

A Continuous-Time Dynamical System Describing both Rate Encoding and Spiking Neurons

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Introduction

- We investigated a two-dimensional nonlinear system, modeling a wide range of dynamic properties of spiking neurons.
- By altering key parameters of this system, its dynamics become identical to those of a time-continuous rate-encoding model.
- Differences of the dynamical properties of single units as well as of network structures under these two regimes can be treated within the same mathematical framework.

Neuron Model

- The model consists of a two-dimensional non-linear system given by

$$\begin{aligned}\tau_x \dot{x} &= f(x) - y \\ \tau_y \dot{y} &= g(x) - y \\ u(x, y) &:= \frac{x + y}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}f(x) &= s\sigma(a(x - s/4)) \\ g(x) &= g_0\sigma(a_g(x - I)) - \Delta_y \\ \sigma(x) &= (1 + \exp(-x))^{-1}\end{aligned}$$

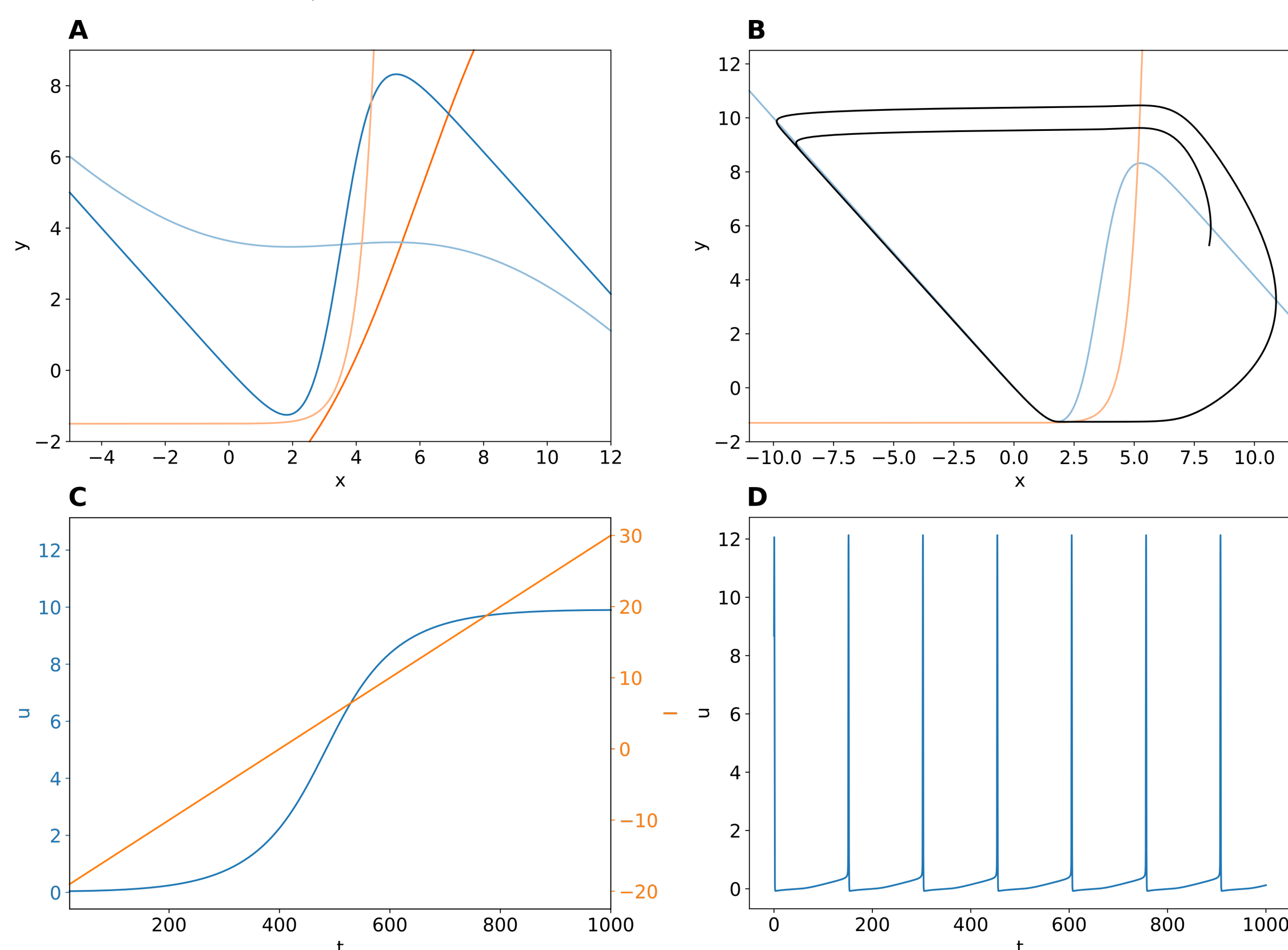


Figure 1: **A:** x/y-nullclines (blue/red) of the dynamical system for parameter sets generating spiking/non spiking behavior. **B:** Phase plane trajectory for the spiking dynamics. **C/D:** Dynamics of readout variable u for the spiking/non-spiking case.

Spiking Neuron Model

- A range of different types of spiking dynamics can be reproduced by this generic system.

