Notes on Homeostasis and Plasticity in a Non-binary Recurrent Network

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1 Description of the Network Model

We investigate a discrete-time, fully connected recurrent network model that includes homeostatic threshold and gain control by using the Kullback-Leibler divergence between a target distribution and neuronal output as a control measure. Standard parameters of the model are given in Table 1. Find the details of the model in the following sections.

1.1 Neuron Model

At discrete times t, each neuron i in our network is characterized by a single activity variable y_i^t. The next state is calculated by:

$$y_i^{t+1} = \sigma\left(x_i^t\right) \tag{1}$$

$$\sigma(x) = \frac{1}{1+\exp\left(-a\left(x-b\right)\right)} \tag{2}$$

$$x_i^t = x_{i,ee}^t + x_{i,eext}^t = \sum_j W_{ij} y_j^t + x_{i,eext}^t \tag{3}$$

$$\tag{4}$$

where W_{ij} is a connectivity matrix whose properties are described in the following section and $x_{i,eext}^t$ is an optional external input.

1.2 Recurrent Network Properties

W_{ij} is a fixed randomly generated matrix, whose entries were drawn from a normal distribution with mean and standard deviation given in Table 1. Autapses

Table 1: Parameters of the network

N	300
$\mathrm{E}[W]$	0
$\mathrm{Std}[W]$	$1/\sqrt{N}$
μ_b	0.001
μ_a	0.001
Initial a	1
Initial b	0

were prohibited, meaning that $W_{ii} = 0$. Note that we did not impose Dale's law onto the signs of the connections.

1.3 Gain and Threshold Control via the Kullback-Leibler Divergence

We define by

$$p_{\mathrm{t}}(y, \lambda_1, \lambda_2) \propto \exp\left(\lambda_1 y + \lambda_2 y^2\right) \tag{5}$$

a family of Gaussian distributions as a target for the neural activity. Note that λ_1 and λ_2 are related to the target mean and variance μ_t, σ_t via $\lambda_1 = \mu_t/\sigma_t^2$ and $\lambda_2 = -1/\left(2\sigma_t^2\right)$. The Kullback-Leibler divergence allows us to define a measure between the target distribution and the actual distribution—though only assessible via sampling of the output—which shall be denoted by $p_{\mathrm{s}}(y)$. The K.-L. divergence is given by

$$D_{KL}\left(p_{\mathrm{s}}||p_{\mathrm{t}}\right) = \int \mathrm{d}y\ p_{\mathrm{s}}(y) \ln\left(\frac{p_{\mathrm{s}}(y)}{p_{\mathrm{t}}(y)}\right) \tag{6}$$

$$= \int \mathrm{d}y\ q_{\mathrm{s}}(x)\left[\ln q_{\mathrm{s}}(x) - \ln \sigma'(x) - \ln p_{\mathrm{t}}(\sigma(x))\right] \tag{7}$$

where $q_{\mathrm{s}}(x)$ denotes the sampled distribution of the synaptic input or "membrane potential". The total derivative with respect to a parameter θ of our transfer function is then given by

$$\frac{\mathrm{d}D}{\mathrm{d}\theta} = -\int \mathrm{d}x\ q_{\mathrm{s}}(x)\sigma'^{-1}(x)\frac{\partial\sigma'}{\partial\theta}(x) - \int \mathrm{d}x\ q_{\mathrm{s}}(x)\frac{p_{\mathrm{t}}'(x)}{p_{\mathrm{t}}(x)}\frac{\partial\sigma}{\partial\theta}(x) \tag{8}$$

$$\equiv \int \mathrm{d}x\ q_{\mathrm{s}}(x)\frac{\partial d(x)}{\partial\theta}\ . \tag{9}$$

We then used this expression to derive on-line local adaptation rules for the gain a and threshold b by steepest decent:

$$a_i^{t+1} = a_i^t + \Delta a_i^t \tag{10}$$

$$b_i^{t+1} = b_i^t + \Delta b_i^t \tag{11}$$

$$\Delta a_i^t = -\epsilon_a \frac{\partial d(x_i^t)}{\partial a_i} = \epsilon_a \frac{1}{a_i^t}\left[1 - \ln\left(\frac{1}{y_i^t} - 1\right)\Theta_i^t\right] \tag{12}$$

$$\Delta b_i^t = -\epsilon_b \frac{\partial d(x_i^t)}{\partial b_i} = \epsilon_b\left(-a_i^t\right)\Theta_i^t \tag{13}$$

$$\Theta_i^t \equiv 1 - 2y_i^t + y_i^t(1 - 2y_i^t)[\lambda_1 + 2\lambda_2 y_i^t] \tag{14}$$

Note that, aside from a scaling factor, gain and threshold dynamics can be expressed solely in terms of output activity and target parameters. This allows us to find dynamical fixed points of a and b in terms of the output activity.

2

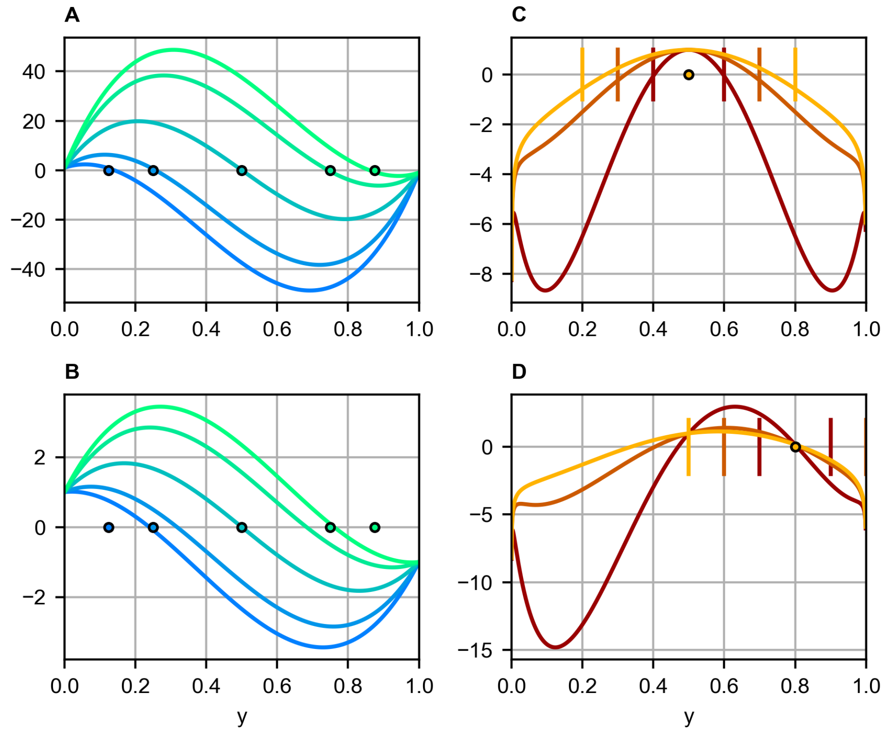

Figure 1: **A**,**B**: Θ as a function of y as given in (**??**).