A Continuous-Time Dynamical System Describing both Rate Encoding and Spiking Neurons

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Ever since the electro-physiological properties of neurons have been investigated, an abundance of dynamical systems was proposed and used to explain neuronal activity. The simplest types of models that exhibit spiking behavior by means of completely differentiable dynamics are nonlinear two-dimensional oscillators with two very distinct time scales, whose dynamics are often derived by simplifying higher-dimensional systems by means of projection or separation of timescales [1, 2]. Furthermore, on an higher level of abstraction, we find neuronal models that encode information on the level of neuronal firing rates [3].

We introduce a two-dimensional nonlinear system, modeling a wide range of dynamic properties of spiking neurons. Yet, by altering key parameters of this system, its dynamics become identical to those of a time-continuous rate encoding model. Thus, our model allows for a continuous transition between spiking dynamics and a canonical form of rate encoding. Thereby, the differences of the dynamical properties of single units as well as of network structures under these two regimes can be treated within the same mathematical framework. In particular, applying this approach to recurrent networks enables us to unify concepts of irregular/regular network behavior typically associated with either of these classes of neuron models [4, 5, 6].

Furthermore, we show how a canonical form of Hebbian plasticity can be incorporated such that well-established formulations appear within the respective parameter range, spanning from spike-timing dependent plasticity to a rate-based mechanism.

Though appearing contradictory, we believe that a *unification* of different neural modeling schemes can help clarifying *differences* between their functional properties, since an analysis of the systems can take place within the same scope of qualitative and quantitative measures.

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