
بسم الله الرحمن الرحيم

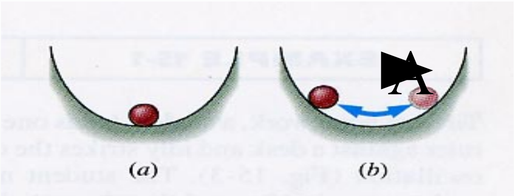
Physics For Engineers NS (110)

Lecture # 11,12

Date: 28th Oct, 2019

Oscillations

Oscillatory motion

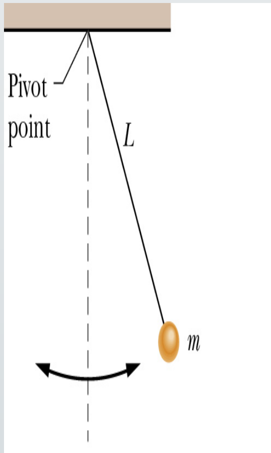


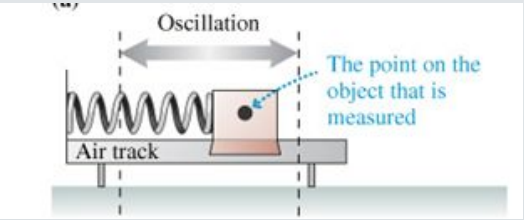
Motion which is periodic in time, that is, motion that repeats itself in time.

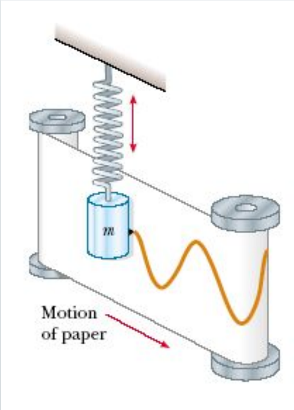
Examples:

- Power line oscillates when the wind blows past it
- Earthquake oscillations move buildings

Sometimes the oscillations are so severe, that the system exhibiting oscillations break apart.







Simple Harmonic Motion

- *Simple harmonic motion (SHM) refers to a certain kind of oscillatory, or wave-like motion that describes the behavior of many physical phenomena:*
  - *a pendulum*
  - *a bob attached to a spring*
  - *low amplitude waves in air (sound), water, the ground*
  - *the electromagnetic field of laser light*
  - *vibration of a plucked guitar string*
  - *the electric current of most AC power supplies*

Simple Harmonic Motion

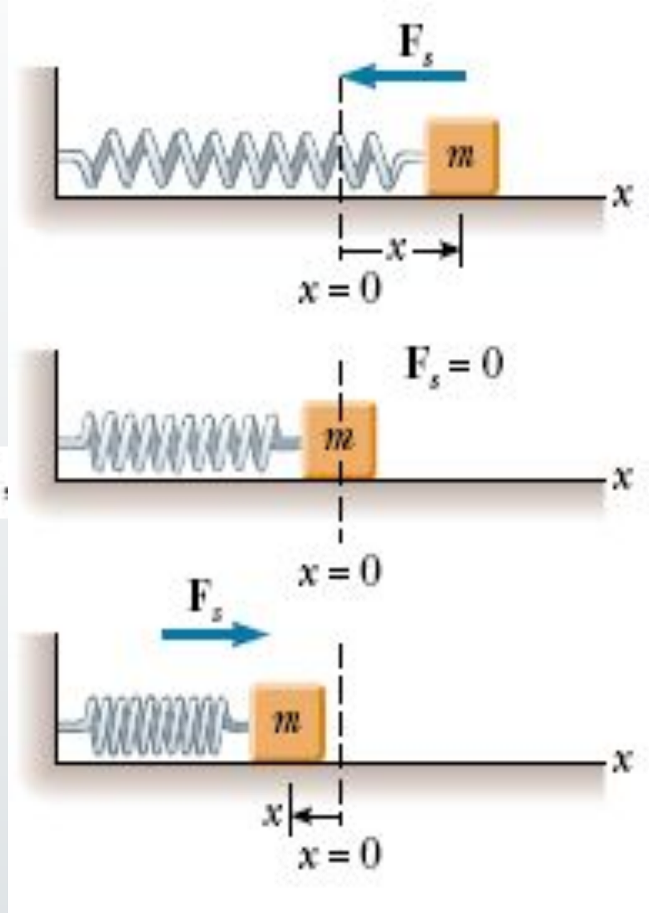
When the block is displaced a small distance x from equilibrium, the spring exerts on the block a force that is proportional to the displacement and given by Hoo ke's law

$$F_s = -kx$$

Applying Newton's second law to the motion of the block,

$$F_s = -kx = ma$$

$$a = -\frac{k}{m}x$$



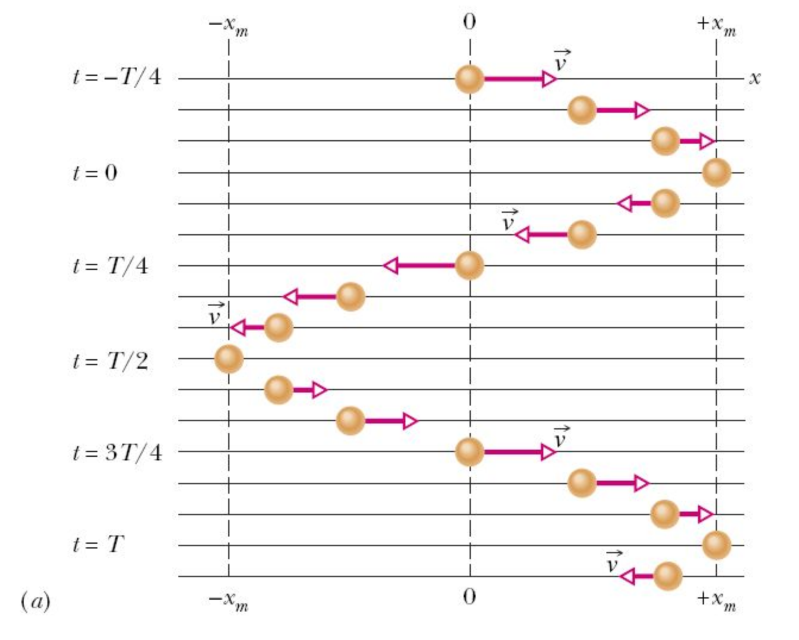
An object moves with simple harmonic motion whenever its acceleration is proportional to its displacement from some equilibrium position and is oppositely directed.

Simple Harmonic Motion

In the figure snapshots of a simple oscillatory system is shown. A particle repeatedly moves back and forth about the point x=0.

The time taken for one complete oscillation is the period, T. In the time of one T, the system travels from $x=+x_m$, to $-x_m$, and then back to its original position x_m.

The velocity vector arrows are scaled to indicate the magnitude of the speed of the system at different times. At $x=\pm x_m$, the velocity is zero.



Simple Harmonic Motion

Frequency of oscillation is the number of oscillations that are completed in each second.

The symbol for frequency is f, and the SI unit is the hertz (abbreviated as Hz).

It follows that

$$T = \frac{1}{f}$$

Simple Harmonic Motion

Any motion that repeats itself is periodic or harmonic.

If the motion is a sinusoidal function of time, it is called simple harmonic motion (SHM).

Mathematically SHM can be expressed as:

$$x(t) = x_m \cos(\omega t + \phi)$$

Here,

- x_m is the amplitude (maximum displacement of the system)
- t is the time
- ω is the angular frequency, and
- φ is the phase constant or phase angle

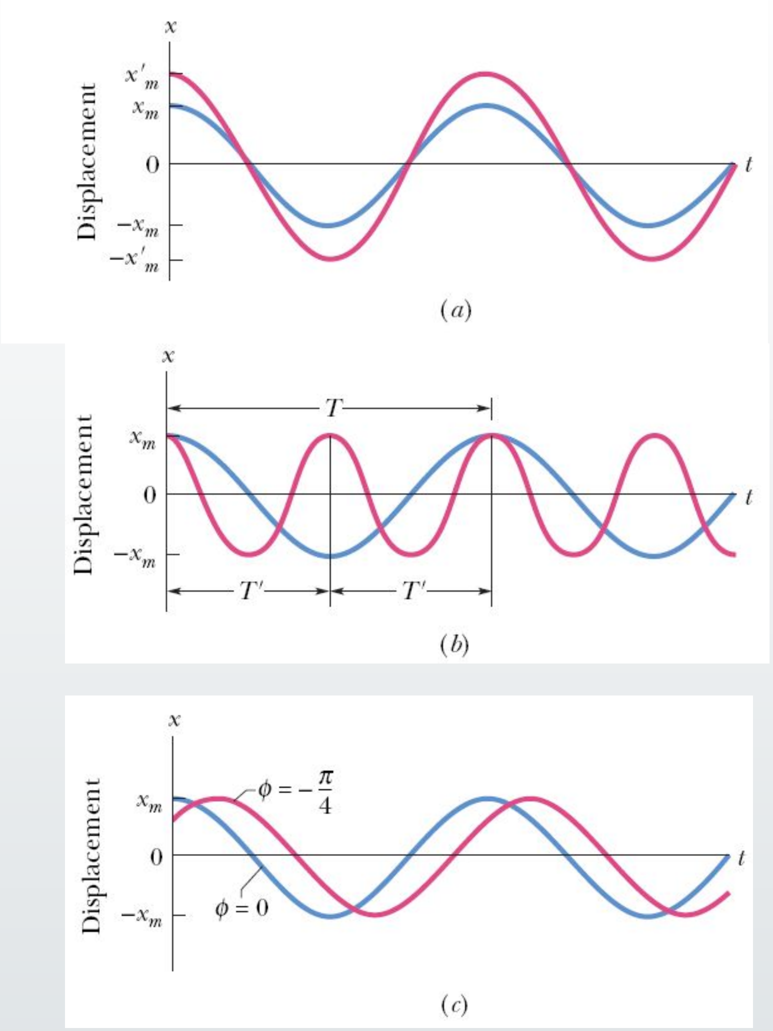
Simple Harmonic Motion

Figure a plots the displacement of two SHM systems that are different in amplitudes, but have the same period.

