NFA TO DFA

NFAs accept the Regular Languages

Equivalence of Machines

Definition for Automata:

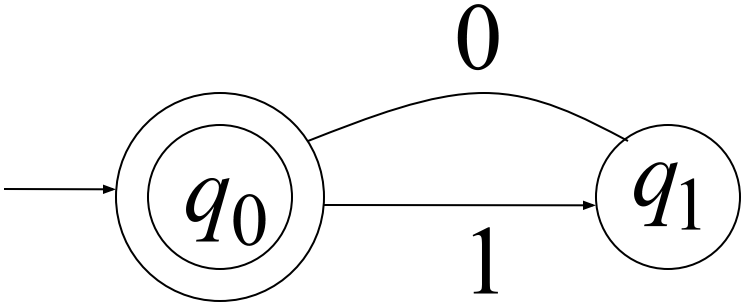
Machine M_1 is equivalent to machine M_2

if $L(M_1) = L(M_2)$

Example of equivalent machines

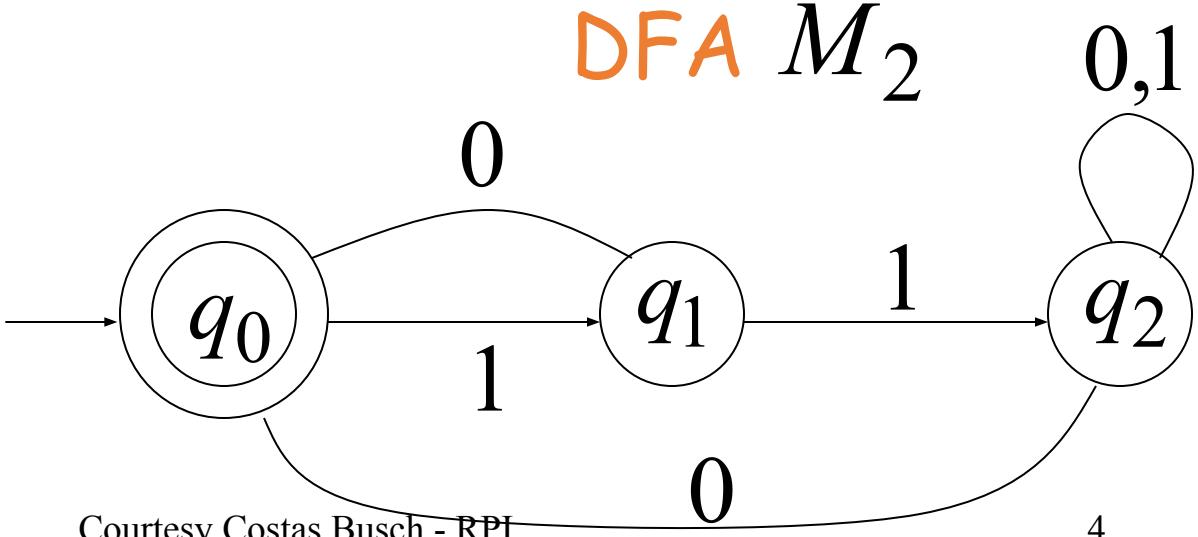
$L(M_1) = \{10\}^*$

NFA M_1

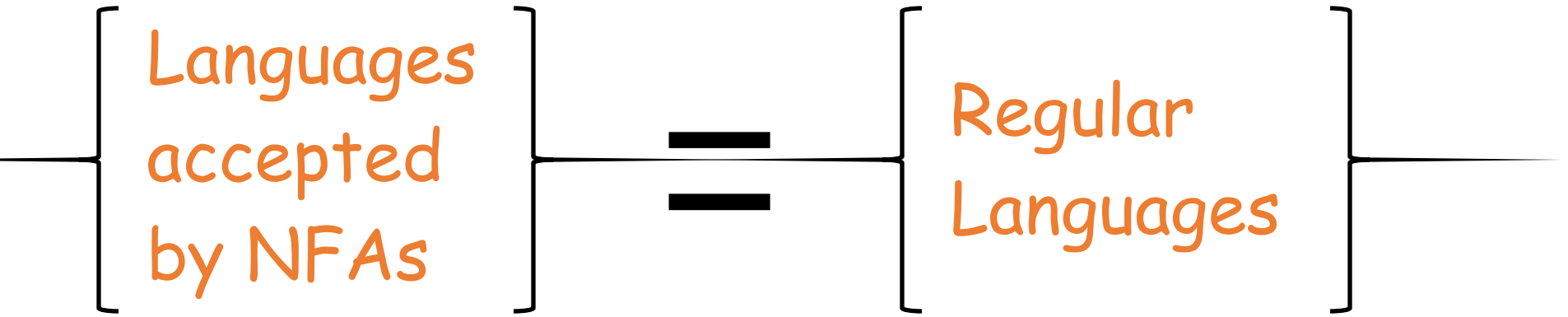


$L(M_2) = \{10\}^*$

DFA M_2



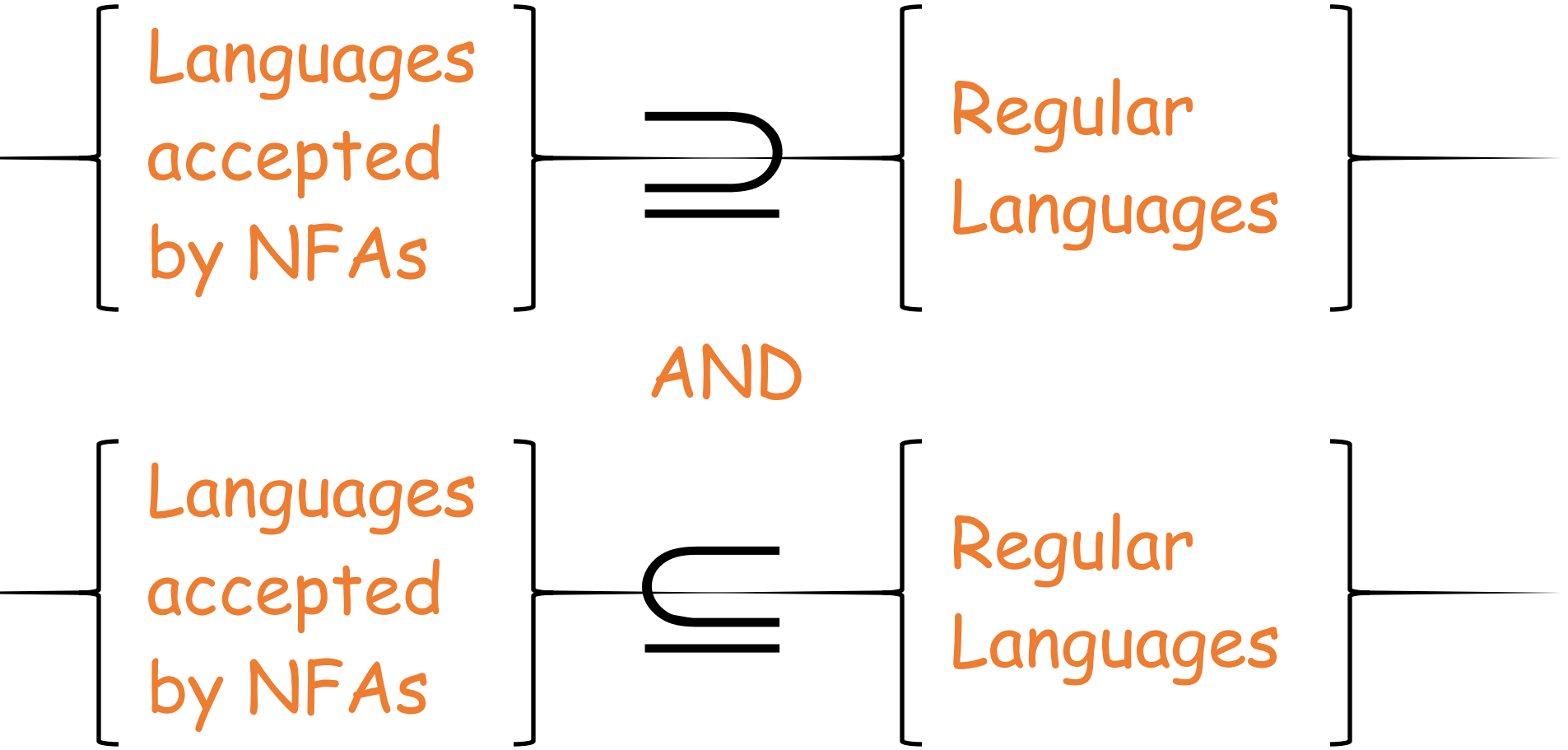
We will prove:



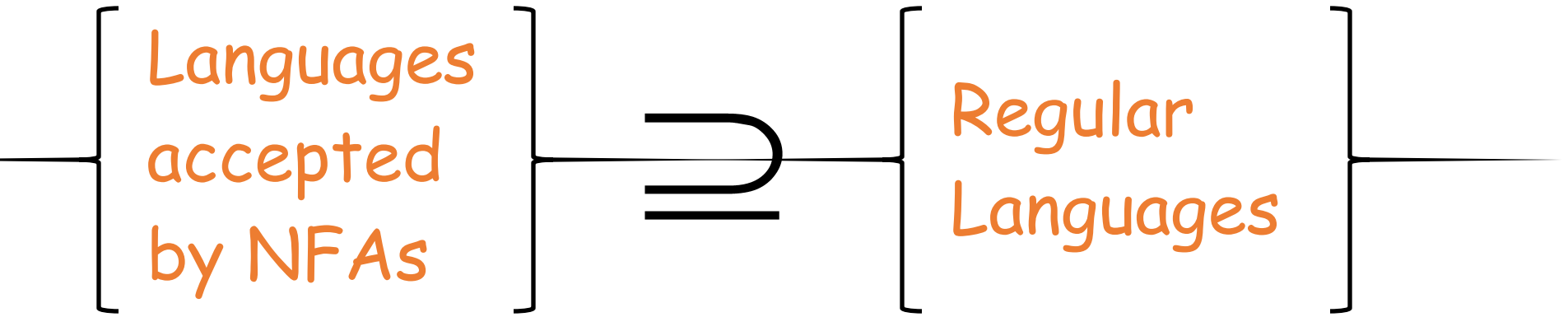
Languages accepted by DFAs

NFAs and DFAs have the same computation power

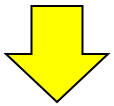
Proof: we only need to show



Step 1

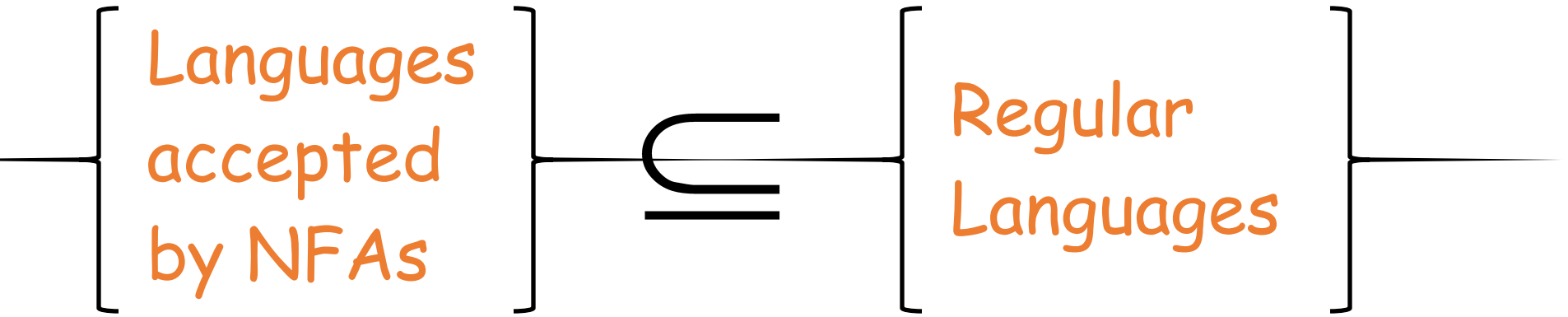


Proof: Every DFA is trivially an NFA

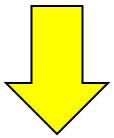


Any language L accepted by a DFA is also accepted by an NFA

Step 2



Proof: Any NFA can be converted to an equivalent DFA



Any language L accepted by an NFA is also accepted by a DFA

Non Deterministic Features of NFA

There are three main cases of non- determinism in NFAs:

1. Transition to a state without consuming any input.
2. Multiple transitions on the same input symbol.
3. No transition on an input symbol.

To convert NFAs to DFAs we need to get rid of non-determinism from NFAs.

