

# A dispatch-based approach to fully allocate costs in optimal power systems

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## Abstract

### Highlights

- The LMP always breaks down into contributions of different cost terms (OPEX, CAPEX, etc.) of the system.
- Breaking the LMP further down into contributions of components (generator, transmission line, etc.) requires assumptions on power flow assignments, where prominent methods as Average Participation or Flow Based Market Coupling can be applied.
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### Nomenclature

$\lambda_{n,t}$	Locational Market Price at bus $n$ and time step $t$ in €/MW
$d_{n,t}$	Electric demand per bus $n$ , demand type $a$ , time step $t$ in MW
$g_{s,t}$	Electric generation of generator $s$ , time step $t$ in MW
$f_{\ell,t}$	Active power flow on line $\ell$ , time step $t$ in MW
$o_s$	Operational price in €/MW
$c_s$	Capital Price in €/MW
$c_\ell$	Capital Price in €/MW for transmission capacity on line $\ell$
$G_s$	Generation capacity in MW
$F_\ell$	Transmission capacity in MW
$K_{n,\ell}$	Incidence matrix

## 1 Economic Context

In long-term operation and investment planning models, the total costs  $\mathcal{TC}$  of a power system is the sum of multiple cost terms. Typically, these include operational expenditures (OPEX)  $\mathcal{O}^G$  for generators, expenditures for emissions  $\mathcal{E}$ , capital expenditures (CAPEX) for generators  $\mathcal{C}^G$ , CAPEX for the transmission system  $\mathcal{C}^F$  and possible other terms, *i.e.*

$$\mathcal{TC} = \mathcal{O}^G + \mathcal{E} + \mathcal{C}^G + \mathcal{C}^F + \dots \quad (1)$$

In turn, each of these terms  $\mathcal{C} = \{\mathcal{O}^G, \mathcal{E}, \mathcal{C}^G, \mathcal{C}^F, \dots\}$  consists of cost associated to the asset  $i$  in the system,

$$\mathcal{C} = \sum_i \mathcal{C}_i \quad (2)$$

where an asset describes any operating part of the network, such as a generator, line, energy storage etc.

In a cost-optimal setup with minimized  $\mathcal{TC}$ , the Locational Marginal Price (LMP) describes the price for an incremental increase of electricity demand  $d_{n,t}$  at node  $n$ . It is given by the derivative of the total system cost  $\mathcal{TC}$  with respect to the local demand  $d_{n,t}$

$$\lambda_{n,t} = \frac{\partial \mathcal{TC}}{\partial d_{n,t}} \quad (3)$$

This leads to a nodal pricing where over the span of optimized time steps  $t$ , the system costs are partially or totally payed back by the consumers

$$\mathcal{TC} - \mathcal{R} = \sum_{n,t} \lambda_{n,t} d_{n,t} \quad (4)$$

depending on the costs  $\mathcal{R}$  which are independent of the nodal demand

$$\frac{\partial \mathcal{R}}{\partial d_{n,t}} = 0 \quad (5)$$

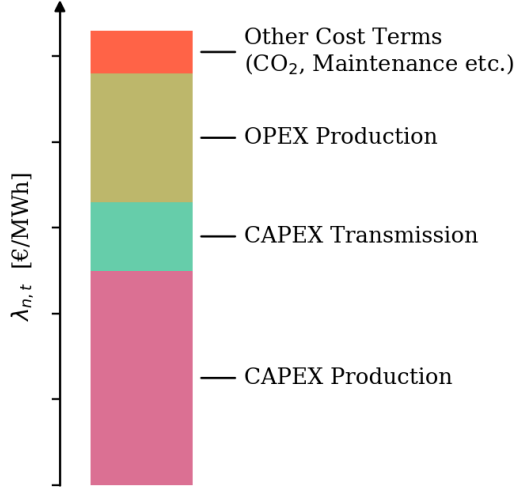


Figure 1: Schematic decomposition of the Locational Market Price  $\lambda_{n,t}$ . In power system model with optimal long-term operation and planning, the total system costs  $\mathcal{TC}$  split into different cost terms, *i.e.* OPEX and CAPEX for production and transmission and possibly other expenditures.

Generally speaking, the cost term  $\mathcal{R}$ , not covered by the consumers, results from additional demands on the network design, such as capacity expansion limits or minimum share of one technology in the power mix. However, in most cases, where  $\mathcal{R} \ll \mathcal{TC}$ , these play a minor role.

From feeding Eq. (1) into Eq. (3) it follows naturally that the LMP splits into contribution to the above mentioned cost terms. This relation, which we schematically show in Fig. 1, was already shown in extensive investigations of the LMP [13]. However the question of how the LMP can be decomposed into contributions of single cost terms  $\mathcal{C}_i$  associated with asset  $i$  remains unanswered. This work aims at presenting and illustrating an intuitive, peer-to-peer cost allocation including all network assets.

## 2 Dispatch-Based Cost Allocation

Let  $\mathcal{C}_i$  denote a general cost term associated with asset  $i$ . Consider a long-term equilibrium in a power system with perfect competition, then, according to the zero-profit condition, each cost term  $\mathcal{C}_i$  can be considered as a cost-weighted sum of the operational state  $s_{i,t}$  of asset  $i$ , *i.e.*

$$\mathcal{C}_i = \sum_t \gamma_{i,t} s_{i,t} \quad (6a)$$

where  $\gamma_{i,t}$  denotes a cost factor in €/MW. If  $\mathcal{C}_i$  describes the OPEX occasioned by asset  $i$ , the cost factor  $\gamma_{i,t}$  is simply given by the marginal operational price  $o_i$ . However, as we will show later, if it describes the CAPEX of asset  $i$ ,  $\gamma_{i,t}$  is a composition of shadow prices  $\mu_{i,t}$  given at the optimum.

Following the implications of Eqs. (3) and (4), we define the cost  $\mathcal{C}_{n \rightarrow i,t}$  that consumers at bus  $n$  have pay to asset  $i$  at time  $t$ , in order to compensate for  $\mathcal{C}_i$ . This leads us to

$$\mathcal{C}_{n \rightarrow i,t} = \gamma_{i,t} \frac{\partial s_{i,t}}{\partial d_{n,t}} d_{n,t} \quad (6b)$$

The derivative on the right hand side is defined through the sensitivity of the operational variable  $s_{i,t}$  at the optimum against changes in the demand.

$$A_{i,n,t} \rightarrow \frac{\partial s_{i,t}}{\partial d_{n,t}} d_{n,t} \quad (6c)$$

... may be interpreted as the amount of power that asset  $i$  supplies demand  $d_{n,t}$  with. It heavily relies on the derivative of the operational state with respect to the nodal demand  $\partial s_{i,t} / \partial d_{n,t}$ . From the natural fact that the sum of all contributions must return the cost term,

$$\mathcal{C}_i = \sum_{n,t} \mathcal{C}_{n \rightarrow i,t} \quad (6d)$$

it follows that  $A_{i,n,t}$  must fulfill

$$s_{i,t} = \sum_n A_{i,n,t} \quad (6e)$$

The last equation states that all power produced or processed by asset  $i$  must be totally consumed by the network demand  $d_{n,t}$ . Finally, the total contribution from node  $n$  at time  $t$  to the cost term  $\mathcal{C}$  amounts

$$\mathcal{C}_{n,t} = \sum_i \mathcal{C}_{n \rightarrow i,t} \quad (6f)$$

Now, assume a network with generators  $s$ , transmission lines  $\ell$  and storage units  $r$ . Each asset  $i = \{s, \ell, r\}$  adds an term for OPEX and a term for CAPEX to the total system cost  $\mathcal{TC}$ .

### 2.1 OPEX Allocation

Let the operational price for an asset  $i$  be given by  $o_i$ . Then, for example the OPEX occasioned by generator  $s$  is given by

$$\mathcal{O}_s^G = \sum_t o_s g_{s,t} \quad (7)$$

	$i$	$\mathcal{C}$	$\mathcal{C}_i$	$\gamma_{i,t}$	$s_{i,t}$
OPEX Production	$s$	$\mathcal{O}^G$	$\sum_t o_s g_{s,t}$	$o_s$	$g_{s,t}$
OPEX Transmission	$\ell$	$\mathcal{O}^F$	$\sum_t o_\ell  f_{\ell,t} $	$o_\ell$	$ f_{\ell,t} $
OPEX Storage	$r$	$\mathcal{O}^E$	$\sum_t o_r g_{r,t}^{\text{dis}}$	$o_r$	$g_{r,t}$
CAPEX Production	$s$	$\mathcal{C}^G$	$c_s G_s$	$\bar{\mu}_{s,t}$	$g_{s,t}$
CAPEX Transmission	$\ell$	$\mathcal{C}^F$	$c_\ell F_\ell$	$(\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t})$	$f_{\ell,t}$
CAPEX Storage	$r$	$\mathcal{C}^E$	$c_r G_r$	$\bar{\mu}_{r,t}^{\text{dis}} - \underline{\mu}_{r,t}^{\text{dis}} + (\eta_r^{\text{dis}})^{-1} \lambda_{r,t}^{\text{ene}}$	$g_{r,t}$
Emission Cost	$s$	$\mathcal{E}$	$\mu_{\text{CO}_2} e_s g_{s,t}$	$\mu_{\text{CO}_2} e_s$	$g_{s,t}$

Table 1: Mapping of different cost terms to the cost allocation scheme given in Eqs. (6). These include OPEX & CAPEX for production, transmission and storage assets in the network, as well as a cost term for the total Green House Gas (GHG) emissions.

where  $g_{s,t}$  denotes its power generation at time  $t$ . As Eq. (7) matches the form of Eq. (6a) which allows us to use the above presented scheme in Eqs. (6). As a result we obtain

$$\mathcal{O}_{n \rightarrow s,t} = o_s A_{s,n,t} \quad (8)$$

which is the contribution of  $d_{n,t}$  to the OPEX at generator  $s$ . The quantity

$$A_{s,n,t} = \frac{\partial g_{s,t}}{\partial d_{n,t}} d_{n,t} \quad (9)$$

can be considered as the power that is produced by generator  $s$  and consumed at node  $n$  at time  $t$ .

In the same manner, we can follow the scheme allocate OPEX for flow  $f_{\ell,t}$  and storage unit dispatch  $g_{r,t}^{\text{dis}}$ . As we assume a bidirectional flow on line  $\ell$ , the OPEX is set proportional to the absolute value of the flow. The upper section in Table 1 shows the mapping of variables to Eqs. (6) in order to define the full OPEX allocation.

