Inter-generational conflict and the declining labor share*

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Abstract

The age structure of the population has received little attention as a determinant of the labor share. This paper argues that demographic dynamics in high-income countries affect the labor share in two different ways: directly through the labor supply and capital accumulation, but also indirectly through public policy. I use an OLG model in which a generational conflict arises because young and old individuals have different income sources and opposite objectives in terms of public policy. The youth face unemployment risk and use their political weight to raise the unemployment benefit, while the old favor health expenditures. This tension over the public budget allocation has consequences for wage bargaining and thus for the labor share. Numerical simulations for France and the United States indicate that the model can replicate the data and that boomers' cohorts have driven the observed decline of the labor share.

Keywords: Labor share, Inter-generational conflict, Wage bargaining, Probabilistic voting.

JEL Codes: E25, J11, J52.

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1 Introduction

The labor income share, its evolution and its distributional implications have been of interest for economists since at least the work of Kaldor (1955).[1] While initially existing evidence indicated that it remained stable for decades, several OECD countries have witnessed a decline since the beginning of the 1970s and a heated debate has emerged trying to understand the reasons for these dynamics; see, for example, Karabarbounis and Neiman (2014) and Elsby et al. (2013). After World War II, these countries also experienced significant changes in the age structure of their population following the birth of the so-called *baby-boomer* cohort. Yet the literature on the labor share has paid virtually no attention to population dynamics. In this paper, I argue that changes in population are a key element, and that, more specifically, the boomers generation drove the decline of the labor share when they were young and continue to drive its dynamics nowadays as they retire.

My argument is based on the idea that the boomers, because of the size of the cohort and hence its political weight, have driven public policy choices over the past decades, with consequences for the labor share. The idea that population dynamics can affect the labor share has been explored by Schmidt and Vosen (2013) whose model shows that an aging population leads to more savings and hence more capital. When capital and labor are gross substitutes, the accumulation of capital lowers the labor share, creating a direct mechanism that takes as given the institutional framework and public policy. Yet, public policy, and particularly labor market institutions, have undergone major changes since the 1970s. My approach hence combines this direct mechanism with an indirect policy effect that results from inter-generational conflict when choosing public policy.

I develop a two-period OLG model with young and old agents that encompasses both mechanisms. The policy effect appears when individuals vote over public policy. An age-related conflict arises because the young and the old have different income sources and

[1] Starting with Blanchard (1997) a growing literature has documented changes in the labor share. A renewed interest on its distributional consequences is largely due to Atkinson (2009), and it has been shown to be a key determinant of the distribution of personal income; see Checchi and García-Peñalosa (2010).

opposite objectives in terms of public policy. Old people receive capital income and favor government health expenditures. In contrast, the young receive wages and face unemployment risk, therefore using their political weight to raise unemployment benefits. As a large cohort of boomers enters the labor market, they vote for pro-labor institutions, and their large political weight results in an increase in the unemployment benefit. The unemployment benefit acts as an outside option in wage bargaining and allows workers to bargain higher wages. When labor and capital are gross substitutes, the representative firm shifts away from labor toward capital leading to the decline of the labor share. Once the boomers retire, the political weight of the young declines and hence pro-labor institutions dwindle. However, the consequent positive effect of employment on the labor share is dampened by the capital accumulation fostered by extensive savings of the boomers when they were young, implying a further decline. The observed shift away from labor toward capital by firms is thus a consequence of changes in labor market institutions endogenously determined by the age structure of the population, a novel mechanism that this paper is the first to identify.

My framework implies that demographic dynamics affect the labor share in two different ways. On the one hand, there is a direct impact through factor accumulation. A young generation that is larger and expects to live longer, such as the boomers, results in a higher labor supply when they are young and a larger capital stock when they retire. On the other hand, there is an indirect effect through public policy, which determines labor market institutions which in turn affect the sharing of income across factors. To determine the respective roles of the direct and indirect mechanisms, I calibrate the model for France and the United States starting in the 1970s. Among the parameters, the elasticity of substitution between capital and labor plays a crucial role in the model. While there is a substantive debate about its value in the literature (see Antràs 2004, Chirinko 2008, León-Ledesma et al. 2010, among others), I use an elasticity greater than one, in line with the literature on the labor share (see Caballero and Hammour 1998, Karabarbounis and Neiman 2014, Piketty 2015). The model is then able to replicate the data and predict future labor share dynamics.

The model predicts that over the next two decades the labor share should slightly increase in France and remain stable in the US due to the retirement of the boomers, before declining again in both countries because of the global aging of the population.

I provide several counterfactual analyses. I quantify through which channels demographic dynamics affect the labor share and find that the policy mechanism is as important as the factor-accumulation mechanism and, therefore, should not be omitted. I also quantify the effects of the two determinants of demographic changes, namely, declining population growth and increasing life expectancy. I show that the life expectancy effect is the main determinant of the demographic dynamics on the labor share when the boomers are young, accounting for 65% of the total effect for France and 74% for the US between 1980 and 2010. Once the boomers retire, the declining population growth becomes the key determinant and accounts for 72% of the impact in France and 62% in the US. I conclude by showing that the boomers are the winners of the age-related conflict despite the labor share declining when they are young because they manage to compensate their labor income losses through redistribution.

The model has important implications concerning an increase in the retirement age. The counterfactual analysis shows that any increase in the age of retirement would lead to a decline in the labor share over the next decades in France, as the current young generation would have expected to be retired longer, leading to an excess of savings and therefore an over accumulation of capital. Meanwhile agents would remain employed longer which increases the political weight of those not retired and fosters more pro-labor institutions. Both effects make firms to shift away from labor. The effect for the US is ambiguous due to the two offsetting effects. Nonetheless, increasing the retirement age would increase the labor share for both countries in the long run, once the over accumulation disappears and demographic dynamics stabilize.

This paper contributes to the literature on the determinants of the labor share. These determinants have been widely studied and debated, ranging from globalization to capital-biased technical change and changing labor market institutions.[2] In line with the two latter mechanisms, Caballero and Hammour (1998) show that pro-labor income institutions are a burden to firms because they limit their ability to optimize inputs but also because they enable workers to obtain a high income share. As a response, firms shift away from labor toward capital through biased technical change. I contribute to this literature in three ways. First, I look upstream of the key mechanism in Caballero and Hammour (1998) and reproduce it without biased technical change, but rather by endogenizing changes in labor market institutions which are determined by the age structure of the population. I hence show that demography is a key determinant of the labor share. Second, I build upon Schmidt and Vosen (2013) by taking into account the indirect policy mechanism in addition to the direct factor accumulation effect they explore. I find that this new mechanism accounts for more than half of the total effect of demographic dynamics on the labor share. Lastly, my results suggest that the boomers' generations are important drivers of the declining labor share in France and the US, a concept that has so far not been put forward.

My work also relates to the literature on the effect of the population aging on different aspects of the economy and the underlying debate about the age of retirement. This literature mainly focuses on the optimal retirement age for either economic growth or the sustainability of pension systems.[3] I contribute to this literature by providing insights on a key indicator that has never been considered by this debate, namely the allocation of income between capital and labor.

This paper is organized as follows. Section 2 presents the theoretical framework. Section

[2]Globalization can shrink the labor share due to capital openness which increases the bargaining strength of capital (Jayadev 2007, Young and Tackett 2018) or incites competing countries to adopt liberalizing labor market reforms (Pica 2010). Autor et al. (2020) argue that globalization allows superstar firms to rise within industries, and those firms are characterized by high markups and low labor share. Karabarbounis and Neiman (2014) argue that the decline of the labor share is due to the decrease in the relative price of investment goods which gives incentives to firms to shift away from labor toward capital through capital-biased technical change; see also Acemoglu (2002) and Acemoglu (2003). Blanchard (1997) argues that labor share dynamics are driven by labor market institutions that play a role in the wage bargaining; see also Bentolila and Saint-Paul (2003) and Bental and Demougin (2010).

[3]For the effect of the retirement age on the economic growth, see Futagami and Nakajima (2001), Soares (2005), Lee and Mason (2010), Gonzalez-Eiras and Niepelt (2012). For its effect on the sustainability of pension systems, see de la Croix et al. (2013), Dedry et al. (2017).

3 provides the quantitative analysis. Section 4 discusses some results of the paper. Section 5 concludes.

2 Theoretical framework

I consider a two-period OLG model in which there are two types of households: young and old. Both types vote each period to determine the tax rate, the unemployment benefits and the health expenditure to set the public policy equilibrium. The inter-generational conflict arises because young and old households have different preferences in terms of public policy; the former is in favor of more unemployment benefits while the latter prefers more health expenditure. At the same time, the representative union and the representative firm bargain over wages, which determines the labor market equilibrium. I assume that voting and bargaining are independent and simultaneous. Households cannot synchronize their vote with the action of the union or the firm, therefore, the union and the firm take the voting outcome as given while the households takes as given the bargaining outcome. These two outcomes, the vote and the bargaining decisions, hence, jointly determine the equilibrium of the economy and therefore the labor share.

2.1 Households

The population consists of N_t^y young and N_t^o old individuals. Demographic dynamics are given by $N_t^y = n_t N_{t-1}^y$ where $n_t > 0$ is the gross rate of population growth, and $N_t^o = p_t N_{t-1}^y$ with $p_t \in (0, 1]$ being the survival rate. The survival rate p_t is an increasing function of the life expectancy and a decreasing function of the retirement age.[4] Both demographic parameters are exogenous and may vary over time. Their variations will generate population dynamics. Thus, the old-age dependency ratio is $N_t^o/N_t^y = p_t/n_t$.

[4] In the model, agents are considered as old once they retire. If the life expectancy and the retirement age grow at the same rate, then the survival rate remains constant. For more details on the measurement of population aging, see Sanderson and Scherbov (2006); Sanderson and Scherbov (2013); D'Albis and Collard (2013).

Each cohort consists of a continuum of agents with identical preferences. Households have logarithmic utility functions and derive utility from consumption. Young households discount the future at factor $\alpha \in (0, 1)$. They face an idiosyncratic longevity risk: with probability p_{t+1} they survive and become old households in period $t + 1$. Due to risk of death, the effective discount factor of young households equals αp_{t+1}. In period t, young households supply labor inelastically and earn a disposable income y_t that they allocate between consumption $c_{1,t}$ and savings s_t. Once old, they receive the net return of their savings $(1 - \tau_{t+1})s_t\hat{R}_{t+1}$, where τ_{t+1} is the tax rate and $\hat{R}_{t+1}$ the gross return on savings of a young household that survives to old age. I suppose a perfect annuities market where savings of young agents who die before becoming old are distributed among their surviving peers. Due to the perfect annuities market $\hat{R}_t = R_t/p_t$ where R_t is the gross return on physical capital. Old households allocate all their capital income to consume $c_{2,t+1}$ and also derive utility from public health spending h_{t+1}. Lastly, old households die at the end of period $t + 1$.

Maximizing expected utility, a household in period t solves the following maximization problem:

$$\begin{aligned}
\max_{c_{1,t},\ c_{2,t+1}} \quad & \ln c_{1,t} + \alpha p_{t+1} \left(\ln c_{2,t+1} + \beta \ln h_{t+1} \right) \\
\text{s.t.} \quad & c_{1,t} + s_t = y_t, \\
& c_{2,t+1} = (1 - \tau_{t+1}) s_t \hat{R}_{t+1},
\end{aligned}$$

where $\beta > 0$ characterizes the preference for health expenditure. The first period disposable income y_t depends on the employment situation of the household. Each young household faces an idiosyncratic unemployment risk with probability $u_t \in [0, 1)$. An employed household earns a net wage $y_t^e = (1 - \tau_t)w_t$ where w_t is the wage rate, while an unemployed one gets the unemployment benefit $y_t^u = b_t$ where b_t are the unemployment benefits.

Solving the household's maximization problem leads to the optimal consumption in both periods and savings in first period, which are

$$c_{1,t} = \frac{1}{1+\alpha p_{t+1}} y_t, \tag{1}$$

$$c_{2,t+1} = \frac{\alpha p_{t+1}}{1+\alpha p_{t+1}} (1-\tau_{t+1}) \hat{R}_{t+1} y_t, \tag{2}$$

$$s_t = \frac{\alpha p_{t+1}}{1+\alpha p_{t+1}} y_t. \tag{3}$$

Since the utility function is logarithmic, savings are a constant proportion of disposable income. Aggregate savings in the economy are the weighted average of all disposable incomes of the young such that

$$S_t = \frac{\alpha p_{t+1}}{1+\alpha p_{t+1}} \Big[(1-u_t)(1-\tau_t) w_t + u_t b_t \Big] N_t^y. \tag{4}$$

I assume that capital fully depreciates between two periods. Thus, equation (4) determines the capital stock next period so that $K_{t+1} = S_t$. This assumption also implies that the gross return on physical capital is equal to the rental rate, i.e. $R_t = r_t$.

2.2 Production

We consider a representative firm with a standard CES production function given by

$$Y_t = A \left[\phi K_t^{\frac{\sigma-1}{\sigma}} + (1-\phi) L_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where K_t is the capital stock, L_t labor, σ the elasticity of substitution between capital and labor, ϕ the factor share parameter capturing the relative importance of inputs in production to vary and A a scale parameter. Rewriting the production function in per-worker terms, we have

$$\frac{Y_t}{L_t} = A \left(\phi k_t^{\frac{\sigma-1}{\sigma}} + 1 - \phi \right)^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where $k_t \equiv K_t/L_t$ is the capital-per-worker. The labor-demand obtained from profit maximization by the representative firm is

$$w_t = (1-\phi)A\left(\phi k_t^{\frac{\sigma-1}{\sigma}} + 1 - \phi\right)^{\frac{1}{\sigma-1}}. \tag{7}$$

The labor share is defined as the ratio between the wage rate and output-per-worker, i.e. $\theta_t \equiv w_t L_t/Y_t$. Using equations (6) and (7), the labor share is given by

$$\theta_t = \left(1 + \frac{\phi}{1-\phi} k_t^{\frac{\sigma-1}{\sigma}}\right)^{-1}. \tag{8}$$

Note that when the capital-labor elasticity of substitution equals unity, then the labor share is constant, i.e. $\theta_t = 1 - \phi$. For the remaining of the paper I do not treat this trivial case.[5] From equation (8), we can also define the labor-to-capital income ratio as

$$\Theta_t \equiv \frac{\theta_t}{1-\theta_t} = \frac{1-\phi}{\phi} k_t^{\frac{1-\sigma}{\sigma}}. \tag{9}$$

The comparative statics of these expressions are straightforward. A higher capital-per-worker increases the wage and output-per-worker, i.e. $\partial w_t/\partial k_t > 0$ and $\partial(Y_t/L_t)/\partial k_t > 0$. However, the impact on the labor share depends on the elasticity of substitution between both factors, with $\partial\theta_t/\partial k_t \lesseqgtr 0$ if $\sigma \gtreqless 1$. In order to have a negative relationship between the capital-per-worker and the labor share without biased technical change, both input factors have to be gross substitutes, i.e. $\sigma > 1$. In such a case, any rise of the capital-per-worker leads to a higher wage that is outweighted by the increase in output-per-worker. Thus, the labor share declines along with the labor-to-capital income ratio.

[5] Neither the cases where the production function is Leontief ($\sigma \to 0$) or linear ($\sigma \to +\infty$).

2.3 Public policy

Government. The government taxes the labor income of the young and saving returns of the old at the same tax rate.[6] The revenue generated from these taxes is allocated between unemployment benefits and health spending. Therefore, the government budget constraint is $\tau_t \Big[w_t(1-u_t)N_t^y + R_t S_{t-1} \Big] = b_t u_t N_t^y + h_t N_t^o$. Since the expression between square brackets corresponds to the total income in the economy Y_t, I rewrite the government budget constraint as

$$\tau_t Y_t = b_t u_t N_t^y + h_t N_t^o. \tag{10}$$

Everything else equal, both types of agents prefer lower taxes as they reduce their disposable income. The youth prefer more unemployment benefits since they face unemployment risk, while the elderly want more health spending because they derive utility from it. Recent empirical evidence shows that people change their public spending preferences over their life cycle (e.g. Busemeyer et al. 2009; Sørensen 2013), which reflects a form of age-related selfishness in public spending preferences.[7]

I model the inter-generational conflict over the public budget allocation with this trade-off between unemployment benefits and health spending for two reasons. First, although health spending is also for the young, the magnitude with respect to the one for old people is much lower.[8] It can be interpreted as an old-age specific health expenditure. Second, replacing health expenditure with pensions would indeed allow for an old-age specific policy instrument. Nonetheless, it would reduce the tractability of the model without any substan-

[6] I consider a common tax rate to simplify the analysis. Young and old agents, both prefer a lower tax rate as it reduces their disposable income. By introducing different labor and capital income tax rates, I would have two sources of inter-generational conflict, adding complexity to the voting process but without providing additional insights.

[7] Sørensen (2013) shows that elderly people desire less spending in education while they are in support of more in health expenditure and pensions. Busemeyer et al. (2009) find sizable age-related differences in public policy preferences. Although these studies disagree on the magnitude of the conflict, they both claim that such a conflict does exist.

[8] Papanicolas et al. (2020) show that the US average per-capita health expenditure in 2015 is about three times larger for individuals above 65 with respect to those between 20 and 64. They also find an average ratio about 3.14 for a sample of 8 OECD countries (excluding the US).

tial gain in the analysis.[9] To summarize, the model can be extended to other types of policy instrument for the old as long as they derive more utility from it than the young, either directly or through their income. The central point is to oppose young and old agents with different returns to policy instruments in utility terms.

