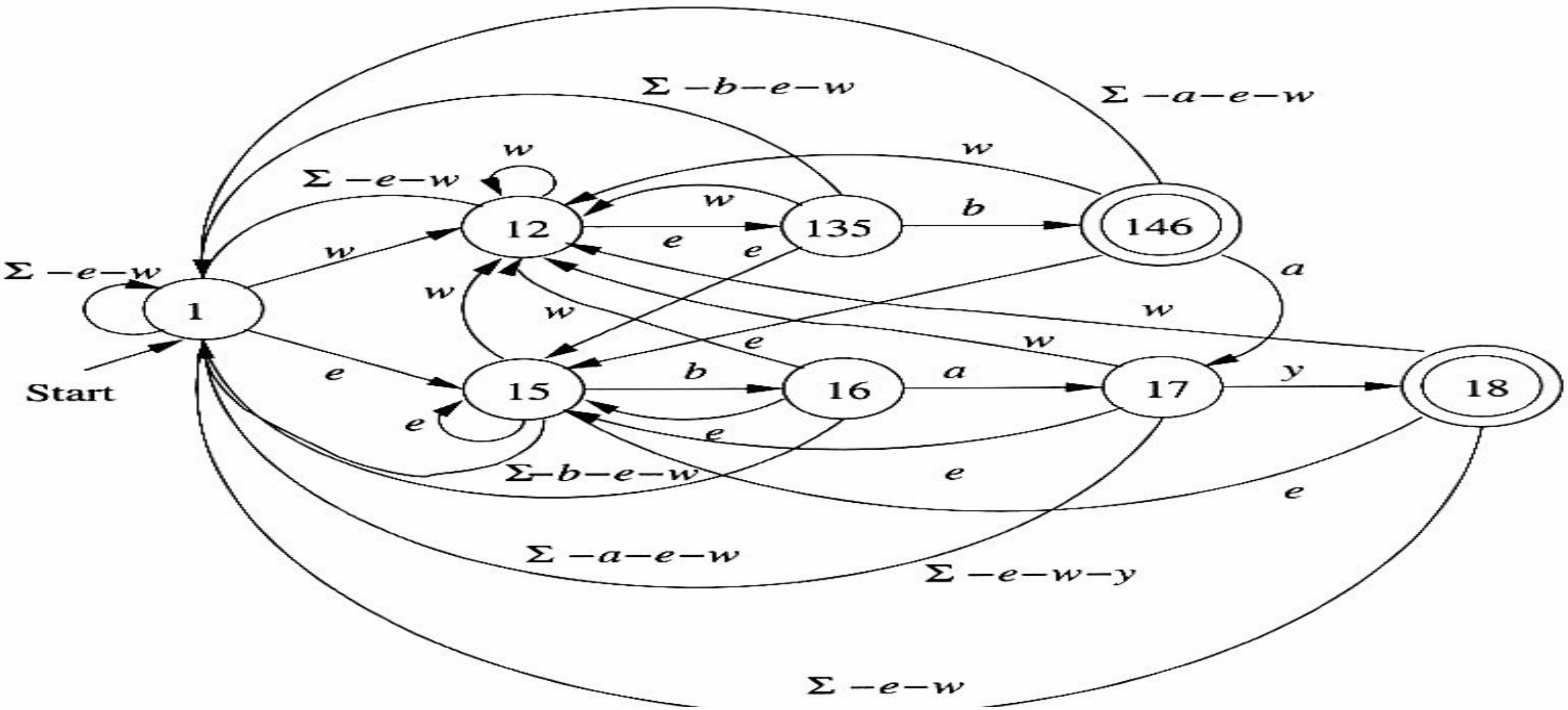
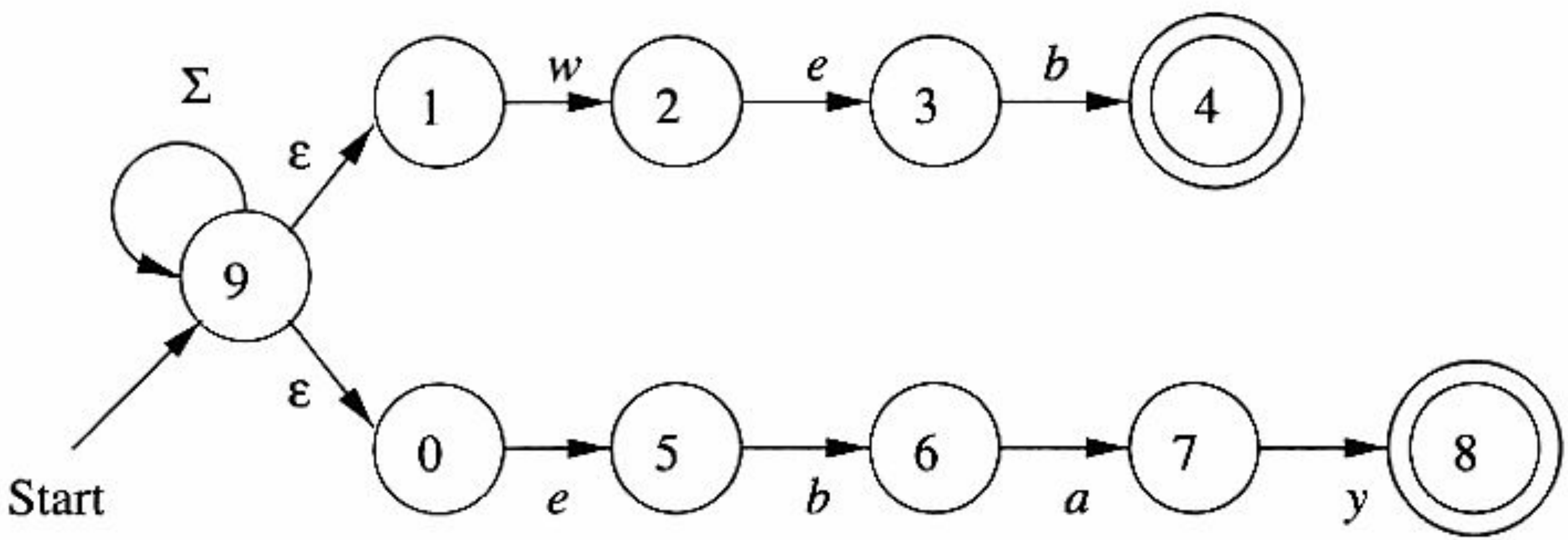
