Flow-Based Cost Allocation in Linear Optimized Power Systems are non-unique but localizable

F. Hofmann, Brown T.

August 11, 2020

Abstract

Highlights

- The LMP always breaks down into contributions of different cost terms (OPEX, CAPEX, etc.) of the system.

- Breaking the LMP further down into contributions of components (generator, transmission line, etc.) requires assumptions on power flow assignments.

- Methods like Average Participation or Flow Based Market Coupling allow for localizing the P2P cost assignments and prioritize assignments within one market zone.

Nomenclature

$\lambda_{n,t}$	Locational Market Price at bus n and time step t in €/MW
$d_{n,t}$	Electric demand per bus n, demand type a, time step t in MW
$g_{s,t}$	Electric generation of generator s, time step t in MW
$f_{\ell,t}$	Active power flow on line ℓ, time step t in MW
o_s	Operational price in €/MW
c_s	Capital Price in €/MW
c_ℓ	Capital Price in €/MW for transmission capacity on line ℓ
G_s	Generation capacity in MW
F_ℓ	Transmission capacity in MW
$K_{n,\ell}$	Incidence matrix

1 Economic Context

In long-term operation and investment planning models, the total costs $\mathcal{TC}$ of a power system is the sum of mutliple cost terms. Typically, these include operational expenditures (OPEX) $\mathcal{O}^G$ for generators, expenditures for emissions $\mathcal{E}$, capital expenditures (CAPEX) for generators $\mathcal{C}^G$, CAPEX for the transmission system $\mathcal{C}^F$ and possible other terms, *i.e.*

$$\mathcal{TC} = \mathcal{O}^G + \mathcal{E} + \mathcal{C}^G + \mathcal{C}^F + ... \tag{1}$$

In turn, each single cost term on the right side $\mathcal{C} = \{\mathcal{O}^G, \mathcal{E}, \mathcal{C}^G, \mathcal{C}^F, ...\}$ consists of cost associated to the asset i in the system,

$$\mathcal{C} = \sum_i \mathcal{C}_i \tag{2}$$

For example the total OPEX, $\mathcal{O}^G = \sum_s \mathcal{O}^G_s$, is the combined OPEX of all generators s.

In a cost-optimal setup with minimized $\mathcal{TC}$, the Locational Marginal Price (LMP) describes the price for an incremental increase of electricity demand $d_{n,t}$ at node n. It is given by the derivative of the total system cost $\mathcal{TC}$ with respect to the local demand $d_{n,t}$

$$\lambda_{n,t} = \frac{\partial\, \mathcal{TC}}{\partial\, d_{n,t}} \tag{3}$$

This leads to a nodal pricing where over the span of optimized timesteps t, the system costs are partially or totally payed back by the consumers

$$\mathcal{TC} - \mathcal{R} = \sum_{n,t} \lambda_{n,t}\, d_{n,t} \tag{4}$$

depending on the costs $\mathcal{R}$ which are independent of the nodal demand

$$\frac{\partial \mathcal{R}}{\partial d_{n,t}} = 0 \tag{5}$$

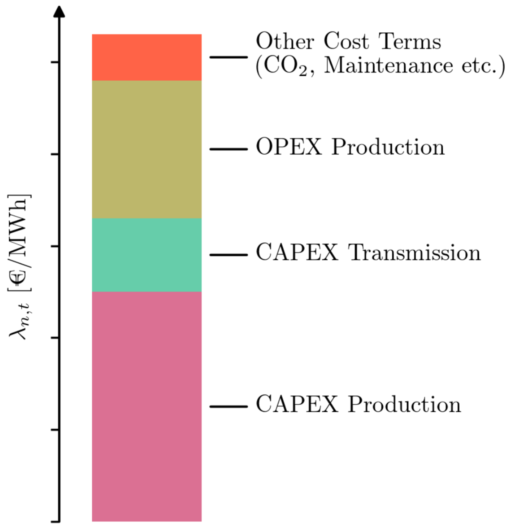


Figure 1: Schematic decomposition of the Locational Market Price $\lambda_{n,t}$. In power system model with optimal long-term operation and planning, the total system costs $\mathcal{TC}$ split into different cost terms, *i.e.* OPEX and CAPEX for production and transmission and possibly other expenditures.

Generally speaking, the cost term $\mathcal{R}$, not covered by the consumers, results from additional demands on the network design, such as capacity expansion limits or minimum share of one technology in the power mix. However, in most cases, where $\mathcal{R} \ll \mathcal{TC}$, these play a minor role. From feeding Eq. (1) into Eq. (3) it follows naturally that the LMP splits into different cost terms assigned to OPEX and CAPEX for production and transmission (and other possible expenditures). This relation, which we schematically show in Fig. 1, was already shown in extensive investigations of the LMP [8].

However the question of how the LMP can be decomposed into contributions of single cost terms $\mathcal{C}_i$ associated with asset i remains unanswered. This work aims at presenting and illustrating a an intuitive, peer-to-peer cost allocation including all network assets.

2 Dispatch-Based Cost Allocation

Let $\mathcal{C}_i$ denote a general cost term associated with asset i. Consider a long-term equilibrium in a power system with perfect competition, then, according to the zero-profit condition, each cost term $\mathcal{C}_i$ can be considered as a cost-weighted sum of the operational state $s_{i,t}$ of asset i, *i.e.*

$$\mathcal{C}_i = \sum_t c_{i,t}\, s_{i,t} \tag{6a}$$

where $c_{i,t}$ denotes a general cost factor in €/MW. For OPEX the cost factor $c_{i,t}$ is simply given by the marginal operational price o_i for asset i. However for CAPEX, $c_{i,t}$ is a composition of shadow prices μ_i, given by the KKT variables of the capacity limit constraints. As we will show later the composition of μ_i must be determined for each cost term individually.