Indirect utility. In order to determine their preferred public policy, agents maximize their indirect utility function. Using the first order conditions from the household maximization problem in equations (1), (2) and (3), we obtain:

$$U_t^{y,i} = \ln\left[\frac{1}{1+\alpha p_{t+1}} y_t^i\right] + \alpha p_{t+1} U_{t+1}^{o,i}, \tag{11}$$

$$U_t^{o,i} = \ln\left[\frac{\alpha p_t}{1+\alpha p_t}(1-\tau_t) y_{t-1}^i \hat{R}_t\right] + \beta \ln h_t, \tag{12}$$

where $U_t^{y,i}$ is the indirect utility of a young household at time t in employment status $i \in \{e, u\}$ and $U_t^{o,i}$ is the indirect utility of an old household at time t who was in employment status i in the previous period. Thus, indirect utilities depend on the first period disposable income, y_t^i, and therefore the employment status.[10]

The assumptions on the timing of the model imply that the young vote before their employment status is revealed. They hence vote on the basis of their expected income, corresponding to the weighted average of both potential incomes. Therefore, the expected indirect utility of a young individual at time t is

$$\mathbb{E}(U_t^y) = \ln\left(\frac{\mathbb{E}(y_t)}{1+\alpha p_{t+1}}\right) + \alpha p_{t+1}\left[\ln\left(\frac{\alpha p_{t+1}}{1+\alpha p_{t+1}}(1-\tau_{t+1})\mathbb{E}(y_t)\hat{R}_{t+1}\right) + \beta \ln h_{t+1}\right], \tag{13}$$

[9] Pensions would introduce the policy instrument within the budget constraint of the old rather than directly in the utility function. From the point of view of the *indirect policy mechanism*, the elderly would still desire more of this instrument. On the side of the *direct factor-accumulation mechanism*, Schmidt and Vosen (2013) reach the same conclusions about the direct effect of aging on the labor share by considering an exogenous pension system. Moreover, additional assumptions would be required about the type of pension system, i.e. pay-as-you-go vs fully funded pension system.

[10] Implicitly, public policy preferences are functions of the economic environment when the individuals are young. In line with the literature on preferences for redistribution, Giuliano and Spilimbergo (2013) show that individuals growing in recession tend to have greater preferences for redistribution; see also Alesina and Giuliano (2011) for a general review of this literature. However, in this model, such a link is canceled by the logarithmic form of the utility function. For instance, the partial derivative of the indirect utility of the old with respect to either τ_t or h_t does not contain the disposable income of the previous period y_{t-1}^i.

where $\mathbb{E}(y_t) = (1-u_t)(1-\tau_t)w_t + u_t b_t$, with $\mathbb{E}$ being the expectation operator. In contrast, the old do not vote on the basis of expected income because there is no uncertainty about the returns of their savings.

Voting. I consider a probabilistic voting setup.[11] With probabilistic voting, all agents vote for a policy platform $g_t = (\tau_t, b_t, h_t)$ represented by opportunistic candidates (or parties). Candidates try to maximize their probability of winning the election. They differ in their popularity and there is an idiosyncratic bias among voters for one candidate or the other. Candidates know about these biases. In equilibrium, all candidates choose the same policy platform $g_t^\star$ that maximizes the political objective function $W_t(g_t)$ defined below. See Lindbeck and Weibull (1987) for more details on the probabilistic-voting setup.

The political objective function depends on the share of each group of voters in the population and their respective sensitivity to policy changes ω^j with $j \in \{y, o\}$, where ω^j denotes the density parameter of the uniform distribution function that characterizes the ideology of the j group. The greater ω^j, the more spread are the ideologies within the j group. Hence, opportunistic candidates prefer targeting less ideological groups, i.e. large ω^j, because they are easier to convince. There are three groups of voters: young households; and old households who are divided in two subgroups according to their employment situation when young. I assume all elderly have the same sensitivity regardless of their employment situation when they were young. The equilibrium public policy $g_t^\star$ maximizes the following political objective function:

$$W_t(g_t) = \frac{N_t^y}{N_t}\omega^y \mathbb{E}\left[U_t^y(g_t)\right] + \frac{N_t^o}{N_t}\omega^o \Big\{ u_{t-1} U_t^{o,u}(g_t) + (1-u_{t-1}) U_t^{o,e}(g_t) \Big\},$$

subject to the government budget constraint from equation (10), where $\mathbb{E}\left[U_t^y(g_t)\right]$ and $U_t^{o,i}(g_t)$ are respectively defined by equations (12) and (13).

[11] The alternative would be a median voter setup. However, the median voter setup would create two extreme regimes with one of them being a gerontocracy. It would also generate large swings in public policy if the median-voter switches from young to old or vice versa. Under probabilistic voting, the equilibrium policy platform is a continuous function of the old-age dependency ratio.

Voting takes place at the same time as the wage bargaining and there is no coordination between both. Therefore, households only care about direct effects of public policy on their utility. They do not consider the indirect effects operating through unemployment, wages and the accumulation of capital. Let $\tilde{U}_t^i$ be the part of the utility which is directly affected by the public policy platform. From equation (12), we have that $\tilde{U}_t^o = \tilde{U}_t^{o,u} = \tilde{U}_t^{o,e}$. Hence, I rewrite the political objective function as

$$W_t(g_t) = \frac{N_t^y}{N_t}\omega^y \mathbb{E}\left[\tilde{U}_t^y(g_t)\right] + \frac{N_t^o}{N_t}\omega^o \tilde{U}_t^o(g_t) + \textit{other terms}$$

where *other terms* encompasses all the terms that are not directly affected by public policy.

Let ω be the *relative ideological spread-out* of the youth with respect to the elderly. The relative ideological spread-out is characterized by the ratio of the sensitivities of voting behavior to policy changes for each group, i.e. $\omega \equiv \omega^y/\omega^o$. I assume this spread-out is constant over time.[12] Using equations (12) and (13), I rewrite the maximization program that characterizes the public policy equilibrium as

$$\max_{\tau_t, b_t, h_t} W_t(\tau_t, b_t, h_t) = \eta_t \ln\left[(1-u_t)(1-\tau_t)w_t + u_t b_t\right] + \ln(1-\tau_t) + \beta\ln(h_t) + \textit{other terms}$$

subject to the government budget constraint from equation (10), where

$$\eta_t = \frac{n_t}{p_t}\omega(1+\alpha p_{t+1}) \tag{14}$$

is the *political weight of the young*. This political weight is the key variable in the model because it is the channel through which the age structure affects public policy. It depends negatively on the old-age dependency ratio p_t/n_t. As expected, the older the population, the lower the political weight of the young in policy determination. On the contrary, it depends

[12] This assumption can be interpreted in two ways: either both relative ideological spread-outs are time invariant or they vary in same proportions. It would be interesting to consider these spread-outs as endogenous or to make them cohort-specific. This goes beyond the scope of this paper.

positively on the relative ideological spread-out ω. The less ideological are the youth, the higher is its political weight because it is easier for the opportunistic candidates to get their votes with an appropriate public policy. As a consequence, candidates pay more attention to them. The political weight of the young is also increasing in the effective discount factor αp_{t+1}. This term appears because public policy at time t also affects the future income of the young generation once they retire.

Partial equilibrium. Focusing on the interior solution of the maximization program, the first order conditions lead to the following public policy preferences:

$$X_t \equiv \frac{b_t}{(1-\tau_t)w_t} = \frac{1-u_t}{u_t}\left(\frac{\eta_t}{\Theta_t} - 1\right), \tag{15}$$

$$\tau_t = 1 - \Big[(1-\theta_t)\,(1+\beta+\eta_t) \Big]^{-1}, \tag{16}$$

$$h_t N_t^o = \frac{\beta}{1+\beta+\eta_t} Y_t, \tag{17}$$

where equation (15) defines the net replacement rate in unemployment, X_t, equation (16) gives the tax rate and equation (17) the aggregate health expenditure. Comparative statics are straightforward. The young generation desires more redistribution and a higher net replacement rate in unemployment because they face an unemployment risk, i.e. $\partial\tau_t/\partial\eta_t > 0$ and $\partial X_t/\partial\eta_t > 0$. Conversely, this generation desires less government health spending because they do not derive any utility from it yet, i.e. $\partial h_t/\partial\eta_t < 0$.[13]

Redistribution. The aggregate net income of young households can be defined as $Y_t^y = [(1-u_t)(1-\tau_t)w_t + u_t b_t]\,N_t^y$. Using equations (15) and (16), I rewrite it as a share of the total income such that $Y_t^y/Y_t = \eta_t/(1+\beta+\eta_t)$. For a given level of total income Y_t, the comparative statics indicate that when the political weight of the young raises, they increase their income share through more redistribution i.e. $\partial(Y_t^y/Y_t)/\partial\eta_t > 0$. Conversely, the

[13] I do not consider any form of explicit altruism in the model. However, the parameter β which is the preference for government health spending captures a form of implicit altruism from young to their elders. The greater the parameter, the more individuals care about government health spending once old. Lastly, a form of explicit altruism from young to old generations would simply soften the age-related conflict without reversing its outcome.

income share of the elderly shrinks and so does the health spending, i.e. $\partial(Y_t^o/Y_t)/\partial\eta_t < 0$ and $\partial(h_t N_t^o/Y_t)/\partial\eta_t < 0$. Furthermore, it is possible to express the after-tax income ratio between young and old households as

$$\frac{Y_t^y}{Y_t^o} = \eta_t. \tag{18}$$

As expected, the greater is the political weight of the young, the greater is the redistribution toward them.

2.4 Wage bargaining

A representative union bargains over the wage rate with the representative firm. I consider a "right-to-manage" model *à la* Nickell and Andrews (1983). The union bargains over wages and the employer retains the prerogative to hire and fire.[14] Consequently, the firm is always on its labor demand curve and equation (7) holds. The union maximizes the workers' utility compared to the unemployed, i.e. $L_t\,(U_t^{y,e} - U_t^{y,u})$, while the firm maximizes its outside option which is $Y_t - w_t L_t$. Thus, I define the maximization program that characterizes the labor market equilibrium as

$$\max_{w_t > 0} \quad \Big[L_t(U_t^{y,e} - U_t^{y,u})\Big]^{\gamma}\Big[Y_t - w_t L_t\Big]^{1-\gamma},$$

subject to

$$U_t^{y,e} - U_t^{y,u} = (1 + \alpha p_{t+1})\ln\left[\frac{(1-\tau_t)w_t}{b_t}\right],$$

and the labor demand from equation (7), where $\gamma \in (0,1)$ is the relative bargaining power of the union.

Solving the maximization program, the labor market equilibrium wage is implicitly de-

[14]Another possibility would have been to consider an "efficient contract" model *à la* McDonald and Solow (1981) where the union bargains over wages and employment. However, this specification does not fit well the data as Bentolila and Saint-Paul (2003) showed. Moreover, it would add a lot of complexity to the model without a substantial gain in the analysis.

fined by

$$X_t = \exp\left(-\frac{1}{\sigma + \tilde{\gamma}\Theta_t}\right), \tag{19}$$

where $\tilde{\gamma} \equiv (1-\gamma)/\gamma + \sigma > 0$, X_t is the net replacement rate in unemployment and Θ_t the labor-to-capital income ratio. Differentiating this equation, I obtain $d\Theta_t/dX_t > 0, \forall\sigma$. Hence, the labor-to-capital income ratio increases along with the net replacement rate in unemployment *regardless of* the value of the elasticity of substitution between labor and capital σ. When the replacement rate increases, so does the workers' outside option. As a result, the union bargains a greater wage and workers get a larger share of the rents. Therefore, the right-to-manage specification of wage bargaining is able to replicate the partial equilibrium effect in Caballero and Hammour (1998). Yet, this partial equilibrium does not take into account the response of the firm, i.e. the adjustment of the labor demand. When both input factors are gross substitutes ($\sigma > 1$), the firm will shift away from labor toward capital to thwart workers appropriation of the rents, i.e. the increase of labor costs.

2.5 Equilibrium

Equilibrium is defined by the two equations giving a relationship between the net replacement rate in unemployment, X_t, and the labor-to-capital income ratio, Θ_t, namely

$$X_t = \frac{1-u_t}{u_t}\left(\frac{\eta_t}{\Theta_t} - 1\right),$$
$$X_t = \exp\left(-\frac{1}{\sigma + \tilde{\gamma}\Theta_t}\right),$$

where the first one is the outcome of the voting and the second one is the bargaining outcome we just obtained. These two equations yield a solution, in the equilibrium capital-per-worker k_t, to the following equation:

$$\ln\left(\frac{\frac{N_t^y}{K_t}k_t - 1}{\frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\eta_t - 1}\right) = \left(\sigma + \tilde{\gamma}\frac{1-\phi}{\phi}k_t^{\frac{1-\sigma}{\sigma}}\right)^{-1}. \tag{20}$$

The aggregate capital stock K_t being predetermined by the savings of the previous period, k_t is only determined by labor L_t. In appendix A, I show that the equilibrium is unique. However, the dynamics are ambiguous and cannot be derived analytically without additional assumptions on parameters values, as detailed in appendix B. Thus, I turn to quantitative analysis.

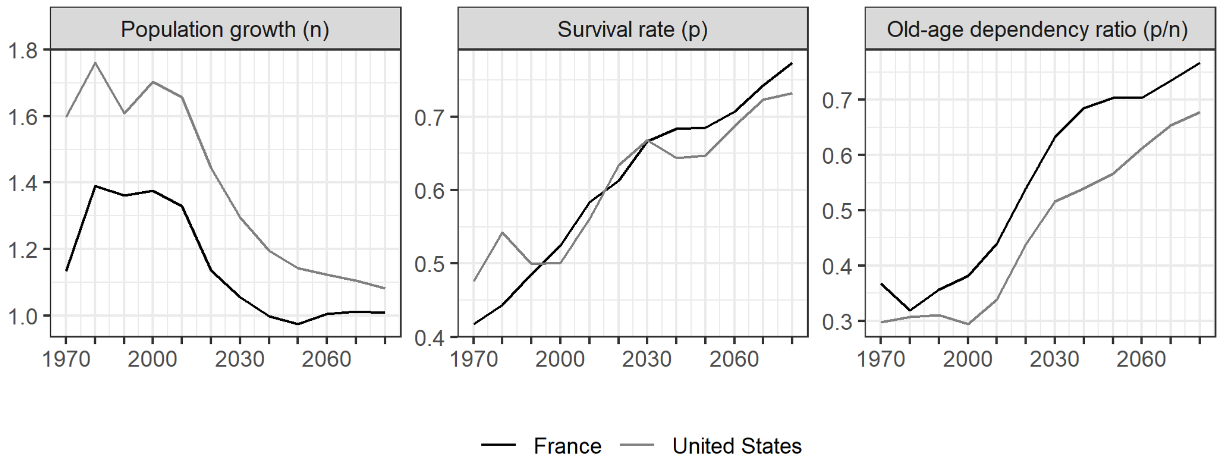
3 Quantitative analysis

This section presents the quantitative analysis of the model with three main objectives: to reproduce the labor share dynamics observed over the period 1970 to 2010; to provide model predictions after 2010; and to understand the transmission channels of demographic effects on the labor share. I compute model predictions for France and the United States. I focus on these two countries because they face important changes in the age structure of the population due to the emergence of the boomers' generation, while having sizeable differences in terms of institutions and public policy such as health and unemployment expenditure.

To simulate the model, I follow the methodology of Gonzalez-Eiras and Niepelt (2012). One period in the model is assumed to correspond to 40 years in the data. Thus, households are considered as young between 20 and 60 years of age and as old thereafter.[15] I compute four sequences of model predictions with a period length of 40 years each. Periods of the first sequence corresponds to 1970, 2010, 2050; for the second sequence to 1980, 2020, 2060; for the third sequence to 1990, 2030, 2070; and for the fourth sequence to 2000, 2040, 2080. When I report time series predictions, I list these four sequences in a single time series. Thus, there are always eight generations living simultaneously, four of them being young and the four others old. Every 10 years, a new generation is born and an old one dies.

[15] An implicit assumption of the model is that the retirement age is constant, see section 4 for further discussion. The average French effective retirement age was 67.8 in 1970 and has declined to 59.3 in 2010. In the US it has gone from 68.4 to 65.6 over the same period (data from the OECD Database, Ageing and Employment Policies - Statistics on average effective age of retirement). I suppose, as an approximation, that agents retire at 60 years-old to match the period lengths of the calibration. Such an assumption should not affect the voting outcome because almost-retired agents may anticipate their future situation once they vote. Nonetheless, a 5-year change remains moderate compared to the 40 years between two periods.

Figure 1: Demographic variables: Time trends.



Notes: The figure displays, for each country every 10 years, the population growth rate, the survival rate and the old-age dependency ratio. Data are from the United Nations World Population Prospects 2017. Projections correspond to the "medium variant" estimates from the same source.

3.1 Data

Demography. I use demographic data from the United Nations World Population Prospects 2017.[16] I start by computing the old-age dependency ratio from the data as the number of old individuals divided by the number of young ones. Then, I compute the population growth rate using the ratio between the number of young individuals relative to the number of young people in the previous period of the sequence, i.e. $n_t = N_t^y / N_{t-1}^y$. Lastly, the survival rate verifies the identity and equals the product of the old-age-dependency ratio and the population growth, i.e. $p_t \equiv N_t^o / N_t^y \times n_t$. Figure 1 plots demographic dynamics for France and the United States, and indicates that both countries face the same demographic context. I distinguish two eras in terms of dynamics which correspond to the life cycle of the boomers' generation: when the boomers are young between 1970 and 2010; when they retire and thereafter. Until 2010, the old-age dependency ratio remains roughly stable due to the massive entry of the boomers in the labor force that offsets the rise in the survival rate due

[16]Demographic data from 1950 to 2010 come from the United Nations World Population Prospects 2017. For future dynamics, I rely on the "medium variant" estimates from the United Nations. Demographic data before 1950 are from http://www.populstat.info. I suppose both rates n_t and p_t converge to unity in the very long run (after the 5th period of the 4th sequence, hence 2160). Nevertheless, I limit my analysis to 3 periods (hence 2080) due to the large degree of uncertainty thereafter.

to increasing life expectancy. Thereafter, as the boomer generation retires, the survival rate continues to grow and population growth declines. As a consequence, the old-age dependency ratio explodes.