The scheme works for all other cost attached to the operational state of an asset  $i$ . Given for example a fix price for emissions  $\mu_{\text{CO}_2}$  in € per tonne- $\text{CO}_2$  equivalents, the cost term for emission adds up to

$$\mathcal{E} = \mu_{\text{CO}_2} \sum_s e_s g_{s,t} \quad (10)$$

where  $e_s$  denotes the emission factor in tonne- $\text{CO}_2$  per MWh<sub>el</sub> of generator  $s$ . The allocated payment for consumers at bus  $n$  at time  $t$  assigned to generator  $s$  is then given by

$$\mathcal{E}_{n \rightarrow s,t} = \mu_{\text{CO}_2} e_s A_{s,n,t} \quad (11)$$

## 2.2 CAPEX Allocation

For the CAPEX allocation, it becomes crucial to look at the individual relations between operational state  $s_{i,t}$  and the capacity limit. For all assets, let the capital price for one unit capacity expansion be denoted by  $c_i$ . All quantities for the CAPEX allocation, which we now discuss in detail, are summarized in the middle section of Table 1.

### 2.2.1 Generators

The nominal capacity  $G_s$  constrains the generation  $g_{s,t}$  in the form of

$$g_{s,t} - \bar{g}_{s,t} G_s \leq 0 \quad \perp \quad \bar{\mu}_{s,t} \quad \forall s, t \quad (12)$$

$$-g_{s,t} \leq 0 \quad \perp \quad \underline{\mu}_{s,t} \quad \forall s, t \quad (13)$$

where  $\bar{g}_{s,t} \in [0, 1]$  is the capacity factor for renewable generators. At a cost-optimum, these two constraints yield the shadow prices  $\bar{\mu}_{s,t}$  and  $\underline{\mu}_{s,t}$ . As shown in [3] and in detailed in Appendix A.3, over the whole time span, the CAPEX for generator  $s$  is payed back by the production  $g_{s,t}$  times the shadow price  $\bar{\mu}_{s,t}$ ,

$$\mathcal{C}_s^G = c_s G_s = \sum_t \bar{\mu}_{s,t} g_{s,t} \quad (14)$$

This representation connects the CAPEX with the operational state of generator  $s$ , *i.e.* matches the form in Eq. (6a) allows for using the cost allocation scheme. The resulting the CAPEX allocation is given by

$$\mathcal{C}_{n \rightarrow s,t}^G = \bar{\mu}_{s,t} A_{s,n,t} \quad (15)$$

How does this allocation behave? According to the polluter pays principle, it differentiates between consumers who are ‘responsible’ for investments and

those who are not. If  $\bar{\mu}_{s,t}$  (in literature often denoted as the Quality of Supply) is bigger than zero, the upper Capacity Constr. (12) is binding. Thus it is these time steps which push investments in  $G_s$ . If  $\bar{\mu}_{s,t} = 0$ , the generation  $g_{s,t}$  is not bound and investments are not necessary. When summing over all CAPEX payments to generator  $s$  in Eq. (15), we can use Eqs. (6e) and (14) to see that each generator retrieves exactly the cost that were spent to build the capacity  $G_s$ .

### 2.2.2 Transmission Lines

The transmission capacity  $F_\ell$  limits the flow  $f_{\ell,t}$  in both directions,

$$f_{\ell,t} - F_\ell \leq 0 \quad \perp \quad \bar{\mu}_{\ell,t} \quad \forall \ell, t \quad (16)$$

$$-f_{\ell,t} - F_\ell \leq 0 \quad \perp \quad \underline{\mu}_{\ell,t} \quad \forall \ell, t \quad (17)$$

which yield the shadow prices  $\bar{\mu}_{\ell,t}$  and  $\underline{\mu}_{\ell,t}$ . Again, we use the result of [3] (for details see Appendix A.4) which derives that over the whole time span, the investment in line  $\ell$  is paid back by the shadow prices times the flow

$$\mathcal{C}_\ell^F = c_\ell F_\ell = \sum_t (\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}) f_{\ell,t} \quad (18)$$

From here, we follow the scheme in Eqs. (6) which finally defines the CAPEX allocation as

$$\mathcal{C}_{n \rightarrow \ell, t}^F = (\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}) A_{\ell, n, t} \quad (19)$$

The quantity

$$A_{\ell, n, t} = \frac{\partial f_{\ell,t}}{\partial d_{n,t}} d_{n,t} \quad (20)$$

can be interpreted as the flow that the demand at node  $n$  and time  $t$  causes on line  $\ell$ . The shadow prices  $\bar{\mu}_{\ell,t}$  and  $\underline{\mu}_{\ell,t}$  again can be seen as a measure for necessity of transmission investments at  $\ell$  at time  $t$ . Hence, the definition of  $\mathcal{C}_{n \rightarrow \ell, t}^F$  states that consumers, which retrieve power flowing on congested lines, yielding a bound Constr. (16) or (17), pay compensations for the resulting investments at  $\ell$ . Again the sum of all CAPEX payments to line  $\ell$  equals the total CAPEX spent. This is seen when summing Eq. (19) over all buses and time steps and using Eqs. (6e) and (18)

### 2.2.3 Storages

In a simplified storage model,  $G_r$  limits the storage dispatch  $g_{r,t}^{\text{dis}}$  and charging  $g_{r,t}^{\text{sto}}$ . Further it limits the maximal storage capacity  $g_{r,t}^{\text{ene}}$  by a fix ratio  $h_r$ , denoting the maximum hours at full discharge. The storage  $r$  dispatches power with efficiency  $\eta_r^{\text{dis}}$ , charges power with efficiency  $\eta_r^{\text{sto}}$  and preserves power from one time step  $t$  to the next,  $t+1$ , with an efficiency of  $\eta_r^{\text{ene}}$ . In Appendix A.5 we formulate the mathematical details. As already shown in [3], the total expenditures at  $r$  are fully paid back by differences of the LMP at which the storage “buys” and “sells” power. Taking only the CAPEX into account the zero profit condition reduces to

$$\begin{aligned} \mathcal{C}^E &= c_r G_r \\ &= \sum_t \left( \bar{\mu}_{r,t}^{\text{dis}} - \underline{\mu}_{r,t}^{\text{dis}} + (\eta_r^{\text{dis}})^{-1} \lambda_{r,t}^{\text{ene}} \right) g_{r,t}^{\text{dis}} \\ &\quad - \sum_t \lambda_{n,t} K_{n,r} g_{r,t}^{\text{sto}} \quad \forall r \end{aligned} \quad (21)$$

where  $\bar{\mu}_{r,t}^{\text{dis}}$  and  $\underline{\mu}_{r,t}^{\text{dis}}$  are the shadow prices of the upper and lower dispatch capacity bound and  $\lambda_{r,t}^{\text{ene}}$  is the shadow price of the energy balance constraint. When applying the cost allocation scheme Eqs. (6), it stands to reason to assume that  $\partial g_{r,t}^{\text{sto}} / \partial d_{n,t} \cdot d_{n,t} = 0$ , implying that the when a storage charges power, it does not supply any demand. Rather it stands with the demand on the consumer side, retrieving power from producing assets. This leaves us with

$$\mathcal{C}_{n \rightarrow r, t}^E = \left( \bar{\mu}_{r,t}^{\text{dis}} - \underline{\mu}_{r,t}^{\text{dis}} + (\eta_r^{\text{dis}})^{-1} \lambda_{r,t}^{\text{ene}} \right) A_{r, n, t} \quad (22)$$

and the power allocation

$$A_{r, n, t} = \frac{\partial g_{r,t}^{\text{dis}}}{\partial d_{n,t}} d_{n,t} \quad (23)$$

The latter only allocates dispatched power of storage  $r$ . Note that this will break Eq. (6d) as the payments to  $r$  surpass the CAPEX by an amount  $\mathcal{R}_r^E$ . It is crucial to note that like this, storage units perform a redistribution of money, and therefore distort the cost allocation. So, certain share of what is allocated to the CAPEX of a storage is in another time step spent by the storage in order to buy power from other assets. This effect scales with the amount of installed capacity.

It is possible to incorporate this redistribution effect into the cost allocation, by replacing the demand  $d_{n,t}$

with the power charge  $g_{r,t}^{\text{sto}}$  in Eqs. (6). Then, the derived payments that a storage unit  $r$  has to pay to asset  $i$  is given by  $\mathcal{C}_{r \rightarrow i}$ . The sum of those payments due to  $r$  will the sum up to  $\mathcal{R}_r^E$ .

## 2.3 Design Constraints

Power system modelling does rarely follow a pure Greenfield approach with unlimited capacity expansion. Rather, today's models are setting various constraints defining socio-political or technical requirements. As mentioned before this will alter the equality of total cost and total revenue, *i.e.* leads to  $\mathcal{R} \neq 0$  in Eq. (4). More precisely, each constraint  $h_j$  (other then the nodal balance constraint) of the form

$$h_j(s_{i,t}, S_i) - K < 0 \quad (24)$$

where  $K$  is any non-zero constant, will result in a cost term contributing to  $\mathcal{R}$  and in some cases alter Eq. (6a) to

$$\mathcal{C}_i - \mathcal{R}_i = \sum_t \gamma_{i,t} s_{i,t} \quad (25)$$

The effective revenue per asset  $i$  associated with cost term  $\mathcal{C}$  can then be defined as

$$\mathcal{C}' = \sum_i \mathcal{C}_i - \mathcal{R}_i \quad (26)$$

Note that  $\mathcal{C}'$  is the amount which is allocated to consumers which, according to the quality of Eq. (24) and the corresponding  $\mathcal{R}_i$ , can either be larger, equal or lower than  $\mathcal{C}$ . In the following we highlight two often used classes of designs constraints in the form of Eq. (24) and show how to incorporate them into the cost allocation.

### 2.3.1 Capacity Expansion Limit

In more realistic setups, generators, lines or other assets can only be built up to a certain limit. This might be due to land use restrictions or social acceptance problems. However, when constraining the capacity  $S_i$  for a subset  $I$  of assets to an upper limit  $\bar{S}$ , in the form of

$$S_i - \bar{S} \leq 0 \perp \bar{\mu}_i^{\text{nom}} \quad \forall i \in I, \quad (27)$$

the zero profit condition alters as soon as the constraint becomes binding. Then, the revenue of asset  $i$  exceeds its total expenditures (OPEX + CAPEX).

More precisely, the allocated CAPEX in Table 1 will surpass the actual CAPEX of asset  $i$  by the cost it has to pay for the scarcity, given by the absolute value of

$$\mathcal{R}_i^{\text{scarcity}} = -\bar{\mu}_i^{\text{nom}} S_i \quad \forall i \in I \quad (28)$$

The costs which consumers at  $n$  have to pay for the scarcity impacting asset  $i$  are given by

$$\mathcal{R}_{n \rightarrow i, t}^{\text{scarcity}} = \frac{\bar{\mu}_i^{\text{nom}}}{c_i + \bar{\mu}_i^{\text{nom}}} \mathcal{C}_{n \rightarrow i, t}^I \quad \forall i \in I \quad (29)$$

where  $\mathcal{C}_{n \rightarrow i, t}^I$  denotes the CAPEX allocation presented above.

