# A Continuous-Time Dynamical System Describing both Rate Encoding and Spiking Neurons

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## Introduction

- We introduce a two-dimensional nonlinear system, modeling a wide range of dynamic properties of spiking neurons.
- By altering key parameters of this system, its dynamics become identical to those of a time-continuous rate-encoding model.
- Differences of the dynamical properties of single units as well as of network structures under these two regimes can be treated within the same mathematical framework.

## Neuron Model

The model consists of a two-dimensional non-linear system given by

$$\tau_x \dot{x} = f(x) - y \qquad f(x) = s\sigma\left(a\left(x - s/4\right)\right)$$
$$\tau_y \dot{y} = g(x) - y \qquad g(x) = g_0\sigma\left(a_g\left(x - I\right)\right) - \Delta_y$$
$$u(x,y) := \frac{x+y}{\sqrt{2}} \qquad \sigma(x) = \left(1+\exp(-x)\right)^{-1}$$

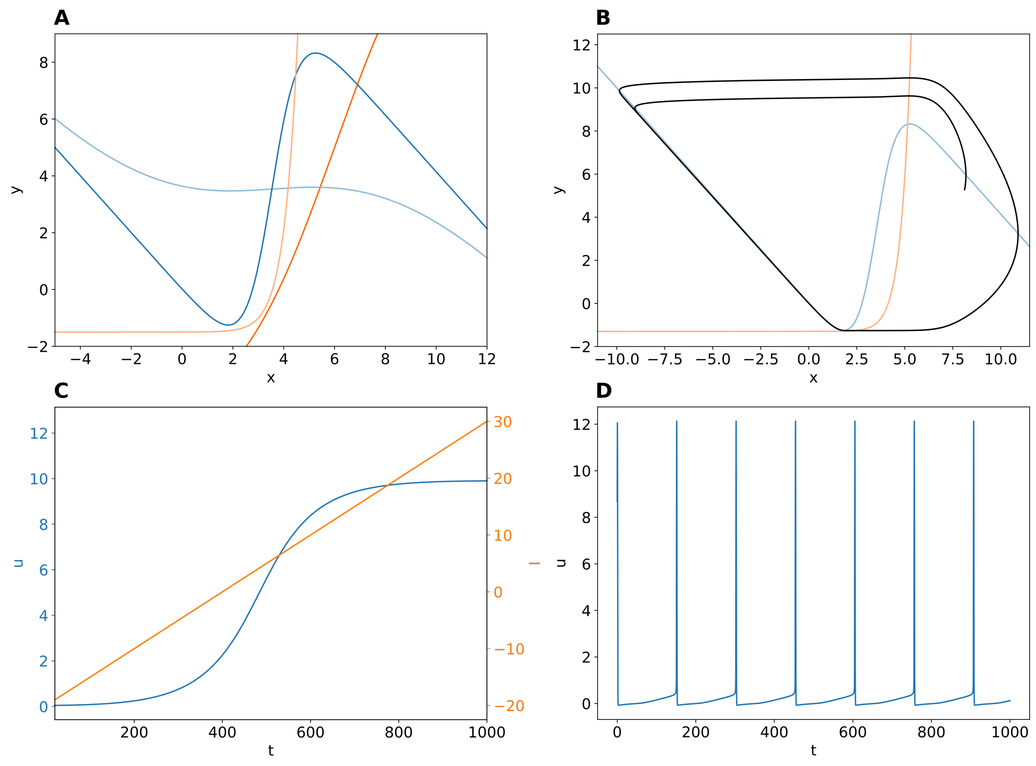


Figure 1: **A**: x/y-nullclines (blue/red) of the dynamical system for parameter sets generating spiking/non spiking behavior. **B**: Phase plane trajectory for the spiking dynamics. **C**/**D**: Dynamics of readout variable $u$ for the spiking/non-spiking case.

## Results – Spiking Neuron Model

- A range of different types of spiking dynamics can be reproduced by this generic system.