Labor share. I use labor share data from the Penn World Table 9.1 (PWT); see Feenstra et al. (2015) for more details on these data. In this dataset, the labor share θ_t corresponds to the share of labor compensation in GDP. As argued by Gollin (2002), the measurement of the labor share is influenced by the adjustment method to take into account self-employed income. In the theoretical framework, workers are young individuals and supply only labor. In line with the model, I consider self-employed income as a labor compensation.

Other variables. I use data from the PWT for the capital stock K_t and the output Y_t. I use, respectively, the capital stock at constant 2011 national prices and the real GDP at constant 2011 national prices. In order to disentangle the effect of changes in the number of hours worked, I adjust both variables by the average annual hours worked by persons engaged from the same data source.

I also use the number of persons in employment from the PWT. In the model, labor supply is inelastic and there is no distinction between unemployed and inactive individuals. The unemployed, in terms of the model specification, correspond to all agents that do not work. However, in high-income countries such as France and the United States, inactive people also benefit from redistribution. Therefore, I treat them as unemployed and the redistribution is captured through unemployment benefit b_t in the model. I compute the unemployment rate such that $u_t = 1 - emp_t/N_t^{15-64}$, where emp_t is the number of persons in employment and N_t^{15-64} is the working age population.[17] Then, I compute labor according to the identity $L_t \equiv (1 - u_t)N_t^y$. Lastly, I use the government revenue as a share of GDP from the OECD Tax Database to proxy the tax rate τ_t.

[17] I consider the whole working age population instead of the young population. Due to the demographic specification of the model, young agents correspond to those between 20 and 60 years old. Data on the number of persons engaged per age groups are not available in PWT. Therefore, taking only N_t^y as denominator would bias downward the unemployment rate. Although there are other sources of population data, I rely on the PWT to have consistency using the same data source for input factors and output.

Table 1: Parameters.

	Parameter	France	United States
ϕ	Capital share in 1970	0.270	0.325
γ	Relative bargaining power of the union	0.500	0.500
α	Discount rate	0.669	0.669
σ	Capital-labor elasticity of substitution	1.279	1.234
ω	Relative ideological spread-out	1.017	0.652
β	Preference for government health expenditure	0.739	0.138
A	Scale parameter of the production function	23.891	22.840

Notes: The table reports the parameters from the calibration of the model for France and the United States. The capital-labor elasticity of substitution σ is obtained with a single-equation estimation from the two first-order conditions of the profit maximization with normalized CES production function; see appendix C for the details.

Normalization. I normalize both input factors K_t and L_t to their 1970 values. As a consequence, the capital-labor ratio k_t is also normalized and is equal to 1 in 1970. Then, N_t^y and N_t^o are normalized such that the unemployment rate u_t matches the one computed for 1970.

3.2 Calibration

Once stock variables are normalized, I calibrate the parameters of the model $\{\phi, \gamma, \alpha, \sigma, \omega, \beta, A\}$. Table 1 summarizes parameters for both countries. The first parameter ϕ corresponds to the capital share in 1970 and is derived from the labor share at the same year. I set the relative bargaining power of the union γ to 0.5, thus, neither the union nor the firm have an advantage in the bargaining apart from their respective outside options.[18] I also set the discount rate α at 0.669, i.e. 0.99 on annual basis.

The main parameter of the model is the elasticity of substitution between capital and

[18] There are two alternatives to this approach. The first one is to calibrate γ to match the net replacement rate in unemployment at a given point in time. However, there is no distinction between the unemployed and the inactive in the model. Thus, it requires additional assumptions to encompass the income of inactive individuals into the computation of the replacement rate. The second approach is to use proxies for γ such as the trade union density and the collective union coverage. Nonetheless, in addition to be likely endogenous, the choice of one proxy, the other, or any aggregation of both is not obvious; notably in the presence of countries such as France where the coverage is high despite the low density. Lastly, changes in the value of the relative bargaining power γ have a small effect on labor share dynamics with respect to changes on either the elasticity of substitution between capital and labor , σ, or both input factors, K_t and L_t.

labor, σ. I estimate this elasticity with a combination of the first order conditions of the profit maximization. Details of the estimation are reported in appendix C. I obtain an elasticity of 1.279 for France and 1.234 for the United States. Therefore, both input factors are gross substitutes. These values are in line with recent estimate in the literature on the labor share such as Karabarbounis and Neiman (2014) who use cross-sectional data on 50 countries over the period 1975-2012 to find an elasticity greater than 1, with an average of 1.28 in their baseline estimates.

To calibrate the three remaining parameters, I match three moments in the data. The relative ideological spread-out ω is set to match the capital-labor ratio k_t in 1970 using equation (20). The parameter being greater in France than in the US, it suggests that young people have inherently more political weight in France compared to the US. The preference for government health expenditure β is set to match the tax rate τ_t in 1970 using equation (16). As expected, the preference for government health expenditure is greater in France than in the United States. Lastly, the scale parameter of the production function A is set to match the average labor share between 2008 and 2012.

In the model, I summarize changes in labor market institutions due to policy changes only through unemployment benefits. Yet, other institutions such as firing costs may play a role in firms' employment decisions. Bentolila and Bertola (1990) argue that firing costs have an effect on firms' propensity to hire and fire, and thus a positive effect on employment levels in the long run. Subsequent to the oil shock, firms faced high firing costs, and reduced employment with attrition.[19] As a result, employment remained higher than it should have during some years, keeping the capital-labor ratio lower than desired. In France, most of the employment security policies were introduced between the 1960s and the beginning of the 1970s. Although I do not integrate any of them it in the model, these protections may be the result of the boomers' generation operating a political pressure to secure employment.

[19] Blanchard (1997) also shows that wages failed to adjust to the productivity slowdown and adverse supply shocks of the 1970s. Thus, wages remained relatively high and so did the labor share. This phenomenon was more pronounced in Continental European countries than in the Anglo-Saxon countries, mainly due to labor market institutions.

France started to implement policies in order to increase the labor market flexibility during the 1980s to alleviate the effects of adverse supply shocks of the 1970s. For instance, short term contracts have been introduced in 1979. To summarize, labor market institutions are different before and after the 1980s. Therefore, structural parameters such as the ability of firms to substitute labor with capital may also have changed. I show in appendix D that a break in the regime of the capital-labor elasticity of substitution has occurred during the 1980s. I separately estimate the elasticity over the periods 1970-1985 and 1986-2010. Before 1985, I find an elasticity of 1.067 but not statistically different from one; while I estimate it about 1.321 after 1985. Thus, for model simulations, I consider an elasticity about 1 in 1970 and 1980 and about 1.321 thereafter.[20]

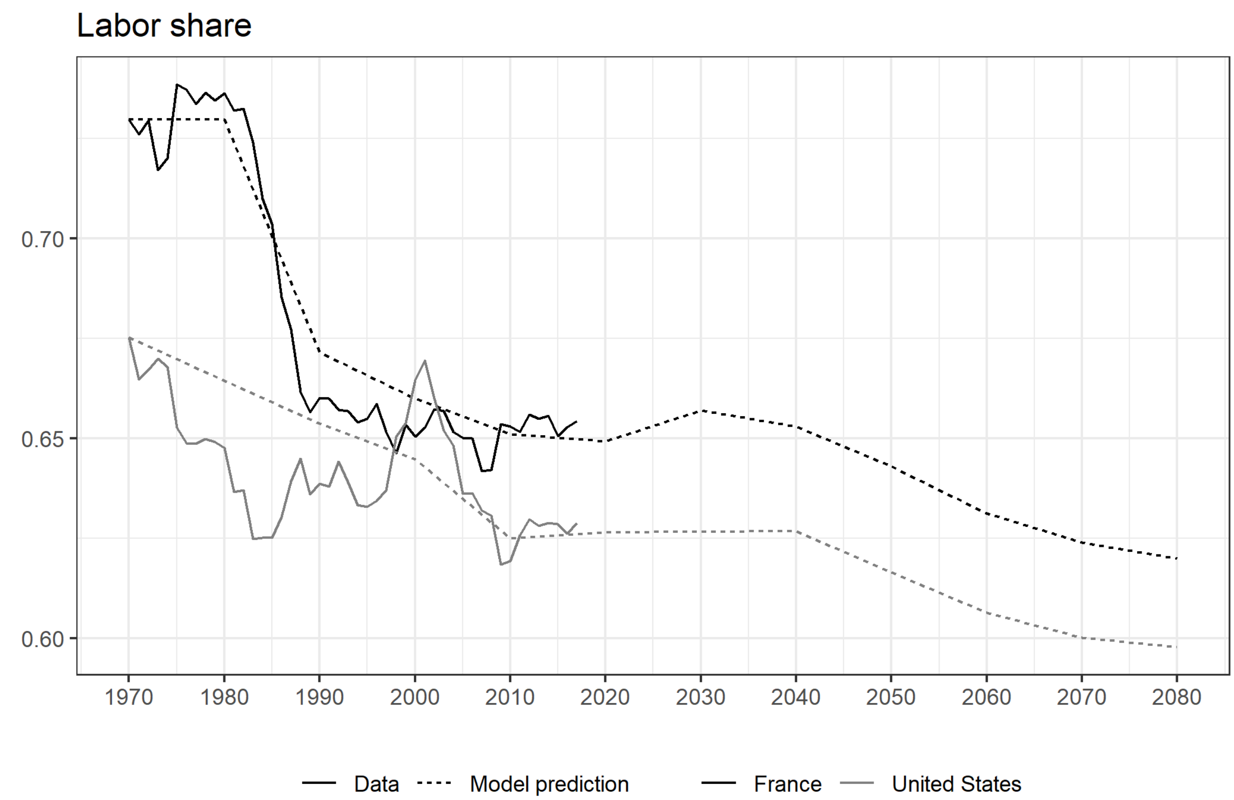
3.3 Model predictions

I simulate the model using the parameters values above. For the remaining of the paper, I refer to this simulation as the benchmark simulation. Figure 2 displays the labor shares predicted by the model. The model reproduces the global trend in the data for both countries until 2010. For the US, the model overestimates the labor share in 1980s and 1990s and underestimates it around 2000. However, it captures the overall trend of the labor share over the period. For France, model predictions are more accurate and reproduce the data since 1970. Looking at the model's predictions after 2010, the labor share should slightly increase up to 66% in France by 2030 and remain stable around 63% in the US until 2040, before declining again in both countries.

The young boomers (1970-2010). The variations of variables help to highlight the mechanisms of the declining labor share until 2010. Figure 3 displays the deviation from the 1970's value of these variables in percentage. The rate of population growth n_t slightly exceeds the increasing survival rate p_t between 1970 and 2000. Thus, the old-age-dependency

[20] Caballero and Hammour (1998) use a relatively high value of the capital-labor elasticity of substitution, about 6.00, to simulate French data. Karabarbounis and Neiman (2014) use cross-sectional data on 50 countries over the period 1975-2012 to find an elasticity around 1.28 in their baseline estimates.

Figure 2: Model predictions of the labor share.

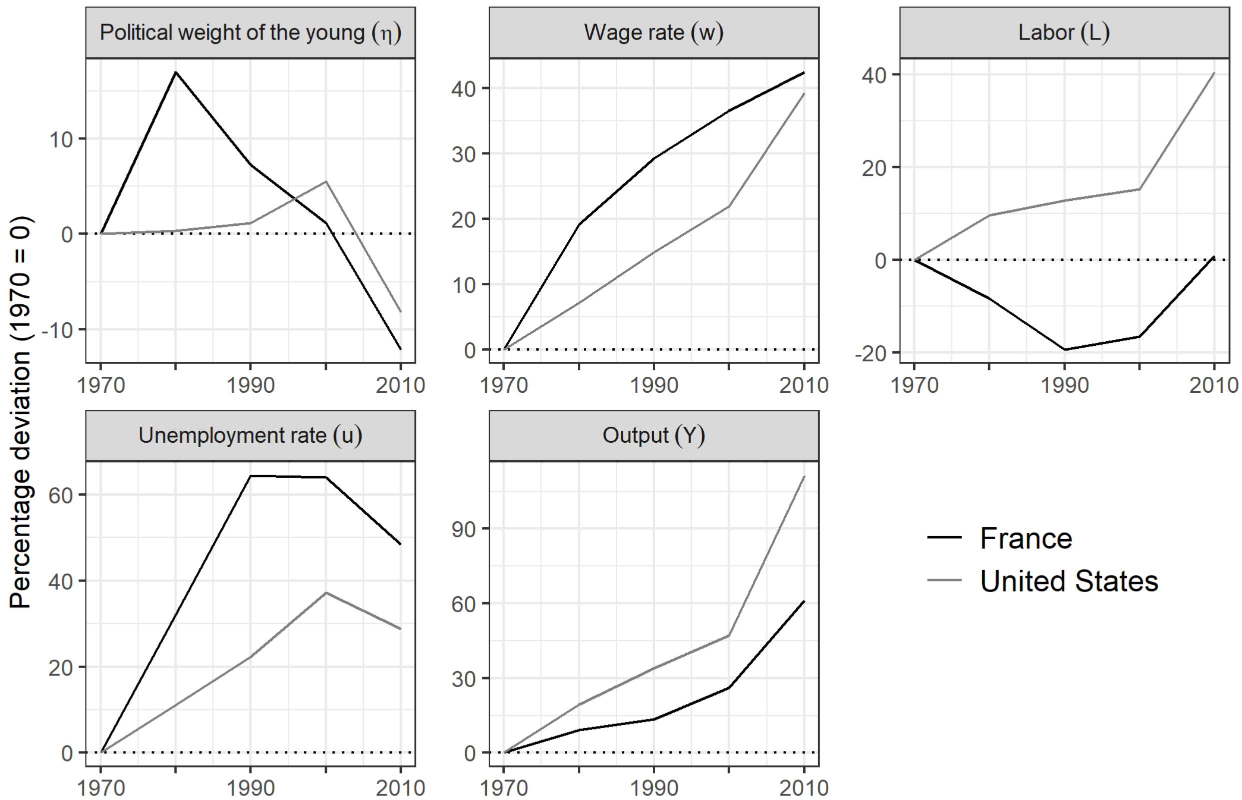


Notes: The figure shows the labor share predictions of the model (dashed lines) and the labor share in the data (solid lines) from 1970 to 2080 for France and the US. Labor share data are from the Penn World Table 9.1 with self-employed income as labor compensation.

ratio p_t/n_t remains roughly stable, although it declines slightly in France between 1980 and 1990 due to the massive entry of the boomers in the labor force. The old-age-dependency ratio starts to increase around 2000 due to a steady population growth combined with a sharply increasing survival rate, the boomers' generation starting to retire. As a result of this demographic context, the political weight of the young η_t is above its 1970's level until 2000 in both countries as depicted in the first panel of the figure.

As the political weight of the young boomers rises, pro-youth policies are implemented due to the opportunistic behavior of political parties. These policies consist of more redistribution, i.e. a greater tax rate and more unemployment benefits, in order to prevent the income losses due to unemployment of the young boomers. Since the unemployment benefits

Figure 3: Variables dynamics over the 1970-2010 period.



Notes: The figure shows the deviation of the variables from their 1970's value (in percentage) for France and the United States over the 1970-2010 period. Solid lines represent the dynamics obtained from the model simulation, whereas the dotted line represents the 0-degree line.

act as an outside option for the workers, they are able to bargain greater wages w_t as shown in the second panel.

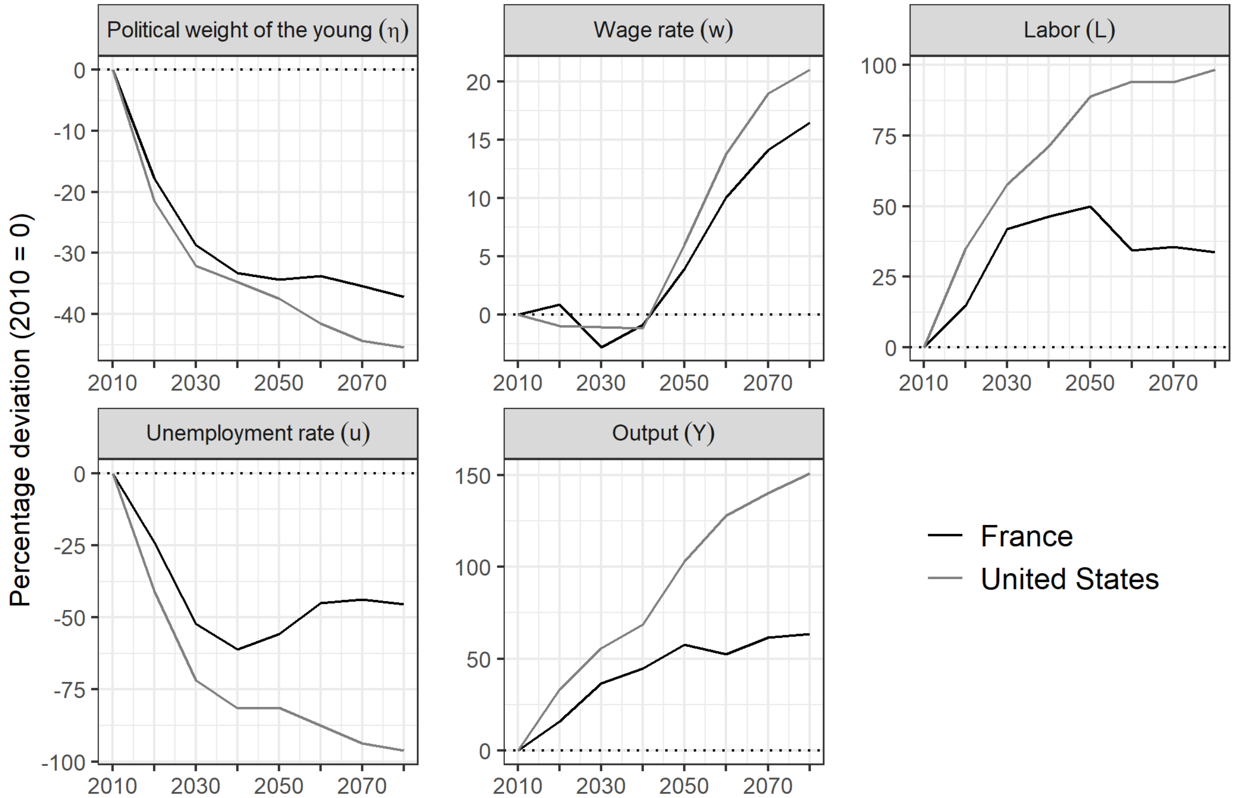
Because the labor cost (i.e. the wage) increases for firms, they shift away from labor. This behavior is permitted by two features of the model. First, the right-to-manage specification of the wage bargaining enables the firm to hire and fire as much as wanted. Second, the capital-labor elasticity of substitution σ is greater than one, thus, both input factors are gross substitutes and the firm is all the more able to substitute labor with capital for a given output level. This behavior leads to a decline of the number of workers L_t in France and a moderate increase in the US, as highlighted in the third panel. The diverging patterns between the two countries are due to the substitution effect being stronger in France than in the US. The higher elasticity of substitution in France combined with a faster growth of the capital stock K_t pushes French firms to substitute relatively more labor with capital. Thus, the number of workers becomes lower than its 1970's level in France, whereas the US manage to slightly increase their labor factor because the increase in wages is not as strong as in France.