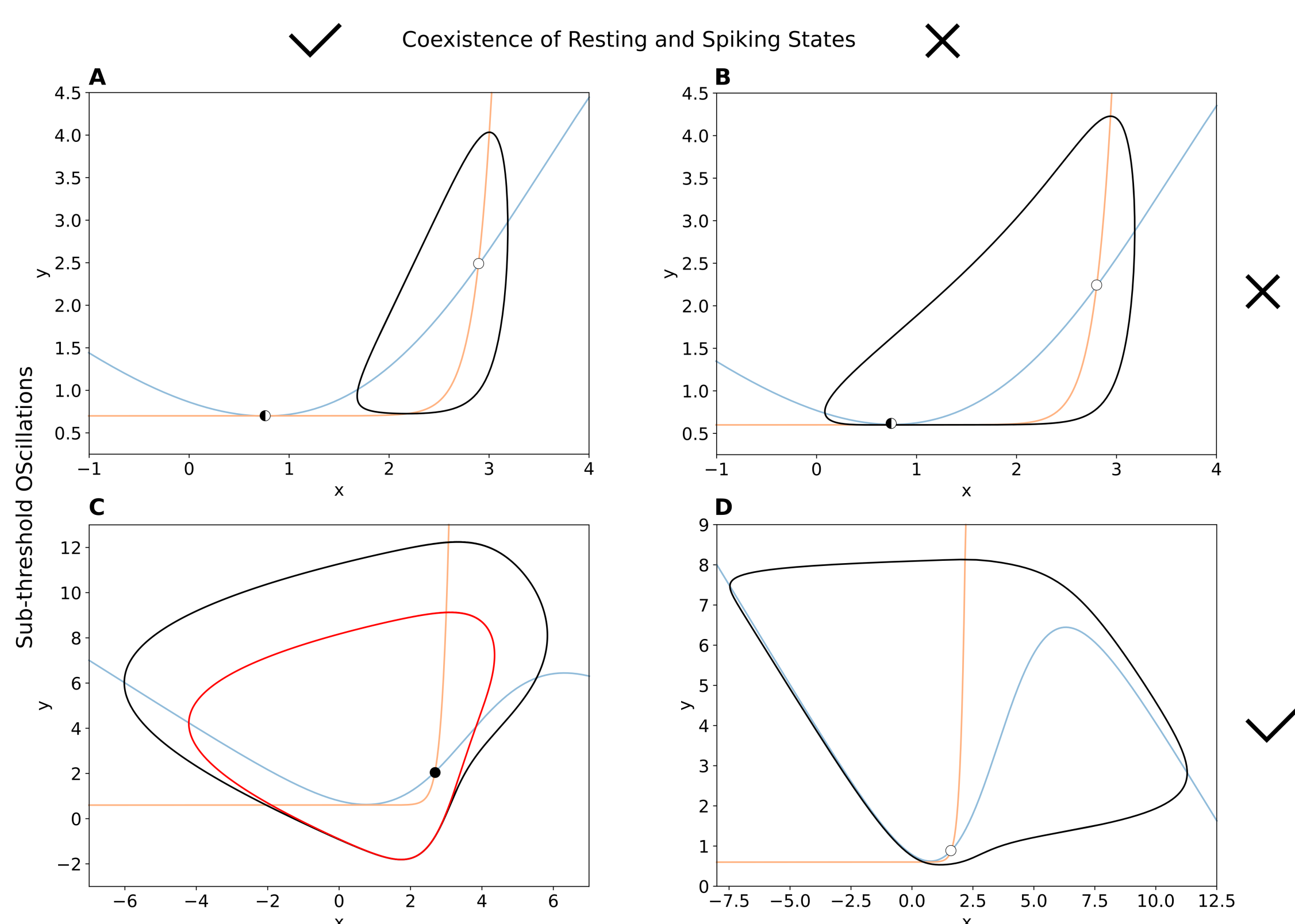


Figure 2: Examples of spiking dynamics/bifurcations observed in the model, ordered by the property of coexistence of resting/spiking states and the possibility of sub-threshold oscillations. **A:** Saddle node bifurcation with coexisting limit cycle. **B:** Saddle node b. on invariant cycle. **C:** Subcritical Hopf b/ Fold b. of limit cycles. **D:** Supercritical Hopf b.