Figure b plots the displacement of two SHM systems which are different in periods but have the same amplitude.

The value of the phase constant term, φ, depends on the value of the displacement and the velocity of the system at time t = 0.

Figure c plots the displacement of two SHM systems having the same period and amplitude, but different phase constants.



Simple Harmonic Motion

For an oscillatory motion with period T,

$$x(t) = x(t+T)$$

The cosine function also repeats itself when the argument increases by 2π. Therefore,

$$\omega(t+T) = \omega t + 2\pi$$

$$\rightarrow \omega T = 2\pi$$

$$\rightarrow \omega = \frac{2\pi}{T} = 2\pi f$$

Here, ω is the angular frequency, and measures the angle per unit time. Its SI unit is radians/second. To be consistent, φ then must be in radians.

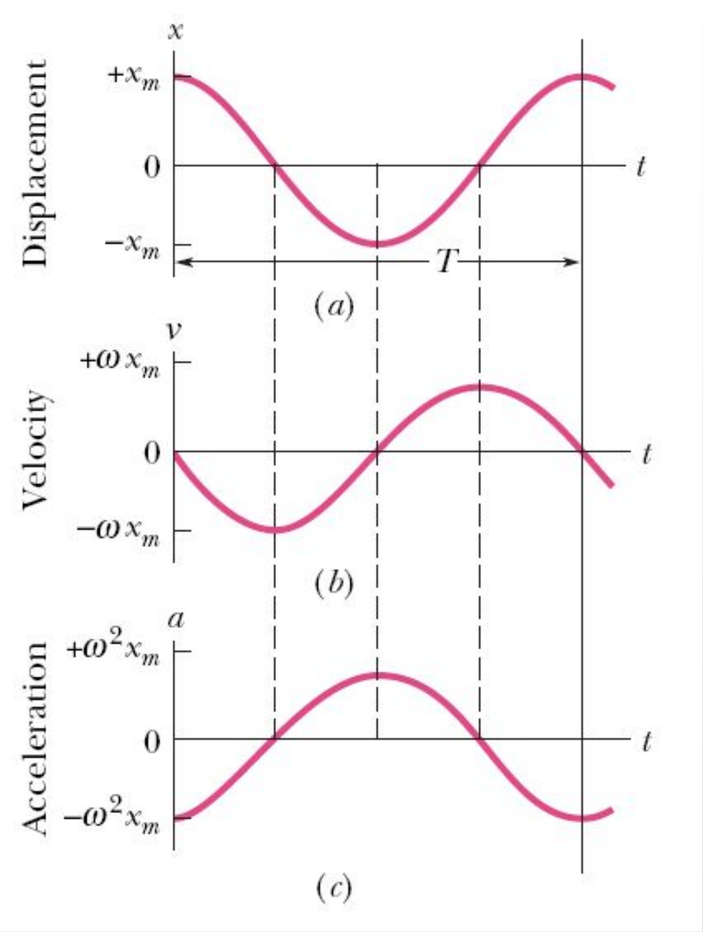
Simple Harmonic Motion

The velocity of SHM:

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right]$$

$$\rightarrow v(t) = -\omega x_m \sin(\omega t + \phi$$

The maximum value (amplitude) of velocity is ωx_m. The phase shift of the velocity is $\pi/2$, making the cosine to a sine function.



The acceleration of SHM is:

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$$

$$\rightarrow a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

$$\rightarrow a(t) = -\omega^2 x(t)$$

The acceleration amplitude is $\omega^2 x_m$.

In SHM a(t) is proportional to the displacement but opposite in sign.

Force Law for SHM

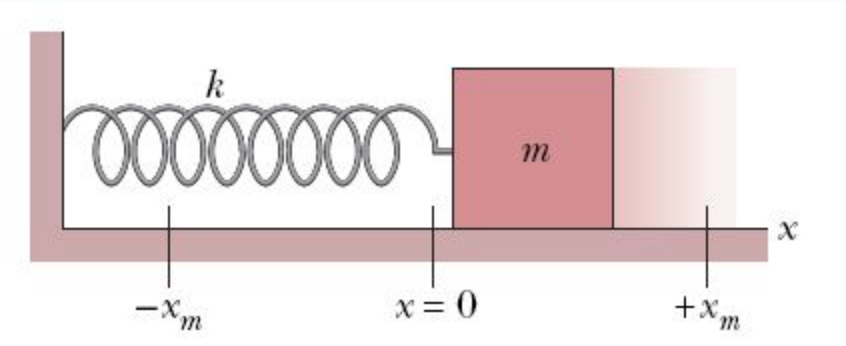
From Newton's 2nd law:

$$F = ma = -(m\omega^2)x = -kx$$

SHM is the motion executed by a system subject to a force that is proportional to the displacement of the system but opposite in sign.

Force Law for SHM

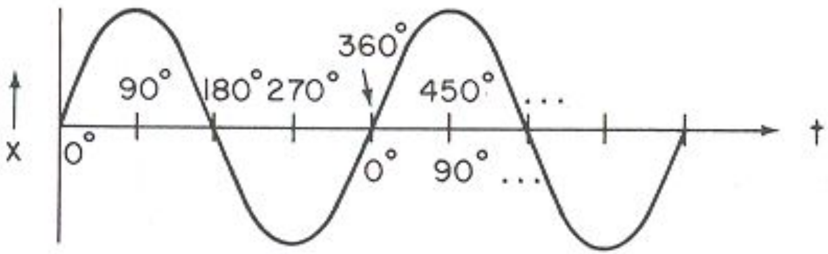
The block-spring system shown on the right forms a linear SHM oscillator.

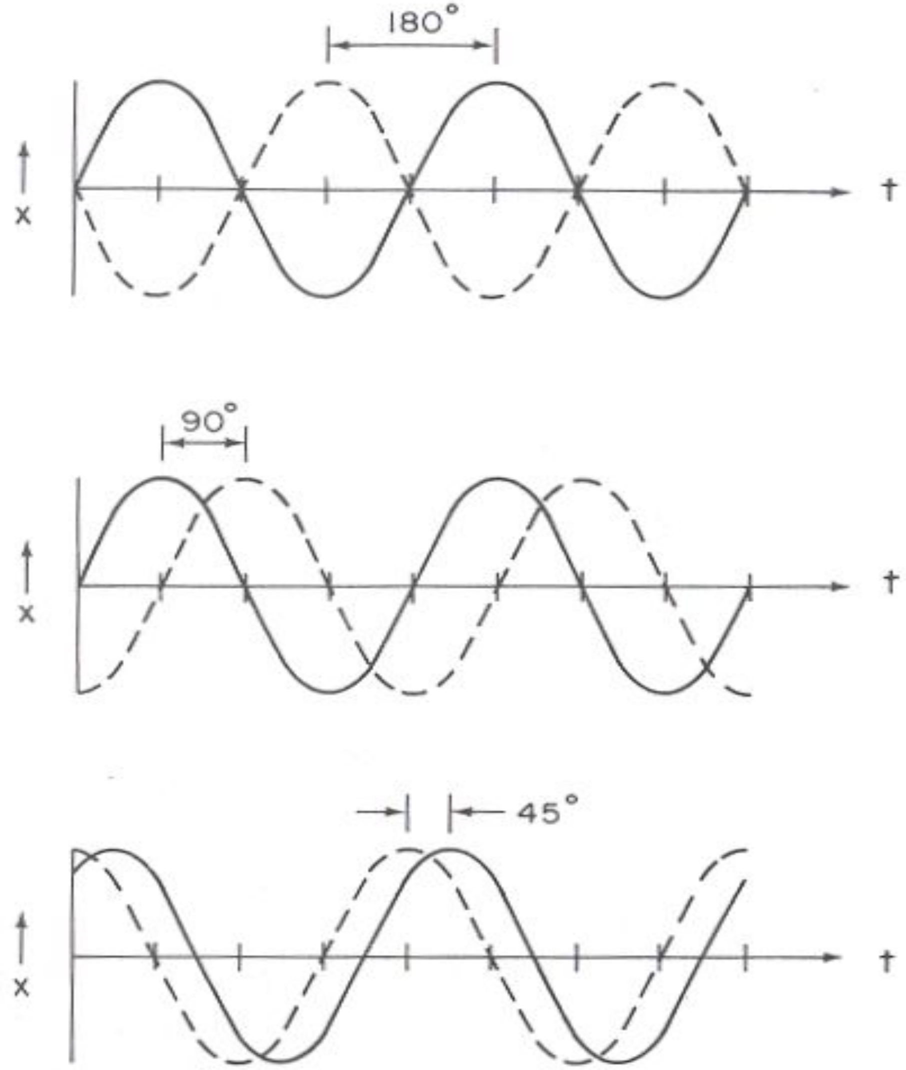


The spring constant of the spring, k, is related to the angular frequency, ω, of the oscillator:

$$\omega = \sqrt{\frac{k}{m}} \rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

