Inter-generational conflict and the declining labor share[*]

Fabien Petit[†][1]

[1]Aix-Marseille University, CNRS, EHESS, Centrale Marseille, AMSE

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Abstract

A large literature has studied multiple determinants of the labor share, but the age structure of the population has received little attention. This paper argues that demographic dynamics in high-income countries affect the labor share in two different ways: directly through the labor supply and the capital accumulation, and indirectly through public policy. I use an OLG model in which a generation conflict arises because young and old individuals have different income sources and opposite objectives in terms of public policy. The youth face an unemployment risk and use their political power to raise the unemployment benefits. This fight over the public budget allocation has consequences for wage bargaining and thus for the labor share. Numerical simulations for France and the United-States indicate that the model can replicate the data and that baby-boomers' cohorts have driven the labor share dynamics.

Keywords: Labor share, Inter-generational conflict, Wage bargaining, Probabilistic voting.

JEL Codes: E25, E62, J11, J58.

1 Introduction

The labor income share is often assumed as constant by the economists. However, it has decreased during the last decades in OECD countries (Karabarbounis and Neiman 2014). The reasons have been widely studied and debated ranging from the globalization (Jayadev 2007; Pica 2010; Young and Tackett 2018; Autor et al. 2020) to the role of institutions (Blanchard 1997; Bentolila and Saint-Paul 2003) and including the capital biased technical change (Acemoglu 2002; Acemoglu 2003; Caballero and Hammour 1998). Although high-income countries such as France and the United States have experienced changes in the age structure of the population due to the aging of a large cohort called the *baby-boomers*, the literature has paid hardly any attention to the impact of population dynamics on the labor share (except Schmidt and Vosen 2013). Yet, the public budget allocation, which may affect labor market institutions along with the labor share, is the ground of inter-generational conflicts (Busemeyer et al. 2009; Sørensen 2013). Thus, the shift away from labor toward capital may reflect changes in labor market institutions endogenously determined by the age structure of the population. To the best of my knowledge, this paper is the first to investigate the long-run relationship between the demographic dynamics and the labor share through the inter-generational conflict.

I start by presenting the theoretical framework which links the age structure of the population to the labor share. I use a two-period overlapping generations (OLG) model with two types of individuals: young and old. An age-related conflict arises because any welfare improvement for a generation is done at the expense of the other generation. Recent empirical studies present evidence of it since the youth and the elderly do not benefit from the same public policy instruments nor have the same income sources (e.g. Busemeyer et al. 2009; Sørensen 2013).[1] Public policy is endogenous and determined through voting, as it is

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[1]Sørensen (2013) shows that elderly people desire less spending in education while they are in support of more in health and pension. But he also claims that these life-cycle effects are quite small. On the contrary, Busemeyer et al. (2009) find sizable age-related differences in public policy preferences. Although these studies disagree on the magnitude of the conflict, they both claim that such a conflict does exist. They also agree on the heterogeneity across countries.

usual in the literature of politico-economic models (e.g. Alesina and Rodrik 1994; Persson and Tabellini 1994; Krusell et al. 1997). The larger is a generation with respect to the other, the stronger is its political power and therefore the closer to its desired public policy is this generation, as in Gonzalez-Eiras and Niepelt (2012).[2] The greater is the political power of the youth, the greater is the redistribution and the unemployment benefits (e.g. Razin et al. 2002; Jäger and Schmidt 2016).[3] These unemployment benefits represent the outside option in the bargaining of the representative labor union with the representative firm over the wages. Thus, greater unemployment benefits enable the firm to bargain greater wages (e.g. Bentolila and Saint-Paul 2003). But, it also reduces the labor demand of the representative firm and thus increases the capital-per-worker and output-per-worker. Since the labor share is defined as the ratio between the wage rate and output-per-worker, the effect on the labor share depends on the value of the capital-labor elasticity of substitution. The equilibrium is determined by the interaction of the voting and the wage bargaining. The total effect of demographic dynamics on the labor share passes through three variables: *i)* the capital stock determined by the savings of the previous young generation; *ii)* the labor supply which is taken into account in the wage bargaining; and *iii)* the youth political power that defines the level of the out-of-work option and thus the ability of workers to increase their wages. However, the total effect is ambiguous and depends on parameter values.

Then, I turn to the quantitative analysis for France and the United-States. The calibration of the parameters leads to an elasticity of substitution between capital and labor greater than one, as Caballero and Hammour (1998).[4] This is consistent with recent estimates sug-

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[2]My approach is closely linked to Gonzalez-Eiras and Niepelt (2012) although applied to the labor share. They distinguish two effects: a direct one through the savings rate, labor supply and capital accumulation, and an indirect one through the age-related conflict to determine taxes, government spending and the retirement age. They predict an increase in the tax rate and the retirement age in OECD countries to offset population aging which should boost per-capita growth. I use the same decomposition into the direct and indirect channels.

[3]Razin et al. (2002) show that an increase in the old-age dependency ratio can lead to lower tax rates. In the same spirit, Jäger and Schmidt (2016) find a negative long-run relationship between population aging and public investment because older individuals discount more future payoffs with respect to young individuals.

[4]Caballero and Hammour (1998) also use a relatively high value of the capital-labor elasticity of substitution, about 6.00, to simulate French data.

gesting that this elasticity may be greater than one (e.g. Karabarbounis and Neiman 2014).[5] Thus, both input factors are gross substitutes which meaning a rise in the price of a factor reduces the firm's demand for this factor while the demand for the other factor increases.

The model is able to replicate the data since 1970 and predicts a slight rise of the labor share due to the aging of the baby-boomers' cohort. This cohort drives the public policy agenda and hence the labor share. When the baby-boomers are young, their massive entrance in the labor market increases the labor supply. They also shape institutions in their favor through the voting process, leading to greater wages. As a response, firms shift away from labor toward capital to thwart the workers' ability to grab part of the income. Thus, the labor share declines. Once this generation becomes old, the mechanism is reversed. However, the expected increase in the labor share is dampened by the capital accumulation fostered by the extensive savings of the baby-boomers when they were young. Hence, the model reproduces the mechanism of Caballero and Hammour (1998) without any biased technical change but with endogenous labor market institutions.[6] I interpret it as the fact that biased technical change is also driven by demographic dynamics through the ability of workers to seize part of the rent. This *grability* arises when a young cohort is sufficiently numerous to shape the labor market institutions in its favor and therefore in favor of labor. I also quantify the role of each determinant and channel. Demographic dynamics are determined by the population growth and the survival rate, i.e. the life expectancy. The rising survival rate is the dominant explanatory factor in both countries. Then, the age structure of the population affects the labor share in two different ways: directly through the labor supply and the capital accumulation, and indirectly through the endogenous public policy. Model predictions suggest that the indirect channel should play a considerable role in the next

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[5]But an other part of the literature has found it below one, particularly for the United-States (e.g. Antràs 2004; Chirinko 2008).

[6]Caballero and Hammour (1998) argue that pro-labor income institutions are a burden to firms because they limit the firms' ability to optimize input factors' allocation but also because they enable the workers to obtain a high income share in the short-run. In their model, they incorporate the long-run response of firms in order to thwart workers empowerment. This response is the substitution of labor with capital through biased technical change.

decades due to the retirement of the baby-boomers.

In the last part of the paper, I discuss two points of the results. First, although the baby-boomers appear as income losers over the last decades because the labor share falls, they are actually the winners once net income is considered due to the implementation of a redistributive public policy. Once they retire, they are still the winners of the public budget allocation's inter-generational conflict because the redistribution works the other way around. Second, I find that an increase of the retirement age in the next decades should lead to a decline of the labor share due to capital over-accumulation because the agents expected to remain longer retired and therefore saved more. Meanwhile, the youth's political power increases and so does the wage. Thus, the firm substitutes labor with the cheap over-accumulated capital leading to the decline of the labor share. However, once the capital comes back to its optimal accumulation path, so in the very long-run, the labor share increases again.

To the best of my knowledge, Schmidt and Vosen (2013) is the only existing paper looking at the impact of population aging on the labor share. They use an OLG model with a pension system to show that an aging population leads to more saving and hence more capital. For a capital-labor elasticity of substitution greater (resp. smaller) than unity, the effect on the labor share is negative (resp. positive). Their paper points out a link between the population aging and the labor share through capital accumulation.[7] My paper includes this mechanism and reaches the same conclusions on it. However, I add a new mechanism through the inter-generational conflict over public budget allocation and show that it accounts for more than the half of the dynamics.

The paper is organized as follows. Section 2 presents the theoretical framework that links demographic changes to labor share dynamics. Section 3 provides the quantitative analysis. It starts with the analysis of the model's predictions and mechanisms. I then perform an

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[7]They also show that population aging reduces the labor share in a small economy with perfect capital mobility, regardless of the value of capital-labor elasticity of substitution. Households invest abroad because domestic interest rates fall. As a result, the increasing net foreign assets income shrinks the labor share.

aging-effect decomposition with counterfactual simulations. Section 4 discusses some results of the paper. Section 5 concludes.

2 Theoretical framework

I consider a two-period OLG model in which there are two types of individuals: young and old. The economy is closed and capital fully depreciates between two periods. Agents vote to establish the public policy through probabilistic voting. Public policy, and more specifically labor market institutions, impact wage bargaining between the representative firm and the representative union. While at the same time, the representative union bargains over wages with the representative firm. The outcome of the bargaining, i.e. the wage rate, determines the level of employment and thus the tax base to fund public policy. Both processes are independent and simultaneous, hence they jointly determine the equilibrium of the economy and therefore the labor share. Thus, I assume that there is no coordination between households and the labor union. Households cannot synchronize their vote with the action of the representative union, neither the opposite. Therefore, the agents who operate within the wage bargaining take the voting outcome as given and vice versa.

2.1 Households

The demographic dynamic of young households is $N_t^y = n_t N_{t-1}^y$ with $n_t > 0$ the gross young population growth. While the demographic dynamic of old households is $N_t^o = p_t N_{t-1}^y$ with $p_t \in (0, 1]$ the survival rate.[8] Both demographic parameters follow deterministic processes. Thus, the old-age dependency ratio is $N_t^o/N_t^y = p_t/n_t$. The whole population is therefore $N_t = N_t^y + N_t^o$.

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[8]The survival rate p_t is an increasing function of the life expectancy and a decreasing function of the retirement age. In the model, agents are considered as old once they retire. If the life expectancy and the retirement age grow at the same rate, then the survival rate remains constant. For more details on the measurement of population aging, see Sanderson and Scherbov (2007); Sanderson and Scherbov (2013); D'Albis and Collard (2013).

Each cohort consists of a continuum of agents with identical preferences. Young households in period t supply labor inelastically, earn a net income, consume and save for retirement. They face an idiosyncratic longevity risk: with probability p_{t+1} they survive to become old households in period $t+1$. Once old, they pay taxes on their saving returns, consume and derive utility from government health spending. Finally, old households die at the end of the period. Savings of young agents who die before reaching old age are distributed among their surviving peers, reflecting a perfect annuities market.

Households derive utility from consumption when young $c_{1,t}$ and old $c_{2,t+1}$. Once old, they also value government health expenditure h_{t+1}. Agents discount the future at factor $\alpha \in (0,1)$. Due to risk of death, the effective discount factor of young households equals αp_{t+1}.[9] I assume that period utility functions are logarithmic. In first period, they earn a net income y_t to allocate between consumption and savings s_t. In second period, they consume the net return of their savings $(1-\tau_{t+1})s_t\hat{R}_{t+1}$, where τ_{t+1} is the tax rate and $\hat{R}_{t+1}$ the gross return on savings of a young household that survives to old age[10]. Maximizing expected utility, a household in period t solves the following maximization problem:

$$\max_{c_{1,t},\ c_{2,t+1}} \ln c_{1,t} + \alpha p_{t+1}\left(\ln c_{2,t+1} + \beta \ln h_{t+1}\right)$$

$$\text{s.t.} \begin{cases} c_{1,t} + s_t = y_t \\ c_{2,t+1} = (1-\tau_{t+1})s_t\hat{R}_{t+1} \end{cases}$$

where $\beta > 0$ characterizes the preference for health expenditure. Solving the household maximization problem, I obtain the optimal consumption in both period and savings in first

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[9]Since the expected survival rate p_{t+1} is an increasing function of the expected life expectancy. The longer an agent expects to live, the less they discounts future.

[10]Due to perfect annuity market $\hat{R}_t = R_t/p_t$ where R_t is the gross return on physical capital.

period for a household of type $i \in \{e, u\}$:

$$c_{1,t}^i = \frac{1}{1 + \alpha p_{t+1}} y_t^i$$

$$c_{2,t+1}^i = \frac{\alpha p_{t+1}}{1 + \alpha p_{t+1}} (1 - \tau_{t+1}) \hat{R}_{t+1} y_t^i$$

$$s_t^i = \frac{\alpha p_{t+1}}{1 + \alpha p_{t+1}} y_t^i$$

where e corresponds to employed young agents and u to the unemployed ones. Since the utility function is logarithmic, this is a standard result where the saving is a constant proportion of the income. Each household faces an idiosyncratic unemployment risk with probability $u_t \in [0, 1)$. The income of an employed young agent is $y_t^e = (1 - \tau_t) w_t$, while an unemployed young agent earns $y_t^u = b_t$, where w_t is the wage and b_t the unemployment benefits per capita. The aggregate saving in the economy is $S_t = (1 - u_t) N_t^y s_t^e + u_t N_t^y s_t^u$. Thus,

$$S_t = \frac{\alpha p_{t+1}}{1 + \alpha p_{t+1}} \left[(1 - u_t)(1 - \tau_t) w_t + u_t b_t \right] N_t^y \tag{1}$$

At the beginning of each period, a voting takes place to determine the public policy. Agents maximize their indirect utility function to determine the tax rate τ_t, the unemployment benefits per capita b_t and the government health spending per capita h_t. The indirect utility of a young agent at time t is:

$$U_t^{y,i} = \ln \left(\frac{1}{1 + \alpha p_{t+1}} y_t^i \right) + \alpha p_{t+1} U_{t+1}^{o,i} \quad \forall i = \{e, u\} \tag{2}$$

While the indirect utility of an old agent at time t depends on their income in $t-1$. Hence,

$$U_t^{o,i} = \ln \left(\frac{\alpha p_t}{1 + \alpha p_t} (1 - \tau_t) y_{t-1}^i \hat{R}_t \right) + \beta \ln h_t \quad \forall i = \{e, u\} \tag{3}$$

Using equation 2, I compute the gap between employed and unemployed people in utility

terms and obtain:

$$U_t^{y,e} - U_t^{y,u} = (1 + \alpha p_{t+1}) \ln \left[\frac{(1 - \tau_t)w_t}{b_t} \right] \tag{4}$$

Young individuals are not aware of their employment status before to vote. Hence, they have to maximize their indirect utility with expectations on their income. The expected income of each young individual is a weighted average of both incomes (labor income and unemployment benefits). Thus, $\mathbb{E}(y_t) = (1 - u_t)(1 - \tau_t)w_t + u_t b_t$, where $\mathbb{E}$ is the expectation operator. Computing the expected utility of a young household at time t, I obtain:

$$\mathbb{E}(U_t^y) = \ln \left(\frac{\mathbb{E}(y_t)}{1 + \alpha p_{t+1}} \right) + \alpha p_{t+1} \left[\ln \left(\frac{\alpha p_{t+1}}{1 + \alpha p_{t+1}} (1 - \tau_{t+1}) \mathbb{E}(y_t) \hat{R}_{t+1} \right) + \beta \ln h_{t+1} \right] \tag{5}$$

2.2 Production

Firms are represented by a representative firm that uses a standard CES production function given by:

$$Y_t = A \left[\phi K_t^{\frac{\sigma-1}{\sigma}} + (1 - \phi) L_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{6}$$

where K_t is the capital stock, L_t the labor, σ the elasticity of substitution between capital and labor, ϕ the factor share parameter allowing the relative importance of inputs in production to vary and A a constant scale parameter. I do not consider any form of technical change, whether neutral or biased. Rewriting the production function in units of labor,

$$\frac{Y_t}{L_t} = A \left(\phi k_t^{\frac{\sigma-1}{\sigma}} + 1 - \phi \right)^{\frac{\sigma}{\sigma-1}} \tag{7}$$

where $k_t \equiv K_t/L_t$ is the capital-per-worker (i.e. capital intensity). Since the economy is closed and capital fully depreciates between two periods, the capital in the economy is determined by the saving of the previous period, i.e. $K_t = S_{t-1}$. The labor-demand equation

obtained from the profit maximization of the representative firm is:

$$w_t = (1 - \phi)A \left(\phi k_t^{\frac{\sigma-1}{\sigma}} + 1 - \phi \right)^{\frac{1}{\sigma-1}} \tag{8}$$

The labor demand elasticity is defined as $\mathcal{E}_t^{L,w} = \frac{\partial L_t}{\partial w_t} \frac{w_t}{L_t}$. Using the equation (8),

$$\mathcal{E}_t^{L,w} = -\sigma \left(1 + \frac{1-\phi}{\phi} k_t^{\frac{1-\sigma}{\sigma}} \right) \tag{9}$$

Using equations (7) and (8), I compute the labor share $\theta_t = \frac{w_t L_t}{Y_t}$ which is:

$$\theta_t = \left(1 + \frac{\phi}{1-\phi} k_t^{\frac{\sigma-1}{\sigma}} \right)^{-1} \tag{10}$$

This equation holds if and only if the representative firm is on its labor demand curve. Notice that when the capital-labor elasticity of substitution equals unity, then $\theta_t = 1 - \phi$. In this case, the production function is Cobb-Douglas and the labor income share is constant in k_t. When $\sigma \to 0$, the production function is Leontief. When $\sigma \to +\infty$, the production function is linear. As long as the representative firm is not constrained in its ability to hire and fire workers, it means that the firm is on its labor demand curve and thereby the labor share only depends on k_t.[11] With the equation (10), I compute the labor-to-capital income ratio $\Theta_t = \frac{\theta_t}{1-\theta_t}$ which is:

$$\Theta_t = \frac{1-\phi}{\phi} k_t^{\frac{1-\sigma}{\sigma}} \tag{11}$$

The comparative statics are straightforward,

$$\left\{ \frac{\partial w_t}{\partial k_t} > 0, \quad \frac{\partial (Y_t/L_t)}{\partial k_t} > 0, \quad \frac{\partial \theta_t}{\partial k_t} \lessgtr 0 \right\}, \quad \sigma \gtrless 1$$

A higher capital-labor ratio increases the wage resulting from the labor demand equation

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[11]Considering a small open economy (SOE), the capital-labor ratio becomes constant. Schmidt and Vosen (2013) show that, in such a case, the labor share dynamics only reflects variations in net foreign assets of the economy.

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and the production-per-worker. However, the impact on the labor share depends on the elasticity of substitution between both input factors. When capital and labor are gross substitutes (i.e. $\sigma > 1$), a higher capital-labor ratio decreases the labor share. While the relationship is positive when both input factors are gross complement (i.e. $\sigma < 1$). Since the labor share can be defined as the ratio between wage and production-per-worker, it means that with gross substitutes input factors, a rise of the capital-per-worker increases more the production-per-worker relatively to the wage rate. Vice versa when $\sigma < 1$. Thus,

$$\left\{ \ \frac{\partial w_t}{\partial k_t} \ \lessgtr \ \frac{\partial (Y_t/L_t)}{\partial k_t} \ \right\}, \ \ \sigma \gtrless 1$$

In order to have a negative relationship between the capital-to-labor ratio and the labor share without the presence of biased technical change, both input factors must be gross substitutes (i.e. $\sigma > 1$).

