

Flow-Based Cost Allocation in Linear Optimized Power Systems are non-unique but localizable

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Abstract

Highlights

- The LMP always breaks down into contributions of different cost terms (OPEX, CAPEX, etc.) of the system.
- Breaking the LMP further down into contributions of components (generator, transmission line, etc.) requires assumptions on power flow assignments.
- Methods like Average Participation or Flow Based Market Coupling allow for localizing the P2P cost assignments and prioritize assignments within one market zone.

Nomenclature

$\lambda_{n,t}$	Locational Market Price at bus n and time step t in €/MW
$d_{n,t}$	Electric demand per bus n , demand type a , time step t in MW
$g_{s,t}$	Electric generation of generator s , time step t in MW
$f_{\ell,t}$	Active power flow on line ℓ , time step t in MW
o_s	Operational price in €/MW
c_s	Capital Price in €/MW
c_ℓ	Capital Price in €/MW for transmission capacity on line ℓ
G_s	Generation capacity in MW
F_ℓ	Transmission capacity in MW
$K_{n,\ell}$	Incidence matrix

1 Economic Context

In long-term operation and investment planning models, the total costs \mathcal{TC} of a power system is the sum of multiple cost terms. Typically, these include operational expenditures (OPEX) \mathcal{O} for generators, expenditures for emissions \mathcal{E} , capital expenditures (CAPEX) for generators \mathcal{C}^G , CAPEX for the transmission system \mathcal{C}^F and possible other terms, *i.e.*

$$\mathcal{TC} = \mathcal{O} + \mathcal{E} + \mathcal{C}^G + \mathcal{C}^F + \dots \quad (1)$$

In turn, each single cost term on the right side $\mathcal{C} = \{\mathcal{O}, \mathcal{E}, \mathcal{C}^G, \mathcal{C}^F, \dots\}$ consists of cost associated to the asset i in the system,

$$\mathcal{C} = \sum_i \mathcal{C}_i \quad (2)$$

For example the total OPEX, $\mathcal{O} = \sum_s \mathcal{O}_s$, is the combined OPEX of all generators s .

Minimizing the total cost \mathcal{TC} of a network, which translates to maximizing the total welfare, will determine the single expenditure \mathcal{C}_i dedicated to asset i . with maximized welfare. Then, the Locational Marginal Price (LMP) describes the price for an incremental increase of electricity demand $d_{n,t}$ at node n . It is given by the derivative of the total system cost \mathcal{TC} with respect to the local demand $d_{n,t}$

$$\lambda_{n,t} = \frac{\partial \mathcal{TC}}{\partial d_{n,t}} \quad (3)$$

This leads to a nodal pricing where over the span of optimized timesteps t , the system costs are partially or totally paid back by the consumers

$$\mathcal{TC} - \mathcal{R} = \sum_{n,a,t} \lambda_{n,t} d_{n,t} \quad (4)$$

depending on the costs \mathcal{R} which are independent of the nodal demand

$$\frac{\partial \mathcal{R}}{\partial d_{n,t}} = 0 \quad (5)$$

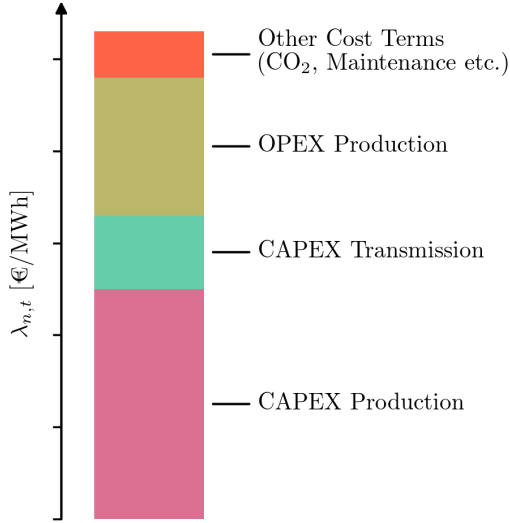


Figure 1: Schematic decomposition of the Locational Market Price $\lambda_{n,t}$. In power system model with optimal long-term operation and planning, the total system costs \mathcal{TC} split into different cost terms, *i.e.* OPEX and CAPEX for production and transmission and possibly other expenditures.

Generally speaking, the cost term \mathcal{R} , not covered by the consumers, results from additional demands on the network design, such as capacity expansion limits in certain regions of the network, or minimum share of one technology in the power mix. However, in most cases, where $\mathcal{R} \ll \mathcal{TC}$, these play a minor role. Feeding Eq. (1) into Eq. (3) illustrates that the LMP splits into different cost terms assigned to OPEX and CAPEX for production and transmission (and other possible expenditures), schematically shown in Fig. 1. Extensive investigations of the LMP as done in [8] already showed this connection. However the question of how the LMP translates to the single cost terms \mathcal{C}_i associated to asset i in the network remains unanswered. This work aims at presenting and illustrating an intuitive scheme how to cost breakdown the cost decomposition to a full peer-to-peer (P2P) cost allocation including all network assets.