This fall in employment raises unemployment in France, the effect being enhanced by the labor force growth due to the number of young boomers. For the US, the moderate increase in labor does not manage to offset population growth. Therefore, the unemployment rate also raises as depicted in the fourth panel. Since both factors are gross substitutes, output Y_t and output-per-worker grow along with capital-per-worker. The increase in output-per-worker exceeds the one of the wage, and as a result, the labor share declines.

The mechanisms until 2010 can be summarized as follows. The young boomers change labor market institutions in their favor due to their relatively high political weight. This raises the outside option of workers and hence their bargaining power, enabling them to bargain greater wages. Labor becoming costly, firms decide to shift away toward capital. This shift away from labor engenders an increase in output-per-worker that exceeds the wage gain; thus, the labor share declines.

The retired boomers (2010-2050) and afterwards (2050-2080). The variations of the same set of variables also help to highlight the mechanisms of the model's predictions for the labor share after 2010. Figure 4 displays the deviation from their 2010's value in percentage. The demographic context over this period is the following: the rate of population growth n_t declines sharply between 2010 and 2050 before stabilizing thereafter. Meanwhile, the survival rate p_t grows by around 4% per decade. Thus, the old-age-dependency ratio sharply increases from 2010 to 2050. Once the rate of population growth becomes stable, the old-age-dependency ratio still grows but at a lower rate. As a result, the political weight of the young, η_t, never returns to its 2010 level and strongly declines until 2050 for both countries as showed in the first panel.

Figure 4: Variables dynamics over the 2010-2080 period.



Notes: The figure shows the deviation of the variables from their 2010's value (in percent) for France and the United States over the 2010-2080 period. Solid lines represent the dynamics obtained from the model simulation, whereas the dotted line represents the 0-degree line. Data are from the benchmark simulation of the model.

As the political weight of the young declines, the reverse of the mechanism that led to the decline of the labor share when the boomers were young is expected. Opportunistic political parties favor the retired boomers and implement pro-elderly public policies, i.e. a lower tax rate and more public health spending. Thus, unemployment benefits decline and so does the outside option of workers. As a result, they concede a wage stagnation inciting firms to hire more, as depicted in the second and third panels. The unemployment rate drops due to higher employment combined with the decline of the rate of population growth, as shown in the fourth panel.

Nonetheless, the labor share never recovers its 1970's level. The dynamics of the labor share are governed by two factors: an increase in employment and a higher capital stock arising from the savings of the boomers. These savings were fostered by the size of the boomer generation; the rising expected life expectancy; and the high level of their wages. While higher employment tends to increase the labor share, the larger stock of capital tends to reduce it, keeping the labor share roughly stable in both countries when the boomers are retired.

Once the boomers pass away, after 2050, the decline in political power of the young slows down in both countries. This slowdown allows workers to bargain greater wages. French firms substitute labor with capital to thwart workers' appropriation of the rents, leading to a decline of the labor factor and so a rise in unemployment. On the other side of the Atlantic, firms in the US manage to hire until almost full-employment due to the sharp increase in capital and the stagnation of the labor supply. However, the wage gains remain lower than the rise in output-per-worker in both countries. Therefore, both labor shares decline to reach 62% in France and 59.8% in the US by 2080, while their respective levels were about 65.1% and 62.5% in 2010.

The mechanisms after 2010 can be summarized as follows. The boomers retire and change the public policy in their favor, reducing taxes and unemployment benefit which raises employment. The effect of employment on the labor share is dampened by capital accumulation due to the extensive savings of the boomers when they were young. Consequently, the labor share slightly increases in France and stabilizes in the US, before declining again by the end of the century due to the aging of the population.

3.4 Counterfactual and decomposition

So far I have highlighted the different mechanisms through which the age structure of the population affects economic variables and therefore the labor share. Demographic changes are due to two exogenous variables in the model: population growth n_t and the survival rate p_t. Their dynamics may affect the labor share through two channels: the direct factor-accumulation effect and the indirect policy-mechanism effect.

Table 2: Demographic variables in 1970.

	Variable	**France**	**United States**
n_{1970}	Population growth rate	1.134	1.597
p_{1970}	Survival rate	0.417	0.476
p_{2010}	Expected survival rate	0.583	0.561
$\frac{p_{1970}}{n_{1970}}$	Old-age dependency ratio	0.368	0.298
η_{1970}	Young political weight of the young	3.846	3.008

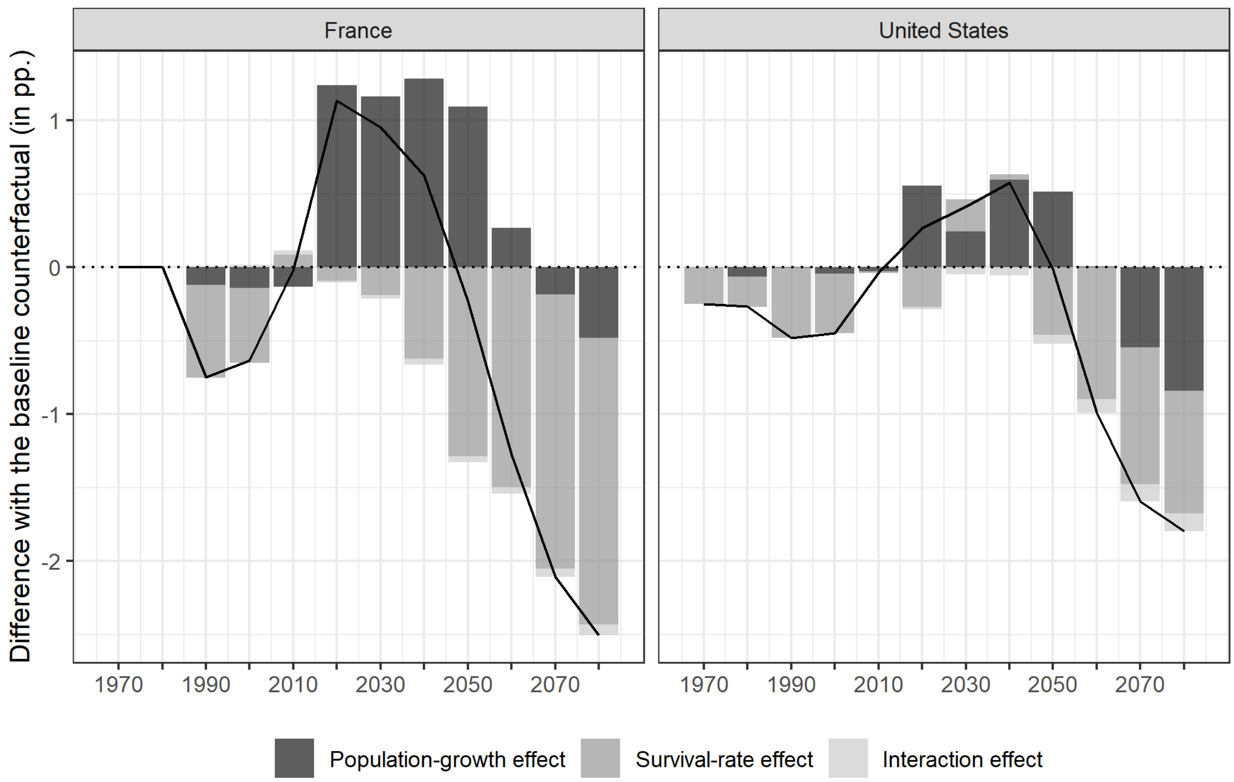
Notes: The table reports the demographic variables in 1970 for France and the United States.

To quantify the respective role of each exogenous variable and channel, I make counterfactual simulations. In these simulations, I neutralize either a determinant of demographic changes or a channel by setting it to its level in 1970. Table 2 summarizes the demographic variables in 1970. Then, I compare counterfactual simulations to the benchmark obtained in section 3.3, thus, I quantify to which extent each determinant/channel affects the labor share. I proceed in two steps. First, I examine the impact of the two determinants of demographic changes, i.e. population growth versus survival rate, then, I investigate through which channels the impact occurs, i.e. factor-accumulation mechanism versus policy mechanism. For more details on the methodology to construct the counterfactual simulations, see appendix E.1.

Population growth versus survival rate. To neutralize the impact of population growth, I suppose that the rate of population growth remains at its level in 1970, i.e. $n'_t = n_{1970}$, which affects the population and political weight dynamics. Next, I neutralize the impact of the survival rate by setting constant the survival rate and the expected survival rate, i.e. $p'_t = p'_{t+1} = p_{1970}$, which affects the dynamics of old households, the political weight and the saving rate. Lastly, I make a last counterfactual simulation to neutralize both effects in which $n'_t = n_{1970}$ and $p'_t = p'_{t+1} = p_{1970}$. I refer to this latter simulation as the baseline counterfactual simulation.

Figure 5 presents the sizes of the population-growth and survival-rate effects on the labor share, derived from the counterfactual simulations, in percentage points. Until 2010,

Figure 5: Decomposition of the determinants of demographic changes.



Notes: The figure shows the decomposition of the effects of the determinants of demographic changes on the labor share. Effects are expressed in percentage point difference with the baseline counterfactual simulation. The baseline counterfactual corresponds to the simulation where all the demographic variables and the political weight of the young remain at their initial levels. The population growth effect accounts for the effect of population growth dynamics on the labor share, while the survival rate effect accounts for the effect of survival rate dynamics. Both effects are obtained by taking the difference between the benchmark labor share and the labor share from the simulation in which the effect is canceled. The interaction effect is defined as the part which is not exclusively explained by both effects independently. The solid line represents the net effect corresponding to the sum of the three effects, that is also the difference between the labor shares from the benchmark and the baseline counterfactual simulation.

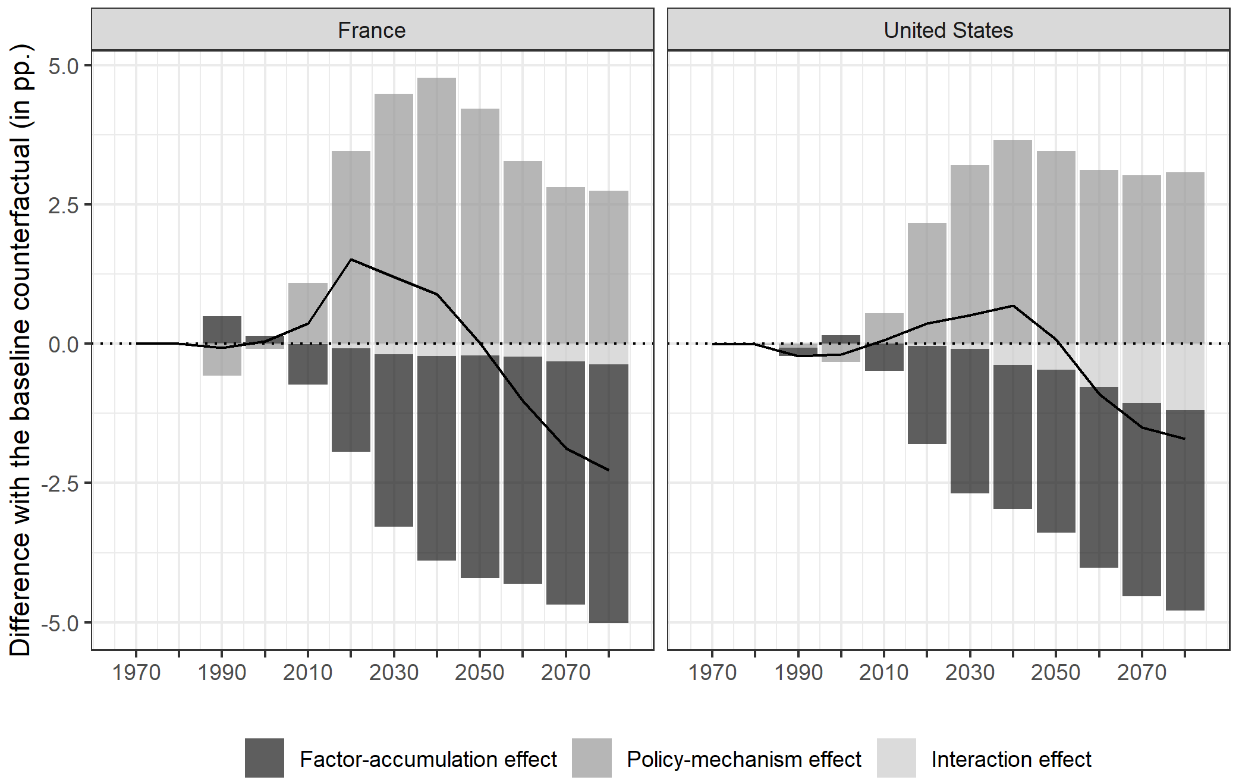
the population growth effect on the labor share is negative. The increasing population growth rate raises the political weight of the young, thus, they raise the net-replacement rate in unemployment through voting to bargain greater wages, which therefore leads the firms to substitute labor with capital. Nonetheless, the effect is small because the increasing number of young individuals reduces the per-capita unemployment benefits which moderates the raise of the net-replacement rate in unemployment. The effect is larger in France than in the US because the population growth rate increases in France while it remains roughly stable in the US over this period. Meanwhile, the survival rate effect has also a negative effect on the labor share. On the one hand, the increasing survival rate thwarts the raising population growth due to the boomers' arrival, which dampens the increase of the old-age dependency ratio and so the one of the political weight of the young. On the other, the increase of the expected survival rate raises the political weight and leads to higher saving rates and therefore more capital. Both effects combined allow workers to bargain greater wages by raising their outside option, i.e. the unemployment benefits, which incites firms to substitute labor with capital as there is more capital available in the economy.

Between 2010 and 2050, when the boomers retire, the population growth rate effect becomes positive as the population growth sharply declines over this period. As a result, the political weight of the young also declines along with the capital stock because fewer people saved, which leads to an increase of the labor share. Nonetheless, the survival rate effect remains mostly negative for the same reasons as when the boomers were young. Note that, for the US, the survival-rate effect is positive in 2020 and 2030 because the survival rate declines over this period. Once the boomers disappear, after 2050, both effects become negative due to the global aging of both population, i.e. the population growth rate and the survival rate converge to unity in the very long run.

Factor accumulation versus policy mechanism. To neutralize the factor accumulation effect, I suppose that all demographic parameters remain at their 1970's level, i.e. $n'_t = n_{1970}$ and $p'_t = p'_{t+1} = p_{1970}$, which affects population dynamics and the saving rate. In this simulation, only the political weight of the young remains identical to the benchmark simulation, i.e. $\eta'_t = \eta_t$. Conversely, I neutralize the policy mechanism effect by setting the political weight of the young to its level in 1970, i.e. $\eta'_t = \eta_{1970}$, while all demographic parameters remain at their benchmark values. Lastly, I make a counterfactual simulation to neutralize both channels. This latter simulation is the baseline counterfactual simulation.

Figure 6 presents the sizes of the factor accumulation effect and the policy mechanism effect, derived from the counterfactual simulations, in percentage points. The factor-

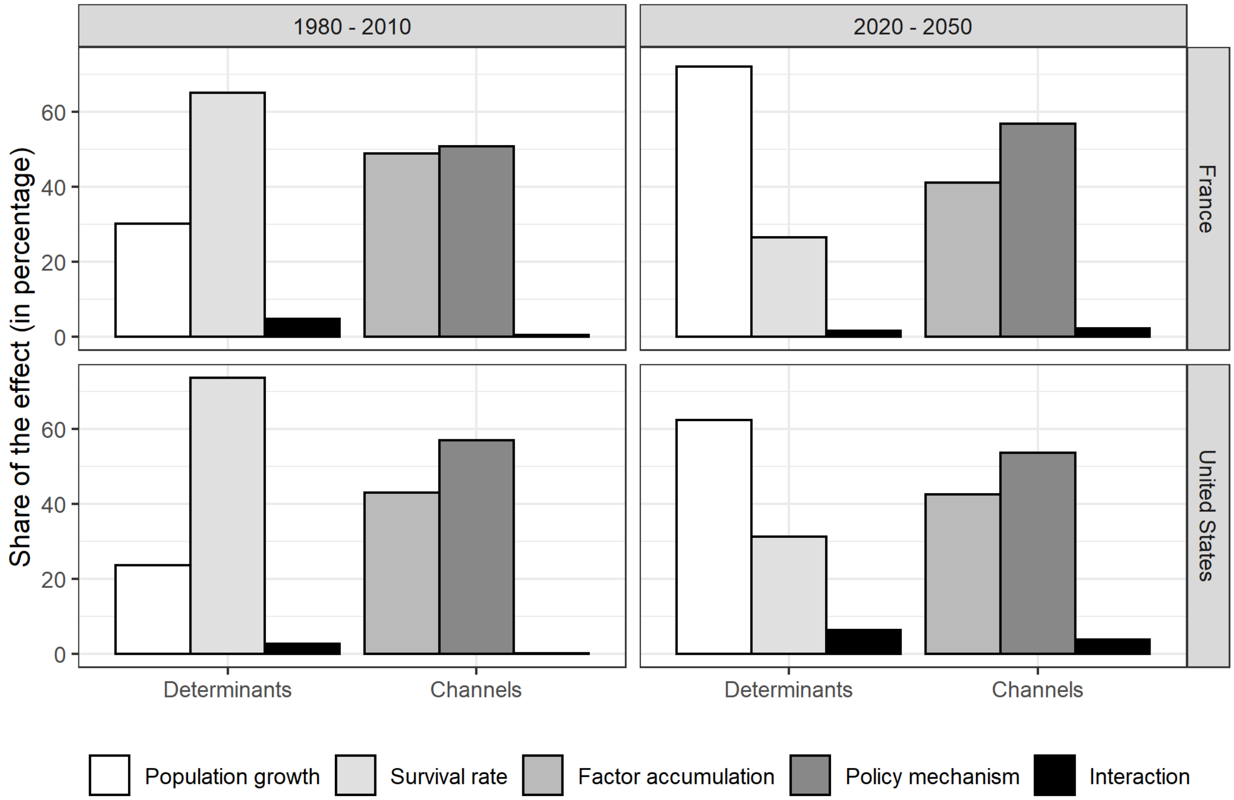
Figure 6: Decomposition of the channels of demographic changes.