- Based on the classification by Izhikevich [1], our model was able to generate class 1 (arbitrary small firing rates) as well as class 2 (all-or-none spiking behavior) firing patterns.
- Due to the relatively simple mathematical form of our system, we could perform analytical analyses and predictions about firing behavior closely matching the simulation results, see Fig. 3.

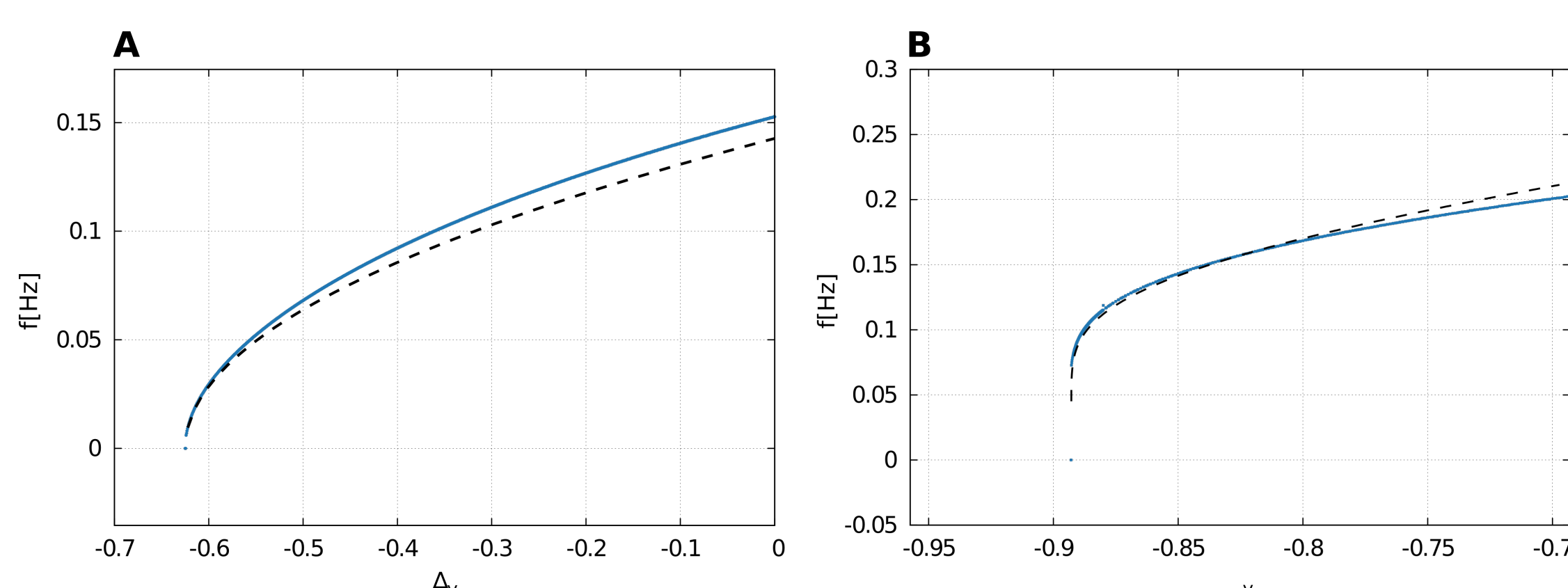


Figure 3: Measured and predicted firing rates as a function of Δ_y under the configurations shown in Fig. 2A/B.