2 Nodal Cost Allocation

In following we show how to disaggregate the cost terms \mathcal{O} , \mathcal{E} , \mathcal{C}^G and \mathcal{C}^F for asset i into contributions of demand $d_{n,t}$ at bus n and time t . That is what amount consumers at (n, t) pay for the operation, investment and other cost-intensive, but welfare

contributing, actions at asset i . We follow the general procedure: For each cost term \mathcal{C} we break down the contributions of asset i , *i.e.* define Eq. (2). According to Eqs. (3) and (4), we define its derivative with respect to $d_{n,t}$ and multiply the result with $d_{n,t}$. This returns $\mathcal{C}_{n \rightarrow i, t}$, the contribution of $d_{n,t}$ to the cost term \mathcal{C}_i . Follows naturally that when summing up all contributions the cost term \mathcal{C}_i must be covered, *i.e.*

$$\mathcal{C}_i = \sum_{n,t} \mathcal{C}_{n \rightarrow i, t} \quad (6)$$

and the total money spent at (n, t) for the cost term \mathcal{C} amounts

$$\mathcal{C}_{n,t} = \sum_i \mathcal{C}_{n \rightarrow i, t} \quad (7)$$

Note that for allocating CAPEX, we will make use of the shadowprices associated to \mathcal{C}_i , given by the dual values at cost optimum μ .

2.1 Allocate Operation Expenditures

The total OPEX is given by the operational price o_s times the generation $g_{s,t}$ of all generators s ,

$$\mathcal{O} = \sum_{s,t} o_s g_{s,t} \quad (8)$$

Hence, the contribution of $d_{n,t}$ to the OPEX at generator s is straightforward given by

$$\mathcal{O}_{n \rightarrow s, t} = o_s \frac{\partial g_{s,t}}{\partial d_{n,t}} d_{n,t} \quad (9)$$

We define the derivative on the right hand side times the nodal demand as

$$A_{s \rightarrow n, t} = \frac{\partial g_{s,t}}{\partial d_{n,t}} d_{n,t} \quad (10)$$

The quantity $A_{s \rightarrow n, t}$ can be considered as a power allocation, that is the power produced by generator s and consumed at node n and time t . By feeding back Eq. (9) into Eq. (8), we see that the power allocation has to fulfill

$$g_{s,t} = \sum_n A_{s \rightarrow n, t} \quad (11)$$

The definition of $\mathcal{O}_{n \rightarrow s, t}$ in Eq. (9) can be interpreted as the following. Consumers at bus n which retrieve power from a generators s , directly compensate its

operational costs. The cost allocation behaves like P2P tradings between producers and consumers for fixed production prices. In this way, generator s retrieves the exact amount of money from consumers that it spends on its operation at time t , thus

$$\sum_n \mathcal{O}_{n \rightarrow s, t} = o_s g_{s, t} \quad (12)$$

2.2 Allocate Emission Expenditures

Given a fix price for emissions in CO₂ equivalents μ_{CO_2} , the cost term for emission adds up to

$$\mathcal{E} = \mu_{\text{CO}_2} \sum_s e_s g_{s, t} \quad (13)$$

where e_s denotes the emission factor in tonne-CO₂ per MWh_{el} for generator s . Similar to the allocated OPEX, the emission cost for consumers at bus n at time t for operation at generator s is allocated through

$$\mathcal{E}_{n \rightarrow s, t} = \mu_{\text{CO}_2} e_s A_{s \rightarrow n, t} \quad (14)$$

2.3 Allocate Capital Expenditures for Generators

The optimization of the generation capacity G_s adds the cost term

$$\mathcal{C}^G = \sum_s c_s G_s \quad (15)$$

to \mathcal{TC} . It combines all capacities times their capital cost c_s . The nominal capacity constraints the generation $g_{s, t}$ in the form of

$$g_{s, t} - \bar{g}_{s, t} G_s \leq 0 \quad \perp \quad \bar{\mu}_{s, t} \quad \forall n, s, t \quad (16)$$

$$-g_{s, t} \leq 0 \quad \perp \quad \underline{\mu}_{s, t} \quad \forall n, s, t \quad (17)$$

where $\bar{g}_{s, t} \in [0, 1]$ is the capacity factor for renewable generators. As shown in [3] and in detailed in Appendix A.3, the capital cost for generator s is payed back by the generation times the shadow price for the upper capacity constraint,

$$c_s G_s = \sum_t \bar{\mu}_{s, t} g_{s, t} \quad (18)$$

With our definition in Eq. (10) the allocated CAPEX can then be defined as

$$\mathcal{C}_{n \rightarrow s, t}^G = \bar{\mu}_{s, t} A_{s \rightarrow n, t} \quad (19)$$

So, how does this allocation behave? According to the polluter pays principle, it differentiates between consumers who are ‘responsible’ for investments and those who are not. If $\bar{\mu}_{s, t}$ (in literature often denoted as the Quality of Supply) is bigger than zero, the upper Capacity Constr. (16) is binding. Thus it is these times steps which push investments in G_s . If $\bar{\mu}_{s, t} = 0$, the generation $g_{s, t}$ is not bound and investments are not necessary. When summing over all CAPEX payments to generator s in Eq. (19) and using Eqs. (11) and (18), we see that each generator retrieves exactly the cost that were spent to build the capacity G_s .

