A Continuous-Time Dynamical System Describing both Rate Encoding and Spiking Neurons

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Introduction

- We introduce a two-dimensional nonlinear system, modeling a wide range of dynamic properties of spiking neurons.
- By altering key parameters of this system, its dynamics become identical to those of a time-continuous rate-encoding model.
- Differences of the dynamical properties of single units as well as of network structures under these two regimes can be treated within the same mathematical framework.

Neuron Model

The model consists of a two-dimensional non-linear system given by

$$
\begin{aligned}
\tau_x \dot{x} &= f(x) - y \qquad & f(x) &= s\sigma\left(a\left(x - s/4\right)\right) \\
\tau_y \dot{y} &= g(x) - y \qquad & g(x) &= g_0\sigma\left(a_g\left(x - I\right)\right) - \Delta_y \\
u(x,y) &:= \frac{x+y}{\sqrt{2}} \qquad & \sigma(x) &= \left(1 + \exp(-x)\right)^{-1}
\end{aligned}
$$

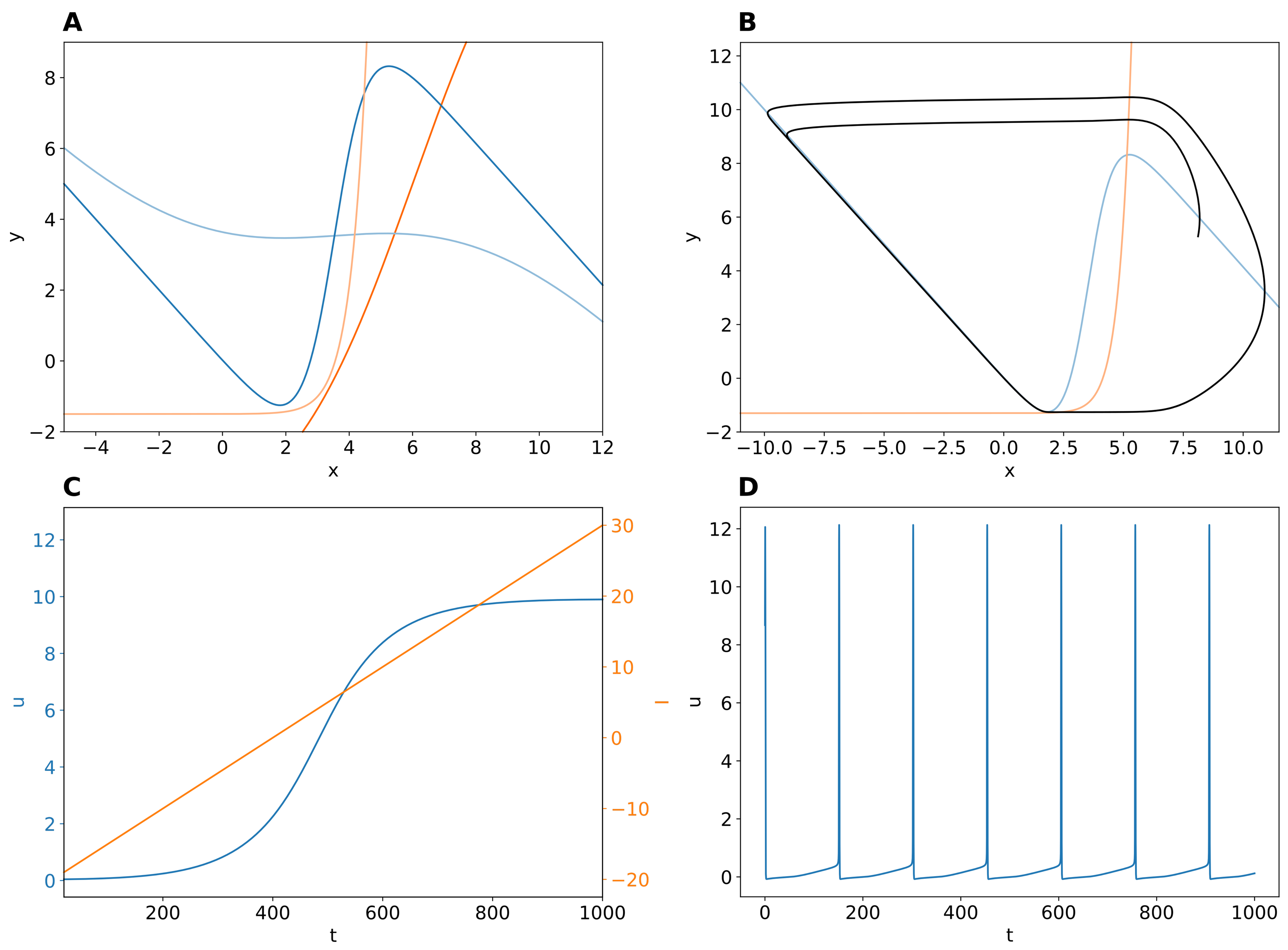

Figure 1: **A**: x/y-nullclines (blue/red) of the dynamical system for parameter sets generating spiking/non spiking behavior. **B**: Phase plane trajectory for the spiking dynamics. **C**/**D**: Dynamics of readout variable u for the spiking/non-spiking case.