Phase (Time) Phase Diffference





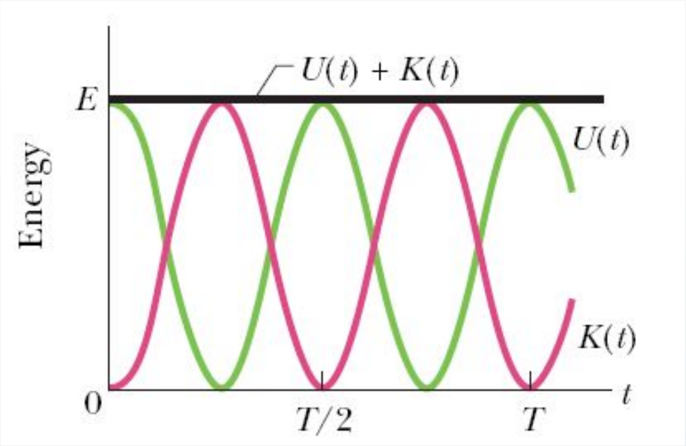
Properties of simple harmonic motion

- The acceleration of the particle is proportional to the displacement but is in the opposite direction. This is the *necessary and sufficient condition for simple harmonic motion*, as opposed to all other kinds of vibration.
- The displacement from the equilibrium position, velocity, and acceleration all vary sinusoidally with time but are not in phase
- The frequency and the period of the motion are independent of the amplitude.

Energy in SHM

The potential energy of a linear oscillator is associated entirely with the spring.

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m{}^2 \cos^2(\omega t + \phi)$$

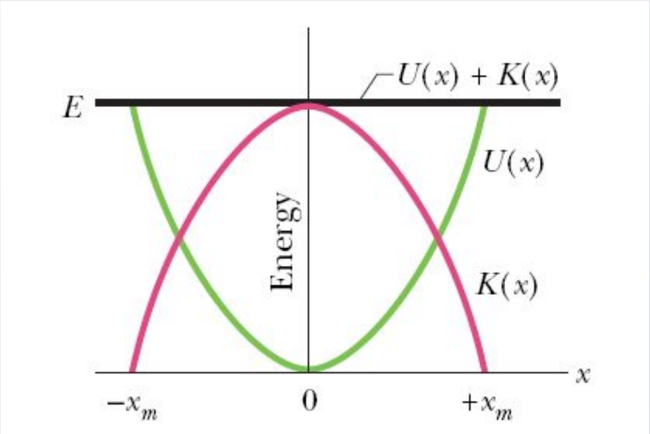


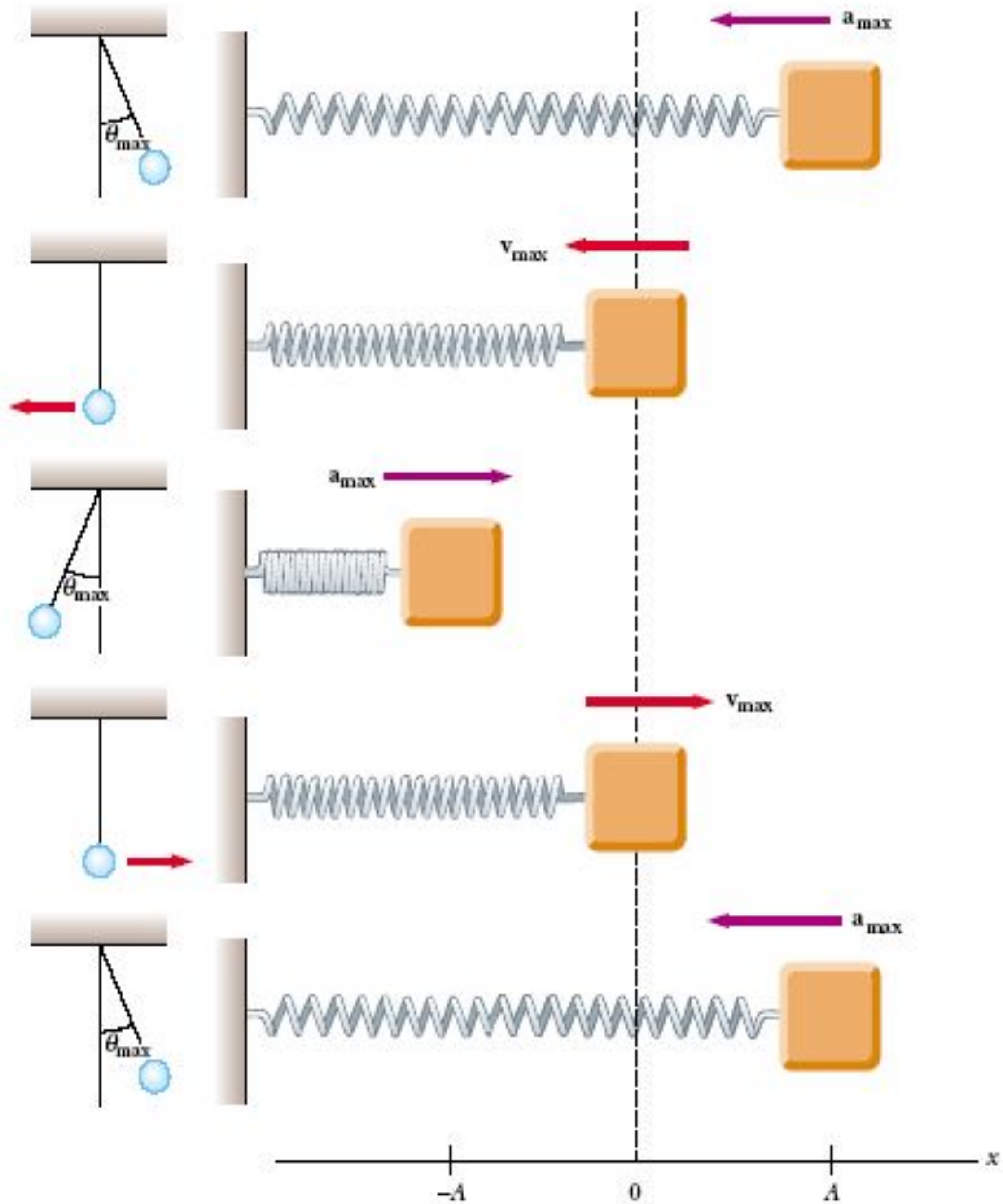
The kinetic energy of the system is associated entirely with the speed of the block.

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_m{}^2 \sin^2(\omega t + \phi) = \frac{1}{2}kx_m{}^2 \sin^2(\omega t + \phi)$$