2.3 Public policy preferences

The government taxes the labor income of young households and the savings return of old households in period t at rate τ_t. Government revenue is spent in unemployment benefits and health expenditure, so that the government budget constraint is given by $\tau_t \left(w_t(1 - u_t)N_t^y + R_t S_{t-1} \right) = b_t u_t N_t^y + h_t N_t^o$. Capital fully depreciates between two periods, thus $R_t = r_t$ and $K_t = S_{t-1}$, where r_t is the rental price of capital, hence the government budget constraint can be written as $\tau_t Y_t = b_t u_t N_t^y + h_t N_t^o$.

Young and old agents vote at the beginning of each period. The youth desire more unemployment benefits since they cover the risk to be unemployed. While the elderly desire more health spending because they derive utility from it. *Everything else equal*, both desire less taxes as it reduces their disposable income. Therefore, young and old agents have different objectives on public policy preferences. I model the inter-generational conflict over public budget allocation with this trade-off but it could be extended to other public policy

instruments. The central point is to oppose young and old individuals with different returns in utility terms of policy instruments.

Sørensen (2013) analyzes cross-section survey data for 22 countries and shows that agents change their public spending preferences over their life-cycle. Thus, he provides evidence for an age-related selfishness in public spending preferences. In line with these results, I consider a probabilistic voting setup in contrast to the median voter setup.[12]

With probabilistic voting, all agents vote for a policy platform $g_t = (\tau_t, b_t, h_t)$ represented by opportunistic candidates (or parties). Candidates try to maximize their probability of winning the election. They differ in their popularity and there is an idiosyncratic bias among voters for one candidate or the other. Candidates know about these biases. At equilibrium, all candidates choose the same policy platform $g_t^\star$ that maximizes the political objective function $W_t(g_t)$.[13]

This function depends on the population share of voters' groups and their respective sensitivity to policy changes ω^j with $j \in (y, o)$.[14] There are three groups of voters: young households; and old households who are divided in two subgroups according to their employment situation when young. I assume that all the elderly have the same sensitivity $\omega^o = \omega^{o,u} = \omega^{o,e}$. Thus, the equilibrium public policy $g_t^\star$ maximizes the following function:

$$W_t(g_t) = \frac{N_t^y}{N_t}\omega^y \mathbb{E}\left[U_t^y(g_t)\right] + \frac{N_t^o}{N_t}\omega^o \left\{u_{t-1}U_t^{o,u}(g_t) + (1 - u_{t-1})U_t^{o,e}(g_t)\right\}$$

Subject to the indirect utilities implied by the equations (3) and (5); and to the government budget constraint. I assume that individuals only care about direct effects of public policy on their utility. They do not consider the indirect effects operating through unemployment,

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[12]The median voter setup would create two extreme regimes with one of them being a gerontocracy. It would also generate large changes in public policy if the median-voter switches from young to old or vice versa. Under probabilistic voting, the equilibrium policy platform is a continuous function of the old-age dependency ratio. This setup smooths public policy changes in line with the literature of recent politico-economic models.

[13]See Lindbeck and Weibull (1987) for more details on the probabilistic-voting setup.

[14]ω^j denotes the density parameter of the uniform distribution function that characterizes the ideology of the j group. The greater ω^j, the more spread are the ideologies withing the j group. Hence, opportunistic candidates prefer targeting less ideological groups, i.e. large ω^j, because they are easier to convince.

wages and the accumulation of capital.[15] Let $\tilde{U}_t^i$ be the part of the utility which is directly affected by the public policy platform. From (3), it is straightforward that $\tilde{U}_t^o = \tilde{U}_t^{o,u} = \tilde{U}_t^{o,e}$. Therefore, the political objective function can be rewritten as:

$$W_t(g_t) = \frac{N_t^o}{N_t}\omega^o\tilde{U}_t^o(g_t) + \frac{N_t^y}{N_t}\omega^y\tilde{U}_t^y(g_t) + \text{other terms}$$

where *other terms* encompasses all the terms that are not directly affected by the public policy. Let ω be the *relative ideological spread-out* of the elderly with respect to the youth. This relative ideological spread-out is characterized by the ratio of the sensitivities of voting behavior to policy changes for each group. Thus, $\omega \equiv \omega^o/\omega^y$. I assume that this spread-out is constant over time.[16] Using equations (3) and (5), I rewrite the maximization program that characterize the public policy equilibrium as:

$$\max_{\tau_t,b_t,h_t} W_t(\tau_t,b_t,h_t) = \ln(1-\tau_t) + \beta\ln(h_t) + \eta_t \ln\left[(1-u_t)(1-\tau_t)w_t + u_tb_t\right] + \text{other terms}$$

$$\text{s.t. } \tau_t Y_t = b_t u_t N_t^y + h_t N_t^o$$

where

$$\eta_t = \frac{n_t}{p_t}\frac{1+\alpha p_{t+1}}{\omega} \tag{12}$$

is the political weight of the youth. This variable is the channel through which the age structure affects the public policy. It depends negatively on the old-age dependency ratio p_t/n_t, the older the population the lower the political weight of the youth within the policy determination. It also depends negatively on the relative ideological spread-out ω. The less ideological is the youth, the higher will be its political weight. If young people are less

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[15]An interpretation of this assumption is that agents are not able to predict what will be the response of the labor market to changes in public policy. Otherwise stated, the investment of effort to be perfectly informed and able to calculate the indirect effects of different public policies is too costly for an individual (or at least a representative share of the electorate). Thus, rationality is limited to the direct effects on the utility function.

[16]Two interpretations for this assumption are plausible: either both relative ideological spread-outs are time invariant; or they vary in same proportions. An interesting point would be to endogenize these spread-outs or to make them cohort-specific. Nevertheless, this goes beyond the scope of this paper.

ideologically tenacious, then it is easier for the opportunistic candidates to get their votes with an appropriated public policy. As a consequence, candidates pay more attention to them.[17] η_t is increasing in αp_{t+1}, corresponding to the fact that young voters are more likely to swing in favor of a pro-labor-income policy platform. The longer they expect to live, the more they will have to save in order to smooth their consumption, therefore, they require a relatively greater first-period income.

Focusing on the interior solution, the first order conditions lead to the following public policy preferences:

$$\frac{b_t}{(1-\tau_t)w_t} = \frac{1-u_t}{u_t}\left(\frac{\eta_t}{\Theta_t} - 1\right), \tag{13}$$

$$\tau_t = 1 - \left[(1-\theta_t)(1+\beta+\eta_t)\right]^{-1}, \tag{14}$$

$$h_t N_t^o = \frac{\beta}{1+\beta+\eta_t}Y_t, \tag{15}$$

where the first equation is the unemployment replacement rate, the second the tax rate and the third the total health expenditure. The comparative statics are straightforward,

$$\left\{\frac{\partial \frac{b_t}{(1-\tau_t)w_t}}{\partial \eta_t} > 0, \ \frac{\partial \tau_t}{\partial \eta_t} > 0, \ \frac{\partial h_t}{\partial \eta_t} < 0\right\}.$$

Young generations desire more redistribution and a higher unemployment replacement rate because they face an unemployment risk. Such a public policy allows them to increase their expected income. However, they vote to reduce the government health spending because they do not derive utility from it yet.[18]

The aggregate income of young households is $Y_t^y = [(1-u_t)(1-\tau_t)w_t + b_t u_t] N_t^y$. Using

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[17]Otherwise stated, the less spread-out is the ideology distribution within a group, the more numerous are the voters that swing after changes in public policy platform.

[18]I do not consider any form of explicit altruism. However, the parameter β which is the preference for government health spending captures a form of implicit altruism from young to their elders. The greater the parameter, the more individuals care about government health spending once old. Finally, a form of explicit altruism from young to old generations would smooth the results and soften the age-related conflict.

equation (13) and (14), it can be written as a share of the total income Y_t such that

$$\frac{Y_t^y}{Y_t} = \frac{\eta_t}{1 + \beta + \eta_t}.$$

Notice that total income is split between the incomes of both groups and health spending. Thus, $Y_t = Y_t^y + Y_t^o + h_t N_t^o$. Hence, using equation (15), the elderly income share is given by

$$\frac{Y_t^o}{Y_t} = \frac{1}{1 + \beta + \eta_t}.$$

For a given level of total income Y_t, the comparative statics are unambiguous. When the youth's political weight raises, young individuals increase their income share with more redistribution, i.e. $\partial(Y_t^y/Y_t)/\partial\eta_t > 0$. Conversely, the elderly income share decreases and so does the government health spending, i.e. $\partial(Y_t^o/Y_t)/\partial\eta_t < 0$ and $\partial(h_t N_t^o/Y_t)/\partial\eta_t < 0$. A higher η_t can be due to two changes: either a decrease of the old-age dependency ratio $(p_t/n_t \searrow)$ or an increase of the survival rate of the current young generation $(p_{t+1} \nearrow)$. Furthermore, it is possible to express the after-tax income ratio between young and old households as:

$$\frac{Y_t^y}{Y_t^o} = \eta_t \tag{16}$$

As expected, the greater is the political weight of the young, the greater the redistribution and the larger the gap between both incomes.

2.4 Wage bargaining

These public policy instruments interact with the outcome of the wage bargaining to determine the equilibrium of the economy. There is a single union that represents the workers. In order to model wage bargaining, I consider a "Right-to-manage" model *à la* Nickell and Andrews (1983). The union bargains only over wages with the employer retaining the pre-

rogative to hire and fire.[19] Consequently, the firm is always on its labor demand curve and equation (8) holds. The union wants to maximize the workers' utility compared to the unemployed, i.e. $L_t \left(U_t^{y,e} - U_t^{y,u} \right)$. The representative firm maximizes its outside option which is $Y_t - w_t L_t$ corresponding to $r_t K_t$. Thus, the outcome of the bargaining process is determined by the following maximization program

$$\max_{w_t > 0} \quad \{ (L_t [U_t^{y,e} - U_t^{y,u}])^\gamma \, (Y_t - w_t L_t)^{1-\gamma} \}$$

$$\text{s.t.} \quad U_t^{y,e} - U_t^{y,u} = (1 + \alpha p_{t+1}) \ln \left[\frac{(1-\tau_t) w_t}{b_t} \right],$$

where $\gamma \in (0,1)$ is the relative bargaining power of the union. The first-order condition can be expressed as

$$\frac{\gamma}{1-\gamma} \left(\mathcal{E}_t^{L,w} \ln \left[\frac{(1-\tau_t) w_t}{b_t} \right] + 1 \right) = \ln \left[\frac{(1-\tau_t) w_t}{b_t} \right] \frac{w_t L_t}{Y_t - w_t L_t},$$

where $\mathcal{E}_t^{L,w}$ is the labor demand elasticity. Using equations (9) and (10), I can express the optimal capital-per-worker as a function of the unemployment replacement rate:

$$k_t(X_t) = \left[\frac{1-\phi}{\phi} \frac{1-\gamma(1-\sigma)}{\gamma} \frac{X_t}{1-\sigma X_t} \right]^{\frac{\sigma}{\sigma-1}} \tag{17}$$

where $X_t = \ln \left[\frac{(1-\tau_t) w_t}{b_t} \right]$ is the value-added of being employed (in utility terms) which only depends on the unemployment replacement rate. Since capital-per-worker has to be positive, it implies that $1 - \sigma X_t > 0$. The maximum capital-labor elasticity of substitution ensuring that this condition is met can be defined as the function $\bar{\sigma}(x_t) = -\log^{-1}(x_t)$, where x_t is the unemployment replacement rate. The lowest unemployment replacement rates for France and the United-States over the period 1970-2010 correspond, respectively, to 0.592

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[19]Another possibility would have been to consider an "Efficient contract" model *à la* McDonald and Solow (1981) where the union bargains over wages and employment. However, this specification does not fit well the data as Bentolila and Saint-Paul (2003) showed. Moreover, it would add a lot of complexity to the model without a substantial gain in the analysis.

and 0.567. Hence, the condition is met for all σ between zero and 1.907 for France or 1.762 for the United-States.[20] In what follows, I assume that $1 > \sigma X_t$ is always satisfied.

The comparative statics with respect to X_t are as follows

$$\left\{ \frac{\partial k_t}{\partial X_t} \gtreqless 0 \Rightarrow \left\{ \frac{\partial w_t}{\partial X_t} \gtreqless 0, \ \ \frac{\partial (Y_t/L_t)}{\partial X_t} \gtreqless 0, \ \ \frac{\partial w_t}{\partial X_t} < \frac{\partial (Y_t/L_t)}{\partial X_t} \right\} \Rightarrow \frac{\partial \theta_t}{\partial X_t} < 0 \right\}, \ \ \sigma \gtreqless 1$$

When the unemployment replacement rate $\frac{b_t}{(1-\tau_t)w_t}$ increases, the value-added of having a job X_t falls. Hence, the labor share θ_t increases. The intuition is that when the unemployment replacement rate increases, it allows the union to bargain a higher wage and so to grab a relatively larger share of income. This mechanism holds whatever the value of the capital-labor elasticity of substitution σ. However, intermediate partial derivatives consider only the partial effect of a change in the unemployment replacement rate.

2.5   Equilibrium

Voting and wage bargaining take place simultaneously and there is no-coordination between the two. As a consequence, both processes take the outcome of the other as given. The interaction determines the equilibrium of the economy. Expressing equations (13) and (17) as function of the capital-labor ratio. We have that equilibrium is given by the solution to the following system:

$$X_t = \ln \left(\frac{\frac{N_t^y}{K_t} k_t - 1}{\frac{\phi}{1-\phi} k_t^{\frac{\sigma-1}{\sigma}} \eta_t - 1} \right), \tag{18}$$

$$X_t = \left(\sigma + \frac{1-\phi}{\phi} \frac{1-\gamma(1-\sigma)}{\gamma} k_t^{\frac{1-\sigma}{\sigma}} \right)^{-1}, \tag{19}$$

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[20]Unemployment replacement rate data are from the Comparative Welfare Entitlements Dataset 2 (CWED 2). These values correspond to a single-type household. With a family-type household, the lowest replacement rates are 0.593 and 0.584, respectively, for France and the United-States. Thus, the condition is met for all σ between 0 and 1.914 for France or 1.859 for the United-States. See Scruggs et al. (2017) for more details.

which yields the equilibrium capital-labor ratio and employment value-added. Note that at time t, the aggregate capital stock K_t is predetermined by the saving decisions at $t-1$, so that k_t is given by L_t.

The Cobb-Douglas case, i.e. $\sigma = 1$, can be solved analytically and leads to the following equilibrium:

$$X_t = \frac{\gamma}{1 - \phi(1-\gamma)}$$
$$k_t = \left[1 + e^{X_t}\left(\frac{\phi}{1-\phi}\eta_t - 1\right)\right]\frac{K_t}{N_t^y}$$

The value-added to have a job is constant, while the capital-per-worker is determined by demographic variables and is proportional to the capital-per-young individual. However, this case is not useful to examine the labor share's dynamics as the labor share is constant with a Cobb-Douglas technology ($\theta_t = 1 - \phi$).

In , I show that the equilibrium is unique for any other value of σ. The

equilibrium is characterized by 13 equations,

$$\eta_t = \frac{n_t}{p_t}\frac{1 + \alpha p_{t+1}}{\omega}, \tag{Pub. η}$$

$$X_t = \ln\left(\frac{\frac{N_t^y}{K_t}k_t - 1}{\frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\eta_t - 1}\right), \tag{Eq. g}$$

$$X_t = \left(\sigma + \frac{1-\phi}{\phi}\frac{1-\gamma(1-\sigma)}{\gamma}k_t^{\frac{1-\sigma}{\sigma}}\right)^{-1}, \tag{Eq. h}$$

$$L_t = \frac{K_t}{k_t}, \tag{Prod. L}$$

$$w_t = A(1-\phi)\left(\phi k_t^{\frac{\sigma-1}{\sigma}} + 1 - \phi\right)^{\frac{1}{\sigma-1}}, \tag{Prod. w}$$

$$Y_t = A\left[\phi K_t^{\frac{\sigma-1}{\sigma}} + (1-\phi)L_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{Prod. Y}$$

$$u_t = 1 - \frac{L_t}{N_t^y}, \tag{Prod. u}$$

$$\theta_t = \left(1 + \frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\right)^{-1}, \tag{Prod. θ}$$

$$\tau_t = 1 - [(1-\theta_t)(1+\beta+\eta_t)]^{-1}, \tag{Pub. τ}$$

$$b_t = (1-\tau_t)w_t e^{-X_t}, \tag{Pub. b}$$

$$h_t = \frac{\tau_t Y_t - b_t u_t N_t^y}{N_t^o}, \tag{Pub. h}$$

$$S_t = \frac{\alpha p_{t+1}}{1 + \alpha p_{t+1}}\left[(1-\tau_t)w_t(1-u_t) + b_t u_t\right]N_t^y, \tag{HH. S}$$

$$K_{t+1} = S_t. \tag{HH. K}$$

Let define, respectively, equations (Eq. g) and (Eq. h) with the functions $g(L_t, K_t, \eta_t, N_t^y; \phi, \sigma)$ and $h(L_t, K_t; \sigma, \phi, \gamma)$, such that:

$$X_t = g(L_t, K_t, \eta_t, N_t^y; \phi, \sigma)$$

$$X_t = h(L_t, K_t; \sigma, \phi, \gamma)$$

Differentiating this system,

$$dX_t = g_L dL_t + g_K dK_t + g_\eta d\eta_t + g_{N^y} dN_t^y,$$

$$dX_t = h_L dL_t + h_K dK_t,$$

where f_x is the partial derivative of f with respect to x. Solving the system,

$$dL_t = \frac{1}{h_L - g_L} \left[(g_K - h_K) dK_t + g_\eta d\eta_t + g_{N^y} dN_t^y \right]$$

The variables η_t and N_t^y are given by the demographic dynamics while the capital stock K_t is given by savings of the previous period. Thus, the variable that determines the equilibrium is the level of employment L_t. Partial derivatives are derived in appendix B. g_η is negative because the greater is the youth's political power, the greater are the unemployment benefits per capita and the smaller is the value-added to have a job. g_{N^y} is positive because the more numerous the young are, the more these unemployment benefits are divided among them and so the smaller is the value-added to have a job. The sign of $g_K - h_K$ depends on the value of σ with $g_K - h_K \lessgtr 0$, $\forall \sigma \gtrless 1$. This is directly related to the fact that both factors are either gross substitutes or gross complements. When labor and capital are gross substitutes, more capital requires less labor for a given of level of output, while when they are complements, the representative firm has to jointly increase labor and capital stock in order to maintain the same production level. Lastly, the sign of $h_L - g_L$ is ambiguous.[21] Due to this ambiguity, I perform a quantitative analysis in section 3.

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[21]When $\sigma < 1$, then g_L and h_L are both positive, thus, $g_L - h_L$ is ambiguous. When $\sigma > 1$, h_L is unambiguously negative. The sign of g_L cannot be determined without additional assumptions. If $\eta_t / \Theta_t > \sigma$, then g_L is negative. Otherwise, g_L is negative if the unemployment rate is lower than a certain threshold, i.e $u_t < \bar{u}_t$ where $\bar{u}_t = \left(2 - \frac{\eta_t}{\Theta_t \sigma} \right)^{-1}$. Thus, when the unemployment rate is high enough such that $u_t > \bar{u}_t$, g_L is unambiguously positive and so is $g_L - h_L$. This is the only case where it is possible to determine unambiguously the impact of variables on L_t. In such a case, $\frac{dL_t}{dK_t}\Big|_{dN_t^y=0}^{d\eta_t=0} < 0$, $\frac{dL_t}{dN_t^y}\Big|_{d\eta_t=0}^{dK_t=0} > 0$ and $\frac{dL_t}{d\eta_t}\Big|_{dN_t^y=0}^{dK_t=0} < 0$. Therefore, except in this special case, $g_L - h_L$ is ambiguous.

