Chapter 5: Linear Model Selection and Regularization

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```
# Libraries
library(ISLR)
summary(Hitters)
```

```
##      AtBat            Hits         HmRun            Runs       
##  Min.   : 16.0   Min.   :  1   Min.   : 0.00   Min.   :  0.00  
##  1st Qu.:255.2   1st Qu.: 64   1st Qu.: 4.00   1st Qu.: 30.25  
##  Median :379.5   Median : 96   Median : 8.00   Median : 48.00  
##  Mean   :380.9   Mean   :101   Mean   :10.77   Mean   : 50.91  
##  3rd Qu.:512.0   3rd Qu.:137   3rd Qu.:16.00   3rd Qu.: 69.00  
##  Max.   :687.0   Max.   :238   Max.   :40.00   Max.   :130.00  
##                                                                
##       RBI             Walks            Years            CAtBat       
##  Min.   :  0.00   Min.   :  0.00   Min.   : 1.000   Min.   :   19.0  
##  1st Qu.: 28.00   1st Qu.: 22.00   1st Qu.: 4.000   1st Qu.:  816.8  
##  Median : 44.00   Median : 35.00   Median : 6.000   Median : 1928.0  
##  Mean   : 48.03   Mean   : 38.74   Mean   : 7.444   Mean   : 2648.7  
##  3rd Qu.: 64.75   3rd Qu.: 53.00   3rd Qu.:11.000   3rd Qu.: 3924.2  
##  Max.   :121.00   Max.   :105.00   Max.   :24.000   Max.   :14053.0  
##                                                                      
##      CHits            CHmRun           CRuns             CRBI        
##  Min.   :   4.0   Min.   :  0.00   Min.   :   1.0   Min.   :   0.00  
##  1st Qu.: 209.0   1st Qu.: 14.00   1st Qu.: 100.2   1st Qu.:  88.75  
##  Median : 508.0   Median : 37.50   Median : 247.0   Median : 220.50  
##  Mean   : 717.6   Mean   : 69.49   Mean   : 358.8   Mean   : 330.12  
##  3rd Qu.:1059.2   3rd Qu.: 90.00   3rd Qu.: 526.2   3rd Qu.: 426.25  
##  Max.   :4256.0   Max.   :548.00   Max.   :2165.0   Max.   :1659.00  
##                                                                      
##      CWalks        League  Division    PutOuts          Assists      
##  Min.   :   0.00   A:175   E:157    Min.   :   0.0   Min.   :  0.0  
##  1st Qu.:  67.25   N:147   W:165    1st Qu.: 109.2   1st Qu.:  7.0  
##  Median : 170.50                    Median : 212.0   Median : 39.5  
##  Mean   : 260.24                    Mean   : 288.9   Mean   :106.9  
##  3rd Qu.: 339.25                    3rd Qu.: 325.0   3rd Qu.:166.0  
##  Max.   :1566.00                    Max.   :1378.0   Max.   :492.0  
##                                                                     
##      Errors          Salary       NewLeague
##  Min.   : 0.00   Min.   :  67.5   A:176    
##  1st Qu.: 3.00   1st Qu.: 190.0   N:146    
##  Median : 6.00   Median : 425.0            
##  Mean   : 8.04   Mean   : 535.9            
##  3rd Qu.:11.00   3rd Qu.: 750.0            
##  Max.   :32.00   Max.   :2460.0            
##                  NA's   :59
```

```
# Best Subset Selection

names(Hitters)
```

```
##  [1] "AtBat"     "Hits"      "HmRun"     "Runs"      "RBI"
##  [6] "Walks"     "Years"     "CAtBat"    "CHits"     "CHmRun"
## [11] "CRuns"     "CRBI"      "CWalks"    "League"    "Division"
## [16] "PutOuts"   "Assists"   "Errors"    "Salary"    "NewLeague"
```

```
dim(Hitters)
```

```
## [1] 322  20
```

```
sum(is.na(Hitters$Salary))
```

```
## [1] 59
```

```
Hitters=na.omit(Hitters)
dim(Hitters)
```

```
## [1] 263  20
```

```
sum(is.na(Hitters))
```

```
## [1] 0
```

```
library(leaps)
regfit.full=regsubsets(Salary~.,Hitters)
summary(regfit.full)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., Hitters)
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##          AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1  ( 1 ) " "   " "  " "   " "  " " " " " "   " "   " "   " "    " "   " "
## 2  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "
## 3  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "
## 4  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "
## 5  ( 1 ) "*"   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "
## 6  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "    " "   " "
```

```
## 7  ( 1 ) " "   "*"  " "   " "  " " "*"   " "   "*"    "*"   "*"    " "
## 8  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   " "    " "   "*"    "*"
##          CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 ) "*"  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 ) "*"  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 ) "*"  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 ) "*"  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 ) "*"  " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 ) "*"  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 ) " "  " "    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 ) " "  "*"    " "     "*"       "*"     " "     " "    " "
```

```
regfit.full=regsubsets(Salary~.,data=Hitters,nvmax=19)
reg.summary=summary(regfit.full)
names(reg.summary)
```

```
## [1] "which"  "rsq"    "rss"    "adjr2"  "cp"     "bic"    "outmat" "obj"
```

```
reg.summary$rsq
```

```
##  [1] 0.3214501 0.4252237 0.4514294 0.4754067 0.4908036 0.5087146 0.5141227
##  [8] 0.5285569 0.5346124 0.5404950 0.5426153 0.5436302 0.5444570 0.5452164
## [15] 0.5454692 0.5457656 0.5459518 0.5460945 0.5461159
```

```
par(mfrow=c(2,2))
plot(reg.summary$rss,xlab="Number of Variables",ylab="RSS",type="l")
plot(reg.summary$adjr2,xlab="Number of Variables",ylab="Adjusted RSq",type="l")
which.max(reg.summary$adjr2)
```