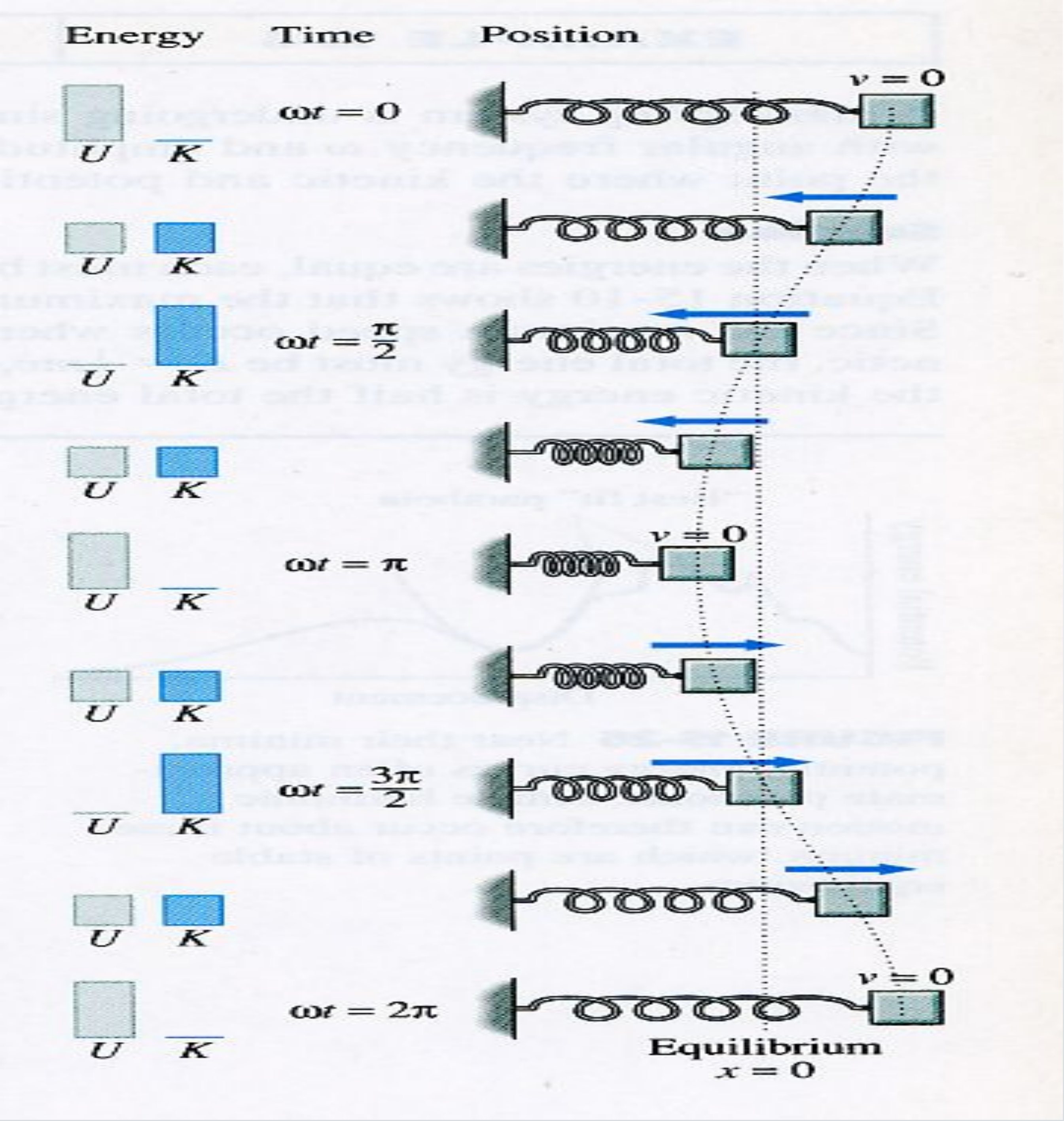
The total mechanical energy of the system:

$$E = U + K = \frac{1}{2}kx_m{}^2$$





t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$T/4$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$T/2$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$3T/4$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$

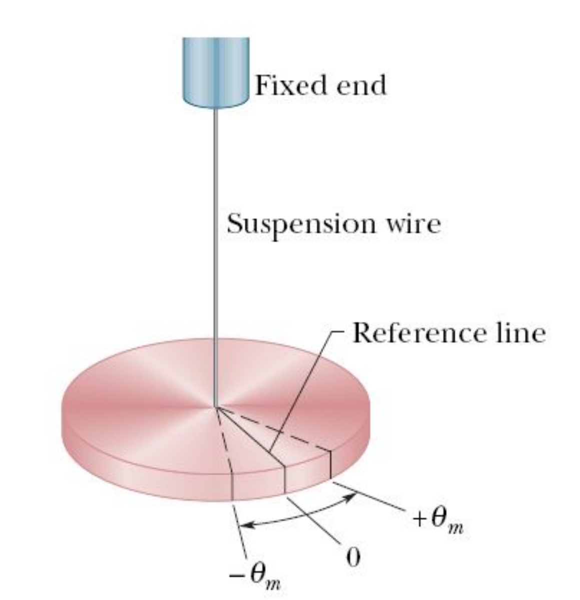


Energy
Time
Position
v = 0
ωt = 0
U
K
U
K
ωt = π/2
U
K
U
K
v = 0
ωt = π
U
K
U
K
ωt = 3π/2
U
K
U
K
v = 0
ωt = 2π
U
K
Equilibrium
x = 0

Simple Harmonic Motion

An angular SHM:

The figure shows a **torsion pendulum,** which involves the twisting of a suspension wire as the disk oscillates in a horizontal plane.



The torque associated with an angular displacement of θ is given by:

$$\tau = -\kappa\theta = I\frac{d^2\theta}{dt^2}$$

κ is the torsion constant, which depends on the length, diameter, and material of the suspension wire, and I is the moment of inertia (rotational inertia) of the disk.

The period, T, is then

$$\omega = \sqrt{\frac{\kappa}{I}} \qquad T = 2\pi\sqrt{\frac{I}{\kappa}}$$

SHM and uniform circular motion

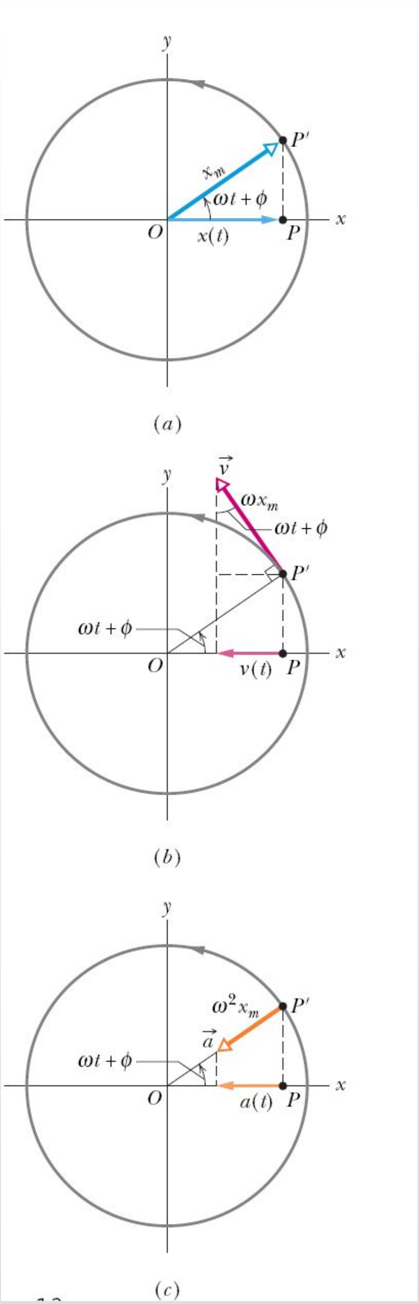
Consider a reference particle P’ moving in uniform circular motion with constant angular speed (w).

The projection of the particle on the x-axis is a point P, describing motion given by:

$$x(t) = x_m \cos(\omega t + \phi).$$

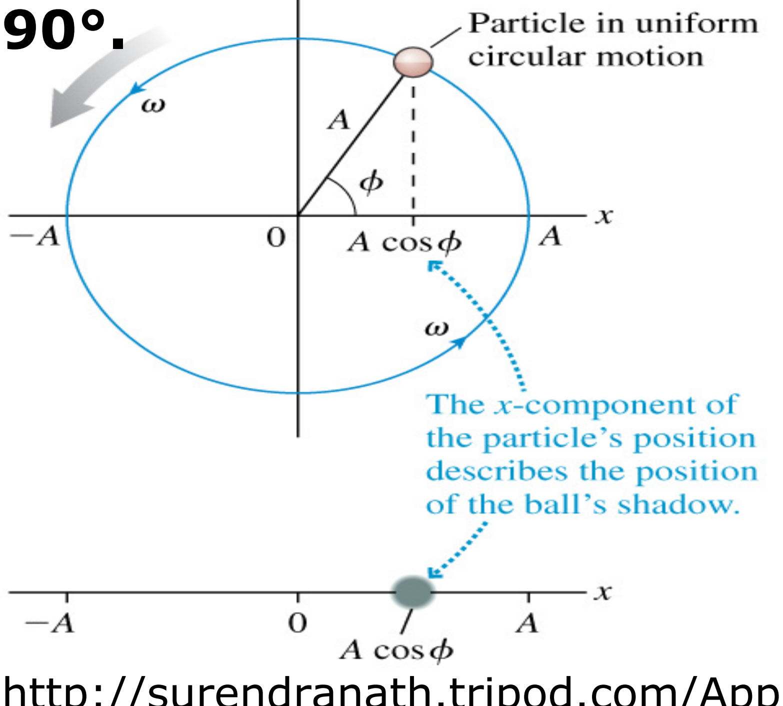
This is the displacement equation of SHM.

