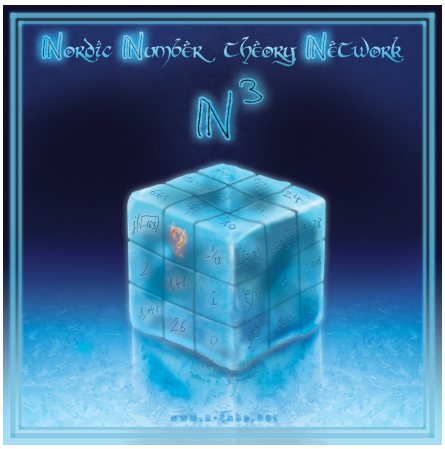
Rational points and Zariski density

University of Copenhagen,
November 15 – November 17, 2017
organised by Sara Checcoli and Fabien Pazuki,
with the support of the Niels Bohr Professorship (Denmark) and
of ANR Gardio (France).



Program

	Wednesday 15.11	Thursday 16.11	Friday 17.11
08:30-09:00	*Welcome*		
09:00-10:00	**Per Salberger**	**Tim Browning**	**David Harari**
10:00-10:30	*Coffee break*	*Coffee break*	*Coffee break*
10:30-11:30	**Damiano Testa**	**Dan Loughran**	**Aaron Levin**
11:30-12:30	*Lunch*	*Lunch*	**Emmanuel Peyre**
12:30-13:00	*Lunch*	*Lunch*	*Lunch*
13:00-14:00	**Lars Halle**	**Sho Tanimoto**	
14:10-15:10	**Aurélien Galateau**	**Tony Várilly-Alvarado**	
15:15-16:00	*Coffee break*	*Note 1*	
16:00-17:00	**Ekaterina Amerik**	*Note 2*	
18:30	*Conference Dinner*		

Welcome: Meet in Auditorium 8 to obtain your name tag and the relevant documents.

Lunch: Various options for lunch on campus.

Coffee breaks: in front of Auditorium 8.

Conference dinner: meet in the room 4.4.19 at 18:30 on Wednesday, November 15.

Note 1: Talk by Karen Vogtmann on Thursday, November 16, from 15:15 to 16:00, before receiving a Honorary Doctorate from Univ Copenhagen on Friday, November 17. Title: Spaces of graphs. Venue: Auditorium 4. You are welcome to attend!

Note 2: Reception in honor of Karen Vogtmann in the room 4.4.19. You are welcome to attend!

Abstracts

Time: Wednesday 15, 9:00-10:00.
Room: Auditorium 8.
Speaker: **Per Salberger** (Chalmers Univ. Göteborg).
Title: *Diophantine approximation and Diophantine equations.*
Abstract: To obtain quantitative results on Diophantine approximation and on Diophantine equations it is common to construct auxiliary polynomials. The aim of our talk is to explain how Chow forms and Mumford's geometric invariant theory can be used in the study of such auxiliary polynomials.

Time: Wednesday 15, 10:30-11:30.
Room: Auditorium 8.
Speaker: **Damiano Testa** (Univ. Warwick).
Title: *Plane quartics and their inflection lines.*
Abstract: Let C be a general plane quartic curve. There are two plane curves associated classically to C: another plane quartic C_4 and a plane sextic C_6 (both in the dual projective plane). The curves C_4 and C_6 meet at points corresponding to the inflection lines of C. We denote by F the rational map associating to a plane curve C the subscheme $C_4 \cap C_6$ of the plane. In joint work with Marco Pacini, we show that over fields of characteristic different from 3, the general fibers of the map F have Zariski-dense sets of rational points.

Time: Wednesday 15, 13:30-14:30.
Room: Auditorium 8.
Speaker: **Lars Halle** (Univ. Copenhagen).
Title: *Kulikov models and rational points.*
Abstract: Let K be the function field of a Dedekind scheme S, and let X be a smooth projective K-variety with trivial canonical bundle. A regular S-model of X is called a Kulikov model if it has trivial relative canonical bundle. The existence of such a model is not guaranteed in general. I will explain how Kulikov models turned out to be useful tools in the study of rational points in two recent projects (with Nicaise, resp. with Bogomolov, Pazuki and Tanimoto).

Time: Wednesday 15, 14:45-15:45.
Room: Auditorium 8.
Speaker: **Aurélien Galateau** (Univ. Franche-Comté).
Title: *Explicit versions of the Manin-Mumford conjecture.*
Abstract: The Manin-Mumford conjecture describes the distribution of torsion points in subvarieties of abelian varieties. It was proven by Raynaud thirty years ago, and some explicit versions were later given by Coleman, Buium or Hrushovski. In a joint work with César Martinez, we give uniform bounds for the distribution of torsion points with essentially sharp dependence on the geometry of the subvariety. The proof combines algebraic interpolation with a theorem of Serre on homotheties in the Galois representation associated to the torsion of abelian varieties.

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Time: Thursday 16, 9:00-10:00.
Room: Auditorium 8.
Speaker: **Tim Browning** (Univ. Bristol).
Title: *Rational curves on hypersurfaces and a geometric circle method.*
Abstract: In the light of work by Ellenberg and Venkatesh (and their collaborators), it has been known for several years that information about the cohomology groups of appropriate varieties over finite fields can be used to answer long-standing questions in analytic number theory over function fields. In an attempt to reverse the flow I will discuss the geometry of the space of rational curves on hypersurfaces using methods that herald from analytic number theory, with the outcome that in certain cases the associated compactly supported cohomology groups can be computed. This is a combination of joint work with Pankaj Vishe and forthcoming joint work with Will Sawin.