Following the implications of Eqs. (3) and (4), we define the cost $\mathcal{C}_{n\rightarrow i,t}$ that consumers at bus n have pay to asset i at time t as

$$\mathcal{C}_{n\rightarrow i,t} = c_{i,t}\, \frac{\partial s_{i,t}}{\partial d_{n,t}} d_{n,t} \tag{6b}$$

Therefore, $\mathcal{C}_{n\rightarrow i,t}$ indicates the contribution of $d_{n,t}$ to the cost term $\mathcal{C}_i$. It heavily relies on the derivative of the operational state with respect to the nodal demand $\partial s_{i,t}/\partial d_{n,t}$. Without further specifying, we define the derivative multiplied with the nodal demand as

$$A_{i,n,t} = \frac{\partial s_{i,t}}{\partial d_{n,t}} d_{n,t} \tag{6c}$$

This quantity may be interpreted as the amount of power that asset i supplies demand $d_{n,t}$ with. From the natural fact that the sum of all contributions must return the cost term,

$$\mathcal{C}_i = \sum_{n,t} \mathcal{C}_{n\rightarrow i,t} \tag{6d}$$

it follows that $A_{i,n,t}$ must fulfill

$$s_{i,t} = \sum_n A_{i,n,t} \tag{6e}$$

Finally, the total contribution from node n at time t to the cost term $\mathcal{C}$ amounts

$$\mathcal{C}_{n,t} = \sum_i \mathcal{C}_{n\rightarrow i,t} \tag{6f}$$

Now, assume a network with generators s, transmission lines ℓ and storage units r. Each asset $i = \{s, \ell, r\}$ adds an term for OPEX and a term for CAPEX to the total system cost $\mathcal{TC}$. The operational price for an asset i is given by o_i, the capital

	i	$\mathcal{C}$	$\mathcal{C}_i$	$c_{i,t}$	$s_{i,t}$
OPEX Production	s	$\mathcal{O}^G$	$\sum_t o_s\, g_{s,t}$	o_s	$g_{s,t}$
OPEX Transmission	ℓ	$\mathcal{O}^F$	$\sum_t o_\ell\, \lvert f_{\ell,t}\rvert$	o_ℓ	$\lvert f_{\ell,t}\rvert$
OPEX Storage	r	$\mathcal{O}^E$	$\sum_t o_r\, g_{r,t}^{\mathrm{dis}}$	o_r	$g_{r,t}$
CAPEX Production	s	$\mathcal{C}^G$	$c_s G_s$	$\bar{\mu}_{s,t}$	$g_{s,t}$
CAPEX Transmission	ℓ	$\mathcal{C}^F$	$c_\ell F_\ell$	$(\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t})$	$f_{\ell,t}$
CAPEX Storage	r	$\mathcal{C}^E$	$c_r G_r$	$(\eta_r^{\mathrm{dis}})^{-1} \lambda_{r,t}^{\mathrm{ene}} - \underline{\mu}_{r,t}^{\mathrm{dis}}$	$g_{r,t}$
Emission Cost	s	$\mathcal{E}$	$\mu_{\mathrm{CO2}}\, e_s\, g_{s,t}$	$\mu_{\mathrm{CO2}}\, e_s$	$g_{s,t}$

Table 1: Mapping of different cost terms to the cost allocation scheme given in Eqs. (6). These include OPEX & CAPEX for production, transmission and storage assets in the network, as well as a cost term for the total Green House Gas (GHG) emissions.

price for one unit capacity expansion by c_i. For example the OPEX occasioned by generator s is given by

$$\mathcal{O}_s^G = \sum_t o_s\, g_{s,t} \tag{7}$$

where $g_{s,t}$ denotes its production at time t. It becomes clear that Eq. (7) matches the form of Eq. (6a) which allows us to use the above presented scheme in Eqs. (6) for a full cost allocation of OPEX on production. As a result we obtain the contribution of $d_{n,t}$ to the OPEX at generator s, given by

$$\mathcal{O}_{n \to s,t} = o_s\, A_{s,n,t} \tag{8}$$

The quantity

$$A_{s,n,t} = \frac{\partial g_{s,t}}{\partial d_{n,t}}\, d_{n,t} \tag{9}$$

can be considered as a power allocation which indicates the power that is produced by generator s and consumed at node n at time t. It follows naturally that, in the same manner, the scheme can be applied to the OPEX on the flow $f_{\ell,t}$ and storage unit dispatch $g_{r,t}^{\mathrm{dis}}$. As we assume a bidirectional flow on line ℓ, the OPEX is set proportional to the absolute value of the flow. The upper section in Table 1 shows the mapping of variables to Eqs. (6) in order to define the full OPEX allocation. Note, in all cases, the allocation $A_{i,n,t}$ remains undefined which we will resolve later.

For the CAPEX allocation, it becomes crucial to look at the individual relations between operational state $s_{i,t}$ and the capacity limit. All quantities for the CAPEX allocation, which we discuss in detail in the following, are summarized in the middle section of Table 1.

Allocate CAPEX on Generators

The nominal capacity G_s constrains the generation $g_{s,t}$ in the form of

$$g_{s,t} - \bar{g}_{s,t} G_s \leq 0 \;\perp\; \bar{\mu}_{s,t} \quad \forall\, s,t \tag{10}$$

$$-g_{s,t} \leq 0 \;\perp\; \underline{\mu}_{s,t} \quad \forall\, s,t \tag{11}$$

where $\bar{g}_{s,t} \in [0,1]$ is the capacity factor for renewable generators. At a cost-optimum, these two constraints yield the shadow prices $\bar{\mu}_{s,t}$ and $\underline{\mu}_{s,t}$. As shown in [3] and in detailed in Appendix A.3, over the whole time span, the CAPEX for generator s is payed back by the production $g_{s,t}$ times the shadow price $\bar{\mu}_{s,t}$,

$$\mathcal{C}_s^G = c_s G_s = \sum_t \bar{\mu}_{s,t}\, g_{s,t} \tag{12}$$

This representation connects the CAPEX with the operational state of generator s and matches the needed form in Eq. (6a) for using the allocation scheme. The resulting the CAPEX allocation is given by

$$\mathcal{C}_{n \to s,t}^G = \bar{\mu}_{s,t}\, A_{s,n,t} \tag{13}$$

How does this allocation behave? According to the polluter pays principle, it differentiates between consumers who are 'responsible' for investments and those who are not. If $\bar{\mu}_{s,t}$ (in literature often denoted as the Quality of Supply) is bigger than zero, the upper Capacity Constr. (10) is binding. Thus it is these times steps which push investments in G_s. If $\bar{\mu}_{s,t} = 0$, the generation $g_{s,t}$ is not bound and investments are not necessary. When summing over all CAPEX payments to generator s in Eq. (13) and using Eqs. (6e) and (12), we see that each generator retrieves exactly the cost that were spent to build the capacity G_s.