Notes: The figure shows the decomposition of the channels of demographic changes on the labor share. Effects are expressed in percentage point difference with the baseline counterfactual simulation. The baseline counterfactual corresponds to the simulation where all the demographic variables and the young political weight remain at their initial levels. The factor-accumulation effect accounts for the effect of demographic changes through the factor-accumulation channel on the labor share, while the policy-mechanism effect accounts for the effect of demographic changes through the policy-mechanism channel. Both effects are obtained by taking the difference between the benchmark labor share and the labor share from the simulation in which the channel is canceled. The interaction effect is defined as the part which is not exclusively explained by both effects independently. The solid line represents the net effect corresponding to the sum of the three effects, that is also the difference between the labor shares from the benchmark and the baseline counterfactual simulation.

accumulation effect on the labor share is mostly positive when the boomers are young, because the increasing labor supply is in favor of firms within the bargaining, keeping wages low which fosters employment. In addition, the saving rate remains low and so does the capital stock. Meanwhile, the policy-mechanism effect has a negative impact on the labor share, owing to the rise of the young boomers' political weight which increases the net replacement rate in unemployment and therefore incites firms to shift away from labor toward capital.

Figure 7: Decomposition of demographic changes on the labor share.



Notes: The figure shows the average percentage share of the determinants and channels of demographic changes on the labor share over the two periods of the boomers generation (young and old), for France and the United States. Data are from the counterfactual simulations of the model and summarize the results from figures 5 and 6.

Once the boomers start to retire, both effects are reversed. The policy-mechanism effect becomes positive because old boomers foster pro-elderly public policy. This change in the policy is done at the cost of labor market insurance. Thus, workers are not able to bargain greater wages which fosters labor demand. Nonetheless, the factor-accumulation effect is negative because of the large amount of available capital stock due to the savings of the boomers when they were young. As a result, the factor-accumulation effect dampens the positive impact on the labor share of the reversal policy-mechanism effect.

Summary of the effects. Figure 7 summarizes the share of each effect for the previous decompositions by period and country. The survival-rate effect is the main determinant of demographic dynamics on the labor share when the boomers are young. On average, this effect accounts for 65.1% of the observed changes in France and 73.6% in the US between 1980 and 2010. Once the boomers retire, this decomposition is reversed and the role of the survival rate falls to 26.4% in France and 31.3% in the US. Therefore, the rise in life expectancy outweighs the increase in population growth as a demographic determinant of the labor share when the boomers are young, while the declining population growth becomes the main determinant once they retire.

These determinants affect the labor share through two channels: the factor-accumulation effect and the policy-mechanism effect. When the boomers are young, both channels play roughly similar roles in France while the policy mechanism effect prevails in the US. On average, the share explained by the policy mechanism effect is about 50.8% in France and 56.9% in the US. Thereafter, between 2020 and 2050, the share of this latter effect rises by 6 pp. in France while it slightly decreases by 3.2 pp. in the US. These results suggest that the policy-mechanism effect accounts for more than half of the effect of demographic dynamics on the labor share.

Schmidt and Vosen (2013) consider only the factor accumulation mechanism and show that this mechanism disappears in a small open economy because capital-per-worker and the wage rate are independent of domestic savings, so that labor share dynamics only reflect changes in net foreign assets. The major advantage of my approach is that the policy mechanism holds in a small open economy. With capital mobility, Pica (2010) argues that competition to attract capital between countries leads to reduced labor market regulation and a lower labor share. Nonetheless, he uses a Cobb-Douglas production function which cancels out the shift away from labor toward capital of firms that is allowed by the CES production function that I employ. In terms of consequences for the labor share, the effect of capital markets integration that occurs through labor market deregulation in an open economy is equivalent to the response of the firms that substitute labor with capital to thwart workers' appropriation of the rents in a closed economy.

4 Discussion

4.1 Age-related conflict: who are the winners ?

The results show that the labor share declines due to the size of the boomers' generation in France and in the US. First, when they are young because they shape labor market institutions in their favor inciting firms to shift away from labor toward capital. Second, when they are old because they have substantially increased the available capital in the economy through their savings, pushing firms to substitute even more. Although it may seem obvious that the boomers are the winners of the age-related conflict when they are old, the results raise the question of whether they were the losers when they were young because the labor share declined a lot over this period. However, the labor share is a gross indicator of the income distribution that does not take into account redistribution, and a more appropriate indicator to determine the winners of the age-related conflict is the net income ratio between young and old.[21]

The *before-tax* young-to-old income ratio corresponds to the labor-to-capital income ratio Θ_t. While equation (18) shows that the *after-tax* young-to-old income ratio is equal to the political weight of the young, i.e. $Y_t^y/Y_t^o = \eta_t$, which includes unemployment benefits. Let $\tilde{\Theta}_t$ and $\tilde{\eta}_t$ be respectively these ratios in per capita terms. For readability purposes, I refer to them as the before-tax ratio and the after-tax ratio. Rewriting both expressions,

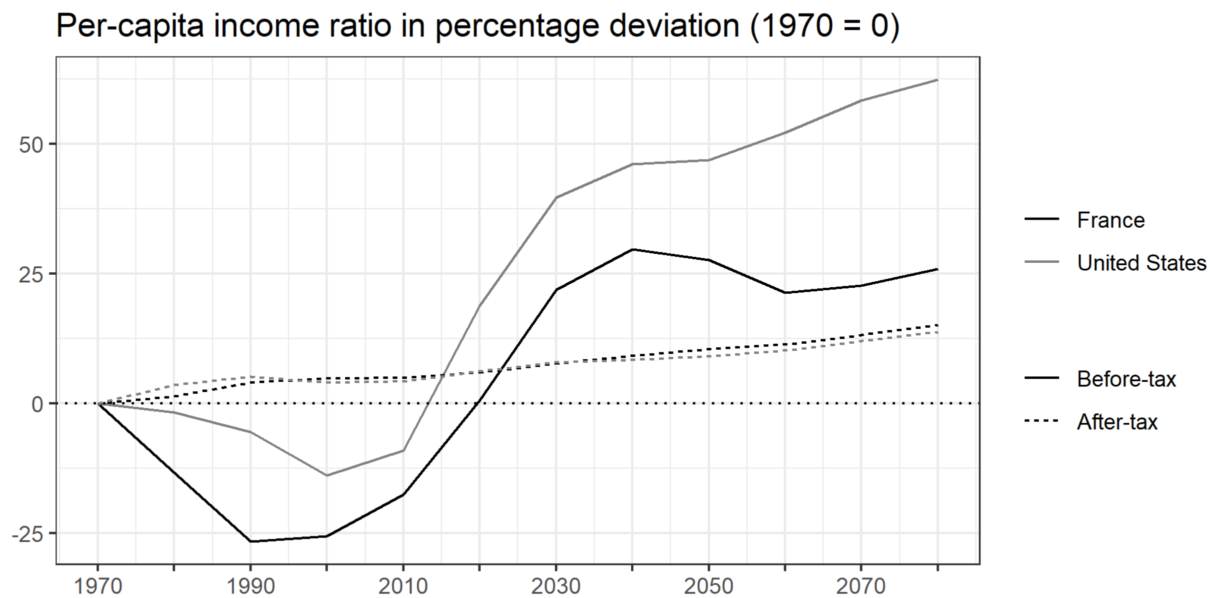
$$\tilde{\Theta}_t \equiv \Theta_t \times \frac{N_t^o}{N_t^y} = \Theta_t \times \frac{p_t}{n_t},$$
$$\tilde{\eta}_t \equiv \eta_t \times \frac{N_t^o}{N_t^y} = \eta_t \times \frac{p_t}{n_t} = \omega(1 + \alpha p_{t+1}).$$

Therefore, changes in $\tilde{\eta}_t$ only reflect changes in the expected survival rate p_{t+1}, while changes in $\tilde{\Theta}_t$ are composed of the changes in the labor-to-capital income ratio Θ_t and in the old-age

[21] I do not consider the difference in life-time utility between generations to assess who are the winners. The shape of the utility function does depend on the date at which a generation appears because the effective discount factor αp_{t+1} depends on life expectancy which varies across generations. Since two generations do not have the same baseline, then utility comparisons do not make much sense.

dependency ratio p_t/n_t.

Figure 8: Per-capita income ratios dynamics.



Notes: The figure shows the before-tax and after-tax per-capita income ratios between young and old for France and the United States in percentage deviation since 1970. The solid line represents the before-tax ratio, whereas the dashed line represents the after-tax one. The dotted line represents the 0-degree line. Data are from the benchmark simulation of the model.

Figure 8 presents the before-tax and after-tax ratios in percentage deviation from their values in 1970. The after-tax ratio steadily increases due to the constant increase in life expectancy. When the boomers are young and earn a labor income, the before-tax ratio falls below its initial level due to the decline of the labor share. Thus, the *before-tax* ratio decreases while the *after-tax* one increases. The young boomers are the winners of the age-related conflict over this period because they manage to recover their labor income losses through redistribution. Once they retire and earn capital income, the before-tax ratio explodes due to the rise of the old-age dependency ratio since the labor share is rather stable over this period. This rise in the before-tax ratio exceeds the steady growth of the after-tax one. As a result, the boomers are also the winners of the age-related conflict when they are old because young people have a lower income after redistribution compared to the their income before redistribution..

4.2 Retirement age

The results on the survival-rate effect only focus on the effect of rising life expectancy on the labor share, keeping the retirement age constant. One might argue that changes in the age of retirement may have implications in this context. The literature has focused on the optimal retirement age for economic growth (Futagami and Nakajima 2001; Soares 2005; Lee and Mason 2010; Gonzalez-Eiras and Niepelt 2012) or the sustainability of pension systems (de la Croix et al. 2013; Dedry et al. 2017), while how it affects the allocation of income between capital and labor has received no attention so far.[22]

To take into account the role of the retirement age on the labor share, I provide counterfactual simulations with different scenarii based on an exogenous change in the age of retirement. The retirement age is captured by the survival rate p_t which depends negatively on it but positively on life expectancy. I do not endogenize this variable for two reasons. First, it would require several assumptions and reduce the tractability of the model by adding an extra policy instrument.[23] Second, the allocation between capital and labor does not seem to be a key indicator in the debate about the optimal age of retirement, as indicated by the lack of interest in it in the debate.

I suppose a positive exogenous shock to the age of retirement between 2020 and 2030.

[22] For instance, Gonzalez-Eiras and Niepelt (2012) predict that the retirement age in OECD countries should increase in response to population aging. They use a model of politico-economic equilibrium in which individuals vote on the age of retirement with perfect foresight. As a result, individuals decide to raise the retirement age as long as population ages. Because they work longer and so accumulate more wealth, it reduces social-security transfers and thereby releases more government spending to public investment which is an engine for growth. On the contrary, Futagami and Nakajima (2001) claim that population aging does not necessarily depress economic growth and may even foster it through savings. Thus, postponing the retirement age would result in a decline of savings and so the economic growth. Lastly, Dedry et al. (2017) also discuss the role of legal retirement age according to the type of pension system in a context of population aging.

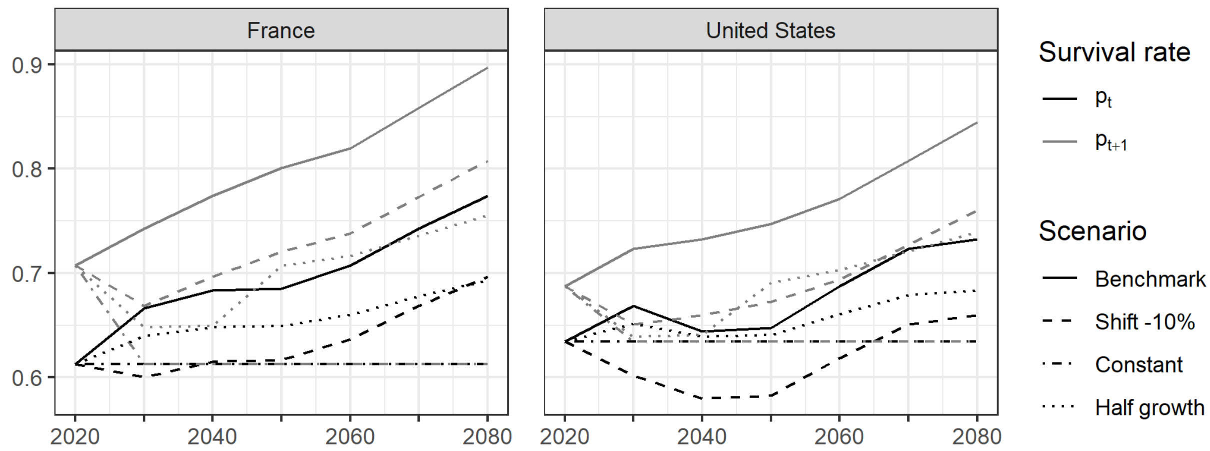
[23] To have an endogenous retirement age, agents should vote on it and thereby implicitly vote on the survival rate. The first question would be to determine whether agents vote on the age of retirement of their elders p_t, their own p_{t+1} or both. Thus, further assumptions on the timing of the model would be required. Then, the perfect annuities market would have a lot of implications on the results. Since savings of young agents who die before reaching old age are distributed among their surviving peers, it means that an agent has an incentive to vote for an increase of the retirement age, i.e. a decline of the survival rate, because fewer peers would reach old age. Therefore, it would be necessary to determine whether or not agents internalize the redistribution due to the perfect annuities market in their voting decisions.

Fewer people reach the old age which is translated into a decline of the survival rate p_t. As in section 3.4, all demographic parameters are affected by this change, because the greater the retirement age, the longer an individual remains young in terms of the model. However, these individuals do not vanish but remain in the labor force. The capital stock after 2030 is not changed as in section 3.4 because it has already been accumulated through savings from previous periods. Therefore, it has no reason to vanish or to be scraped. In appendix E.2, I provide the details on the methodology for these counterfactual simulations. The underlying assumption of this whole exercise is that the change in retirement age has no consequence on life expectancy.[24] Despite the presence of a potential co-integration, I suppose that the resulting exogenous change in the survival rate takes this co-integration into account. In line with the model in which there are perfect forecasts on demographic variables and therefore life expectancy.

I consider three scenarii for the exogenous change in the age of retirement between 2020 and 2030, in addition to the benchmark simulation. Figure 9 summarizes the survival rate dynamics according to the scenario for both countries. First, the retirement age increases in such a way that the survival rate falls by 10% in 2030 and continues to grow as in the benchmark simulation thereafter. This first scenario can be summarized as a *one-shot shift by -10%* of the survival rate. Then, I suppose the retirement age increases in such a way that the survival rate grows at a fraction $\zeta \in [0, 1]$ of the benchmark simulation's growth rate. The new survival rate p'_t after 2030 is described as $p'_t = \zeta(p_t - \bar{p}) + \bar{p}$, where $\bar{p}$ is the value of the survival rate in 2030. This can be interpreted as a gradual increase of the retirement age that is proportional to improvement in life expectancy. The second and third scenarii are derived from this specification. In the second one, I suppose that variations in retirement age are fully proportional to the ones in life expectancy. Thus, $\zeta = 0$ and the survival rate remains *constant* after 2030, i.e. $p'_t = \bar{p}$. In the third scenario, the retirement age increases

[24] Some authors argue that increasing the retirement age has a negative impact on health and so the life expectancy (Insler 2014 for the United States; Coe and Zamarro 2011 for Europe). Thereby, the negative effect on the survival rate may be all the more important due to the declining life expectancy.

Figure 9: Survival rate dynamics with changing retirement age specifications.

