# A Continuous-Time Dynamical System Describing both Rate Encoding and Spiking Neurons

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#### Introduction

- ► We investigated a two-dimensional nonlinear system, modeling a wide range of dynamic properties of spiking neurons.
- ▶ By altering key parameters of this system, its dynamics become identical to those of a time-continuous rate-encoding model.
- ▶ Differences of the dynamical properties of single units as well as of network structures under these two regimes can be treated within the same mathematical framework.

#### **Neuron Model**

► The model consists of a two-dimensional non-linear system given by

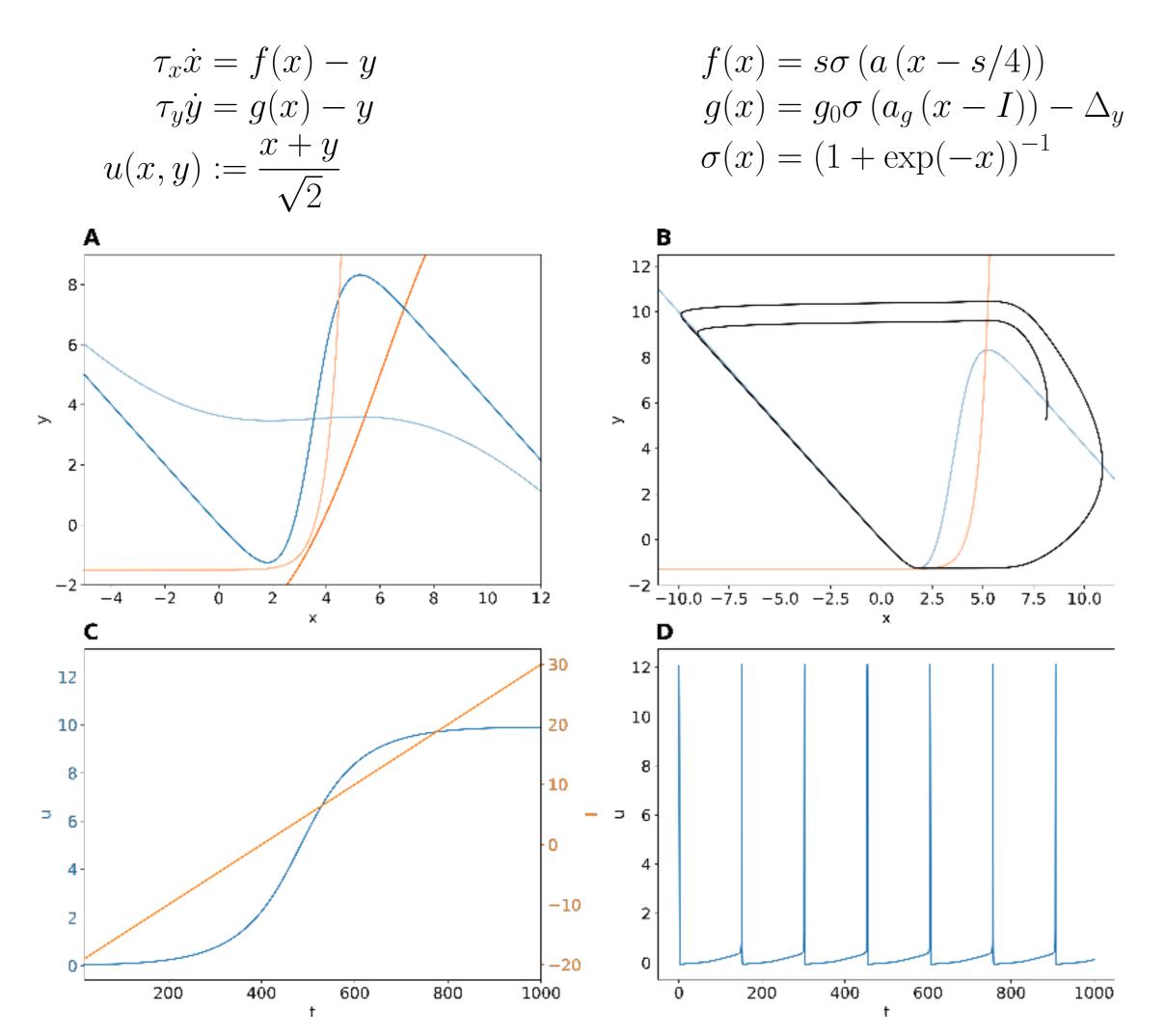


Figure 1: **A**: x/y-nullclines (blue/red) of the dynamical system for parameter sets generating spiking/non spiking behavior. **B**: Phase plane trajectory for the spiking dynamics. **C**/**D**: Dynamics of readout variable u for the spiking/non-spiking case.

## **Spiking Neuron Model**

► A range of different types of spiking dynamics can be reproduced by this generic system.