Allocate CAPEX on Transmission Lines

The transmission capacity F_ℓ limits the flow $f_{\ell,t}$ in both directions,

$$f_{\ell,t} - F_\ell \leq 0 \;\perp\; \bar{\mu}_{\ell,t} \quad \forall\, \ell, t \tag{14}$$
$$-f_{\ell,t} - F_\ell \leq 0 \;\perp\; \underline{\mu}_{\ell,t} \quad \forall\, \ell, t \tag{15}$$

which yield the shadow prices $\bar{\mu}_{\ell,t}$ and $\underline{\mu}_{\ell,t}$. Again, we use the result of [3] (for details see Appendix A.4) which derives that over the whole time span, the investment in line ℓ is payed back by the shadow prices times the flow

$$\mathcal{C}^F_\ell = c_\ell F_\ell = \sum_t \left(\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}\right) f_{\ell,t} \tag{16}$$

Again, we follow the scheme in Eqs. (6) which defines the CAPEX allocation as

$$\mathcal{C}^F_{n\to\ell,t} = \left(\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}\right) A_{\ell,n,t} \tag{17}$$

The quantity

$$A_{\ell,n,t} = \frac{\partial f_{\ell,t}}{\partial d_{n,t}}\, d_{n,t} \tag{18}$$

can be interpreted as the flow that the demand at (n,t) causes on line ℓ. The shadowprices $\bar{\mu}_{\ell,t}$ and $\underline{\mu}_{\ell,t}$ again can be seen as a measure for necessity of transmission investments at ℓ at time t. Hence, the definition of $\mathcal{C}^F_{n\to\ell,t}$ states that consumers, which retrieve power flowing on congested lines, yielding a bound Constr. (14) or (15), pay compensations for the resulting investments at ℓ. Again the sum of all CAPEX payments to line ℓ equals the total CAPEX spent. This is seen when summing Eq. (17) over all buses and time steps and using Eqs. (6e) and (16)

Allocate CAPEX on Storages

We use the result of Appendix B.3 in [3] which shows that a storage recovers its capital (and operational) costs from aligning dispatch and charging to the LMP, thus

$$\sum_t o_r\, g^{\text{dis}}_{r,t} + c_r\, G_r = \sum_t \lambda_{n,t} K_{n,r} \left(g^{\text{dis}}_{r,t} - g^{\text{sto}}_{r,t}\right) \tag{19}$$

where $K_{n,r}$ is one if storage s is at node n and zero otherwise. Using Eq. (A.25), we can reformulate to

$$c_r\, G_r = \sum_t K_{n,r} \left(\left(\eta^{\text{dis}}_r\right)^{-1} \lambda^{\text{ene}}_{r,t} - \underline{\mu}^{\text{dis}}_{r,t}\right) g^{\text{dis}}_{r,t} \tag{20}$$

Allocate Emission Expenditures

Given a fix price for emissions in CO_2 equivalents μ_{CO2}, the cost term for emission adds up to

$$\mathcal{E} = \mu_{\text{CO2}} \sum_s e_s\, g_{s,t} \tag{21}$$

where e_s denotes the emission factor in tonne-CO_2 per $\text{MWh}_{\text{el}}$for generator s. We use the same set of equation as given through Eqs. (6) and **??** but update the cost factor to $c_{i,t} \Rightarrow \mu_{\text{CO2}}\, e_s$. For consumers at bus n at time t this leads to the emission cost of

$$\mathcal{E}_{n\to s,t} = \mu_{\text{CO2}}\, e_s\, A_{s,n,t} \tag{22}$$

associated with generator s.

3 Assumptions on Power Allocations

The presented cost allocation suits for any type of topology and network setup. But so far, the question of how $A_{i,n,t}$ for generators s, lines ℓ and storages r are defined was left open. We recap that all rely on derivatives of $g_{s,t}$, $f_{\ell,t}$ and $g^{\text{dis}}_{r,t}$ with respect to the nodal demand $d_{n,t}$ (see Eq. (6e)).

Let $g_{m,t}$ denote the nodal power generation which combines the power production of all producing assets, in this case generators S and storages R, at node n and time t. It is given by

$$g_{m,t} = \sum_{i\in\{S,R\}} K_{m,i}\, s_{i,t} \tag{23}$$

with $K_{m,i}$ being 1 if asset i is attached to bus m and zero otherwise. Further, let $A_{m\to n,t}$ collect the power produced by assets at node m and consumed at n, given by

$$A_{m\to n,t} = \sum_{i\in\{S,R\}} K_{m,i}\, A_{i,n,t} \tag{24}$$

Now, let $H_{\ell,n}$ denote Power Transfer Distribution Factors (PTDF) giving the changes in the flow on line ℓ for one unit (typically one MW) of net power production at bus n. The linear power flow equation can be written as

$$f_{\ell,t} = \sum_m H_{\ell,m}\left(g_{m,t} - d_{m,t}\right) \tag{25}$$

Note that for transport models or mixed AC-DC networks, $H_{\ell,n}$ can be artificially calculated using the formulation presented in [6]. Taking the derivative with respect to the demand,

$$A_{\ell,n,t} = \frac{\partial f_{\ell,t}}{\partial d_{n,t}} d_{n,t} = \sum_m H_{\ell,m} \left(A_{m\to n,t} - \delta_{n,m} d_{n,t} \right), \tag{26}$$

shows that $A_{\ell,n,t}$ is fully determined through the peer-to-peer allocation $A_{m\to n,t}$. In other words, we only need to know how much power produced at node m is consumed at node n in order to derive the allocated flow $A_{\ell,n,t}$. Further we can breakdown $A_{m\to n,t}$ to $A_{s,n,t}$ for generators and $A_{r,n,t}$ for storages proportionally to their contribution to the nodal generation $g_{m,t}$. Unfortunately, the solution for $A_{m\to n,t}$ is non-unique and requires further assumptions. Established flow allocation schemes approach this problem from different directions. Principally two options exist *what* is allocated

1. gross power injections
2. net power injections

Further it is important *what assumptions* define the allocation, *i.e.* what method is used to define the pairs of sources and sinks. The three suitable approaches we present here are

a. Equivalent Bilateral Exchanges (EBE) [4] which assumes that every producer supplies every consumer proportional to its share in the total consumption.

b. Average Participation (AP) [2, 1] which traces the flow from producer to consumer following the law of proportional sharing.

c. Flow Based Market Coupling (FBMC) which uses zonal PTDF for allocating power within predefined regions. The interregional exchange is only allocating net power decifit or excess of the regions.