SHM, therefore, is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

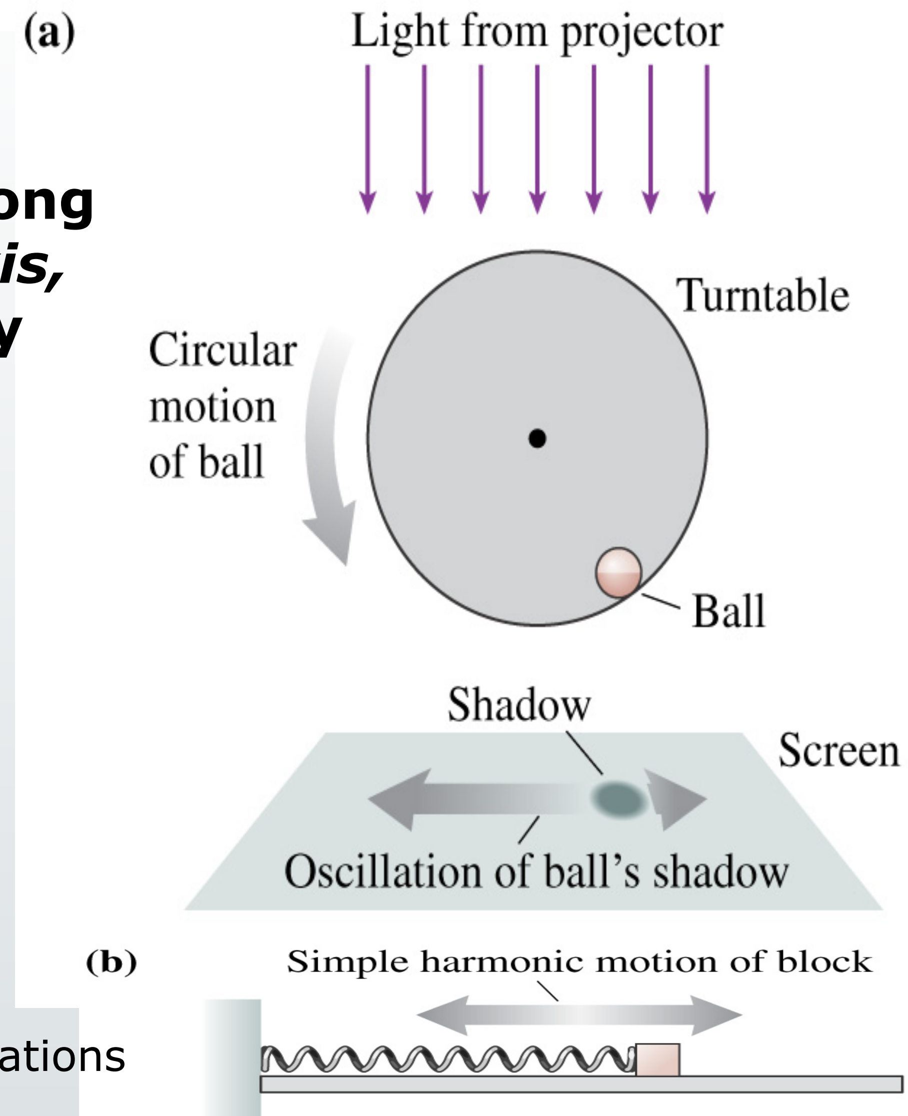


COMPARING SIMPLE HARMONIC MOTION WITH UNIFORM CIRCULAR MOTION

Uniform circular motion can be considered a combination of two simple harmonic motions, one along the *x axis and one along the y axis, with the two* differing in phase by 90°.



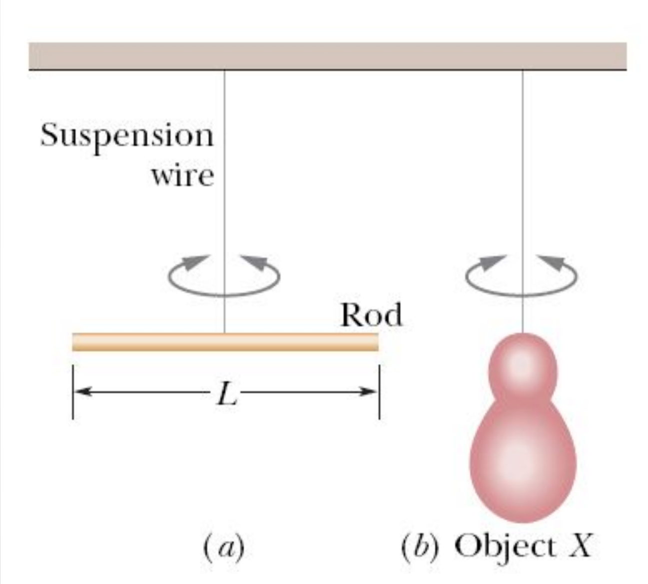
http://surendranath.tripod.com/Applets/Oscillations/SHM/SHM.html



simple harmonic motion along a straight line can be represented by the projection of uniform circular motion along a diameter of a reference circle.

Example, angular SHM:

Figure *a shows a thin rod whose length L is 12.4 cm* and whose mass *m is 135 g, suspended at its midpoint* from a long wire. Its period T_a *of angular SHM is measured* to be 2.53 s. An irregularly shaped object, which we call object *X, is then hung from the same wire, as in Fig. b, and its period* T_b *is found to be 4.76 s. What is the* rotational inertia of object *X about its suspension axis?*



Example, angular SHM:

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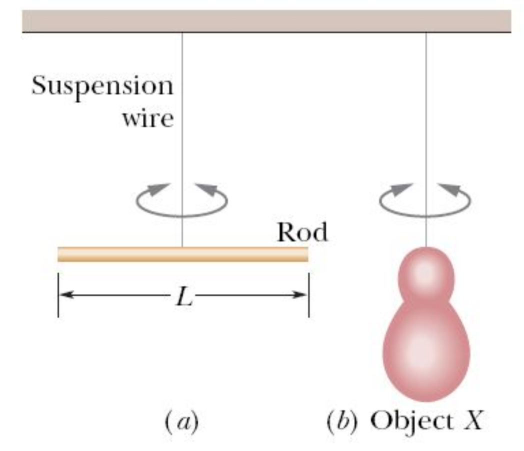


Answer: The rotational inertia of either the rod or object *X is related to the measured period.* The rotational inertia of a thin rod about a perpendicular axis through its midpoint is given as 1/12 mL^2.*Thus, we have, for the rod in Fig. a,*

$$I_a = \tfrac{1}{12}mL^2 = (\tfrac{1}{12})(0.135\ \text{kg})(0.124\ \text{m})^2$$
$$= 1.73 \times 10^{-4}\ \text{kg}\cdot\text{m}^2.$$

Now let us write the periods, once for the rod and once for object *X:*

$$T_a = 2\pi\sqrt{\frac{I_a}{\kappa}} \quad \text{and} \quad T_b = 2\pi\sqrt{\frac{I_b}{\kappa}}.$$

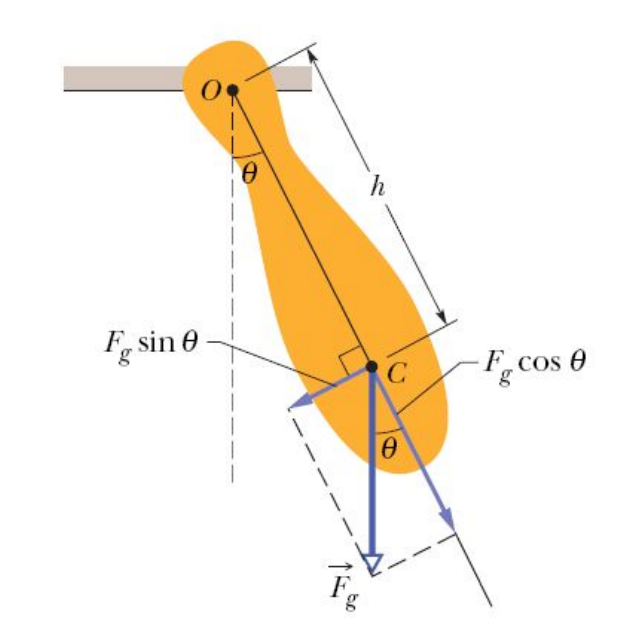
The constant κ, which is a property of the wire, is the same for both figures; only the periods and the rotational inertias differ.

Let us square each of these equations, divide the second by the first, and solve the resulting equation for I_b. *The result is*

$$I_b = I_a\frac{T_b^2}{T_a^2} = (1.73 \times 10^{-4}\ \text{kg}\cdot\text{m}^2)\frac{(4.76\ \text{s})^2}{(2.53\ \text{s})^2}$$
$$= 6.12 \times 10^{-4}\ \text{kg}\cdot\text{m}^2. \qquad \text{(Answer)}$$

Pendulums

A physical pendulum can have a complicated distribution of mass. If the center of mass, C, is at a distance of h from the pivot point (figure), then for *small angular amplitudes*, the motion is simple harmonic.



The period, T, is:

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Here, I is the rotational inertia of the pendulum about O.

Pendulums

In a *simple pendulum*, a particle of mass m is suspended from one end of an unstretchable massless string of length L that is fixed at the other end.