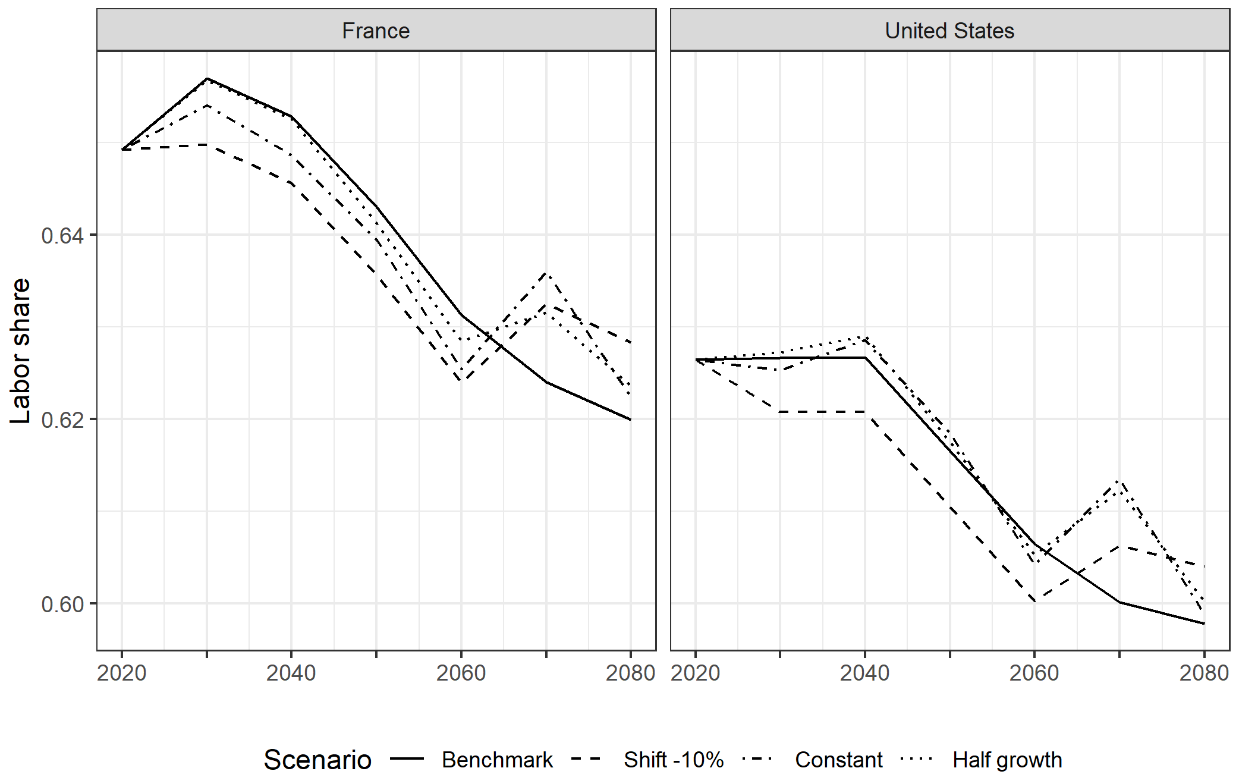


Notes: The figure shows the survival rate and the expected survival rate for France and the United States according to the scenario of increase in the age of retirement between 2020 and 2030. The solid line represents the survival rate dynamics as in the benchmark simulation. The dashed line represents the scenario with a -10% shift of the survival rate in 2030. The dash-dotted line represents the scenario with a constant survival rate after 2030. The dotted line represents the scenario in which the survival rate grows at half the speed of the one in the benchmark simulation.

in such a way that the survival rate *grows at half the speed* of the benchmark simulation. Thus, $\zeta = 0.5$ and the survival rate becomes the average of the benchmark simulation and the second scenario, i.e. $p'_t = (p_t + \bar{p})/2$. The benchmark simulation corresponds to the case where $\zeta = 1$, i.e. $p'_t = p_t$.

Figure 10 presents model predictions of the labor share following the change in retirement age, according to the scenario. The raise of the age of retirement in France leads to a decline of the labor share with respect to the benchmark simulation in a first time. In a second time, the very long-run, the labor share is relatively greater than in the benchmark case. This result holds regardless of the scenario. In the United States, the impact on the labor share depends on the scenario. As in France, the -10% shift scenario generates a sharp decline of the labor share before to exceed the benchmark simulation's one in the very long-run. However, both scenarii of diminished growth does not affect much the labor share because survival rate dynamics are rather stable over this period in the US. They both exceed the benchmark labor share in the very long-run. Therefore, I argue that changes in the retirement

Figure 10: Model predictions of the labor share with changing retirement age specifications.



Notes: The figure shows the labor share predictions of the model following a change in the retirement age between 2020 and 2030. The solid line represents the benchmark simulation. The dashed line represents the scenario with a -10% shift of the survival rate in 2030. The dash-dotted line represents the scenario with a constant survival rate after 2030. The dotted line represents the scenario in which the survival rate grows at half the speed of the one in the benchmark simulation.

age may have different impact on the labor share according to the scenario and the country. Nonetheless, increasing the retirement age increases the labor share for both countries in the very long run regardless of the scenario.

The underlying mechanisms are related to those detailed in section 3.4, except that the capital stock does not immediately adjust. The young generation at previous period expected to be retired longer because the expected survival rate p_{t+1} was much greater than the after-reform one p'_{t+1}. It leads to an excess of savings and therefore an over accumulation of capital. Meanwhile, the agents remain young longer in terms of the model which increases the political weight of the young and fosters more pro-labor income institutions. The cost of labor factor increases inciting the firms to hire less. Both effects combined push the firms to shift away from labor toward capital. Therefore, the labor share is smaller than the benchmark one.[25] In the very long run, the over-accumulation disappears because the savings adjust to the after-reform survival rate dynamics. The jumps of the labor share between 2060 and 2070 in the counterfactual simulations are due to the methodology of the four sequences because the capital stock awaits 40 years to adjust. Allowing the capital stock to partially adjust over these 40 years would smooth the dynamics, although it would not change the qualitative results.

5 Conclusion

A vast literature emphasizes the role of biased technical change and institutions to explain the shift from labor toward capital and therefore the decline of the labor share. This paper focuses upstream of these determinants and highlights the role of demography as a force that shapes labor market institutions and hence the allocation of factor incomes. These institutions define the rules of the game for wage bargaining between firms and workers. When a particular generation, such as the boomers, is able to change institutions in its favor, then these rules also change, which affects the allocation of income between capital and labor. This mechanism accounts for the *indirect effect* of demographic changes on the labor share that results from the inter-generational conflict when choosing public policy. Besides, the age structure of the population also has a *direct effect* that occurs through factor accumulation of the labor supply and capital. Both effects combined help understand the role of the boomers' cohort in the decline of the labor share in France and in the United States.

This paper shows that it is important to take into account changes in institutions, that are endogenously determined by the age structure of the population, in order to understand macroeconomic dynamics in the long run. Decomposing the direct factor-accumulation effect

[25]Except in 2040 and 2050 in the US because the survival rate is below its level in 2030.

and the indirect policy-mechanism effect, I find that the latter accounts for more than half of the total effect of demographic dynamics on the labor share. Thus, omitting this indirect mechanism, and more broadly supposing that institutions do not change in the long run, leads to underestimate of the role of demography on the factor income distribution. In this regard, my results provide a new conceptual framework to examine demographic dynamics and institutions in future work.

These results have implications in terms of policy debate. On the one hand, several high-income countries experienced an aging of their population which has led to a debate about the optimal public policy, and more specifically about the optimal age of retirement. In this respect, my results shed some light on the consequences of changing the retirement age on the allocation of income between capital and labor. On the other hand, developing countries are witnessing large demographic changes and may give birth to a generation such as the boomers' cohort, which would change their institutions along with factor shares, and therefore, may have consequences on their development.

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Appendixes

A Uniqueness of the equilibrium

In this appendix, I provide proofs that there is a unique equilibrium. The equilibrium is given by the solution to equation (20), namely

$$\ln\left(\frac{\frac{N_t^y}{K_t}k_t-1}{\frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\eta_t-1}\right)=\left(\sigma+\frac{1-\gamma(1-\sigma)}{\gamma}\frac{1-\phi}{\phi}k_t^{\frac{1-\sigma}{\sigma}}\right)^{-1},$$

which yields the equilibrium capital-labor ratio k_t, given K_t. Let us define the LHS and RHS equations respectively with the functions $g(k_t, N_t^y, K_t, \eta_t, \phi, \sigma)$ and $h(k_t, \gamma, \phi, \sigma)$ such as

$$g(k_t, N_t^y, K_t, \eta_t, \phi, \sigma)=\ln\left(\frac{\frac{N_t^y}{K_t}k_t-1}{\frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\eta_t-1}\right), \tag{21}$$

$$h(k_t, \gamma, \phi, \sigma)=\left(\sigma+\frac{1-\gamma(1-\sigma)}{\gamma}\frac{1-\phi}{\phi}k_t^{\frac{1-\sigma}{\sigma}}\right)^{-1}. \tag{22}$$

The g function has two vertical asymptotes depending on whether the numerator or the denominator within the logarithm is equal to zero. Let k_1 (resp. k_2) be the vertical asymptote associated to the numerator (resp. denominator). Thus, $k_1 = K_t/N_t^y$ and $k_2 = [(1-\phi)/\phi/\eta_t]^{\frac{\sigma}{\sigma-1}}$. Rewriting the function with these vertical asymptotes, we have

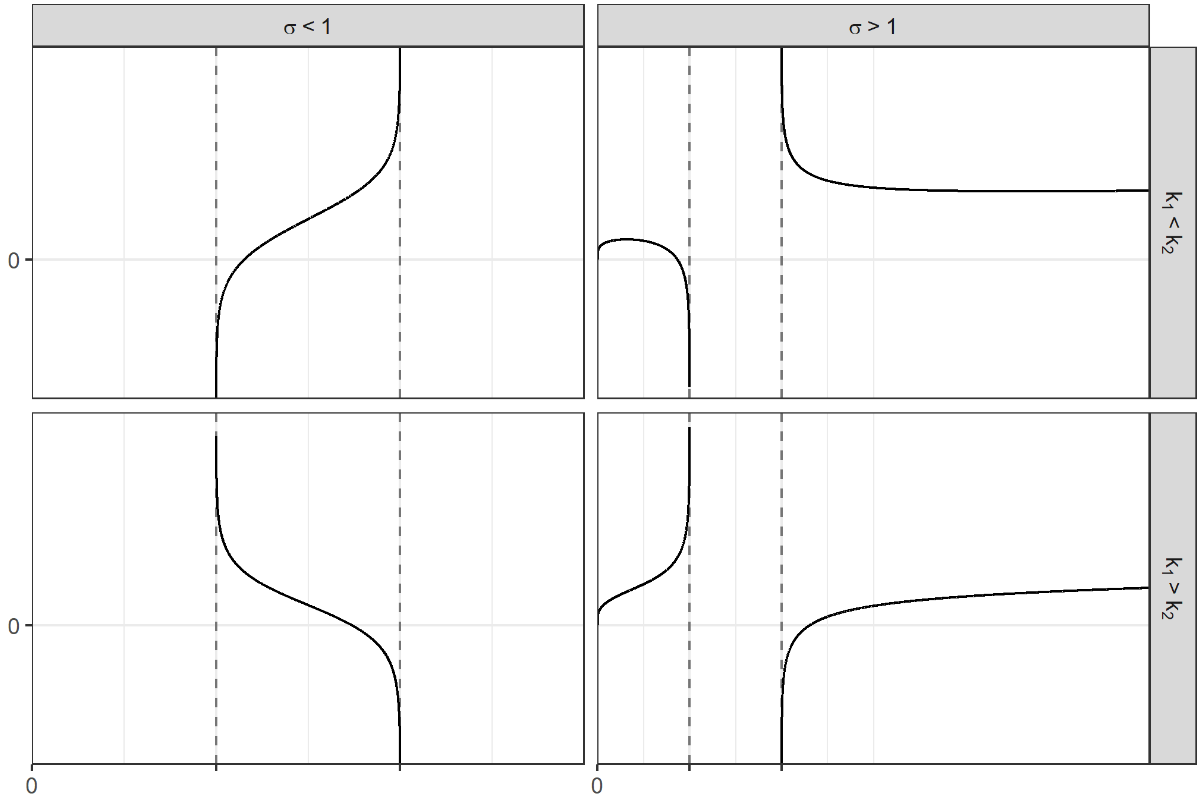
$$g(k_t, k_1, k_2, \sigma)=\ln\left(\frac{\frac{k_t}{k_1}-1}{\left(\frac{k_t}{k_2}\right)^{\frac{\sigma-1}{\sigma}}-1}\right).$$

This g function can exhibit four different patterns according to the value of σ and the relative magnitude of k_1 and k_2. Figure 11 plots these patterns. The h function has two different patterns according to the value of σ. When $\sigma < 1$, the h function is strictly decreasing, whereas the function is strictly increasing when $\sigma > 1$. The upper limit of h in k_t is $1/\sigma$ and its lower limit is 0. This leads to proposition 1.

Proposition 1 *If $k_t \leq k_1$ at the equilibrium, then there is a unique equilibrium with full employment where $k_t = k_1$ and the net wage equals the unemployment benefits, i.e. $(1-\tau_t)w_t = b_t$.*

Proof. The numerator of the g function is positive if and only if $k_t > k_1 \Leftrightarrow L_t < N_t^y$, i.e. the number of workers is smaller than the young population. This condition is always

Figure 11: Shapes of the g function.



Notes: The figure shows the four different patterns of the g function (solid line) according to the value of the capital-labor elasticity of substitution σ and the vertical asymptotes $k1$ and $k2$ (dashed line). The x-axis corresponds to k_t.

satisfied when there is unemployment. However, if this condition is not satisfied, then the labor demand exceeds the labor force, i.e. $L_t^d > N_t^y$. Therefore, the economy is in full-employment, i.e. $L_t = N_t^y$ and capital-per-worker is $k_t = k_1$. In such a case, the net replacement rate in unemployment X_t tends to $+\infty$. However, as long as the unemployment benefit does not exceed the net wage, the upper bound of X_t is 1. Even though I consider a model with inelastic labor supply, no agent would work for a wage lower than unemployment benefits. This assumption can be considered as an incentive constraint. Thus, if $k_t \leq k_1 \implies k_t = k_1 \implies u_t = 0 \implies X_t = 1 \implies b_t = (1-\tau_t)w_t$. ■

I normalize k_t by the vertical asymptote k_1. Let $\tilde{k}_t = k_t/k_1 > 0$. Then $k_t \gtreqless k_1$ when $\tilde{k}_t \gtreqless 1$. Similarly, let $\nu = k_2/k_1 > 0$, so that $k_2 \gtreqless k_1$ when $\nu \gtreqless 1$. To simplify the notation, let $\rho = \frac{\sigma-1}{\sigma} \in (-\infty, 1)$. When both input factors are gross complements, the elasticity of substitution is $\sigma \in (0,1)$ and the corresponding interval for ρ is $(-\infty, 0)$. When both are gross substitutes, the elasticity of substitution is $\sigma \in (0, +\infty)$ and the corresponding interval for ρ is $(0,1)$. Using this specification, I rewrite the g and h functions such that

$$g(\tilde{k}_t;\nu,\rho) = \ln\left(\frac{\tilde{k}_t - 1}{\tilde{k}_t^{\rho} - \nu^{\rho}}\right) + \rho\ln(\nu),$$

$$h(\tilde{k}_t;\tilde{\gamma},\rho) = \left(\frac{1}{1-\rho} + \hat{\gamma}\tilde{k}_t^{-\rho}\right)^{-1},$$

where $\hat{\gamma} \equiv \frac{1-\phi}{\phi}\frac{1-\gamma(1-\sigma)}{\gamma}k_1^{-\rho} > 0$. These functions are more convenient to prove that there is a unique equilibrium, first, when both input are gross complements ($\sigma < 1$); and second, when they are gross substitutes.

A.1 Under gross-complementarity

The behavior of the g function depends on the value of ν. If $\nu > 1$, then $g(\tilde{k}_t)$ is defined, continuous and strictly increasing on $(1,\nu)$. The limits in 1 and ν are respectively $-\infty$ and $+\infty$. If $\nu < 1$, then $g(\tilde{k}_t)$ is defined, continuous and strictly decreasing on $(\nu, 1)$. The limits in ν and 1 are respectively $-\infty$ and $+\infty$. Meanwhile, the behavior of the h function does not depend on ν. As long as $\rho < 0$, $h(\tilde{k}_t)$ is defined, continuous and strictly decreasing on $\mathbb{R}_+$. The limits in 0 and $+\infty$ are respectively $1-\rho$ and 0. These properties lead to lemmas 1 and 2. Using these intermediate results with proposition 1, I can prove proposition 2.

Lemma 1 *If $\rho < 0$ and $\nu > 1$, then there exists a unique equilibrium.*

Proof. Let $\rho < 0$ and $\nu > 1$. The g function is defined, continuous and strictly increasing in $\tilde{k}_t \in (1,\nu)$ with two infinite vertical asymptotes of opposite signs. The h function is defined, continuous and strictly decreasing in $\tilde{k}_t \in \mathbb{R}_+ \supset (1,\nu)$ with two finite horizontal asymptotes $1-\rho$ and 0. Therefore, both functions intersect in only one point. Hence, there is uniqueness of the equilibrium. ■

Lemma 2 *If $\rho < 0$ and $\nu < 1$, then there exists at least one equilibrium.*

Proof. Let $\rho < 0$ and $\nu < 1$. The g function is defined, continuous and strictly decreasing in $\tilde{k}_t \in (\nu, 1)$ with two infinite vertical asymptotes of opposite signs. The h function is defined, continuous and strictly decreasing in $\tilde{k}_t \in \mathbb{R}_+ \supset (\nu, 1)$ with two finite horizontal asymptotes $1-\rho$ and 0. Therefore, both functions intersect in at least one point. Hence, there is at least one equilibrium. ■

Proposition 2 *If $\rho < 0$, then there exists a unique equilibrium.*

Proof. Lemma 1 implies that there is a unique equilibrium when $\nu > 1$. Lemma 2 asserts that there is at least one equilibrium when $\nu < 1$. Yet, proposition 1 states that any equilibrium such that $k_t < k_1$ (or equivalently $\tilde{k}_t < 1$) leads to the unique equilibrium with full-employment. Therefore, if $\rho < 0$ there is a unique equilibrium. ■

A.2 Under gross-substituability

As in the previous case, the behavior of the g function depends on the value of ν. If $\nu > 1$, then $g(\tilde{k}_t)$ is defined and continuous on $(0, 1) \cap (\nu, +\infty)$. If $\nu < 1$, then $g(\tilde{k}_t)$ is defined, continuous and strictly increasing on $(0, \nu) \cap (1, +\infty)$. The limits in 1 and ν are respectively 0 and $+\infty$ regardless of the value of ν with respect to 1. Lastly, if $\nu = 1$, then $g(\tilde{k}_t)$ is defined, continuous and strictly increasing on $\mathbb{R}_+$. Meanwhile, the behavior of the h function does not depend on ν. As long as $\rho \in (0, 1)$, $h(\tilde{k}_t)$ is defined, continuous and strictly increasing on $\mathbb{R}_+$. The limits in 0 and $+\infty$ are respectively 0 and $1 - \rho$.