We show the mathematical formulation for all combinations a1 - c2 in Appendices A.8 to A.11. Principally, type 2 leads to less P2P trades then type 1 as power from a bus m with $g_{m,t} \leq d_{m,t}$ is not assigned to other buses, only to m. Further, as literature has often pointed out, the EBE principal a does not suit for large networks where remote buses would interconnect in the same way as buses in close vicinity [5]. The AP based type b tackles this problem by restricting P2P trades to those which are traceable when applying the proportional sharing principal. Therefore $A_{s,n,t}$ denotes that part of power produced by bus m which, when only following in the direction of $f_{\ell,t}$, ends up at bus n. Type c further allows to control the regions or market zones which are netted out in a first step. If in a region R the generation undercuts the demand, $\sum_{n\in R} g_{n,t} \leq \sum_{n\in R} d_{n,t}$, none of the inner-regional generation is assigned to other regions.

3.1 Numerical Example

In the following, we illustrate the cost allocation and explain its functionality by taking the example of a two bus system with one optimized time step, shown in Fig. 2.

The two buses are connected via one transmission line, each has one generator. Whereas generator 1 (at bus 1) has an operational price of 50 €/MWh$_{\text{el}}$, generator 2 (at bus 2) has a higher operational price of 200 €/MWh$_{\text{el}}$. For both the CAPEX rate is set to 500 €/MW and the maxmimal capacity is limited to $\bar{G}_s = 100$ MW. The transmission line has a CAPEX rate of 100 €/MW and no upper capacity limit. With a demand of 60 MW at bus 1 and 90 MW at bus 2, the optimization expands the cheaper generator at bus 1 to its full limit of 100 MW. The 40 MW excess power, not consumed at bus 1, flows to bus 2 where the generator is built with only 50 MW.

Allocating Gross Power using EBE (1a)

Figure 3 shows the allocated transactions on basis of equivalently traded gross power (type 1a) for both buses 1 & 2 separately. The resulting P2P payments are given in Fig. 4. The upper graph Fig. 3a shows that $A_{1\to 1} = 40$ MW at bus 1 are self-sustained. With only one generator at bus 1, consumers at bus 1 consequently pay 2k € OPEX and 22k € CAPEX to the generator 1. The remaining 20 MW come from bus 2 and induce a subflow on line 1 of $A_{\ell=1,1} = -20$. As this flow is in contrary direction to the total flow, it is relieving the transmission system. This translates to a congestion reward for consumers at bus 1 of $c_{\ell=1} A_{\ell=1,1} = 2$k € which is exactly the cost that had to be spent on the transmission system if bus 1 didn't induce a relieving flow, see again Fig. 4.

The lower graph Fig. 3b illustrates the impact of consumption at bus 2. As d_2 is higher than d_1 the

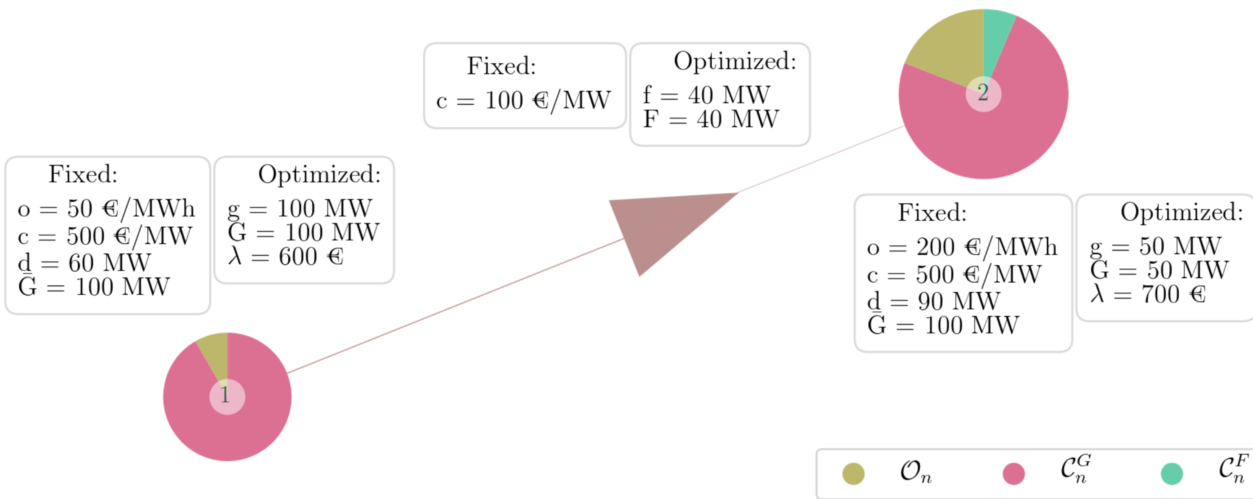
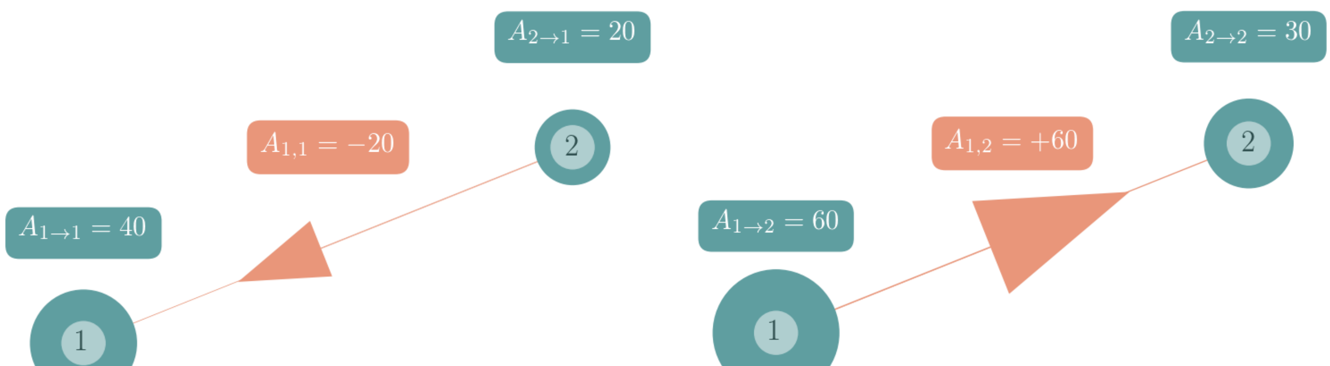


Figure 2: Illustrative example of a 2-bus network with one optimized time step. Fixed prices and constraining values are given in the left box for each bus and the transmission line. Optimized values are given in the right boxes. Bus 1 has a cheaper operational price o, capital prices are the same for both. As both generator capacities are constraint to 100 MW, the optimization also deploys the generator at bus 2. The resulting electricity prices λ are then a composition of all prices for operation and capital investments.



