Parameter Identification of DC Motors

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Advantages of the Parameter Identification Method

- Saving time and money in the testing process: no mechanical coupling necessary
- Full information: Entire characteristic curve and values of any motor

DC-motors are used in a wide variety of applications touching our daily lives, where they serve to relieve us of much work. However, the large number of DC-motors used is attended by a large amount of time and resources devoted to inspecting them at the end of their production cycle. The time needed for this process should be kept as brief as possible so that the inspection procedure is not the slowest part of the production process. Due to the increasing mass production of these motors, procedures for inspecting them have been developed which are able to deter-mine the test objects' characteristic curves within seconds. Such procedures are known as Parameter Identification procedures. They determine the parameters without applying any external load, simply by measuring the current and voltage. The time and apparatus required for attaching a load and for aligning the test object with a loading mechanism can thus be totally omitted.

1 Introduction

The dynamic behaviour of d.c. motor can be described using two equations.

The first equation describes the electrical behavior

$$u = R \cdot i + k \cdot \omega + L \cdot \frac{di}{dt} \qquad (1)$$

The second equation describes the mechanical behavior

$$J \frac{d\omega}{dt} = k \cdot i - k_r \cdot \omega - M_L \qquad (2)$$

where the algebraic symbols represent the following:

Symbol	Unit	Definition
u	V	electric terminal voltage
i	A	electric armature current
ω	1/s	rotational freqency
R	Ω	Ohmic ferrule resistor
k	Vs	generator constant
L	H	inductivity
J	kgm²	moment of inertia
k_r	Nms	sliding friction
M_L	Nm	load

Note: The load also reflects the moment of static friction inherent in the system. Equations (1) and (2) can now be summarized in a single equivalent electromechanic circuit.

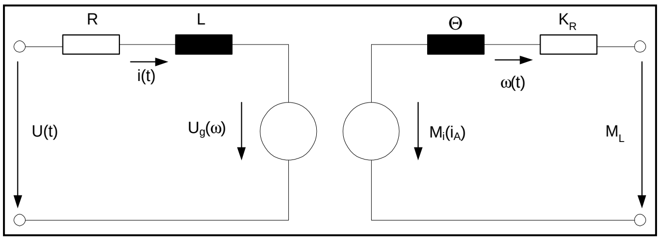

Fig. 1: Electromechanic equivalent circuit

1.1 Energy model

To better interpret Equations (1) and (2), we now proceed to model the energy relationships.

Thus, Equation (1) is multiplied with the current i and integrated over an indefinite interval.

$$\int u \cdot i dt = R \int i^2 dt + k \int \omega \cdot i dt + L \int i \frac{di}{dt} dt \qquad (3)$$

The first term in Equation (3) describes the energy transferred electrically, the second the Ohmic losses, the third the mechanical energy contained in the system and the last the energy stored as inductivity.

Equation (2) is multiplied by the rotational frequency ω and integrated over an indefinite interval.

$$J \int \frac{d\omega}{dt} \cdot \omega dt = k \int i \cdot \omega dt - k_r \int \omega^2 dt - \int M_L \cdot w dt \quad (4)$$

The first term in Equation (4) describes the rotation energy stored in the mechanical system, the second term the mechanical energy, the third the speed-proprotional energy losses and the last the mechanical energy delivered including wasted energy due to static friction.

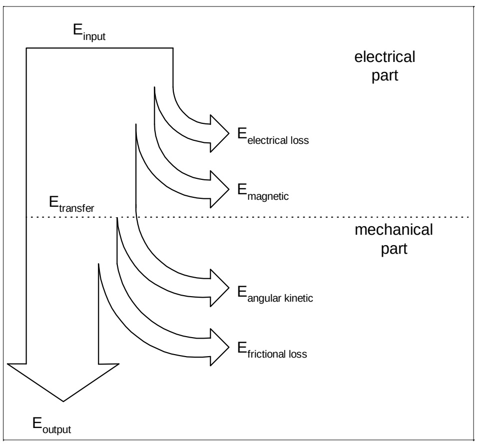

Fig. 2 : Diagram of energy distribution

1.2 Transfer function of a DC motor

Since only the terminal quantities voltage and armature current are used in identifying a DC motor's parameters, the course of the spectrum of the armature current-to-terminal voltage ratio is of particular interest. On the basis of this transfer function it is possible to make statements about at which point in the frequency range excitation is useful, since relevant parameter changes take effect in that frequency range.