Time: Thursday 16, 10:30-11:30.
Room: Auditorium 8.
Speaker: **Dan Loughran** (Univ. Manchester).
Title: *Pseudo-split varieties and arithmetic surjectivity.*
Abstract: A classical theorem of Ax and Kochen (1965) states that every homogeneous form in sufficiently many variables admits a non-trivial p-adic zero, apart from possibly finitely many primes which only depend on the degree. This theorem was originally proved using tools from model theory, however Denef, following a strategy suggested by Colliot-Thélène, recently found a purely geometric proof that moreover gives results for more general families of varieties. In this talk we build upon Denef's work and give a criterion that completely classifies those families of varieties for which an analogue of the Ax–Kochen theorem holds, using the new notion of a "pseudo-split variety". This work is joint with Arne Smeets and Alexei Skorobogatov.

Time: Thursday 16, 13:30-14:30.
Room: Auditorium 8.
Speaker: **Sho Tanimoto** (Univ. Copenhagen).
Title: *The space of rational curves and Manin's conjecture.*
Abstract: Manin's conjecture predicts the growth rate of rational points of bounded height on Fano varieties over number fields or global function fields after removing some contribution from exceptional sets. With Brian Lehmann, we developed a study of birational geometry on exceptional sets using the minimal model program and the boundedness of log Fano varieties. In this talk, I would like to explain how the study of exceptional sets has applications to questions regarding the space of rational curves on Fano varieties. This is joint work with Brian Lehmann.

Time: Thursday 16, 14:45-15:45.
Room: Auditorium 8.
Speaker: **Ekaternia Amerik** (Univ. Paris-Sud Orsay).
Title: *On the characteristic foliation.*
Abstract: Let X be an irreducible holomorphic symplectic manifold, that is, a simply-connected manifold such that the space of holomorphic two-forms on X is generated by a symplectic form σ. If Y is a smooth hypersurface on X, the kernel of the restriction of σ defines a smooth foliation in curves on Y. One can ask when is a general leaf of this foliation Zariski-dense. If Y is uniruled it is easy to see that the leaves are rational curves. The second obvious case when the leaves are degenerate is when X admits a holomorphic lagrangian fibration and Y is the inverse image of a hypersurface on its base. I shall explain two results in this direction: the first one, joint with F. Campana, affirms that a general leaf is not an algebraic curve unless Y is uniruled, and the second one, joint with L. Guseva, concerns the case of dimension 4 and states that the general leaves are Zariski-dense unless Y is uniruled or is the inverse image of a hypersurface on the base of a Lagrangian fibration.

Time: Thursday 16, 16:15-17:15.
Room: Auditorium 8.
Speaker: **Tony Várilly-Alvarado** (Rice Univ.).
Title: *Vojta's conjecture and uniform boundedness of full-level structures on abelian varieties over number fields.*
Abstract: In 1977, Mazur proved that the torsion subgroup of an elliptic curve over $\mathbb{Q}$ is, up to isomorphism, one of only 15 groups. Before Merel gave a qualitative generalization of this

result to arbitrary number fields, it was known that variants of the *abc* conjecture would imply uniform boundedness of torsion on elliptic curves over number fields of bounded degree. In this talk, I will explain how, using Vojta's conjecture as a higher-dimensional generalization of the *abc* conjecture, one can deduce similar uniform boundedness statements for full-level structures on abelian varieties of fixed dimension over number fields. This is joint work with Dan Abramovich and Keerthi Madapusi-Pera.

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Time: Firday 17, 9:00-10:00.
Room: Auditorium 8.
Speaker: **David Harari** (Univ. Paris-Sud Orsay).
Title: *Local-global principles for tori over function fields.*
Abstract: Let K be the function field of a curve over a field k. Let T be a K-torus and let X be a principal homogeneous space of X. One discusses local-global principles for the existence of a rational point on X when k is an "arithmetic" field (p-adic field, number field).

Time: Firday 17, 10:30-11:30.
Room: Auditorium 8.
Speaker: **Aaron Levin** (Univ. Michigan).
Title: *Greatest common divisors and Diophantine approximation.*
Abstract: In 2003, Bugeaud, Corvaja, and Zannier gave an (essentially sharp) upper bound for the greatest common divisor $\gcd(a^n - 1, b^n - 1)$, where a and b are fixed integers and n varies over the positive integers. In contrast to the elementary statement of their result, the proof required deep results from Diophantine approximation. I will discuss a higher-dimensional generalization of their result and some related problems, including relations with Vojta's conjectures.

Time: Firday 17, 11:30-12:30.
Room: Auditorium 8.
Speaker: **Emmanuel Peyre** (Univ. Grenoble Alpes).
Title: *Heights and beyond.*
Abstract: Slopes à la Bost give new invariants refining heights for rational points on varieties. The aim of this talk is to describe the links between thin accumulating subsets which are obstructions to the equidistribution of rational points of bounded height and these invariants. These links are rooted in the analogies between rational points and rational curves.