### 2.3.2 Brownfield Constraints

In order to take already built infrastructure into account, the capacity  $S_i$  can be constrained to a minimum required capacity  $\underline{S}$ . Mathematically this translates to

$$\underline{S} - S_i \leq 0 \perp \underline{\mu}_i^{\text{nom}} \quad \forall i \in I \quad (30)$$

Again, such a setup alters the zero profit condition of asset  $i$ , as soon as the constraint becomes binding. In that case, asset  $i$  does not collect enough revenue in order to match the CAPEX. The difference, given by

$$\mathcal{R}_i^{\text{subsidy}} = \underline{\mu}_i^{\text{nom}} S_i \quad \forall i \quad (31)$$

has to be subsidized by governments or communities. It is rather futile wanting to allocate these cost to consumers as assets may not gain any revenue for their operational state, *i.e.* where  $\mathcal{C}^I = \mathcal{R}_i^{\text{subsidy}}$ .

## 3 Assumptions on Power Allocations

The presented cost allocation suits for any type of topology and network setup. But so far, the question of how  $A_{i,n,t}$  for generators  $s$ , lines  $\ell$  and storages  $r$  are defined was left open. We recap that all rely on derivatives of  $g_{s,t}$ ,  $f_{\ell,t}$  and  $g_{r,t}^{\text{dis}}$  with respect to the nodal demand  $d_{n,t}$  (see Eq. (6e)).

Let  $g_{m,t}$  denote the nodal power generation which combines the power production of all producing assets, in this case generators  $S$  and storages  $R$ , at node  $n$  and time  $t$ . It is given by

$$g_{m,t} = \sum_{i \in \{S, R\}} K_{m,i} s_{i,t} \quad (32)$$

with  $K_{m,i}$  being 1 if asset  $i$  is attached to bus  $m$  and zero otherwise. Further, let  $A_{m \rightarrow n,t}$  collect the power produced by assets at node  $m$  and consumed at  $n$ , given by

$$A_{m \rightarrow n,t} = \sum_{i \in \{S,R\}} K_{m,i} A_{i,n,t} \quad (33)$$

Now, let  $H_{\ell,n}$  denote Power Transfer Distribution Factors (PTDF) giving the changes in the flow on line  $\ell$  for one unit (typically one MW) of net power production at bus  $n$ . The linear power flow equation can be written as

$$f_{\ell,t} = \sum_m H_{\ell,m} (g_{m,t} - d_{m,t}) \quad (34)$$

Note that for transport models or mixed AC-DC networks,  $H_{\ell,n}$  can be artificially calculated using the formulation presented in [9]. Taking the derivative with respect to the demand,

$$A_{\ell,n,t} = \frac{\partial f_{\ell,t}}{\partial d_{n,t}} d_{n,t} = \sum_m H_{\ell,m} (A_{m \rightarrow n,t} - \delta_{n,m} d_{n,t}), \quad (35)$$

shows that  $A_{\ell,n,t}$  is fully determined through the peer-to-peer allocation  $A_{m \rightarrow n,t}$ . In other words, we only need to know how much power produced at node  $m$  is consumed at node  $n$  in order to derive the allocated flow  $A_{\ell,n,t}$ . Further we can breakdown  $A_{m \rightarrow n,t}$  to  $A_{s,n,t}$  for generators and  $A_{r,n,t}$  for storages proportionally to their contribution to the nodal generation  $g_{m,t}$ . Unfortunately, the solution for  $A_{m \rightarrow n,t}$  is non-unique and requires further assumptions. Established flow allocation schemes approach this problem from different directions. Principally two options exist *what* is allocated

1. gross power injections
2. net power injections

Further it is important *what assumptions* define the allocation, *i.e.* what method is used to define the pairs of sources and sinks. The three suitable approaches we present here are

- a. Equivalent Bilateral Exchanges (EBE) [7] which assumes that every producer supplies every consumer proportional to its share in the total consumption.

- b. Average Participation (AP) [2, 1] which traces the flow from producer to consumer following the law of proportional sharing.
- c. Flow Based Market Coupling (FBMC) which uses zonal PTDF for allocating power within predefined regions. The interregional exchange is only allocating net power deficit or excess of the regions.

We show the mathematical formulation for all combinations **a1** - **c2** in Appendices B.1 to B.4. Principally, type **2** leads to less P2P trades than type **1** as power from a bus  $m$  with  $g_{m,t} \leq d_{m,t}$  is not assigned to other buses, only to  $m$ . Further, as literature has often pointed out, the EBE principal **a** does not suit for large networks where remote buses would interconnect in the same way as buses in close vicinity [8]. The AP based type **b** tackles this problem by restricting P2P trades to those which are traceable when applying the proportional sharing principal. Therefore  $A_{s,n,t}$  denotes that part of power produced by bus  $m$  which, when only following in the direction of  $f_{\ell,t}$ , ends up at bus  $n$ . Type **c** further allows to control the regions or market zones which are netted out in a first step. If in a region  $R$  the generation undercuts the demand,  $\sum_{n \in R} g_{n,t} \leq \sum_{n \in R} d_{n,t}$ , none of the inner-regional generation is assigned to other regions. However, it relies on further assumptions such as the Generation Shift Keys determining the production which are deployed for the inter-regional exchange.

From this point of view, we restrict the focus of this research to **net** injection allocation on the basis of **AP**, type **2b**, as locality and no further dependency on external decisions are strong arguments for a transparent cost allocation. We emphasize that the AP based allocation does not imply traceability of the resulting flows, as only the *source-sink* relations  $A_{m \rightarrow n,t}$  are determined by it, the flow allocation still follows the power flow equation given as shown in Eq. (35). In the following example, we illustrate the functionality of the cost allocation by an example of a small artificial network.

### 3.1 Numerical Example

Consider a two bus system, as shown in Fig. 2, with one transmission line and one generator per bus. Whereas generator 1 (at bus 1) has a cheap operational price of 50 €/MWh<sub>el</sub>, an expensive opera-



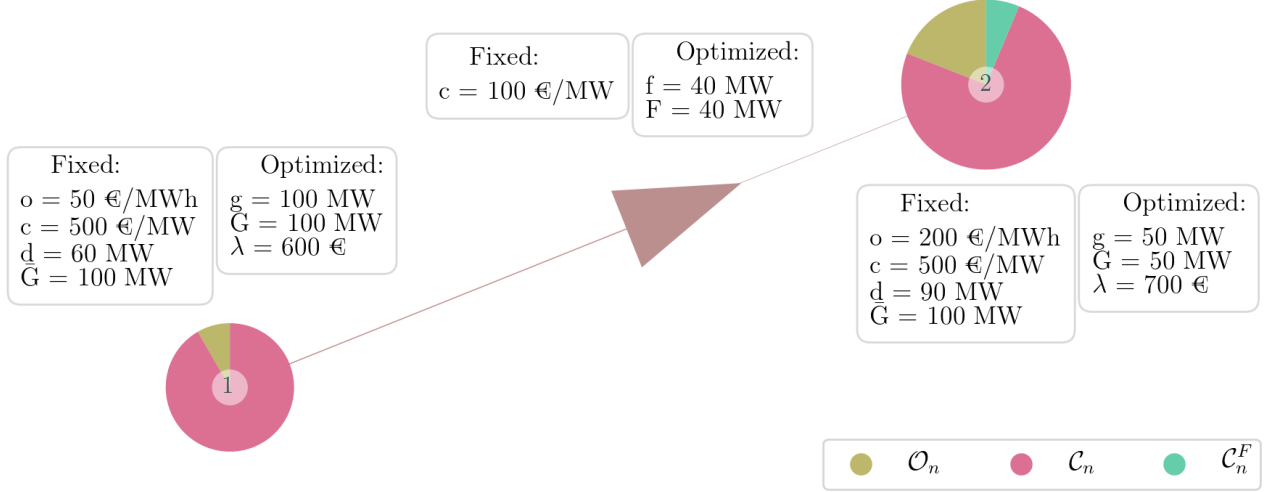


Figure 2: Illustrative example of a 2-bus network with one optimized time step. Fixed prices and constraining values are given in the left box for each bus and the transmission line. Optimized values are given in the right boxes. Bus 1 has a cheaper operational price  $o$ , capital prices are the same for both. As both generator capacities are constraint to 100 MW, the optimization also deploys the generator at bus 2. The resulting electricity prices  $\lambda$  are then a composition of all prices for operation and capital investments.

tional price of 200 €/MWh<sub>e</sub> is set to generator 2 (at bus 2). For both the capital investments amount 500 €/MW and the maximal capacity is limited to  $\bar{G}_s = 100$  MW. The transmission line has a capital price of 100 €/MW and no upper capacity limit. With a demand of 60 MW at bus 1 and 90 MW at bus 2, the optimization expands the cheaper generator at bus 2, the optimization expands the cheaper generator at bus 1 to its full limit of 100 MW. The 40 MW excess power, not consumed at bus 1, flows to bus 2 where the generator is built with only 50 MW.

Figure 3 shows the allocated transactions on the basis of allocation 2b for both buses 1 & 2 separately. Note, the “sum” of the two figures give to the actual dispatch. The corresponding P2P payments are given in Fig. 4.

The left graph Fig. 3a shows that  $d_1$  is with 60 MW totally supplied by the local production. Consequently consumers at bus 1 pay 3k € OPEX, which is the operational price of 50 € times the 60 MW. Further they pay 33k € for the CAPEX at generator 1. Note that 3k € of these account for the scarcity imposed by the upper expansion limit  $\bar{G}_s$ . The rest makes up 60% of the CAPEX spent at generator 1, exactly the share of power allocated to  $d_1$ . Consumers at bus 1 don’t pay any transmission CAPEX as no flow is associated with their demand. The right graph Fig. 3b shows the power allocations

to  $d_2$ . We see that 50 MW are self-supplied whereas the remaining 40 MW are imported from bus 1. Thus, consumers have to pay for the local OPEX and CAPEX as well as the expenditures at bus 1 and the transmission system. As the capacity at generator 2 does not hit the expansion limit  $\bar{G}_s$ , no scarcity cost are assigned to it. The allocated CAPEX exactly matches the actual CAPEX of generator 2. However in the case of generator 1, 2k € of the 22k € CAPEX allocation are associated to the scarcity cost. The paid congestion revenue of 4k € is exactly the CAPEX of the transmission line (zero-profit condition). The sum of all values in the payoff matrix in Fig. 4 yield  $\mathcal{TC} - \mathcal{R}^{\text{scarcity}}$ , the total system cost minus the scarcity cost (which in turn is negative). The sum of a column yields the total revenue per the asset  $i$ . These values match their overall spending plus the cost of scarcity. The sum of a row returns the total payment of consumers at bus  $n$ . For example the sum of payments of consumers at bus 1 is 36k €. This is exactly the electricity price of 600 €/MW times the consumption of 60 MW,  $\lambda_1 d_1$ .