A simple example should serve to affirm this:

To identify the parameters of an electric low-pass with a cutoff frequency of 10 kHz, it isn't a practical approach to excite it with a 10 Hz oscillating quantity, since, considering the measurement precision, the input and output signals are approximately the same (transfer factor approx. 1, phase-shift between input and output signals approx. 0 degrees). Only if the excitation approaches the cutoff frequency do the filter parameters become noticeable and is there a chance of determining them with reasonably high accuracy.

To determine the transfer function, Equations (1) and (2) are transferred into the Laplace domain.

$$U(s) = R \cdot I(s) + k \cdot \Omega(s) + L \cdot s \cdot I(s) \qquad (5)$$

$$J \cdot s \cdot \Omega(s) = k \cdot I(s) - k_r \cdot \Omega(s) - M_L \qquad (6)$$

From Equation (5) we thus find the rotational frequency

$$\Omega(s) = \frac{k \cdot I(s) - M_L}{k_r + s \cdot J} \qquad (7)$$

By using (7) in (5) and making some rearrangements, we obtain the equation

$$U(s)=\frac{LJs^2+s(RJ+Lkr)+RK_r+k^2}{k_r+sJ}I(s)-\frac{kM_L}{J}\frac{1}{\left(s+\frac{k_r}{J}\right)s} \qquad (8)$$

Equation (8) describes the relationship between the terminal quantities U and I and to the load ML.

From Equation (8), it follows that:

$$\frac{I(s)}{U(s)+\frac{kM_L}{J}\frac{1}{\left(s+\frac{k_r}{J}\right)s}}=\frac{1}{L}\frac{s+\frac{k_r}{J}}{s^2+s\left(\frac{R}{L}+\frac{k_r}{J}\right)+\frac{R}{L}\left(\frac{k_r}{J}+\frac{k^2}{RJ}\right)} \qquad (9)$$

We now introduce the following identities:

Symbol	Unit	Definition
τ_{ele}	s	electrical time constant
τ_{mech}	s	mechanical time constant
k_A	1/s	run-down constant
V	A/Vs	gain
$U^{\#}(s)$	V	$U(s)+\frac{kM_L}{J}\frac{1}{\left(s+\frac{k_r}{J}\right)s}$

and inserting these abbreviations results in the following transfer equation

$$\frac{I(s)}{U^{\#}(s)}=V\frac{s+k_A}{s^2+s\left(\frac{1}{\tau_{ele}}+k_A\right)+\frac{1}{\tau_{ele}}\left(k_A+\frac{1}{\tau_{mech}}\right)} \qquad (10)$$

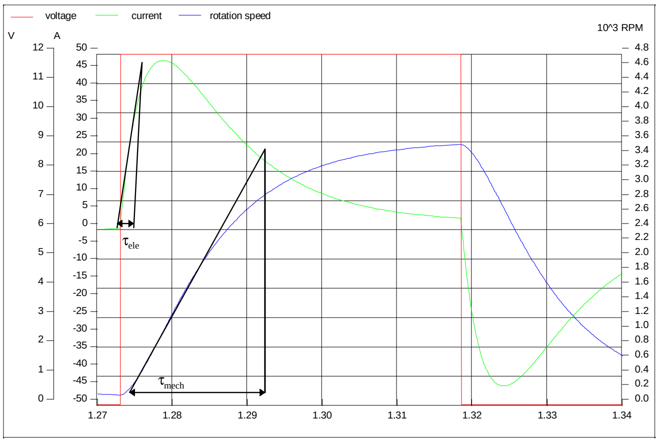

Fig. 3 : Electrical and mechanical time constants of the electrical DC-motor

The constant τ_{ele} is referred to as the electrical time constant of the DC motor. The electrical time constant is a measure of the current's reaction time upon change in the terminal voltage.

