

Notes on Homeostasis and Plasticity in a Non-binary Recurrent Network

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1 Initial Motivation

Recurrent neural networks are a common tool for sequence prediction tasks and have been extensively studied within the realm of machine learning [1, 2].

2 Description of the Network Model

We investigate a discrete-time, fully connected recurrent network model that includes homeostatic threshold and gain control by using the Kullback-Leibler divergence between a target distribution and neuronal output as a control measure. Standard parameters of the model are given in Table 1. Find the details of the model in the following sections.

2.1 Neuron Model

At discrete times t , each neuron i in our network is characterized by a single activity variable y_i^t . The next state is calculated by:

$$y_i^{t+1} = \sigma(x_i^t) \quad (1)$$

$$\sigma(x) = \frac{1}{1 + \exp(-a(x - b))} \quad (2)$$

$$x_i^t = x_{i,ee}^t + x_{i,ext}^t = \sum_j W_{ij} y_j^t + x_{i,ext}^t \quad (3)$$

$$(4)$$

Table 1: Parameters of the network

N	300
$E[W]$	0
$\text{Std}[W]$	$1/\sqrt{N}$
μ_b	0.001
μ_a	0.001
Initial a	1
Initial b	0

where W_{ij} is a connectivity matrix whose properties are described in the following section and $x_{i,ext}^t$ is an optional external input.

2.2 Recurrent Network Properties

W_{ij} is a fixed randomly generated matrix, whose entries were drawn from a normal distribution with mean and standard deviation given in Table 1. Autapses were prohibited, meaning that $W_{ii} = 0$. Note that we did not impose Dale's law onto the signs of the connections.

2.3 Gain and Threshold Control via the Kullback-Leibler Divergence

We define by

$$p_t(y, \lambda_1, \lambda_2) \propto \exp(\lambda_1 y + \lambda_2 y^2) \quad (5)$$

a family of Gaussian distributions as a target for the neural activity. Note that λ_1 and λ_2 are related to the target mean and variance μ_t, σ_t via $\lambda_1 = \mu_t/\sigma_t^2$ and $\lambda_2 = -1/(2\sigma_t^2)$. The Kullback-Leibler divergence allows us to define a measure between the target distribution and the actual distribution—though only assessible via sampling of the output—which shall be denoted by $p_s(y)$. The K.-L. divergence is given by

$$D_{KL}(p_s||p_t) = \int dy p_s(y) \ln \left(\frac{p_s(y)}{p_t(y)} \right) \quad (6)$$

$$= \int dx q_s(x) [\ln q_s(x) - \ln \sigma'(x) - \ln p_t(\sigma(x))] \quad (7)$$

where $q_s(x)$ denotes the sampled distribution of the synaptic input or “membrane potential”. The total derivative with respect to a parameter θ of our transfer function is then given by

$$\frac{dD}{d\theta} = - \int dx q_s(x) \sigma'^{-1}(x) \frac{\partial \sigma'}{\partial \theta}(x) - \int dx q_s(x) \frac{p'_t(x)}{p_t(x)} \frac{\partial \sigma}{\partial \theta}(x) \quad (8)$$

$$\equiv \int dx q_s(x) \frac{\partial d(x)}{\partial \theta}. \quad (9)$$

We then used this expression to derive on-line local adaptation rules for the gain a and threshold b by steepest decent:

$$a_i^{t+1} = a_i^t + \Delta a_i^t \quad (10)$$

$$b_i^{t+1} = b_i^t + \Delta b_i^t \quad (11)$$

$$\Delta a_i^t = -\epsilon_a \frac{\partial d(x_i^t)}{\partial a_i} = \epsilon_a \frac{1}{a_i^t} \left[1 - \ln \left(\frac{1}{y_i^t} - 1 \right) \Theta_i^t \right] \quad (12)$$

$$\Delta b_i^t = -\epsilon_b \frac{\partial d(x_i^t)}{\partial b_i} = \epsilon_b (-a_i^t) \Theta_i^t \quad (13)$$

$$\Theta_i^t \equiv 1 - 2y_i^t + y_i^t(1 - 2y_i^t)[\lambda_1 + 2\lambda_2 y_i^t] \quad (14)$$

Note that, aside from a scaling factor, gain and threshold dynamics can be expressed solely in terms of output activity and target parameters. This allows us to find dynamical fixed points of a and b in terms of the output activity.

