FAST- National University of Computer and Emerging Sciences, Karachi.

FAST School of Computing

Assignment 1, Fall 2021

Course Code: CS1005	**Course Name: Discrete Structures**
Instructor Name: Safia Baloch	
Student Roll No:	**Section: 3B & 3D**

Instruction:

1. Submission is in hard copy.
2. Due date to submit is **Friday 24th December 2021**
3. **ONLY Attempt** Number of questions as defined with topic in brackets (Choice based). The total number of questions will be 35.

- **Propositional Logic *[Any 5 questions]***
  1. Using basic known equivalences, show that $(\neg p \wedge q) \vee \neg(p \vee q)$ is logically equivalent to $\neg p$
  2. Use known logical equivalences to show that $\neg(\neg p \rightarrow \neg q) \vee (p \wedge q)$ is logically equivalent to q.
  3. Find all truth values for p, q and r for which $(p \rightarrow \neg q) \leftrightarrow r$ is true
  4. Use known logical equivalences to show that $q \rightarrow (p \rightarrow q)$ is a tautology.
  5. Use a truth table to determine whether $(p \leftrightarrow \neg q) \rightarrow (\neg p \vee \neg q)$ is a tautology.
  6. Suppose the statement $p \leftrightarrow q$ is false. Find all combinations of truth values for p, q, and r such that the statement $(\neg r \leftrightarrow p) \vee (r \wedge \neg q)$ is true.
  7. Use a truth table to determine whether $(p \rightarrow \neg q) \leftrightarrow \neg(p \vee \neg q)$ is a contradiction.

- **Quantifiers and Written Proofs *[All questions are compulsory]***

  8. Let A, B and C be sets. Give a counterexample to show that the statement “If $A \cup C = B \cup C$ then A = B.” is false.
  9. Write the following argument in symbolic form, and then use known logical equivalences and inference rules to show that it is valid. Please clearly indicate which letters correspond to which statements. Justify each step.

I am not wearing a pink tie or I am wearing a red shirt.
If it is not Saturday, then I am wearing a pink tie.
I am not wearing a red shirt

$\therefore$ It is Saturday

- **Set Theory: *[Any 2 questions]***
  10. Let A, B and C be sets.
      a. Prove that $A \setminus (B \cap C^c) \subseteq (A \cap B^c) \cup (A \cap C)$.
      b. Let $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $b|c$ then $a|c$.
  11. Let A and B be sets. Use any method except a Venn diagram to prove the identity $(A \cap B)^c = A^c \cup B^c$. Hint: there is a short argument that uses set-builder notation.

12. Suppose that in a group of 50 motorcyclists, 30 own a Triumph and 32 own a Honda. If 15 motorcyclists in the group own neither type of motorcycle, how many own a motorcycle of each type?
13. In a group of 35 ex-athletes, 17 play golf, 20 go cycling, and 12 do yoga. Exactly 8 play golf and go cycling, 8 play golf and do yoga, 7 go cycling and do yoga, and 4 do all three activities. How many of the ex-athletes do none of these activities?

- **Number Theory, Combinatorics and Discrete Probability:** ***[Any 10 questions]***

14. Use the Euclidean Algorithm to find d = gcd(3142, 900) and then use your work to find integers x and y such that 3142x + 900y = d.
15. Use the Euclidean Algorithm to show that gcd(2017, 122) = 1 and then use your work to find integers x and y such that 2017x + 122y = 1.
16. Use the Euclidean Algorithm to find d = gcd(578, 442) and then use your work to find integers x and y such that 578x + 442y = d.
17. Consider the integers m = 900 and n = 189 = 337. Fill in the blanks:
    a. The prime factorization of m is _______________
    b. gcd(m, n) = and lcm(m, n) = __________________
    c. The smallest multiple k of 189 such that gcd(m, k) = 45 is ___________
18. Suppose k ≡ 3 (mod 5). Find the remainder when 11k 3 + 12 is divided by 5.