Subset Construction Method

Using Subset construction method to convert NFA to DFA involves the following steps:

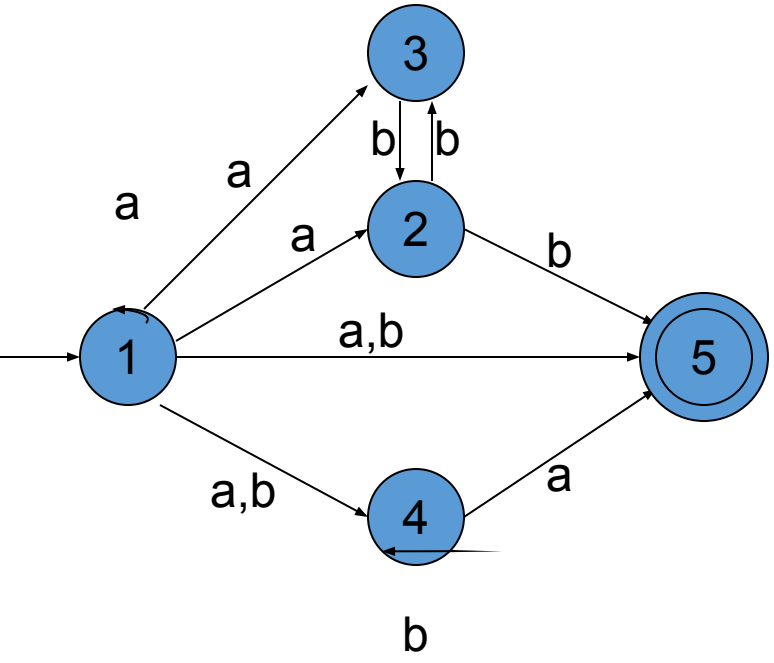
- For every state in the NFA, determine all *reachable states* for every input symbol.
- The set of reachable states constitute a *single state* in the converted DFA (Each state in the DFA corresponds to a subset of states in the NFA).
- Find *reachable states* for each new DFA state, until no more new states can be found.

Subset Construction Method

Fig1. NFA without λ-transitions

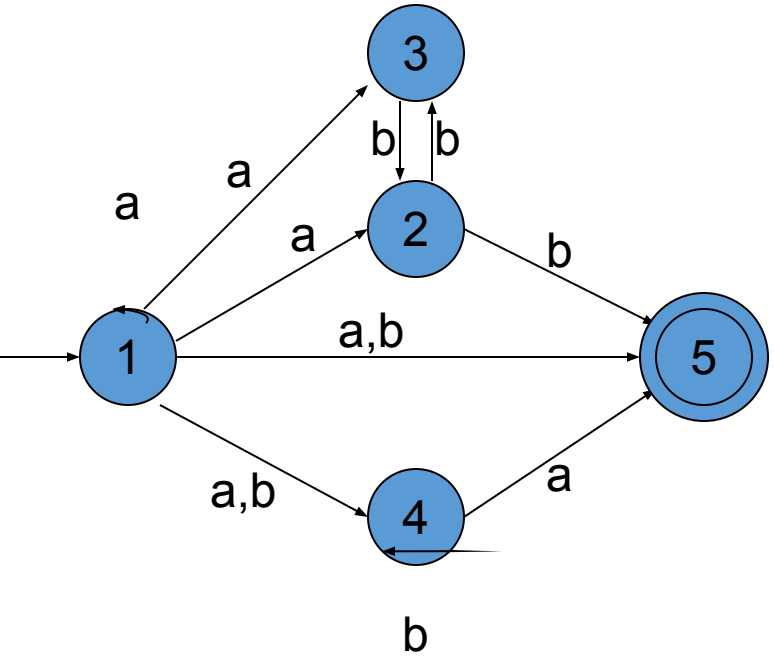
Subset Construction Method

Fig1. NFA without λ-transitions



Subset Construction Method

Fig1. NFA without λ-transitions



Step1

Construct a transition table showing all reachable states for every state for every input signal.

Subset Construction Method

Fig1. NFA without λ-transitions

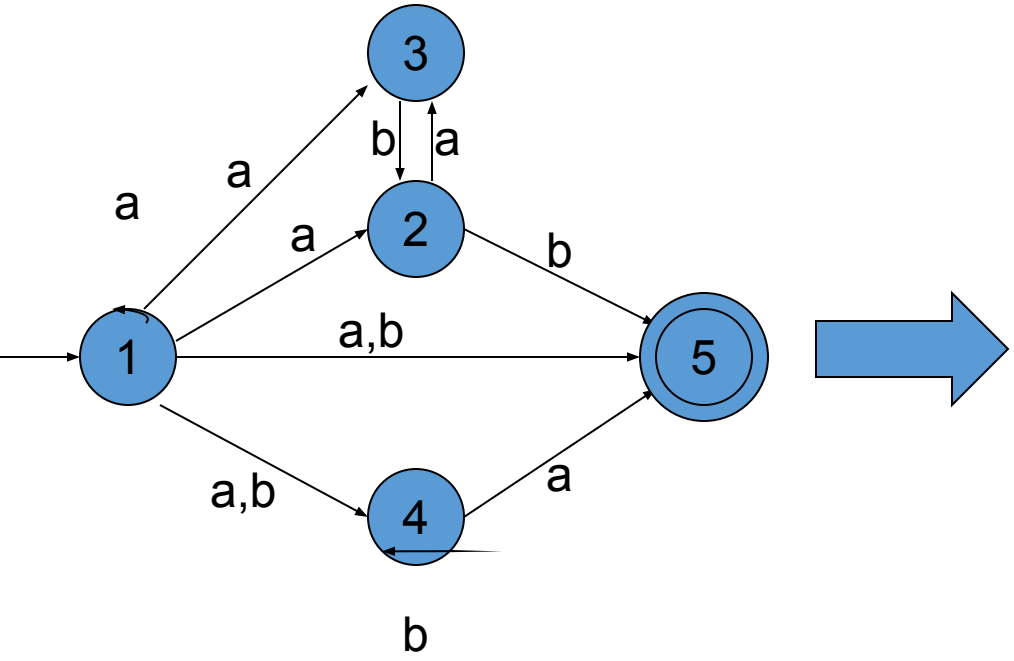


Fig2. Transition table

Subset Construction Method

Fig1. NFA without λ-transitions

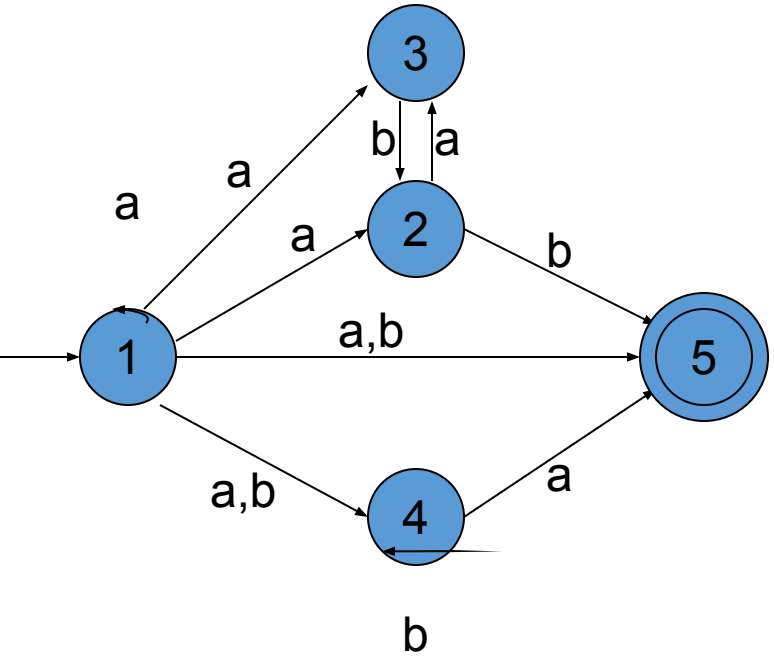


Fig2. Transition table

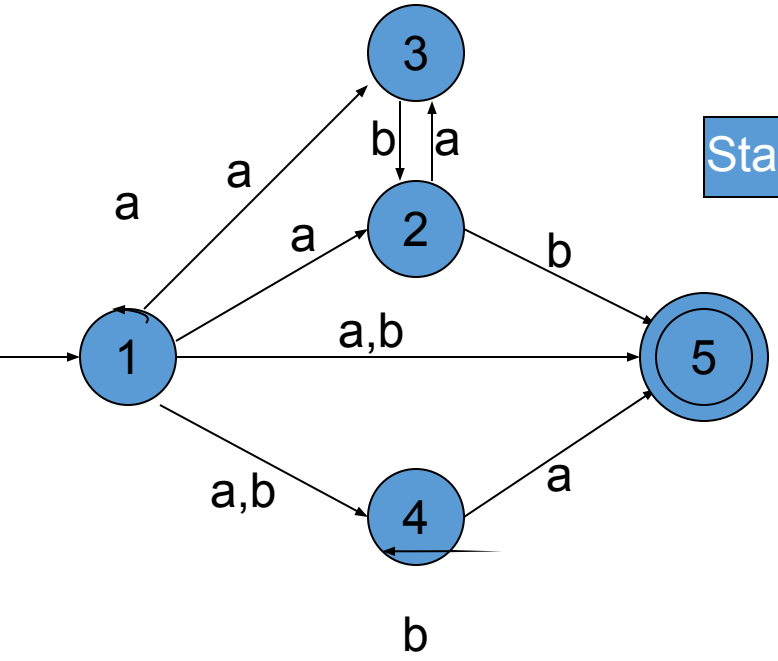
q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

Subset Construction Method

Transition from state q with input a

Transition from state q with input b

Fig1. NFA without λ-transitions



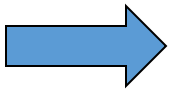
Starts here

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

Subset Construction Method

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅



Step2

The set of states resulting from every transition function constitutes a new state. Calculate all reachable states for every such state for every input signal.

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

Starts with Initial state

Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

Starts with Initial state

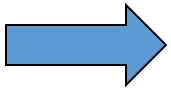


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}		
{4,5}		

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

Starts with Initial state

Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}		
{4,5}		

Step3

Repeat this process(step2) until no more new states are reachable.

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

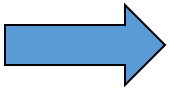


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}		
{2,4,5}		

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

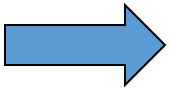


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}		
5		
4		

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

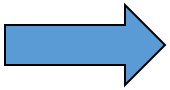


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4, 5}
5		
4		
{3,5}		

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

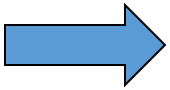


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4,5}
5	∅	∅
4		
{3,5}		
∅		

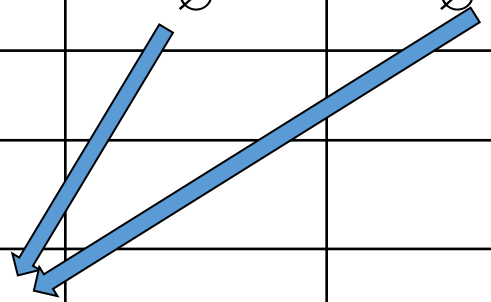
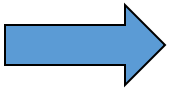


Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅



We already got 4 and 5.
So we don't add them again.

Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4,5}
5	∅	∅
4	5	4
{3,5}		

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

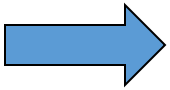


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4,5}
5	∅	∅
4	5	4
{3,5}	∅	2
∅		
2		

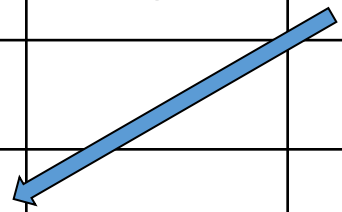


Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

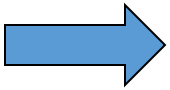


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4,5}
5	∅	∅
4	5	4
{3,5}	∅	2
∅	∅	∅
2		

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

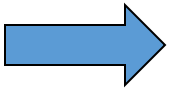


Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4,5}
5	∅	∅
4	5	4
{3,5}	∅	2
∅	∅	∅
2	3	5
3		

Fig2. Transition table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
2	{3}	{5}
3	∅	{2}
4	{5}	{4}
5	∅	∅