The constant τ_{mech} is referred to as the mechanical time constant of the DC motor. The mechanical time constant is a measure of the RPM's reaction time upon change in the terminal voltage. Using Equation (9), the DC-motor's frequency response can be shown for fixed parameters.

The figures below show the frequency responses in terms of both magnitude and phase, as well as the characteristic curve for a motor with given parameters.

Parameter	Unit	Value
R	Ω	0.19
L	H	0.0005
k	Vs	0.0323
J	kgm²	7.5e-5
kr	Nms	2e-5

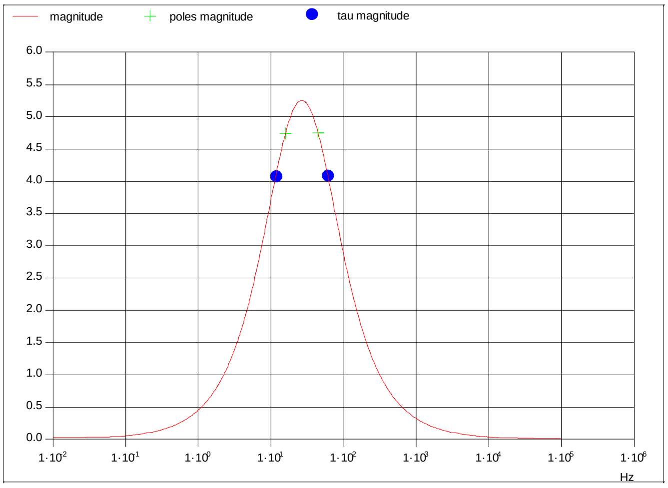

Fig. 4: Magnitude frequency response of a DC motor

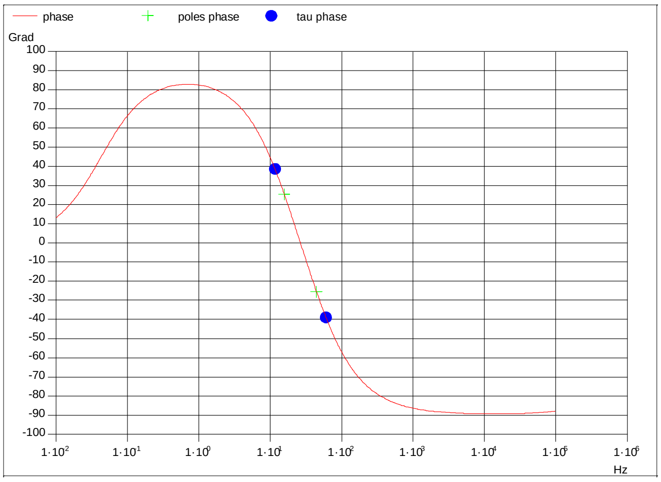

Fig. 5 : Phase frequency response of a DC motor

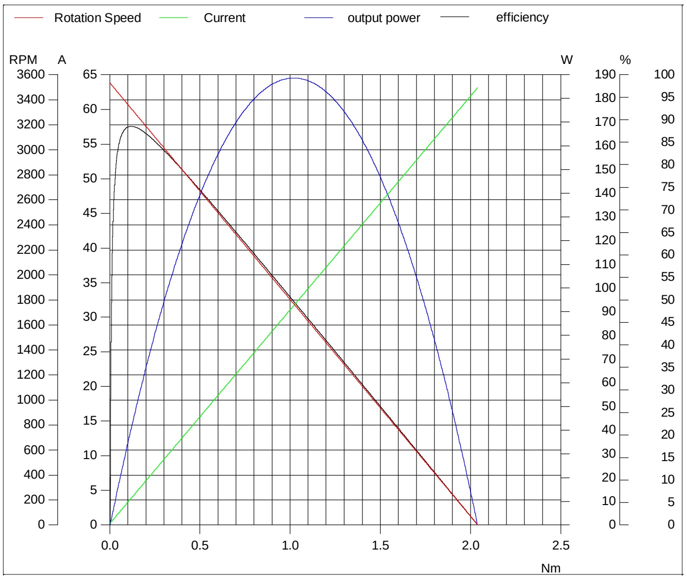

Fig. 6: Characteristic curve of a DC motor

In addition to the curves of the respective frequency responses, the frequencies which are proportional to the mechanical and electric time constants, as well as the frequencies of the transfer function's poles are plotted.