Results – Spiking Neuron Model

- A range of different types of spiking dynamics can be reproduced by this generic system.

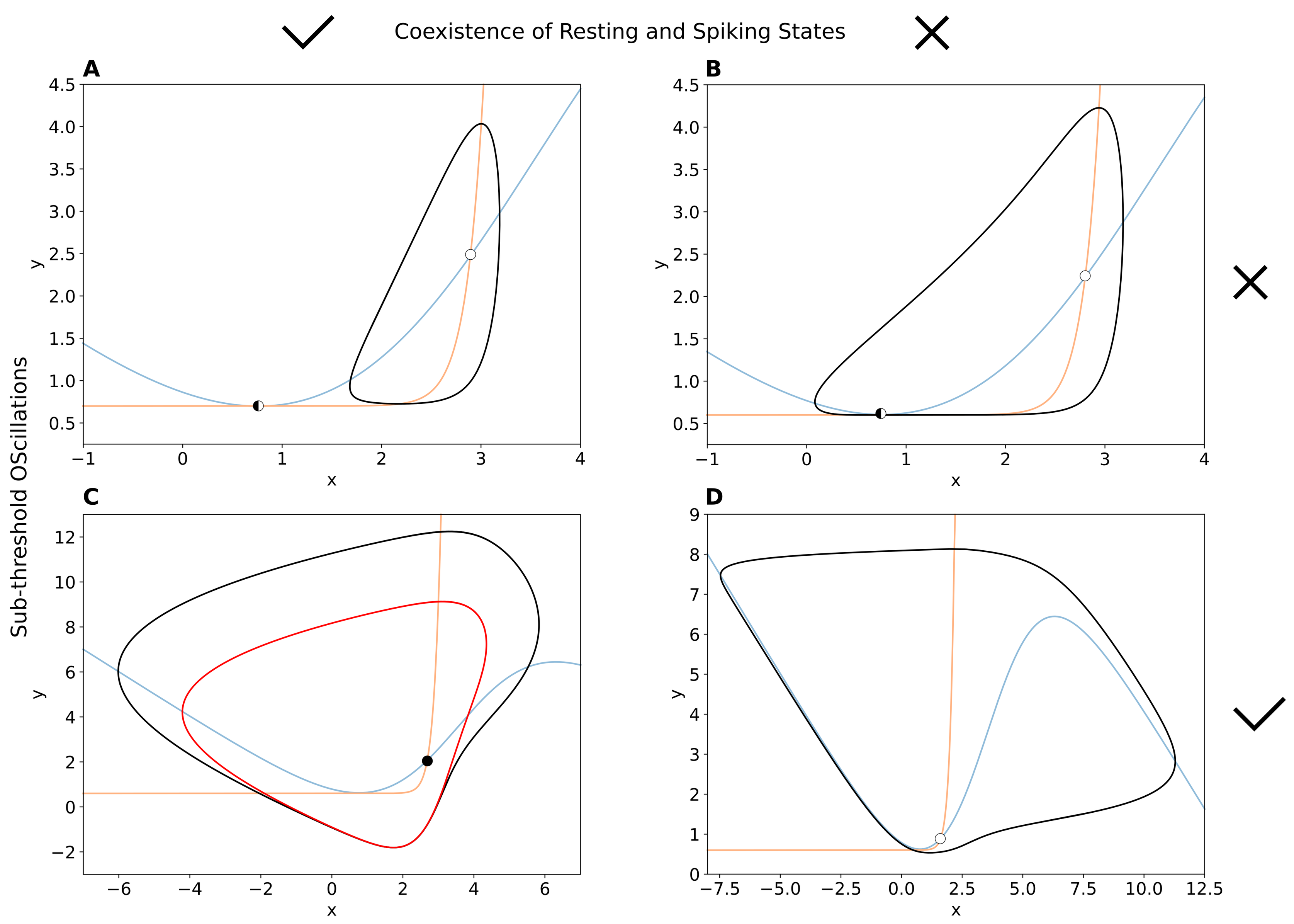

Figure 2: Examples of spiking dynamics/bifurcations observed in the model, ordered by the property of coexistence of resting/spiking states and the possibility of sub-threshold oscillations. A: Saddle node bifurcation with coexisting limit cycle. B: Saddle node b. on invariant cycle. C: Subcritical Hopf b/ Fold b. of limit cycles. C: Supercritical Hopf b.