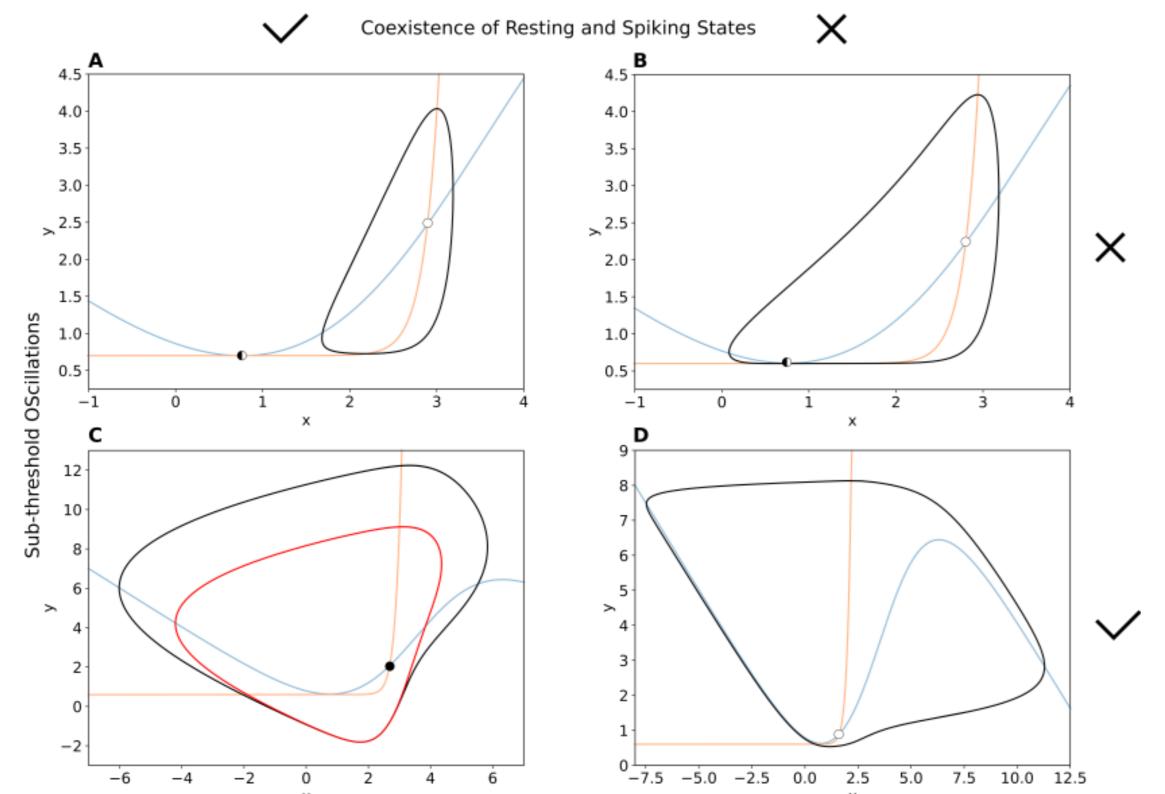


Figure 2: Examples of spiking dynamics/bifurcations observed in the model, ordered by the property of coexistence of resting/spiking states and the possibility of sub-threshold oscillations. A: Saddle node bifurcation with coexisting limit cycle. B: Saddle node b. on invariant cycle. C: Subcritical Hopf b/ Fold b. of limit cycles. C: Supercritical Hopf b.

- ► Based on the classification by Izhikevich [1], our model was able to generate class 1 (arbitrary small firing rates) as well as class 2 (all-or-none spiking behavior) firing patterns.
- ▶ Due to the relatively simple mathematical form of our system, we could perform analytical analyses and predictions about firing behavior closely matching the simulation results, see Fig. 3.