(a) (b)

Figure 3: Power allocations of equivalently allocated gross power, type 1a, for bus 1 (a) and bus 2 (b) of the example network in Fig. 2. Bus 1 retrieves 40 MW from itself and 20 MW from bus 2. The latter in turn retrieves 60 MW from bus 1 and self supplies 30 MW. The sum of both net flows equals the resulting flow of $f_1 = 40$ MW.

retrievements from both generators are proportionately increased as well as the OPEX and CAPEX allocations to the generators. But instead of a relieving flow, consumers at bus 2 drive the burdening flow in direction of congestion. Hence the payoff to the transmission system is positive and much higher than for bus 1.

The sum of all rows in the payoff matrix in Fig. 4 yields the revenues of the assets m, ℓ. These values match their overall spendings, *e.g.* the total revenue of the transmission line is 4k € which equals the cost for investments $c_1 F_1$. The sum of all columns yields the total payment of consumers at bus n. For example the sum of payments of consumers at bus 1 is 36k €. This is exactly the electricity price of 600 €/MW times the consumption of 60 MW, $\lambda_1 d_1$.

The fact that OPEX and CAPEX allocations are proportional to the total consumption at a bus results from optimizing one time step only. In larger

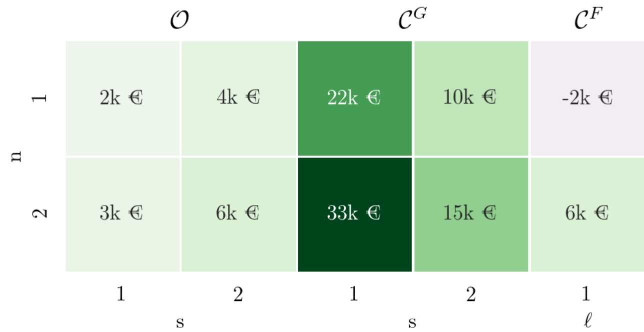


Figure 4: Full P2P cost allocation for the example setup shown in Fig. 2. The payments are derived on the basis of Eqs. (8), (13) and (17). Consumers at bus n have to pay each generator proportional to their consumption. As we only consider one time step the proportionality applies for OPEX $\mathcal{O}_{n\to s,t}$ and CAPEX $\mathcal{C}_{n\to i}$. As bus 1 induces a relieving flow an line 1 and therefore "prevents" further tranmission expansion, it is rewarded proportional to the relief.

optimization problems with multiple time steps the CAPEX allocation takes effect only for time steps in which one or more of the capacity contraints Constrs. (10), (14) and (15) become binding.

Allocating Net Power using EBE (2a)

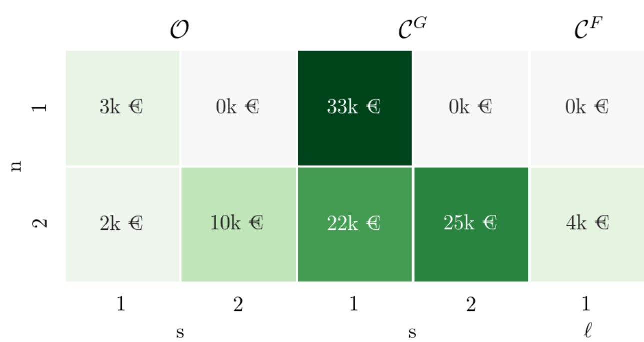


Figure 5: Full P2P cost allocation for the example setup shown in Fig. 2 when equivalently allocation net power injection (scheme a2).

In contrast the to equivalent allocation of gross power production, netting out injections for each bus leads to less P2P payments. The resulting payment given in Fig. 5 builds on the allocated power flow shown Fig. A.1 in Appendix A.12. As bus 2 does not produce excess power, none of its power production is assigned to bus 1 and thus no payment of bus 1 to bus 2 allocated. Neither has bus 1 to pay fee to the transmission system as it only exports power. So all its consumers pay to its local generators. Bus 2 in constrast bear all CAPEX for the transmission system as well as CAPEX and OPEX for generators at bus 1. Again the cummulative payments per bus meet the nodal spendings $\lambda_{n,t}\, d_{n,t}$. The cummulative revenues per generator and transmission line meet the all CAPEX and OPEX.

As the other schemes b1 - c2 apart from the EBE scheme don't distinguishably modify allocations within the presented example, we move on to a more realistic setup where systemic and local effects show up.

4 Application Case

A Appendix

A.1 LMP from Optimization

The nodal balance constraint ensures that the amount of power that flows into a bus equals the power that flows out of a bus, thus reflects the Kirchhoff Current Law (KCL)

$$\sum_{l} K_{n,\ell}\, f_{\ell,t} - g_{n,t} + d_{n,t} = 0 \;\perp\; \lambda_{n,t} \quad \forall\, n,t \tag{A.1}$$

Its shadow price mirrors the Locational Marginal Prizes (LMP) $\lambda_{n,t}$ per bus and time step. In a power market this is the €/MWh$_{\text{el}}$-price which a consumer has to pay.

A.2 Full Lagrangian

$$\begin{aligned}
&\mathcal{L}\left(g_{s,t}, f_{\ell,t}, G_s, F_\ell, \boldsymbol{\lambda}, \boldsymbol{\mu}\right) = \\
&\quad \sum_{n,s} c_s G_s + \sum_{n,s,t} o_s g_{s,t} + \sum_{\ell} c_\ell\, F_\ell \\
&\quad + \sum_{n,t} \lambda_{n,t} \left(\sum_{\ell} K_{n,\ell}\, f_{\ell,t} - \sum_{s} K_{n,s}\, g_{s,t} + d_{n,t} \right) \\
&\quad + \sum_{\ell,c,t} \lambda_{c,t}\, C_{\ell,c}\, x_\ell\, f_{\ell,t} \\
&\quad + \sum_{n,s,t} \bar{\mu}_{s,t} \left(g_{s,t} - \bar{g}_{s,t} G_s\right) - \sum_{n,s,t} \underline{\mu}_{s,t} g_{s,t} \\
&\quad + \sum_{\ell,t} \bar{\mu}_{\ell,t} \left(f_{\ell,t} - F_\ell\right) - \sum_{\ell,t} \underline{\mu}_{\ell,t} \left(f_{\ell,t} + F_\ell\right)
\end{aligned} \tag{A.2}$$

