Lecture no 2

Differentiation

Week no 2

Chapter no 2

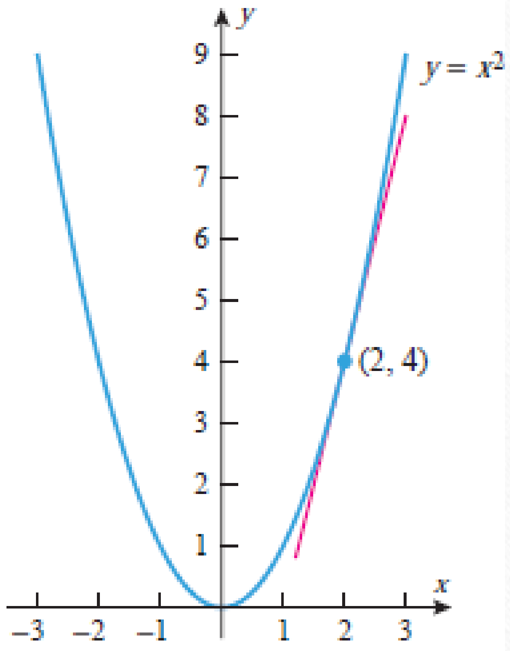
2.2 THE DERIVATIVE FUNCTION

2.2.1 DEFINITION The function f' defined by the formula

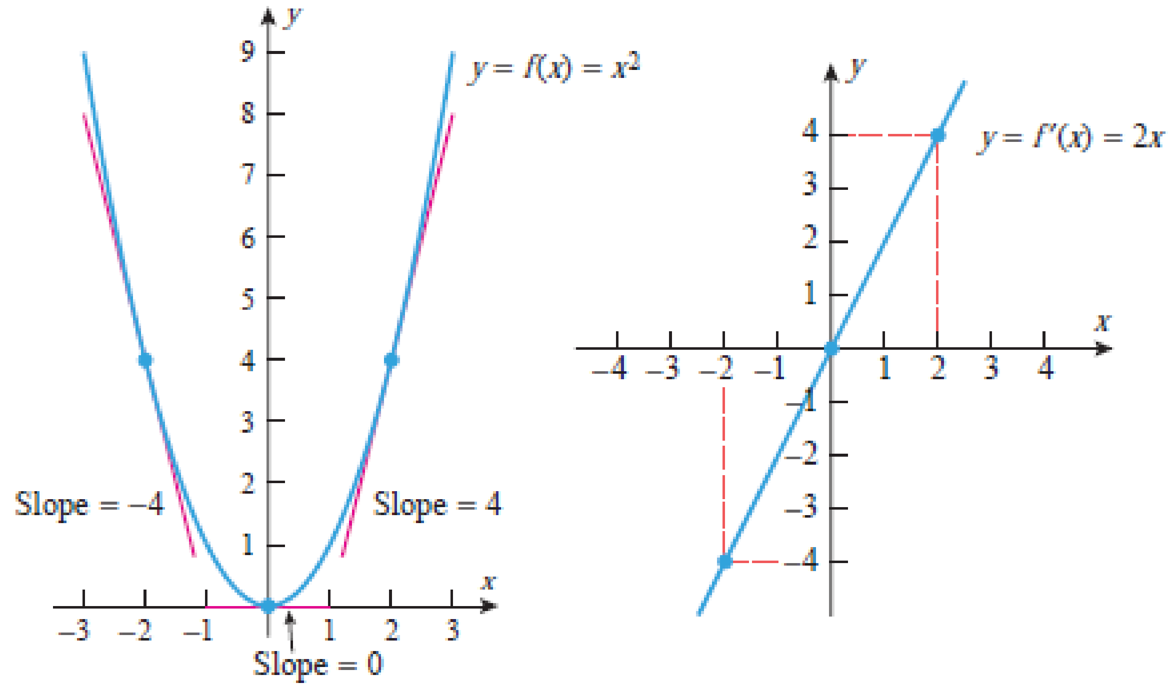
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (2)$$

is called the ***derivative of f with respect to x***. The domain of f' consists of all x in the domain of f for which the limit exists.