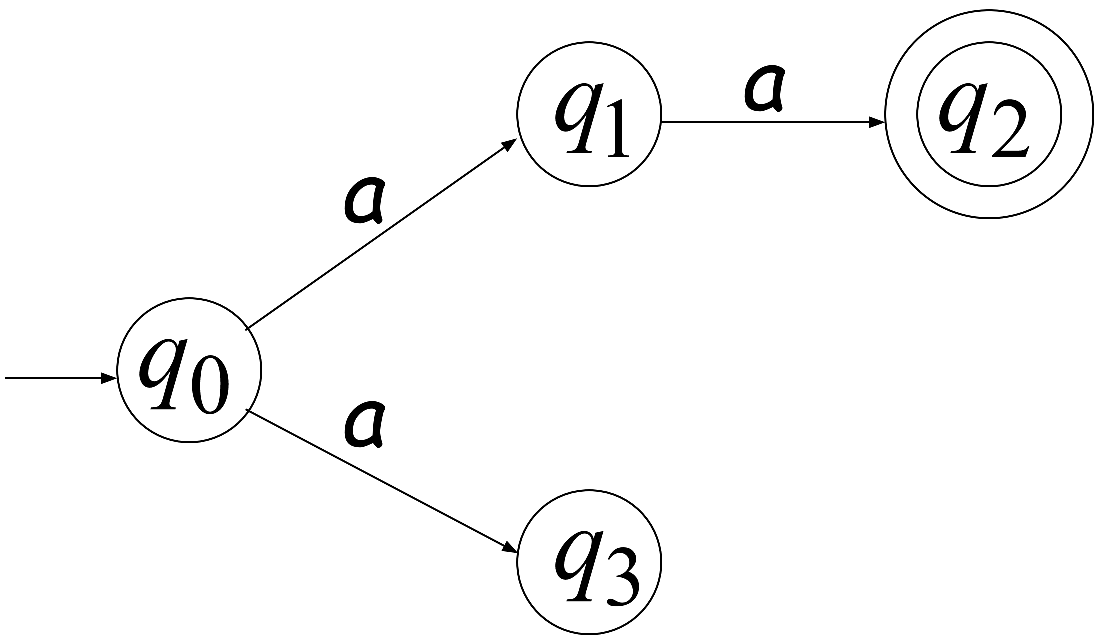