where $\boldsymbol{\lambda} = \{\lambda_{n,t}, \lambda_{c,t}\}$ and $\boldsymbol{\mu} = \{\bar{\mu}_{s,t}, \underline{\mu}_{s,t}, \bar{\mu}_{\ell,t}, \underline{\mu}_{\ell,t}\}$ denote the set of related KKT variables. The global maximum of the Lagrangian requires stationarity with respect to all variables. The stationarity of the generation capacity variable leads to

$$\frac{\partial \mathcal{L}}{\partial G_s} = 0 \;\rightarrow\; c_s = \sum_t \bar{\mu}_{s,t}\, \bar{g}_{s,t} \quad \forall\, n,s \tag{A.3}$$

the stationarity of the transmission capacity to

$$\frac{\partial \mathcal{L}}{\partial F_\ell} = 0 \;\rightarrow\; c_\ell = \sum_t \left(\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}\right) \quad \forall\, \ell \tag{A.4}$$

and the stationarity of the generation to

$$\frac{\partial \mathcal{L}}{\partial g_{s,t}} = 0 \;\rightarrow\; o_s = K_{n,s}\, \lambda_{n,t} - \bar{\mu}_{s,t} + \underline{\mu}_{s,t} \quad \forall\, n,s \tag{A.5}$$

$$0 = \frac{\partial \mathcal{L}}{\partial f_{\ell,t}} \tag{A.6}$$

$$0 = \sum_n K_{n,\ell}\lambda_{n,t} + \lambda_{c,t} C_{\ell,c} x_\ell - \bar{\mu}_{\ell,t} + \underline{\mu}_{\ell,t} \quad \forall\, n,s \tag{A.7}$$

A.3 Proof Generation Capacity Payback

As stated in [3] the capical cost of a generator s is equal to the shadowprice $\bar{\mu}_{s,t}$ times the upper availibility $\bar{g}_{s,t}$ of all time steps

$$c_s = \sum_t \bar{\mu}_{s,t}\, \bar{g}_{s,t} \tag{A.8}$$

Constrs. (10) and (11) which yield the KKT variables $\bar{\mu}_{s,t}$ and $\underline{\mu}_{s,t}$ imply the complementary slackness,

$$\bar{\mu}_{s,t} \left(g_{s,t} - \bar{g}_{s,t}\, G_s\right) = 0 \quad \forall\, n,s,t \tag{A.9}$$

$$\underline{\mu}_{s,t}\, g_{s,t} = 0 \quad \forall\, n,s,t \tag{A.10}$$

Multiplying both sides of Eq. (A.3) with G_s and using Eq. (A.9) leads to

$$c_s\, G_s = \sum_t \bar{\mu}_{s,t}\, g_{s,t} \tag{A.11}$$

The zero-profit rule for generators is obtained by multiplying Eq. (A.5) with $g_{s,t}$ and using Eqs. (A.10) and (A.11) which results in

$$c_s\, G_s + \sum_t o_s g_{s,t} = \sum_t \lambda_{n,t} K_{n,s} g_{s,t} \tag{A.12}$$

It states that over the whole time span, all OPEX and CAPEX for generator s (left hand side) are payed back by its revenue (right hand side).

A.4 Proof Flow Capacity Payback

The yielding KKT variables $\bar{\mu}_{\ell,t}$ and $\underline{\mu}_{\ell,t}$ are only non-zero if $f_{\ell,t}$ is limited by the trasmission capacity in positive or negative direction, i.e. Constr. (14) or Constr. (15) are binding. For flows below the thermal limit, the complementary slackness

$$\bar{\mu}_{\ell,t} \left(f_{\ell,t} - F_\ell\right) = 0 \quad \forall\, \ell,t \tag{A.13}$$

$$\underline{\mu}_{\ell,t} \left(f_{\ell,t} - F_\ell\right) = 0 \quad \forall\, \ell,t \tag{A.14}$$

sets the respective KKT to zero. When multipying Eq. (A.4) with F_ℓ and using the complementary slackness Eqs. (A.13) and (A.14) we obtain

$$c_\ell\, F_\ell = \sum_t \left(\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}\right)\, f_{\ell,t} \tag{A.15}$$

Againg we can use this to formulate the zero-profit rule for transmission lines. We mulitply Eq. (A.7) with $f_{\ell,t}$, which finally leads us to

$$c_\ell \, F_\ell = \sum_n K_{n,\ell} \, \lambda_{n,t} \, f_{\ell,t} + \lambda_{c,t} \, C_{\ell,c} \, x_\ell \, f_{\ell,t} \quad \text{(A.16)}$$

It states that the congestion revenue of a line (first term right hand side) substracted by the cost for cycle constraint exactly matches its CAPEX.

A.5 CO_2 constraint

Imposing an additional CO_2 constraint limiting the total emission to K,

$$\sum_{n,s,t} e_s \, g_{s,t} \leq \mathrm{K} \;\perp\; \mu_{\mathrm{CO2}} \quad \text{(A.17)}$$

with e_s being the emission factor in tonne-CO_2 per $\mathrm{MWh_{el}}$, returns an effective CO_2 price μ_{CO2} in €/tonne-CO_2. As shown in ... the constraint can be translated in a dual price which shift the operational price per generator

$$o_s \rightarrow o_s + e_s \, \mu_{\mathrm{CO2}} \quad \text{(A.18)}$$

A.6 Storage Units

For an simplified storage model, the upper capacity G_r limits the discharging dispatch $g^{\mathrm{dis}}_{r,t}$, the storing power $g^{\mathrm{sto}}_{r,t}$ and state of charge $g^{\mathrm{ene}}_{r,t}$ of a storage unit r by

$$g^{\mathrm{dis}}_{r,t} - G_r \leq 0 \quad \forall\, r,t \;\perp\; \bar{\mu}^{\mathrm{dis}}_{r,t} \quad \text{(A.19)}$$
$$g^{\mathrm{sto}}_{r,t} - G_r \leq 0 \quad \forall\, r,t \;\perp\; \bar{\mu}^{\mathrm{sto}}_{r,t} \quad \text{(A.20)}$$
$$g^{\mathrm{ene}}_{r,t} - h_r \, G_r \leq 0 \quad \forall\, r,t \;\perp\; \bar{\mu}^{\mathrm{ene}}_{r,t} \quad \text{(A.21)}$$

where we assume a fixed ratio between dispatch and storage capacity of h_r. From stationarity we obtain