2.4 Allocate Capital Expenditures for Transmission Lines

The CAPEX for the transmission system is given by

$$\mathcal{C}^F = \sum_\ell c_\ell F_\ell \quad (20)$$

The transmission capacity variable F_ℓ limits the flow $f_{\ell, t}$ in both directions, such that

$$f_{\ell, t} - F_\ell \leq 0 \quad \perp \quad \bar{\mu}_{\ell, t} \quad \forall \ell, t \quad (21)$$

$$-f_{\ell, t} - F_\ell \leq 0 \quad \perp \quad \underline{\mu}_{\ell, t} \quad \forall \ell, t \quad (22)$$

We use the result of [3] (for details see Appendix A.4) which derives that the investment in line ℓ is payed back by the shadowprices times the flow

$$c_\ell F_\ell = \sum_t (\bar{\mu}_{\ell, t} - \underline{\mu}_{\ell, t}) f_{\ell, t} \quad (23)$$

We insert this into Eq. (20) and define the individual cost terms as

$$\mathcal{C}_{n \rightarrow \ell, t}^F = (\bar{\mu}_{\ell, t} - \underline{\mu}_{\ell, t}) \frac{\partial f_{\ell, t}}{\partial d_{n, t}} d_{n, t} \quad (24)$$

Again, we define the derivative on the right side times the demand as

$$A_{\ell, n, t} = \frac{\partial f_{\ell, t}}{\partial d_{n, t}} d_{n, t} \quad (25)$$

which from inserting into Eq. (23) has to fulfill

$$f_{\ell, t} = \sum_n A_{\ell, n, t} \quad (26)$$

$A_{\ell, n, t}$ can be interpreted as the flow that the demand at (n, t) induces on line ℓ . The shadowprices $\bar{\mu}_{\ell, t}$ and $\underline{\mu}_{\ell, t}$ again can be seen as a measure for necessity

of transmission investments at ℓ at time t . Hence, the definition of $\mathcal{C}_{n \rightarrow \ell, t}^F$ states that consumers, which retrieve power flowing on congested lines, yielding a bound Constr. (21) or (22), pay compensations for the resulting investments at ℓ . Again the sum of all CAPEX payments to line ℓ equals the total CAPEX spent. This is seen when summing Eq. (24) over all buses and time steps and using Eqs. (23) and (26)

3 Assumptions on Power Allocations

The presented cost allocation suits for any type of topology and network setup. But so far, the question of how $A_{s \rightarrow n, t}$ and $A_{\ell, n, t}$ are defined was left open. We recap that both rely on derivatives of $g_{s, t}$ and $f_{\ell, t}$ with respect to the nodal demand $d_{n, t}$ fulfilling Eqs. (11) and (26).

In fact, it turns out that ones, the peer-to-peer allocations $A_{s \rightarrow n, t}$ are defined, the power flow allocations $A_{\ell, n, t}$ are determined by the linear power flow equations. In order to show this, let $H_{\ell, n}$ denote Power Transfer Distribution Factors (PTDF) giving the changes in the flow on line ℓ for one unit (typically one MW) of net power production at bus n . Further let $K_{n, s}$ be 1 if generator s is attached to bus n and zero otherwise. The linear power flow equals the

$$f_{\ell, t} = \sum_m H_{\ell, m} (g_{m, t} - d_{m, t}) \quad (27)$$

where the nodal generation $g_{m, t} = \sum_s K_{m, s} g_{s, t}$ combines the production of all generators attached to bus m . Taking the derivative with respect to the demand, leads to

$$A_{\ell, n, t} = \frac{\partial f_{\ell, t}}{\partial d_{n, t}} d_{n, t} = \sum_m H_{\ell, m} (A_{m \rightarrow n, t} - \delta_{n, m} d_{n, t}) \quad (28)$$

where $A_{m \rightarrow n, t} = \sum_s K_{n, s} A_{s \rightarrow n, t}$ combines all power allocations of the source m . Equation (28) shows that the peer-to-peer allocation $A_{s \rightarrow n, t}$ directly determines the flow allocation on line ℓ .

This leaves us with only solving Eq. (11) for $A_{s \rightarrow n, t}$. In fact, we have to answer the question of how much power supply does generator s deliver to bus n . Unfortunately, the solution is non-unique and requires further assumptions. Established flow allocation schemes approach this problem from different

directions. Principally two options exist *what* is allocated

1. gross power injections
2. net power injections

Further it is important *what assumptions* define the allocation, *i.e.* what method is used to define the pairs of sources and sinks. The three suitable approaches we present here are

- a. Equivalent Bilateral Exchanges (EBE) [4] which assumes that every producer supplies every consumer proportional to its share in the total consumption.
- b. Average Participation (AP) [2, 1] which traces the flow from producer to consumer following the law of proportional sharing.
- c. Flow Based Market Coupling (FBMC) which uses zonal PTDF for allocating power within predefined regions. The interregional exchange is only allocating net power deficit or excess of the regions.