```
## [1] 11
```

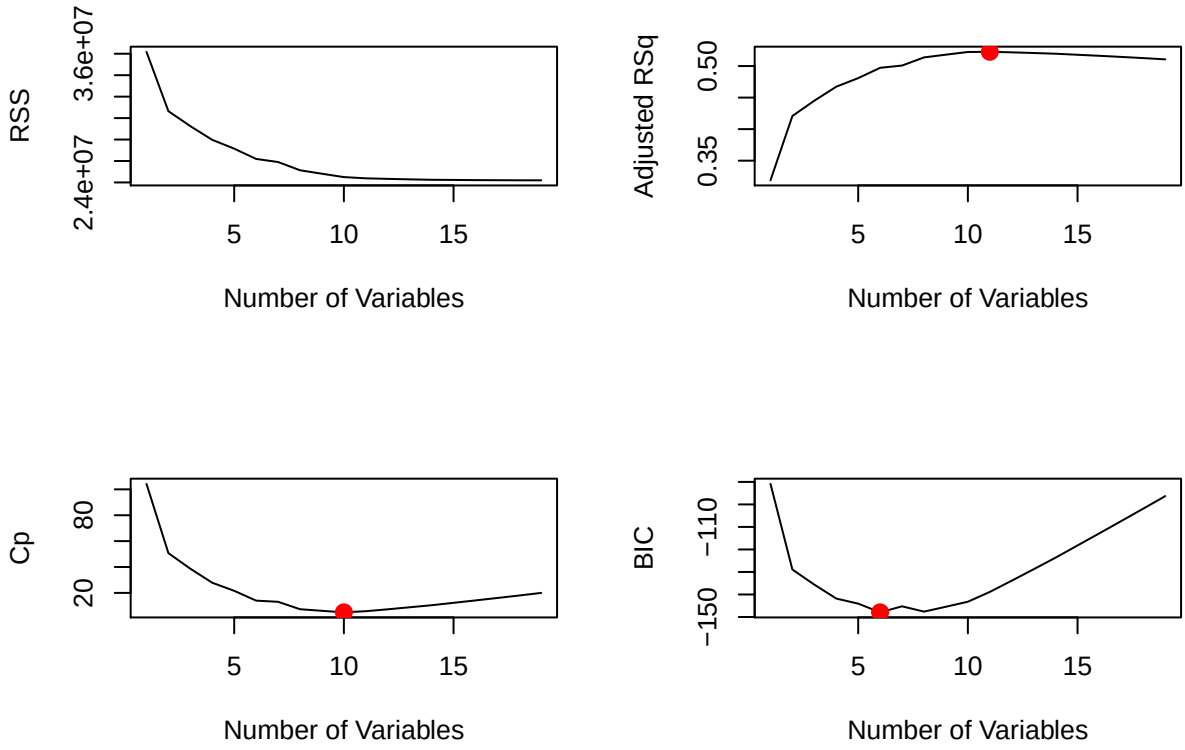
```
points(11,reg.summary$adjr2[11], col="red",cex=2,pch=20)
plot(reg.summary$cp,xlab="Number of Variables",ylab="Cp",type='l')
which.min(reg.summary$cp)
```

```
## [1] 10
```

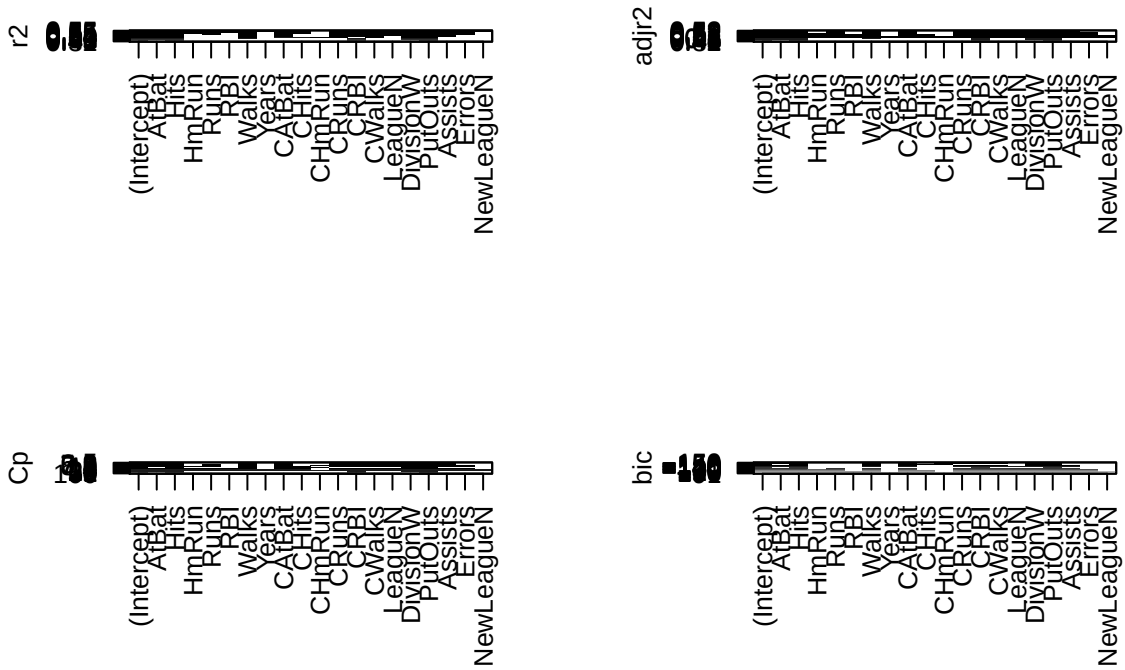
```
points(10,reg.summary$cp[10],col="red",cex=2,pch=20)
which.min(reg.summary$bic)
```