Fitting Experimental Data

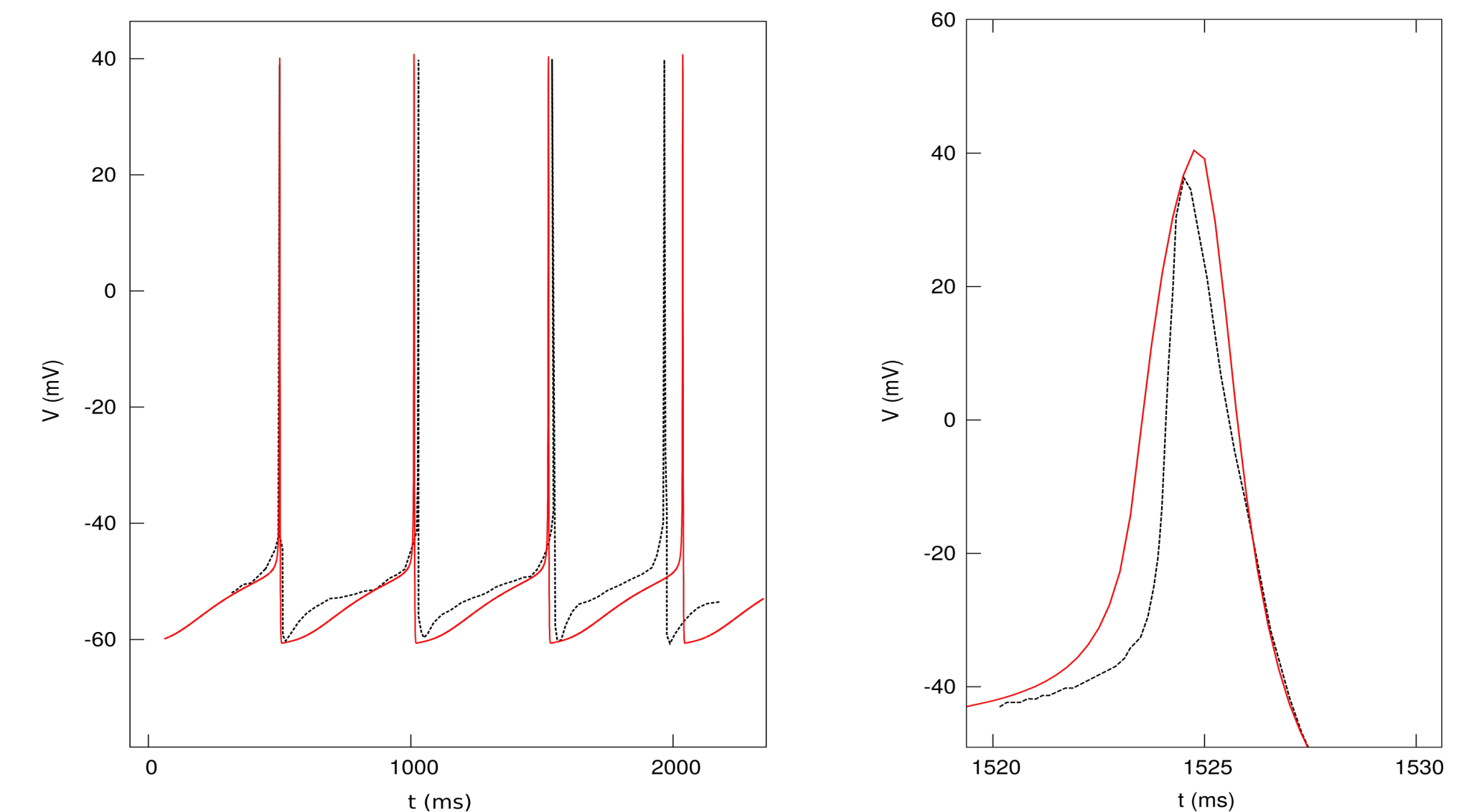


Figure 4: Fit of the dynamical system onto measurements of a dopaminergic neuron in the Substantia Nigra of a mouse brain [2].