The magnitude frequency response of the DC motor corresponds to a band-pass filter with a center frequency lying between the motor's two time constants.

The phase frequency response of the DC motor corresponds to the phase frequency response of a band-pass. The zero in the transfer function's numerator causes the phase to approach zero at low frequencies.

The DC motor in the example described has two real poles in the transfer function. These poles appear in the frequency response as simple but separate poles. Conversely, there are also motors with conjugated complex poles which can be represented in the frequency response as double-poles at the band-pass center frequency.

Taking a look at the transfer function, it is now possible to determine certain regions in which the DC motor's parameters can be identified. As indicated above, this results in a dependence on the numerator polynomial's zero-crossing in the lower range of the transfer function; the zero-crossing in turn depends on the run-down constant. In addition, in this range the magnitude frequency is almost zero, making estimation basically impossible and the run-down constant not to be determined with adequate precision.

In the region of the DC-motor's time constants, sufficiently large amplitudes result for the purpose of estimating the system's poles. Then, the DC-motor's parameters can be calculated from the poles.

From Equation (10), we see that for the zero-crossings:

$$s_{01,2} = \frac{\frac{1}{\tau_{ele}} + k_A}{2} \pm \sqrt{\left(\frac{\frac{1}{\tau_{ele}} + k_A}{2}\right)^2 - \frac{1}{\tau_{ele}}\left(k_A + \frac{1}{\tau_{mech}}\right)} \quad (11)$$

The system's poles become purely real if the square root expression is positive, from which follows:

$$\tau_{mech} > 4\frac{\tau_{ele}}{\left(1 - k_A \cdot \tau_{ele}\right)^2} \quad (12)$$

All DC motors meeting the conditions in Equation (12) have purely real poles.

1.3 Estimating the transfer function's poles

For all motors for which Equation (12) applies, the following approach to determining the parameters is available:

$$\frac{I(s)}{U^{\#}(s)} = V\frac{s + k_A}{\left(1 - sT_1\right)\left(1 - sT_2\right)} \quad (13)$$

this equation can be multiplied out to yield

$$\frac{I(s)}{U^{\#}(s)} = \frac{V}{T_1 T_2}\frac{s + k_A}{s^2 + s\frac{T_1 + T_2}{T_1 T_2} + \frac{1}{T_1 T_2}} \quad (14)$$

By comparing coefficients with Equation (9), we obtain the following conditional equations for the DC-motor's parameters.

$$L = \frac{T_1 T_1}{V} \quad (15)$$

$$R = \left(\frac{T_1 + T_2}{T_1 T_2} - k_A\right)L \quad (16)$$

$$\frac{k^2}{J} = \left(\frac{L}{T_1 T_2} - Rk_A\right) \quad (17)$$

If the numerator polynomial's poles have been determined and the motor's run-down constant is additionally known, these can be used to find the motor's parameters. The last actual purpose of the estimate is to determine the numerator polynomial's poles.

Upon closer inspection of the transfer function and of the frequency response, it is seen that the system can be divided into a high-pass and a low-pass filter.

$$\frac{I(s)}{U^{\#}(s)} = V\frac{s + \frac{k_r}{J}}{(1 + sT_1)(1 + sT_2)} = V\frac{s + \frac{k_r}{J}}{1 + sT_2}\frac{1}{1 + sT_1} \quad (18)$$

The first term in Equation (18) represents a high-pass filter, the second a low-pass.