The fact that OPEX and CAPEX allocations are proportional to each other results from the fact of optimizing one time step only. This coherence breaks

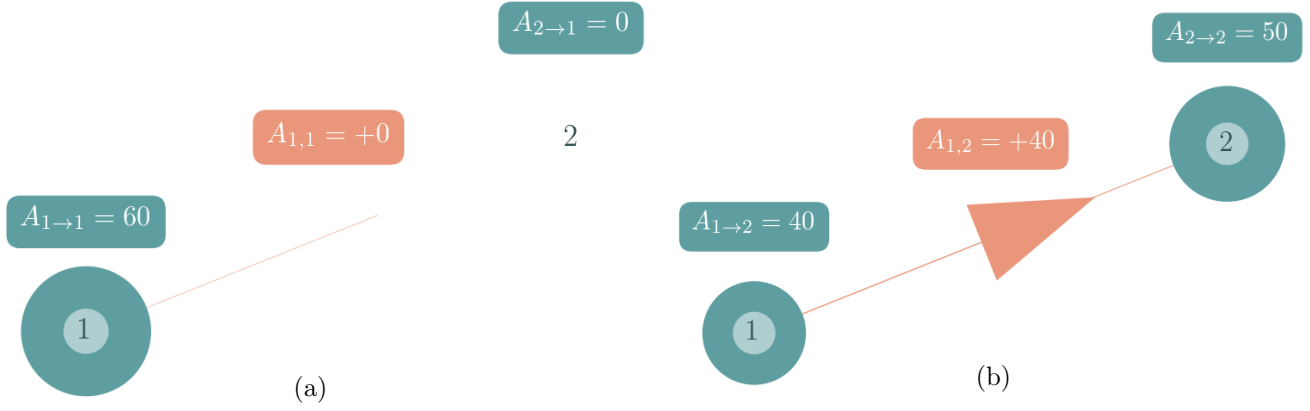


Figure 3: Power allocations of net power injections using the AP scheme, type **2b**, in the example network, Fig. 2. Consumers at bus 1, figure (a), are, with 60 MW, totally supplied by the local generator and do not consume any imported power from bus 2. In contrast consumer at bus 2, figure (b), retrieve 40 MW from generator 1, induce a flow of 40 MW on the transmission line and consume 50 MW from the local generator.

	$\mathcal{O}$		$\mathcal{C}^G$		$\mathcal{C}^F$
1	3k €	0k €	33k €	0k €	0k €
2	2k €	10k €	22k €	25k €	4k €
	1	2	1	2	1
	s		s		$\ell$

Figure 4: Full P2P cost allocation on the basis of power allocations shown in Fig. 3. Consumers compensate OPEX and CAPEX of the generators they retrieve from, see Fig. 3. As bus 1 is totally self-supplying, all its payment is assigned to the local generator. As bus 2 imports power from bus 1 and thus induces a flow on line 1, it not only compensates local expenditures but also OPEX and CAPEX at bus 1 and CAPEX for the transmission.

for for larger optimization problems with multiple time steps. Then CAPEX allocation takes effect only for time steps in which one or more of the capacity constraints Constrs. (12), (16) and (17) become binding.

The example shows that allocating net power injection leads to intuitive and transparent cost allocations.

## 4 Application Case

We showcase the behavior of the cost allocation in a more complex system, by applying it to a cost-optimized German power system model with 50 nodes and one year time span with hourly resolution. The model builds on the PyPSA-EUR workflow [11] with technical details and assumptions reported in [10].

We follow a brownfield approach where transmission lines can be expanded starting from today's capacity values, originally retrieved from the ENTSO-E Transmission System Map [6]. Pre-installed wind and solar generation capacity totals of the year 2017 were distributed in proportion to the average power potential at each site excluding those with an average capacity factor of 10%. Further, wind and solar capacity expansion are limited by land use restriction. These consider agriculture, urban, forested and protected areas based on the CORINE and NATURA2000 database [4, 5]. Pumped Hydro Storages (PHS) and Run-of-River power plants are fixed to today's capacities with no more expansion allowed. Additionally, unlimited expansion of batteries and  $H_2$ -storages and Open-Cycle Gas Turbines (OCGT) are allowed at each node. We impose an effective carbon price of 120 € per tonne- $CO_2$  which, for OCGT, adds an effective price of 55 €/MWh<sub>el</sub> (assuming a gross emission of 180 kg/MWh and an efficiency of 39%). All cost assumptions on operational costs  $o_i$  and annualized capital cost  $c_i$  are summarized in detail in Table 2.



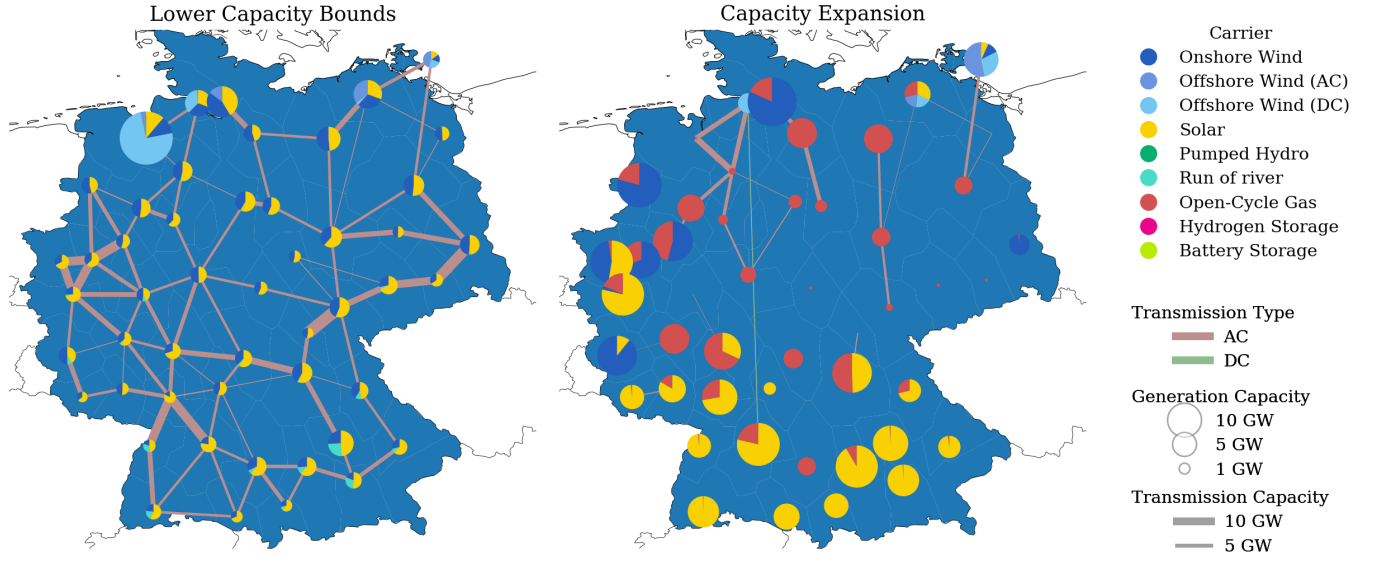


Figure 5: Brownfield optimization of the German power system. The left side shows existent renewable capacities, matching the total capacity for the year 2017, which serve as lower capacity limits for the optimization. The right side shows the capacity expansion of renewables as well as installation of backup gas power plants. The effective CO<sub>2</sub> price is set to 120 €/per tonne CO<sub>2</sub> emission.

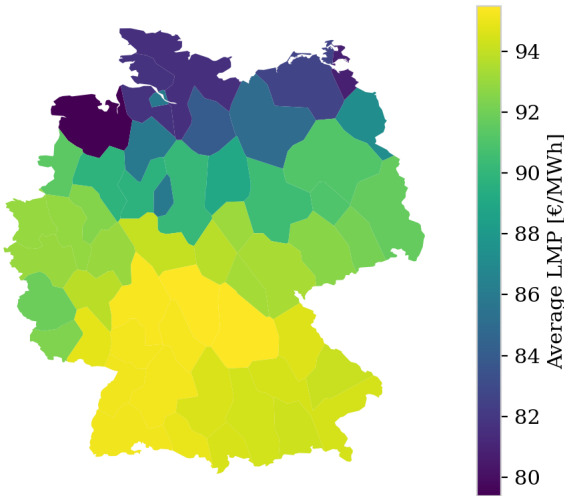


Figure 6: Average electricity price  $\langle \lambda_{n,t} \rangle_t$  per region as a result from the optimization of the German power system. Regions in the middle and south of Germany have high prices whereas electricity in the North with a strong wind, transmission and OCGT infrastructure is cheaper.

The optimized network is shown in Fig. 5. On the left we find the lower capacity bounds for renewable generators and transmission infrastructure, on the right the capacity expansion for generation, storage and transmission. The optimization expands solar capacities in the south, onshore and offshore wind in the upper north and most west. Open-Cycle Gas Turbines (OCGT) are built within the broad middle of the network. Transmission lines are amplified in along the North-South axis, including one large DC link, associated with the German Süd-Link, leading from the coastal region to the South-West. The total annualized cost of the power system roughly sums up to 42 billion €.

Figure 6 displays the average electricity price  $\lambda_{n,t}$  per region. We observe a relatively strong gradient from south (at roughly 92 €/MWh) to north (80 €/MWh). Regions with little pre-installed capacity and capacity expansion, especially for renewables, tend to have higher prices. The node with the lowest LMP in the upper North-West, stands out through high pre-installed offshore capacities.

The costs are allocated according to type 2b. For allocating flows on DC-lines we use the method described in [9]. In Fig. 7 we show the total of all “allocatable” costs  $\mathcal{C}'$ . Due to the design constraints, Section 2.3, and storage units, Section 2.2.3, these are

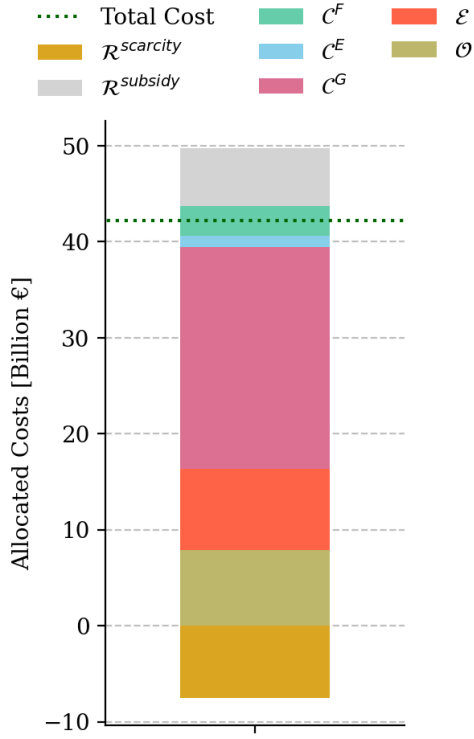


Figure 7: Total allocated payments of the system.

different from the original cost terms  $\mathcal{C}$ . The figure also includes costs for scarcity  $\mathcal{R}^{\text{scarcity}}$  and subsidy  $\mathcal{R}^{\text{subsidy}}$ . Note that the sum of all contributions in Fig. 7 equals the total cost  $\mathcal{TC}$ .

