of generation capacities $G_{n,s}$, the widths of the transmission lines are proportional to their thermal limit F_ℓ. (b) shows the total nodal payments according to the cost allocation.

Relaxed CO_2 Constraint



(a) (b)

Figure 3: Similar to 2a and 2b but without CO_2 constraint.

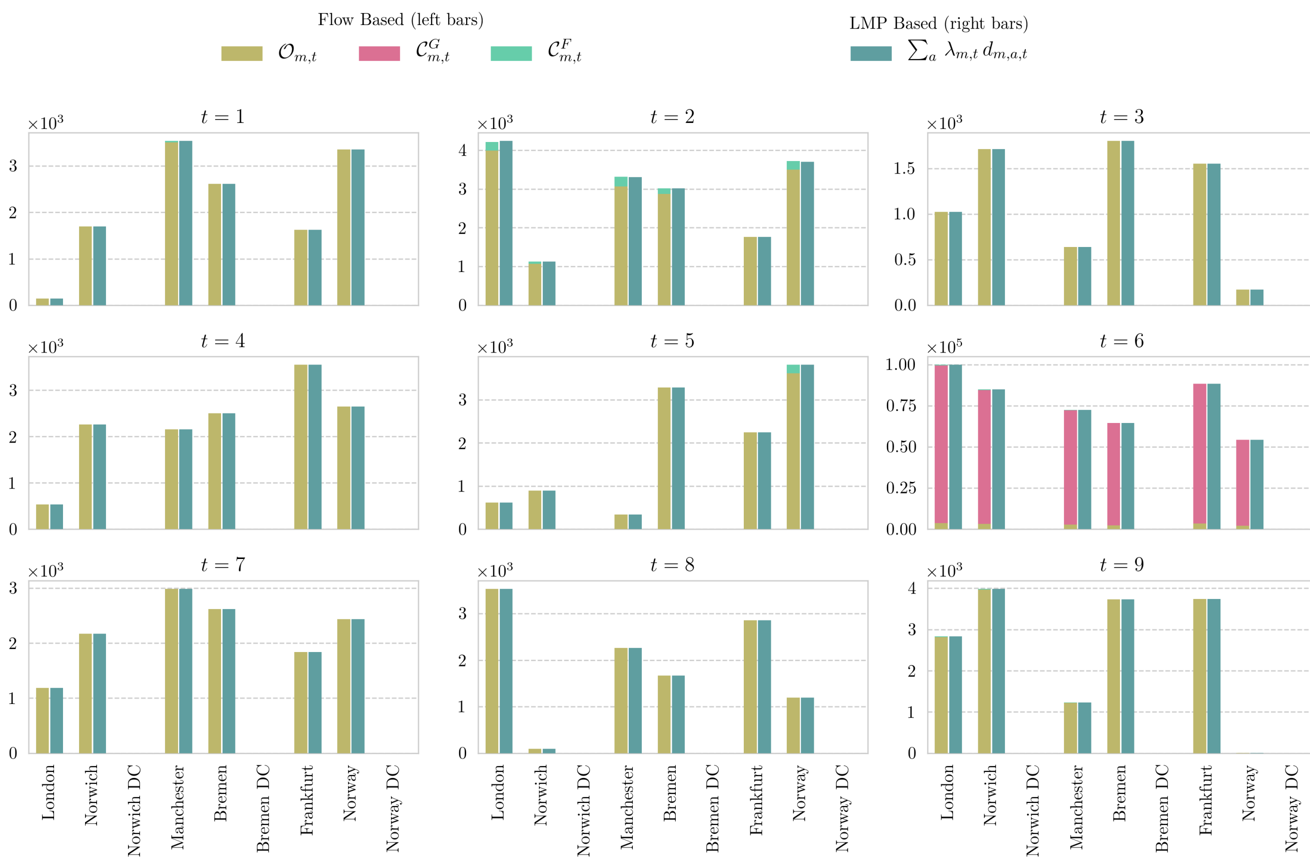


Figure 4: Comparison of the stacked flow based cost allocation with the LMP based cost per consumer for each time step t without CO_2 10. Only one time-step $t = 6$ determines the allocation of generator CAPEX $\mathcal{C}^G_{m,t}$, as for all other time-steps 2 is not binding. Again note the cost scale difference between time step 6 and all others.