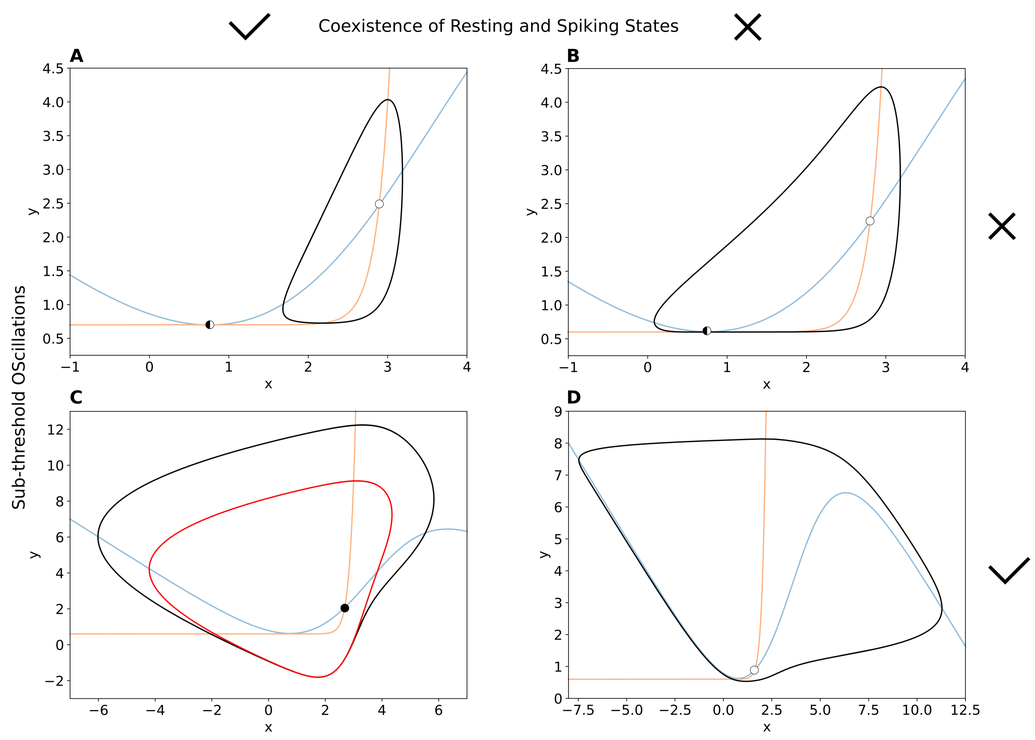


Figure 2: Examples of spiking dynamics/bifurcations observed in the model, ordered by the property of coexistence of resting/spiking states and the possibility of sub-threshold oscillations. A: Saddle node bifurcation with coexisting limit cycle. B: Saddle node b. on invariant cycle. C: Subcritical Hopf b/ Fold b. of limit cycles. C: Supercritical Hopf b.

- Based on the classification by Izhikevich [1], our model was able to generate class 1 (arbitrary small firing rates) as well as class 2 (all-or-none spiking behavior) firing patterns.

## Results – Fitting Experimental Data

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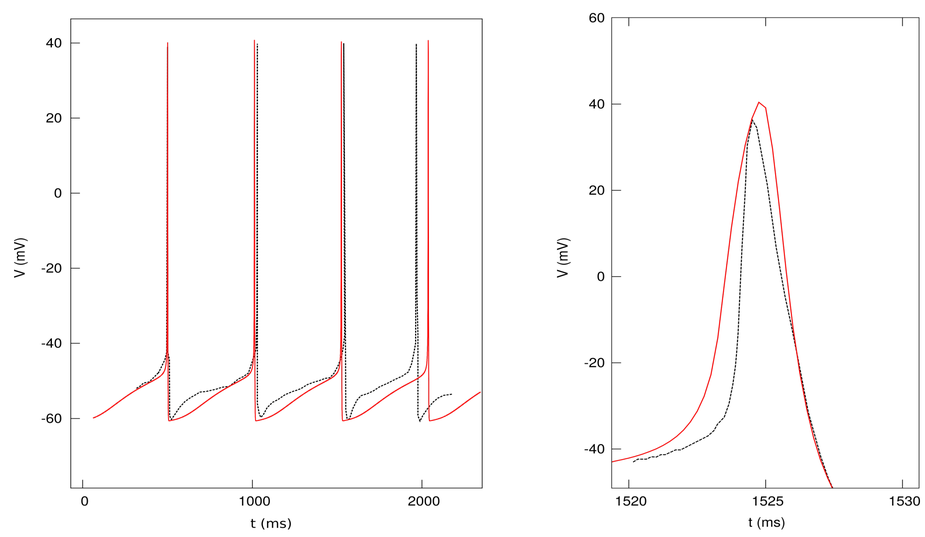


[1] E. M. Izhikevich. *Dynamical Systems in Neuroscience*. The MIT Press, Cambridge Massachusetts, 2007.