The largest proportion of the payments is associated with CAPEX for generators, transmission system and storage units in decreasing order. Taking the different technologies into account, we observe a fundamental difference between the controllable OCGT and the variable renewable resources: As shows in detail in Fig. C.6, more than half of all investments in OCGT is determined in one specific hour. At this time (morning, end of February), the system hits the highest mismatch between low renewable power potential and high demand. The necessity for backup generators manifests in high allocation of CAPEX and consequently high LMP. With a few exceptions in the South West and East, all consumers receive power from OCGT at this time, thus all pay high amounts for the needed backup infrastructure (operational state of the system is in detailed shown in Fig. C.7). Note that this is the most extreme event, which ensures backup infrastructure for other inferior extreme events where backup power is needed. The total CAPEX allocation for OCGT infrastructure is

depicted in Fig. C.1d which correlates with our findings from the extreme event.

Contrary to this, CAPEX for renewable infrastructure are allocated evenly throughout several thousands of hours. As for onshore and offshore wind farms, the produced power deeply penetrates the network, see Fig. C.4, thus it is not only local, but also remote consumers which cover the CAPEX, see Fig. C.1. This in turn benefits local consumers, which profit from the cheap power which explains why these regions end up with a low average LMP.

Together with the emission cost  $\mathcal{E}$ , the total OPEX  $\mathcal{O}$  amount around 16 billion €. As to expect, 99.97% are dedicated to OCGT alone. For a detailed regional distribution of payments per MWh and the resulting revenues for generator, see Figs. C.2 and C.3. Compared to the CAPEX allocation for OCGT, the OPEX distributes in a more local manner. This results from the fact that during “standard” demand peaks, it is rather local OCGT generators which serve as backup generators. In the average power mix per region, see Fig. C.4, we observe that regions with strong OCGT capacities predominately have high shares of OCGT power. The OPEX allocations for renewables play with 0.2 €/MWh in average an inferior role.

The negative segment in Fig. 7 is associated with the scarcity costs  $\mathcal{R}^{\text{scarcity}}$ , caused by land use constraints for renewables and the transmission expansion limit. These sum up approximately to 7.5 b€. Note again, that according to Eq. (28) this term is negative and is part of the allocated CAPEX payments. It translates to the cost that consumers pay “too much” for assets limited in their capacity expansion. In the real world this money would be used to pay for augmented land costs or civic participation in the dedicated areas. In Fig. C.9 we give a detailed insight of how the scarcity cost manifest in the average price  $\langle \lambda_{n,t} \rangle_t$ . The scarcity for wind and solar are relatively low and impact the average price by around 2€/MWh. They mostly affect regions with strong investments and those in close vicinity. Remarkably, the scarcity cost per MWh for run-of-river power plants amounts up to 16€. This high impact is due to the steady power potential from run-off water and the strong limitation of capacity expansion. However, as these power plants in particular are already amortized and scarcity cost should be reconsidered and removed from a final cost allocation. The scarcity cost for the transmission system, resulting

from the limited transmission volume expansion of 25% mostly influence regions in the middle of Germany, where in average 8€/MWh account for transmission scarcity, see Fig. C.8. This illustrates the strong impact that the line volume constraint has on the optimization. Regions in the North are not touched by the constraint.

The last cost term in Fig. 7, is caused by lower capacity constraints for pre-existing assets. These violate the optimal design and are not compensated through the nodal payments  $d_{n,t} \lambda_{n,t}$ .

## 5 Conclusion

In this work we presented a new cost-allocation scheme based on peer-to-peer dispatch allocations from asset to consumer. Starting from an optimized long-term investment model, we were able to decompose single cost terms into contributions of single consumers. For three typical classes of assets, namely generators, transmission lines and storage units, we showed how operational prices and shadow prices must be weighted with the dispatch allocation in order to allocate all system costs. Further we highlighted the impact of additional design constraints, that is constraints with a non-zero constant on the right hand side, such as lower and upper capacity expansion limits. These alter the locational marginal price and therefore distort the cost allocations. In the case of upper capacity expansion limits, this leads to an additional charge for consumers which have to compensate for *e.g.* land use restrictions. For lower capacity investment bounds, the cost have to be subsidized by the overall system operator or the government, as those cannot be allocated.

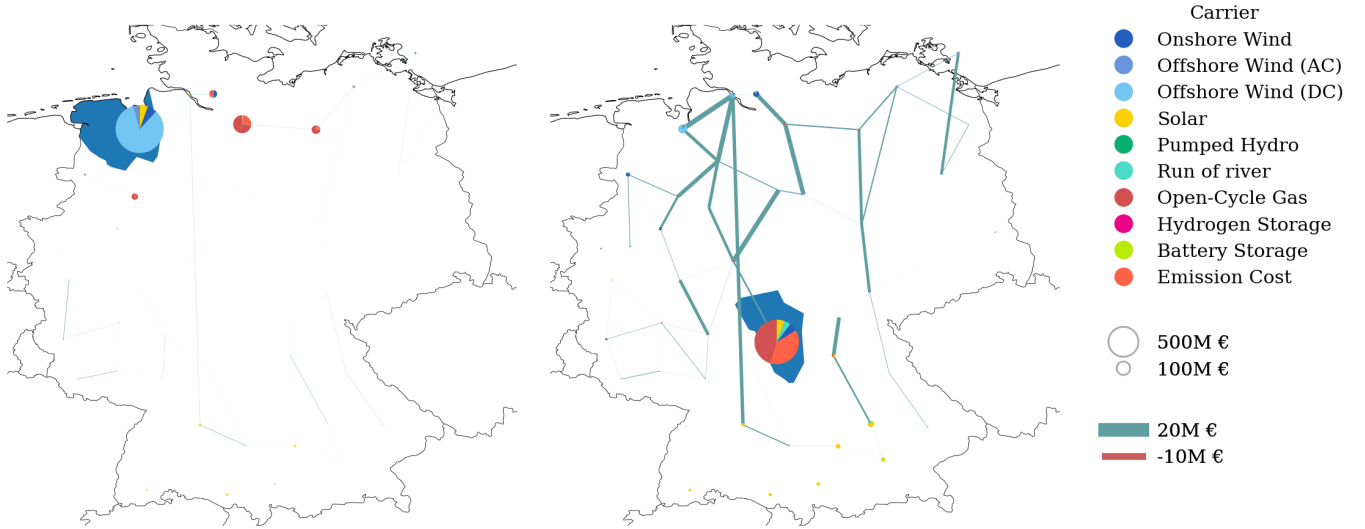


Figure 8: Comparison of payments of the node with the **lowest LMP (left)** and the node with the **highest LMP (right)**. The region of the paying bus is colored in dark blue. The circles indicate where to which bus and technology combined OPEX and CAPEX payments. Further the thickness of the lines indicates the dedicated amount of payments. The cheap prices in the North...

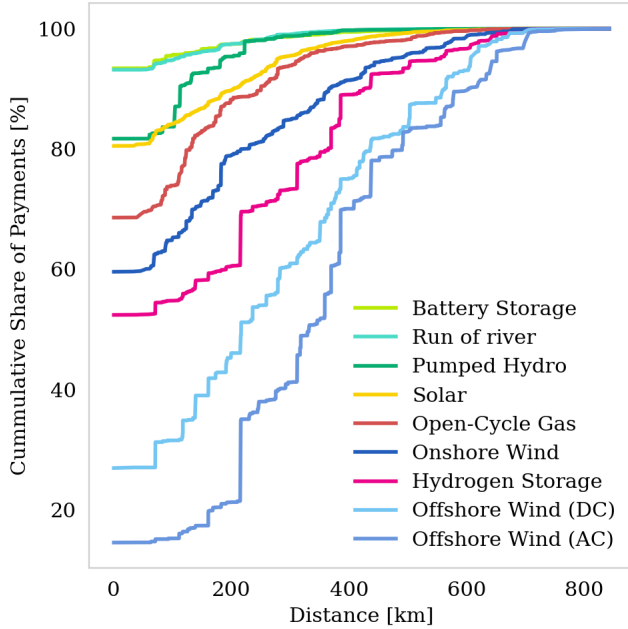


Figure 9: Average distance between payer and receiver for different technologies and shares of the total production.

## A Network Optimization

### A.1 LMP from Optimization

The nodal balance constraint ensures that the amount of power that flows into a bus equals the power that flows out of a bus, thus reflects the Kirchhoff Current Law (KCL). Alternatively, we can the demand  $d_{n,t}$  has to be supplied by the attached as-sets,

$$\sum_i K_{n,i} s_{i,t} = d_{n,t} \perp \lambda_{n,t} \quad \forall n, t \quad (\text{A.1})$$

where  $K_{n,i}$  is +1 if  $i$  is attached to  $n$  and a positive operation  $s_{i,t}$  delivers power to  $n$ , -1 if is attached to  $n$  and a positive operation retrieves power from  $n$  and zero else (note that for lines this results in the negative of the conventional Incidence Matrix). The shadow price of the nodal balance constraint mirrors the Locational Marginal Prizes (LMP)  $\lambda_{n,t}$  per bus and time step. In a power market this is the €/MWh<sub>el</sub>-price which a consumer has to pay.

### A.2 Full Lagrangian

The Lagrangian for the investment model can be condensed to the following expression

$$\begin{aligned} \mathcal{L}(s_{i,t}, S_i, \lambda_{n,t}, \mu_j) = & \sum_{i,t} o_i s_{i,t} + \sum_i c_i S_i \\ & + \sum_{n,t} \lambda_{n,t} \left( d_{n,t} - \sum_i K_{n,i} s_{i,t} \right) \\ & + \sum_j \mu_j h_j(s_{i,t}, S_i) \end{aligned} \quad (\text{A.2})$$

where  $h_j(s_{i,t}, S_i)$  denotes all inequality constraints attached to  $s_{i,t}$  and  $S_i$ . In order to impose the Kirchhoff Voltage Law (KVL) for the linearized AC flow, the term

$$\sum_{\ell,c,t} \lambda_{c,t} C_{\ell,c} x_\ell f_{\ell,t} \quad (\text{A.3})$$

can be added to  $\mathcal{L}$ , with  $x_\ell$  denoting the line's impedance and  $C_{\ell,c}$  being 1 if  $\ell$  is part of the cycle  $c$  and zero otherwise.