3 Quantitative analysis

This section provides the quantitative analysis of the model with three main objectives: *i)* to reproduce the labor share dynamics observed over the period 1970 to 2010; *ii)* to produce model predictions after 2010; and *iii)* to understand the transmission channels of demographic effects on the labor share. Based on analytical results, I compute quantitative predictions for France and the United-States.

3.1 Calibration

One period in the model corresponds to 40 years in the data. Following the methodology of Gonzalez-Eiras and Niepelt (2012) to simulate an OLG model, I compute four sequences of model predictions with a period length of 40 years each. Periods of the first sequence corresponds to 1970, 2010, 2050, ...; for the second sequence to 1980, 2020, 2060, ...; for the third sequence to 1990, 2030, 2070, ...; for the fourth sequence to 2000, 2040, 2080, ... When reporting time series predictions, I list the four sequences in a single time series. With a period length of 40 years, agents are considered as young between 20 and 60 years of age and as elderly thereafter.[22] I compute the old-age dependency ratio from the data as the number of old people divided by the number of young people (i.e. $p_t/n_t \equiv N_t^o/N_t^y$). Then, the population growth rate at time t is computed using the ratio between the number of young households in period t relative to the number of young households in the previous period of the sequence (i.e. $n_t = N_t^y/N_{t-1}^y$). Finally, the survival rate at time t is the product

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[22]An implicit assumption of the model is that the retirement age is constant (this assumption is discussed in section 4). The average French effective retirement age was 67.8 in 1970 and has declined to 59.3 in 2010. In the US it has gone from 68.4 to 65.6 over the same period (data from the OECD Database, Ageing and Employment Policies - Statistics on average effective age of retirement). I suppose, as an approximation, that agents retire at 60 years-old to match the period lengths of the calibration. Such an assumption should not affect the voting outcome because almost-retired agents may anticipate their future situation once they vote. However, there could be implications in terms of labor supply. Nonetheless, a 5-year approximation remains marginal compared to the 40 years between the two periods.

of the old-age-dependency ratio and the population growth.[23] Figure 1 plots demographic variables for both countries.

[Figure 1 about here.]

Economic variables data $(K_t, emp_t, Y_t, \theta_t)$ are taken from the Penn World Table 9.1.[24] When considering data on the labor share, I use the first adjustment method of Feenstra et al. (2015).[25] I use government revenue as a share of GDP from the OECD database as proxy for the tax rate τ_t. In the model, labor supply is inelastic and there is no distinction between unemployed and inactive young individuals. Unemployed agents in terms of the model specification correspond to all agents that do not work. However, in high-income countries, such as France and United States, inactive individuals also benefits from redistribution. Therefore, I treat inactive individuals as unemployed and the redistribution is captured through unemployment benefits b_t in the model. I compute the unemployment rate such that $u_t = 1 - emp_t/N_t^{15-64}$, where N_t^{15-64} is the working age population.[26] Then, I compute labor such that $L_t = (1 - u_t)N_t^y$. I normalize K_t and L_t to their 1970 values. As a consequence, the capital-to-labor ratio k_t is also normalized and is equal to 1 in 1970. Then, N_t^y and N_t^o are normalized such that u_t matches the data in 1970.

Once all stock variables are normalized, I calibrate the seven remaining parameters $\{\phi, \sigma, \gamma, \alpha, \omega, \beta, A\}$. Table 1 summarizes parameters for both countries.

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[23]Demographic data from 1950 are taken from the United Nations World Population Prospects 2017. Demographic data before 1950 are from http://www.populstat.info. Demographic data between 2010 and 2100 are the "medium variant" estimates from the United Nations. Both rates n_t and p_t converges to unity in the very long run (after the 5th period of the 4th sequence, hence 2160). However, I limit my analysis to 3 periods (hence 2080) due to high uncertainty above 100 years.

[24]K is the capital stock at constant 2011 national prices, emp the number of persons engaged, Y the real GDP at constant 2011 national prices and θ the share of labor compensation in GDP. K and Y are adjusted with the average annual hours worked by persons engaged. For further details the reader is referred to Feenstra et al. (2015).

[25]Gollin (2002) argues that it is necessary to take into account self-employed income. In the model, workers are only young individuals and provide only labor supply. Therefore, I assume that self-employed individuals earn an income that is characterized as a compensation.

[26]I consider the whole working age population instead of the young population in order to compute the unemployment rate. Due to demographic specification of the model, young agents correspond to those between 20 and 60 years old. The number of persons engaged (emp) per age groups are not available in PWT 9.1. Therefore, taking only N_t^y as denominator would bias downward the unemployment rate. Results are robust to different specifications of the unemployment rate.

[Table 1 about here.]

The first parameter ϕ corresponds to the capital share in 1970. Because the capital-labor ratio is normalized (i.e. $k_{1970} = 1$), the labor share θ in 1970 is equal to $1 - \phi$, allowing me to determine ϕ. I set the relative bargaining power of the union γ to 0.5. Hence, I assume that neither the representative union nor the representative firm have any other advantage in the bargaining apart from their respective outside options. I also set the discount rate at 0.669 in line with the literature.

The main parameter of the model is the elasticity of substitution between capital and labor σ. I estimate it using a combination of the first order conditions of the profit maximization. Details of the estimation in appendix C. I obtain 1.279 and 1.234, respectively, for France and the United States. According to the estimation, both input factors are gross substitute. These estimates are in line with Karabarbounis and Neiman (2014) who use cross-sectional data on 50 countries over the period 1975-2012 to find an elasticity greater than 1 and on average around 1.28 in their baseline estimates.

Then, I have three remaining parameters that are deduced. The relative ideological spread-out ω is set such that the model prediction matches the capital-to-labor ratio k in 1970. The preference for government health expenditure β is set to match the tax rate τ in 1970. Lastly, the scale parameter of the production function A is set with grid-search to match the average labor share between 2008 and 2012 for each country.

Due to initial labor share values, the parameter distribution ϕ which correspond to the capital share in 1970 is larger in the United States than in France. Moreover, the relative per-capita political influence of old households ω is also greater. This deduced parameter suggests that young generations have less political weight in the United States compared to France. Notice that ω is lower than one for France. It implies that French youth voting behavior is more sensitive to policy changes compared to elderly. French households have a much greater preference for government health expenditure β. Finally, the scale parameter A is ranged between 20 and 30 for both countries.

For France, I show in appendix D that there is a a change in the regime of capital-labor elasticity of substitution. I find that such a break has occurred during the 1980s in France. I model the workers outside option within the wage bargaining using only unemployment benefits and the tax rate but other labor market institutions may have played a role in firms' employment policies during this period. For example, in my framework, there is no firing cost. Bentolila and Bertola (1990) argue that firing costs have an effect on firms' propensity to hire and fire, and thus on employment levels. Further, they also find that firing costs slightly increase average long-run employment. Subsequent to the oil shock, firms were not able to fire as they would have due to high firing costs.[27] They reduced employment with attrition, thus the process took time and the employment remained higher than it should during some years. In addition, most of the employment security policies were introduced between the 1960s and the beginning of the 1970s. Here again, these protections may be the result of the baby-boomers generation operating a political pressure to secure employment. Although, I do not integrate it in the model. Then, France during the 1980s has started to implement policies in order to make the labor market more flexible. For instance, short term contracts have been introduced in 1979. To summarize, the labor market institutions in France were tremendously different before and after the 1980s. Therefore, structural parameters such as the ability of firms to substitute labor with capital may also have changed. Another way to think about it, in terms of the model, is to suppose that the capital-labor elasticity of substitution was much smaller before 1980.[28] Therefore, the representative firm was more constrained and not able to substitute labor by capital as nowadays.

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[27]Blanchard (1997) also shows that wages failed to adjust to the productivity slowdown and adverse supply shocks of the 1970s. Thus, wages remained relatively high and so did the labor share. This phenomenon was more pronounced in Continental European countries than in the Anglo-Saxon countries, mainly due to labor market institutions.

[28]Perhaps even lower than 1, meaning that an increase in k_t leads to an increase in θ_t.

3.2 Model predictions over the period 1970-2010

Figure 2 displays the labor share over the period 1970-2010, depicting the model predictions and the data.

[Figure 2 about here.]

The model reproduces the trend in the data for both countries. For the United States, the model tends to overestimate the labor share in 1980 and 1990 and underestimate it in 2000. However, model predictions capture the global trend of the labor share over the period. For France, model predictions are more accurate and reproduce the data since 1970. To highlight the mechanisms of the declining labor share, it is necessary to look at the variations of determinant variables. Figure 3 displays the deviation from the 1970's value of determinant variables in percentage.

[Figure 3 about here.]

The equilibrium of the economy is determined by the interaction of the wage bargaining and the voting. Both processes take place simultaneously. The demographic context over this period is the following: the population growth n_t slightly exceeds the increasing survival rate p_t between 1970 and 2000. Thus, the old-age-dependency ratio p_t/n_t remains roughly constant. With a decline in France in 1980 due to the massive entry of the baby-boomers in the labor force. The old-age-dependency ratio starts to increase between 2000 and 2010 due to a steady population growth but a sharply increasing survival rate. This increase is also due to the baby-boomers' cohort starting to retire. As a result of this demographic context, the youth political weight η_t was above its 1970's level until 2000 in both countries.

As the youth's political weight rises and because they desire more redistribution, the tax rate τ_t increases. Due to the opportunistic behavior of political parties, pro-youth policies are implemented. In the model, pro-youth policies correspond to pro-labor policies and are translated in terms of unemployment benefits in order to insure the unemployment risk.

Thus, the unemployment benefits share in government revenue increases at the cost of the government health spending share in government revenue.

Due to the ability of workers to bargain a greater wage, the representative firm shifts away from labor. This behavior is permitted by two features of the model. First, the right-to-manage specification of the wage bargaining enables the firm to hire and fire as much as wanted. Second, the capital-labor elasticity of substitution σ is greater than unity for both countries. Thus, both input factors are gross substitutes. The firm is able to substitute labor with capital for a given output level. This behavior leads to a decline of the number of workers L_t because the labor cost (i.e. the wage) has increased. Notice that the substitution effect is more vigorous in France because of the relatively higher elasticity of substitution σ. The French economy has a faster growth in capital stock K_t, while French firms are able to substitute relatively more labor with capital. Thus, the number of workers becomes lower than its 1970's level in France. While the United-States manage to slightly increase its labor factor. As a consequence, the US output Y_t grows faster.

The slowdown in employment has consequences on the labor market. This slowdown in employment directly raises unemployment in France by reducing the number of workers. Furthermore, the effect is all the more tremendous because the labor force N_t^y grows due to the baby-boomers' arrival. The United-States also know a growing population but the impact on unemployment is much weaker due to the increasing number of workers, as discussed above. The dynamics of the production function affects the determinants of the labor share. Due to the fact that both input factors are gross substitute, the output-per-worker Y_t/L_t increases when the capital-per-worker k_t does so. This increase exceeds the one of the wage. As a result, the labor share declines.

Let us summarize the mechanisms over this period. The baby-boomers change labor market institutions in their favor due to their relatively high political weight. It raises the outside option of workers, giving more bargaining power to the union who is able to bargain greater wages. The cost of labor becoming too high, the firms decide to shift away from

labor. The shift away from labor engenders an increase in output per worker which thwarts and exceeds the wage gain. Thus, the labor share declines.

3.3   Model predictions over the period 2010-2080

Figure 4 displays the labor share predicted by the model for the period 2010-2080.

[Figure 4 about here.]

The model predicts that the labor share should continue to decline after 2010 in both countries. However, France should face a slight rise in 2030 up to 66.1% before shrinking again. While the US labor share should remain stable around 62.6 % between 2010 and 2040 and then follows the same pattern. Figure 5 displays the deviation from the 2010's value of determinant variables in percentage.

[Figure 5 about here.]

The demographic context over the period 2010-2080 is roughly similar for France and the United-States in terms of dynamics, although the magnitudes are not. The population growth n_t faces a sharp decline between 2010 and 2050 before stabilizing thereafter. Meanwhile, the survival rate p_t grows by around 4% per decade. Thus, the old-age-dependency ratio increases up to 2050, more in France than in the United-States. Once the population growth becomes stable, the ratio still grows but at a lower rate. This aging of the population is mainly due to the baby-boomers' retirement. As a result, the youth political weight never gets back to its 2010's level and will strongly decline up to 2050 for both countries.

The youth's political weight η_t sharply declines due to the aging of the population. Thus, opportunistic political candidates favor old individuals who desire less redistribution, i.e. $\partial \tau_t / \partial \eta_t > 0$. Pro-elderly public policies are implemented. In terms of the model, it is translated by an increase of the health spending share in government revenue at the cost of the unemployment benefits one. These policy changes interact with the wage bargaining because they affect the outside option of young households.

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As the wage decreases between 2010 and 2050, the representative firm has incentive to hire. Thus, the labor L_t does increase over this period. Meanwhile, the capital stock K_t has increased due to savings S_t of the previous periods. Thus, both factors increase and so does the output Y_t. After 2050, France and the United-States diverge in their trajectories. On one hand, the US capital stock rises due to high savings of the previous periods.[29] This rise is strong enough to foster the representative firm to hire workers. Thus, the substitution mechanism does not operate. On the other hand, the increase of the French capital stock does not follow the same exploding pattern. The french representative firm, as during the end of the twentieth century, shifts away from labor and decreases the number of workers. This divergence also appears on the labor market.

The labor force N_t^y is roughly constant over the whole period and France is even below its 2010's level. Thus, variations in the number of workers mainly drives the unemployment ones. The unemployment rate decreases in the United-States over the whole period. It falls down to 1.69% by 2080 against 44.6% in 2010. Here again, the sharp increase of the capital stock fosters employment until almost full-employment. The French unemployment rate decreases until 2050 before to slightly rise a bit and stabilize itself around 26.9% in 2080.

French and US workers experience a minor reduction of their wage w_t between 2010 and 2050 and so does the output per worker Y_t/L_t. The latter being greater than the former, the French labor share slightly increases. The US labor share remains constant because both variations compensate. After 2050, both labor shares decline to reach 62% and 59.8%, respectively for France and the United-States. While their respective levels in 2010 were about 65.1% and 62.5%.

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[29]This increase in savings owes to the baby-boomers cohort when they were young. Their massive entry on the labor market, the rising expected life expectancy and the higher wages have fostered savings and therefore the capital available in the economy once old.

3.4 Counterfactual and aging effect decomposition

So far, I have highlighted the different mechanisms through which the age structure of the population affects economic variables and therefore the labor share. I now consider the previous model's prediction as the benchmark prediction. To summarize, demographic changes are due to two determinant variables in the model: the population growth n_t and the survival rate p_t. These variations may affect the labor share through two channels: the direct cohort effect and the indirect cohort effect η_t.

I make counterfactual predictions in order to quantity the respective role of each channel. The idea is to neutralize either a determinant of the demographic change or a channel through which the labor share is affected. The intuition behind the counterfactual is to observe what would have happened in terms of model predictions if this effect/channel was neutralized. By comparing a counterfactual simulation to the benchmark one, I can quantify its extent. I proceed in two steps. First, I examine the impact of the different determinants of demographic changes (i.e. n_t versus p_t). Second, I investigate through which channels it occurs (i.e. direct versus indirect).

3.4.1 Survival rate and population growth effects

To neutralize the impact of the survival rate p_t, I assume that it remains at its 1970's level. Thus, $p_t = p_{1970}$ and $p_{t+1} = p_{2010}$. Population size is recalculated such that $N_t^{o'} = N_t^o \times \frac{p_{1970}}{p_t}$. Moreover, $\eta_t = \frac{n_t}{p_{1970}} \frac{1+\alpha p_{2010}}{\omega}$. The other demographic variables (i.e. n_t and N_t^y) follow the time series of the benchmark simulation. The initial capital stocks of the four sequences are also recalculated such that $K_0' = \frac{1+\alpha p_t}{p_t} \frac{p_{1970}}{1+\alpha p_{1970}} K_0$.[30] The methodology is analogous to neutralizing the impact of the population growth but with $n_t = n_{1970}$,

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[30]Setting constant the survival rate implies changes in the saving rate through the expected survival rate p_{t+1}. Thus, $K_0 \equiv S_{-1} = \frac{\alpha p_0}{1+\alpha p_0} \left[(1-\tau_{-1})w_{-1}(1-u_{-1}) + b_{-1}u_{-1} \right] N_{-1}^y$. In order to assess the true impact of the survival rate, it is also necessary to consider saving rate changes in the counterfactual simulation. Thus, the initial capital stocks for the first periods of the four sequences becomes $K_0' = \frac{1+\alpha p_t}{p_t} \frac{p_{1970}}{1+\alpha p_{1970}} K_0$. Notice that a change in the survival rate p_0 should also affect the aggregated disposable income of young households in $t = -1$, i.e. $(1-\tau_{-1})w_{-1}(1-u_{-1}) + b_{-1}u_{-1}$. However, I do not consider this source of change. Notice also that the term N_{-1}^y does not change because $N_{-1}^y = \frac{N_0^o}{p_0} = \frac{N_0^{o'}}{p_{1970}}$.

$N_t^{y\prime} = N_t^y \times \frac{n_{1970}}{n_t}$ and $\eta_t = \frac{n_{1970}}{p_t} \frac{1+\alpha p_{t+1}}{\omega}$.[31] Finally, a last counterfactual is made to neutralize both effects. So, $p_t = p_{1970}$, $p_{t+1} = p_{2010}$, $n_t = n_{1970}$ and $\eta_t = \eta_{1970}$. As before, population sizes and initial capital stock values are recalculated. Here, the old-age-dependency ratio remains constant. Table 2 summarizes the demographic variables in 1970.

[Table 2 about here.]

Figure 6 displays the model predictions of the labor share with counterfactual specifications.

[Figure 6 about here.]

To interpret the role played by each demographic variable, I compare each counterfactual time series to the benchmark one. For instance, let us start with the impact of the survival rate in France. I look at the dash-dotted curve with respect to the solid one. This curve corresponds to the counterfactual where the survival rate p_t and p_{t+1} remain at their 1970's levels. So, if the survival rate would had not changed, then the labor share would have followed the dash-dotted curve pattern. The dash-dotted curve lies below the solid one. Therefore, the survival rate dynamics have a positive impact on the labor share in both countries. The way to interpret the other counterfactuals is similar. The dashed curve remains above the solid one until 2010 for France and 2000 for the United-States. Until these years, the population growth dynamics have a negative impact on the labor share and a positive one thereafter. Lastly, the dotted curve which corresponds to the counterfactual where both effects are neutralized is below the benchmark simulation. Since 1970, all demographic dynamics have led to increase the labor share with respect to what it would have been without these dynamics. However, this representation is tediously legible. Therefore, I compute the distance between the benchmark labor share and each counterfactual labor share. Using the Chasles' relation within an affine space at each point in time, it is possible

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[31]In this specification, the initial capital stocks are not changed because n_0 does not affect S_{-1}.

to isolate the extent of each effect.[32] Figure 6 displays the labors share's gap between the benchmark and the counterfactuals in percentage points.

[Figure 7 about here.]

As mentioned above, the survival rate effect has kept the labor share relatively high in both countries. Another way to think about that is to consider demographic dynamics in terms of cohort sizes. Until 2010, if the survival rate would had been held constant, then the massive increase in the population of young households due to the baby-boomers' presence would have generated an even larger decline of the labor share. However, the aging population (i.e. the increase of the survival rate) has thwarted part of this fall. Notice that the difference with the counterfactual is smaller in the case of the United-States. It suggests that the aging phenomenon over this period is a larger determinant of the labor share in France compared to the United-States. The analysis of the determinants only reveals part of the mechanism. It is also necessary to look at the channels that affect the labor share.