```
## [1] 6
```

```
plot(reg.summary$bic,xlab="Number of Variables",ylab="BIC",type='l')
points(6,reg.summary$bic[6],col="red",cex=2,pch=20)
```




```
plot(regfit.full,scale="r2")
plot(regfit.full,scale="adjr2")
plot(regfit.full,scale="Cp")
plot(regfit.full,scale="bic")
```




```
coef(regfit.full,6)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI
##   91.5117981   -1.8685892    7.6043976    3.6976468    0.6430169
##    DivisionW      PutOuts
## -122.9515338    0.2643076
```

```
# Forward and Backward Stepwise Selection

regfit.fwd=regsubsets(Salary~.,data=Hitters,nvmax=19,method="forward")
summary(regfit.fwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
## 19 Variables  (and intercept)
##           Forced in Forced out
## AtBat         FALSE      FALSE
## Hits          FALSE      FALSE
## HmRun         FALSE      FALSE
## Runs          FALSE      FALSE
## RBI           FALSE      FALSE
## Walks         FALSE      FALSE
## Years         FALSE      FALSE
## CAtBat        FALSE      FALSE
## CHits         FALSE      FALSE
## CHmRun        FALSE      FALSE
## CRuns         FALSE      FALSE
```

```
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1  ( 1 )  " "   " "  " "   " "  " " " " " "   " "   " "   " "   " "    " "
## 2  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "   " "   " "    " "
## 3  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "   " "   " "    " "
## 4  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "   " "   " "    " "
## 5  ( 1 )  "*"   "*"  " "   " "  " " " " " "   " "   " "   " "   " "    " "
## 6  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "   " "    " "
## 7  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "   " "    " "
## 8  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "   " "    "*"
## 9  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   "*"   " "   " "    "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"   " "   " "    "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"   " "   " "    "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"   " "   " "    "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"   " "   " "    "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"   " "   " "    "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"   "*"   " "    "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"   "*"   " "    "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"   "*"   " "    "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"   "*"   " "    "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"   "*"   "*"    "*"
##           CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  "*"  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  "*"  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  "*"  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  "*"  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 )  "*"  " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 )  "*"  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"  "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```
regfit.bwd=regsubsets(Salary~.,data=Hitters,nvmax=19,method="backward")
summary(regfit.bwd)
```

```
## Subset selection object
```

```
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "backward")
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: backward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1  ( 1 )  " "   " "  " "   " "  " " " " " "   " "   " "   " "   " "    "*"
## 2  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "   " "   " "    "*"
## 3  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "   " "   " "    "*"
## 4  ( 1 )  "*"   "*"  " "   " "  " " " " " "   " "   " "   " "   " "    "*"
## 5  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "   " "    "*"
## 6  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "   " "    "*"
## 7  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "   " "    "*"
## 8  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "   " "    "*"
## 9  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   "*"   " "   " "    "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"   " "   " "    "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"   " "   " "    "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"   " "   " "    "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"   " "   " "    "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"   " "   " "    "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"   "*"   " "    "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"   "*"   " "    "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"   "*"   " "    "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"   "*"   " "    "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"   "*"   "*"    "*"
##           CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "  " "    " "     " "       "*"     " "     " "    " "
## 5  ( 1 )  " "  " "    " "     " "       "*"     " "     " "    " "
## 6  ( 1 )  " "  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  " "  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"  "*"    " "     "*"       "*"     " "     " "    " "
```

```
## 10  ( 1 ) "*"  "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"  "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```
coef(regfit.full,7)
```

```
##  (Intercept)         Hits        Walks       CAtBat        CHits
##   79.4509472    1.2833513    3.2274264   -0.3752350    1.4957073
##       CHmRun    DivisionW      PutOuts
##    1.4420538 -129.9866432    0.2366813
```

```
coef(regfit.fwd,7)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI
##  109.7873062   -1.9588851    7.4498772    4.9131401    0.8537622
##       CWalks    DivisionW      PutOuts
##   -0.3053070 -127.1223928    0.2533404
```

```
coef(regfit.bwd,7)
```

```
##  (Intercept)        AtBat         Hits        Walks        CRuns
##  105.6487488   -1.9762838    6.7574914    6.0558691    1.1293095
##       CWalks    DivisionW      PutOuts
##   -0.7163346 -116.1692169    0.3028847
```

```
# Choosing Among Models

set.seed(1)
train=sample(c(TRUE,FALSE), nrow(Hitters),rep=TRUE)
test=(!train)
regfit.best=regsubsets(Salary~.,data=Hitters[train,],nvmax=19)
test.mat=model.matrix(Salary~.,data=Hitters[test,])
val.errors=rep(NA,19)
for(i in 1:19){
   coefi=coef(regfit.best,id=i)
   pred=test.mat[,names(coefi)]%*%coefi
   val.errors[i]=mean((Hitters$Salary[test]-pred)^2)
}
val.errors
```

```
##  [1] 220968.0 169157.1 178518.2 163426.1 168418.1 171270.6 162377.1
##  [8] 157909.3 154055.7 148162.1 151156.4 151742.5 152214.5 157358.7
## [15] 158541.4 158743.3 159972.7 159859.8 160105.6
```

```
which.min(val.errors)
```

```
## [1] 10
```

```
coef(regfit.best,10)
```

```
## (Intercept)       AtBat        Hits       Walks      CAtBat       CHits
## -80.2751499  -1.4683816   7.1625314   3.6430345  -0.1855698   1.1053238
```