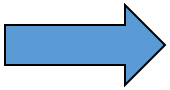


Fig3. Subset Construction table

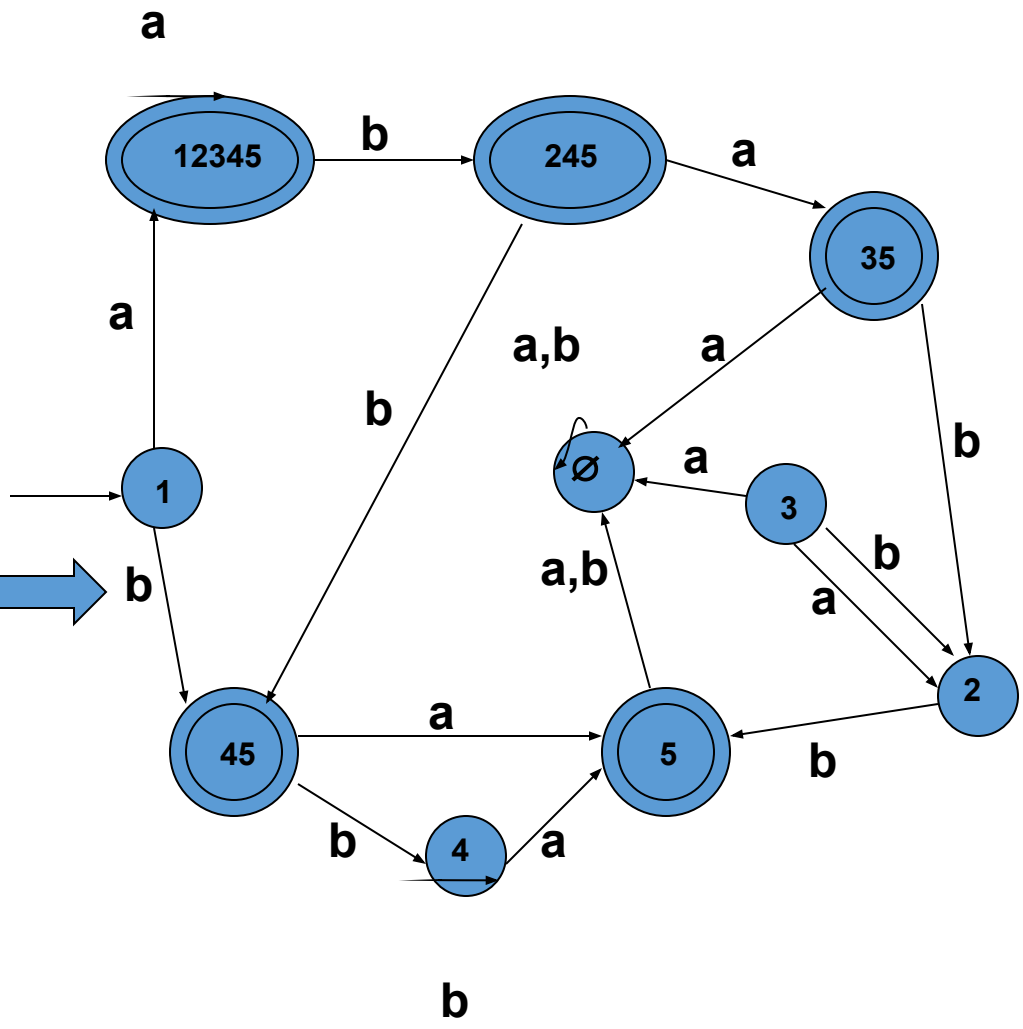
q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4,5}
5	∅	∅
4	5	4
{3,5}	∅	2
∅	∅	∅
2	3	5
3	∅	2

Stops here as there are
no more reachable states

Fig3. Subset Construction table

q	δ(q,a)	δ(q,b)
1	{1,2,3,4,5}	{4,5}
{1,2,3,4,5}	{1,2,3,4,5}	{2,4,5}
{4,5}	5	4
{2,4,5}	{3,5}	{4,5}
5	∅	∅
4	5	4
{3,5}	∅	2
∅	∅	∅
2	3	5
3	∅	2

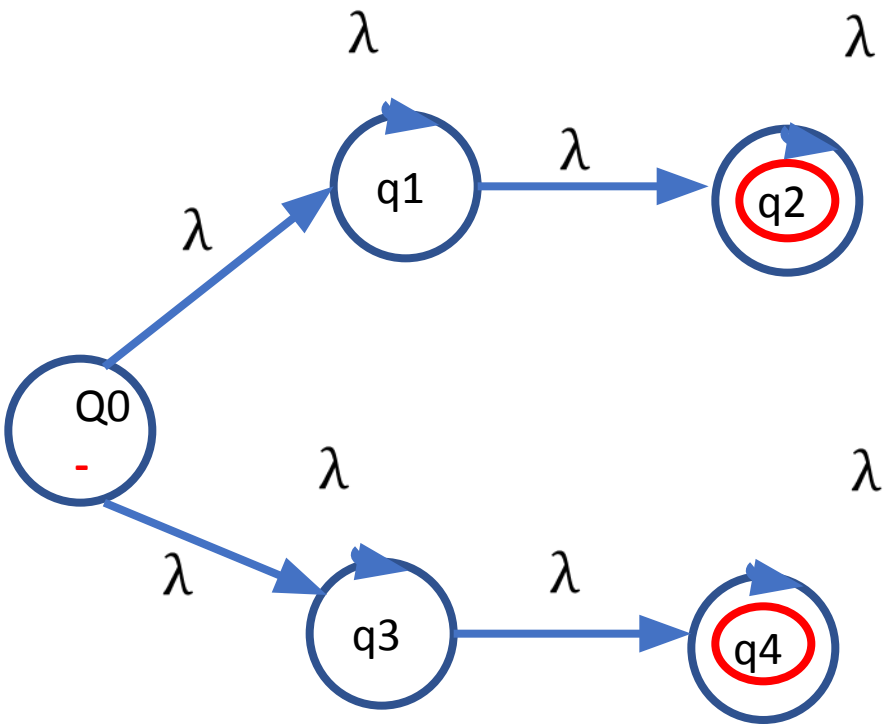
Fig4. Resulting FA after applying Subset Construction to fig1