3.4.2 Direct and indirect cohort effects

To neutralize the indirect cohort effect, I fix the youth political weight η_t to its 1970's value but I keep the demographic variables' time series n_t, p_t and p_{t+1} as in the benchmark simulation. Thus, $\eta_t = \eta_{1970}$. To neutralize the direct cohort effect, I assume that all the demographic variables remain at their 1970's level. Thus, $p_t = p_{1970}$, $p_{t+1} = p_{2010}$, $n_t = n_{1970}$ and the population sizes along with the initial capital stocks are recalculated. But the youth political weight time series η_t is the one of the benchmark simulation. Finally, a last counterfactual is made where I neutralize both effects.[33]

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[32]Let r (resp. b, g, p) be the labor share from the benchmark (resp. n_{1970}, p_{1970}, n_{1970}, p_{1970}) curve on figure 7 For a given year, $2\vec{rp} = \vec{rb} + \vec{bp} + \vec{rg} + \vec{gp} \Leftrightarrow \vec{rp} = \vec{rb} + \vec{rg} + \text{int.}$, where int. $\equiv \vec{bp} + \vec{gp} - \vec{rp} = \vec{gp} + \vec{br} = \vec{bp} + \vec{gr}$ is the interaction of both effects defined as the part which is not exclusively explained by both effects independently.

[33]This last specification is the same as in the survival rate and population growth effects decomposition. So the dotted curve in figure 6.

Figure 8 displays the model predictions of the labor share with counterfactual specifications.

[Figure 8 about here.]

The methodology to interpret this figure is similar to the one of the figure 6. The dashed curve corresponds to the direct cohort effect. For both countries, this curve is below the benchmark curve until 2010 and above thereafter. It means that the demographic dynamics' net impact through the direct channel is positive on the labor share until 2010 and negative thereafter. Looking at the dash-dotted curve, it is slightly above before 2000 and largely below thereafter. The net impact of the indirect cohort effect is negative up to 2000 and positive subsequently. However, this representation is also tediously legible. Therefore, I isolate the extent of each channel with the same methodology as before. Figure 9 displays the labor share's gap between the benchmark and the counterfactuals in percentage points.

[Figure 9 about here.]

The direct cohort effect is positive when the baby-boomers are young and after becomes negative. While the indirect cohort effect has the opposite pattern. The direct cohort effect dominates the indirect one until 2010 before to become less influential. Until 2010, if the demographic variables would had been held constant, then the relatively high political power of the youth due to the baby-boomers' presence would have generated an even larger decline of the labor share. However, the aging population was mainly driven by the increasing survival rate which has thwarted part of the fall. After 2010, so once the baby-boomers' cohort retires, the model predicts that the indirect cohort effect (i.e. the decline of the youth political power) should exceed the direct one. This is directly related to the mechanism analysis of section 3.2. The baby-boomers and more generally the old households becomes the winners of the age-related conflict within the public policy over this period. Thereby, they reduce the workers' outside option which leads to a wage cut or more precisely to a wage stagnation. The labor cost remains constant while the capital available in the economy

accumulates. It generates an incentive for the representative firm to hire more workers. As a result, the labor share increases.

3.4.3 Summary

Figure 10 summarizes the aging-effect decomposition by period and country.

[Figure 10 about here.]

The main determinant of the aging phenomenon is the survival rate. On average, it explains 54.5% for France and 78.1% for the United-States of the impact on the labor share over the period 1980-2010. This decomposition persists thereafter for France while the role of the survival rate shrinks for the United-States. Between 1980 and 2010, demographic dynamics affect the labor share mainly through the direct channel in France. While the indirect one is dominant in the United-States. On average, the share explained by the direct cohort effect is about 53.1% for France and 38.2% for the United-States. However, over the next period, the direct cohort effect is not the dominant channel anymore in France. It falls to 20.3%, suggesting that the indirect cohort effect becomes the main carrier of the demographic dynamics on the labor share. In contrast, it remains relatively stable in the United-States, at around 35.7%.

4 Discussion

4.1 Age-related conflict: who are the winners ?

So far, the labor share has been declining due to the baby-boomers generation in both countries. First, when they were young because they shaped the public policy and so the labor market institutions in their favor. The firms answered to that by substituting labor to capital. Second, when they were old because they have considerably increased the available capital in the economy through savings allowing the firms to substitute even more. However,

the labor income share is a gross indicator of inequalities. A more appropriate indicator may be the income ratio between young and old after redistribution.

From equation (16), I have that the after-tax young-to-old income ratio is equal to the young political weight (i.e. $Y_t^y/Y_t^o = \eta_t$). While the before-tax young-to-old income ratio corresponds to the labor-to-capital ratio (i.e. $\Theta_t = \frac{w_t L_t}{r_t K_t}$). Figure 11 displays these income ratios in deviation from the 1970 steady-state.

[Figure 11 about here.]

While the labor share and so the labor-to-capital ratio (i.e. before-tax young-to-old income ratio) sharply decline between 1970 and 2000, the after-tax young-to-old income ratio lies over its 1970's level during the same period. Even if I include the government health spending as part of the old households income, the young-to-old income ratio remains over its initial level until 2000.[34] This model prediction holds for both countries. The baby-boomers gross income share (i.e. labor income share) has declined due to the mechanisms previously mentioned when they were young. However, they also have spurred political parties to implement redistributive public policies.[35] This redistribution is characterized, in terms of the model, through a raise of the tax rate and an increase of the unemployment spending share within the government revenue.[36] Thus, they have been able to seize part of their elders income through redistribution. Although the baby-boomers appeared as income losers over this period because the labor share was falling, they are actually the winners once net income is considered.

Another way to determine the winners of the age-related conflict within the public policy is to look how the GDP is allocated in the economy. Figure 12 displays the income allocation after redistribution.

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[34]In terms of deviation, the curve is combined with the dashed one.

[35]This is the result of the probabilistic voting specification. The opportunist behavior of political parties lead them to favor this generation in order to maximize their probability to win the election.

[36]Notice that the increase of the unemployment spending share is driven and accentuated by the raising unemployment due to factor substitution.

[Figure 12 about here.]

The dashed curve corresponds to the labor share, so the before-tax young income share. As long as this curve lies in the area of the government unemployment spending share, it means that old agents fund unemployment spending through their taxes. On the contrary, when the dashed curve lies in the area of the government health spending share, the young agents fund government health spending for their elders. The US share of GDP which is allocated to health spending is quite small compared to the French one. This is directly related to the preference parameter for government health spending β.[37] The US young generations are the winners of the age-related conflict from 1970 to 2020 which corresponds to the active period of the baby-boomers. Once the US population ages and therefore the baby-boomers, old households dominates the public policy conflict. However, the size of the government revenue declines due to the fact that the United-States converges to full employment in the long run.[38] In France, the baby-boomers also dominate the public policy conflict between 1990 and 2010 when they are active. Once they retire, so over the period 2010-2040, they are still the winners of this conflict and their children fund government health spending. Notice that the size of the welfare state corresponds to the sum of the unemployment benefits share in GDP and the health spending share in GDP.

4.2 Retirement age

So far, I have not discussed the retirement age and its implications on the labor share. The relationship between aging and economic growth has received much more attention than the one with the labor share. Gonzalez-Eiras and Niepelt (2012) predict that the retirement age in OECD countries should increase in response to population aging. In their model of politico-economic equilibrium, individuals vote with perfect foresight and decide to raise retirement age as long as population ages. Agents work longer and so accumulate more

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[37]$\beta_{\mathrm{FR}} = 0.739$ and $\beta_{\mathrm{US}} = 0.138$.

[38]Thus, there is no need for unemployment benefits. Moreover, since the preference parameter for the government health spending β is quite low in the US, the size of the welfare state shrinks.

wealth, it reduces social-security transfers and thereby releases more government spending to public investment which is an engine for growth.[39] On the contrary, Futagami and Nakajima (2001) claim that population aging does not necessarily depress economic growth and may even foster it through savings. Thus, postponing the retirement age would result in a decline of savings and so the economic growth. Dedry et al. (2017) also discuss the role of legal retirement age according to the type of pension system in a context of population aging due to either declining fertility or increasing longevity.

In order to take into account the role of retirement age, I perform counterfactual predictions with different scenarii based on an exogenous change. The retirement age is captured within the variable p_t which depends negatively on it but positively on the life expectancy. I do not endogenize this variable due to the limited form of rationality and to the several assumptions that would be required.[40] In the public debate, it is often argued that the legal retirement age should change in the future, usually upward, as claimed by Gonzalez-Eiras and Niepelt (2012). Between 2020 and 2030, I suppose a positive exogenous shock on the age of retirement. Meaning that less individuals would reach the old age which is translated into a decline of the survival rate, i.e. p_t. As in section 3.4, other demographic variables have to be changed for the period following the shock of each simulation's sequence (i.e. 2030, 2040, 2050 and 2060). However, the implications for demographic dynamics are not identical to this previous exercise. Because the greater the retirement age, the longer an individual remains young in terms of the model. Thus, these individuals do not vanish but just remain

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[39]However, this result contrasts with Jäger and Schmidt (2016) who find out that the share of elderly people and public investment are negatively co-integrated. They use panel data of 19 OECD countries from 1971 to 2007. This gap may be due to two reasons: *i)* some public policy instruments which are not considered in the model of Gonzalez-Eiras and Niepelt (2012) might invert the relationship between aging and public investment; *ii)* the perfect foresight assumption might be too strong. Both potential explanations may reconcile these diverging results.

[40]To have an endogenous retirement age, agents should vote on it and thereby vote on the survival rate. The first question would be to determine whether agents vote on p_t, p_{t+1} or both. Then, the perfect annuity market would have a lot of implications on the results. Since savings of young agents who die before reaching old age are distributed among their surviving peers, it means that an agent has an incentive to vote for a decline of the survival rate because fewer peers would reach old age. Thus, it would increase its income and so its utility. Therefore, it would be necessary to determine whether or not agents internalize the perfect annuity market in their voting decisions.

longer in the labor force.[41] Therefore, conjointly with the decline of the survival rate, there is a share of the young population that does remain young. From the identity $\frac{N_t^o}{N_t^y} \equiv \frac{p_t}{n_t}$, I obtain:

$$\frac{\dot{N}_t^o}{N_t^o} - \frac{\dot{N}_t^y}{N_t^y} = \frac{\dot{p}_t}{p_t} - \frac{\dot{n}_t}{n_t} \tag{20}$$

where the upper dotted variables correspond to the variables' variation, e.g. $\dot{N}_t^o = N_t^{o\prime} - N_t^o$ where $N_t^{o\prime}$ is the new value for N_t^o. This equation has to be satisfied. The exogenous shock on p_t affects all other demographic variables. First, the size of the old population varies as much as the survival rate does, i.e. $\frac{\dot{N}_t^o}{N_t^o} = \frac{\dot{p}_t}{p_t}$. Second, the variations of the young population's size are inversely proportional to the ones of the old population size, i.e. $\frac{\dot{N}_t^y}{N_t^y} = -\frac{\dot{N}_t^o}{N_t^o}$. Third, by taking into account the two previous points and the fact that equation (20) has to be satisfied, it implies that $\frac{\dot{n}_t}{n_t} = -\frac{\dot{p}_t}{p_t}$. Therefore, the exogenous shock on p_t affects the other variables with the same magnitude. Hence,

$$\dot{N}_t^o = \frac{\dot{p}_t}{p_t} N_t^o \implies N_t^{o\prime} = \left(1 + \frac{\dot{p}_t}{p_t}\right) N_t^o$$

$$\dot{N}_t^y = -\frac{\dot{p}_t}{p_t} N_t^y \implies N_t^{y\prime} = \left(1 - \frac{\dot{p}_t}{p_t}\right) N_t^y$$

$$\dot{n}_t = -\frac{\dot{p}_t}{p_t} n_t \implies n_t{}' = \left(1 - \frac{\dot{p}_t}{p_t}\right) n_t$$

where $\frac{\dot{p}_t}{p_t} = \frac{p_t' - p_t}{p_t} \implies p_t' = p_t + \dot{p}_t$. Thus, the new demographics variables $N_t^{o\prime}, N_t^{y\prime}, p_t', n_t'$ are computed for the years 2030, 2040, 2050 and 2060. For the years 2070 and 2080: n_t' follows the benchmark time series and population sizes are computed with $N_t^{o\prime} = p_t' N_{t-1}^{y\prime}$ and $N_t^{y\prime} = n_t' N_{t-1}^{y\prime}$. Notice that new values of the expected survival rate p_{t+1}' changes according to p_t'. Moreover, $\eta_t' = \frac{n_t'}{p_t'} \frac{1 + \alpha p_{t+1}'}{\omega}$ is also recomputed.

Two types of scenario are possible for a change in the age of retirement between 2020 and 2030. First, I consider that the retirement age increases in such a way that the survival

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[41]In the same way, the capital stock of these periods is not changed as in section 3.4 because it has already been accumulated through savings from previous periods. It has no reason to vanish or to be scraped.

rate is negatively affected by 10% in 2030.[42] Thereafter, the survival rate grows at the same growth rate as in the benchmark simulation.[43] This first scenario can be summarized as a one-shot shift by 10% of the survival rate. As a second scenario, I consider that retirement age increases in such a way that the survival rate grows at a fraction $\zeta \in [0, 1]$ of the benchmark simulation's growth rate. The new values of the survival rate p'_t after 2030 are described as follow:

$$p'_t = \zeta(p_t - \bar{p}) + \bar{p}$$

where $\bar{p}$ is the value of the survival rate in 2030. This can be interpreted as a gradual increase of the retirement age that is proportional to improvement in life expectancy.[44] Such dynamics imply perfect forecasts on life expectancy which are assumed in this model.

Figure 13 displays the survival rate dynamics with changing retirement age specifications for both countries.

[Figure 13 about here.]

The dashed curves correspond to the first scenario mentioned above where the survival rate is shocked by -10% in 2030 and keeps its growth rate thereafter. The dash-dotted and dotted curves are two special cases of the second scenario. The dashed-dotted curves refers to the case where $\zeta = 0$, so the survival rate remains constant after 2030. It means that variations in retirement age are fully proportional to those in life expectancy. While the dotted curves coincide to the case where $\zeta = .5$. Thus, the survival rate p_t grows at half of the speed of the benchmark simulation.

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[42]The underlying (strong) assumption of this whole exercise is that the change in retirement age has no consequences on life expectancy. Some authors argue that increasing the retirement age has a negative impact on health and so the life expectancy (see, for example, Insler (2014) for the United-States; Coe and Zamarro (2011) for Europe). Thereby, the negative effect on the survival rate may be all the more tenacious due to the declining life expectancy. Despite the presence of a potential co-integration of the retirement age and the life expectancy, I believe that the qualitative impact is not affected in the sense that the simulation is a lower bound.

[43]I assume that the shock does not change the future growth path of the survival rate.

[44]When $\zeta = 1$, the survival rate dynamic corresponds to the one of the benchmark simulation, i.e. $p'_t = p_t$. Thus, variations are no longer proportional to changes in life expectancy. While when $\zeta = 0$, the survival rate remains constant, i.e. $p'_t = \bar{p}$ after 2030. In such a case, variations in retirement age are fully proportional to the ones in life expectancy.

Figure 14 displays model predictions of the labor share according to the scenario.

[Figure 14 about here.]

The raise of the age of retirement in France leads to a decline of the labor share with respect to the benchmark simulation in a first phase. In a second phase, the very long-run, the labor share is relatively greater than in the benchmark case. This result holds regardless of the scenario. In the United-States, the impact on the labor share depends on the scenario. As in France, the -10% shift scenario generates a sharper decline of the labor share before to exceed the benchmark simulation's one in the very long-run. However, both scenarii of diminished growth of the survival rate have roughly the same pattern of the benchmark labor share. But there are still above the labor share in the very long-run. Therefore, changing the retirement age may have different impact on the labor share according to the country.

The underlying mechanisms are related to those detailed in section 3. However, there is a particularity to this change of the retirement age. The capital stock does not immediately adjust in the model. Because it has been determined by the savings of the previous period of each sequence, where the expected survival rate p_{t+1} was much greater than the after-reform one p'_{t+1}. This specificity cancels part of the direct cohort effect. This relatively high amount of available capital stock plays in favor of the firm within the wage bargaining. Because the firm is all the more able to substitute labor with capital. On the indirect cohort effect's side, the agents remain young longer due to the increase of the retirement age. This is translated in terms of the model with a decrease of the number of old households and an increase of the number of young households. Thus, the youth has more political weight than in the benchmark case. With this political strength, they raise their outside option through pro-young public policies which allows them to bargain greater wages. As a response, the firm shift away from labor and hire relatively less workers than in the benchmark case. As mentioned above, the relatively high amount of available capital stock due to the over-accumulation enables the firm to substitute all the more.

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5 Conclusion

The literature on the labor share has emphasized the role of factors such as biased technical change or labor market institutions in shaping its changes over time. In this paper, I explore an alternative mechanism focusing on inter-generational conflict and policy choices. I build an OLG model in which labor market institutions are endogenously determined through public policy and affect the wage bargaining. Using this model, I analyze the impact of demographic dynamics on labor share's long-term dynamics in France and the United-States. Numerical simulations are able to replicate the data for both countries since the 1970s.

Model predictions suggest that the decline of the labor share during the last decades was driven by cohort-size effects. For France, the baby-boomers cohort seems to drive public policy and thus the way in which national income is allocated between labor and capital. When the baby-boomers cohort enters on the labor market, they shape labor market institutions in their favor because they face an unemployment risk, as opportunistic political candidates implement the policy desired by this cohort. These more protective labor market institutions raise workers' outside option, enables unions to bargain greater wages, and induce firms to use more capital as long as factors are gross substitutes. The unemployment rate rises and so does the output per worker, offsetting the increase in the wage rate. Therefore, the labor share declines over the end of the twentieth century. Thereafter, the baby-boomers retire and trigger the opposite mechanism. Although the public policy shrinks the outside option of workers within the wage bargaining which should reduce wages and raise employment. The capital stock has sharply increased due to massive savings of the baby-boomers when they were young. The important available capital stocks partially thwart the reverse mechanism and the labor share does not recover its past level. The model predicts a slight resurgence of the labor share for the next decades in France and a stagnation in the United-States.

Counterfactual decomposition suggest that the survival rate dynamics have a larger impact on the labor share than the population growth dynamics. I also decompose this impact between two channels: the direct cohort-size effect and the indirect cohort-size effect.

The former dominates for France when the baby boomers are young, while the latter takes the lead once this cohort retires. Although the labor share declined due to public policies implemented by baby boomers, they managed to increase their after-tax income through redistribution. The model predicts that an increase of the retirement age should decrease the labor share in the following decades due to the over-accumulation of capital and the raising political power of the youth. However, the labor share is expected to be greater in the very long-run.

The two main hypothesis of the paper concern the elasticity of substitution between capital and labor and the right-to-manage specification for the wage bargaining. Recent estimates suggest that this elasticity may be greater than one (Karabarbounis and Neiman 2014) but others have found it below one, particularly for the United-States (Antràs 2004; Chirinko 2008). Moreover, I do not include any form of biased technical change within the model. This is voluntary in order to develop an other theory on the labor share's decline based on demographic dynamics. It could be the case that biased technical change is also driven by demographic dynamics through the *grability* of workers to seize part of the rent. This grability may be generated by some cohorts which are sufficiently numerous to shape labor market institutions in their favor and therefore in favor of labor. I leave the investigation of a potential endogenous biased technical change induced by demographic dynamics for further research.