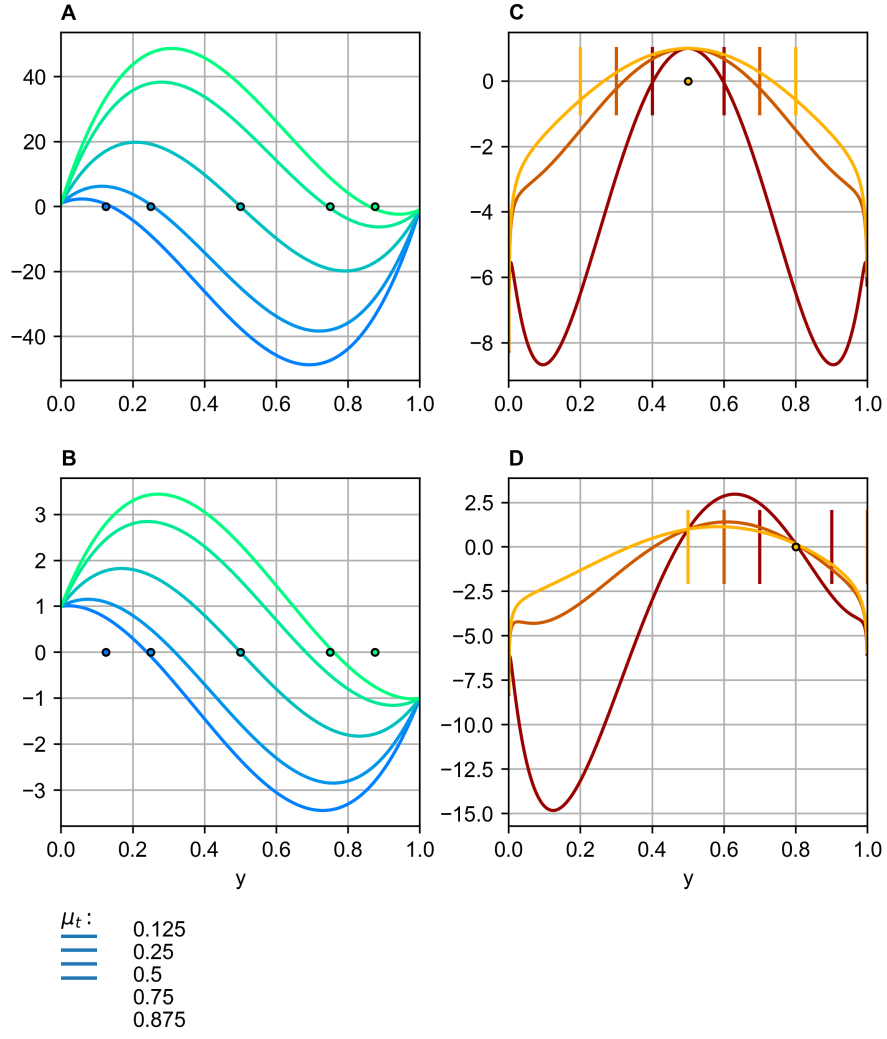


Figure 1: **A,B:** Θ as a function of y as given in (14).

References

- [1] David E. Rumelhart, Geoffrey E. Hinton, and Ronald J. Williams. Neuro-computing: Foundations of Research. chapter Learning Representations by Back-propagating Errors, pages 696–699. MIT Press, Cambridge, MA, USA, 1988.
- [2] J. L. Elman. Finding Structure in Time. *Cognitive Science*, 14(2):179–211, 1990.