```
##      CHmRun      CWalks     LeagueN   DivisionW     PutOuts
##   1.3844863  -0.7483170  84.5576103 -53.0289658   0.2381662
```

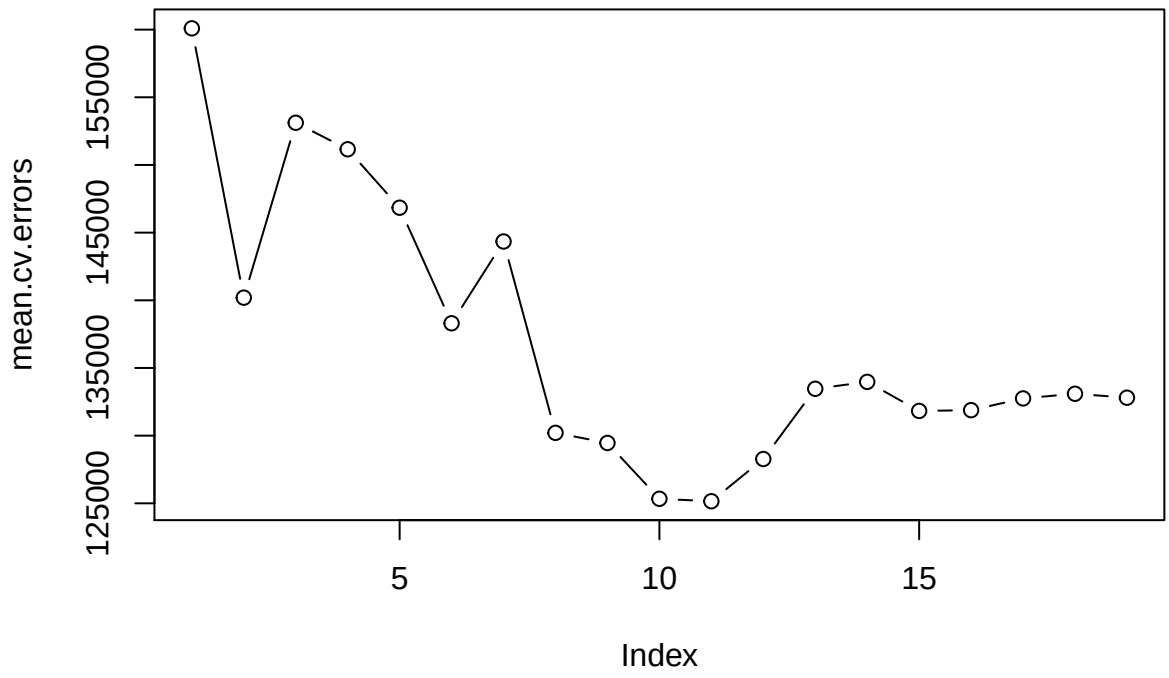
```
predict.regsubsets=function(object,newdata,id,...){
  form=as.formula(object$call[[2]])
  mat=model.matrix(form,newdata)
  coefi=coef(object,id=id)
  xvars=names(coefi)
  mat[,xvars]%*%coefi
  }
regfit.best=regsubsets(Salary~.,data=Hitters,nvmax=19)
coef(regfit.best,10)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat
##  162.5354420   -2.1686501    6.9180175    5.7732246   -0.1300798
##        CRuns         CRBI       CWalks    DivisionW      PutOuts
##    1.4082490    0.7743122   -0.8308264 -112.3800575    0.2973726
##      Assists
##    0.2831680
```

```
k=10
set.seed(1)
folds=sample(1:k,nrow(Hitters),replace=TRUE)
cv.errors=matrix(NA,k,19, dimnames=list(NULL, paste(1:19)))
for(j in 1:k){
  best.fit=regsubsets(Salary~.,data=Hitters[folds!=j,],nvmax=19)
  for(i in 1:19){
    pred=predict(best.fit,Hitters[folds==j,],id=i)
    cv.errors[j,i]=mean( (Hitters$Salary[folds==j]-pred)^2)
    }
  }
mean.cv.errors=apply(cv.errors,2,mean)
mean.cv.errors
```

```
##        1        2        3        4        5        6        7        8
## 160093.5 140196.8 153117.0 151159.3 146841.3 138302.6 144346.2 130207.7
##        9       10       11       12       13       14       15       16
## 129459.6 125334.7 125153.8 128273.5 133461.0 133974.6 131825.7 131882.8
##       17       18       19
## 132750.9 133096.2 132804.7
```

```
par(mfrow=c(1,1))
plot(mean.cv.errors,type='b')
```




```
reg.best=regsubsets(Salary~.,data=Hitters, nvmax=19)
coef(reg.best,11)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat
##  135.7512195   -2.1277482    6.9236994    5.6202755   -0.1389914
##        CRuns         CRBI       CWalks      LeagueN    DivisionW
##    1.4553310    0.7852528   -0.8228559   43.1116152 -111.1460252
##      PutOuts      Assists
##    0.2894087    0.2688277
```

```
x=model.matrix(Salary~.,Hitters)[,-1]
y=Hitters$Salary

# Ridge Regression

library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loading required package: foreach
```

```
## Loaded glmnet 2.0-16
```

```
grid=10^seq(10,-2,length=100)
ridge.mod=glmnet(x,y,alpha=0,lambda=grid)
dim(coef(ridge.mod))
```

```
## [1]  20 100
```

```
ridge.mod$lambda[50]
```

```
## [1] 11497.57
```

```
coef(ridge.mod)[,50]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
## 407.356050200   0.036957182   0.138180344   0.524629976   0.230701523
##           RBI         Walks         Years        CAtBat         CHits
##   0.239841459   0.289618741   1.107702929   0.003131815   0.011653637
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##   0.087545670   0.023379882   0.024138320   0.025015421   0.085028114
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
##  -6.215440973   0.016482577   0.002612988  -0.020502690   0.301433531
```