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A Uniqueness of the equilibrium

Let us define, respectively, equations (18) and (19) with the functions $g(k_t, N_t^y, K_t, \eta_t; \phi, \sigma)$ and $h(k_t; \gamma, \phi, \sigma)$ such as:

$$g(k_t, N_t^y, K_t, \eta_t; \phi, \sigma) = \ln \left(\frac{\frac{N_t^y}{K_t} k_t - 1}{\frac{\phi}{1-\phi} k_t^{\frac{\sigma-1}{\sigma}} \eta_t - 1} \right)$$

$$h(k_t; \gamma, \phi, \sigma) = \left(\sigma + \frac{1-\phi}{\phi} \frac{1-\gamma(1-\sigma)}{\gamma} k_t^{\frac{1-\sigma}{\sigma}} \right)^{-1}$$

Due to the logarithm, we have two vertical asymptotes depending on whether the numerator or the denominator within the logarithm is equal to zero. The first vertical asymptote is the one associated to the numerator $k_1 = \frac{K_t}{N_t^y}$ and the second vertical asymptote is associated to the denominator $k_2 = \left(\frac{1-\phi}{\phi} \frac{1}{\eta_t} \right)^{\frac{\sigma}{\sigma-1}}$. Rewriting the g function with these vertical asymptotes, I have:

$$g(k_t, k_1, k_2; \sigma) = \ln \left(\frac{\frac{k_t}{k_1} - 1}{\left(\frac{k_t}{k_2} \right)^{\frac{\sigma-1}{\sigma}} - 1} \right)$$

This function has four different shapes according to the value of σ and both vertical asymptotes (k_1 and k_2). Figure 15 plots these shapes.

[Figure 15 about here.]

The h function has three different shapes according to the value of σ. Figure 16 plots these shapes.

[Figure 16 about here.]

Proposition 1 *If $k_t \leq k_1$ at the equilibrium, then there is a unique equilibrium with full employment where $k_t = k_1$ and the net wage equals the unemployment benefits, i.e. $(1 - \tau_t)w_t = b_t$.*

Proof. The numerator of the g function is positive if and only if $k_t > k_1 \Leftrightarrow L_t < N_t^y$ (i.e. the number of worker is smaller than the young population size). This condition is always satisfied when there is unemployment. However, if this condition is not satisfied, labor demand exceeds the labor force (i.e. $L_t^d > N_t^y$). Therefore, the economy is in full-employment (i.e. $L_t = N_t^y$) and capital-per-worker is $k_t = k_1$. In such a case, X_t tends to $-\infty$. However, the lower bound of X_t is 0. Because if X_t is negative, the unemployment

benefits would exceed the net wage. I consider that such a case is not possible.[45] Thus, if $k_t \leq k_1 \implies k_t = k_1 \implies u_t = 0 \implies X_t = 0 \implies (1 - \tau_t)w_t = b_t.$ ∎

Therefore, the equilibrium with unemployment requires $k_t > k_1$. Hence, any equilibrium in the case represented in the bottom-left panel of figure 15 leads to the equilibrium with full-employment.

I normalize k_t to the vertical asymptote k_1. Let $\nu = k_2/k_1$ with $\nu > 0$. It implies that $k_2 \gtreqqless k_1$ when $\nu \gtreqqless 1$. Let $\tilde{k}_t = k_t/k_1$ with $\tilde{k}_t > 0$. As for ν, if $\tilde{k}_t$ is greater than unity, then $k_t > k_1$ and vice-versa. To simplify the notation, let $\rho = \frac{\sigma - 1}{\sigma} \in (-\infty, 1)$.[46] Using this specification, it is possible to rewrite g such that:

$$g(\tilde{k}_t; \nu, \rho) = \ln \left(\frac{\tilde{k}_t - 1}{\tilde{k}_t^{\rho} - \nu^{\rho}} \right) + \rho \ln (\nu)$$

Let also rewrite the h function such that:

$$h(\tilde{k}_t; \tilde{\gamma}, \rho) = \left(\frac{1}{1 - \rho} + \tilde{\gamma} \tilde{k}_t^{-\rho} \right)^{-1}$$

where $\tilde{\gamma} \equiv \frac{1 - \phi}{\phi} \frac{1 - \gamma(1 - \sigma)}{\gamma} k_1^{-\rho} > 0$.

A.1   Under gross-complementarity

$g(\tilde{k}_t)$ is define and continuous between both vertical asymptotes within the logarithm, so 1 and ν.[47] When ν is greater than unity, $g(\tilde{k}_t)$ is defined and continuous on $(1, \nu)$. While the function is defined and continuous on $(\nu, 1)$ when $\nu < 1$. Finally, when $\nu = 1$, $\tilde{k}_t$ can take a unique value which corresponds to 1. This definition domain is due to the properties of the logarithm. Both parts of the product within the logarithm must have the same sign in order to remain defined. The g function has two possible shapes according to the value of ν with respect to unity:

1. if $\nu > 1$:

   - $g(\tilde{k}_t)$ is strictly increasing in $\tilde{k}_t$,
   - $\lim_{\tilde{k}_t \to 1} g(\tilde{k}_t) = -\infty$ and $\lim_{\tilde{k}_t \to \nu} g(\tilde{k}_t) = +\infty$.

---

[45] Even though I consider a model with inelastic labor supply, no agent would work for a wage which is lower than unemployment benefits. This assumption can be considered as an incentive constraint.

[46] When both input factors are gross complement, the elasticity of substitution is $\sigma \in (0, 1)$ and the corresponding interval for ρ is $(-\infty, 0)$. However, when both are gross substitute, the elasticity of substitution is $\sigma \in (0, +\infty)$ and the corresponding interval for ρ is $(0, 1)$.

[47] These vertical asymptotes correspond to k_1 and k_2 before the normalization.

2. if $\nu < 1$:

- $g(\tilde{k}_t)$ is strictly decreasing in $\tilde{k}_t$,
- $\lim_{\tilde{k}_t \to 1} g(\tilde{k}_t) = +\infty$ and $\lim_{\tilde{k}_t \to \nu} g(\tilde{k}_t) = -\infty$.

When $\rho < 0$, the h function is defined and continuous on $\mathbb{R}_+$ and has the following properties:

1. $h(\tilde{k}_t)$ is strictly decreasing in $\tilde{k}_t$,

2. $\lim_{\tilde{k}_t \to 0} h(\tilde{k}_t) = 1 - \rho$ and $\lim_{\tilde{k}_t \to +\infty} h(\tilde{k}_t) = 0$.

These properties leads to lemmas 1 and 2. Using these intermediate results with proposition 1, I can prove proposition 2.

Lemma 1 *if $\rho < 0$ and $\nu > 1$, then it exists a unique equilibrium.*

Proof. Let $\rho < 0$ and $\nu > 1$. The g function is defined and continuous in $\tilde{k}_t \in (1, \nu)$, strictly increasing and has two infinite vertical asymptotes of opposite signs. The h function is defined and continuous in $\tilde{k}_t \in \mathbb{R}_+ \supset (1, \nu)$, strictly decreasing and has two finite horizontal asymptotes $1/\sigma$ and 0. Therefore, both functions intersect in only one point. Hence, there is uniqueness of the equilibrium. ■

Lemma 2 *if $\rho < 0$ and $\nu < 1$, then it exists at least one equilibrium.*

Proof. Let $\rho < 0$ and $\nu > 1$. The g function is defined and continuous in $\tilde{k}_t \in (\nu, 1)$, strictly decreasing and has two infinite vertical asymptotes of opposite signs. The h function is defined and continuous in $\tilde{k}_t \in \mathbb{R}_+ \supset (\nu, 1)$, strictly decreasing and has two finite horizontal asymptotes $1/\sigma$ and 0. Therefore, both functions intersect in at least one point. Hence, there is at least one equilibrium. ■

Proposition 2 *if $\rho < 0$, then it exists a unique equilibrium.*

Proof. Lemma 1 claims that there is a unique equilibrium when $\nu > 1$. There is also a unique equilibrium when $\nu = 1$. Lemma 2 asserts that there is at least one equilibrium when $\nu < 1$. Yet, proposition 1 states that any equilibrium when $\nu > 1$ leads to the unique equilibrium with full-employment. Therefore, if $\rho < 0$ there is a unique equilibrium. ■

A.2 Under gross-substituability

Contrary to the previous case, $g(\tilde{k}_t)$ is defined and continuous on $\mathbb{R}_+$ but outside of both vertical asymptotes within the logarithm, so 1 and ν. When ν is greater than unity, $g(\tilde{k}_t)$ is defined and continuous on $(0, 1) \cap (\nu, +\infty)$. While the function is defined and continuous on $(0, \nu) \cap (1, +\infty)$ when $\nu < 1$. Finally, when $\nu = 1$, the function is defined and continuous on $\mathbb{R}_+$ but has no longer infinite discontinuity. Regardless of the value of ν, $\lim_{\tilde{k}_t \to 0} = 0$ and $\lim_{\tilde{k}_t \to +\infty} = +\infty$. Both vertical asymptotes correspond to $\lim_{\tilde{k}_t \to 1} g(\tilde{k}_t) = -\infty$ and $\lim_{\tilde{k}_t \to \nu} g(\tilde{k}_t) = +\infty$. The g function has two possible shapes according to the value of ν with respect to unity.[48] Moreover, $g(\tilde{k}_t)$ is strictly increasing in $\tilde{k}_t$ when $\nu < 1$. When $\rho > 1$, the h function is defined and continuous on $\mathbb{R}_+$ and has the following properties:

1. $h(\tilde{k}_t)$ is strictly increasing in $\tilde{k}_t$,

2. $\lim_{\tilde{k}_t \to 0} h(\tilde{k}_t) = 0$ and $\lim_{\tilde{k}_t \to +\infty} h(\tilde{k}_t) = 1 - \rho$.

I plot both functions with numerical computation for feasible values of σ according to the model conditions as detailed in section ref(wage-bargaining). The parameters γ and ϕ are set according to the calibration in section 3.1, thus $\gamma = 0.5$ and $\phi = 0.3$.[49] Figure 17 shows both functions in the case where $\nu < 1$.

[Figure 17 about here.]

Looking at the behavior of both functions, they do intersect in only one point beyond 1. Therefore, I make the following conjecture:

Conjecture 1 *if $\rho \in (0, 1)$ and $\nu < 1$, then it exists a unique equilibrium.*

This unique equilibrium is the one with unemployment since it lies beyond 1 and therefore beyond k_1 without normalization. I also do the numerical computation in the case where $\nu > 1$, with the same values for γ and ϕ. Figure 17 plots the result.

[Figure 18 about here.]

Looking at the behavior of both functions, they do intersect two times before 1. Therefore, I make the following conjecture:

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[48]Excluding the case where $\nu = 1$.

[49]The exact values for ϕ are 0.27 for France and 0.325 for the United-States. I use 0.3 for this numerical computation as an approximation of the mean.

Conjecture 2 *if $\rho \in (0,1)$ and $\nu > 1$, then it exists at least one equilibrium lying below unity.*

Using these two conjectures with proposition 1, I can prove that there is a unique equilibrium for all $\rho \in (0,1)$.[50]

Proposition 3 *if $\rho \in (0,1)$, then it exists a unique equilibrium.*

Proof. Conjecture 1 claims that there is a unique equilibrium when $\nu < 1$. Conjecture 2 asserts that there is at least one equilibrium when $\nu > 1$ lying below unity. Thus, there is at least one equilibrium such that $k_t < k_1$. Yet, proposition 1 states that any equilibrium with $k_t \leq k_1$ leads to the unique equilibrium with full-employment. Therefore, if $\rho \in (0,1)$ there is a unique equilibrium. ∎

B Derivatives

I derive the partial derivative of both functions g and h that determine the equilibrium.

B.1 The g function's derivatives

$$g(L_t, K_t, \eta_t, N_t^y; \phi, \sigma) = \ln \left[\frac{\frac{N_t^y}{L_t} - 1}{\frac{\phi}{1-\phi} \left(\frac{K_t}{L_t} \right)^{\frac{\sigma-1}{\sigma}} \eta_t - 1} \right]$$

The partial derivative of the g function with respect to η_t is:

$$g_\eta = -\frac{\frac{\phi}{1-\phi} k_t^{\frac{\sigma-1}{\sigma}}}{\frac{\phi}{1-\phi} k_t^{\frac{\sigma-1}{\sigma}} \eta_t - 1} = -(\eta_t - \Theta_t)^{-1} < 0$$

where $\eta_t / \Theta_t - 1$ corresponds to the denominator within the logarithm of the g function. At the equilibrium with unemployment, the denominator must be positive. Therefore $\eta_t - \Theta_t > 0 \implies g_\eta < 0$. The partial derivative of the g function with respect to N_t^y is:

$$g_{N^y} = (N_t^y - L_t)^{-1} > 0$$

At the equilibrium with unemployment, the young population size exceeds the number of workers. Thus, $N_t^y - L_t > 0 \implies g_{N^y} > 0$. The partial derivative of the g function with

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[50] Except in the case where $\nu = 1$, i.e. $k_1 = k_2$. In such a case, the infinite discontinuity disappears and so does the equilibrium.

respect to K_t is:

$$g_K = -\frac{1}{K_t}\frac{\sigma-1}{\sigma}\frac{\frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\eta_t}{\frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\eta_t - 1}$$

$$g_K = -\frac{1}{K_t}\frac{\sigma-1}{\sigma}\left(1 - \Theta_t/\eta_t\right)^{-1} \lessgtr 0, \ \forall\sigma \gtrless 1$$

The sign of the derivative depends on the value of the elasticity of substitution between capital and labor with respect to 1. Finally, the partial derivative of the g function with respect to L_t, after some simplifications, is:

$$g_L = -\frac{1}{N_t^y - L_t} - \frac{1}{\sigma L_t}\left(\frac{\phi}{1-\phi}k_t^{\frac{\sigma-1}{\sigma}}\eta_t - \sigma\right)$$

$$g_L = -\frac{1}{N_t^y - L_t} - \frac{1}{\sigma L_t}\left(\eta_t/\Theta_t - \sigma\right)$$

At the equilibrium with unemployment, $\eta_t/\Theta_t > 1$. $\eta_t/\Theta_t - \sigma$ is always positive for $\sigma < 1$, thus $g_L < 0 \ \forall\sigma < 1$. However, the sign cannot be deduced without additional assumptions for $\sigma > 1$. A more restrictive condition is required, i.e. $\eta_t/\Theta_t > \sigma$. In such a case, the partial derivative is unambiguously negative. If this condition is not met, g_L can still be negative provided that the unemployment rate is lower than a threshold $\bar{u}_t = \left(1 - \frac{\eta_t}{\Theta_t\sigma}\right)^{-1}$. Otherwise, g_L is positive.

B.2 The h function's derivatives

$$h(L_t, K_t; \sigma, \phi, \gamma) = \left[\sigma + \frac{1-\phi}{\phi}\frac{1-\gamma(1-\sigma)}{\gamma}\left(\frac{K_t}{L_t}\right)^{\frac{1-\sigma}{\sigma}}\right]^{-1}$$

The partial derivative of the h function with respect to K_t is:

$$h_K = \frac{1}{K_t}\frac{\sigma-1}{\sigma}\frac{\frac{1-\phi}{\phi}\frac{1-\gamma(1-\sigma)}{\gamma}k_t^{\frac{1-\sigma}{\sigma}}}{\left(\sigma + \frac{1-\phi}{\phi}\frac{1-\gamma(1-\sigma)}{\gamma}k_t^{\frac{1-\sigma}{\sigma}}\right)^2} \gtrless 0, \ \forall\sigma \gtrless 1$$

The partial derivative of the h function with respect to L_t is:

$$h_L = -\frac{1}{L_t}\frac{\sigma-1}{\sigma}\frac{\frac{1-\phi}{\phi}\frac{1-\gamma(1-\sigma)}{\gamma}k_t^{\frac{1-\sigma}{\sigma}}}{\left(\sigma + \frac{1-\phi}{\phi}\frac{1-\gamma(1-\sigma)}{\gamma}k_t^{\frac{1-\sigma}{\sigma}}\right)^2} \lessgtr 0, \ \forall\sigma \gtrless 1$$

C Estimation of the capital-labor elasticity

The CES production function with biased technical change as defined by David and van de Klundert (1965) is:

$$Y_t = A \left[\left(E_t^K K_t \right)^{\frac{\sigma-1}{\sigma}} + \left(E_t^L L_t \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where E_t^K and E_t^L represent the efficiency levels of both input factors. I assume linear growth rates of efficiency levels, so $E_t^i = E_0^i e^{a_i(t-t_0)}$ where a_i denotes growth in technical progress associated with factor $i \in \{K, L\}$ and represents a linear time trend. To normalize the production function, I follow the specification of Klump et al. (2007),

$$E_0^K = \frac{Y_0}{K_0} \left(\frac{1}{\phi_0} \right)^{\frac{\sigma}{1-\sigma}} \quad \text{and} \quad E_0^L = \frac{Y_0}{L_0} \left(\frac{1}{1-\phi_0} \right)^{\frac{\sigma}{1-\sigma}}$$

Normalization of the CES production function requires that factor shares are not biased by the growth of factor efficiencies at the fixed point. At time $t = t_0$, $e^{a_i(t-t_0)} = 1$. Thus, they are just equal to the initial distribution parameters ϕ_0 and $1 - \phi_0$. Assuming that the firm is on its labor demand curve, as in the model, the labor share at time t is :

$$\theta_t \equiv \frac{w_t L_t}{Y_t} = \left[1 + \left(\frac{E_t^K}{E_t^L} \frac{K_t}{L_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{-1}$$

Substituting with both efficiency levels, the labor share becomes:

$$\theta_t = \left[1 + \frac{\phi_0}{1-\phi_0} \left(\frac{K_t}{K_0} \frac{L_0}{L_t} e^{(a_K - a_L)(t-t_0)} \right)^{\frac{\sigma-1}{\sigma}} \right]^{-1}$$

Let $k_t \equiv K_t/L_t$ be the capital-to-labor ratio. Thus, the labor-to-capital income ratio is:

$$\Theta_t \equiv \frac{\theta_t}{1-\theta_t} = \frac{1-\phi_0}{\phi_0} \left(\frac{k_t}{k_0} e^{(a_K - a_L)(t-t_0)} \right)^{\frac{1-\sigma}{\sigma}}$$

Rewriting the above equation and taking logs,

$$\ln \Theta_t = \alpha + \beta \ln \tilde{k}_t + \gamma \left(t - t_0 \right) \tag{21}$$

where $\tilde{k}_t$ is the normalized capital-to-labor ratio, α a constant term, $\beta = \frac{1-\sigma}{\sigma}$ and $\gamma = \frac{1-\sigma}{\sigma}(a_K - a_L)$. With this form, it is not possible to identify each technical change growth rate (i.e. a_K and a_L). However, it captures the overall bias in technical change (i.e. $a_K - a_L$).

I estimate equation (21) using OLS for France and United States over the period 1970-2010.[51] I use data and variables specifications as described in section 3.1. I consider four specifications. The first one is the RAW estimation, without biased technical change nor hours worked correction. Then, I only control for the average hours worked in the HWC estimation and for the biased technical change in the BTC estimation. Finally, both are used as control in the HWC-BTC estimation. Standard errors are relatively high due to the lack of observations. Equations are estimated by country with 41 observations for each one. When the coefficient associated to the line $\frac{1-\sigma}{\sigma}$ is not significant it means that the estimated elasticity is not statistically different from 1, i.e. the Cobb-Douglas case. Table 3 summarizes the results for France and the United States, respectively.

[Table 3 about here.]

For France, worked hours seems to play a role in the estimation of the elasticity when I compare RAW to HWC. Not considering the hours worked correction may bias the elasticity of substitution toward relatively high values (2.593 against 1.887). I compare RAW to BTC and conclude that biased technical change has also to be taken into account. Even though the σ associated coefficient is not significant, the biased technical change's coefficient is significant. Finally, controlling for both in HWC-BTC leads to an elasticity about 1.279. But it is only significant at a 90 % confidence level. This is the result of a break in the regime as explained in appendix D. The biased technical change coefficient is also significant as in BTC. It indicates that biased technical change has to be jointly considered with the hours worked for France.

For the United States, hours worked correction does matter by comparing RAW to HWC. Once I control for it, the elasticity of substitution is about 1.234. However, the technical change does not seem to be biased since both associated coefficients in BTC and HWC-BTC are not significant. The coefficient associated to the elasticity are below unity but not statistically significant. Thus, controlling for biased technical change is not necessary for the United States. I only have to control for average worked hours.