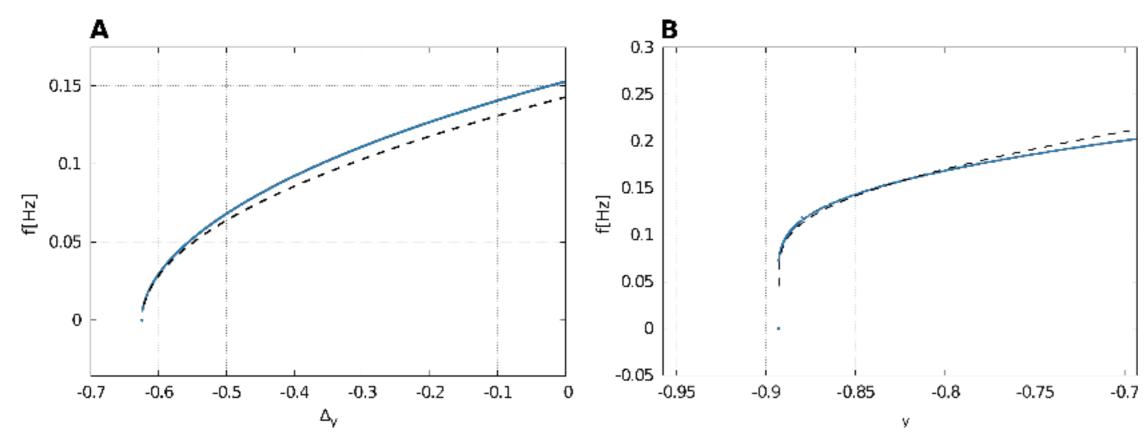


Figure 3: Measured and predicted firing rates as a function of  $\Delta_y$  under the configurations shown in Fig. 2A/B.

#### Fitting Experimental Data

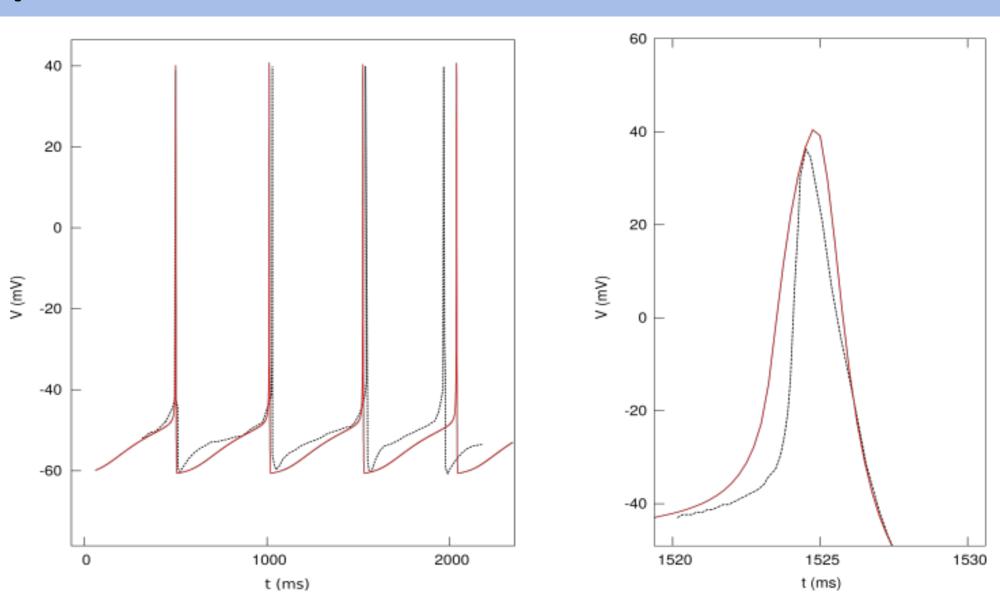


Figure 4: Fit of the dynamical system onto measurements of a dopaminergic neuron in the Substantia Nigra of a mouse brain [2].

► The parameters of the dynamical systems allow to easily control key features of the voltage trace, such as spike height, spike width and threshold potential.

$$u_{\rm spikeheight} \approx s/\sqrt{2}$$

$$\Delta t_{\rm spike} \propto g_O^{-1}$$

$$u_{\rm threshold} \approx 1/\sqrt{2a}$$

## Rate Encoding Model

- ► The system's parameters can be tuned such that dynamics transition from a spiking behavior to a relaxation towards an input-dependent fixed point.
- ightharpoonup The projection of this fixed point onto u follows a sigmoidal as input increases, see Fig. 1C.

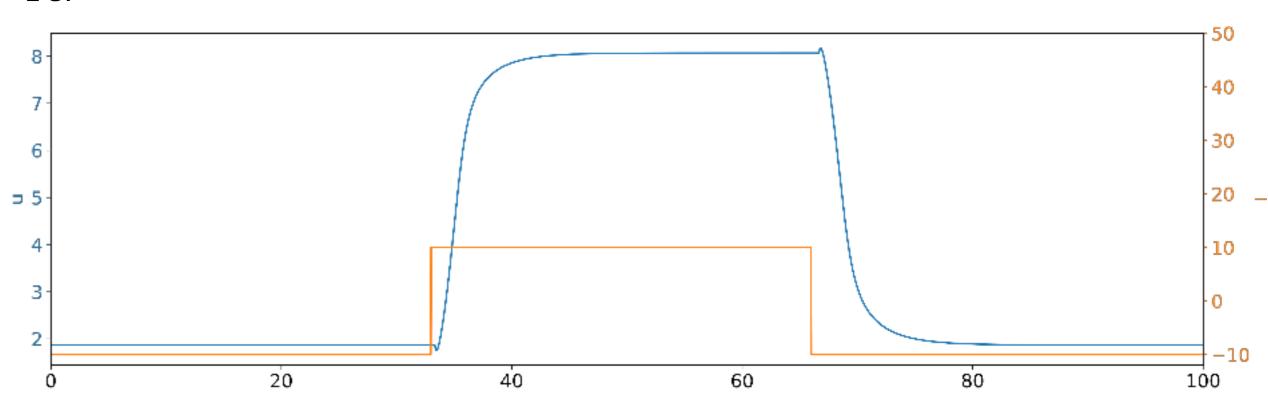


Figure 5: Rate-dynamics response of the readout variable u upon external step input.