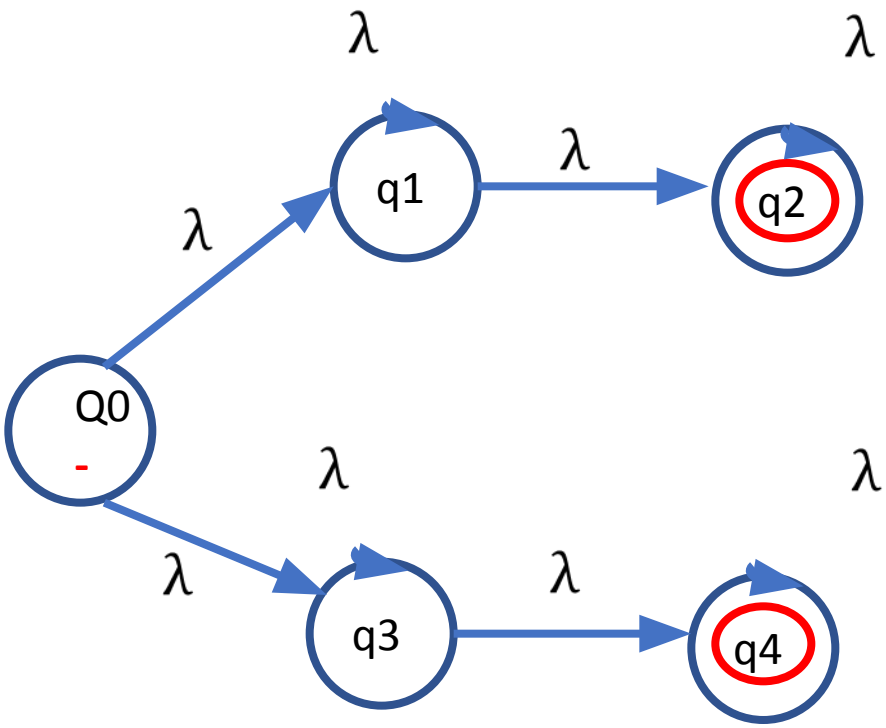


ε-NFA to DFA

Teachers

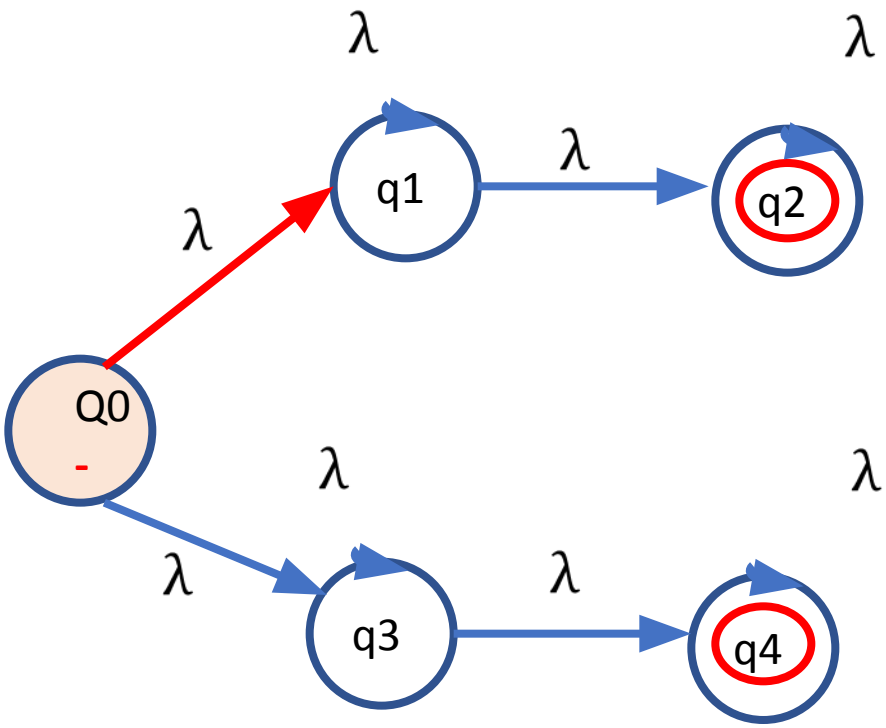
Mr. Musawar and Ms. Bakhtawar


λ
λ
λ
λ
q1
q2
λ
Q0
-
λ
λ
λ
λ
q3
q4



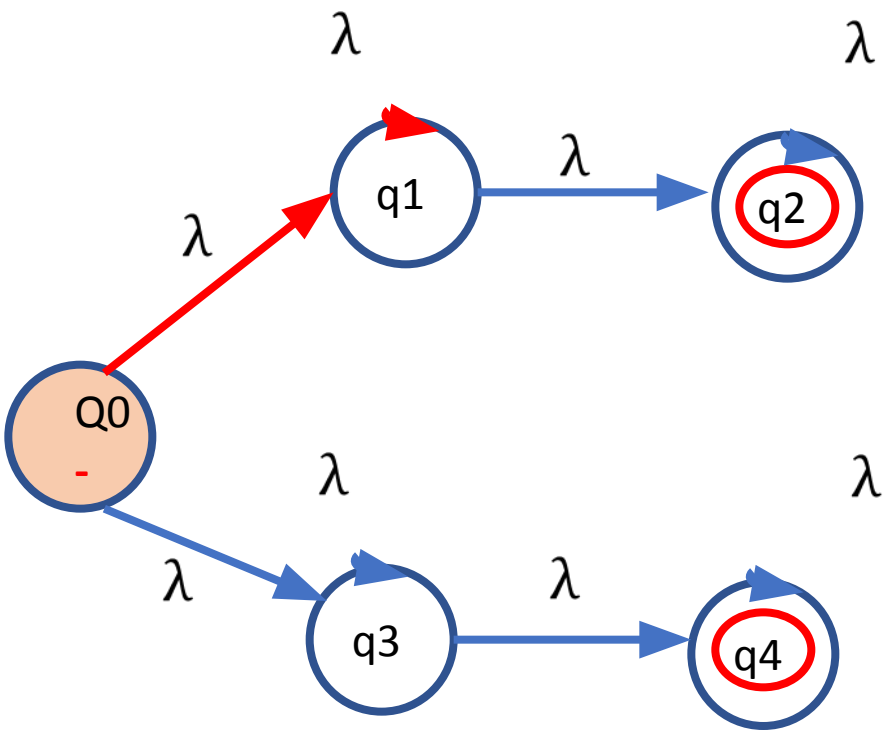
Transition Table

States	a	b



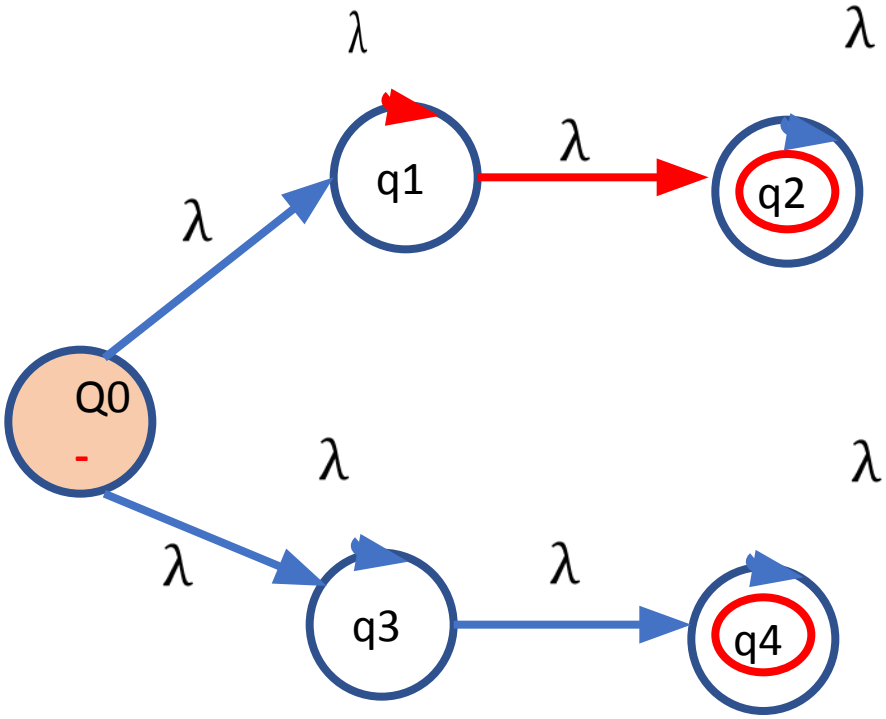
Transition Table

States	a	b
q_o		



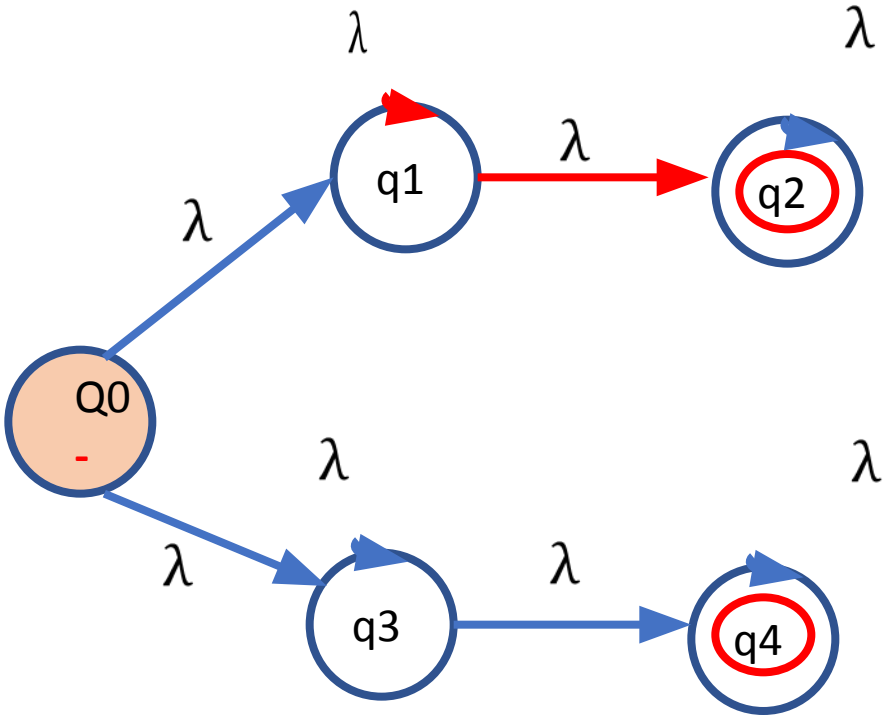
Transition Table

States	a	b
q_o	$\{q_1\}$	



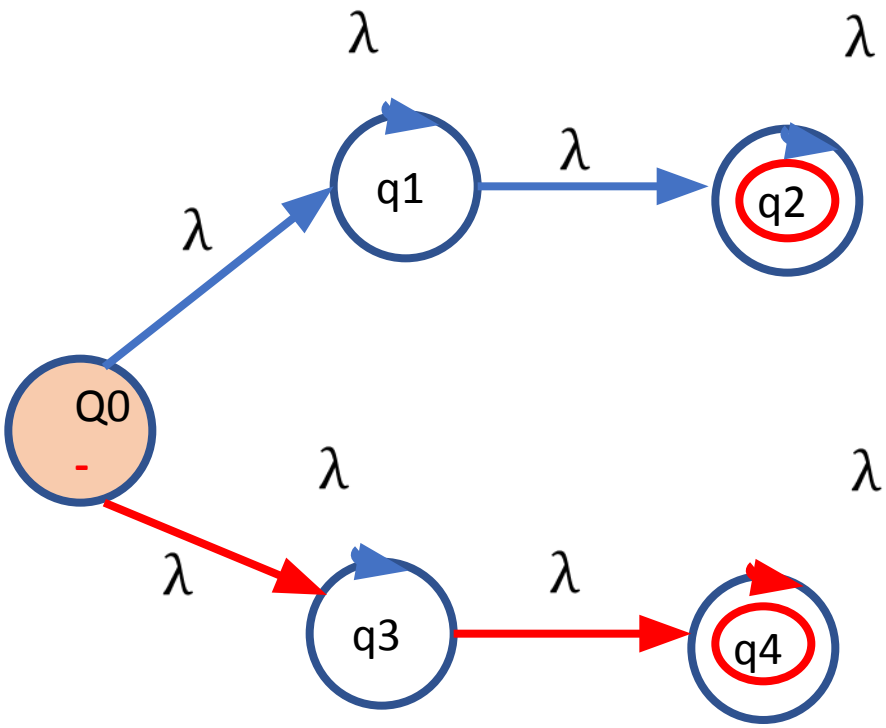
Transition Table

States	a	b
q_o	$\{q_1,\}$	



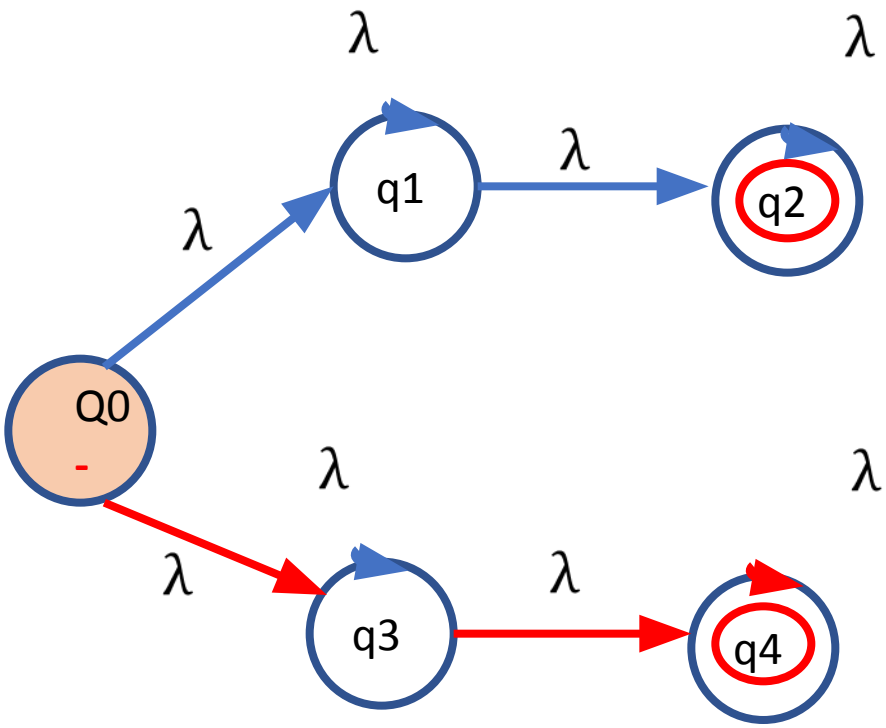
Transition Table

States	a	b
q_o	$\{q_1, q_2\}$	



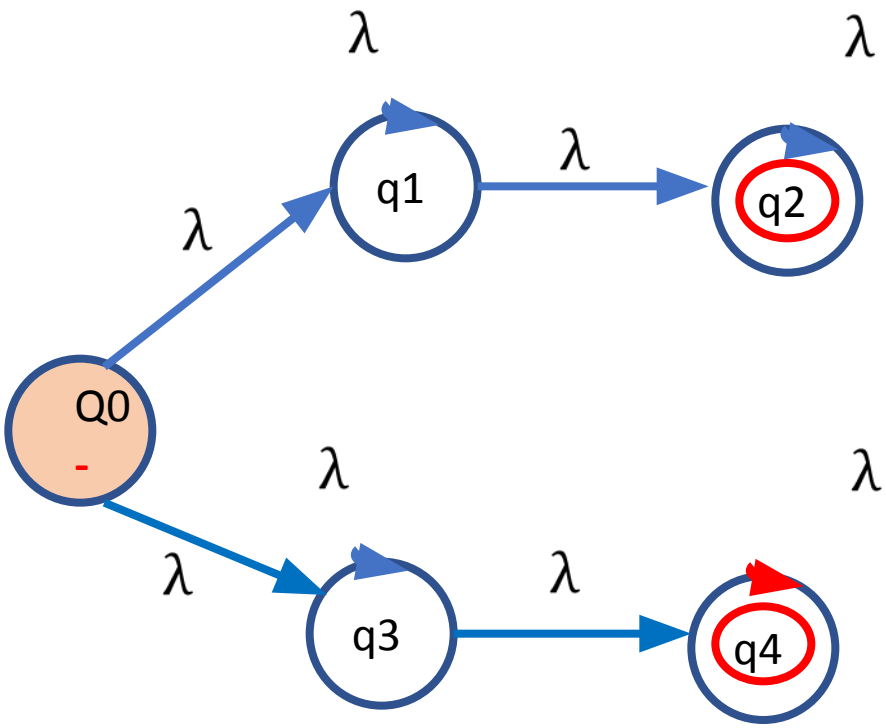
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	



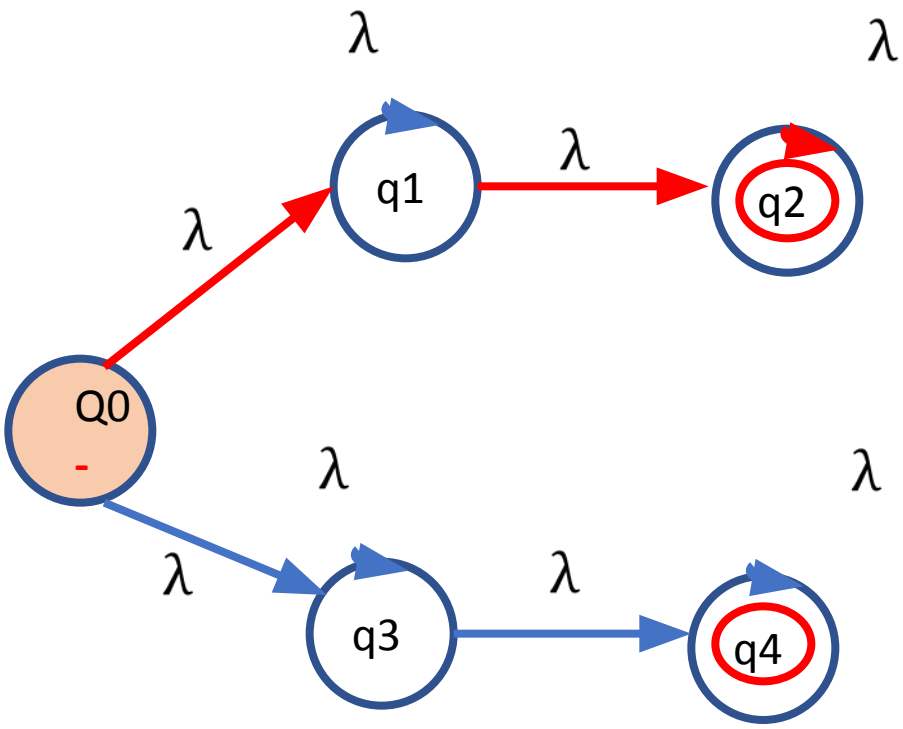
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	



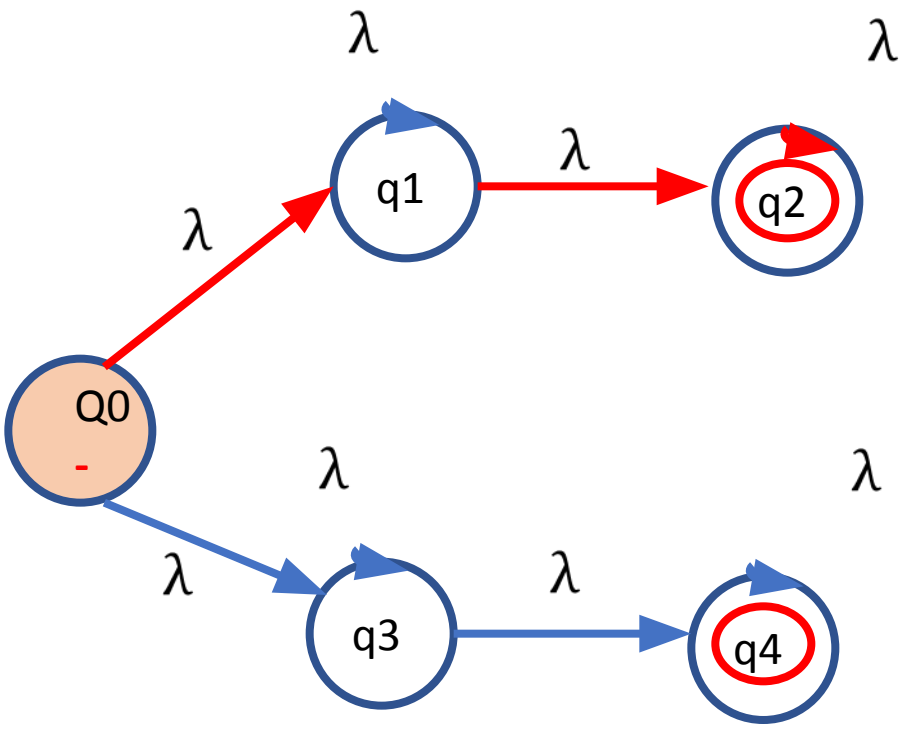
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	