Therefore, I consider a capital-labor elasticity of substitution about 1.279 for France and 1.234 for the United States. These σ values are used to compute the deduced parameter values (i.e. ω and β) in section 3.1.

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[51]This is a single-equation estimation from the two first-order conditions of the profit maximization. As Klump et al. (2007) discussed, single-equation or two-equations estimations can be biased due to endogeneity. They recommend to use supply-side system estimation. However, the aim of this estimation is only to obtain a value to simulate the model. Therefore, the elasticity of substitution I obtain is the one within the model specification.

D   Two capital-labor elasticity regimes in France

Considering the estimation methodology in appendix C, I obtain a capital-labor elasticity of substitution around 1.279 for France over the period 1970-2010. However, the estimated coefficient is only significant at a 90% confidence level. Such a result may be due to a break in the regime of σ. The relationship between the labor-to-capital income ratio Θ_t and the capital-per-worker k_t depends on the value of σ with respect to unity. From the comparatives statics derived in section 2, it can be shown that :

$$\frac{\partial \Theta_t}{\partial k_t} \gtreqqless 0 \quad \text{when} \quad \sigma \lesseqqgtr 1$$

When both input factors are gross complement (i.e. $\sigma < 1$), the labor-to-capital income ratio and the capital-per-worker vary in the same direction. While they vary in opposite ways under gross substituability. The elasticity of substitution σ is estimated using the equation (21). Inspecting the time series of Θ_t and $\tilde{k}_t$, I suspect that there is a change of regime in the link between them. Figure 19 plots these time series.

[Figure 19 about here.]

The figure suggests that the correlation between both variables is different before and after the beginning of the 80's. Suppose that the change of regime in the relationship has occurred in 1982. The correlation between both variables is about 0.499 before 1982 and -0.787 after.[52] Such a difference in the correlation suggests that there are two regimes. However, the biased technical change has not been considered so far. Therefore, I have to detrend both variables in order to check whether the regime change is not driven by it. Using the Frisch-Waugh-Lovell theorem, I regress each variable on a linear time trend and extract the residuals as detrended variables. Both regressions correspond to the following:

$$\ln \Theta_t = \varphi_0^{\Theta} + \varphi_1^{\Theta} \times t + \varepsilon_t^{\Theta}$$
$$\ln \tilde{k}_t = \varphi_0^{\tilde{k}} + \varphi_1^{\tilde{k}} \times t + \varepsilon_t^{\tilde{k}}$$

Figure 19 displays both detrended variables $\varepsilon_t^{\tilde{k}}$ and ε_t^{Θ} on the right-hand side panel. The vertical dashed lines correspond to a potential break year in the regime in 1985. Looking at the variables behavior before and after the three thresholds, it suggests that there are still two regimes. Therefore, this result remains even when controlling for linear biased technical change.

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[52]The year 1982 is included in the first sub-sample. Including it in the sub-sample after the break leads to the correlations of 0.530 and -0.815 for before and after respectively.

For instance, figure 20 plots the regression lines before and after an elasticity regime change in 1985 for the raw and detrended variables. On the right-hand side panel, the slope is positive between 1970 and 1985, so the elasticity would be smaller than 1 during this period and greater thereafter. All years between 1980 and 1990 are candidate to be the break year.

[Figure 20 about here.]

To find which year corresponds to the most efficient break year, I run multiple regressions by changing the break year $\bar{t}$ for each regression. The estimated equation is:

$$\varepsilon_t^\Theta = \nu_1 \mathbb{1}_{\{t \le \bar{t}\}} + \nu_2 \mathbb{1}_{\{t > \bar{t}\}} + \rho_1 \mathbb{1}_{\{t \le \bar{t}\}} \varepsilon_t^{\tilde{k}} + \rho_2 \mathbb{1}_{\{t > \bar{t}\}} \varepsilon_t^{\tilde{k}} + \zeta_t$$

where $\mathbb{1}$ is a dummy variable which is equal to 1 when the condition under brackets is satisfied, 0 otherwise. Table 4 presents the four coefficients (i.e. ν_1, ν_2, ρ_1, ρ_2) and the R^2 of each regression. The regression with a change in the regime in 1985 (i.e. $\bar{t} = 1985$) has the greatest R^2 and all its coefficients are significant. Thus, I assume that the break in the regime has occurred in 1985.

[Table 4 about here.]

I have to estimate a value of σ for 1970-1985 and another one for 1986-2010. I estimate the equation (21) for each sub-sample with and without biased technical change. Table 5 summarizes the coefficients.

[Table 5 about here.]

Biased technical change coefficients are not significant in both sub-sample regressions, neither are the coefficients associated to the elasticity. Therefore, I take into account the estimate where the technical change is not biased. Between 1970 and 1985, I set the capital-labor elasticity of substitution to unity, since the coefficient is not statistically different from 0.[53] However, it is significant between 1986 and 2010 and the elasticity is about 1.321.

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[53]This is mainly due to the lack of observations. The sub-sample has only 16 observations and therefore 14 degrees of freedom.

List of Figures

Figure 1: Demographic variables time trends

Notes: Demographic data from the United Nations World Population Prospects 2017. Demographic data between 2010 and 2100 are the "medium variant" estimates from the United Nations.

Figure 2: Model prediction of the labor share

Notes: Labor share data from the Penn World Table 9.1 with adjustment method 1.

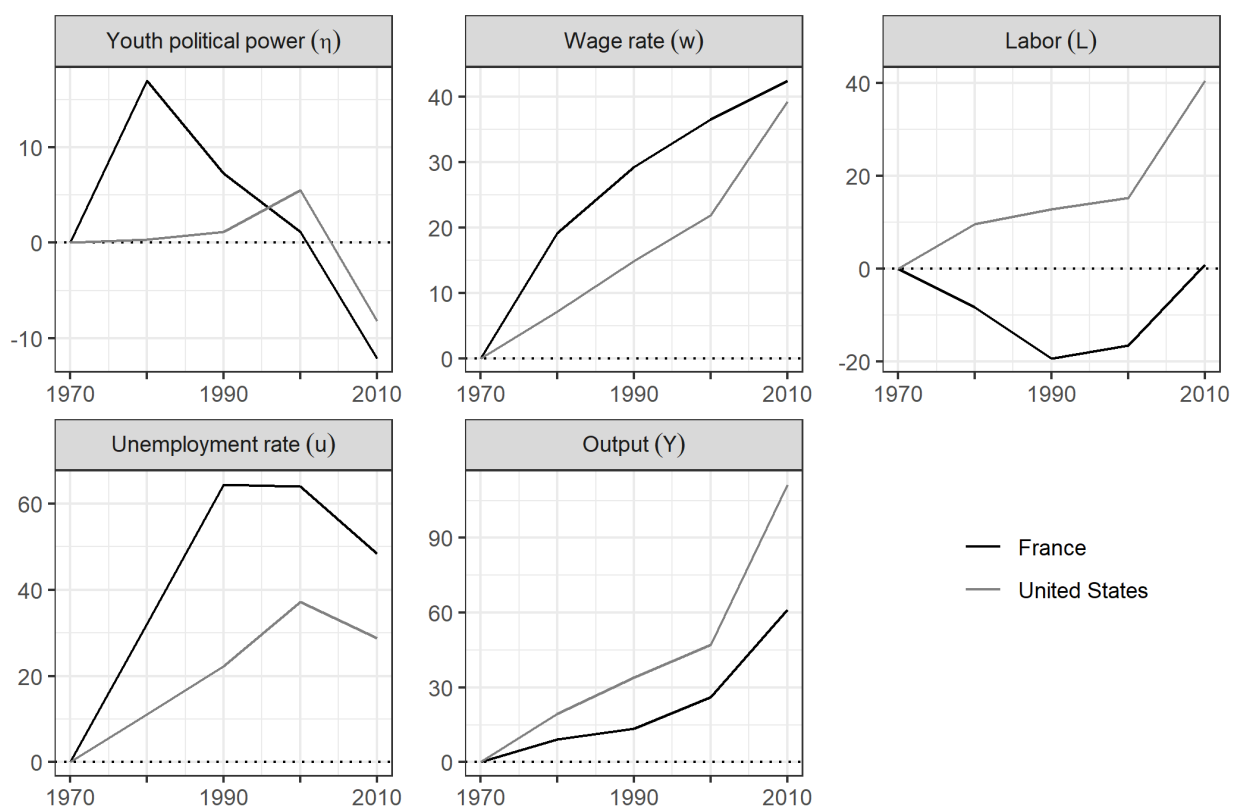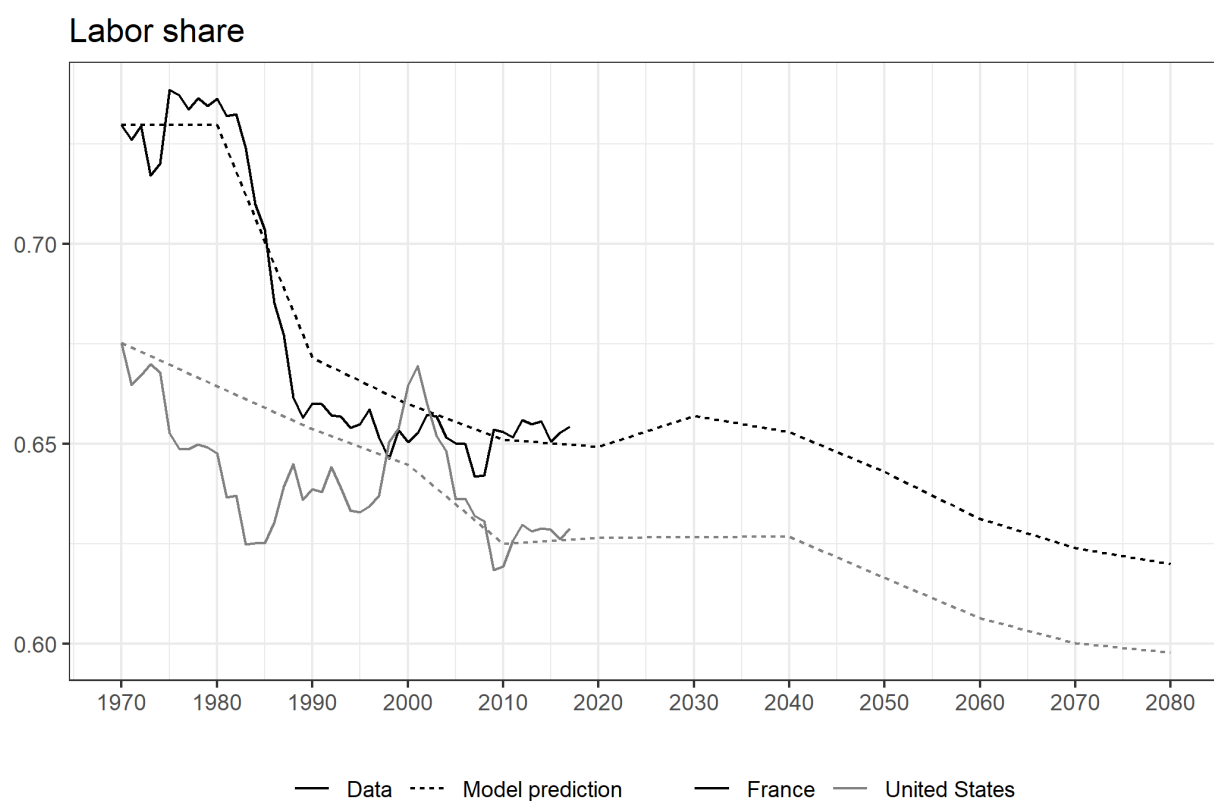Figure 3: Deviation from the 1970's value of determinant variables (in percentage)

Figure 4: Model prediction of the labor share

Labor share

Legend: — Data ···· Model prediction — France — United States

Figure 5: Deviation from the 2010's value of determinant variables (in percent)
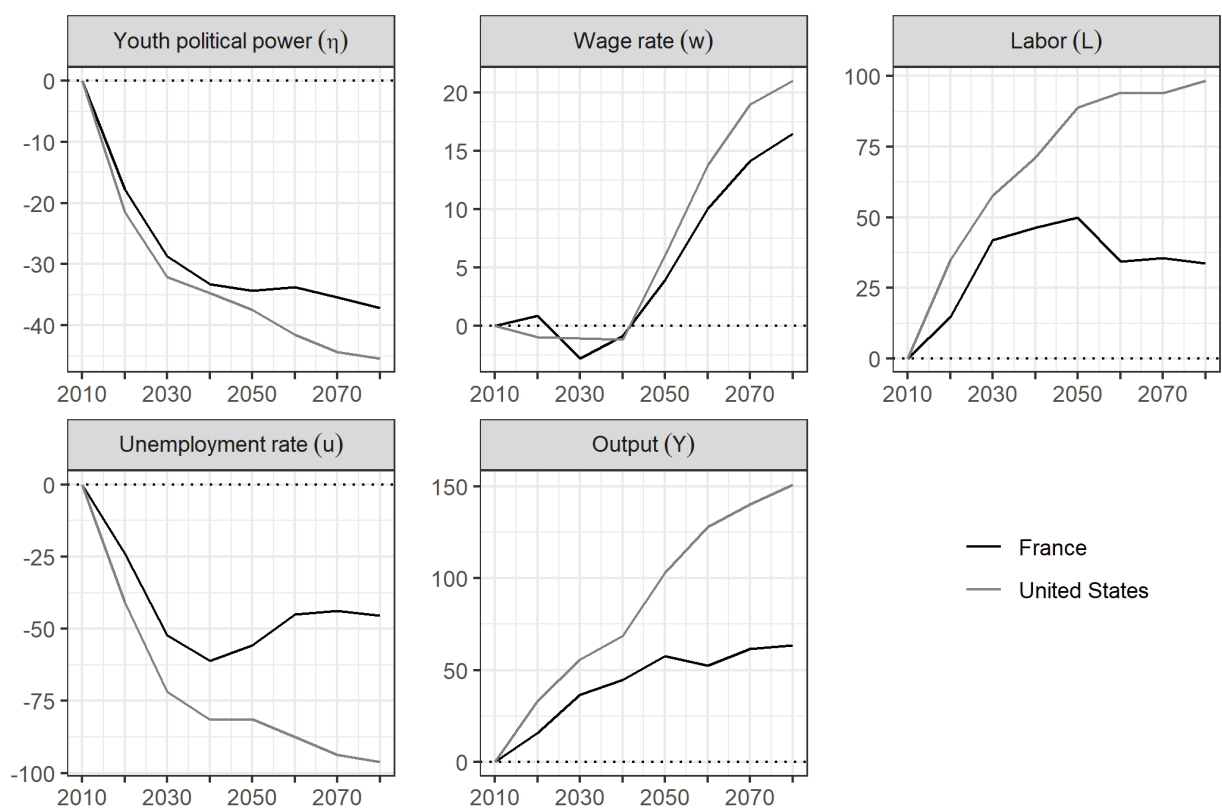
Figure 6: Model predictions of the labor share with counterfactual specifications on the determinant variables
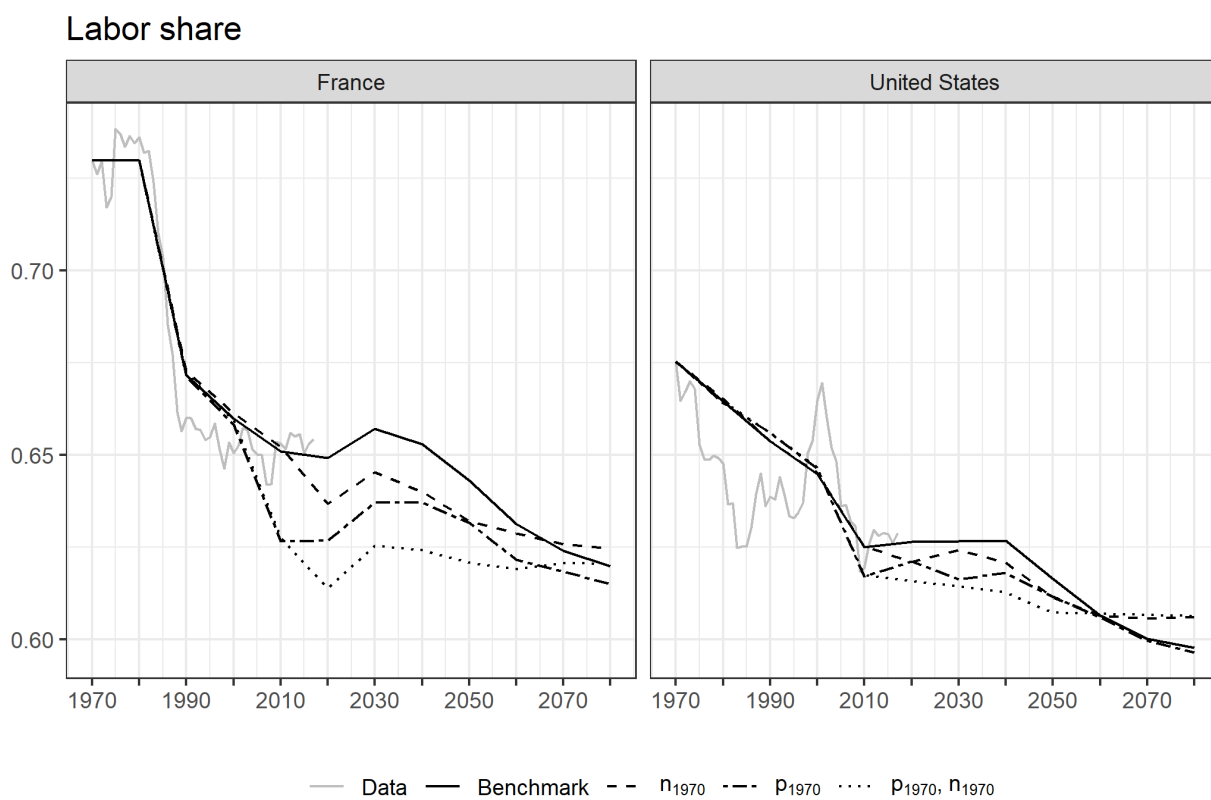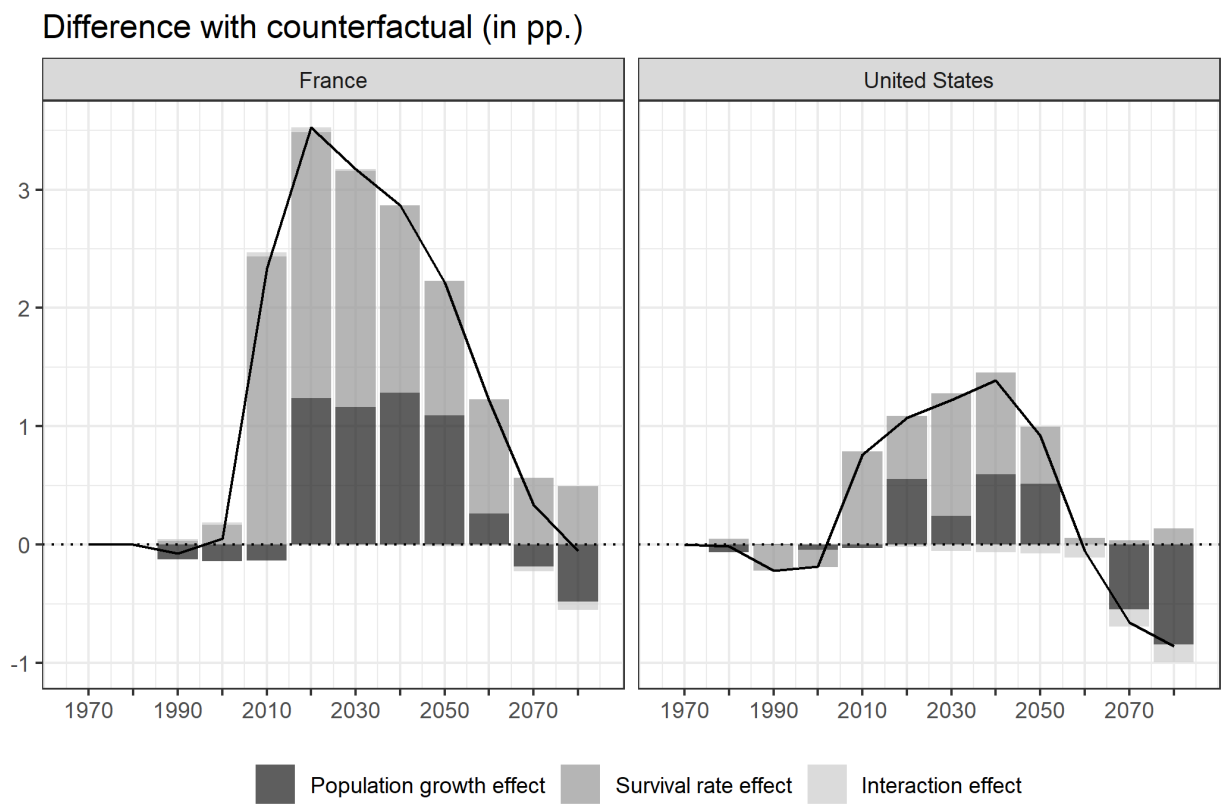

Labor share

Figure 7: Aging-effect decomposition by determinant

Difference with counterfactual (in pp.)