The dynamics of u approximately follow those of 1d-system of the form  $\tau \dot{u} = -u + \frac{s}{\sqrt{2}} \sigma \, (c \cdot I)$ . The time constant  $\tau$  is then described by  $\tau = -\frac{1}{\langle \lambda \rangle}$ , where  $\langle \lambda \rangle$  is the mean of the eigenvalues of the fixed point's Jacobian.

#### **Usage in Networks**

► Combining our model with a simple model of synaptic transmission resulted in stable network dynamics for both spiking and rate encoding dynamics.

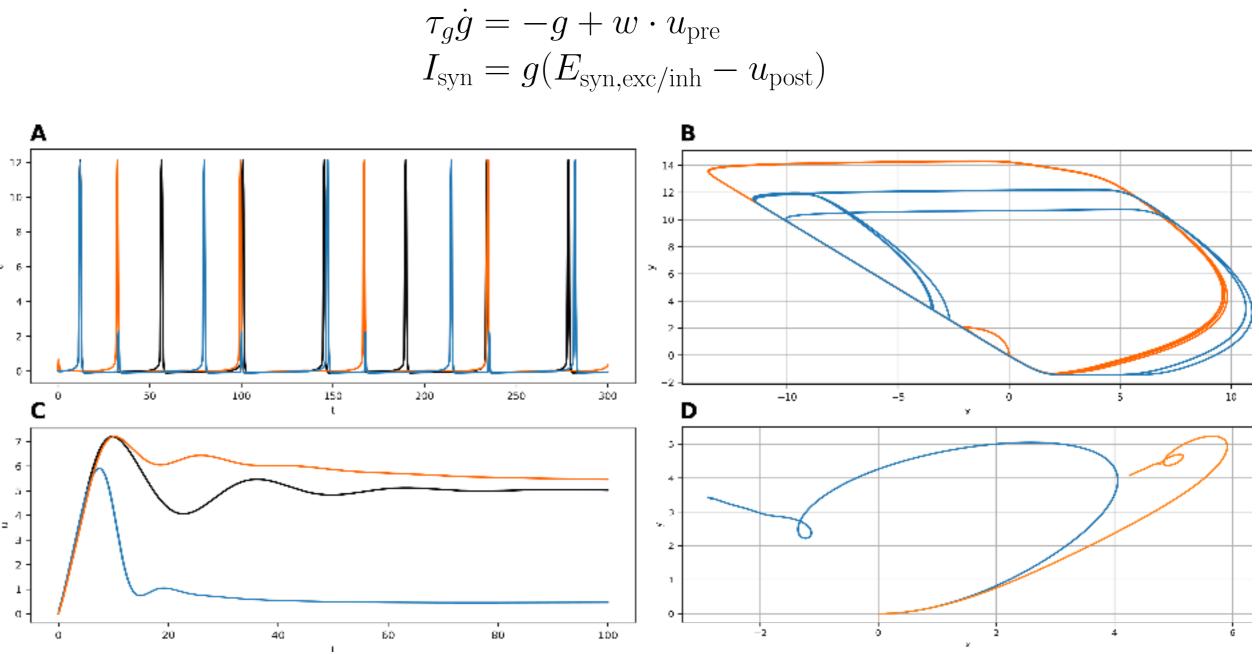


Figure 6: Dynamics of an exc.(blue) and inh.(red) neuron mutually coupled, including additional input driving the exc. neuron. Black traces in A/C represent an isolated reference neuron driven by the same external input A/B: The network in its spiking state. C/D: Rate-encoding state, same coupling strengths.

# Conclusions

- ► The model allows for the reproduction of a wide range of dynamical properties of spiking neurons.
- ► Characteristic properties such as the spike height and width can be controlled very well by certain model parameters.
- Good usability as a rate encoding model with a sigmoid activation function.
- ► Further investigations should compare how spiking and rate network dynamics map onto each other under the same synaptic coupling.
- [1] E. M. Izhikevich. *Dynamical Systems in Neuroscience*. The MIT Press, Cambridge Massachusetts, 2007.
- [2] B. P. Bean. The action potential in mammalian central neurons. *Nature Reviews, Neuroscience*, 8, June 2007.