▸ **Example 1** Find the derivative with respect to x of $f(x) = x^2$, and use it to find the equation of the tangent line to $y = x^2$ at $x = 2$.



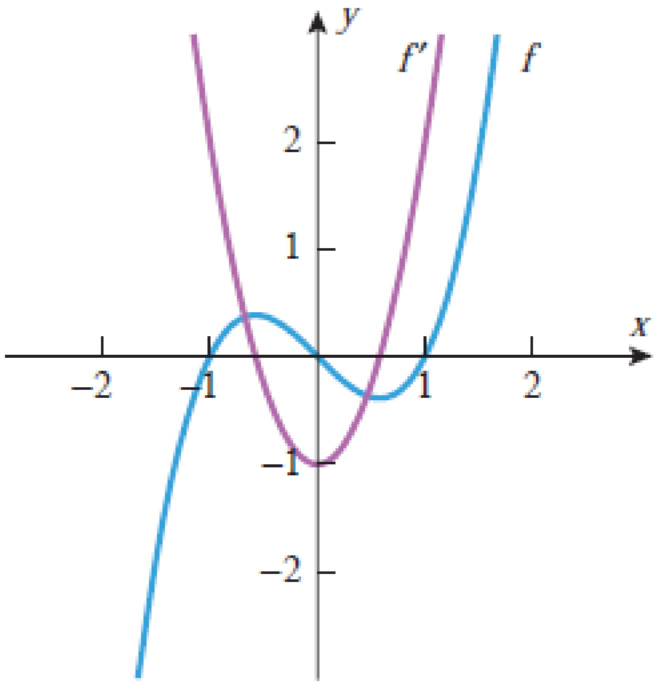
▲ **Figure 2.2.1**



▸ Example 2

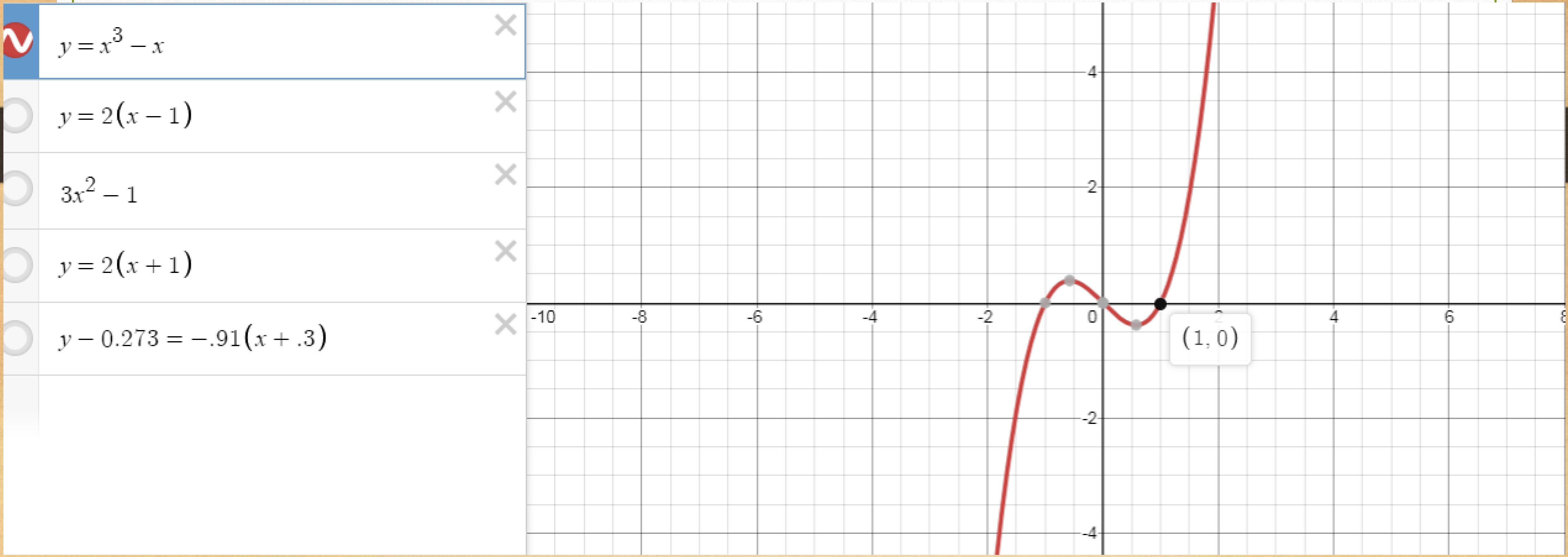
(a) Find the derivative with respect to x of $f(x) = x^3 - x$.