The restoring torque acting on the mass when its angular displacement is θ, is:

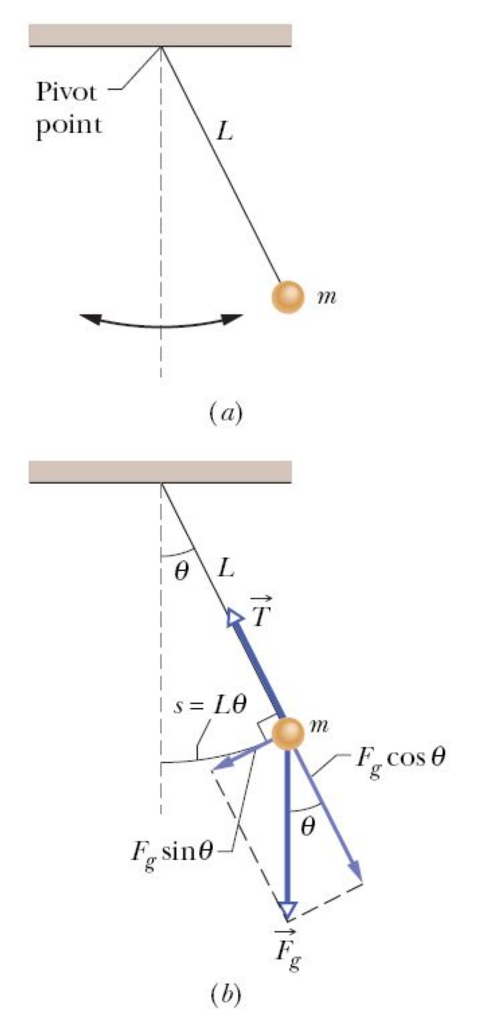
$$\tau = -L(F_g \sin\theta) = I\alpha$$

α is the angular acceleration of the mass. Finally,

$$\alpha = -\frac{mgL}{I}\theta, and$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

This is true for *small angular displacements*, θ.



Pendulums

In the **small-angle approximation** we can assume that $\theta << 1$ and use the approximation $\sin\theta \cong \theta$. Let us investigate up to what angle θ is the approximation reasonably accurate?

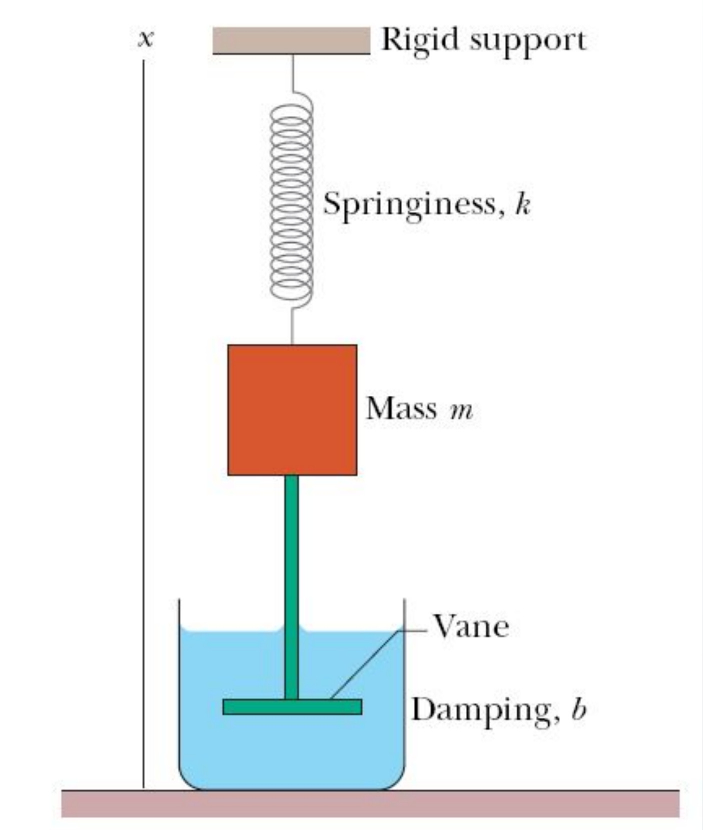
θ (degrees)	θ (radians)	$\sin\theta$
5	0.087	0.087
10	0.174	0.174
15	0.262	0.259 (1% off)
20	0.349	0.342 (2% off)

Conclusion: If we keep $\theta < 10°$ we make less than 1 % error.

Damped Oscillations

In a damped oscillation, the motion of the oscillator is reduced by an external force.

Example: A block of mass m oscillates vertically on a spring with spring constant k.
From the block a rod extends to a vane which is submerged in a liquid.
The liquid provides the external damping force, F_d.

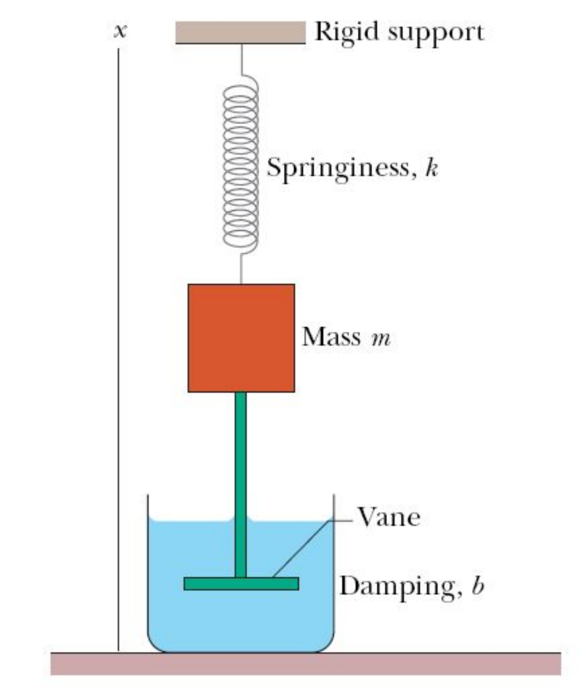


Damped Oscillations

Often the damping force, F_d, is proportional to the 1st power of the velocity v. That is,

$$F_d = -\gamma v$$

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$



The solution is:

$$x(t) = x_0 e^{\frac{-\gamma t}{2m}} \cos(\omega' t + \varphi)$$

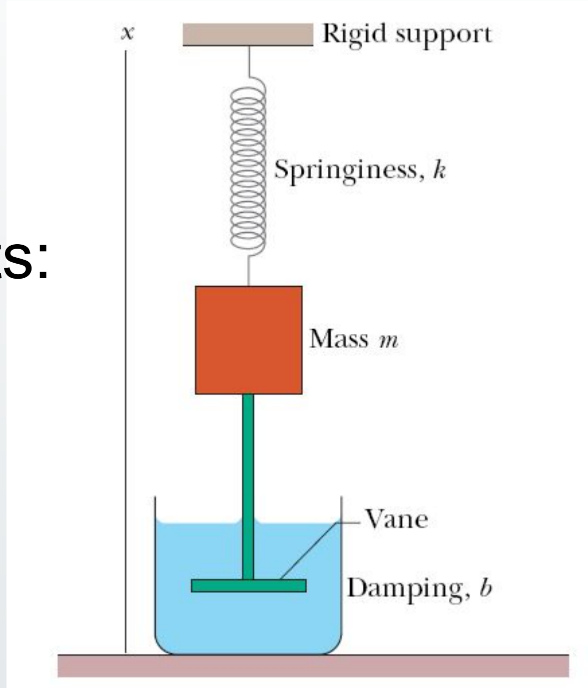
$$\omega' = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2m}\right)^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Damped SHM

Often the damping force, F_d, is proportional to the 1st power of the velocity v. That is,

$$F_d = -bv$$



From Newton's 2nd law, the following DE results:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

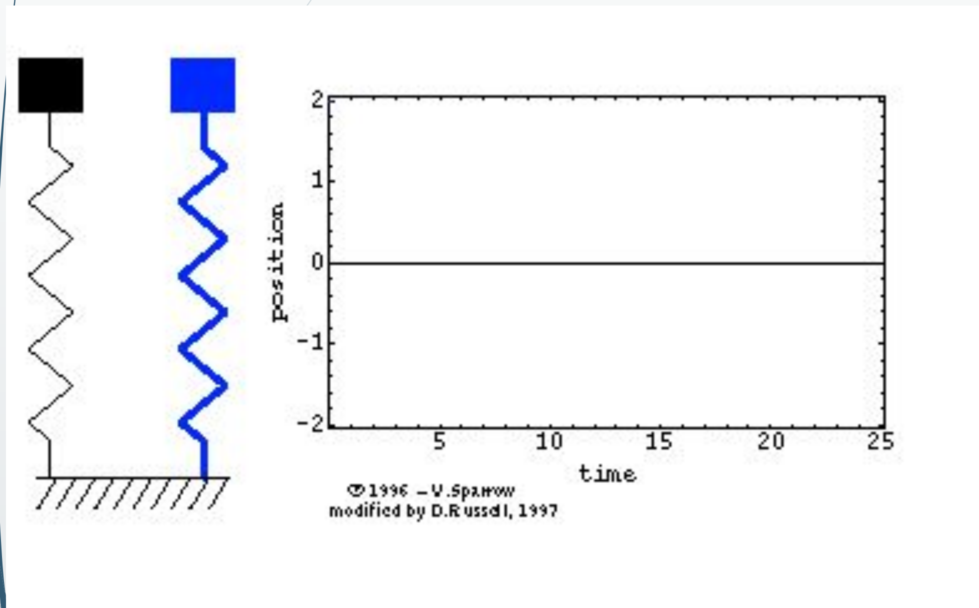
The solution is:

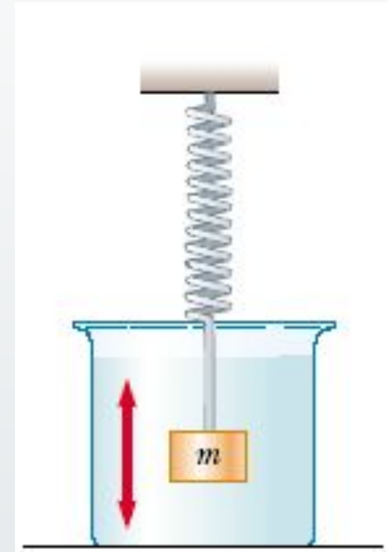
$$x(t) = x_m e^{\frac{-bt}{2m}} \cos(\omega' t + \phi)$$

Here ω' is the angular frequency, and is given by:

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

DAMPED OSCILLATIONS

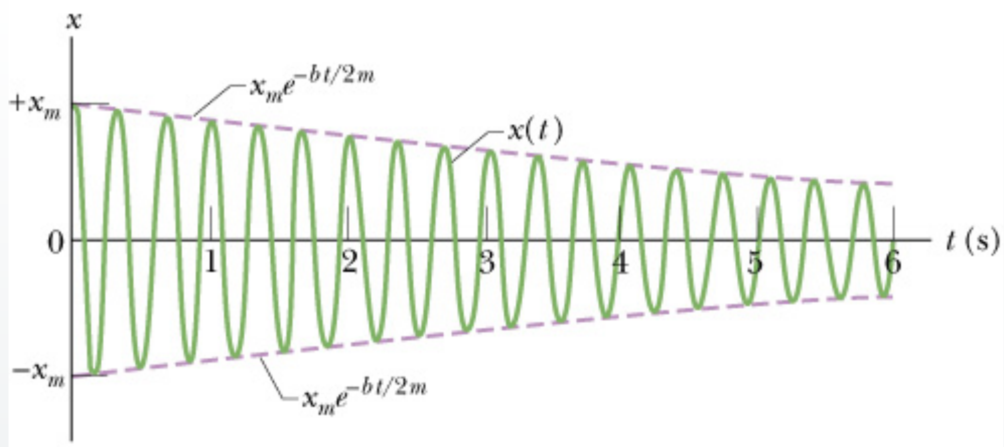




http://www.lon-capa.org/~mmp/applist/damped/d.htm

15.5 Damped Oscillations

$$x(t) = x_0 e^{\frac{-\gamma t}{2m}} \cos(\omega' t + \varphi)$$

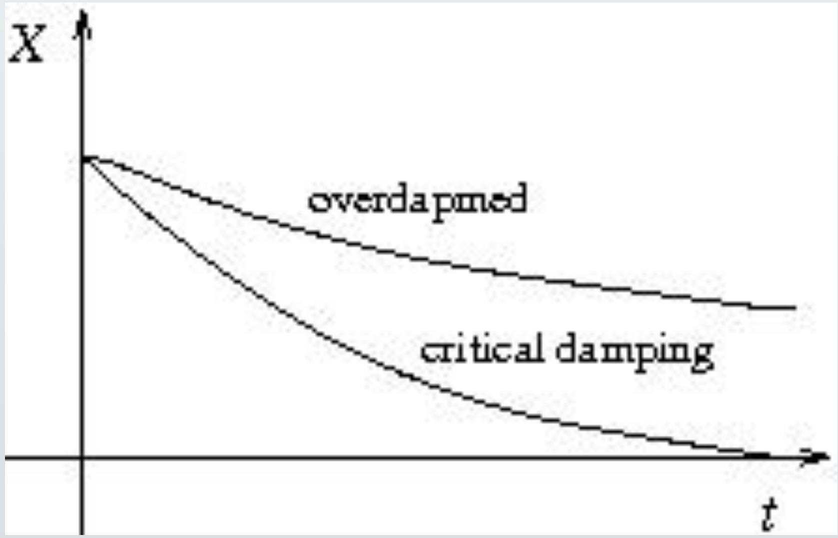


The above figure shows the displacement function *x(t)* for the damped oscillator described before.

The amplitude decreases as $x_0 \exp(-\gamma t / 2m)$ with time.

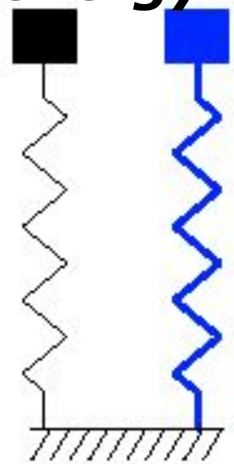
The above is for $\gamma < 2m\omega_0$ (underdapmed).

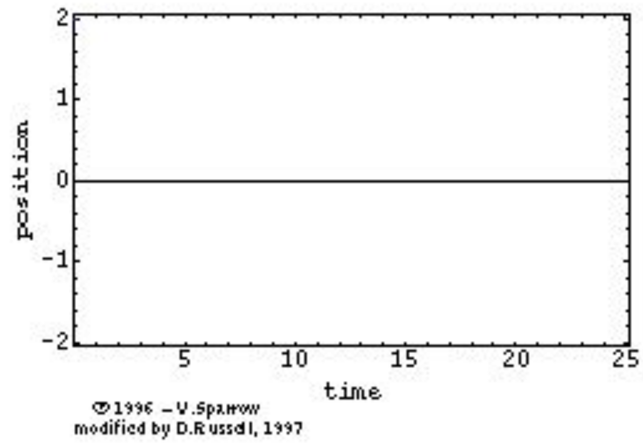
For $\gamma > 2m\omega_0$ (overdapmed) and $\gamma = 2m\omega_0$ (critical damping), the oscillation goes like the right figure.



DAMPED OSCILLATIONS

In many real systems, dissipative forces, such as friction, retard the motion.
Consequently, the mechanical energy of the system diminishes in time, and the motion is said to be Damped.
Retarding force $\mathbf{R} = -b\mathbf{v}$





, we can write Newton's second law as

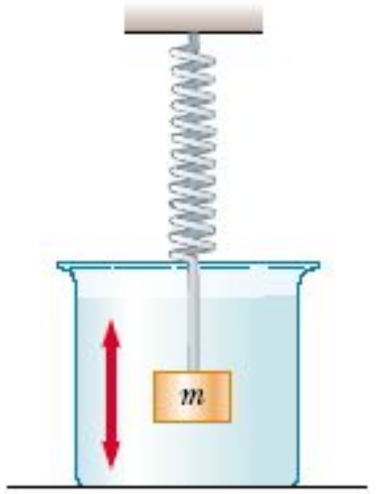
$$\sum F_x = -kx - bv = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

The solution of this equation $x = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$

where the angular frequency of oscillation is

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

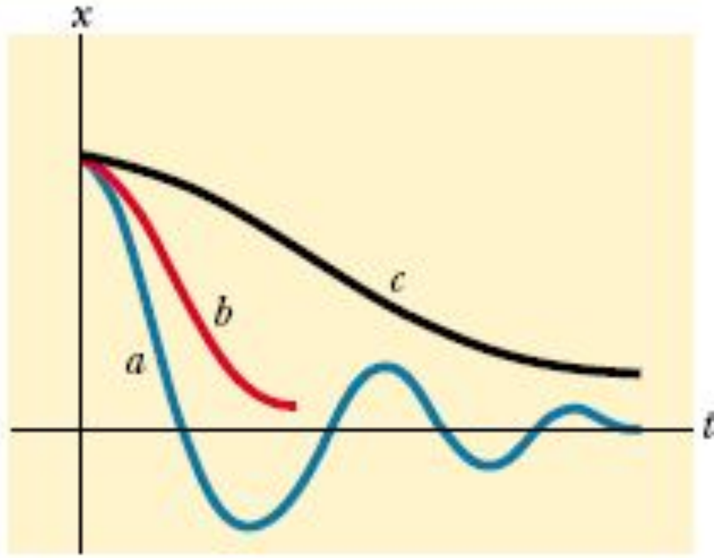


http://www.lon-capa.org/~mmp/applist/damped/d.htm

DAMPED OSCILLATIONS

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2m}\right)^2}$$

where $\omega_0 = \sqrt{k/m}$ represents the angular frequency in the absence of a retarding force (the undamped oscillator) and is called the **natural frequency**



Graphs of displacement versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

Forced Oscillations and Resonance

When the oscillator is subjected to an external force that is periodic, the oscillator will exhibit forced/driven oscillations.