We show the mathematical formulation for all combinations **a1** - **c2** in Appendices A.8 to A.11. Principally, type **2** leads to less P2P trades than type **1** as power from a bus n with $g_{n, t} \leq d_{n, t}$ is not assigned to other buses, only to n . Further, as literature has often pointed out, the EBE principal **a** does not suit for large networks where remote buses would interconnect in the same way as buses in close vicinity [5]. The AP based type **b** tackles this problem by restricting P2P trades to those which are traceable when applying the proportional sharing principal. Therefore $A_{s \rightarrow n, t}$ denotes that part of power produced by bus m which, when only following in the direction of $f_{\ell, t}$, ends up at bus n . Type **c** further allows to control the regions or market zones which are netted out in a first calculation. If in a region R the generation undercuts the demand, $\sum_{n \in R} g_{s, t} \leq \sum_{n \in R} d_{n, t}$, none of the inner-regional generation is assigned to other regions.

3.1 Numerical Example

In the following we showcase a two bus system with one optimized time step and its resulting allocated payments, illustrated in Fig. 2.

The two buses are connected via one transmission

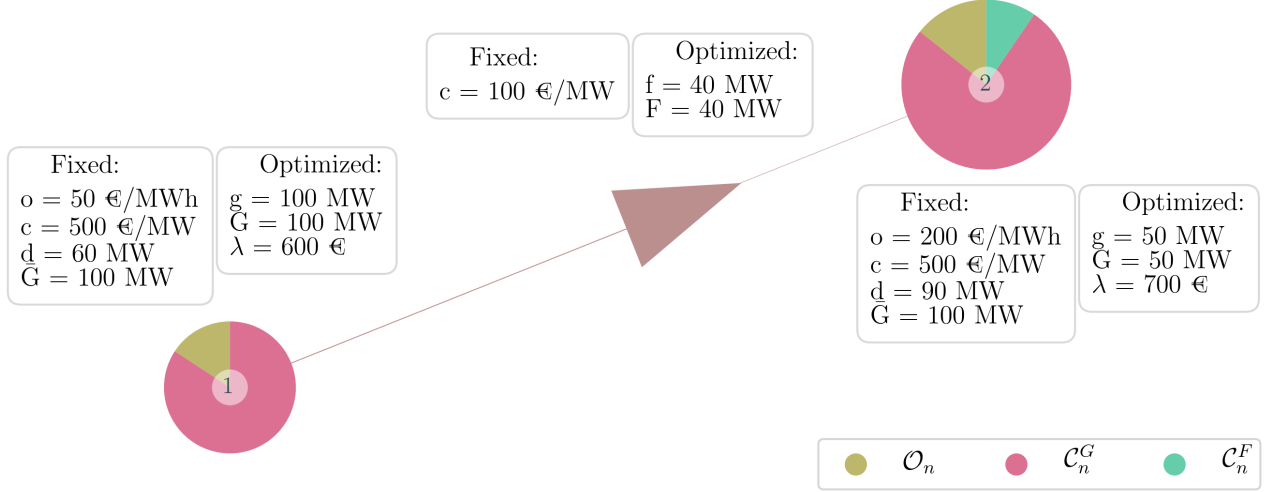


Figure 2: Illustrative example of a 2-bus network with one optimized time step. Fixed prices and constraining values are given in the left box for each bus and the transmission line. Optimized values are given in the right boxes. Bus 1 has a cheaper operational price o , capital prices are the same for both. As both generator capacities are constraint to 100 MW, the optimization also deploys the generator at bus 2. The resulting electricity prices λ are then a composition of all prices for operation and capital investments.

line, each has one generator. Whereas the generator at bus 1 has an operational price of 50 €/MWh_{el}, the generator at bus 2 has a higher operational price of 200 €/MWh_{el}. For both the CAPEX rate is set to 500 €/MW and the maximal capacity is limited to $\bar{G}_s = 100$ MW. The transmission line has a CAPEX rate of 100 €/MW and no upper capacity limit. With a demand of 60 MW at bus 1 and 90 MW at bus 2, the optimization expands the cheaper generator at bus 1 to its full limit of 100 MW. The 40 MW excess power not consumed at bus 1, flow to bus 2 where the generator is built with only 50 MW.

Allocating Gross Power using EBE (1a)

Figure 3 shows the allocated transactions on basis of equivalently traded gross power (type 1a) for both buses 1 & 2 separately. The resulting P2P payments are given in Fig. 4. The upper graph Fig. 3a shows that $A_{1,1} = 40$ MW at bus 1 are self-sustained. Consumers at bus 1 consequently pay 2k € OPEX and 22k € CAPEX to the generator at bus 1. The remaining 20 MW come from bus 2 and induce a sub-flow on line 1 of $A_{2 \rightarrow 1,1} = -20$. As this flow is in contrary direction to the total flow, it is relieving the transmission system. This translates to a congestion reward for consumers at bus 1 of $c_{\ell=1} A_{2 \rightarrow 1,1} = 2k$ € which is exactly the cost that had to be spent on the