19. Let 9. a_1, a_2, . . . be the sequence recursively defined by a_1 = 4, and for all n ≥ 2, a_n = 7a_n−1 + 4.. Calculate a_2, a_3, and a_4.
20. An airline surveyed 200 passengers and recorded the following information about the type of non-alcoholic beverages they liked: 78 passengers liked coffee, 70 liked tea, and 66 liked orange juice; 35 liked both coffee and tea, 30 liked both tea and orange juice, 15 liked both coffee and orange juice, and 10 liked all three types of beverages. How many passengers liked only orange juice or none of the three beverages?
21. A bicycle shop surveyed 50 customers and found that 30 own a mountain bike, 35 own a road bike, and 15 did not currently own a bicycle. How many of the 50 customers own both a mountain bike and a road bike?
22. In a group of 348 students, 321 liked Math 122, and 286 liked Math 101. Sixteen students liked neither of these courses.
    a. How many of the students liked both courses?
    b. How many of the 348 students did not like Math 101?
23. Shahid Afridi is very fond of playing Cricket. He has collected tennis ball in a Basket. He's not sure exactly how many he has today, so when he
    aligns them in rows of 19 balls, 5 balls are left.
    aligns them in rows of 17 balls, 3 balls are left;
    aligns them in rows of 7 balls, 2 balls are left;

    He's positive that there are fewer than 700 balls but how many does he have?
    Hint: In this problem, you are supposed to use the following Theorems:
    a. Chinese Remainder Theorem
    b. The Euclidean Algorithm Lemma
    c. Bézout's Theorem
    d. Linear congruences

24. Once Shahid Afridi has counted all balls he encrypted the number word using Ceaser Cipher. You need to help him. Hint: you have to convert the number in word counting. i.e. Three hundred and twenty.

25. Use Fermat's' theorem, what is the value of a^302 (mod 7), where a is the sum of all the digits of the number obtained in question 19.
26. Afridi purchases a book entitled "Discrete Mathematics and Its Applications". Suppose that ISBN-10 of the textbook is 125973128Q. How can Afridi find the check digit to validate the originality of the book?
27. Shahid Afridi is also serving as the president of Sportics society at LUMS. He is purchasing footballs for FUTSAL tournament. The probability of purchasing defective footballs is 15%. What is the probability of not purchasing defective footballs?

- **Functions and Relations:** ***[Any 5 questions]***

28. Let f : A → B and g : B → C be functions. Prove that if g ◦ f is one-to-one then f is one-to-one.
29. . Let f : R → R be the function defined by f(x) = 4 + |2x + 3|.
    a. Give reasons why f is neither one-to-one nor onto.
    b. Explain how to replace the target R of f with a set B ⊆ R so that the function g : R → B, defined by g(x) = f(x) for all x ∈ R, is onto.

30. Let ~ be the relation on N defined by a ~ b if and only if a/ (b+2) ≥ b/ (a+2) . Prove that ~ is reflexive and anti-symmetric.
31. Let ~ be the relation on the set of all subsets of {1, 2, 3, 4} by X ~ Y ⇔ the smallest element of X equals the smallest element of Y . Prove that ~ is an equivalence relation.
32. Give an example of antisymmetric relations R and S on A = {1, 2, 3} such that R ∪ S is not antisysmmetric.
33. Let ~ be the relation on Z defined by a ~ b ⇔ 6 | a^2 − b^2 . Prove that ~ is an equivalence relation.
34. Consider the relation R defined on the set Z of integers by (a, b) ∈ R if and only if a − b ≤ 5. Consider the statements below. If a statement is true, prove it. If it is false, give a counterexample.
    a. R is reflexive.
    b. R is symmetric
    c. R is antisymmetric.
    d. R is transitive

- **Binomial theorem, Trees and Proofs:** ***[Any 4 questions]***

35.

[redacted] Let $b_0, b_1, \dots$ be the sequence defined by $b_0 = 2,\ b_1 = 1$ and $b_n = b_{n-1} + 2b_{n-2}$ for $n \geq 2$. Use induction to prove that $b_n = 2^n + (-1)^n$ for all $n \geq 0$.

36.

[redacted] Use induction to prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$, for all integers $n \geq 1$.

37.

Let $b_0, b_1, \ldots$ be the sequence defined by $b_0 = 2,\ b_1 = 5$ and $b_n = 5b_{n-1} - 6b_{n-2}$ for $n \geq 2$. Use induction to prove that $b_n = 2^n + 3^n$ for all $n \geq 0$.

38.

$$\text{Expand } \left[\frac{2}{3}x + 1\right]^3$$

39.

$$\text{Find the coefficient of } \frac{1}{y^{10}} \text{ in the expansion of } \left(y^3 + \frac{a^7}{y^5}\right)^{10}$$

40. In the expansion of (x − y)^15, calculate the coefficients of x^3y^12 and x^2y^13.

41. Let n be a positive integer. Prove that

$$\left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil = n.$$

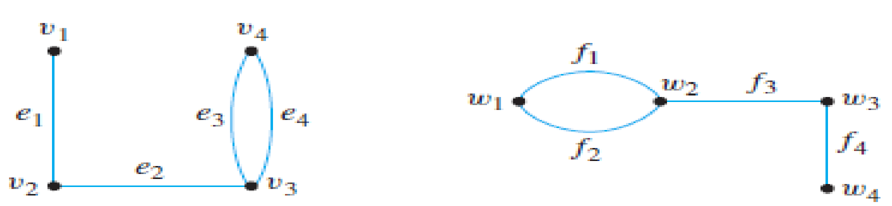
Hint: Consider the cases of n even and n odd.