```
sqrt(sum(coef(ridge.mod)[-1,50]^2))
```

```
## [1] 6.360612
```

```
ridge.mod$lambda[60]
```

```
## [1] 705.4802
```

```
coef(ridge.mod)[,60]
```

```
##  (Intercept)        AtBat         Hits        HmRun         Runs
##  54.32519950   0.11211115   0.65622409   1.17980910   0.93769713
##          RBI        Walks        Years       CAtBat        CHits
##   0.84718546   1.31987948   2.59640425   0.01083413   0.04674557
##       CHmRun        CRuns         CRBI       CWalks      LeagueN
##   0.33777318   0.09355528   0.09780402   0.07189612  13.68370191
##    DivisionW      PutOuts      Assists       Errors   NewLeagueN
## -54.65877750   0.11852289   0.01606037  -0.70358655   8.61181213
```

```
sqrt(sum(coef(ridge.mod)[-1,60]^2))
```

```
## [1] 57.11001
```

```
predict(ridge.mod,s=50,type="coefficients")[1:20,]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
##  4.876610e+01 -3.580999e-01  1.969359e+00 -1.278248e+00  1.145892e+00
##           RBI         Walks         Years        CAtBat         CHits
##  8.038292e-01  2.716186e+00 -6.218319e+00  5.447837e-03  1.064895e-01
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##  6.244860e-01  2.214985e-01  2.186914e-01 -1.500245e-01  4.592589e+01
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
## -1.182011e+02  2.502322e-01  1.215665e-01 -3.278600e+00 -9.496680e+00
```

```
set.seed(1)
train=sample(1:nrow(x), nrow(x)/2)
test=(-train)
y.test=y[test]
ridge.mod=glmnet(x[train,],y[train],alpha=0,lambda=grid, thresh=1e-12)
ridge.pred=predict(ridge.mod,s=4,newx=x[test,])
mean((ridge.pred-y.test)^2)
```

```
## [1] 101036.8
```

```
mean((mean(y[train])-y.test)^2)
```

```
## [1] 193253.1
```

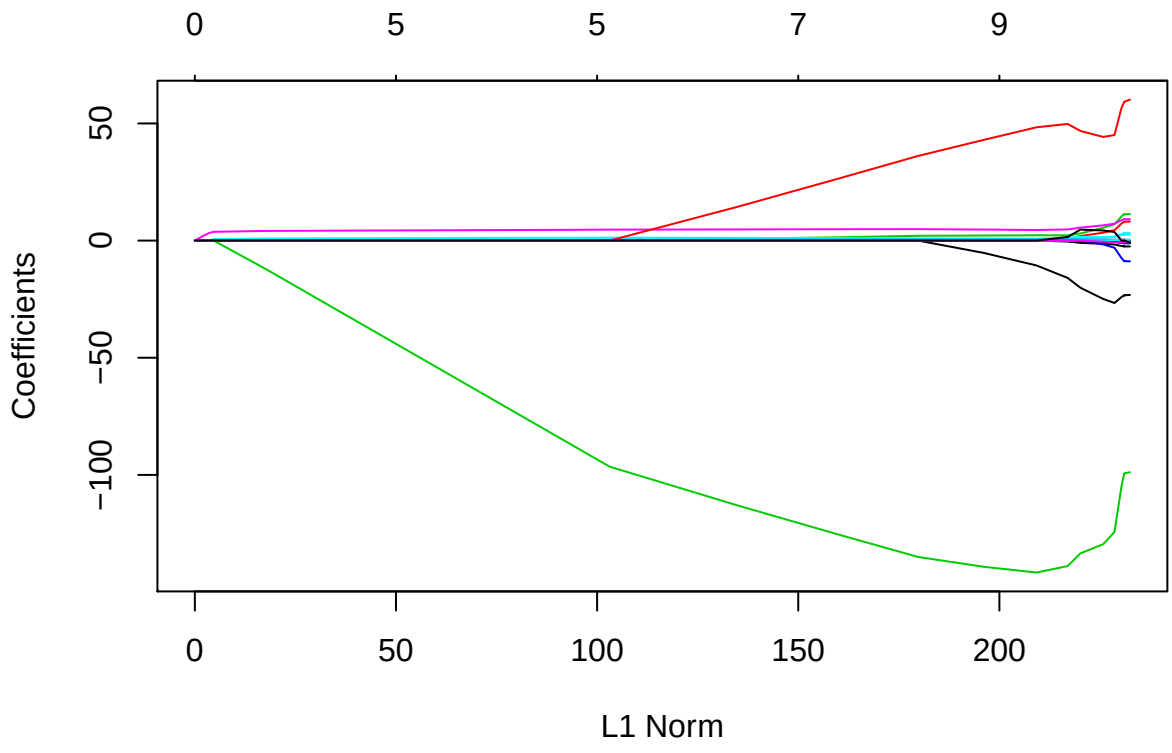
```
ridge.pred=predict(ridge.mod,s=1e10,newx=x[test,])
mean((ridge.pred-y.test)^2)
```