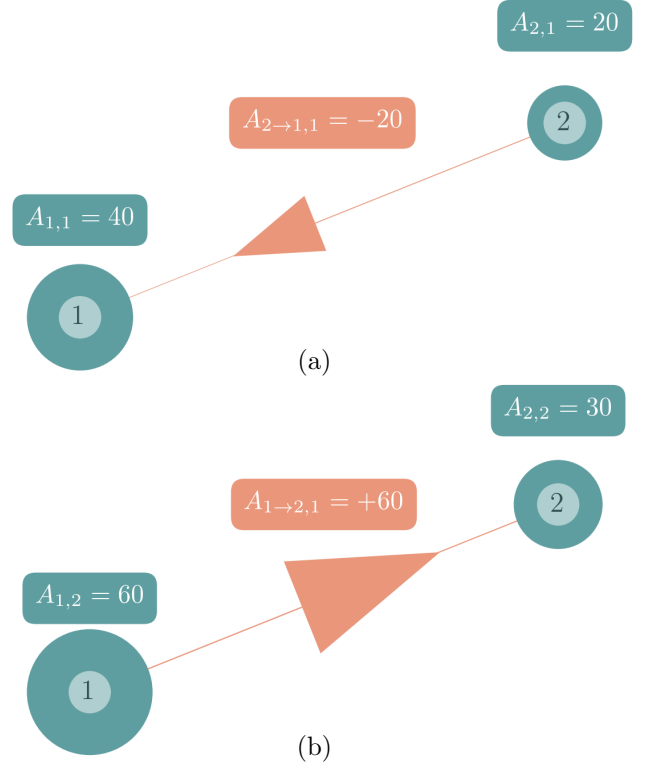


Figure 3: Power allocations for bus 1 (a) and bus 2 (b) of the example network in Fig. 2. Bus 1 retrieves 40 MW from itself and 20 MW from bus 2. The latter in turn retrieves 60 MW from bus 1 and self supplies 30 MW. The sum of both net flows equals the resulting flow of $f = 40$ MW.

		\mathcal{O}_n		\mathcal{C}_n^G		\mathcal{C}_n^F
n	1	2k €	4k €	22k €	10k €	-2k €
	2	3k €	6k €	33k €	15k €	6k €
		1	2	1	2	1
		m		m		ℓ

Figure 4: Full P2P cost allocation for the example setup shown in Fig. 2. The payments are derived on the basis of Eqs. (9), (19) and (24). Consumers at bus n have to pay each generator proportional to their consumption. As we only consider one time step the proportionality applies for OPEX \mathcal{O}_n and CAPEX \mathcal{C}_n . As bus 1 induces a relieving flow on line 1 and therefore “prevents” further transmission expansion, it is rewarded proportional to the relief.

transmission system if bus 1 didn’t induce a relieving flow, see again Fig. 4.

The lower graph Fig. 3b illustrates the impact of consumption at bus 2. As d_2 is higher than d_1 the retrievments from both generators are proportionately increased as well as the OPEX and CAPEX allocations to the generators. But instead of a relieving flow, consumers at bus 2 drive the burdening flow in direction of congestion. Hence the payoff to the transmission system is positive and much higher than for bus 1.

The sum of all rows in the payoff matrix in Fig. 4 yields the revenues of the assets m, ℓ . These values match their overall spendings, *e.g.* the total revenue of the transmission line is 4k € which equals the cost for investments $c_1 F_1$. The sum of all columns yields the total payment of consumers at bus n . For example the sum of payments of consumers at bus 1 is 36k €. This is exactly the electricity price of 600 €/MW times the consumption of 60 MW, $\lambda_1 d_1$.

The fact that OPEX and CAPEX allocations are proportional to the total consumption at a bus results from optimizing one time step only. In larger optimization problems with multiple time steps the CAPEX allocation takes effect only for time steps in which one or more of the capacity constraints Constrs. (16), (21) and (22) become binding.

Allocating Net Power using EBE (2a)

In contrast the to equivalent allocation of gross power production, netting out injections for each bus leads to less P2P payments. The resulting payment given in Fig. 5 builds on the allocated power flow shown Fig. A.1 in Appendix A.12. As bus 2 does not produce excess power, none of its power production is assigned to bus 1 and thus no payment of bus 1 to bus 2 allocated. Neither has bus 1 to pay fee to the transmission system as it only exports power. So all its consumers pay to its local generators. Bus 2 in constrast bear all CAPEX for the transmission system as well as CAPEX and OPEX for generators at bus 1.

		\mathcal{O}_n		\mathcal{C}_n^G		\mathcal{C}_n^F
n	1	3k €	0k €	33k €	0k €	0k €
	2	2k €	10k €	22k €	25k €	4k €
		1	2	1	2	1
		m		m		ℓ

Figure 5: Full P2P cost allocation for the example setup shown in Fig. 2 when equivalently allocation net power injection (scheme a2).

Again the cummulative payments per bus meet the nodal spendings $\lambda_{n,t} d_{n,t}$. The cummulative revenues per generator and transmission line meet the all CAPEX and OPEX.

As the other schemes b1 - c2 apart from the EBE scheme don’t distinguishably modify allocations within the presented example, we move on to a more realistic setup where systemic and local effects show up.

4 Application Case

A Appendix

A.1 LMP from Optimization

The nodal balance constraint ensures that the amount of power that flows into a bus equals the power that flows out of a bus, thus reflects the Kirchhoff Current Law (KCL)

$$\sum_l K_{n,\ell} f_{\ell,t} - g_{n,t} + d_{n,t} = 0 \quad \forall n, t \quad (\text{A.1})$$

Its shadow price mirrors the Locational Marginal Prizes (LMP) $\lambda_{n,t}$ per bus and time step. In a power market this is the €/MWh_{el}-price which a consumer has to pay.