42.

Let $n = (d_3 d_2 d_1 d_0)_{10}$. Prove that $3 \mid n$ if and only if $3 \mid (d_3 + d_2 + d_1 + d_0)$.

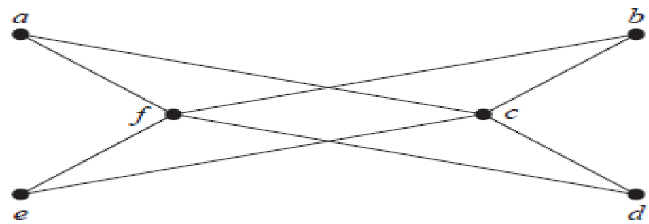
- **Relations and Graph Theory:** ***[All are Compulsory]***

43. **For a given pair of graph G and G/. Determine whether G and G/ are isomorphic. If they are, give function g: V (G) →V (G/) that defines the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.**

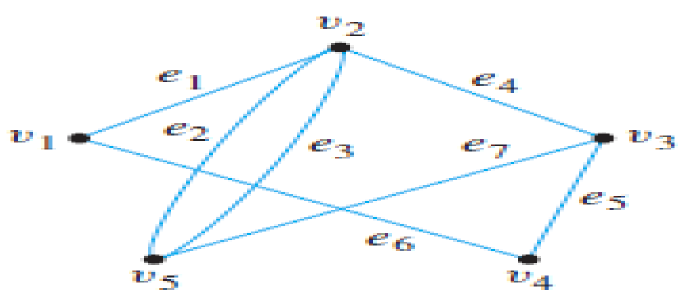


G G'

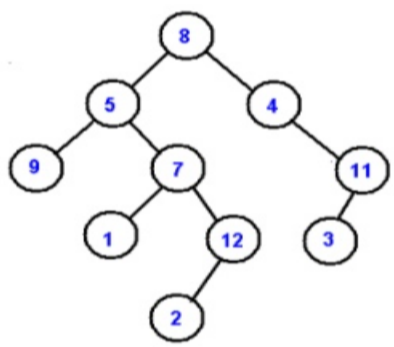
44. Find if the given graph is bipartite or not. Redraw the bipartite graph so its bipartite nature is evident.



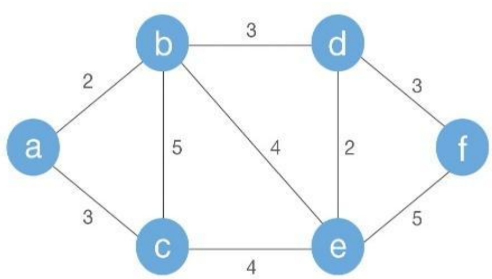
45. Use an incidence matrix to represent the graph shown below.



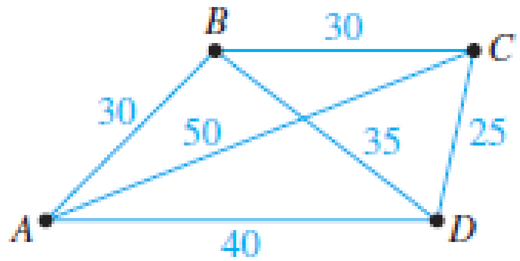
46. Determine the order in which preorder and postorder traversal visits the vertices of the given rooted tree.



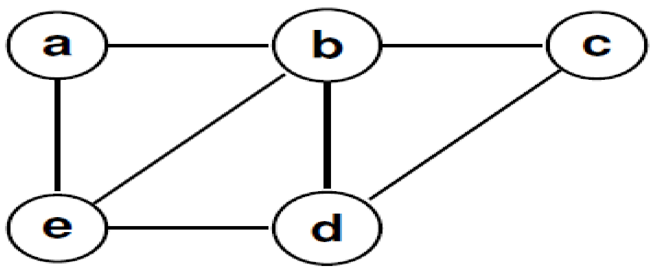
47. Find all minimum spanning trees for the given graph using Prim's algorithm starting at vertex a. Indicate the order in which edges are added to form each tree.



48. Imagine that the drawing below is a map showing four cities and the distances in kilometers between them. Suppose that a salesman must travel to each city exactly once, starting and ending in city A. Which route from city to city will minimize the total distance that must be traveled?



49. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



50. End