Population growth effect   Survival rate effect   Interaction effect

Figure 8: Model predictions of the labor share with counterfactual specifications on the transmission channels

Labor share
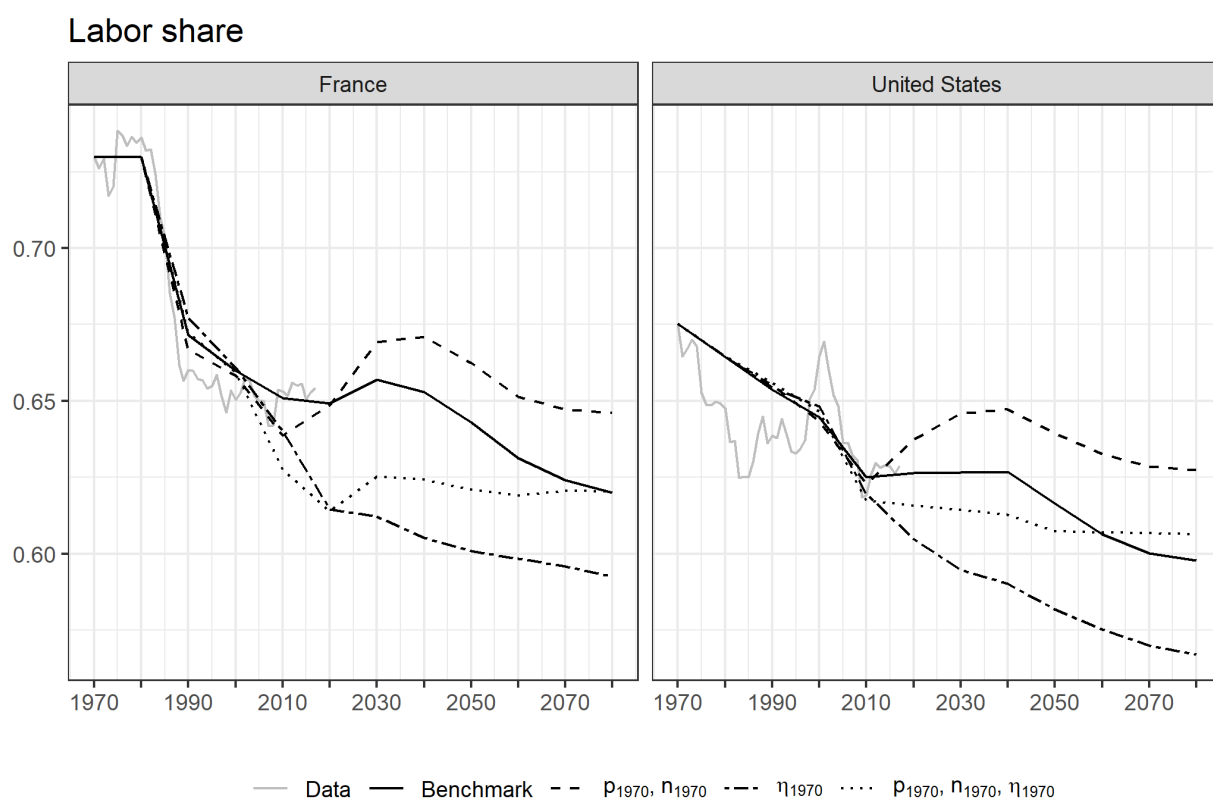
Figure 9: Aging-effect decomposition by channel

Difference with counterfactual (in pp.)
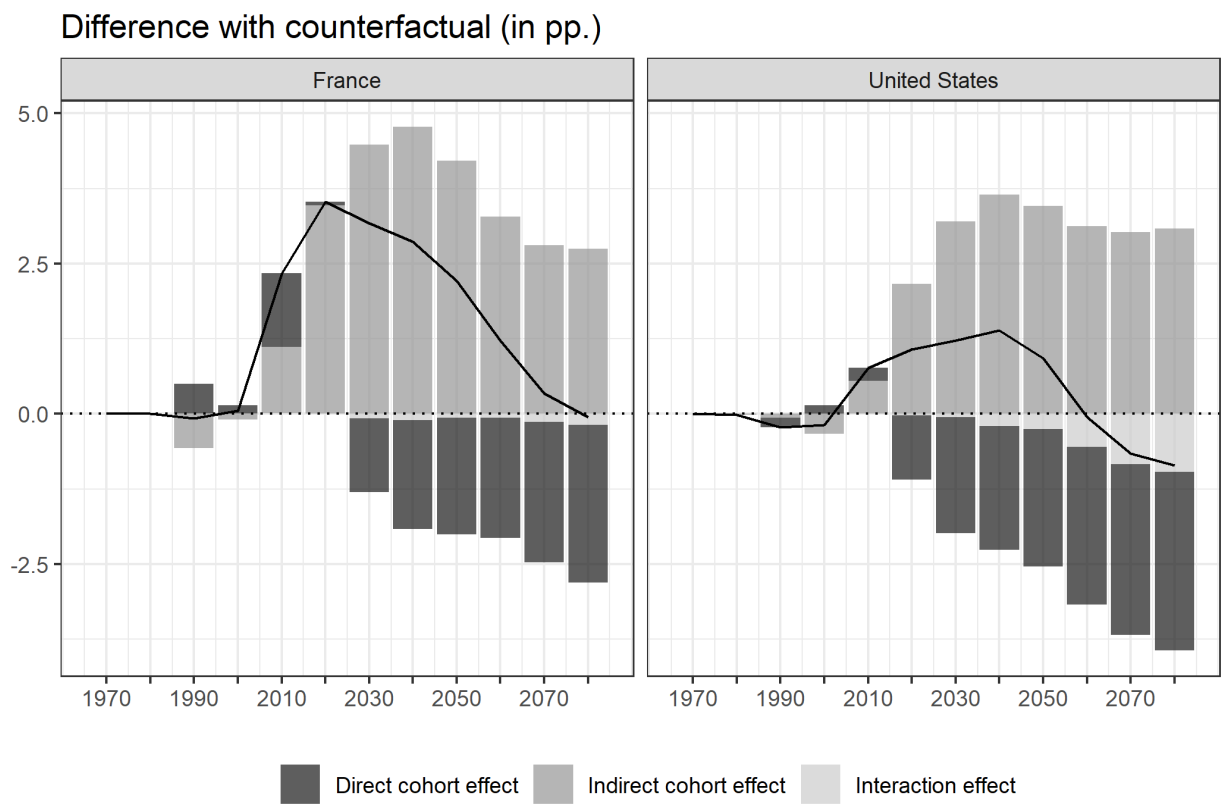
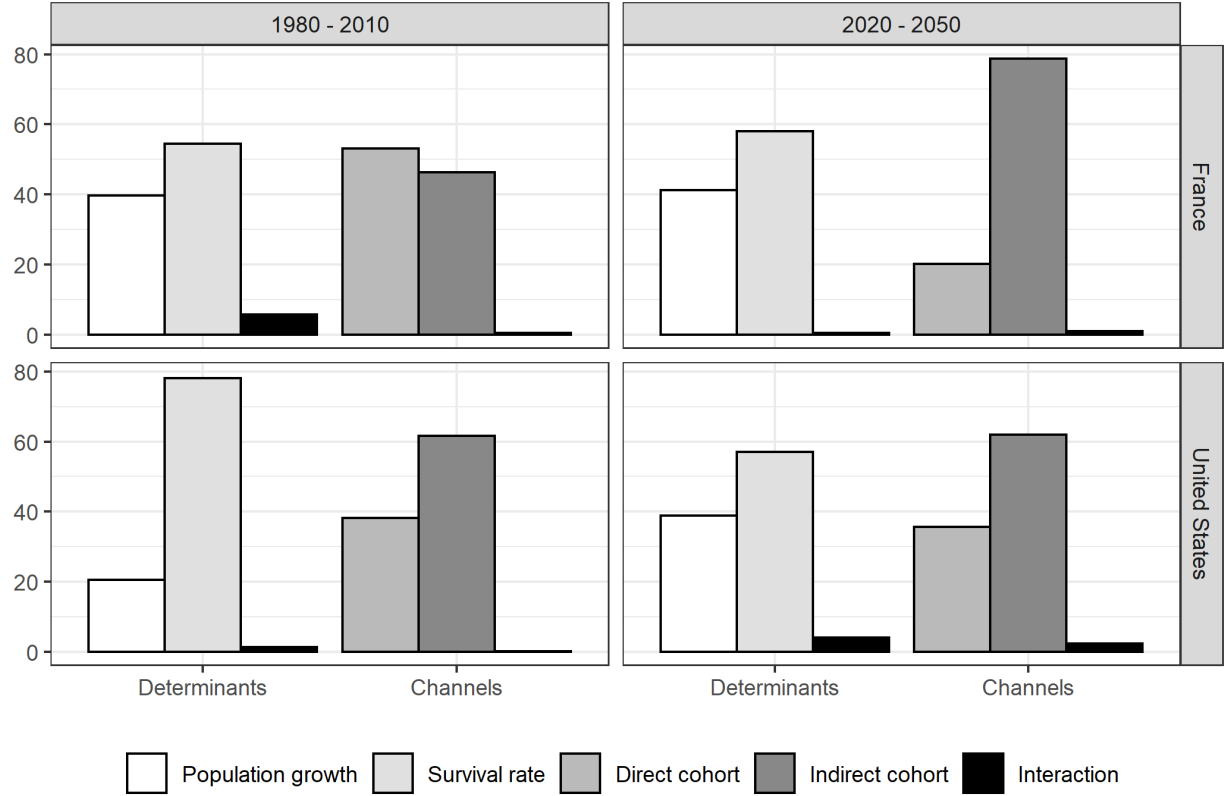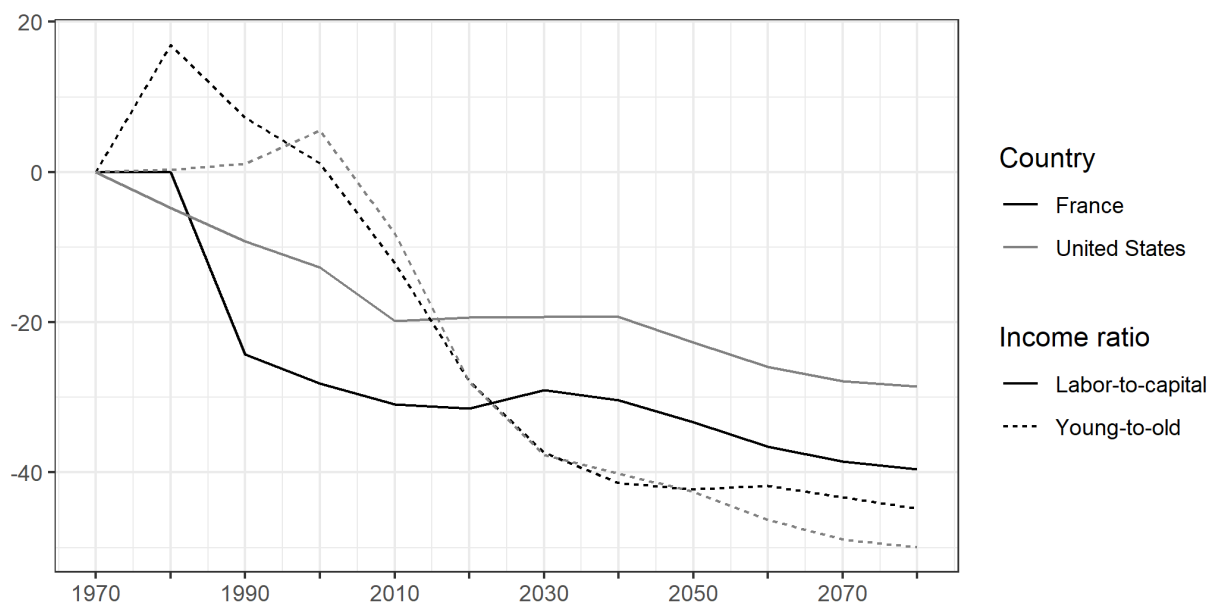Figure 10: Aging-effect decomposition by period and country

Figure 11: Deviation from 1970's value of the labor-to-capital income ratio and the young-to-old income ratio

Notes: The solid curve corresponds to the before-tax young-to-old income Θ_t ratio also named the labor-to-capital income ratio $\frac{w_t L_t}{r_t K_t}$ and the dashed curve is the after-tax young-to-old income ratio Y_t^y/Y_t^o which is equal to the youth political power η_t. Deviation is expressed in percentage.
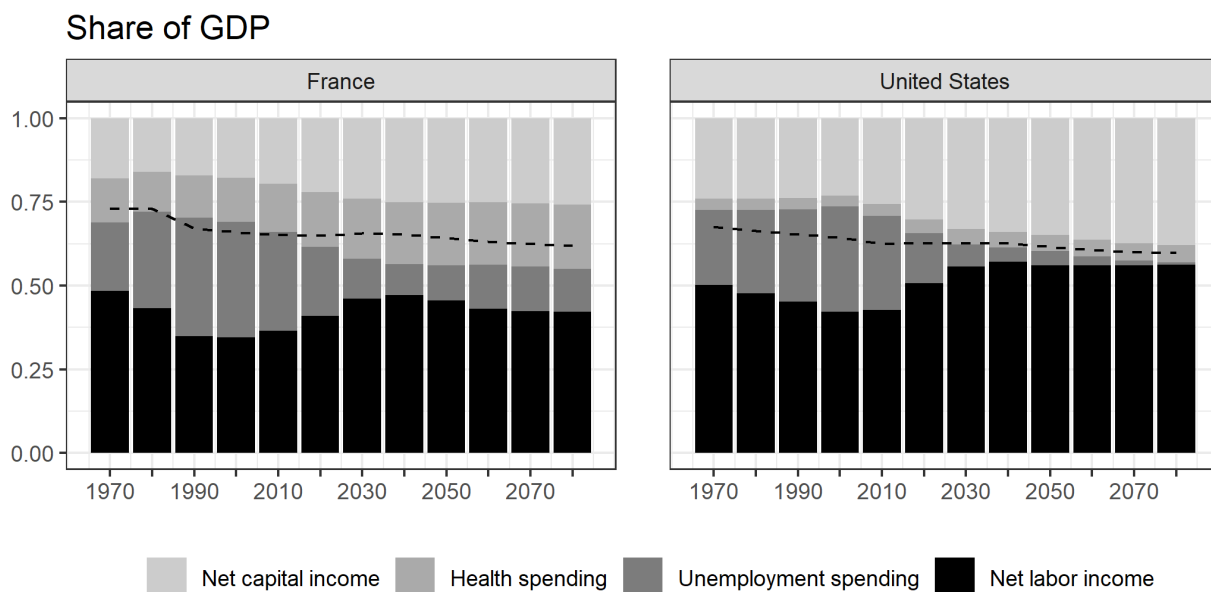
Figure 12: After-tax income share allocation

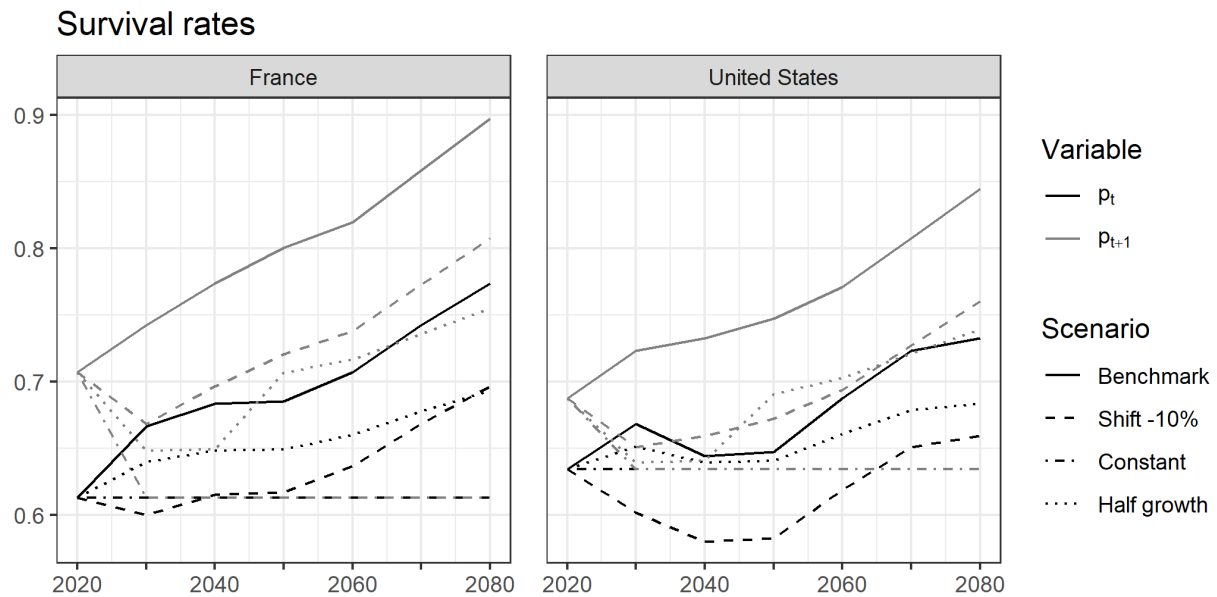Figure 13: Survival rate dynamics with changing retirement age specifications