A.2 Full Lagrangian

$$\begin{aligned} \mathcal{L}(g_{s,t}, f_{\ell,t}, G_s, F_\ell, \boldsymbol{\lambda}, \boldsymbol{\mu}) = & \sum_{n,s} c_s G_s + \sum_{n,s,t} o_s g_{s,t} + \sum_{\ell} c_\ell F_\ell \\ & + \sum_{n,t} \lambda_{n,t} \left(\sum_{\ell} K_{n,\ell} f_{\ell,t} - \sum_s K_{n,s} g_{s,t} + d_{n,t} \right) \\ & + \sum_{\ell,c,t} \lambda_{c,t} C_{\ell,c} x_\ell f_{\ell,t} \\ & + \sum_{n,s,t} \bar{\mu}_{s,t} (g_{s,t} - \bar{g}_{s,t} G_s) - \sum_{n,s,t} \underline{\mu}_{s,t} g_{s,t} \\ & + \sum_{\ell,t} \bar{\mu}_{\ell,t} (f_{\ell,t} - F_\ell) - \sum_{\ell,t} \underline{\mu}_{\ell,t} (f_{\ell,t} + F_\ell) \end{aligned} \quad (\text{A.2})$$

where $\boldsymbol{\lambda} = \{\lambda_{n,t}, \lambda_{c,t}\}$ and $\boldsymbol{\mu} = \{\bar{\mu}_{s,t}, \underline{\mu}_{s,t}, \bar{\mu}_{\ell,t}, \underline{\mu}_{\ell,t}\}$ denote the set of related KKT variables. The global maximum of the Lagrangian requires stationarity with respect to all variables. The stationarity of the generation capacity variable leads to

$$\frac{\partial \mathcal{L}}{\partial G_s} = 0 \rightarrow c_s = \sum_t \bar{\mu}_{s,t} \bar{g}_{s,t} \quad \forall n, s \quad (\text{A.3})$$

the stationarity of the transmission capacity to

$$\frac{\partial \mathcal{L}}{\partial F_\ell} = 0 \rightarrow c_\ell = \sum_t (\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}) \quad \forall \ell \quad (\text{A.4})$$

and the stationarity of the generation to

$$\frac{\partial \mathcal{L}}{\partial g_{s,t}} = 0 \rightarrow o_s = K_{n,s} \lambda_{n,t} - \bar{\mu}_{s,t} + \underline{\mu}_{s,t} \quad \forall n, s \quad (\text{A.5})$$

$$0 = \frac{\partial \mathcal{L}}{\partial f_{\ell,t}} \quad (\text{A.6})$$

$$0 = \sum_n K_{n,\ell} \lambda_{n,t} + \lambda_{c,t} C_{\ell,c} x_\ell - \bar{\mu}_{\ell,t} + \underline{\mu}_{\ell,t} \quad \forall n, s \quad (\text{A.7})$$

A.3 Proof Generation Capacity Payback

As stated in [3] the capital cost of a generator s is equal to the shadowprice $\bar{\mu}_{s,t}$ times the upper availability $\bar{g}_{s,t}$ of all time steps

$$c_s = \sum_t \bar{\mu}_{s,t} \bar{g}_{s,t} \quad (\text{A.8})$$

Constrs. (16) and (17) which yield the KKT variables $\bar{\mu}_{s,t}$ and $\underline{\mu}_{s,t}$ imply the complementary slackness,

$$\bar{\mu}_{s,t} (g_{s,t} - \bar{g}_{s,t} G_s) = 0 \quad \forall n, s, t \quad (\text{A.9})$$

$$\underline{\mu}_{s,t} g_{s,t} = 0 \quad \forall n, s, t \quad (\text{A.10})$$

Multiplying both sides of Eq. (A.3) with G_s and using Eq. (A.9) leads to

$$c_s G_s = \sum_t \bar{\mu}_{s,t} g_{s,t} \quad (\text{A.11})$$

The zero-profit rule for generators is obtained by multiplying Eq. (A.5) with $g_{s,t}$ and using Eqs. (A.10) and (A.11) which results in

$$c_s G_s + \sum_t o_s g_{s,t} = \sum_t \lambda_{n,t} K_{n,s} g_{s,t} \quad (\text{A.12})$$

It states that over the whole time span, all OPEX and CAPEX for generator s (left hand side) are payed back by its revenue (right hand side).

A.4 Proof Flow Capacity Payback

The yielding KKT variables $\bar{\mu}_{\ell,t}$ and $\underline{\mu}_{\ell,t}$ are only non-zero if $f_{\ell,t}$ is limited by the transmission capacity in positive or negative direction, i.e. Constr. (21) or Constr. (22) are binding. For flows below the thermal limit, the complementary slackness

$$\bar{\mu}_{\ell,t} (f_{\ell,t} - F_\ell) = 0 \quad \forall \ell, t \quad (\text{A.13})$$

$$\underline{\mu}_{\ell,t} (f_{\ell,t} + F_\ell) = 0 \quad \forall \ell, t \quad (\text{A.14})$$

sets the respective KKT to zero. When multiplying Eq. (A.4) with F_ℓ and using the complementary slackness Eqs. (A.13) and (A.14) we obtain

$$c_\ell F_\ell = \sum_t (\bar{\mu}_{\ell,t} - \underline{\mu}_{\ell,t}) f_{\ell,t} \quad (\text{A.15})$$

Again we can use this to formulate the zero-profit rule for transmission lines. We multiply Eq. (A.7) with $f_{\ell,t}$, which finally leads us to

$$c_{\ell} F_{\ell} = \sum_n K_{n,\ell} \lambda_{n,t} f_{\ell,t} + \lambda_{c,t} C_{\ell,c} x_{\ell} f_{\ell,t} \quad (\text{A.16})$$

It states that the congestion revenue of a line (first term right hand side) subtracted by the cost for cycle constraint exactly matches its CAPEX.