(b) Graph f and f' together, and discuss the relationship between the two graphs.

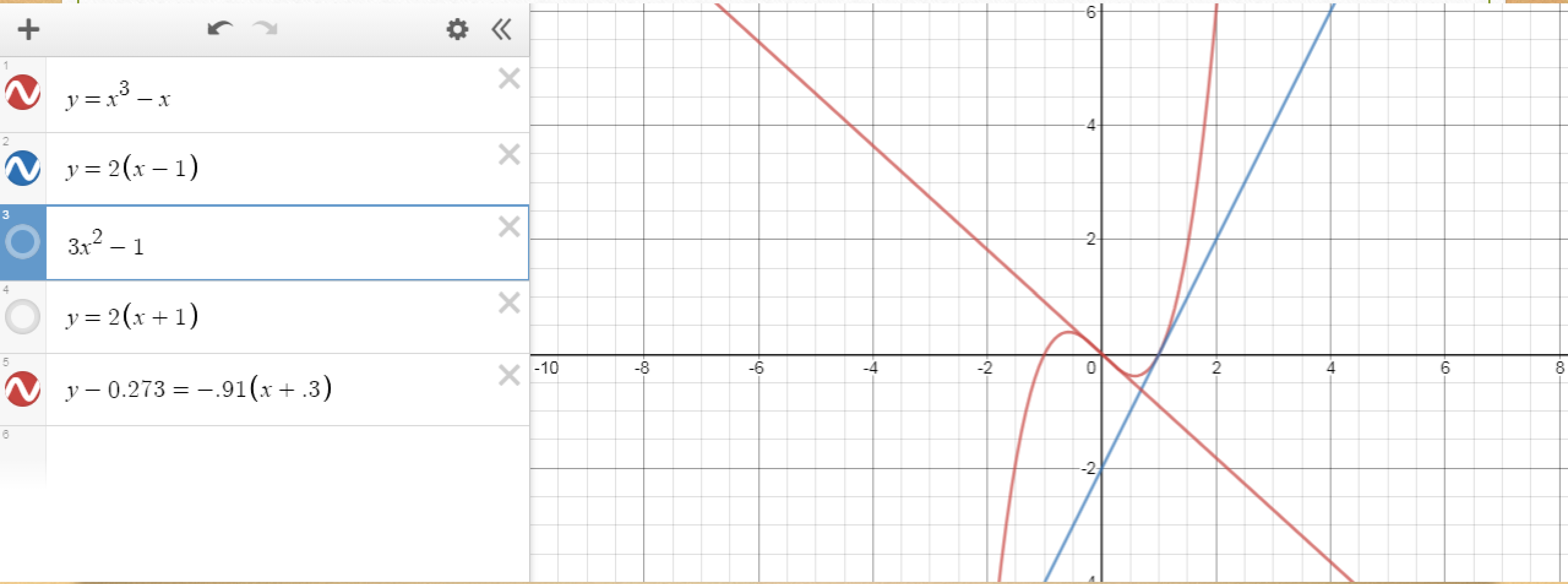


▲ Figure 2.2.3

- *f'(x)* is positive where the tangent line has positive slope,
- *f'(x)* is negative where the tangent line has negative slope,
- *f'(x)* is zero where the tangent line is horizontal.