The global maximum of the Lagrangian requires stationarity with respect to all variables:

$$\frac{\partial \mathcal{L}}{\partial s_{i,t}} = \frac{\partial \mathcal{L}}{\partial S_i} = 0 \quad (\text{A.4})$$

### A.3 Zero Profit Generation

Constrs. (12) and (13), which yield the KKT variables  $\bar{\mu}_{s,t}$  and  $\underline{\mu}_{s,t}$ , imply the complementary slackness,

$$\bar{\mu}_{s,t} (g_{s,t} - \bar{g}_{s,t} G_s) = 0 \quad \forall n, s, t \quad (\text{A.5})$$

$$\underline{\mu}_{s,t} g_{s,t} = 0 \quad \forall n, s, t \quad (\text{A.6})$$

The stationarity of the generation capacity variable leads to

$$\frac{\partial \mathcal{L}}{\partial G_s} = 0 \rightarrow c_s = \sum_t \bar{\mu}_{s,t} \bar{g}_{s,t} \quad \forall n, s \quad (\text{A.7})$$

and the stationarity of the generation to

$$\frac{\partial \mathcal{L}}{\partial g_{s,t}} = 0 \rightarrow o_s = K_{n,s} \lambda_{n,t} - \bar{\mu}_{s,t} + \underline{\mu}_{s,t} \quad \forall n, s \quad (\text{A.8})$$

Multiplying both sides of Eq. (A.7) with  $G_s$  and using Eq. (A.5) leads to

$$c_s G_s = \sum_t \bar{\mu}_{s,t} g_{s,t} \quad (\text{A.9})$$

The zero-profit rule for generators is obtained by multiplying Eq. (A.8) with  $g_{s,t}$  and using Eqs. (A.6) and (A.9) which results in

$$c_s G_s + \sum_t o_s g_{s,t} = \sum_t \lambda_{n,t} K_{n,s} g_{s,t} \quad (\text{A.10})$$

It states that over the whole time span, all OPEX and CAPEX for generator  $s$  (left hand side) are payed back by its revenue (right hand side).

### A.4 Zero Profit Transmission System

The yielding KKT variables  $\bar{\mu}_{\ell,t}$  and  $\underline{\mu}_{\ell,t}$  are only non-zero if  $f_{\ell,t}$  is limited by the transmission capacity in positive or negative direction, i.e. Constr. (16) or Constr. (17) are binding. For flows below the thermal limit, the complementary slackness

$$\bar{\mu}_{\ell,t} (f_{\ell,t} - F_\ell) = 0 \quad \forall \ell, t \quad (\text{A.11})$$

$$\underline{\mu}_{\ell,t} (f_{\ell,t} - F_\ell) = 0 \quad \forall \ell, t \quad (\text{A.12})$$

sets the respective KKT to zero.

The stationarity of the transmission capacity to

$$\frac{\partial \mathcal{L}}{\partial F_\ell} = 0 \rightarrow c_\ell = \sum_t (\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}) \quad \forall \ell \quad (\text{A.13})$$

and the stationarity with respect to the flow to

$$0 = \frac{\partial \mathcal{L}}{\partial f_{\ell,t}} \quad (\text{A.14})$$

$$0 = - \sum_n K_{n,\ell} \lambda_{n,t} + \lambda_{c,t} C_{\ell,c} x_{\ell} - \bar{\mu}_{\ell,t} + \underline{\mu}_{\ell,t} \quad \forall n, s \quad (\text{A.15})$$

When multiplying Eq. (A.13) with  $F_{\ell}$  and using the complementary slackness Eqs. (A.11) and (A.12) we obtain

$$c_{\ell} F_{\ell} = \sum_t (\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}) f_{\ell,t} \quad (\text{A.16})$$

Again we can use this to formulate the zero-profit rule for transmission lines. We multiply Eq. (A.15) with  $f_{\ell,t}$ , which finally leads us to

$$c_{\ell} F_{\ell} = - \sum_n K_{n,\ell} \lambda_{n,t} f_{\ell,t} + \lambda_{c,t} C_{\ell,c} x_{\ell} f_{\ell,t} \quad (\text{A.17})$$

It states that the congestion revenue of a line (first term right hand side) reduced by the cost for cycle constraint exactly matches its CAPEX.

### A.5 Zero Profit Storage Units

For an simplified storage model, the upper capacity  $G_r$  limits the discharging dispatch  $g_{r,t}^{\text{dis}}$ , the storing power  $g_{r,t}^{\text{sto}}$  and state of charge  $g_{r,t}^{\text{ene}}$  of a storage unit  $r$  by

$$g_{r,t}^{\text{dis}} - G_r \leq 0 \quad \forall r, t \perp \bar{\mu}_{r,t}^{\text{dis}} \quad (\text{A.18})$$

$$g_{r,t}^{\text{sto}} - G_r \leq 0 \quad \forall r, t \perp \bar{\mu}_{r,t}^{\text{sto}} \quad (\text{A.19})$$

$$g_{r,t}^{\text{ene}} - h_r G_r \leq 0 \quad \forall r, t \perp \bar{\mu}_{r,t}^{\text{ene}} \quad (\text{A.20})$$

where we assume a fixed ratio between dispatch and storage capacity of  $h_r$ . The state of charge must be consistent throughout every time step according to what is dispatched and stored,

$$g_{r,t}^{\text{ene}} - \eta_r^{\text{ene}} g_{r,t-1}^{\text{ene}} - \eta_r^{\text{sto}} g_{r,t}^{\text{sto}} + (\eta_r^{\text{dis}})^{-1} g_{r,t}^{\text{dis}} = 0 \quad \perp \lambda_{r,t}^{\text{ene}} \quad \forall r, t \quad (\text{A.21})$$

We use the result of Appendix B.3 in [3] which shows that a storage recovers its capital (and operational) costs from aligning dispatch and charging to the LMP, thus

$$\sum_t o_r g_{r,t}^{\text{dis}} + c_r G_r = \sum_t \lambda_{n,t} K_{n,r} (g_{r,t}^{\text{dis}} - g_{r,t}^{\text{sto}}) \quad (\text{A.22})$$

The stationarity of the dispatched power leads us to

$$\frac{\partial \mathcal{L}}{\partial g_{r,t}^{\text{dis}}} = 0 \quad o_r - \lambda_{n,t} K_{n,r} - \underline{\mu}_{r,t}^{\text{dis}} + \bar{\mu}_{r,t}^{\text{dis}} + (\eta_r^{\text{dis}})^{-1} \lambda_{r,t}^{\text{ene}} = 0 \quad (\text{A.23})$$

which we can use to define the revenue which compensates the CAPEX at  $r$ ,

$$c_r G_r = \sum_t \left( \bar{\mu}_{r,t}^{\text{dis}} - \underline{\mu}_{r,t}^{\text{dis}} + (\eta_r^{\text{dis}})^{-1} \lambda_{r,t}^{\text{ene}} \right) g_{r,t}^{\text{dis}} - \sum_t \lambda_{n,t} K_{n,r} g_{r,t}^{\text{sto}} \quad \forall r \quad (\text{A.24})$$

### A.6 Emission Constraint

Imposing an additional CO<sub>2</sub> constraint limiting the total emission to K,

$$\sum_{n,s,t} e_s g_{s,t} \leq K \perp \mu_{\text{CO}_2} \quad (\text{A.25})$$

with  $e_s$  being the emission factor in tonne-CO<sub>2</sub> per MWh<sub>el</sub>, returns an effective CO<sub>2</sub> price  $\mu_{\text{CO}_2}$  in €/tonne-CO<sub>2</sub>. As shown in ... the constraint can be translated in a dual price which shift the operational price per generator

$$o_s \rightarrow o_s + e_s \mu_{\text{CO}_2} \quad (\text{A.26})$$

## B Allocation Schemes

### B.1 Allocating Gross Injections with EBE

The allocation of gross generation to demands  $d_{n,t}$  is straightforwardly obtained by a proportional distribution of the generation, *i.e.*

$$A_{s,n,t} = \frac{g_{s,t}}{\sum_s g_{s,t}} d_{n,t} \quad (\text{B.27})$$

### B.2 Allocating Net Injections with EBE

Allocating net power injections using the EBE methods leads to the same result as the Marginal Participation (MP) [12] algorithm when allocating to consumers only, see [9] for further insight. We calculate it by setting

$$A_{m \rightarrow n,t} = \delta_{m,n} p_{m,t}^{\circ} + \gamma_t p_{n,t}^{-} p_{m,t}^{+} \quad (\text{B.28})$$

where



- $p_{n,t}^+ = \min(g_{n,t} - d_{n,t}, 0)$  denotes the nodal net production
- $p_{n,t}^- = \min(d_{n,t} - g_{n,t}, 0)$  denotes the nodal net consumption
- $p_{n,t}^\circ = \min(p_{n,t}^+, p_{n,t}^-)$  the denotes nodal self-consumption. That is the power generated and at the same time consumed at node  $n$  and
- $\gamma_t = (\sum_n p_{n,t}^+)^{-1} = (\sum_n p_{n,t}^-)^{-1}$  is the inverse of the total injected/extracted power at time  $t$ .

The allocation  $A_{s,n,t}$  from generator  $s$  to  $n$ , is given by multiplying  $A_{m \rightarrow n,t}$  with the nodal share  $g_{s,t}/g_{n,t}$ .

### B.3 Allocating Net Power using AP

Allocating net injections using the AP method is derived from [1]. In a lossless network the downstream and upstream formulations result in the same P2P allocation which is why we restrict ourselves to the downstream formulation only. In a first step we define a time-dependent auxiliary matrix  $\mathcal{J}_t$  which is the inverse of the  $N \times N$  with directed power flow  $m \rightarrow n$  at entry  $(m,n)$  for  $m \neq n$  and the total flow passing node  $m$  at entry  $(m,m)$  at time step  $t$ . Mathematically this translates to

$$\mathcal{J}_t = (\text{diag}(p^+) + \mathcal{K}^- \text{diag}(f) K)_t^{-1} \quad (\text{B.29})$$

where  $\mathcal{K}^-$  is the negative part of the directed Incidence matrix  $\mathcal{K}_{n,\ell} = \text{sign}(f_{\ell,t}) K_{n,\ell}$ . Then the distributed slack for time step  $t$  is given by

$$A_{m \rightarrow n,t} = \mathcal{J}_{m,n,t} p_{m,t}^+ p_{n,t}^- \quad (\text{B.30})$$

### B.4 Allocating Gross Power using AP

We use the same allocation as in Appendix B.3 but replace the net nodal production  $p_{n,t}^+$  by the gross nodal production  $g_{n,t}$  which leads to

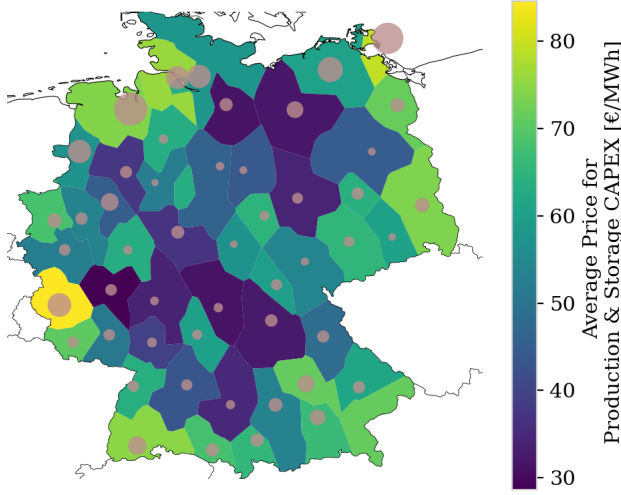
$$\mathcal{J}_t = (\text{diag}(g) + \mathcal{K}^- \text{diag}(f) K)_t^{-1} \quad (\text{B.31})$$