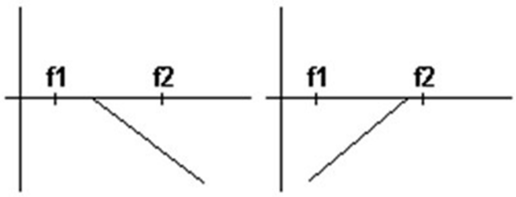

Fig. 7: Representation of the DC-motor's individual frequency responses

If the motor is now driven at frequencies f1 and f2 in succession, the factors in Equation (18) can be separated and calculated individually. As Figure 3 clearly shows, when driven at ω1, the low-pass returns a magnitude factor of approx. 1 and a phase shift of zero degrees, so that the low-pass can be neglected at first approximation. Thus we obtain the following transfer function:

$$\frac{I(\omega_1)}{U^{\#}(\omega_1)} \approx V_1\frac{j\omega_1 + \frac{k_r}{J}}{1 + j\omega_1 T_2} \quad (19)$$

By multiplying out Equation (19) and splitting it into its real and imaginary parts, we find the system of equations (where the indices I and R denote the real and imaginary component of the respective quantity, e.g. U_R= Re{ $U^{\#}(j\omega)$ }.

$$\begin{bmatrix} I_R - \omega_1 T_2 I_I = V_1 U_R^{\ *} \\ I_I + w_1 I_R T_2 = V_1 U_I^{\ *} \end{bmatrix} \quad (20)$$

The following identities are used

$$\begin{bmatrix} U_R^* = U_R \dfrac{k_r}{J} - \omega_1 U_I \\ U_I^* = U_I \dfrac{k_r}{J} + w_1 U_R \end{bmatrix} \qquad (21)$$

and the equations for determining T_2 and V_1 result.

$$\begin{bmatrix} T_2 = \dfrac{I_R U_I^* - I_I U_R^*}{w_1 (I_R U_R^* + I_I U_I^*)} \\ V_1 = \dfrac{I_R^2 + I_I^2}{U_R^* I_R + U_I^* I_I} \end{bmatrix} \qquad (22)$$

Thus we obtain the first value estimate for determining a pole of the transfer function T_2 and of the gain factor V_1.

These value estimates can now be used in estimating the second parameter. For this purpose, Equation (18) is rearranged as follows:

$$I(w_2)(1 + j\omega_2 T_1) = V_2 U(w_2) \frac{jw_2 + \dfrac{k_r}{J}}{1 + jw_2 T_2} \qquad (23)$$

We now solve for T_1 and V_2, which, after separating the real and imaginary components in Equation (24), are determined by the conditional equations:

$$\begin{bmatrix} T_1 = \dfrac{I_R U_I^\circ - I_I U_R^\circ}{w_2 (I_R U_R^\circ + I_I U_I^\circ)} \\ V_2 = \dfrac{I_R^2 + I_I^2}{U_R^\circ I_R + U_I^\circ I_I} \end{bmatrix} \qquad (24)$$

using the following identities

$$\begin{bmatrix} U_R^\circ = \dfrac{U_R \dfrac{k_r}{J} - U_I \omega_2 + \omega_2 T_2 (\omega_2 U_R + U_I \dfrac{k_r}{J})}{1 + \omega_2^2 T_2^2} \\ U_I^\circ = \dfrac{U_I \dfrac{k_r}{J} + U_R \omega_2 + \omega_2 T_2 (\omega_2 U_I - U_R \dfrac{k_r}{J})}{1 + \omega_2^2 T_2^2} \end{bmatrix} \qquad (25)$$

Using Equations (22) and (24), the motor parameters can now be determined by iteration. To do this, the results from (22) are applied to (24) until the iteration algorithm converges. Then the test object's parameters can be found using Equations (15) through (17).