$y = x^3 - x$
$y = 2(x - 1)$
$3x^2 - 1$
$y = 2(x + 1)$
$y - 0.273 = -.91(x + .3)$
(1, 0)

$$f(x) = x^3 - x$$
$$f'(x) = 3x^2 - 1$$



1 $y = x^3 - x$
2 $y = 2(x - 1)$
3 $3x^2 - 1$
4 $y = 2(x + 1)$
5 $y - 0.273 = -.91(x + .3)$
6
-10 -8 -6 -4 -2 0 2 4 6 8
6 4 2 -2

DIFFERENTIABILITY

2.2.2 DEFINITION A function f is said to be ***differentiable at*** x_0 if the limit

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \tag{5}$$

exists. If f is differentiable at each point of the open interval (a, b), then we say that it is ***differentiable on*** (a, b), and similarly for open intervals of the form $(a, +\infty)$, $(-\infty, b)$, and $(-\infty, +\infty)$. In the last case we say that f is ***differentiable everywhere***.

THE RELATIONSHIP BETWEEN DIFFERENTIABILITY AND CONTINUITY

2.2.3 THEOREM *If a function f is differentiable at x_0, then f is continuous at x_0.*

2.6.1 THEOREM (*The Chain Rule*) *If g is differentiable at x and f is differentiable at $g(x)$, then the composition $f \circ g$ is differentiable at x. Moreover, if*

$$y = f(g(x)) \quad \textit{and} \quad u = g(x)$$

then $y = f(u)$ and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \tag{1}$$

▶ **Example 1** Find dy/dx if $y = \cos(x^3)$.

AN ALTERNATIVE VERSION OF THE CHAIN RULE

$$\frac{d}{dx}[f(g(x))] = (f \circ g)'(x) = f'(g(x))g'(x)$$

▶ Example 4

$$\frac{d}{dx}[\tan^2 x] =$$

Exercise 2.6

FOCUS ON CONCEPTS

5. Given the following table of values, find the indicated derivatives in parts (a) and (b).

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
3	5	–2	5	7
5	3	–1	12	4

(a) $F'(3)$, where $F(x) = f(g(x))$
(b) $G'(3)$, where $G(x) = g(f(x))$

6. Given the following table of values, find the indicated derivatives in parts (a) and (b).

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–1	2	3	2	–3
2	0	4	1	–5

(a) $F'(-1)$, where $F(x) = f(g(x))$
(b) $G'(-1)$, where $G(x) = g(f(x))$

[

Implicit Differentiation:

▶ **Example 2** Use implicit differentiation to find dy/dx if $5y^2 + \sin y = x^2$.

Assignment No 1

Exercise	Questions
2.3	12-18
2.4	16-24
2.5	1-18
2.6	21-26,35-40
3.1	3-18
5.2	43-46
5.3	1-12, 15-30

3.5 LOCAL LINEAR APPROXIMATION; DIFFERENTIALS

for values of x near x_0 we can approximate values of $f(x)$ by

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

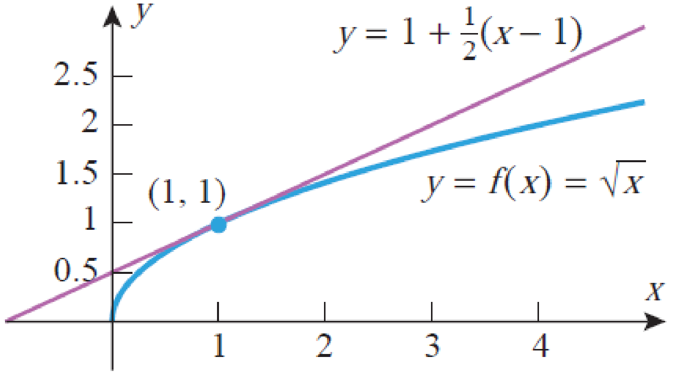
This is called the ***local linear approximation*** of f at x_0. This formula can also be expressed in terms of the increment $\Delta x = x - x_0$ as

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x \tag{2}$$

▶ Example 1

(a) Find the local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 1$.