Figure 12 plots numerical examples of the two functions for feasible values of σ. I set both parameters γ and ϕ according to the calibration in section 3.2, thus $\gamma = 0.5$ and $\phi = 0.3$. Exact values for ϕ are 0.27 for France and 0.325 for the US. I use 0.3 for this numerical computation as an approximation of the mean. The upper panels show both functions when $\nu < 1$. In these cases, they only intersect once beyond 1. The lower panels display both functions when $\nu > 1$. Here, they intersect twice before 1. These properties lead to conjectures 1 and 2.

Conjecture 1 *If $\rho \in (0, 1)$ and $\nu < 1$, then there exists a unique equilibrium.*

Conjecture 2 *If $\rho \in (0, 1)$ and $\nu > 1$, then there exists at least one equilibrium.*

Using both conjectures combined with proposition 1 leads to conjecture 3.

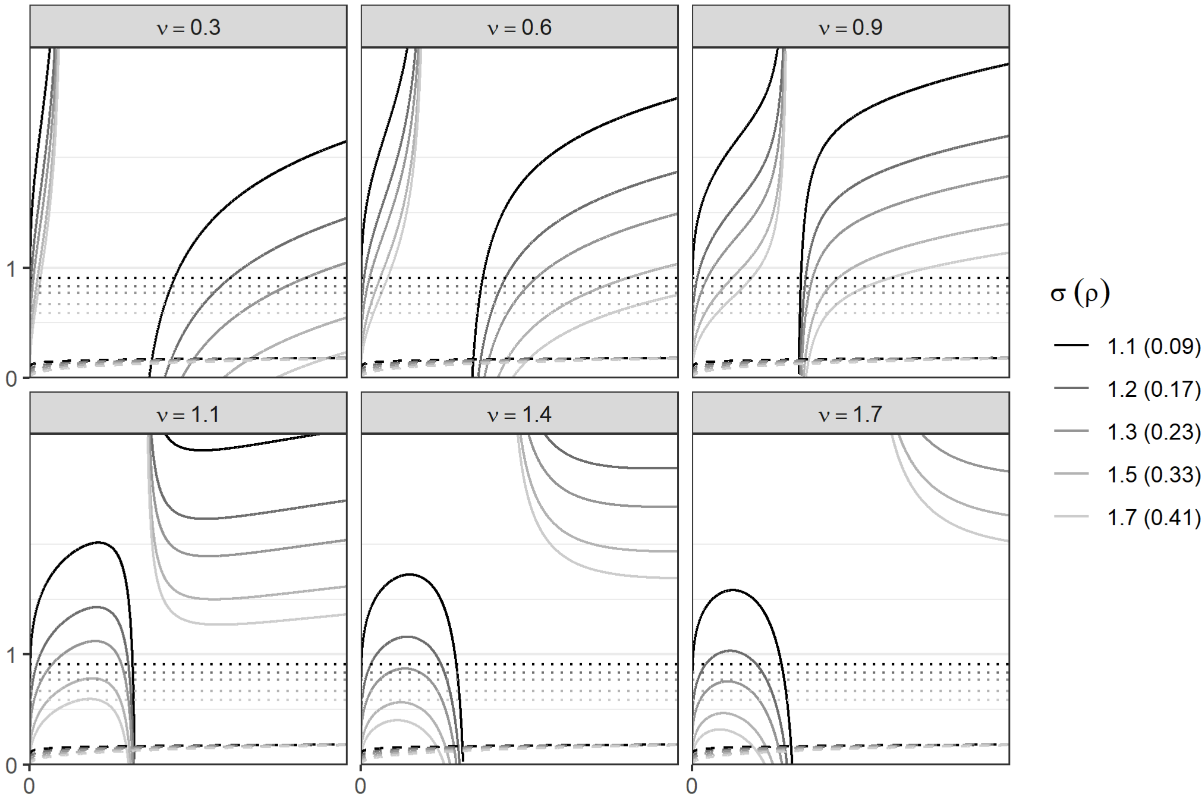
Conjecture 3 *If $\rho \in (0, 1)$, then there exists a unique equilibrium.*

Proof. Conjecture 1 claims that there is a unique equilibrium when $\nu < 1$. Conjecture 2 asserts that there is at least one equilibrium when $\nu > 1$, all lying below unity. Thus, there is at least one equilibrium such that $k_t < k_1$ (or equivalently $\tilde{k}_t < 1$). Yet, proposition 1 states that any equilibrium with $k_t \leq k_1$ leads to the unique equilibrium with full-employment. Therefore, if $\rho \in (0, 1)$ there is a unique equilibrium. ■

B Derivatives

In this appendix, I derive the partial derivatives of the g function and the h function that determine the equilibrium. I focus on the equilibrium with unemployment. Then, I provide conditions on the dynamics as a qualitative analysis.

Figure 12: Numerical simulation of $g(\tilde{k}_t)$ and $h(\tilde{k}_t)$.



Notes: The figure shows the numerical simulations of the g function (solid line) and h function (dashed line) according to the value of the capital-labor elasticity of substitution σ, or equivalently ρ, and the vertical asymptote ν. The dotted lines represent the upper limit of the h function equals to $1-\rho$. The x-axis corresponds to $\tilde{k}_t$.

Derivatives of the g function. The g functions is defined as the LHS of equation (20). Thus,

$$g(L_t, K_t, \eta_t, N_t^y; \phi, \sigma) = \ln\left[\frac{\frac{N_t^y}{L_t} - 1}{\frac{\phi}{1-\phi}\left(\frac{K_t}{L_t}\right)^{\frac{\sigma-1}{\sigma}}\eta_t - 1}\right].$$

The partial derivative with respect to η_t is $g_\eta = 1/(\Theta_t - \eta_t) < 0$. Since $\eta_t/\Theta_t - 1$ corresponds to the denominator within the logarithm of the g function. At an equilibrium with unemployment, the denominator must be positive. Therefore $\eta_t/\Theta_t - 1 > 0$, so that $g_\eta < 0$. The partial derivative with respect to N_t^y is $g_{N^y} = 1/(N_t^y - L_t) > 0$. At the equilibrium an unemployment, $N_t^y > L_t$, thus $g_{N^y} > 0$. The partial derivative with respect to K_t is

$$g_K = -\frac{1}{K_t}\frac{\sigma-1}{\sigma}\left(1-\Theta_t/\eta_t\right)^{-1} \lesseqgtr 0, \ \forall\sigma \gtreqless 1.$$

The sign of the derivative depends on the value of the elasticity of substitution between capital and labor. Lastly, the partial derivative with respect to L_t is

$$g_L = -\frac{1}{N_t^y - L_t} - \frac{1}{\sigma L_t}\left(\eta_t/\Theta_t - \sigma\right)$$

At an equilibrium with unemployment, $\eta_t/\Theta_t > 1 \implies \eta_t/\Theta_t - \sigma > 0,\ \forall \sigma < 1$. Thus, $g_L < 0\ \forall \sigma < 1$. When $\sigma > 1$, the sign cannot be deduced without additional assumptions. A more restrictive condition is required, i.e. $\eta_t/\Theta_t > \sigma$. In such a case, the partial derivative is also unambiguously negative. If this condition is not met, g_L can still be negative provided that the unemployment rate is lower than a threshold $\bar{u}_t = \sigma\Theta_t/(2\sigma\Theta_t - \eta_t)$. Otherwise, g_L is positive.

Derivatives of the h function. The h function is defined as the RHS of equation (20). Thus,

$$h(L_t, K_t; \sigma, \phi, \tilde{\gamma}) = \left[\sigma + \tilde{\gamma}\frac{1-\phi}{\phi}\left(\frac{K_t}{L_t}\right)^{\frac{1-\sigma}{\sigma}}\right]^{-1}$$

with $\tilde{\gamma} = \frac{1-\gamma(1-\sigma)}{\gamma} > 0$. The partial derivative with respect to K_t is

$$h_K = \frac{1}{K_t}\frac{\sigma - 1}{\sigma}\frac{\frac{1-\phi}{\phi}\tilde{\gamma}k_t^{\frac{1-\sigma}{\sigma}}}{\left(\sigma + \frac{1-\phi}{\phi}\tilde{\gamma}k_t^{\frac{1-\sigma}{\sigma}}\right)^2} \gtrless 0,\ \forall \sigma \gtrless 1$$

The partial derivative with respect to L_t is

$$h_L = -\frac{1}{L_t}\frac{\sigma - 1}{\sigma}\frac{\frac{1-\phi}{\phi}\tilde{\gamma}k_t^{\frac{1-\sigma}{\sigma}}}{\left(\sigma + \frac{1-\phi}{\phi}\tilde{\gamma}k_t^{\frac{1-\sigma}{\sigma}}\right)^2} \lesseqgtr 0,\ \forall \sigma \gtrless 1$$

Dynamics at the equilibrium. Writing both functions as a system of equations, I differentiate the system and solve it to obtain

$$dL_t = \frac{1}{h_L - g_L}\Big[(g_K - h_K)dK_t + g_\eta d\eta_t + g_{N^y}dN_t^y\Big].$$

When $\sigma < 1$, h_L and g_L are both positive and therefore $h_L - g_L$ is ambiguous. When $\sigma > 1$, h_L is negative and g_L is also negative as long as the unemployment rate is below $\bar{u}_t$. In this case, $h_L - g_L$ is also ambiguous. The only case where $h_L - g_L$ is unambiguously negative and therefore the dynamics of the model can be solved analytically correspond to the one where $\sigma > 1$ and $u_t > \bar{u}_t$. Thus, $dL_t/dK_t > 0$, $dL_t/dN_t^y < 0$ and $dL_t/d\eta_t > 0$. However, these dynamics do not match those obtained from the simulation in section 3 which suggests that the unemployment rate remains below the threshold $\bar{u}_t$.

C Estimation of the capital-labor elasticity

In this appendix, I estimate the elasticity of substitution between capital and labor with a combination of the first order conditions of the profit maximization. The CES production function with biased technical change as defined by David and van de Klundert (1965) is

$$Y_t = A\left[\left(E_t^K K_t\right)^{\frac{\sigma-1}{\sigma}} + \left(E_t^L L_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where E_t^K and E_t^L represent the efficiency levels of both input factors. I assume linear growth rates of efficiency levels, so that $E_t^i = E_0^i e^{a_i(t-t_0)}$ where a_i denotes growth in technical progress associated with factor $i \in \{K, L\}$ and represents a linear time trend. To normalize the production function, I follow the specification of Klump et al. (2007), hence

$$E_0^K = \frac{Y_0}{K_0}\left(\frac{1}{\phi_0}\right)^{\frac{\sigma}{1-\sigma}} \quad \text{and} \quad E_0^L = \frac{Y_0}{L_0}\left(\frac{1}{1-\phi_0}\right)^{\frac{\sigma}{1-\sigma}}.$$

Normalization of the CES production function requires that factor shares are not biased by the growth of factor efficiencies at the fixed point. At time $t = t_0$, $e^{a_i(t-t_0)} = 1$. Thus, they are just equal to the initial distribution parameters ϕ_0 and $1-\phi_0$. Assuming that the firm is on its labor demand curve, as in the model, the labor share is

$$\theta_t \equiv \frac{w_t L_t}{Y_t} = \left[1 + \left(\frac{E_t^K}{E_t^L}\frac{K_t}{L_t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{-1}.$$

Substituting with both efficiency levels, the labor share becomes

$$\theta_t = \left[1 + \frac{\phi_0}{1-\phi_0}\left(\frac{K_t}{K_0}\frac{L_0}{L_t}e^{(a_K-a_L)(t-t_0)}\right)^{\frac{\sigma-1}{\sigma}}\right]^{-1}.$$

Let $k_t \equiv K_t/L_t$ be the capital-labor ratio. Thus, the labor-to-capital income ratio is

$$\Theta_t \equiv \frac{\theta_t}{1-\theta_t} = \frac{1-\phi_0}{\phi_0}\left(\frac{k_t}{k_0}e^{(a_K-a_L)(t-t_0)}\right)^{\frac{1-\sigma}{\sigma}}.$$

Table 3: Estimation of the capital-labor elasticity of substitution.

	Linear regression							
	France				United States			
	(RAW)	(HWC)	(BTC)	(HWC-BTC)	(RAW)	(HWC)	(BTC)	(HWC-BTC)
α	1.130^{***}	1.123^{***}	1.081^{***}	1.080^{***}	0.636^{***}	0.648^{***}	0.643^{***}	0.649^{***}
	(0.032)	(0.031)	(0.040)	(0.038)	(0.017)	(0.018)	(0.019)	(0.017)
β	-0.614^{***}	-0.470^{***}	-0.273	-0.218	-0.177^{***}	-0.189^{***}	0.155	-0.647^{**}
	(0.051)	(0.039)	(0.186)	(0.144)	(0.061)	(0.055)	(0.405)	(0.286)
γ			-0.007^{*}	-0.007^{*}			-0.004	0.006
			(0.004)	(0.004)			(0.005)	(0.004)
σ	2.593	1.887	1.375	1.279	1.215	1.234	0.866	2.835
R^2	0.789	0.792	0.808	0.809	0.178	0.236	0.193	0.286
Adj. R^2	0.784	0.787	0.798	0.798	0.157	0.216	0.150	0.248
Num. obs.	41	41	41	41	41	41	41	41

Notes: $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$. Standard errors between parentheses.

Rewriting the above equation and taking logs lead to

$$\ln \Theta_t = \alpha + \beta \ln \tilde{k}_t + \gamma \left(t - t_0\right). \tag{23}$$

where $\tilde{k}_t$ is the normalized capital-to-labor ratio, α a constant term, $\beta = (1-\sigma)/\sigma$ and $\gamma = (a_K - a_L)(1-\sigma)/\sigma$. With this form, I cannot identify the technical change growth rates, i.e. a_K and a_L. However, I capture the overall bias in technical change, i.e. $a_K - a_L$.

I estimate equation (23) using OLS for France and the US over the period 1970-2010. This is a single-equation estimation from the two first-order conditions of the profit maximization. Klump et al. (2007) argues that single-equation or two-equations estimations can be biased due to endogeneity and recommend to use supply-side estimation. However, the aim of this estimation is only to obtain a value to simulate the model. Therefore, the elasticity of substitution I obtain is the one within the model specification.

I use data and variables specifications as described in section 3.2. I consider four specifications. The first one is the RAW estimation, without biased technical change nor hours worked correction. Then, I only control for the average hours worked in the HWC estimation and for the biased technical change in the BTC estimation. Finally, both are used as control in the HWC-BTC estimation. Table 3 summarizes the results. Each country has 41 observations. Standard errors are relatively high due to the lack of observations. When the β is not significant, it means that the estimated elasticity is not statistically different from 1, i.e. the Cobb-Douglas case.

For France, worked hours seems to play a role in the estimation of the elasticity when comparing the column (RAW) to (HWC). Not considering the hours worked correction may bias the elasticity of substitution toward relatively high values, i.e. 2.593 against 1.887. Comparing columns (RAW) and (BTC) show that biased technical change has also to be taken into account. Although the coefficient associated to σ is not significant, the coefficient associated to the biased technical change is significant. Lastly, controlling for both in column (HWC-BTC) leads to an elasticity about 1.279 although not significant. In appendix D, I show that this confidence level is due to a break in the regime. The biased technical change coefficient is also significant. Therefore, the biased technical change has to be considered along with the hours worked for France.

For the United States, comparing columns (RAW) and (HWC) shows that hours worked correction does matter. Once I control for it, the elasticity of substitution is about 1.234. However, the technical change does not seem to be biased since both associated coefficients in columns (BTC) and (HWC-BTC) are not significant. The coefficients associated to the elasticity are below one but not statistically significant. Thus, controlling for biased technical change is not necessary for the US. I only have to control for average worked hours. Therefore, I consider a capital-labor elasticity of substitution about 1.279 for France and 1.234 for the United States over the 1970-2010 period in order to calibrate the model.

D Two capital-labor elasticity regimes in France

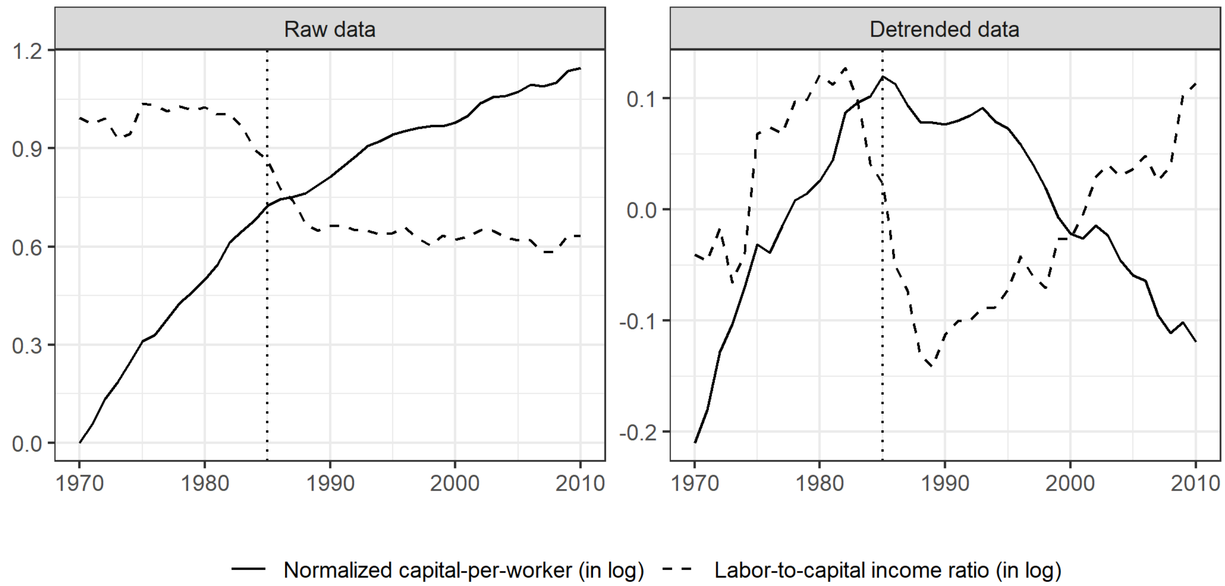
In this appendix, I estimate the elasticity of substitution between capital and labor, σ, for France and show that a break in the regime occurs between 1980 and 1990. Using the estimation methodology in appendix C, I obtain a capital-labor elasticity of substitution of 1.279 for France over the period 1970-2010. However, the estimated coefficient is not significant. Such a result may be due to a break in the regime of σ.