A.5 CO₂ constraint

Imposing an additional CO₂ constraint limiting the total emission to K,

$$\sum_{n,s,t} e_s g_{s,t} \leq K \perp \mu_{\text{CO}_2} \quad (\text{A.17})$$

with e_s being the emission factor in tonne-CO₂ per MWh_{el}, returns an effective CO₂ price μ_{CO_2} in €/tonne-CO₂. As shown in ... the constraint can be translated in a dual price which shift the operational price per generator

$$o_s \rightarrow o_s + e_s \mu_{\text{CO}_2} \quad (\text{A.18})$$

A.6 Storage Units

A.7 Brownfield Optimization and Capacity Restrictions

Constraining the capacities G_s for a subset S of generators to lower or upper limits in the form of

$$G_s \geq \underline{G}_s \perp \underline{\mu}_s^{\text{nom}} \quad \forall n, s \in S \quad (\text{A.19})$$

$$G_s \leq \bar{G}_s \perp \bar{\mu}_s^{\text{nom}} \quad \forall n, s \in S \quad (\text{A.20})$$

alters the objective value as soon as one of those become bounding. The complementary slackness for both are

$$\bar{\mu}_s^{\text{nom}} (G_s - \bar{G}_s) = 0 \quad \forall n, s \in S \quad (\text{A.21})$$

$$\underline{\mu}_s^{\text{nom}} (G_s - \underline{G}_s) = 0 \quad \forall n, s \in S \quad (\text{A.22})$$

The CAPEX paybacks for generators and transmission lines Eqs. (A.11) and (A.15) change to

$$\frac{\partial \mathcal{L}}{\partial G_s} = 0 \rightarrow \quad (\text{A.23})$$

$$c_s = \sum_t \bar{\mu}_{s,t} \bar{g}_{s,t} + \underline{\mu}_s^{\text{nom}} - \bar{\mu}_s^{\text{nom}} \quad \forall n, s \in S \quad (\text{A.24})$$

for generators. Multiplying Eq. (A.24) by G_s leads us to

$$c_s G_s - \mathcal{R}_s = \sum_t \bar{\mu}_{s,t} g_{s,t} \quad \forall n, s \in S \quad (\text{A.25})$$

where we define the cost resulting from the capacity expansion limits as

$$\mathcal{R}_s = (\underline{\mu}_s^{\text{nom}} - \bar{\mu}_s^{\text{nom}}) G_s \quad \forall s \quad (\text{A.26})$$

As $\underline{\mu}_s^{\text{nom}} \geq 0$ and $\bar{\mu}_s^{\text{nom}} \leq 0$ the latter represents a net positive cost term.

Multiplying Eq. (A.5), which is still valid, with $g_{s,t}$ and inserting Eq. (A.25) will bring us to the zero profit rule for generators with capacity expansion limits,

$$c_s G_s + \sum_t o_s g_{s,t} - \mathcal{R}_s = \sum_t \lambda_{n,t} K_{n,s} g_{s,t} \quad \forall s \in S \quad (\text{A.27})$$

We see that the revenue of generator s will not fully pay back its full CAPEX and OPEX. The exogenous constraint shifts the zero-profit equations such that some of the expenditures for $s \in S$ cannot directly be allocated to consumers. The collected payments of all consumers will not cover the costs induced by Eqs. (A.19) and (A.20),

$$c_s G_s - \mathcal{R}_s = \sum_{n,t} \mathcal{C}_{n \rightarrow s,t}^G \quad \forall s \in S \quad (\text{A.28})$$

Doing likewise with a subset L of transmission lines,

$$F_{\ell} \geq \underline{F}_{\ell} \perp \underline{\mu}_{\ell}^{\text{nom}} \quad \forall \ell \in L \quad (\text{A.29})$$

$$F_{\ell} \leq \bar{F}_{\ell} \perp \bar{\mu}_{\ell}^{\text{nom}} \quad \forall \ell \in L \quad (\text{A.30})$$

results in a shifted zero-profit rule for transmission line in the form

$$c_{\ell} F_{\ell} - \mathcal{R}_{\ell} = \sum_{n,t} \mathcal{C}_{n \rightarrow \ell,t}^F \quad \forall \ell \in L \quad (\text{A.31})$$

where \mathcal{R}_{ℓ} is given by

$$\mathcal{R}_{\ell} = (\underline{\mu}_{\ell}^{\text{nom}} - \bar{\mu}_{\ell}^{\text{nom}}) F_{\ell} \quad \forall \ell \in L \quad (\text{A.32})$$