(b) Use the local linear approximation obtained in part (a) to approximate $\sqrt{1.1}$, and compare your approximation to the result produced directly by a calculating utility.

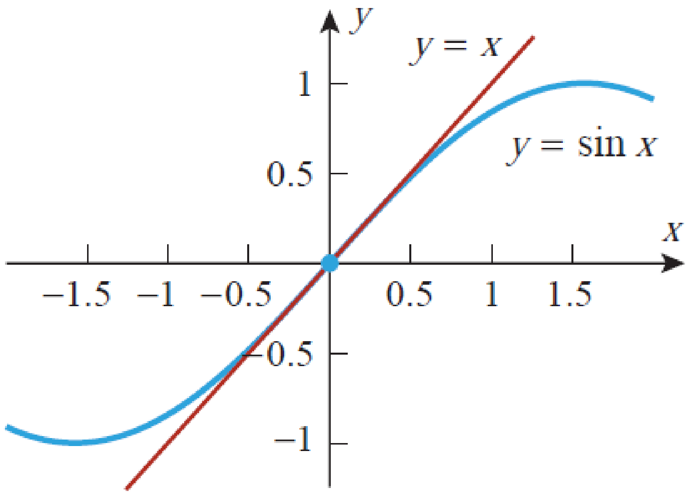


▲ Figure 3.5.2

▶ Example 2

(a) Find the local linear approximation of $f(x) = \sin x$ at $x_0 = 0$.

(b) Use the local linear approximation obtained in part (a) to approximate $\sin 2^\circ$, and compare your approximation to the result produced directly by your calculating device.



▲ **Figure 3.5.3**

11–16 Confirm that the stated formula is the local linear approximation of f at $x_0 = 1$, where $\Delta x = x - 1$.

11. $f(x) = x^4$; $(1 + \Delta x)^4 \approx 1 + 4\Delta x$

12. $f(x) = \sqrt{x}$; $\sqrt{1 + \Delta x} \approx 1 + \frac{1}{2}\Delta x$

13. $f(x) = \dfrac{1}{2 + x}$; $\dfrac{1}{3 + \Delta x} \approx \dfrac{1}{3} - \dfrac{1}{9}\Delta x$

14. $f(x) = (4 + x)^3$; $(5 + \Delta x)^3 \approx 125 + 75\Delta x$

15. $\tan^{-1} x$; $\tan^{-1}(1 + \Delta x) \approx \dfrac{\pi}{4} + \dfrac{1}{2}\Delta x$

16. $\sin^{-1}\left(\dfrac{x}{2}\right)$; $\sin^{-1}\left(\dfrac{1}{2} + \dfrac{1}{2}\Delta x\right) \approx \dfrac{\pi}{6} + \dfrac{1}{\sqrt{3}}\Delta x$

Practice Questions:
Pg no 217 Exercise 3.5 Q no 1-18

3.6 L'HÔPITAL'S RULE; INDETERMINATE FORMS

INDETERMINATE FORMS OF TYPE 0/0

7. $\lim_{x \to 0} \dfrac{e^x - 1}{\sin x}$

INDETERMINATE FORMS OF TYPE ∞/∞

13. $\lim\limits_{x \to +\infty} \dfrac{\ln x}{x}$

INDETERMINATE FORMS OF TYPE $0 \cdot \infty$

Example 4 Evaluate

(a) $\lim_{x \to 0^+} x \ln x$ (b) $\lim_{x \to \pi/4} (1 - \tan x) \sec 2x$

INDETERMINATE FORMS OF TYPE $\infty - \infty$

34. $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{\cos 3x}{x^2} \right)$

INDETERMINATE FORMS OF TYPE 0^0, ∞^0, 1^∞

41. $\lim_{x \to 0^+} \left[-\frac{1}{\ln x}\right]^x$

42. $\lim_{x \to +\infty} x^{1/x}$

28. $\lim_{x \to 0} (1 + 2x)^{-3/x}$

Practice Questions:

Pg no 227 Exercise 3.6 Q no 7-45