```
## [1] 193253.1
```

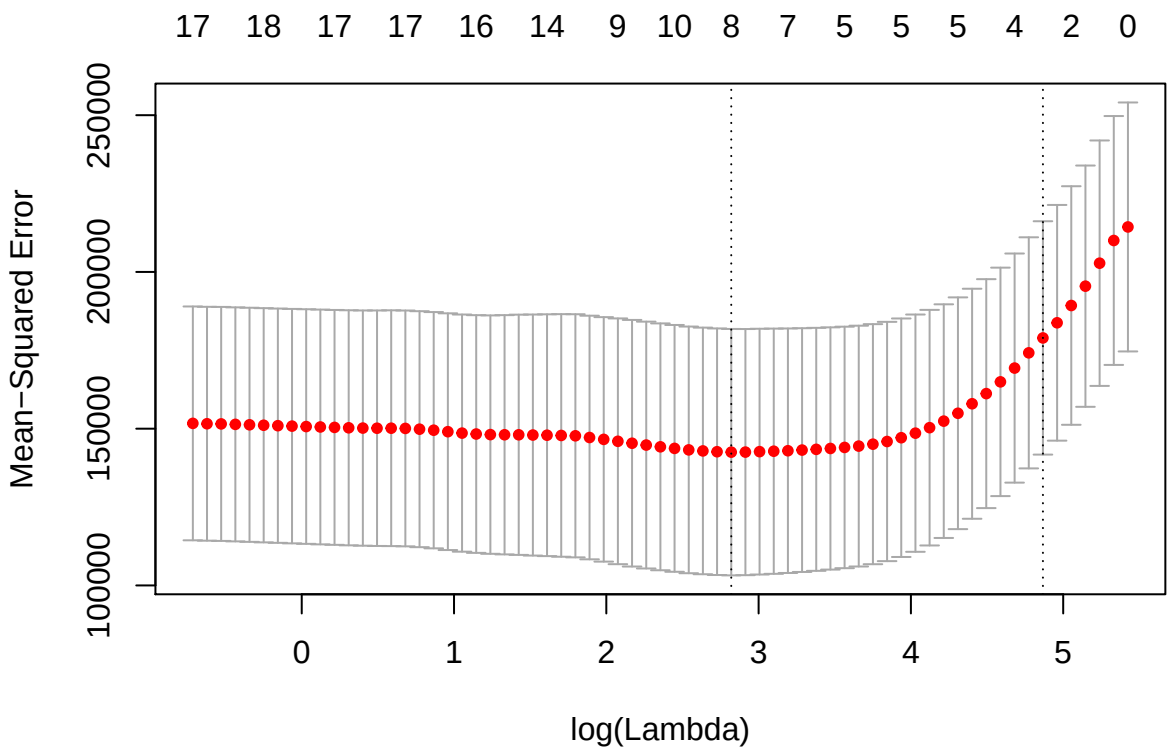
```
#ridge.pred=predict(ridge.mod,s=0,newx=x[test,],exact=T)
#mean((ridge.pred-y.test)^2)
#lm(y~x, subset=train)
#predict(ridge.mod,s=0,exact=T,type="coefficients")[1:20,]
#set.seed(1)
#cv.out=cv.glmnet(x[train,],y[train],alpha=0)
#plot(cv.out)
#bestlam=cv.out$lambda.min
#bestlam
#ridge.pred=predict(ridge.mod,s=bestlam,newx=x[test,])
#mean((ridge.pred-y.test)^2)
#out=glmnet(x,y,alpha=0)
#predict(out,type="coefficients",s=bestlam)[1:20,]
```

```
# The Lasso

lasso.mod=glmnet(x[train,],y[train],alpha=1,lambda=grid)
plot(lasso.mod)
```



```
set.seed(1)
cv.out=cv.glmnet(x[train,],y[train],alpha=1)
plot(cv.out)
```




```
bestlam=cv.out$lambda.min
lasso.pred=predict(lasso.mod,s=bestlam,newx=x[test,])
mean((lasso.pred-y.test)^2)
```

```
## [1] 100743.4
```

```
out=glmnet(x,y,alpha=1,lambda=grid)
lasso.coef=predict(out,type="coefficients",s=bestlam)[1:20,]
lasso.coef
```

```
##  (Intercept)        AtBat         Hits        HmRun         Runs
##   18.5394844    0.0000000    1.8735390    0.0000000    0.0000000
##          RBI        Walks        Years       CAtBat        CHits
##    0.0000000    2.2178444    0.0000000    0.0000000    0.0000000
##       CHmRun        CRuns         CRBI       CWalks      LeagueN
##    0.0000000    0.2071252    0.4130132    0.0000000    3.2666677
##    DivisionW      PutOuts      Assists       Errors   NewLeagueN
## -103.4845458    0.2204284    0.0000000    0.0000000    0.0000000
```

```
lasso.coef[lasso.coef!=0]
```

```
##  (Intercept)         Hits        Walks        CRuns         CRBI
##   18.5394844    1.8735390    2.2178444    0.2071252    0.4130132
##      LeagueN    DivisionW      PutOuts
```

```
##    3.2666677 -103.4845458    0.2204284
```

```
# Principal Components Regression

library(pls)
```

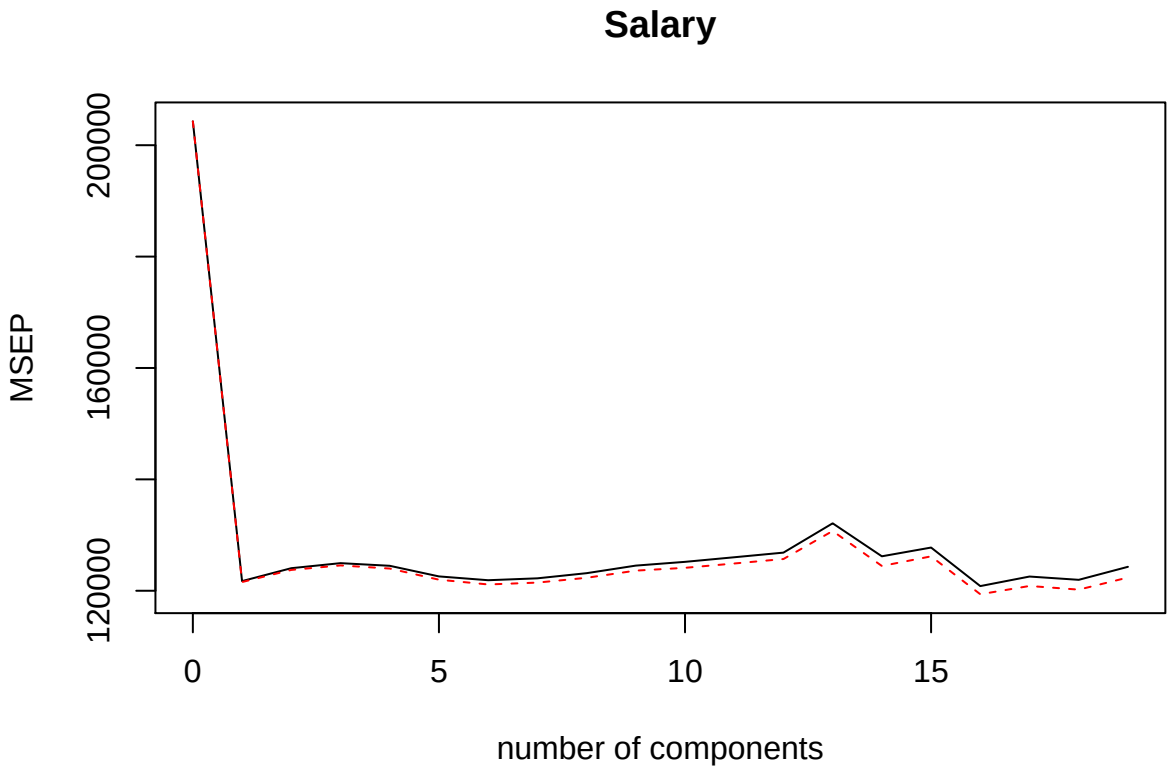
```
##
## Attaching package: 'pls'
```