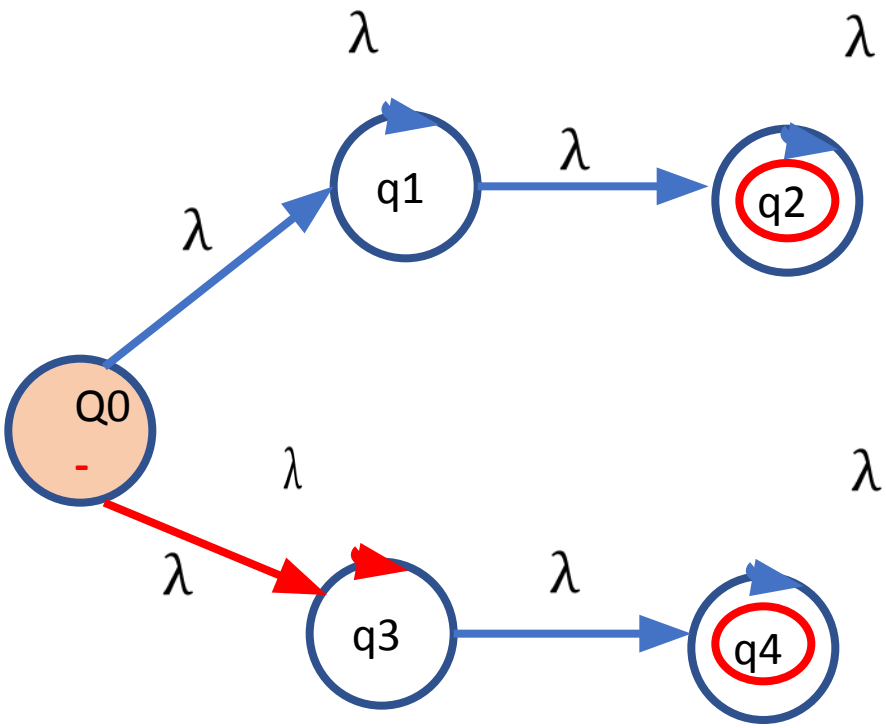
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	



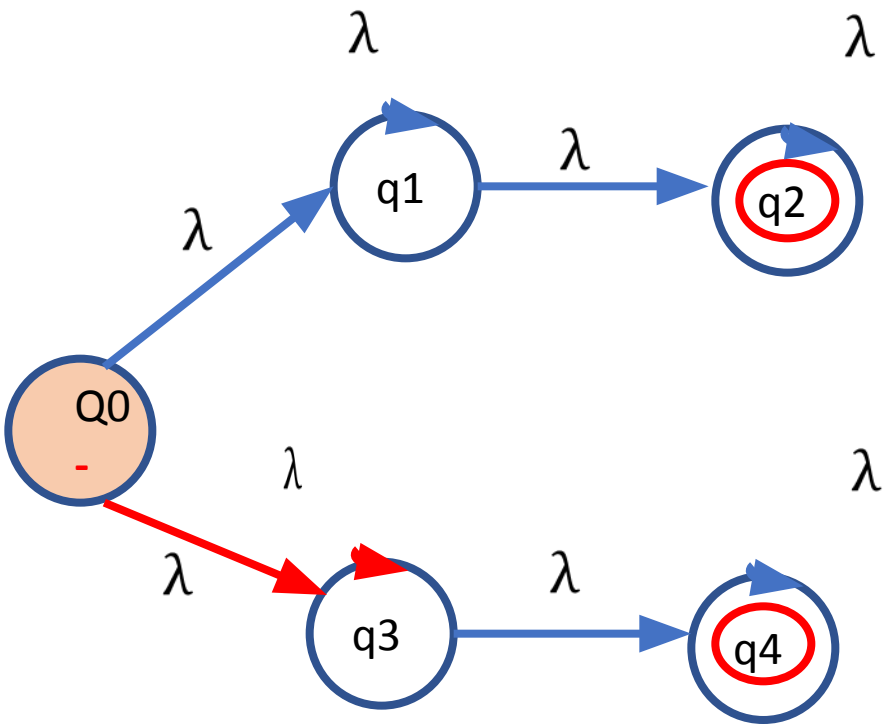
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2$



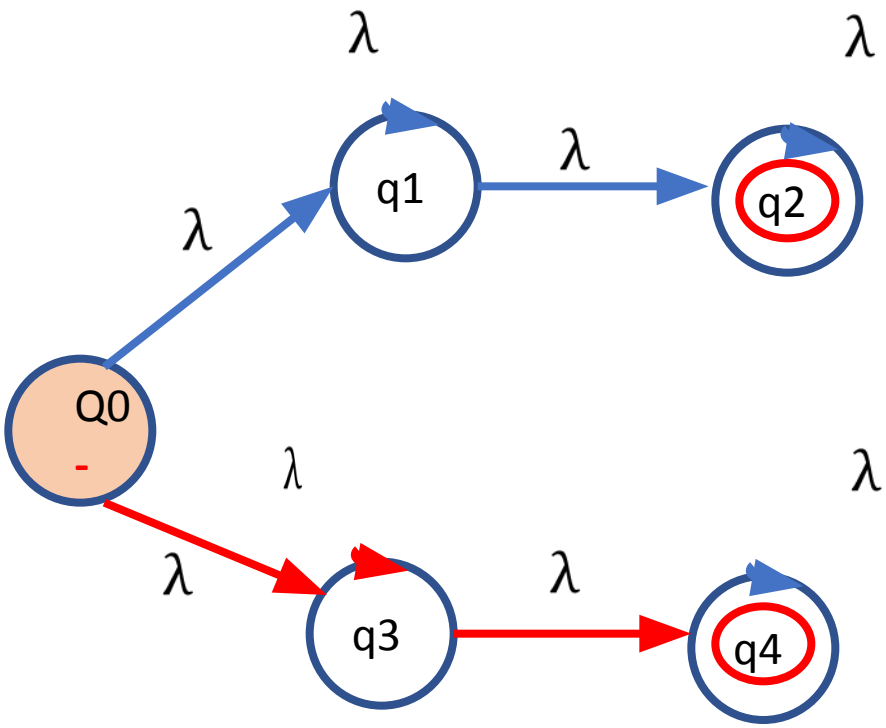
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2,$



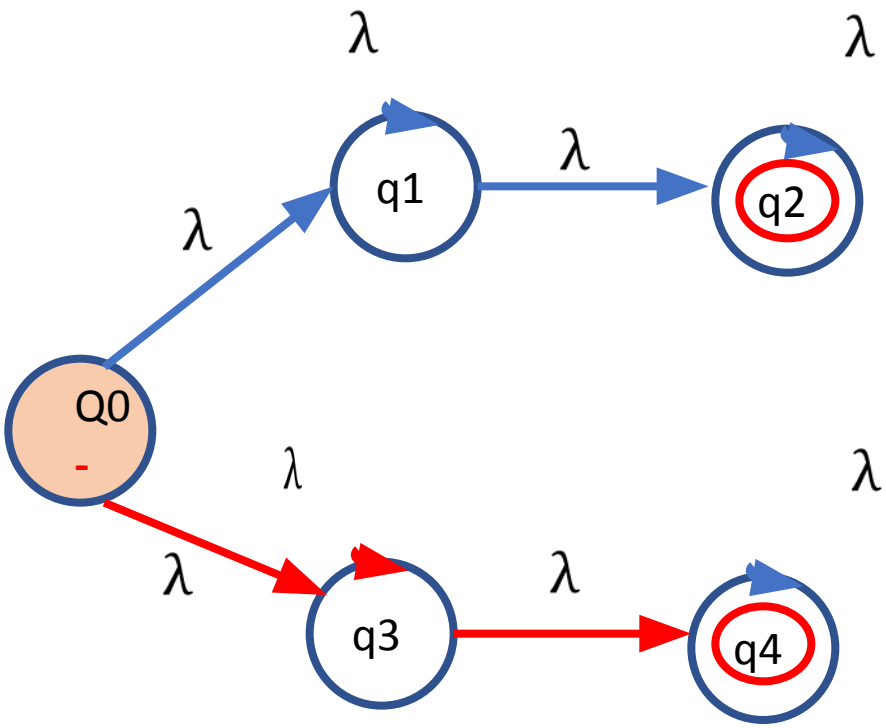
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3$



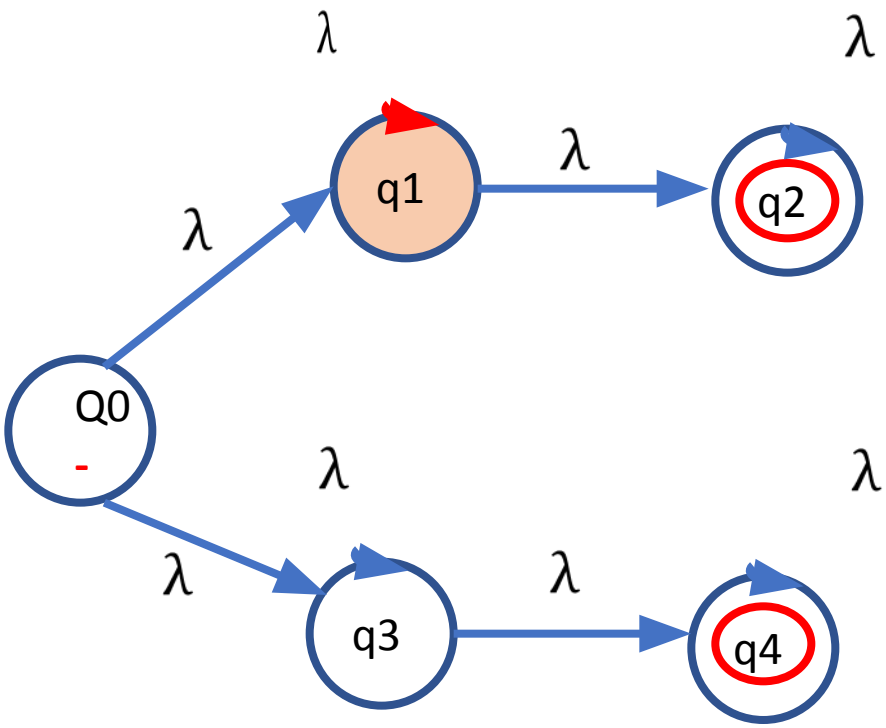
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3$



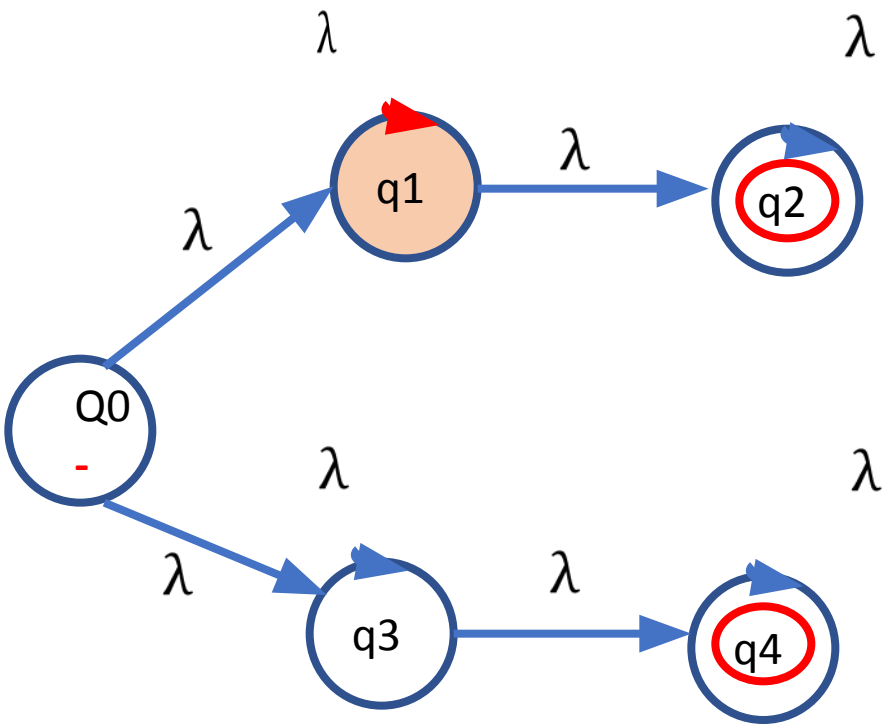
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$