The distributed slack is for time step  $t$  is then given by

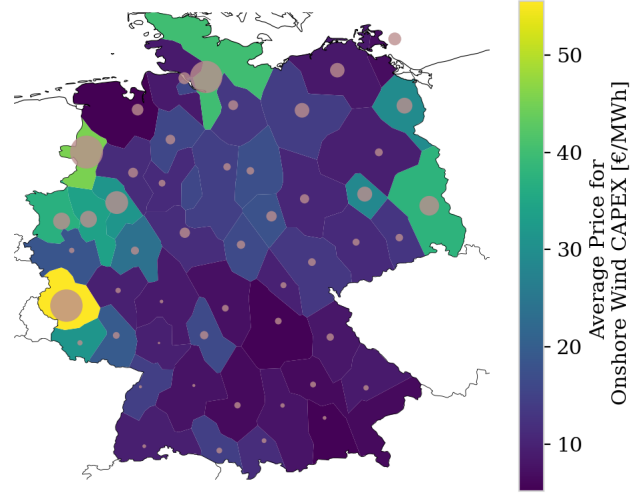
$$A_{s \rightarrow m,t} = \mathcal{J}_{m,n} g_{s,t} d_{n,t} \quad (\text{B.32})$$

## C Working Example

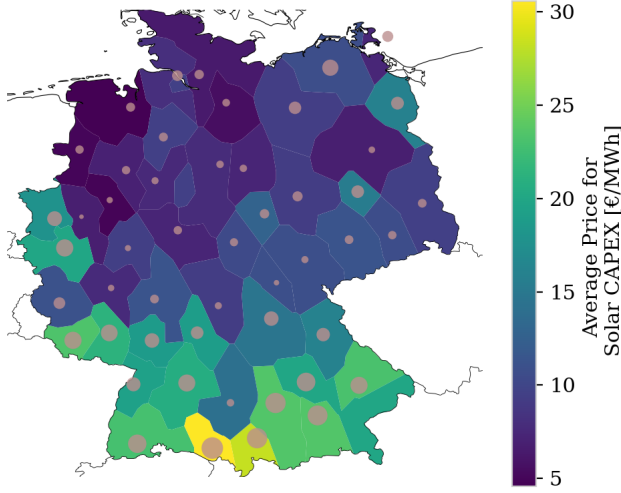
The following figures contain more detailed information about the peer-to-peer cost allocation. The cost or prices paid by consumers are indicated by the region color. The dedicated revenue is displayed in proportion to the size of cycles (for assets attached to buses) or to the thickness of transmission branches.



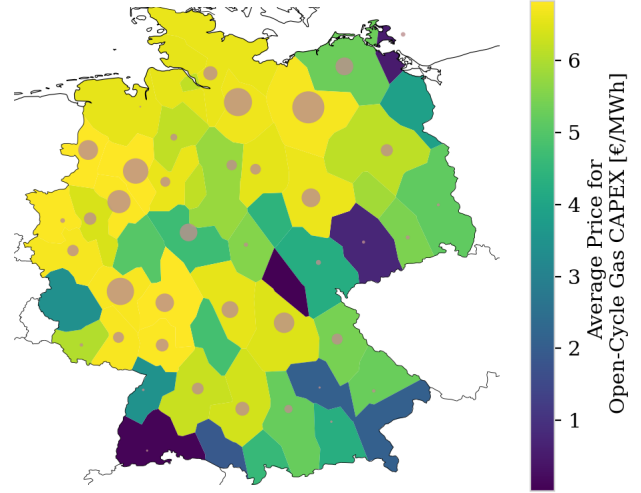
(a) All production and storage technologies



(b) Onshore Wind



(c) Solar



(d) OCGT

Figure C.1: Average **CAPEX allocation** per MWh,  $\sum_t c_{n \rightarrow s,t}^G / \sum_t d_{n,t}$  and  $\sum_t c_{n \rightarrow r,t}^E / \sum_t d_{n,t}$ , for all production and storage assets (a), onshore wind (b), solar (c) and OCGT (d). Average Allocated CAPEX per MWh within the regions are indicated by the color, the revenue per production asset is given by the size of the circles at the corresponding bus.

		o [€/MWh]	c [k€/MW]*
carrier			
Generator	Open-Cycle Gas	120.718	47.235
	Offshore Wind (AC)	0.015	204.689
	Offshore Wind (DC)	0.015	230.532
	Onshore Wind	0.015	109.296
	Run of river		270.941
	Solar	0.01	55.064
Storage	Hydrogen Storage		224.739
	Pumped Hydro		160.627
	Battery Storage		133.775
Line	AC		0.038
	DC		0.070

Table 2: Operational and capital price assumptions for all type of assets used in the working example. The capital price for transmission lines are given in [k€/MW/km]. The cost assumptions are retrieved from the PyPSA-EUR model [11].

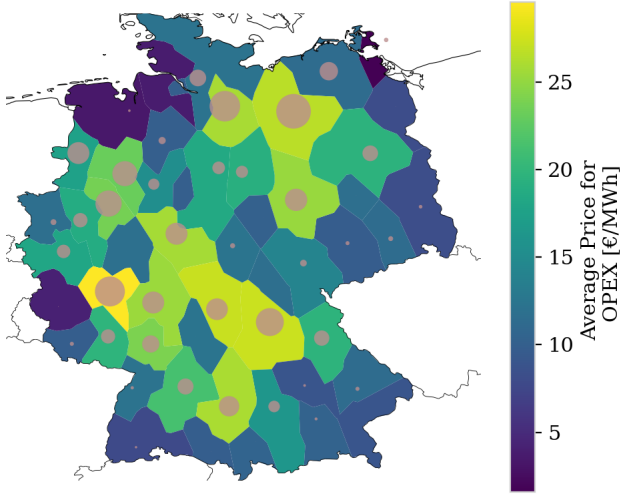


Figure C.2: Average **OPEX allocation** per consumed MWh,  $\sum_t \mathcal{O}_{n \rightarrow s, t} / \sum_t d_{n, t}$ . The effective prices for OPEX are indicated by the color of the region, the size of the circles are set proportional to the revenue per regional generators and storages. As OCGT is the only allowed fossil based technology, the drawn allocation is proportional to OPEX allocation of OCGT generators.

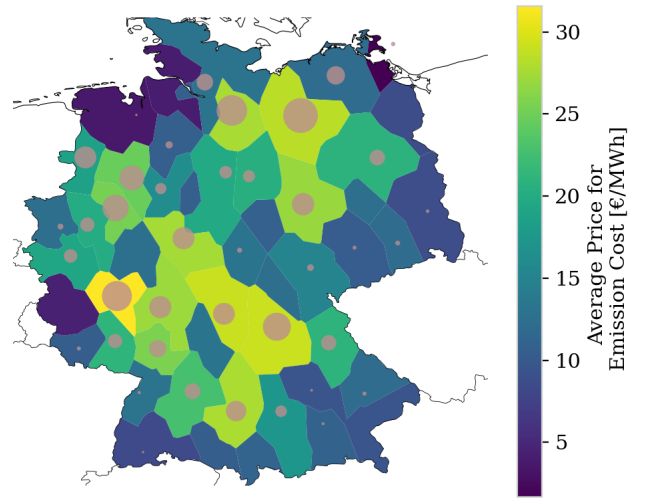


Figure C.3: Average **allocated emission cost**,  $\sum_t \mathcal{E}_{n \rightarrow s, t}$ , per consumed MWh. The effective prices are indicated by the color of the region, the size of the circles are set proportional to the revenue per regional generators.

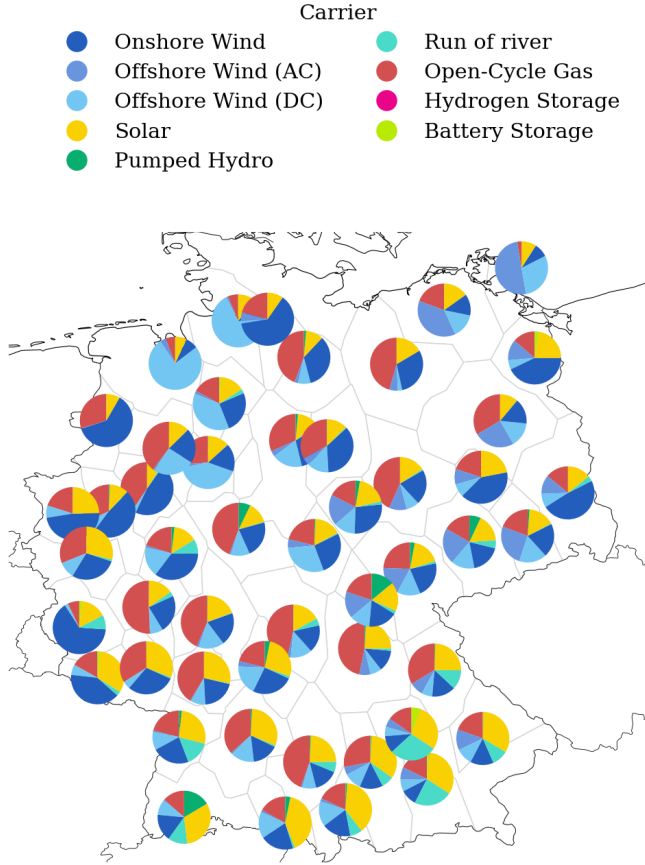


Figure C.4: Average power mix per region calculated by Average Participation **b**. Coastal regions are mainly supplied by local offshore and onshore wind farms. Their strong power injections additionally penetrate the network up to the southern border. In the middle and South, the supply is dominated by a combination of OCGT and solar power.