```
## The following object is masked from 'package:stats':
##
##     loadings
```

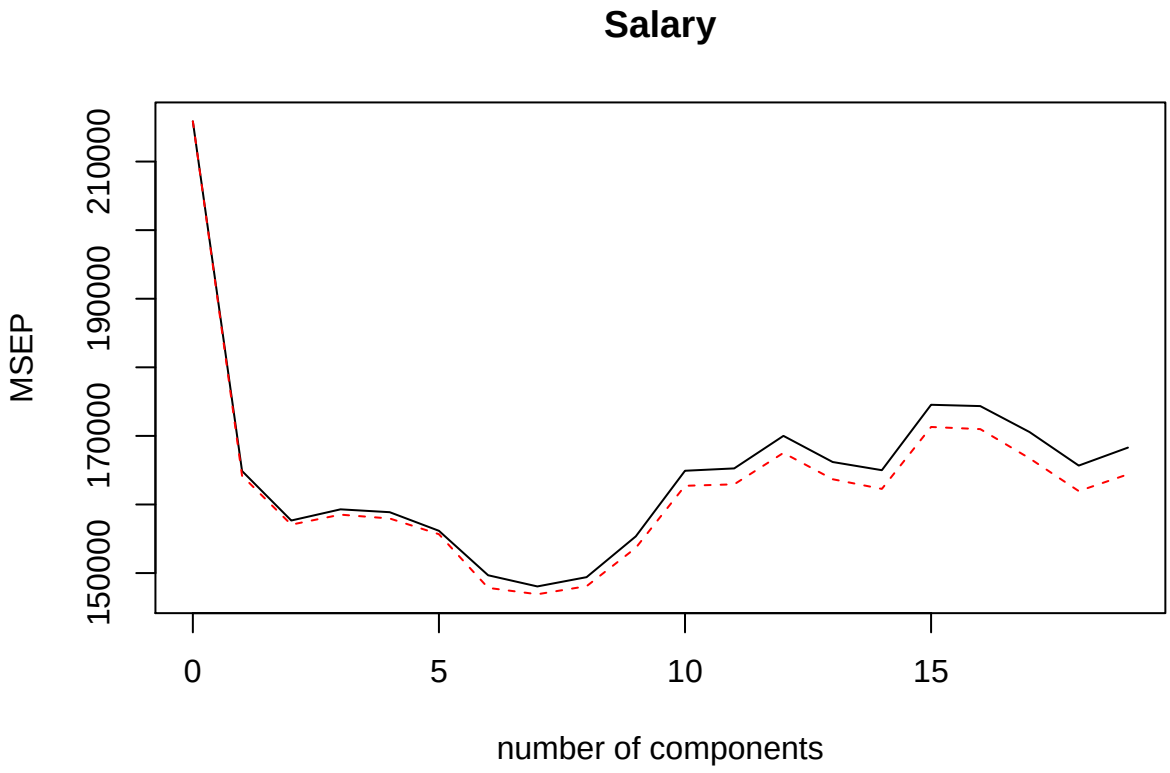
```
set.seed(2)
pcr.fit=pcr(Salary~., data=Hitters,scale=TRUE,validation="CV")
summary(pcr.fit)
```

```
## Data:    X dimension: 263 19
##  Y dimension: 263 1
## Fit method: svdpc
## Number of components considered: 19
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV             452    348.9    352.2    353.5    352.8    350.1    349.1
## adjCV          452    348.7    351.8    352.9    352.1    349.3    348.0
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV       349.6    350.9    352.9     353.8     355.0     356.2     363.5
## adjCV    348.5    349.8    351.6     352.3     353.4     354.5     361.6
##        14 comps  15 comps  16 comps  17 comps  18 comps  19 comps
## CV        355.2     357.4     347.6     350.1     349.2     352.6
## adjCV     352.8     355.2     345.5     347.6     346.7     349.8
##
## TRAINING: % variance explained
##         1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X         38.31    60.16    70.84    79.03    84.29    88.63    92.26
## Salary    40.63    41.58    42.17    43.22    44.90    46.48    46.69
##         8 comps  9 comps  10 comps  11 comps  12 comps  13 comps  14 comps
## X         94.96    96.28     97.26     97.98     98.65     99.15     99.47
## Salary    46.75    46.86     47.76     47.82     47.85     48.10     50.40
##         15 comps  16 comps  17 comps  18 comps  19 comps
## X          99.75     99.89     99.97     99.99    100.00
## Salary     50.55     53.01     53.85     54.61     54.61
```