- Based on the classification by Izhikevich [1], our model was able to generate class 1 (arbitrary small firing rates) as well as class 2 (all-or-none spiking behavior) firing patterns.

Results – Fitting Experimental Data

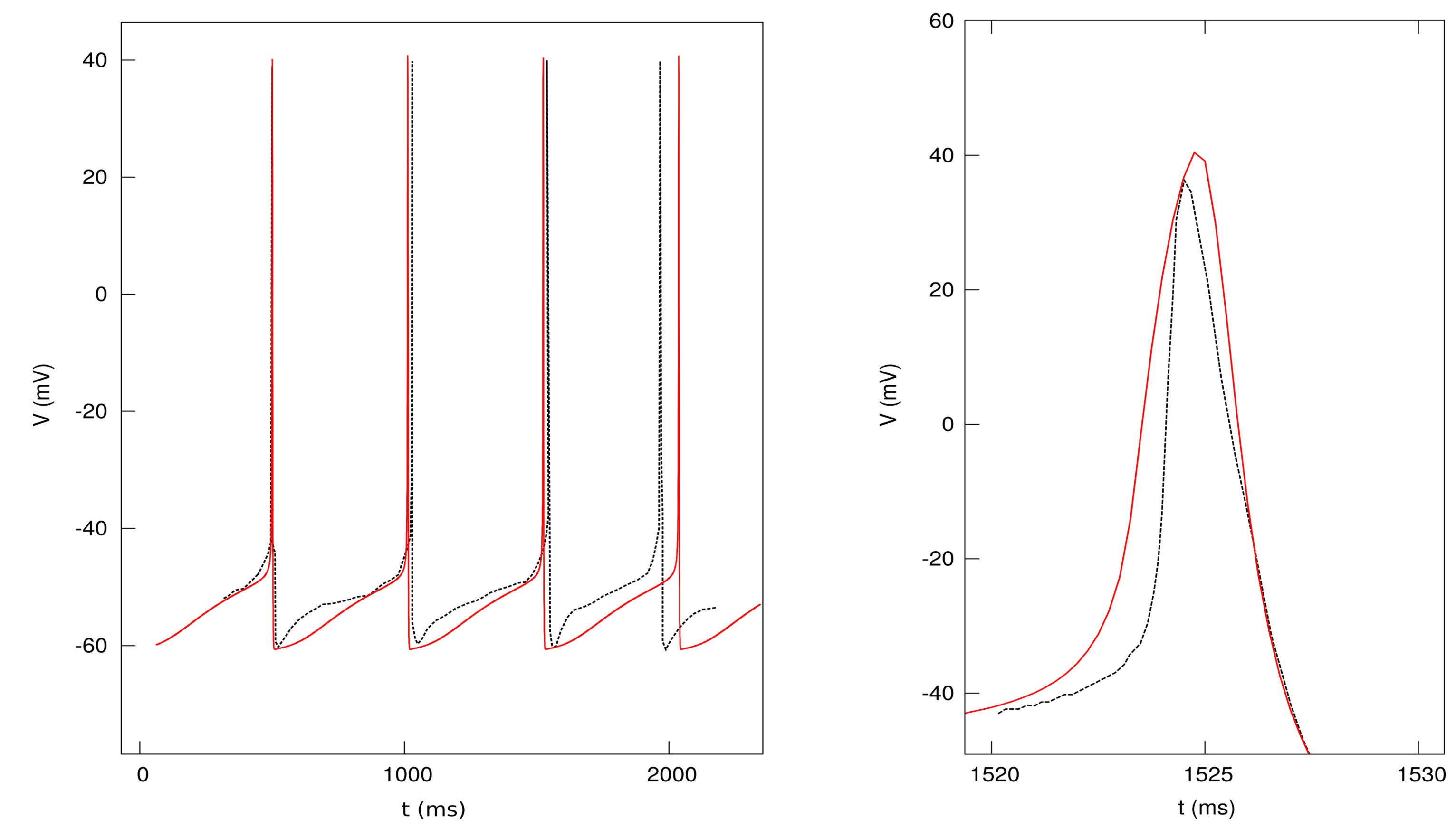

Figure 3: Fit of the dynamical system onto measurements of a dopaminergic neuron in the Substantia Nigra of a mouse brain [2].

- The parameters of the dynamical systems allow to easily control key features of the voltage trace, such as spike height, spike width and resting and threshold potential.

Results – Rate Encoding Model

- The system's parameters can be tuned such that dynamics transition from a spiking behavior to a relaxation towards an input-dependent fixed point.
- The projection of this fixed point onto u follows a sigmoidal as input increases, see Fig. 1C.

[1] E. M. Izhikevich. *Dynamical Systems in Neuroscience*. The MIT Press, Cambridge Massachusetts, 2007.

[2] B. P. Bean. The action potential in mammalian central neurons. *Nature Reviews, Neuroscience*, 8, June 2007.