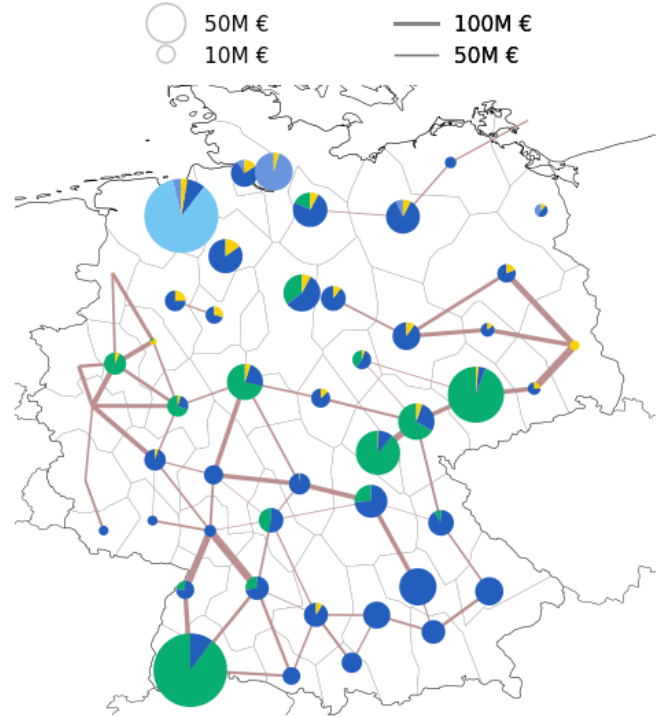


Figure C.5: Total costs for subsidy  $\mathcal{R}^{\text{subsidy}}$  resulting from lower capacity expansion bounds (brown-field constraints). The figure shows the built infrastructure that does not gain back its CAPEX from its market revenue, but is only build due to lower capacity limits.

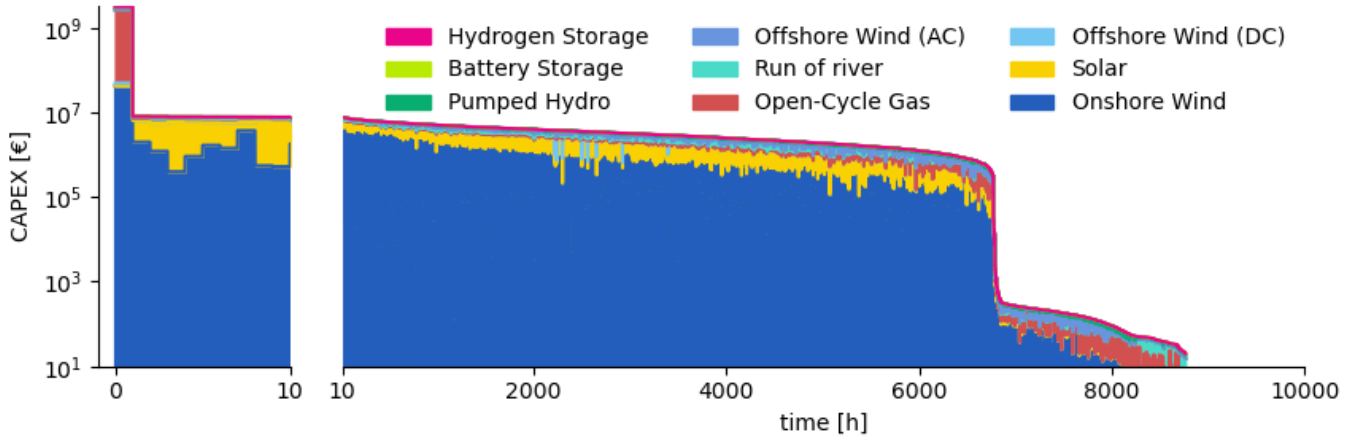


Figure C.6: Duration curve of the CAPEX allocation for production and storage technologies. Hours are sorted by the total amount of allocated expenditure. With 2.7 bn € the first value pushes investments extraordinarily high. Due to low renewable potentials, it is dominated by CAPEX for OCGT which receives 92% of the payments. This hour alone occasions about the half of all OCGT CAPEX. ?? gives a detailed picture of the operational state at this time-step. The following 7000 time-steps are dominated by revenues for onshore wind and reveal a rather even distribution. In hours of low CAPEX allocation (after the second drop) spendings for OCGT start to increase again. These time-steps however play a minor role.

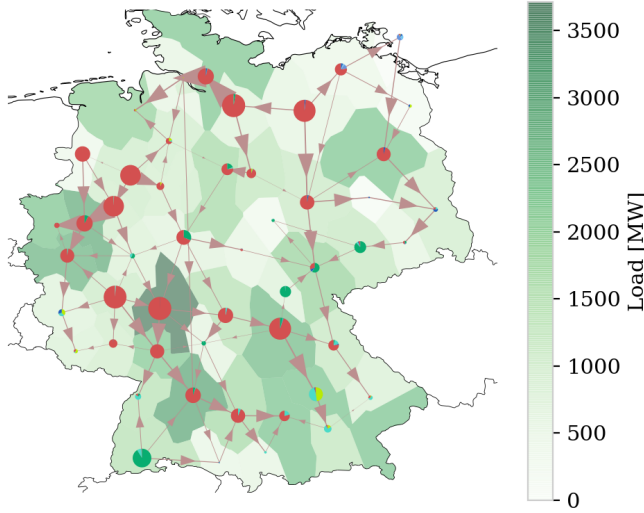


Figure C.7: Production, flow and consumption in the system at the hour with the highest allocated expenditures. The size of the circles are proportional to the power production at a node. Size of arrows are proportional to the flow on the transmission line. The depicted hour corresponds to the first value in the duration curve in ??.

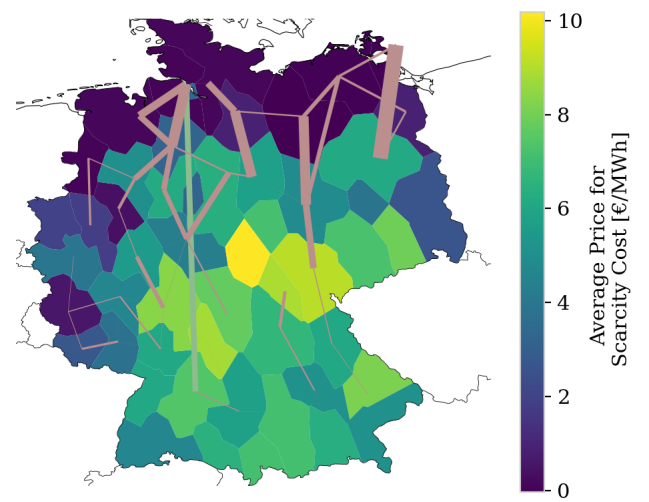
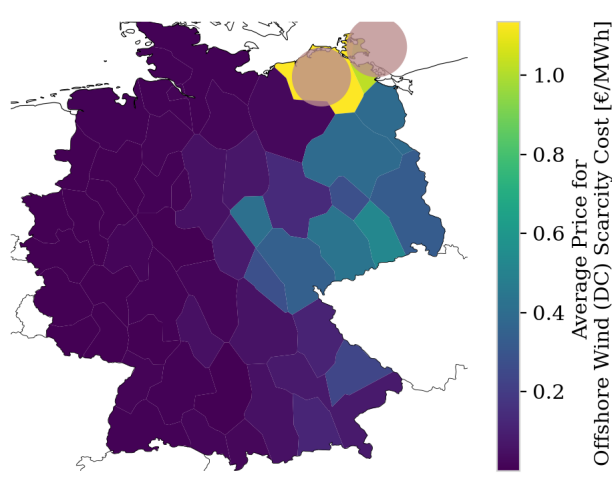
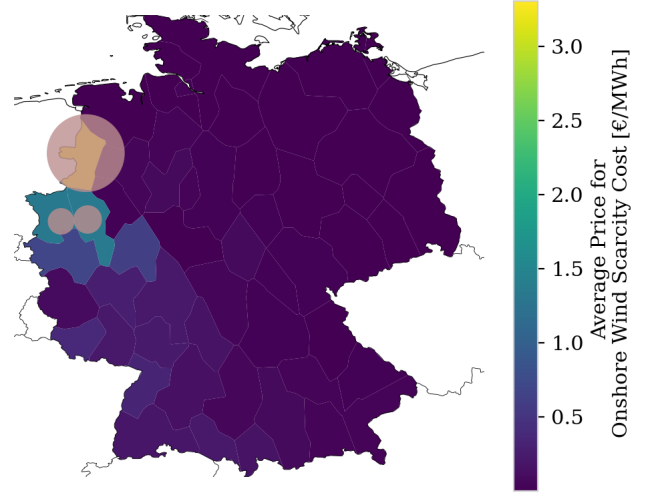


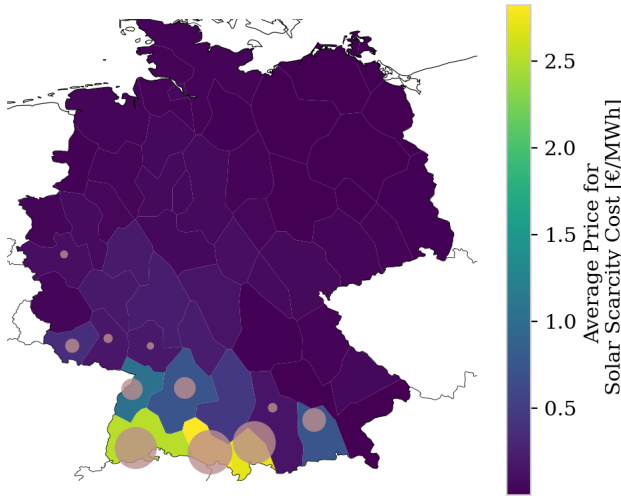
Figure C.8: Average **allocated transmission scarcity cost** per consumed MWh,  $\sum_t \mathcal{R}_{n \rightarrow \ell, t}^{\text{scarcity}} / \sum_t d_{n, t}$ . This scarcity cost results from the upper transmission expansion limit of 25%. The costs are indicated by the regional color. The lines are drawn in proportion to revenue dedicated to scarcity cost.



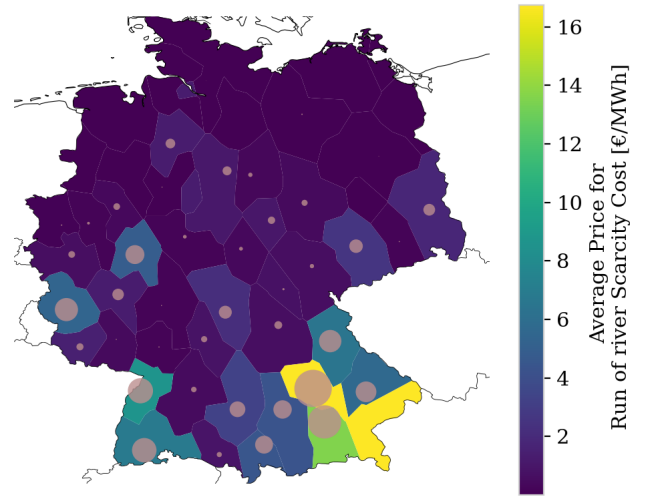
(a) Offshore Wind



(b) Onshore Wind



(c) Solar



(d) Run-of-River

Figure C.9: Average **allocated scarcity cost** per consumed MWh,  $\sum_t \mathcal{R}_{n \rightarrow i, t}^{\text{scarcity}} / \sum_t d_{n, t}$ . These cost result from land use restrictions for offshore wind, onshore wind, solar, run-of-river. The cost per MWh are indicated by the color of a region. The revenue per production asset is given by the size of the circle at the corresponding bus.



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