$$\frac{\partial \mathcal{L}}{\partial G_r} = 0 \quad \text{(A.22)}$$
$$c_r = \sum_t \left(\bar{\mu}^{\mathrm{dis}}_{r,t} + \bar{\mu}^{\mathrm{sto}}_{r,t} + h_r \bar{\mu}^{\mathrm{ene}}_{r,t} \right) \quad \text{(A.23)}$$

$$\frac{\partial \mathcal{L}}{\partial g^{\mathrm{dis}}_{r,t}} = 0 \quad \text{(A.24)}$$
$$o_r + \lambda_{n,t} + \underline{\mu}^{\mathrm{dis}}_{r,t} - \bar{\mu}^{\mathrm{dis}}_{r,t} - (\eta^{\mathrm{dis}}_r)^{-1} \lambda^{\mathrm{ene}}_{r,t} = 0 \quad \text{(A.25)}$$

$$\frac{\partial \mathcal{L}}{\partial g^{\mathrm{sto}}_{r,t}} = 0 \quad \text{(A.26)}$$
$$-\lambda_{n,t} + \underline{\mu}^{\mathrm{sto}}_{r,t} - \bar{\mu}^{\mathrm{sto}}_{r,t} + \eta^{\mathrm{sto}}_r \lambda^{\mathrm{ene}}_{r,t} = 0 \quad \text{(A.27)}$$

A.7 Brownfield Optimization and Capacity Restricitions

Constraining the capacities G_s for a subset S of generators to lower or upper limits in the form of

$$G_s \geq \underline{G}_s \;\perp\; \underline{\mu}^{\mathrm{nom}}_s \quad \forall n, s \in S \quad \text{(A.28)}$$
$$G_s \leq \bar{G}_s \;\perp\; \bar{\mu}^{\mathrm{nom}}_s \quad \forall n, s \in S \quad \text{(A.29)}$$

alters the objective value as soon as one of those become bounding. The complementary slackness for both are

$$\bar{\mu}^{\mathrm{nom}}_s \left(G_s - \bar{G}_s \right) = 0 \quad \forall\, n, s \in S \quad \text{(A.30)}$$
$$\underline{\mu}^{\mathrm{nom}}_s \left(G_s - \underline{G}_s \right) = 0 \quad \forall\, n, s \in S \quad \text{(A.31)}$$

The CAPEX paybacks for generators and transmission lines Eqs. (A.11) and (A.15) change to

$$\frac{\partial \mathcal{L}}{\partial G_s} = 0 \;\rightarrow \quad \text{(A.32)}$$
$$c_s = \sum_t \bar{\mu}_{s,t} \, \bar{g}_{s,t} + \underline{\mu}^{\mathrm{nom}}_s - \bar{\mu}^{\mathrm{nom}}_s \quad \forall\, n, s \in S \quad \text{(A.33)}$$

for generators. Multiplying Eq. (A.33) by G_s leads us to

$$c_s \, G_s - \mathcal{R}_s = \sum_t \bar{\mu}_{s,t} \, g_{s,t} \quad \forall\, n, s \in S \quad \text{(A.34)}$$

where we define the cost resulting from the capacity expansion limits as

$$\mathcal{R}_s = \left(\underline{\mu}^{\mathrm{nom}}_s - \bar{\mu}^{\mathrm{nom}}_s \right) G_s \quad \forall\, s \quad \text{(A.35)}$$

As $\underline{\mu}^{\mathrm{nom}}_s \geq 0$ and $\bar{\mu}^{\mathrm{nom}}_s \leq 0$ the latter represents a net positive cost term.

Multiplying Eq. (A.5), which is still valid, with $g_{s,t}$ and inserting Eq. (A.34) will bring us to the zero profit rule for generators with capacity expansion limits,

$$c_s G_s + \sum_t o_s g_{s,t} - \mathcal{R}_s = \sum_t \lambda_{n,t} K_{n,s} g_{s,t} \quad \forall\, s \in S \quad \text{(A.36)}$$

We see that the revenue of generator s will not fully pay back its full CAPEX and OPEX. The exogenous constraint shifts the zero-profit equations such that some of the expenditures for $s \in S$ cannot directly be allocated to consumers. The collected payments of all consumers will not cover the costs induced by Eqs. (A.28) and (A.29),

$$c_s G_s - \mathcal{R}_s = \sum_{n,t} \mathcal{C}^G_{n\to s,t} \quad \forall\ s \in S \tag{A.37}$$

Doing likewise with a subset L of transmission lines,

$$F_\ell \geq \underline{F}_\ell \ \perp\ \underline{\mu}^{\mathrm{nom}}_\ell \qquad \forall \ell \in L \tag{A.38}$$

$$F_\ell \leq \bar{F}_\ell \ \perp\ \bar{\mu}^{\mathrm{nom}}_\ell \qquad \forall \ell \in L \tag{A.39}$$

results in a shifted zero-profit rule for transmission line in the form

$$c_\ell F_\ell - \mathcal{R}_\ell = \sum_{n,t} \mathcal{C}^F_{n\to \ell,t} \quad \forall\ \ell \in L \tag{A.40}$$

where $\mathcal{R}_\ell$ is given by

$$\mathcal{R}_\ell = \left(\underline{\mu}^{\mathrm{nom}}_\ell - \bar{\mu}^{\mathrm{nom}}_\ell\right)\ F_\ell \quad \forall\ \ell \in L \tag{A.41}$$

A.8 Allocating Gross Injections with EBE

The allocation of gross generation to demands $d_{n,t}$ is straigthforwardly obained by a proportional distribution of the generation, *i.e.*

$$A_{s,n,t} = \frac{g_{s,t}}{\sum_s g_{s,t}} d_{n,t} \tag{A.42}$$

A.9 Allocating Net Injections with EBE

Allocating net power injections using the EBE methods leads to the same result as the Marginal Participation (MP) [7] algorithm when allocating to consumers only, see [6] for further insight. We calculate it by setting

$$A_{m\to n,t} = \delta_{m,n}\, p^\circ_{m,t} + \gamma_t\, p^-_{n,t}\, p^+_{m,t} \tag{A.43}$$

where

- $p^+_{n,t} = \min\left(g_{n,t} - d_{n,t}, 0\right)$ denotes the nodal net production
- $p^-_{n,t} = \min\left(d_{n,t} - g_{n,t}, 0\right)$ denotes the nodal net consumption
- $p^\circ_{n,t} = \min\left(p^+_{n,t}, p^-_{n,t}\right)$ the denotes nodal self-consumption. That is the power generated and at the same time consumed at node n and
- $\gamma_t = \left(\sum_n p^+_{n,t}\right)^{-1} = \left(\sum_n p^-_{n,t}\right)^{-1}$ is the inverse of the total injected/extracted power at time t.