There are two frequencies involved in a forced oscillator:

I. w_0, the natural angular frequency of the oscillator, without the presence of any external force, and

II. w_e, the angular frequency of the applied external force.

The equation of motion is like the following:

$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F_0\cos(\omega_e t)$$

Forced Oscillations and Resonance

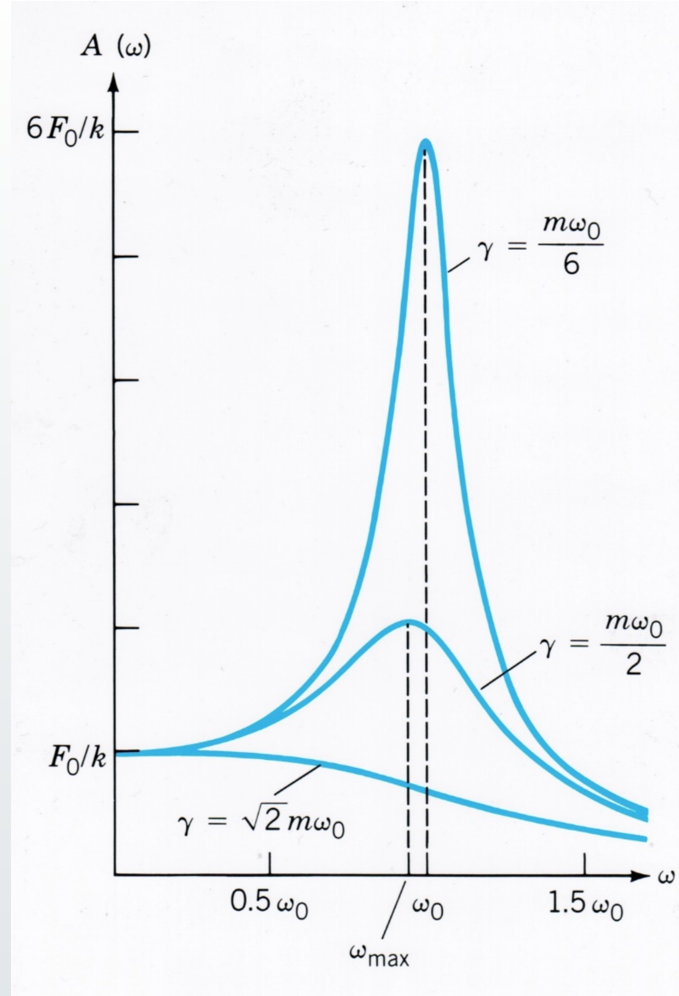
$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F_0\cos(\omega_e t)$$

The *steady state* solution is

$$x(t)=A\cos(\omega_e t+\delta)$$

$$A=\frac{F_0/m}{\sqrt{(\omega_0^2-\omega_e^2)^2+\left(\frac{\gamma}{m}\omega_e\right)^2}}$$

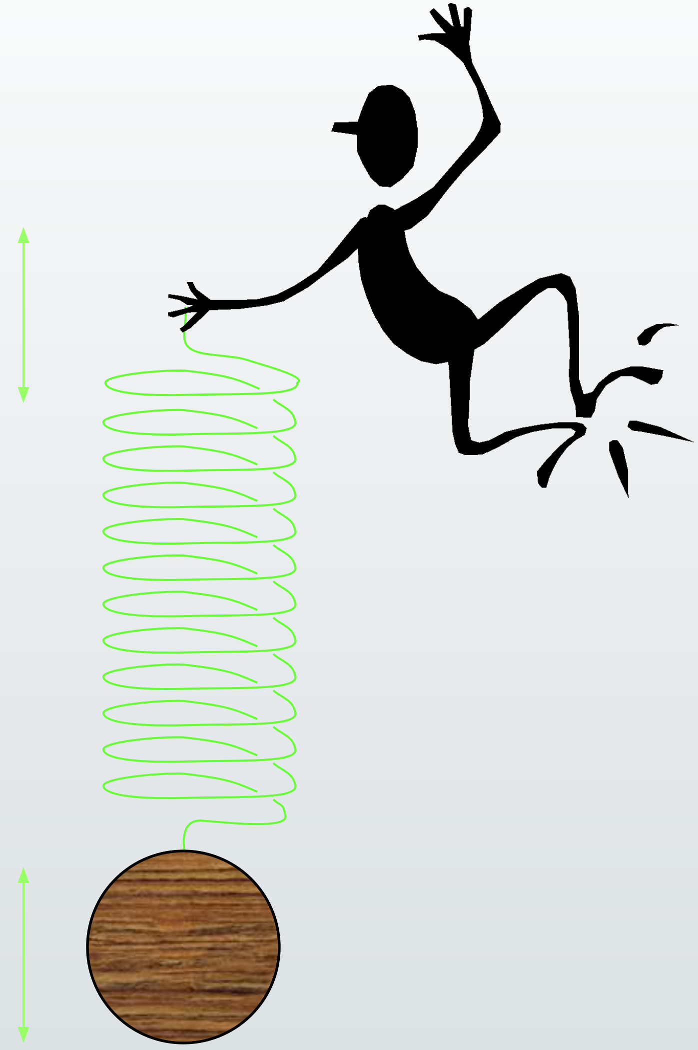
$$\tan\delta=\frac{\gamma}{m}\frac{\omega_e}{\omega_0^2-\omega_e^2} \qquad \omega_0=\sqrt{\frac{k}{m}}$$



<u>**Example**</u> (Mass-Spring System)

Periodic driving force of freq. ***f***

Oscillating with natural freq. f_o



Resonance

When a system is disturbed by a periodic driving force which frequency is ***equal to the natural frequency (fo)*** of the system, the system will oscillate with ***LARGE amplitude***.

Resonance is said to occur.

http://www.acoustics.salford.ac.uk/feschools/waves/shm3.htm

Example 1

Breaking Glass

System : ***glass***

Driving Force :

sound wave

Example 2

Collapse of the Tacoma Narrows suspension bridge in America in 1940

System : ***bridge***

Driving Force :

strong wind

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

KEY IDEA The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by Eq. 15-12:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\ \text{N/m}}{0.68\ \text{kg}}} = 9.78\ \text{rad/s}$$

$$\approx 9.8\ \text{rad/s}. \qquad \text{(Answer)}$$

The frequency follows from Eq. 15-5, which yields

$$f = \frac{\omega}{2\pi} = \frac{9.78\ \text{rad/s}}{2\pi\ \text{rad}} = 1.56\ \text{Hz} \approx 1.6\ \text{Hz}. \qquad \text{(Answer)}$$

The period follows from Eq. 15-2, which yields

$$T = \frac{1}{f} = \frac{1}{1.56\ \text{Hz}} = 0.64\ \text{s} = 640\ \text{ms}. \qquad \text{(Answer)}$$

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(b) What is the amplitude of the oscillation?

KEY IDEA With no friction involved, the mechanical energy of the spring–block system is conserved.

Reasoning: The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system at a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm:

$$x_m = 11 \text{ cm.} \qquad \text{(Answer)}$$

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

KEY IDEA The maximum speed v_m is the velocity amplitude ωx_m in Eq. 15-6.

Calculation: Thus, we have

$$v_m = \omega x_m = (9.78 \text{ rad/s})(0.11 \text{ m})$$
$$= 1.1 \text{ m/s}. \qquad \text{(Answer)}$$

This maximum speed occurs when the oscillating block is rushing through the origin; compare Figs. 15-4*a* and 15-4*b*, where you can see that the speed is a maximum whenever $x = 0$.

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(d) What is the magnitude a_m of the maximum acceleration of the block?

KEY IDEA The magnitude a_m of the maximum acceleration is the acceleration amplitude $\omega^2 x_m$ in Eq. 15-7.

Calculation: So, we have

$$a_m = \omega^2 x_m = (9.78 \text{ rad/s})^2(0.11 \text{ m})$$
$$= 11 \text{ m/s}^2. \qquad \text{(Answer)}$$

This maximum acceleration occurs when the block is at the ends of its path. At those points, the force acting on the block has its maximum magnitude; compare Figs. 15-4*a* and 15-4*c*, where you can see that the magnitudes of the displacement and acceleration are maximum at the same times.

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(e) What is the phase constant ϕ for the motion?

Calculations: Equation 15-3 gives the displacement of the block as a function of time. We know that at time $t = 0$, the block is located at $x = x_m$. Substituting these *initial conditions*, as they are called, into Eq. 15-3 and canceling x_m give us

$$1 = \cos\phi. \tag{15-14}$$

Taking the inverse cosine then yields

$$\phi = 0 \text{ rad.} \qquad \text{(Answer)}$$

(Any angle that is an integer multiple of 2π rad also satisfies Eq. 15-14; we chose the smallest angle.)