- The parameters of the dynamical systems allow to easily control key features of the voltage trace, such as spike height, spike width and threshold potential.

$$\begin{aligned}u_{\text{spikeheight}} &\approx s/\sqrt{2} \\ \Delta t_{\text{spike}} &\propto g_0^{-1} \\ u_{\text{threshold}} &\approx 1/\sqrt{2}a\end{aligned}$$

Rate Encoding Model

- The system's parameters can be tuned such that dynamics transition from a spiking behavior to a relaxation towards an input-dependent fixed point.
- The projection of this fixed point onto u follows a sigmoidal as input increases, see Fig. 1C.

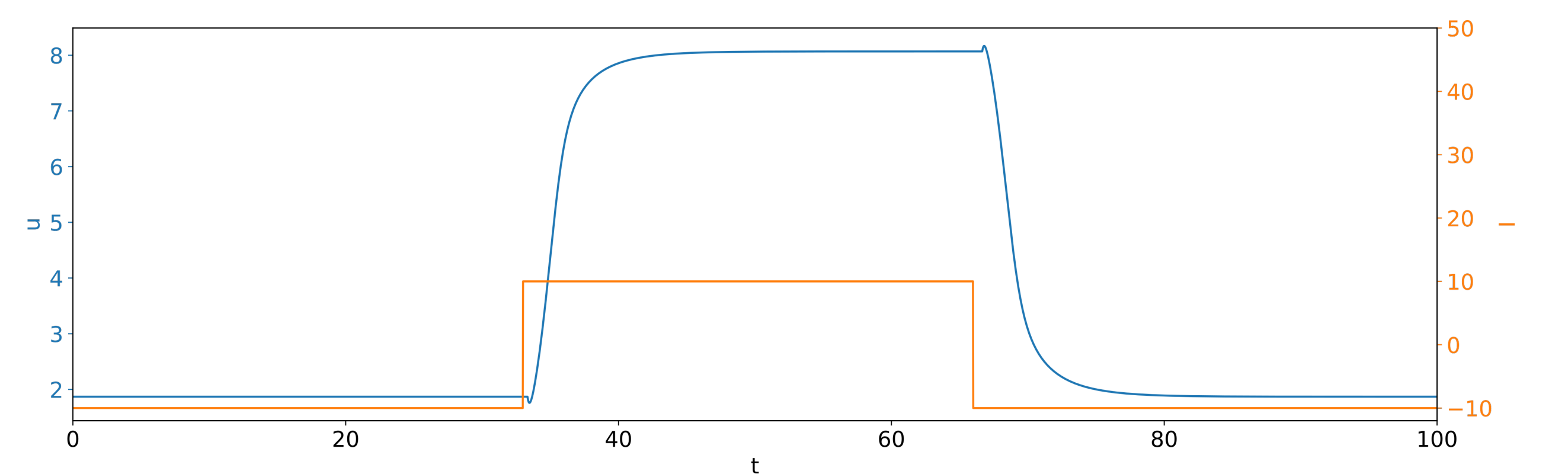


Figure 5: Rate-dynamics response of the readout variable u upon external step input.

- The dynamics of u approximately follow those of 1d-system of the form $\tau \dot{u} = -u + \frac{s}{\sqrt{2}}\sigma(c \cdot I)$. The time constant τ is then described by $\tau = -\frac{1}{\langle \lambda \rangle}$, where $\langle \lambda \rangle$ is the mean of the eigenvalues of the fixed point's Jacobian.