The allocation $A_{s,n,t}$ from generator s to n, is given by multiplying $A_{m\to n,t}$ with the nodal share $g_{s,t}/g_{n,t}$.

A.10 Allocating Net Power using AP

Allocating net injections using the AP method is derived from [1]. In a lossless network the downstream and upstream formulations result in the same P2P allocation which is why we restrict ourselves to the downstream formulation only. In a first step we define a time-dependent auxiliary matrix $\mathcal{J}_t$ which is the inverse of the $N \times N$ with directed power flow $m \to n$ at entry (m, n) for $m \neq n$ and the total flow passing node m at entry (m, m) at time step t. Mathematically this translates to

$$\mathcal{J}_t = \left(\mathrm{diag}\left(p^+\right) + \mathcal{K}^- \,\mathrm{diag}\left(f\right)\ K\right)^{-1}_t \tag{A.44}$$

where $\mathcal{K}^-$ is the negative part of the directed Incidence matrix $\mathcal{K}_{n,\ell} = \mathrm{sign}\left(f_{\ell,t}\right) K_{n,\ell}$. Then the distributed slack for time step t is given by

$$A_{m\to n,t} = \mathcal{J}_{m,n,t}\, p^+_{m,t}\, p^-_{n,t} \tag{A.45}$$

A.11 Allocating Gross Power using AP

We use the same allocation as in Appendix A.10 but replace the net nodal production $p^+_{n,t}$ by the gross nodal production $g_{n,t}$ which leads to

$$\mathcal{J}_t = \left(\mathrm{diag}\left(g\right) + \mathcal{K}^- \,\mathrm{diag}\left(f\right)\ K\right)^{-1}_t \tag{A.46}$$

The distributed slack is for time step t is then given by

$$A_{s\to m,t} = \mathcal{J}_{m,n}\, g_{s,t}\, d_{n,t} \tag{A.47}$$

A.12 Example: Power Flow Allocations of different Types

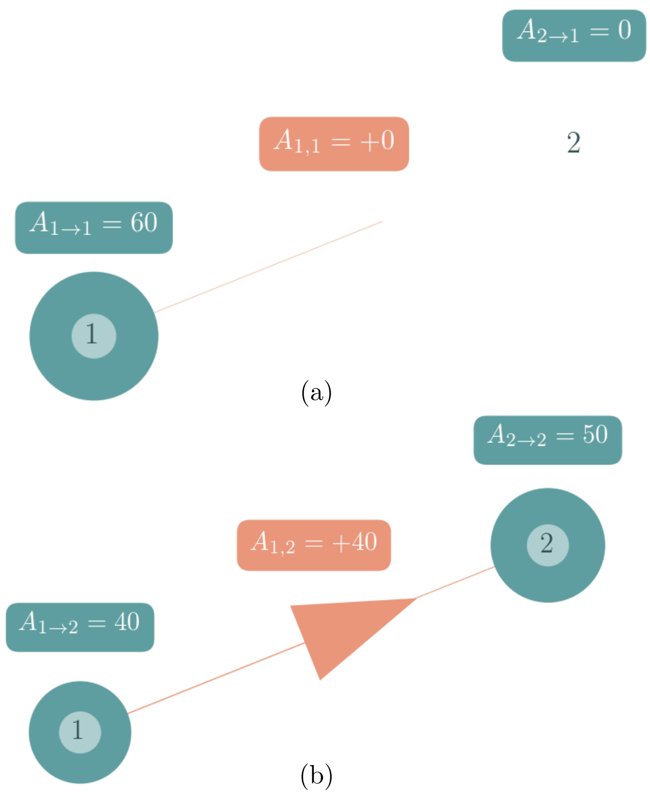


Figure A.1: Power allocations for bus 1 (a) and bus 2 (b) of the example network in Fig. 2 using equivalently allocated net power injections (scheme 2a). Bus 1 retrieves 60 MW from itself and nothing from bus 2. The latter in turn retrieves 40 MW from bus 1 and self supplies 50 MW. The P2P trades are less in number and more intuitive then with allocating gross flow.

References

[1] Chira Achayuthakan et al. "Electricity Tracing in Systems With and Without Circulating Flows: Physical Insights and Mathematical Proofs". en. In: *IEEE Transactions on Power Systems* 25.2 (May 2010), pp. 1078–1087. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2009.2037506.

[2] J. Bialek. "Tracing the Flow of Electricity". en. In: *IEE Proceedings - Generation, Transmission and Distribution* 143.4 (1996), p. 313. ISSN: 13502360. DOI: 10.1049/ip-gtd:19960461.

[3] T. Brown and L. Reichenberg. "Decreasing Market Value of Variable Renewables Is a Result of Policy, Not Variability". In: *arXiv:2002.05209 [econ, math, q-fin]* (Feb. 2020). arXiv: 2002.05209 [econ, math, q-fin].

[4] F.D. Galiana, A.J. Conejo, and H.A. Gil. "Transmission Network Cost Allocation Based on Equivalent Bilateral Exchanges". en. In: *IEEE Transactions on Power Systems* 18.4 (Nov. 2003), pp. 1425–1431. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2003.818689.

[5] H.A. Gil, F.D. Galiana, and A.J. Conejo. "Multiarea Transmission Network Cost Allocation". en. In: *IEEE Transactions on Power Systems* 20.3 (Aug. 2005), pp. 1293–1301. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2005.851951.

[6] Fabian Hofmann et al. "Flow Allocation in Meshed AC-DC Electricity Grids". en. In: *Energies* 13.5 (Mar. 2020), p. 1233. ISSN: 1996-1073. DOI: 10.3390/en13051233.

[7] H. Rudnick, R. Palma, and J.E. Fernandez. "Marginal Pricing and Supplement Cost Allocation in Transmission Open Access". In: *IEEE Transactions on Power Systems* 10.2 (May 1995), pp. 1125–1132. ISSN: 08858950. DOI: 10.1109/59.387960.

[8] Fred C. Schweppe et al. *Spot Pricing of Electricity*. en. Boston, MA: Springer US, 1988. ISBN: 978-1-4612-8950-0 978-1-4613-1683-1. DOI: 10.1007/978-1-4613-1683-1.