A.8 Allocating Gross Injections with EBE

The allocation of gross generation to demands $d_{n,t}$ is straightforwardly obtained by a proportional distribution of the generation, *i.e.*

$$A_{s \rightarrow n,t} = \frac{g_{s,t}}{\sum_s g_{s,t}} d_{n,t} \quad (\text{A.33})$$

A.9 Allocating Net Injections with EBE

Allocating net power injections using the EBE methods leads to the same result as the Marginal Participation (MP) [7] algorithm when allocating to consumers only, see [6] for further insight. We calculate it by setting

$$A_{m \rightarrow n, t} = \delta_{m, n} p_{m, t}^{\circ} + \gamma_t p_{n, t}^{-} p_{m, t}^{+} \quad (\text{A.34})$$

where

- $p_{n, t}^{+} = \min(g_{n, t} - d_{n, t}, 0)$ denotes the nodal net production
- $p_{n, t}^{-} = \min(d_{n, t} - g_{n, t}, 0)$ denotes the nodal net consumption
- $p_{n, t}^{\circ} = \min(p_{n, t}^{+}, p_{n, t}^{-})$ the denotes nodal self-consumption. That is the power generated and at the same time consumed at node n and
- $\gamma_t = (\sum_n p_{n, t}^{+})^{-1} = (\sum_n p_{n, t}^{-})^{-1}$ is the inverse of the total injected/extracted power at time t .

The allocation $A_{s \rightarrow n, t}$ from generator s to n , is given by multiplying $A_{m \rightarrow n, t}$ with the nodal share $g_{s, t} / g_{n, t}$.

A.10 Allocating Net Power using AP

Allocating net injections using the AP method is derived from [1]. In a lossless network the downstream and upstream formulations result in the same P2P allocation which is why we restrict ourselves to the downstream formulation only. In a first step we define a time-dependent auxiliary matrix \mathcal{J}_t which is the inverse of the $N \times N$ with directed power flow $m \rightarrow n$ at entry (m, n) for $m \neq n$ and the total flow passing node m at entry (m, m) at time step t . Mathematically this translates to

$$\mathcal{J}_t = (\text{diag}(p^{+}) + \mathcal{K}^{-} \text{diag}(f) K)_t^{-1} \quad (\text{A.35})$$

where \mathcal{K}^{-} is the negative part of the directed Incidence matrix $\mathcal{K}_{n, \ell} = \text{sign}(f_{\ell, t}) K_{n, \ell}$. Then the distributed slack for time step t is given by

$$A_{m \rightarrow n, t} = \mathcal{J}_{m, n, t} p_{m, t}^{+} p_{n, t}^{-} \quad (\text{A.36})$$

A.11 Allocating Gross Power using AP

We use the same allocation as in Appendix A.10 but replace the net nodal production $p_{n, t}^{+}$ by the gross nodal production $g_{n, t}$ which leads to

$$\mathcal{J}_t = (\text{diag}(g) + \mathcal{K}^{-} \text{diag}(f) K)_t^{-1} \quad (\text{A.37})$$

The distributed slack for time step t is then given by

$$A_{s \rightarrow m, t} = \mathcal{J}_{m, n} g_{s, t} d_{n, t} \quad (\text{A.38})$$

A.12 Example: Power Flow Allocations of different Types

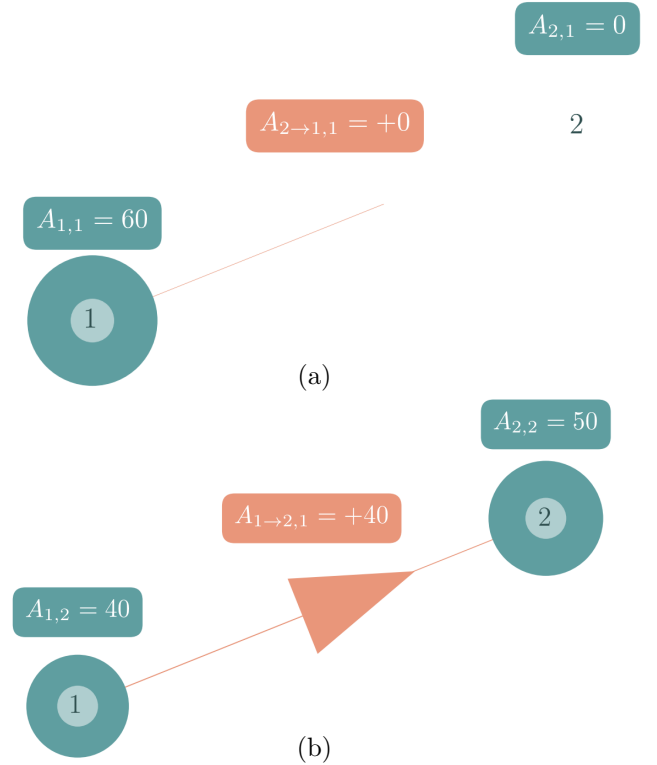


Figure A.1: Power allocations for bus 1 (a) and bus 2 (b) of the example network in Fig. 2 using equivalently allocated net power injections (scheme 2a). Bus 1 retrieves 60 MW from itself and nothing from bus 2. The latter in turn retrieves 40 MW from bus 1 and self supplies 50 MW. The P2P trades are less in number and more intuitive than with allocating gross flow.

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