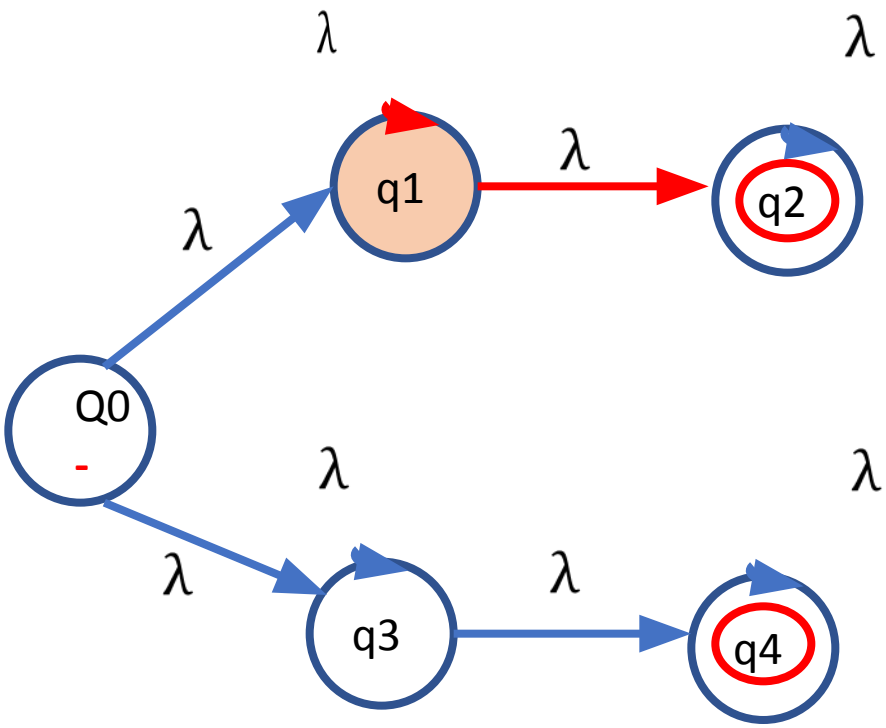
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1		



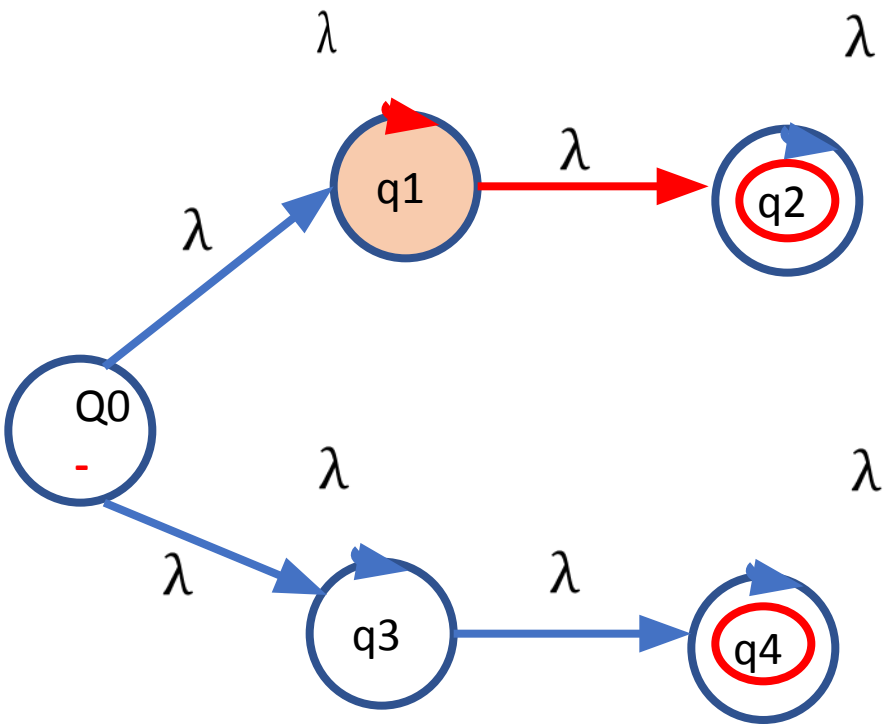
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1$	



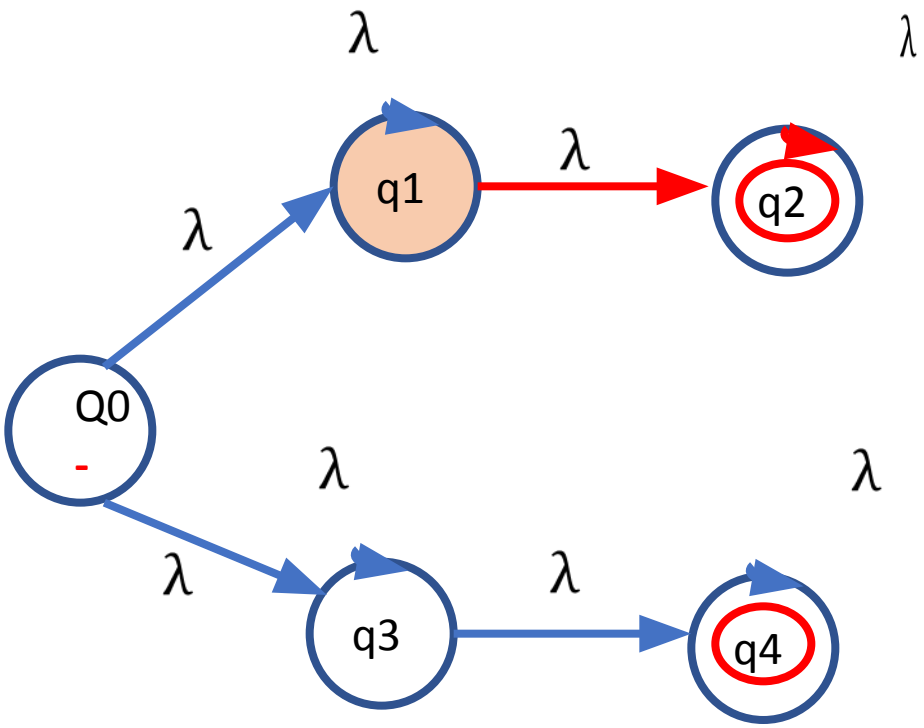
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1,\}$	



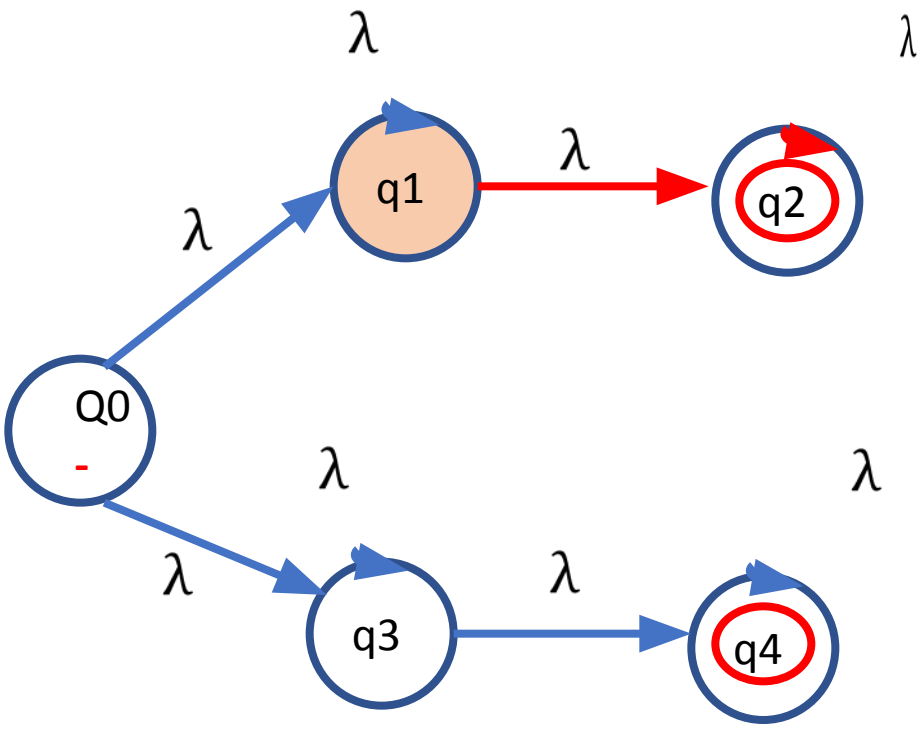
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	



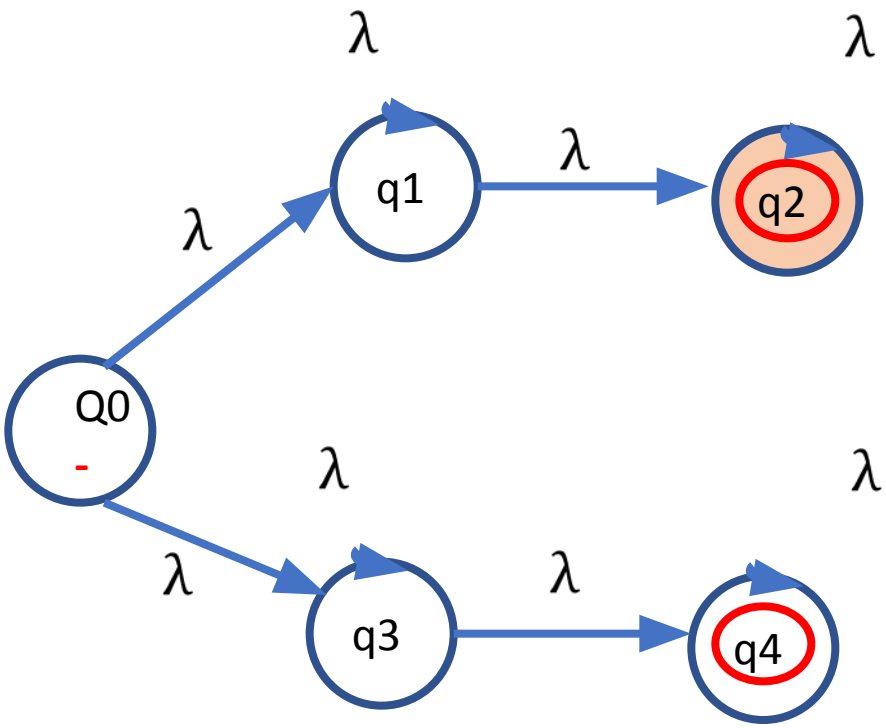
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	



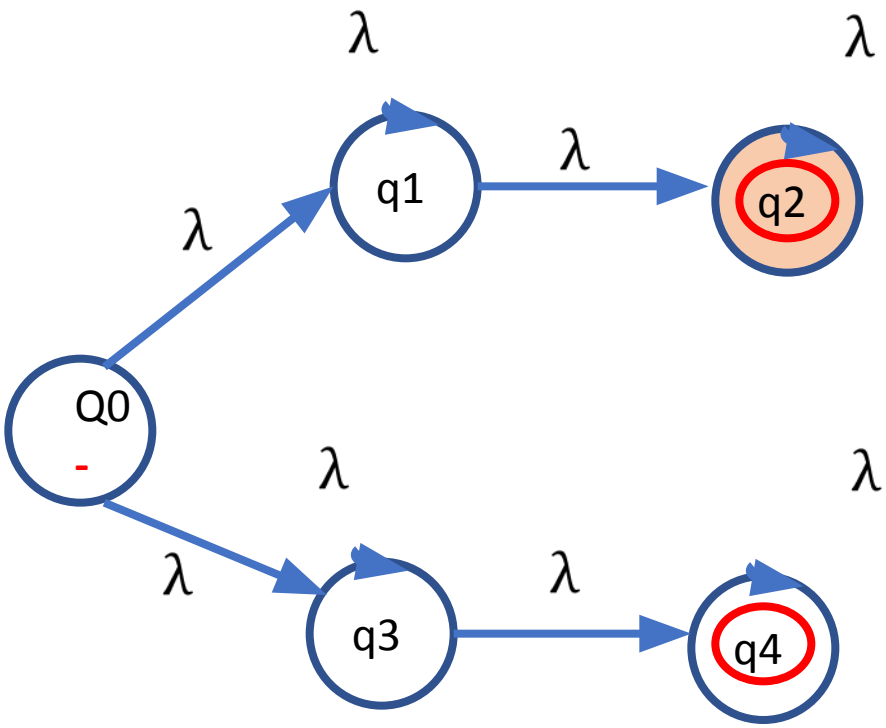
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$