```
validationplot(pcr.fit,val.type="MSEP")
```




```
set.seed(1)
pcr.fit=pcr(Salary~., data=Hitters,subset=train,scale=TRUE, validation="CV")
validationplot(pcr.fit,val.type="MSEP")
```




```
pcr.pred=predict(pcr.fit,x[test,],ncomp=7)
mean((pcr.pred-y.test)^2)
```

```
## [1] 96556.22
```

```
pcr.fit=pcr(y~x,scale=TRUE,ncomp=7)
summary(pcr.fit)
```

```
## Data:    X dimension: 263 19
##  Y dimension: 263 1
## Fit method: svdpc
## Number of components considered: 7
## TRAINING: % variance explained
##    1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X    38.31    60.16    70.84    79.03    84.29    88.63    92.26
## y    40.63    41.58    42.17    43.22    44.90    46.48    46.69
```

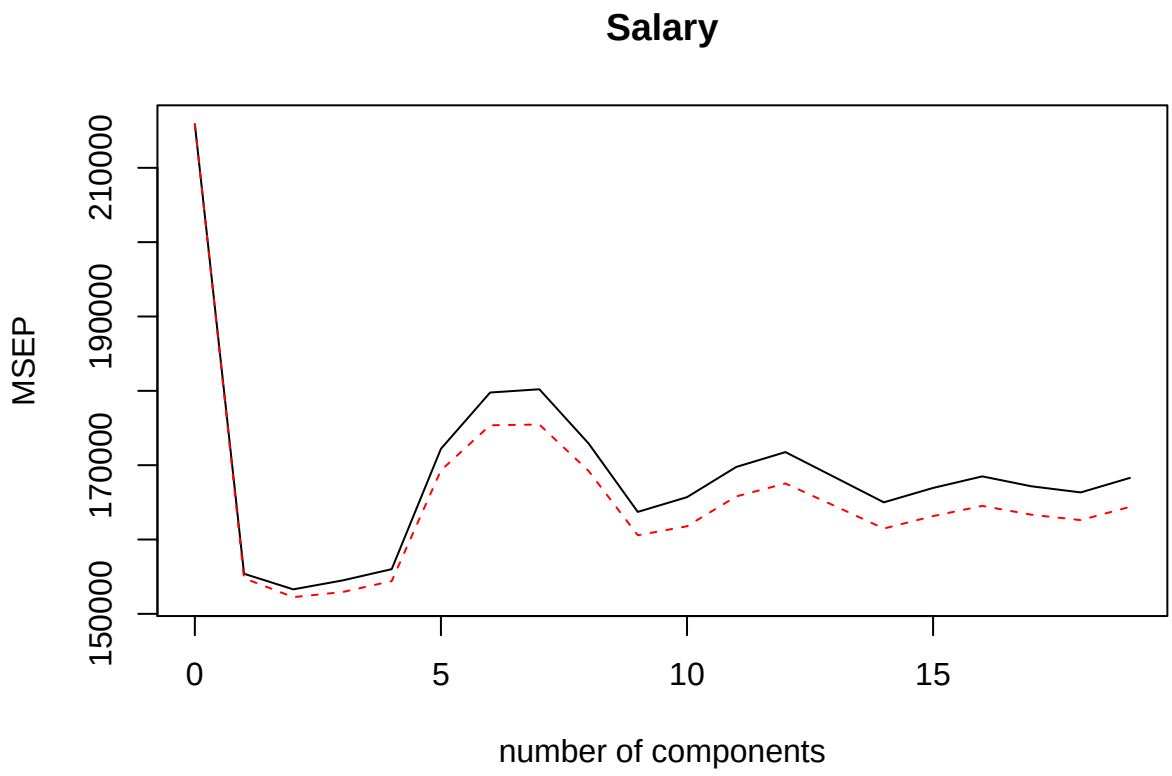
```
# Partial Least Squares

set.seed(1)
pls.fit=plsr(Salary~., data=Hitters,subset=train,scale=TRUE, validation="CV")
summary(pls.fit)
```

```
## Data:    X dimension: 131 19
##  Y dimension: 131 1
## Fit method: kernelpls
## Number of components considered: 19
##
```

```
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV           464.6    394.2    391.5    393.1    395.0    415.0    424.0
## adjCV        464.6    393.4    390.2    391.1    392.9    411.5    418.8
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV       424.5    415.8    404.6     407.1     412.0     414.4     410.3
## adjCV    418.9    411.4    400.7     402.2     407.2     409.3     405.6
##        14 comps  15 comps  16 comps  17 comps  18 comps  19 comps
## CV        406.2     408.6     410.5     408.8     407.8     410.2
## adjCV     401.8     403.9     405.6     404.1     403.2     405.5
##
## TRAINING: % variance explained
##         1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X         38.12    53.46    66.05    74.49    79.33    84.56    87.09
## Salary    33.58    38.96    41.57    42.43    44.04    45.59    47.05
##         8 comps  9 comps  10 comps  11 comps  12 comps  13 comps  14 comps
## X         90.74    92.55     93.94     97.23     97.88     98.35     98.85
## Salary    47.53    48.42     49.68     50.04     50.54     50.78     50.92
##         15 comps  16 comps  17 comps  18 comps  19 comps
## X          99.11     99.43     99.78     99.99    100.00
## Salary     51.04     51.11     51.15     51.16     51.18
```

```
validationplot(pls.fit,val.type="MSEP")
```




```
pls.pred=predict(pls.fit,x[test,],ncomp=2)
mean((pls.pred-y.test)^2)
```

```
## [1] 101417.5
```

```
pls.fit=plsr(Salary~., data=Hitters,scale=TRUE,ncomp=2)
summary(pls.fit)
```

```
## Data:    X dimension: 263 19
##  Y dimension: 263 1
## Fit method: kernelpls
## Number of components considered: 2
## TRAINING: % variance explained
##         1 comps  2 comps
## X         38.08    51.03
## Salary    43.05    46.40
```