The estimate of the elasticity is based on the relationship between the labor-to-capital income ratio Θ_t and the normalized capital-per-worker $\tilde{k}_t$. The preferred specification controls for average hours worked, thus I consider both variables corrected for hours worked. I start by detrending them to examine whether the change in the regime is driven by biased technical change or not. I regress each variable in logarithm on a linear time trend such that

$$\ln \Theta_t = \varphi_0^{\Theta} + \varphi_1^{\Theta} \times t + \varepsilon_t^{\Theta},$$
$$\ln \tilde{k}_t = \varphi_0^{\tilde{k}} + \varphi_1^{\tilde{k}} \times t + \varepsilon_t^{\tilde{k}}.$$

I extract the residuals ε_t^{Θ} and $\varepsilon_t^{\tilde{k}}$ as detrended variables. Figure 13 plots the raw and

Figure 13: Capital-per-worker and labor-to-capital income ratio in France.



Notes: The figure shows the capital-per-worker and labor-to-capital income ratio time series in France over the period 1970-2010. The first panel presents the data with hours worked correction only, whereas the second panel presents the same data but detrended to account for the biased technical change. The solid line represents the capital-per-worker normalized to 1970 in log, whereas the dashed line represents the labor-to-capital income ratio in log. The dotted line represents the year 1985.

detrended data between 1970 and 2010. On the left-hand side panel, the relationship between both time series seems to be different before and after the vertical line that corresponds to the year 1985. On the right-hand side panel, once I also control for biased technical change, this change in the relationship is all the more striking. Before 1985, capital-per-worker increases along with the labor-to-capital income ratio, while they vary in opposite directions after 1985. This evidence suggests a change in the regime of the elasticity between 1980 and 1990.

To find which year corresponds to the most appropriate break year, I rely on the methodology of Bai and Perron (2003) to estimate breaks in time series regression models; see the original paper for more details. Out of all potential break points, the regression with a change in the regime in 1985 leads to the lower residual sum of squares and rejects the null hypothesis. Thus, I assume that the break in the regime has occurred in 1985.

Lastly, I estimate the elasticity of substitution between capital and labor over the two periods separately, according to equation (23) from appendix C. I consider the hours worked correction (HWC) as the baseline estimate and I provide an estimate in which I also control for the biased technical change (HWC-BTC). Table 4 presents the coefficients. In both (HWC-BTC) columns, the coefficients associated to the biased technical change γ and to the elasticity β are not significant in both sub-sample regressions. Therefore, I rely on the

Table 4: Estimation of the capital-labor elasticity of substitution for France with a break in the regime in 1985.

	Linear regression			
	1970-1985		1986-2010	
	(HWC)	(HWC-BTC)	(HWC)	(HWC-BTC)
α	1.007^{***}	0.970^{***}	0.877^{***}	0.916^{***}
	(0.026)	(0.033)	(0.046)	(0.160)
β	-0.063	1.002	-0.243^{***}	-0.322
	(0.058)	(0.648)	(0.048)	(0.321)
γ		-0.050		0.001
		(0.031)		(0.005)
σ_{FR}	1.067	0.499	1.321	1.476
R^2	0.077	0.236	0.527	0.529
Adj. R^2	0.011	0.119	0.507	0.486
Num. obs.	16	16	25	25

Notes: $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$. Standard errors between parentheses.

coefficients from the (HWC) columns. Before 1985, I set the elasticity equal to one because the associated coefficient is not statistically different from zero. After 1985, the elasticity is set to 1.321.

E Methodology for counterfactual simulations

E.1 Counterfactual and decomposition

In this appendix, I provide details on the methodology for the simulations and decompositions in section 3.4. The benchmark simulation is the one obtained in section 3.3. In what follows, a variable with a prime denotes the new value of this variable that is used in the counterfactual simulation.

Population growth counterfactual simulation. I neutralize the effect of population growth by setting the rate of population growth at its level in 1970, i.e. $n'_t = n_{1970}$. Thus, the number of young households in the first period of each sequence, i.e. from 1970 to 2000, is recalculated to be consistent, such that

$$N_t^{y\prime} = \frac{n'_t}{n_t} \times N_t^y.$$

This change affects demographic dynamics, which are therefore recalculated for the second and third periods of each sequence, i.e. from 2010 to 2080, such that

$$N_t^{y\prime} = n_t' N_{t-1}^{y\prime} \quad \text{and} \quad N_t^{o\prime} = p_t N_{t-1}^{y\prime}.$$

For every year, the political weight of the young is also recalculated such that

$$\eta_t = \frac{p_t}{n_t'} \omega(1 + \alpha p_{t+1}).$$

Survival rate counterfactual simulation. I neutralize the effect of the survival rate to quantify the role of its dynamics on the labor share since the 1970. Therefore, I set the survival rate at its level in 1970 , i.e. $p_t' = p_{1970}$. The expected survival rate p_{t+1} of one generation is the survival rate p_t once this generation becomes old, therefore it implies that $p_{t+1}' = p_t' = p_{1970}$. The number of old households and the capital stock in the first period of each sequence, i.e. from 1970 to 2000, are recalculated such that

$$N_t^{o\prime} = \frac{p_t'}{p_t} \times N_t^o,$$
$$K_t' = \frac{1 + \alpha p_t}{\alpha p_t} \frac{\alpha p_t'}{1 + \alpha p_t'} K_t.$$

The initial capital stocks are recalculated because setting constant the survival rate implies changes in the saving rate as

$$K_t \equiv S_{t-1} = \frac{\alpha p_t}{1 + \alpha p_t} Y_{t-1}^y,$$

where Y_{t-1}^y is the aggregate net income of young households. Thus, not taking into account the change in the saving rate would bias the interpretation of the effect of survival rate dynamics by leaving behind part of the effect that occurs through capital accumulation.

These changes do not affect the dynamics of young households, but only the ones of the old which are therefore recalculated for the second and third periods of each sequence, i.e. from 2010 to 2080, such that

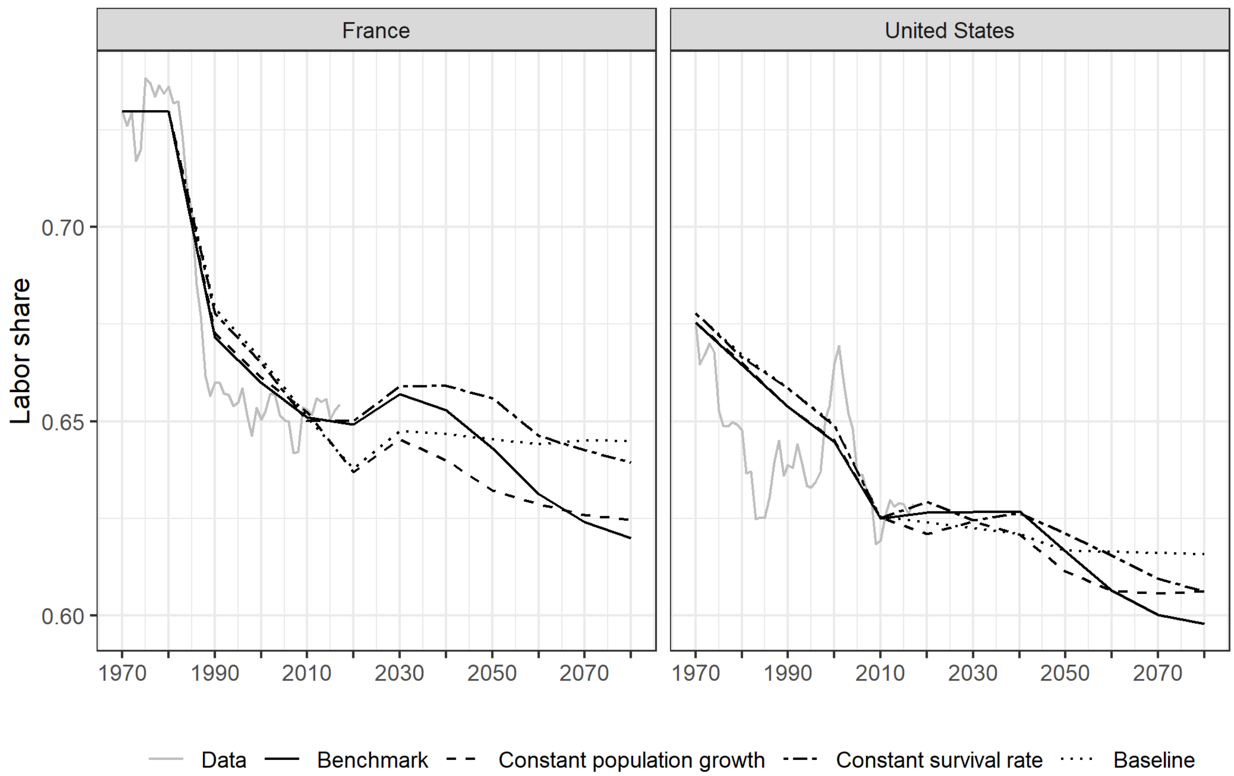
$$N_t^{o\prime} = p_t' N_{t-1}^{y\prime}.$$

For every year, the political weight of the young is also recalculated such that

$$\eta_t' = \frac{p_t'}{n_t} \omega(1 + \alpha p_{t+1}').$$

Baseline counterfactual simulation. I neutralize both effects, therefore, I set $n_t = n_{1970}$, $p_t' = p_{t+1}' = p_{1970}$. This simulation is the combination of the two previous ones. As

Figure 14: Counterfactual simulations of the effects of demographic changes.



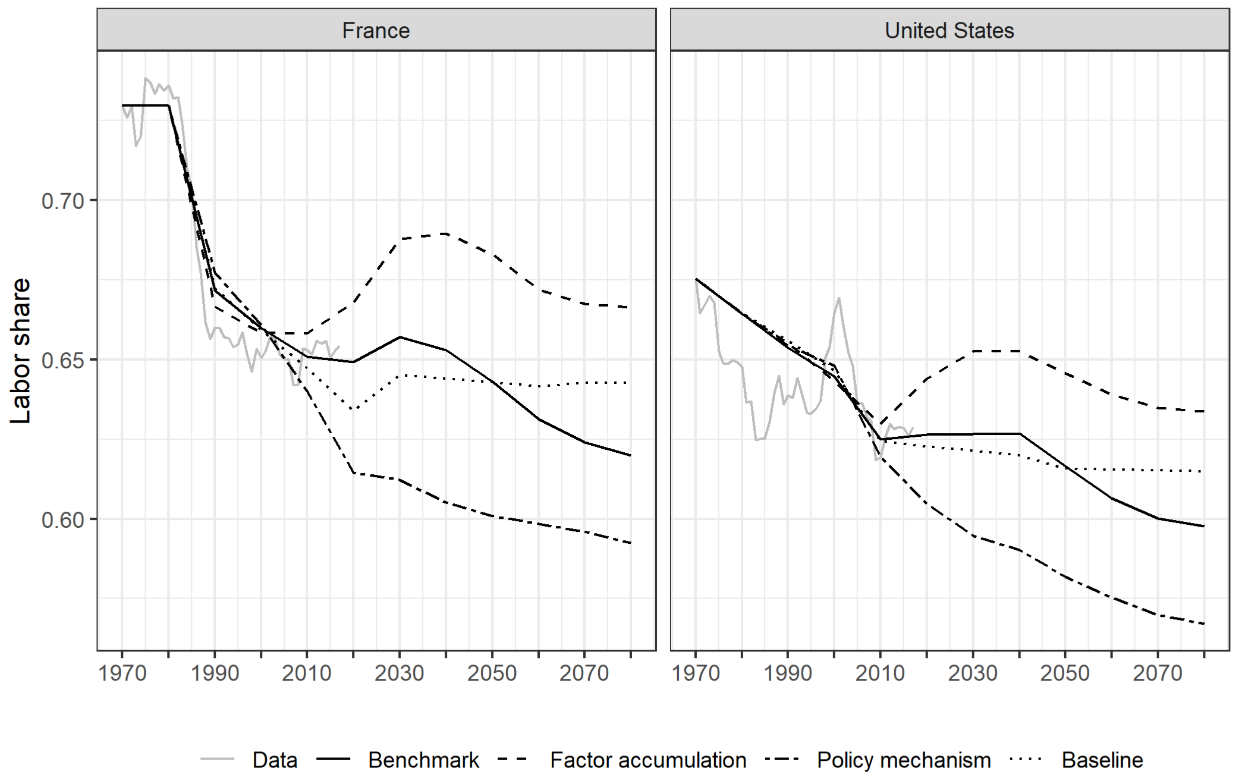
Notes: The figure shows the counterfactual simulations of the effects of the determinants of demographic changes on the labor share. Labor share data are from the Penn World Table 9.1 with self-employed income as labor compensation. The benchmark labor share corresponds to the benchmark predictions of the model. The constant population growth simulation refers to the labor share of the counterfactual simulation in which the population growth rate remains at its level in 1970. The constant survival rate simulation refers to the labor share of the counterfactual simulation in which the survival rate remains at its level in 1970. The baseline labor share corresponds to the predictions when both, the population growth rate and the survival, rate are held constant.

before, the number of young and old households along with the capital stock at first period of each sequence, i.e. from 1970 to 2000, are recalculated. These changes affect the dynamics of young and old households which are therefore recalculated for the second and third periods of each sequence, i.e. from 2010 to 2080. For every year, the political weight of the young remains at its level in 1970.

Population growth versus survival rate. Figure 14 presents the labor share from the four counterfactual simulations, as detailed above. From this figure, I derive the decomposition of the effects of demographic changes, see figure 5.

Factor accumulation versus policy mechanism. I neutralize the factor accumulation effect by reproducing the same changes as in the baseline counterfactual simulation, except that the political weight of the young is not recalculated and therefore remains identi-

Figure 15: Counterfactual simulations of the channels of demographic changes.



Notes: The figure shows the counterfactual simulations of the channels of demographic changes on the labor share. Labor share data are from the Penn World Table 9.1 with self-employed income as labor compensation. The benchmark labor share corresponds to the benchmark predictions of the model. The factor-accumulation simulation refers to the labor share of the counterfactual simulation in which the factor-accumulation channel is neutralized. The policy-mechanism simulation refers to the labor share of the counterfactual simulation in which the policy-mechanism channel is neutralized. The baseline labor share corresponds to the predictions when both channels are neutralized.

cal to the benchmark simulation, i.e. $\eta'_t = \eta_t$. Conversely, I neutralize the policy mechanism effect by setting only the political weight of the young to its level in 1970, i.e. $\eta'_t = \eta_{1970}$ and keeping all demographic parameters and dynamics as in the benchmark simulation. Lastly, the baseline counterfactual simulation is identical to the one above.

Figure 15 presents the labor share from the four counterfactual simulations, as detailed above. From this figure, I derive the decomposition of the channels of demographic changes, see figure 6.

E.2 Retirement age

In this appendix, I provide details on the methodology for counterfactual simulations from section 4.2. From the identity $N_t^o/N_t^y \equiv p_t/n_t$, I have

$$\dot{N}_t^o - \dot{N}_t^y = \dot{p}_t - \dot{n}_t, \tag{24}$$

where the upper dot variables indicate a rate of change, e.g. $\dot{N}_t^o = N_t^{o\prime}/N_t^o - 1$, where $N_t^{o\prime}$ is the new value for N_t^o. Equation (24) has to be satisfied along with the fact that individuals do not vanish, leading to three implications. First, the size of the old population varies as much as the survival rate does, i.e. $\dot{N}_t^o = \dot{p}_t$. Second, variations of the young population are inversely proportional to those of the old population, i.e. $\dot{N}_t^y = -\dot{N}_t^o$. Third, by taking into account the two previous points and that equation (24) holds, it implies that $\dot{n}_t = -\dot{p}_t$. Therefore, the exogenous shock on p_t affects the other demographic parameters with the same magnitude, i.e. $\dot{p}_t = -\dot{n}_t = \dot{N}_t^o = -\dot{N}_t^y$.

The demographics variables are recalculated for the years 2030, 2040, 2050 and 2060 according to:

$$\begin{aligned} p_t' &= (1+\dot{p}_t)p_t, \\ n_t' &= (1-\dot{p}_t)n_t, \\ N_t^{o\prime} &= (1+\dot{p}_t)N_t^o, \\ N_t^{y\prime} &= (1-\dot{p}_t)N_t^y. \end{aligned}$$

For the remaining years 2070 and 2080, n_t' follows the benchmark time series and population sizes are recalculated with $N_t^{o\prime} = p_t' N_{t-1}^{y\prime}$ and $N_t^{y\prime} = n_t' N_{t-1}^{y\prime}$. New values of the expected survival rate p_{t+1}' changes according to p_t'. Moreover, the political weight of the young is also recalculated, i.e. $\eta_t' = n_t'/p_t' \times \omega(1+\alpha p_{t+1}')$.