Usage in Networks

- Combining our model with a simple model of synaptic transmission resulted in stable network dynamics for both spiking and rate encoding dynamics.

$$\begin{aligned}\tau_g \dot{g} &= -g + w \cdot u_{\text{pre}} \\ I_{\text{syn}} &= g(E_{\text{syn,exc/inh}} - u_{\text{post}})\end{aligned}$$

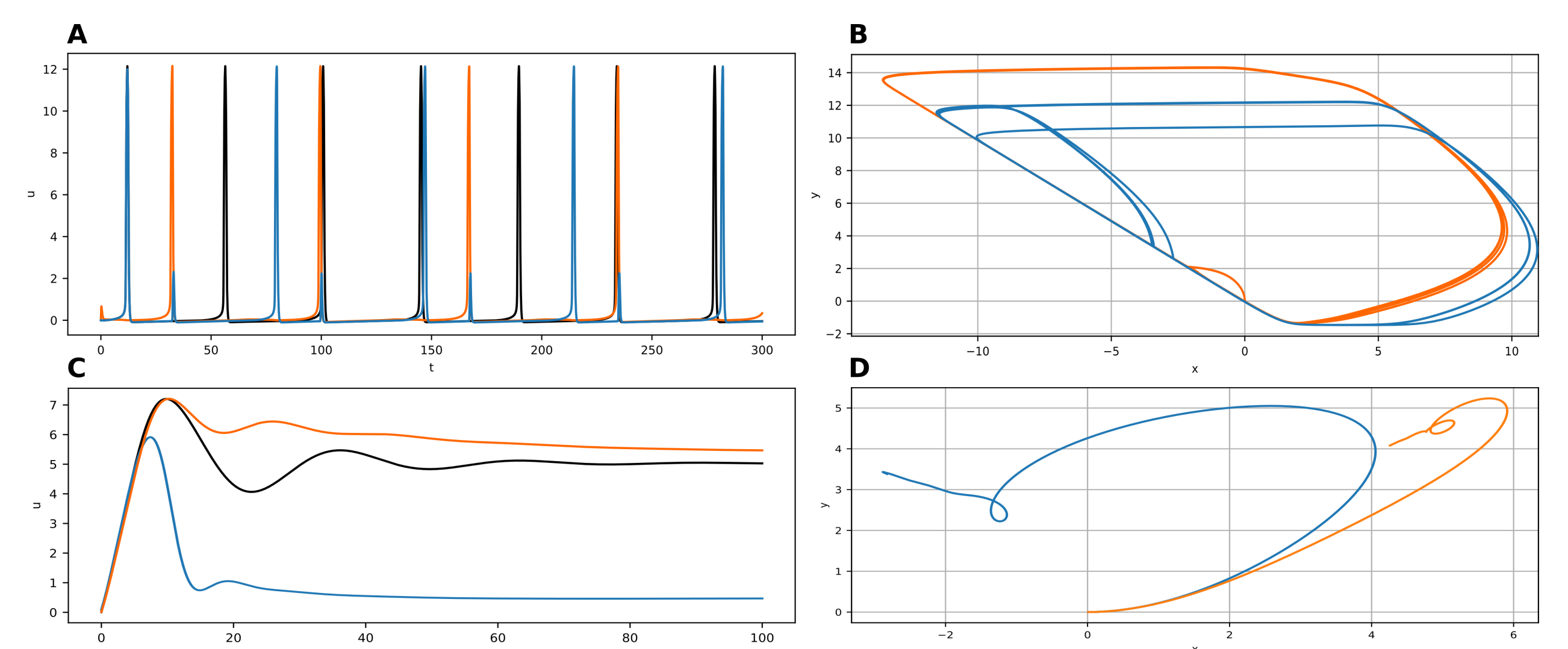


Figure 6: Dynamics of an exc.(blue) and inh.(red) neuron mutually coupled, including additional input driving the exc. neuron. Black traces in A/C represent an isolated reference neuron driven by the same external input A/B: The network in its spiking state. C/D: Rate-encoding state, same coupling strengths.

Conclusions

- The model allows for the reproduction of a wide range of dynamical properties of spiking neurons.
- Characteristic properties such as the spike height and width can be controlled very well by certain model parameters.
- Good usability as a rate encoding model with a sigmoid activation function.
- Further investigations should compare how spiking and rate network dynamics map onto each other under the same synaptic coupling.

- [1] E. M. Izhikevich. *Dynamical Systems in Neuroscience*. The MIT Press, Cambridge Massachusetts, 2007.
- [2] B. P. Bean. The action potential in mammalian central neurons. *Nature Reviews, Neuroscience*, 8, June 2007.