2 Identifying a sample motor's parameters

For the example of a real motor described above, the following transfer function results:

$$\frac{I(s)}{U^\#(s)} = \frac{1}{0.0005} \frac{s + \dfrac{2 \cdot 10^{-5}}{7.5 \cdot 10^{-5}}}{s^2 + s\left(\dfrac{0.19}{0.0005} + \dfrac{2 \cdot 10^{-5}}{7.5 \cdot 10^{-5}}\right) + \dfrac{0.19}{0.0005}\left(\dfrac{2 \cdot 10^{-5}}{7.5 \cdot 10^{-5}} + \dfrac{0.0323^2}{0.19 \cdot 7.5 \cdot 10^{-5}}\right)} \qquad (26)$$

which solves as

$$\frac{I(s)}{U^\#(s)} = 2000 \frac{s + 0.2666}{s^2 + 380.2666s + 27922.4} \qquad (27)$$

and implies the following values for characteristic quantities:

$$\tau_{ele} = 2.63ms$$
$$\tau_{mech} = 13.66ms$$
$$\frac{k_r}{J} = 0.2666$$

Typical plots of current and voltage with the inverse values of the electrical and mechanical time constants as the excitation frequencies are displayed in Figure 8.

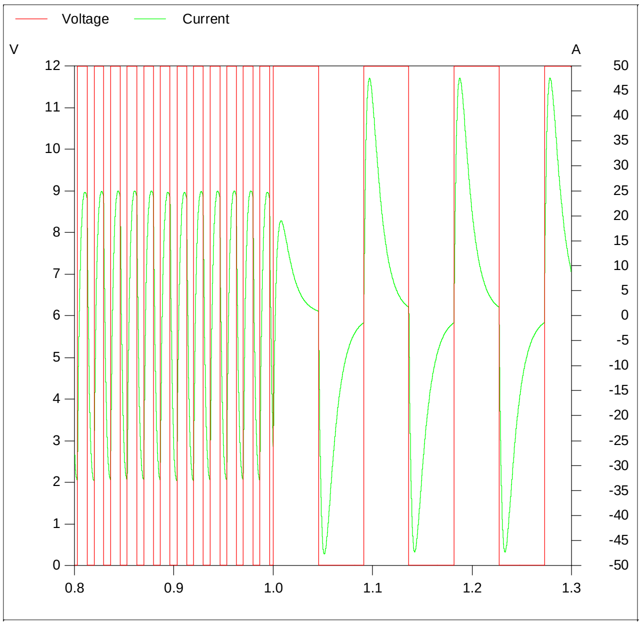

Fig. 8: Plot of current and voltage for determining the parameters of a sample motor

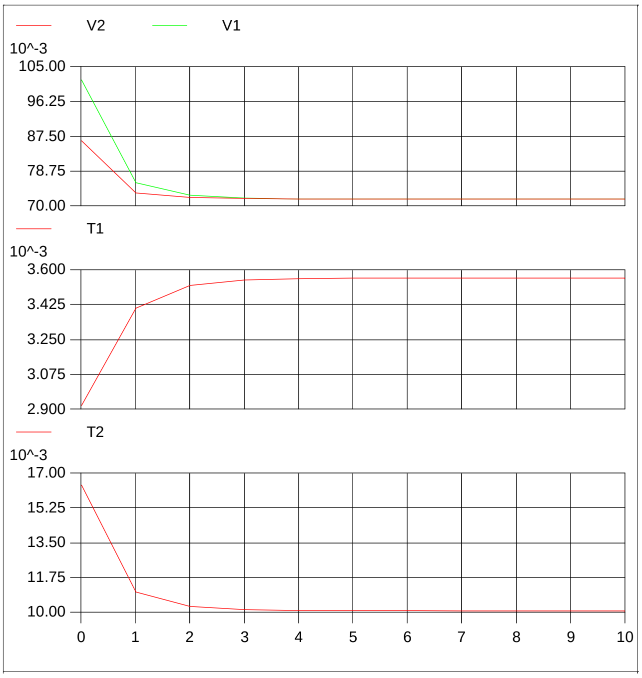

Fig. 9: Plot of identified parameters T1, T2, V1, V2

Figure 9 shows a plot of the parameters T1, T2, V1 and V2. The convergence produced by this procedure is clear to see.

The iteration ultimately returns the following parameters:

T1=0.00356063
T2=0.0100582
V1=0.0716225
V2=0.0716479

This translates to the following parameters for the test object:

R= 0.1904
L= 0.000501
k= 0.03233

Thus the accuracy of the parameters determined is within 0.2%.