

Figure 3.1. Simplified inflow and outflow wind profiles over a mountain barrier (United States Weather Bureau, 1961a)

equivalent), respectively, in mm; Y is the horizontal distance in m; and ΔP_1 and ΔP_2 are the inflow and outflow pressure differences, respectively, in hPa.

Equation 3.1 is a storage equation: that is, precipitation equals inflow of water vapour minus outflow of water vapour. It may be derived as follows. Consider the mass transport through the slice of space bounded by two identical vertical planes, as in Figure 3.2, a short horizontal distance *s* apart. The storage equation for water vapour is:

$$M_r = (M_v)_1 - (M_v)_2 \tag{3.2}$$

where M_r is the rate of conversion of water vapour to precipitation in g/s; $(M_v)_1$ is the rate of inflow of water vapour in g/s; and $(M_v)_2$ is the rate of outflow of water vapour in g/s.

The values of these terms are given by:

$$W_r = RYs\rho \tag{3.3}$$

$$(M_v)_1 = V_1 W_1 s \rho \tag{3.4}$$

$$(M_{\nu})_2 = V_2 W_2 s \rho \tag{3.5}$$

where ρ is the density of water, which is 1.0 g/cm³, and *s* is in cm. The mass of air flowing in equals the mass flowing out if no allowance is made for the mass of precipitation which falls, which is relatively very small and may be neglected. The continuity equation is expressed by:

$$V_1 \Delta P_1 = V_2 \Delta P_2 \tag{3.6}$$

Combining Equations 3.2–3.6 and solving for *R* yields Equation 3.1.

3.2.2.2 Multiple-layer laminar flow model

Greater precision requires dividing the air into several layers of flow, as in Figure 3.3, rather than treating it as a single layer. Equation 3.1 applies to each of these layers. Total precipitation is then obtained by adding the rates from all layers. With several layers, it is more convenient to use the storage equation in the following form:

$$R = \frac{\overline{V}_1 \Delta P_1 \left(\overline{q}_1 - \overline{q}_2\right)}{Y} \frac{1}{g\rho}$$
 (3.7)

where \overline{V}_1 and ΔP_1 refer to the inflow in a particular layer; and \overline{q}_1 and \overline{q}_2 are the mean specific humidities in g/kg at inflow and outflow, respectively. The mixing ratio w is often substituted for specific humidity q. The terms g and ρ refer to the acceleration due to gravity in cm/s² and the density of water in g/cm³, respectively.

Equation 3.7 is derived from the relation between specific humidity and precipitable water:

$$W = \frac{\overline{q}\Delta P}{g\rho} \tag{3.8}$$

Substituting this relation into Equation 3.1 yields:

$$R = \frac{\overline{q}_1 \Delta P_1}{g \rho} - \frac{\overline{q}_2 \Delta P_2}{g \rho} \frac{\Delta P_1}{\Delta P_2}$$
 (3.9)

This reduces to Equation 3.7.