Figure 14: Model predictions of the labor share with changing retirement age specifications
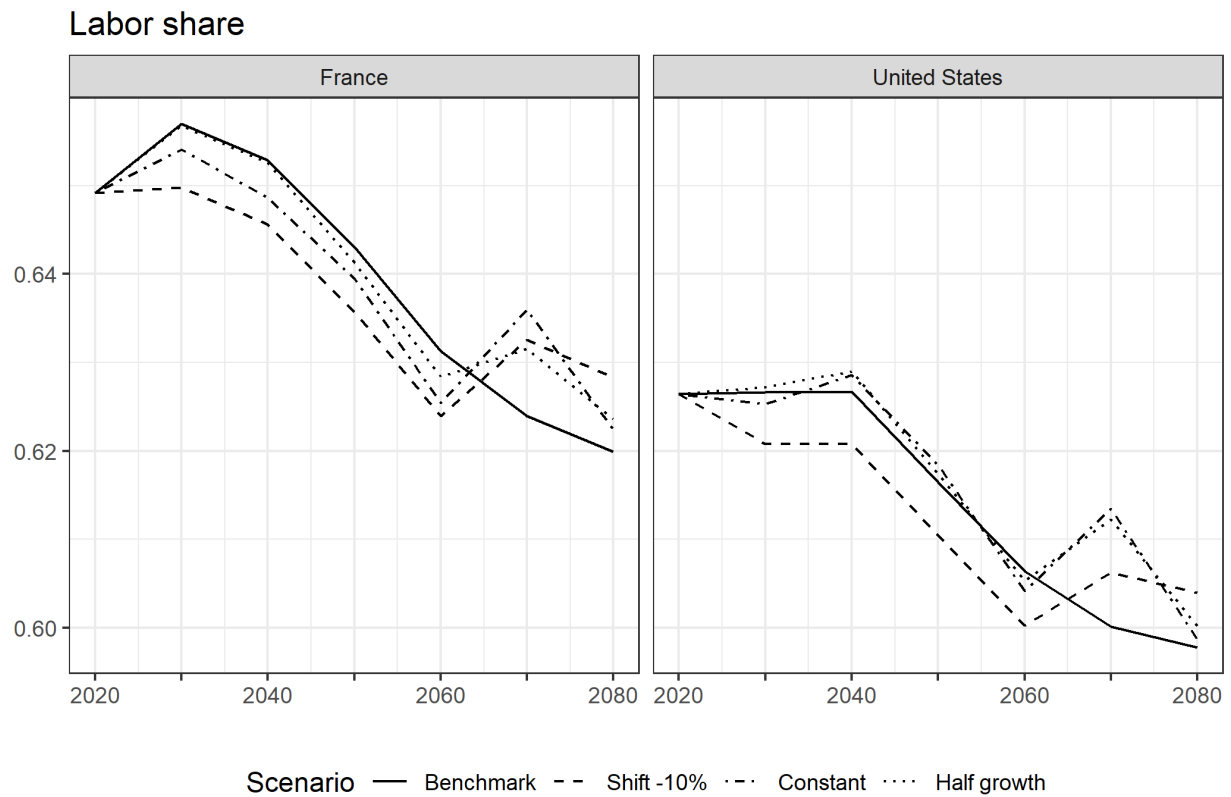
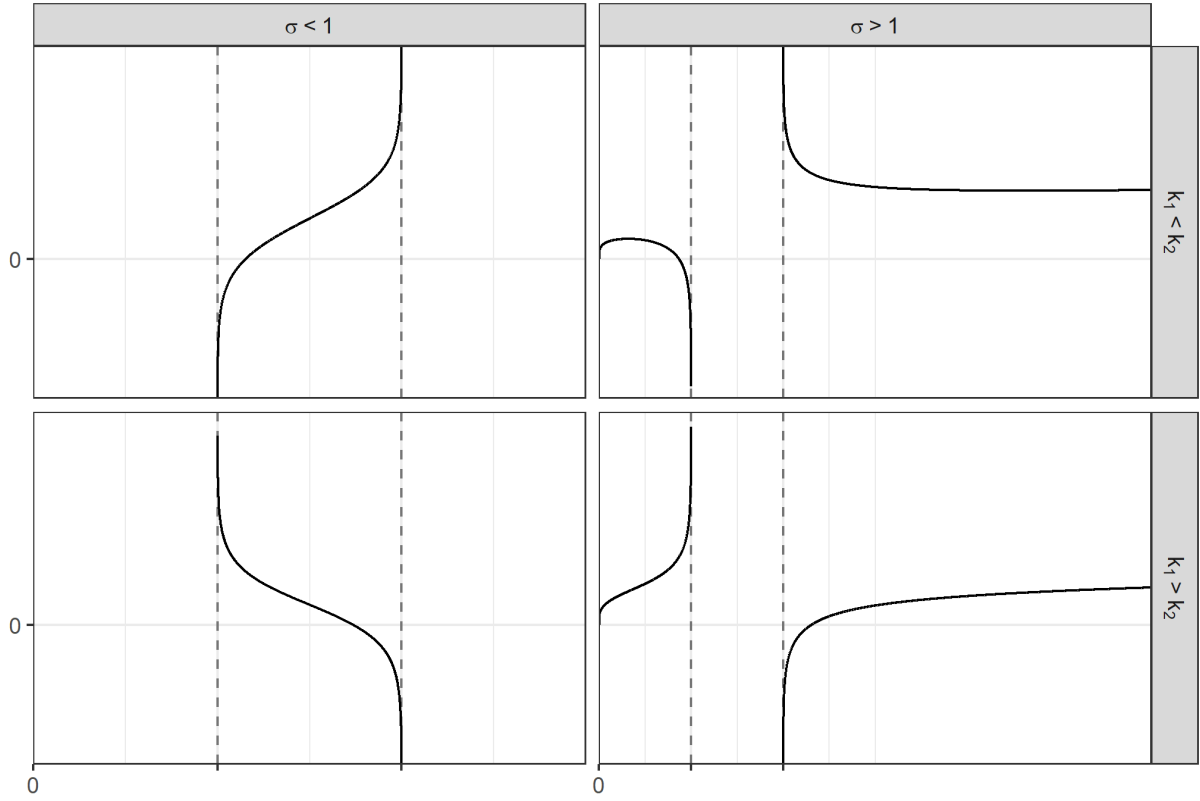Figure 15: Different possible shapes of the g function, according to the values of $sigma$, k_1 and k_2

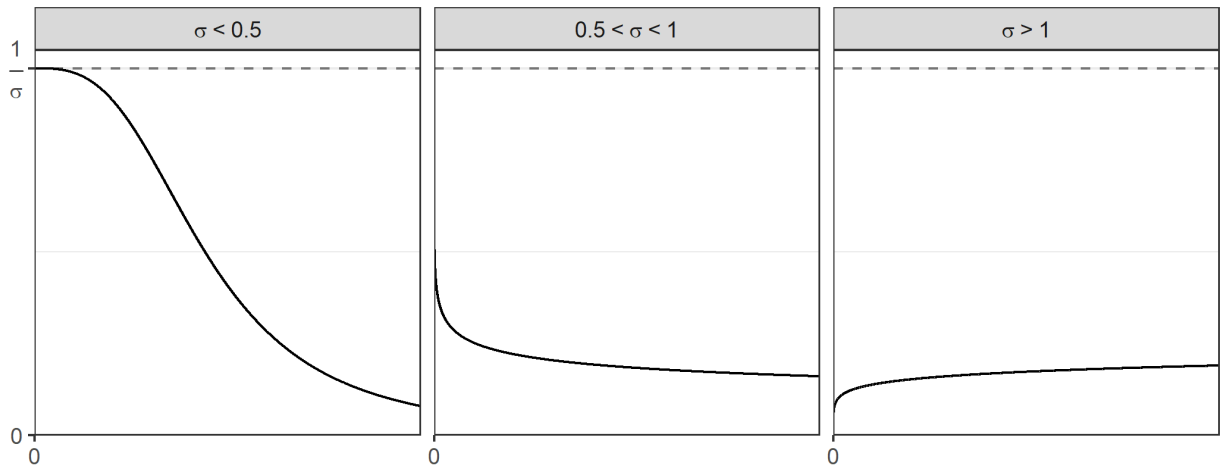Figure 16: Different possible shapes of the h function, according to the values of $sigma$

Figure 17: Numerical simulation of g (solid line) and h (dashed line), according to the values of σ and ν
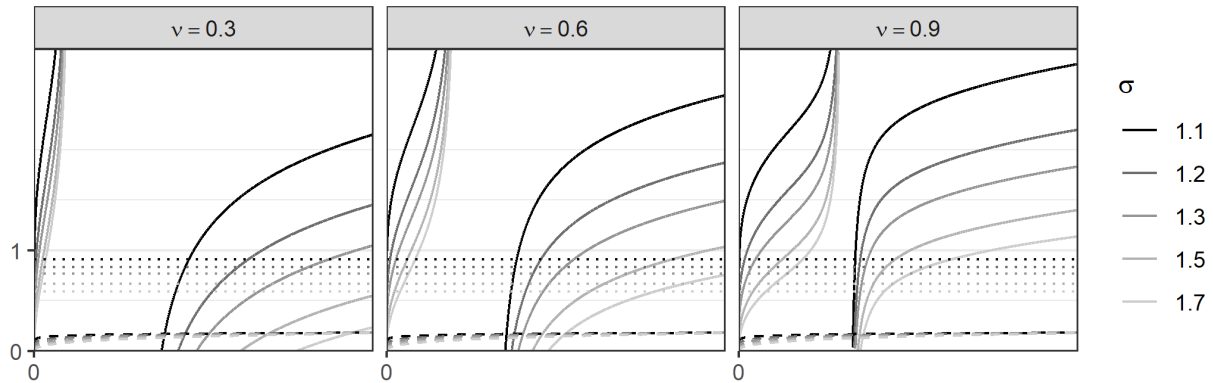
Figure 18: Numerical simulation of g (solid line) and h (dashed line), according to the values of σ and ν
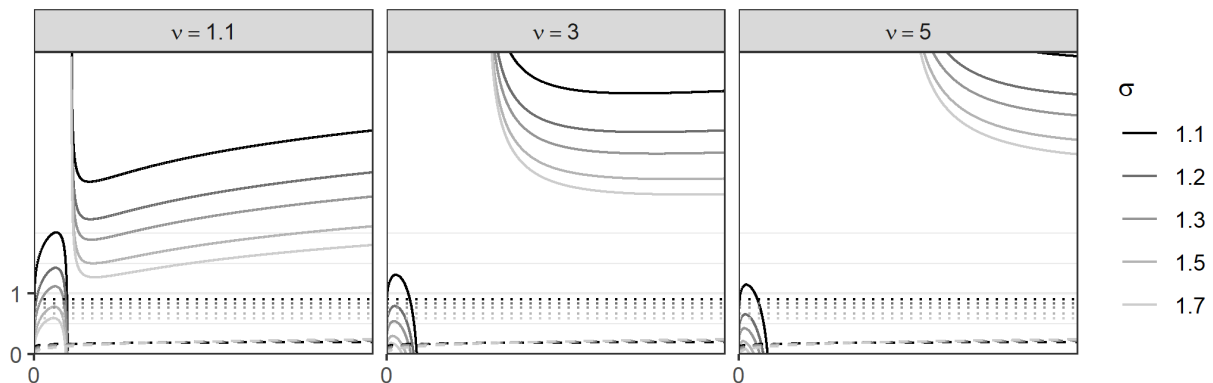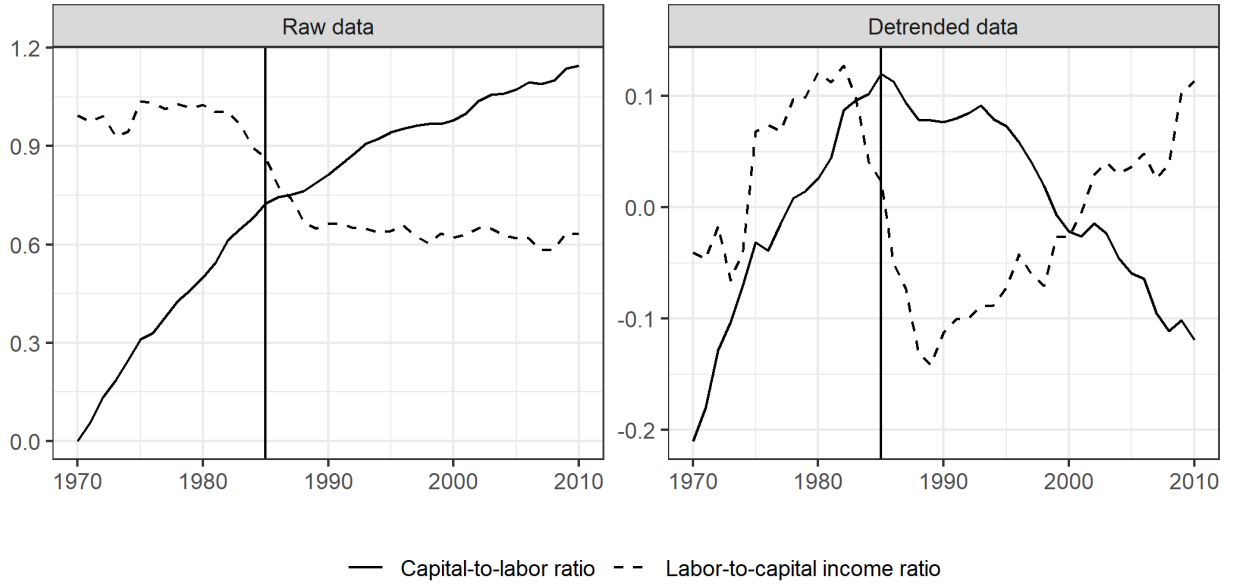
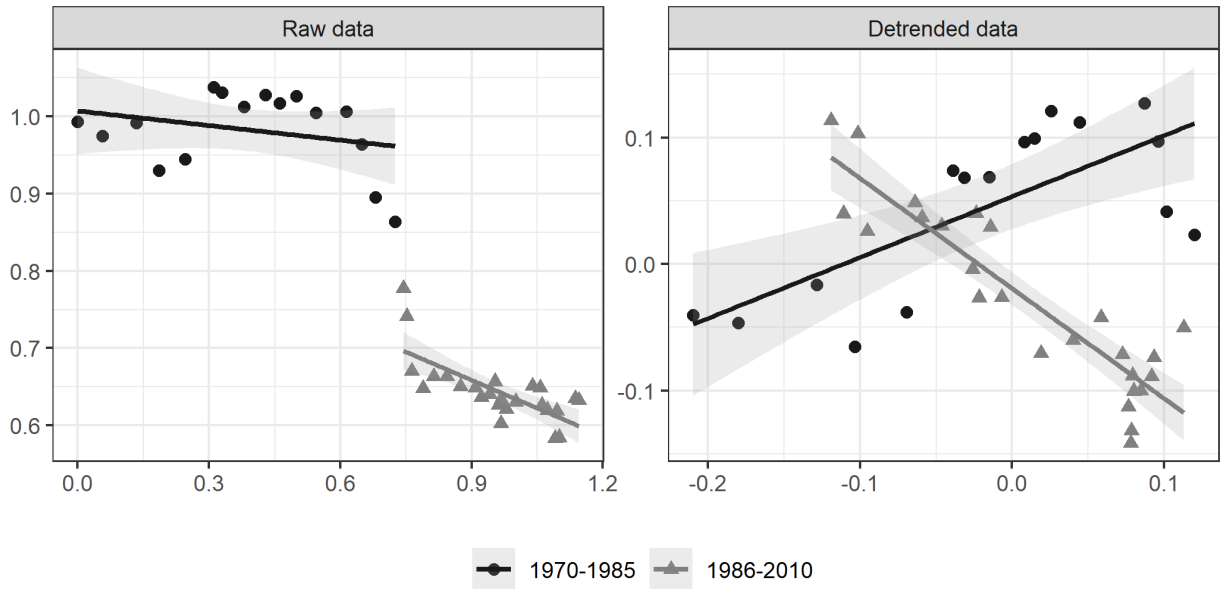Figure 19: $\ln \tilde{k}_t$ and $\ln \Theta_t$ for France over the period 1970-2010

Figure 20: Relationship between k_t and Θ_t (y-axis) for France over the period 1970-2010

The gray area corresponds to the confidence interval of the associated regression. The level of the confidence interval is set to 0.95.

List of Tables

Table 1: Parameters

	Parameter	France	United States
ϕ	Capital share in 1970	0.270	0.325
γ	Relative bargaining power of the union	0.500	0.500
α	Discount rate	0.669	0.669
σ	Capital-labor elasticity of substitution	1.321	1.234
ω	Relative ideological spread-out	0.983	1.533
β	Preference for government health expenditure	0.739	0.138
A	Scale parameter of the production function	23.891	22.840

Notes: Single-equation estimation of σ from the two first-order conditions of the profit maximization with normalized CES production function. σ estimates are significant at $p < 0.1$ for France and $p < 0.05$ for the United States. Details in appendix C.

Table 2: Demographic variables in 1970

	Variable	France	United-States
p_{1970}	Survival rate in 1970	0.417	0.476
n_{1970}	Population growth in 1970	1.134	1.597
p_{2010}	Expected survival rate in 2010	0.583	0.561
$\frac{p_{1970}}{n_{1970}}$	Old-age-dependency ratio in 1970	0.368	0.298
η_{1970}	Youth political power in 1970	3.846	3.008

Table 3: Estimation of the capital-labor elasticity of substitution

	(RAW)	(HWC)	(BTC)	(HWC-BTC)
α	1.130***	1.123***	1.081***	1.080***
	(0.028)	(0.027)	(0.035)	(0.033)
$\alpha \times$ US	−0.494***	−0.475***	−0.438***	−0.430***
	(0.035)	(0.035)	(0.042)	(0.040)
β	−0.614***	−0.470***	−0.273	−0.218*
	(0.045)	(0.033)	(0.165)	(0.125)
$\beta \times$ US	0.437***	0.281***	0.428	−0.429
	(0.085)	(0.075)	(0.502)	(0.370)
γ			−0.007**	−0.007**
			(0.003)	(0.003)
$\gamma \times$ US			0.003	0.014**
			(0.007)	(0.006)
σ_{FR}	2.593	1.887	1.375	1.279
σ_{US}	1.215	1.234	0.866	2.835
R^2	0.811	0.817	0.823	0.830
Adj.R^2	0.803	0.810	0.811	0.819
Num.obs.	82	82	82	82

Table 4: Estimation of the capital-labor elasticity of substitution for France with break year

$\bar{t}$	ν_1	ν_2	ρ_1	ρ_2	R^2
1980	0.080***	0.002	0.765***	−0.516***	0.385
	(0.025)	(0.012)	(0.251)	(0.158)	
1981	0.079***	−0.002	0.762***	−0.533***	0.461
	(0.021)	(0.012)	(0.217)	(0.148)	
1982	0.077***	−0.007	0.736***	−0.607***	0.596
	(0.016)	(0.010)	(0.168)	(0.130)	
1983	0.071***	−0.012	0.677***	−0.690***	0.705
	(0.013)	(0.009)	(0.131)	(0.113)	
1984	0.062***	−0.015*	0.580***	−0.764***	0.741
	(0.011)	(0.008)	(0.114)	(0.108)	
1985	0.053***	−0.019**	0.482***	−0.870***	0.780
	(0.010)	(0.008)	(0.098)	(0.104)	
1986	0.043***	−0.022**	0.365***	−0.928***	0.713
	(0.011)	(0.009)	(0.106)	(0.124)	
1987	0.034***	−0.023**	0.282**	−0.956***	0.635
	(0.012)	(0.010)	(0.116)	(0.145)	
1988	0.024*	−0.022*	0.202	−0.931***	0.507
	(0.013)	(0.012)	(0.133)	(0.173)	
1989	0.016	−0.019	0.131	−0.893***	0.391
	(0.014)	(0.014)	(0.145)	(0.198)	
1990	0.009	−0.018	0.082	−0.871***	0.321
	(0.015)	(0.015)	(0.151)	(0.216)	

Table 5: Estimation of the capital-labor elasticity of substitution for France with break in the regime in 1986

	1970-1985		1986-2010	
	(HWC)	(HWC-BTC)	(HWC)	(HWC-BTC)
α	1.007***	0.970***	0.877***	0.916***
	(0.026)	(0.033)	(0.046)	(0.160)
β	−0.063	1.002	−0.243***	−0.322
	(0.058)	(0.648)	(0.048)	(0.321)
γ		−0.050		0.001
		(0.031)		(0.005)
σ_{FR}	1.067	0.499	1.321	1.476
R^2	0.077	0.236	0.527	0.529
Adj.R^2	0.011	0.119	0.507	0.486
Num.obs.	16	16	25	25