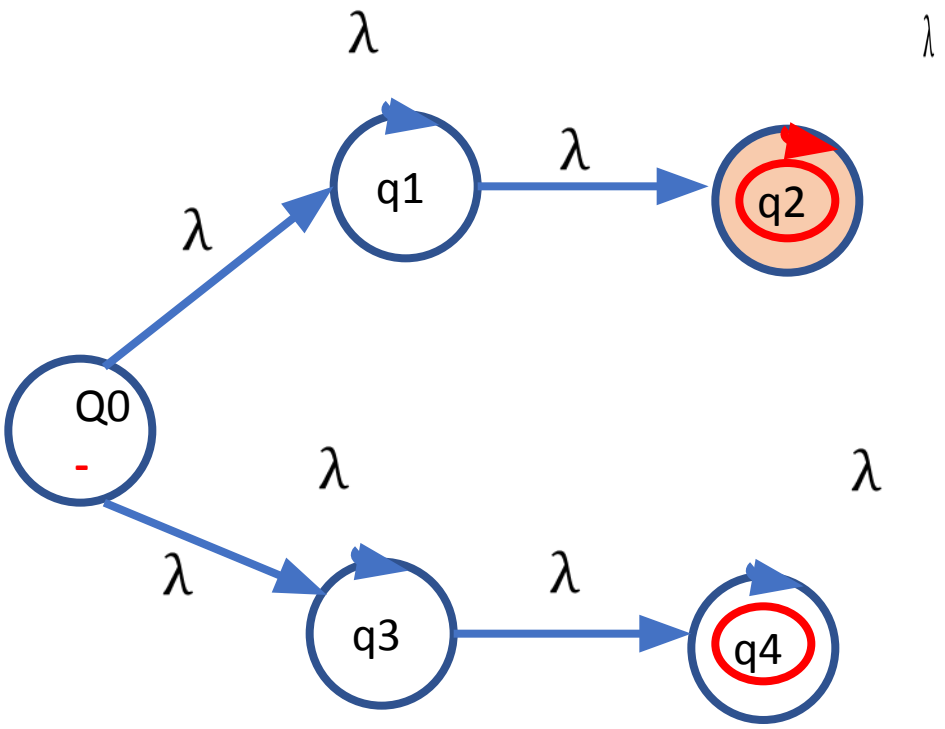
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2		



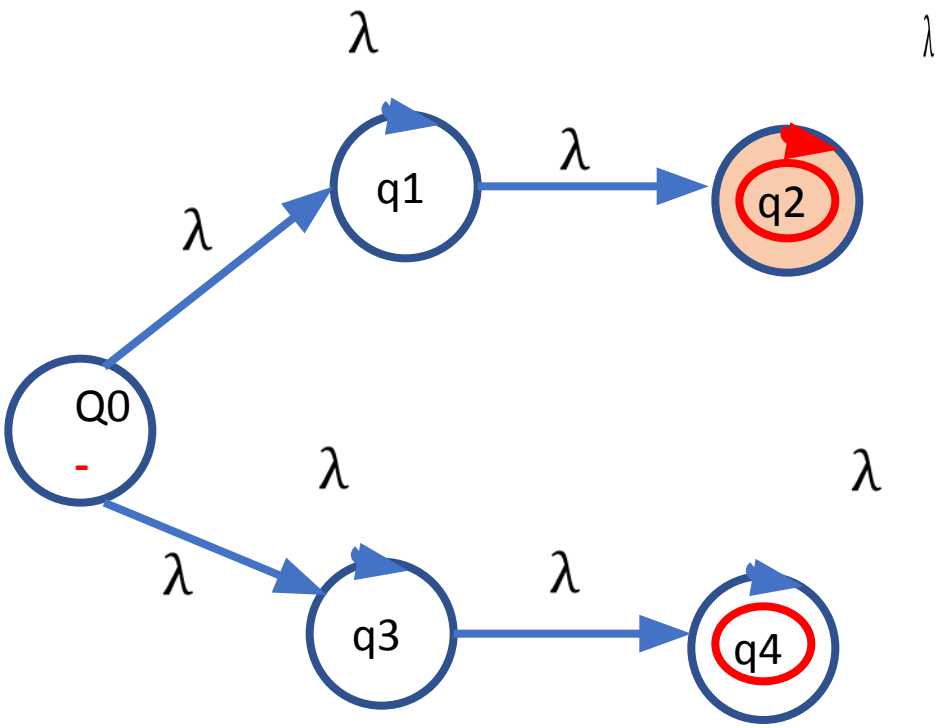
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	



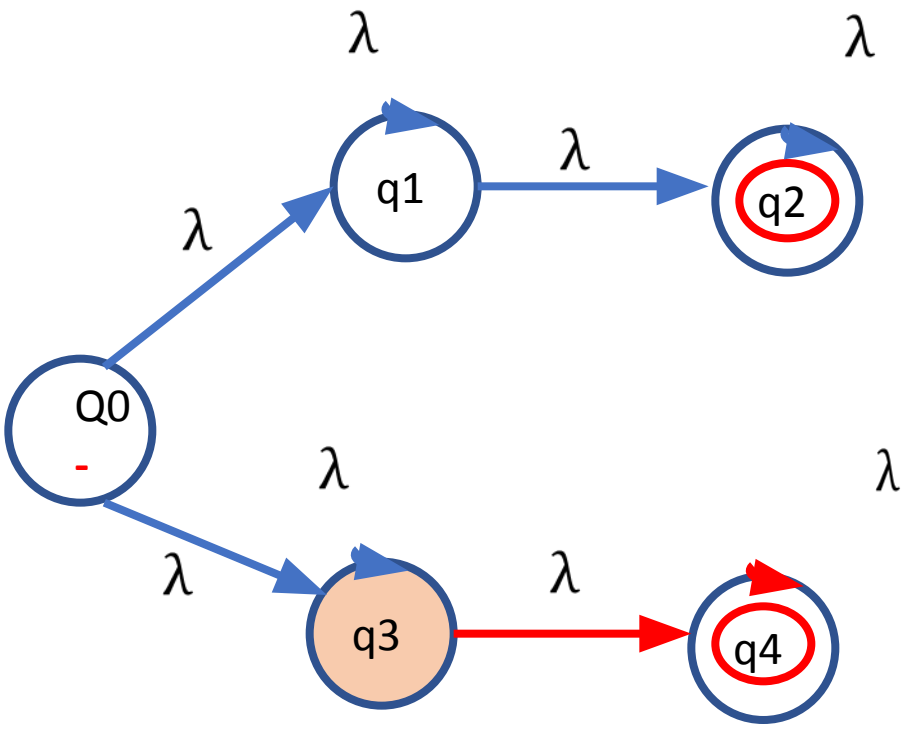
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	



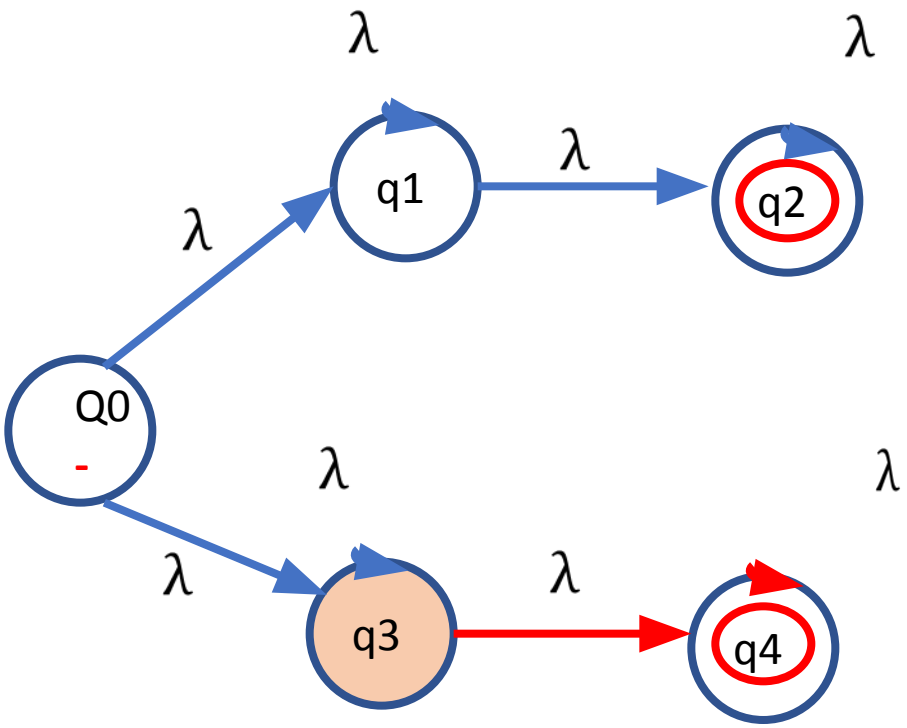
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$



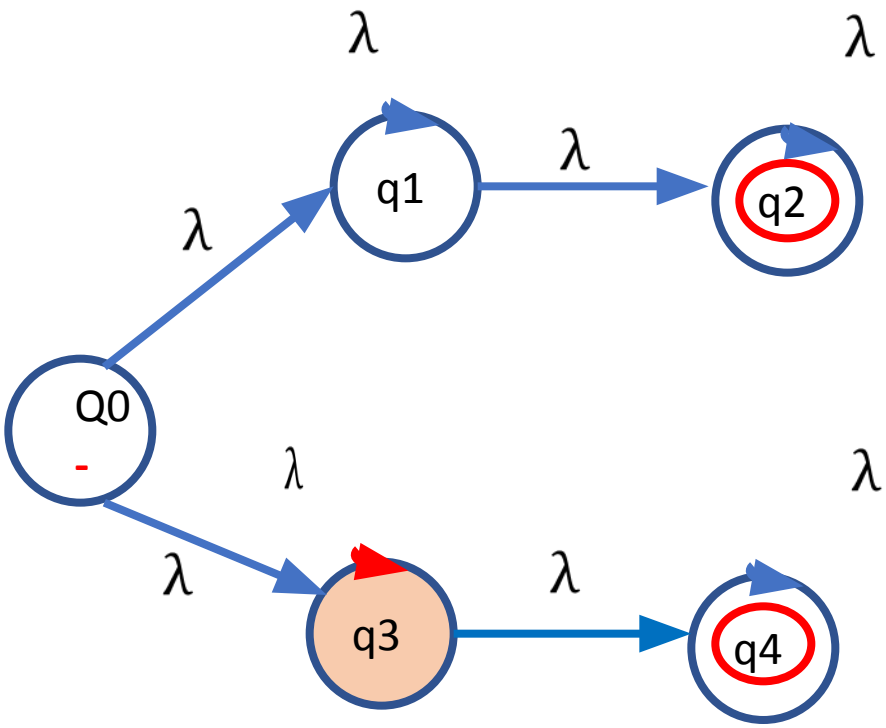
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3		



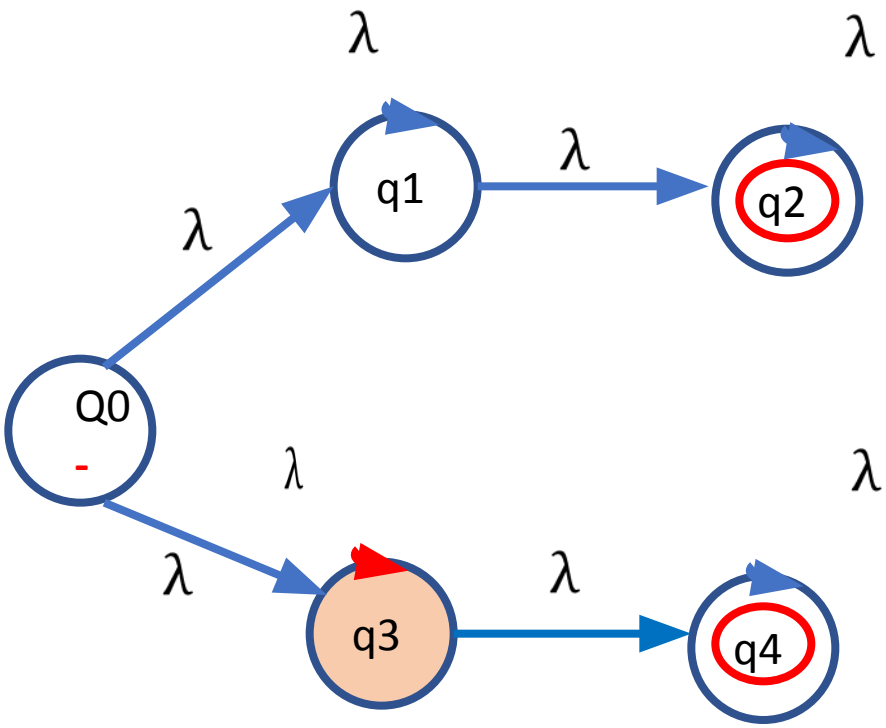
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	



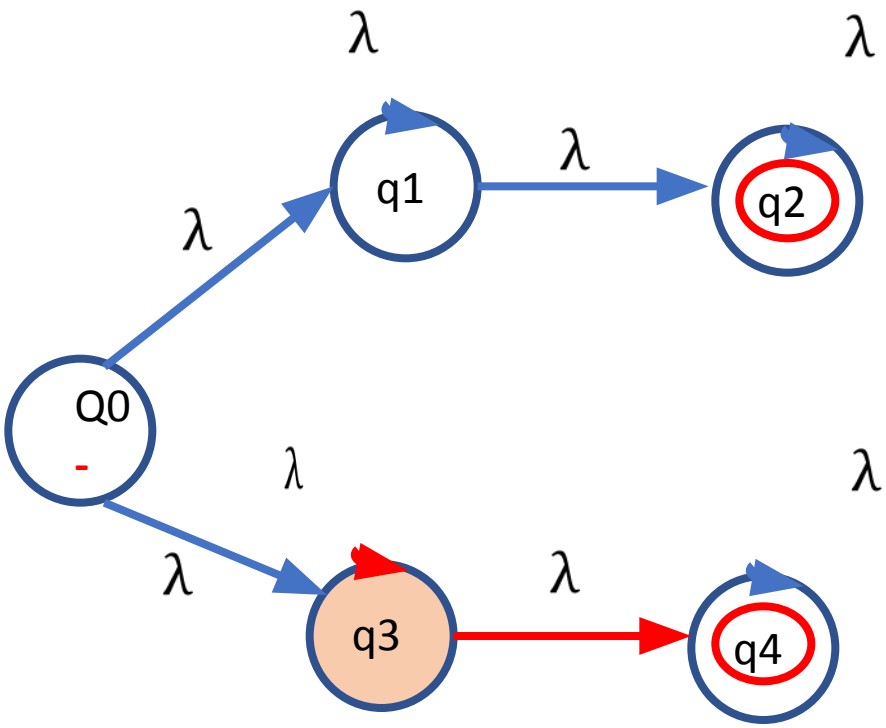
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	



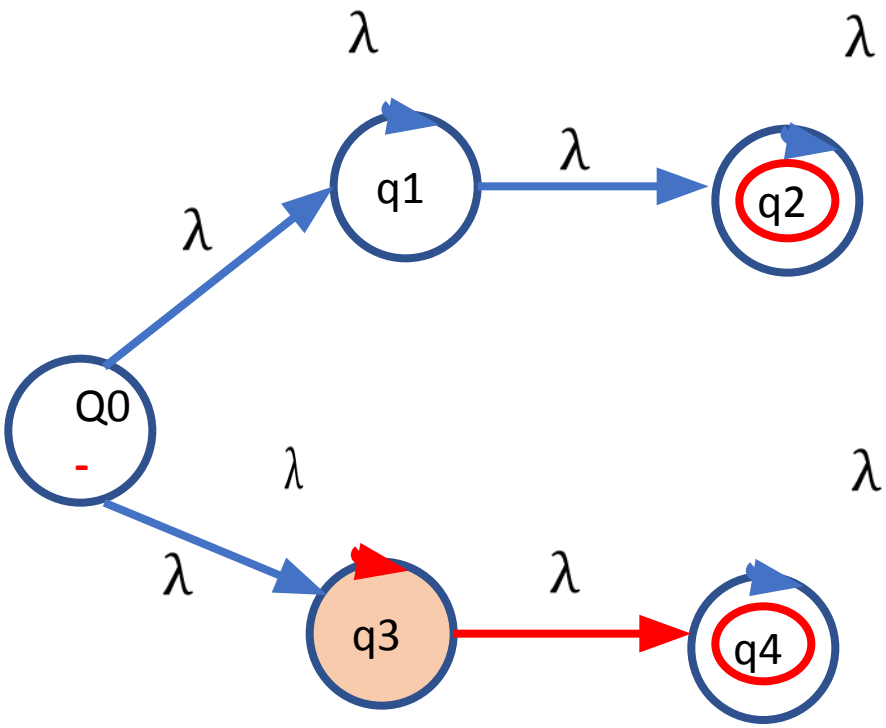
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	$\{q_3$



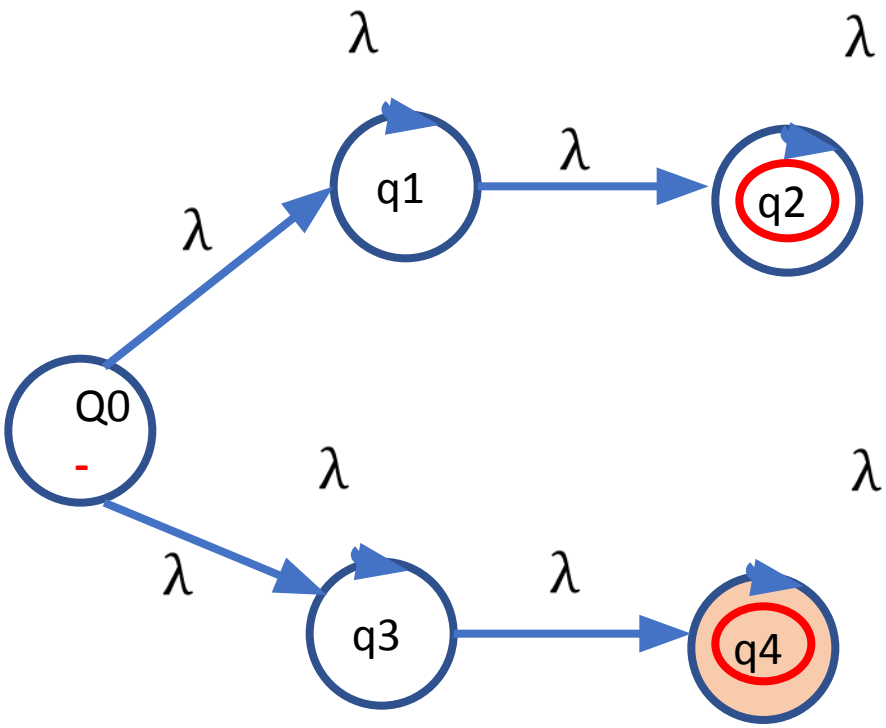
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	$\{q_3,\}$



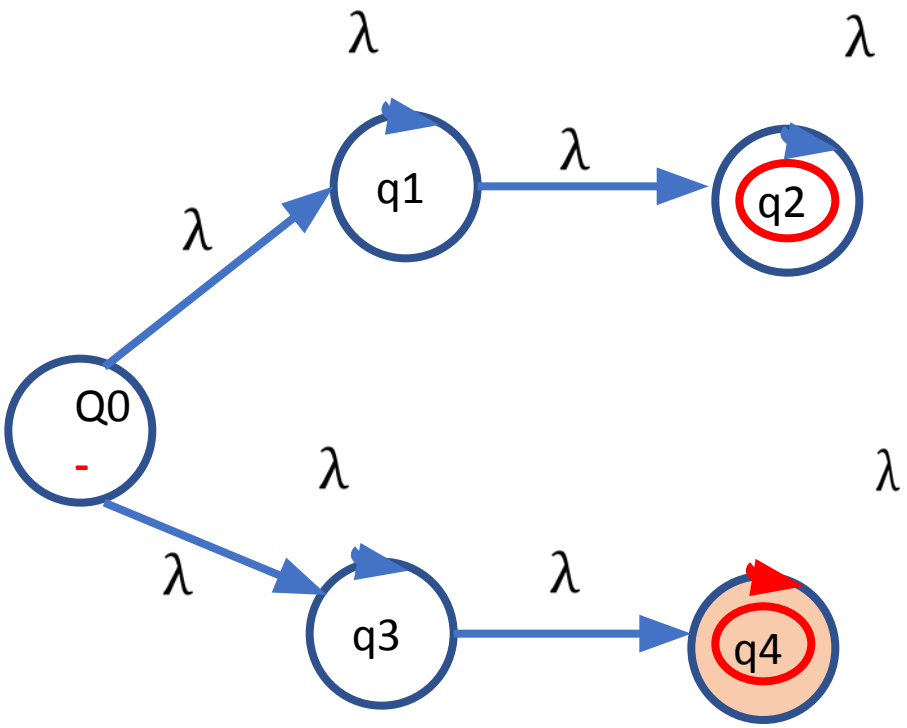
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	$\{q_3, q_4\}$



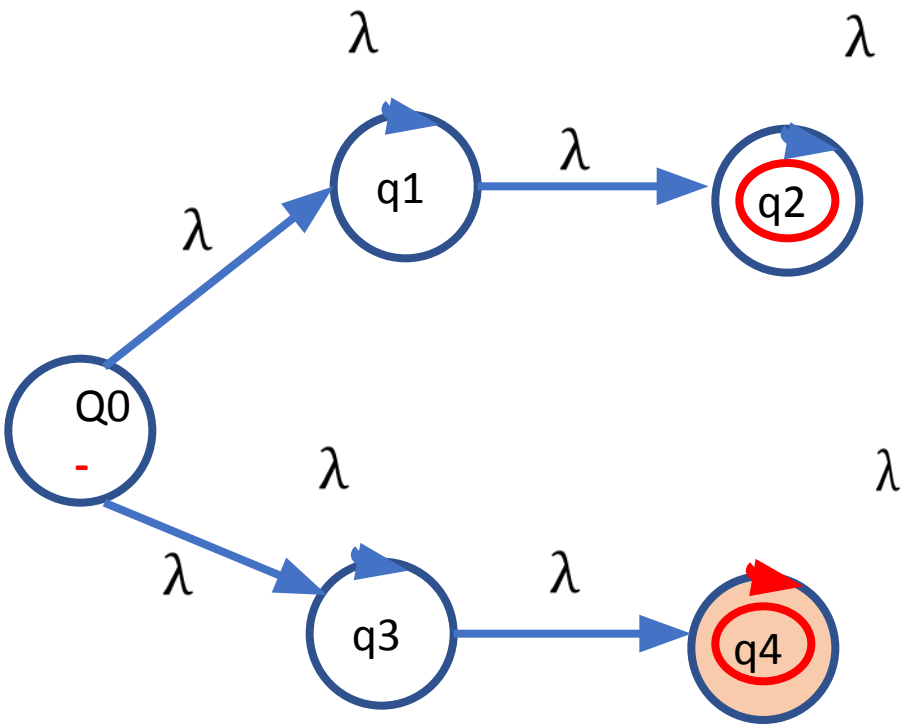
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	$\{q_3, q_4\}$
q_4		



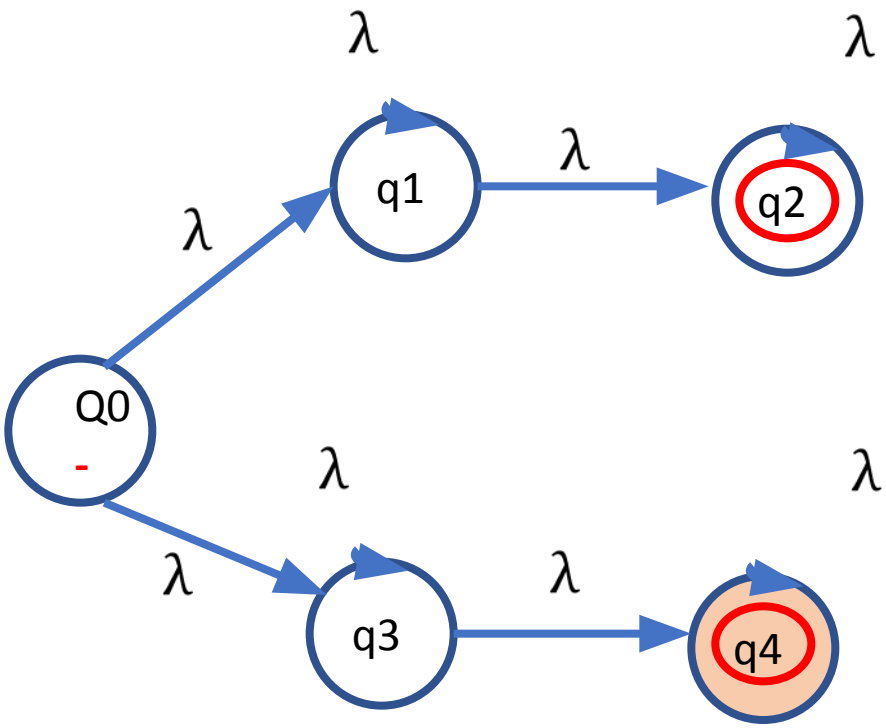
Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	$\{q_3, q_4\}$
q_4		



Transition Table

States	a	b
q_o	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
q_3	$\{q_4\}$	$\{q_3, q_4\}$
q_4	$\{q_4\}$	



Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$
$\{q_2, q_3, q_4\}$	$\{q_4\}$	$\{q_2, q_3, q_4\}$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$
$\{q_2, q_3, q_4\}$	$\{q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_2\}$	$\emptyset$	$\{q_2\}$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$
$\{q_2, q_3, q_4\}$	$\{q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_2\}$	$\emptyset$	$\{q_2\}$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$
$\{q_2, q_3, q_4\}$	$\{q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_2\}$	$\emptyset$	$\{q_2\}$
$\{q_4\}$	$\{q_4\}$	$\emptyset$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$
$\{q_2, q_3, q_4\}$	$\{q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_2\}$	$\emptyset$	$\{q_2\}$
$\{q_4\}$	$\{q_4\}$	$\emptyset$

Transition Table

States	a	b
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2\}$
$+q_3$	$\{q_4\}$	$\{q_3, q_4\}$
$+q_4$	$\{q_4\}$	$\emptyset$

Transition Table DFA

States	a	B
$-q_o$	$\{q_1, q_2, q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$	$\{q_2\}$
$\{q_2, q_3, q_4\}$	$\{q_4\}$	$\{q_2, q_3, q_4\}$
$\{q_2\}$	$\emptyset$	$\{q_2\}$
$\{q_4\}$	$\{q_4\}$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$