Σ −b−e−w
Σ −a−e−w
w
Σ −e−w
w
b
135
146
12
Σ −e−w
1
Start
15
16
17
18
a
y
Σ−b−e−w
Σ −a−e−w
Σ −e−w−y
Σ −e−w


Σ
Start
ε
ε
1
2
3
4
9
0
5
6
7
8
w
e
b
e
b
a
y

Non-Deterministic Finite Automata

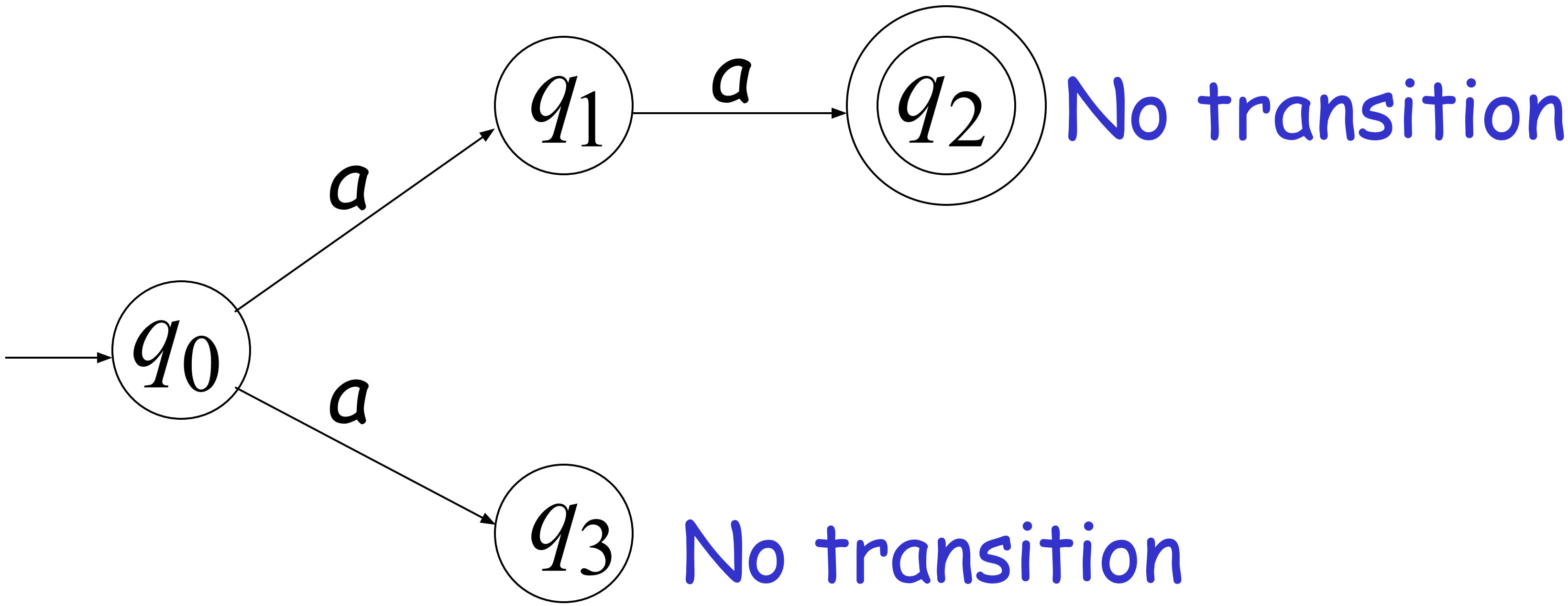
Nondeterministic Finite Accepter (NFA)

Alphabet set = $\{a\}$



Nondeterministic Finite Accepter (NFA)

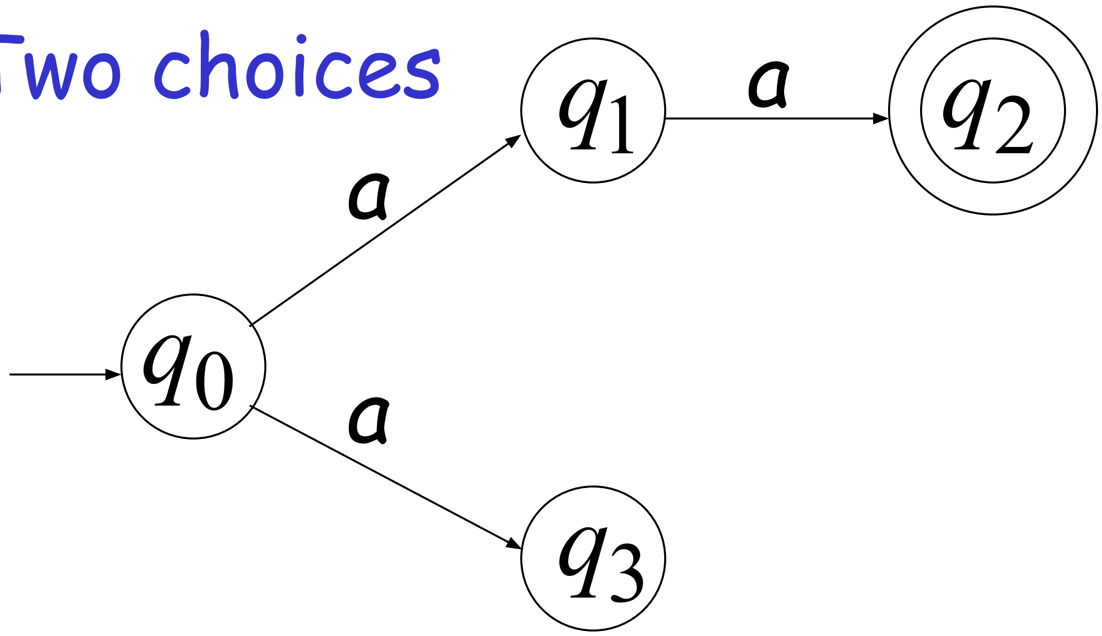
Alphabet set = $\{a\}$



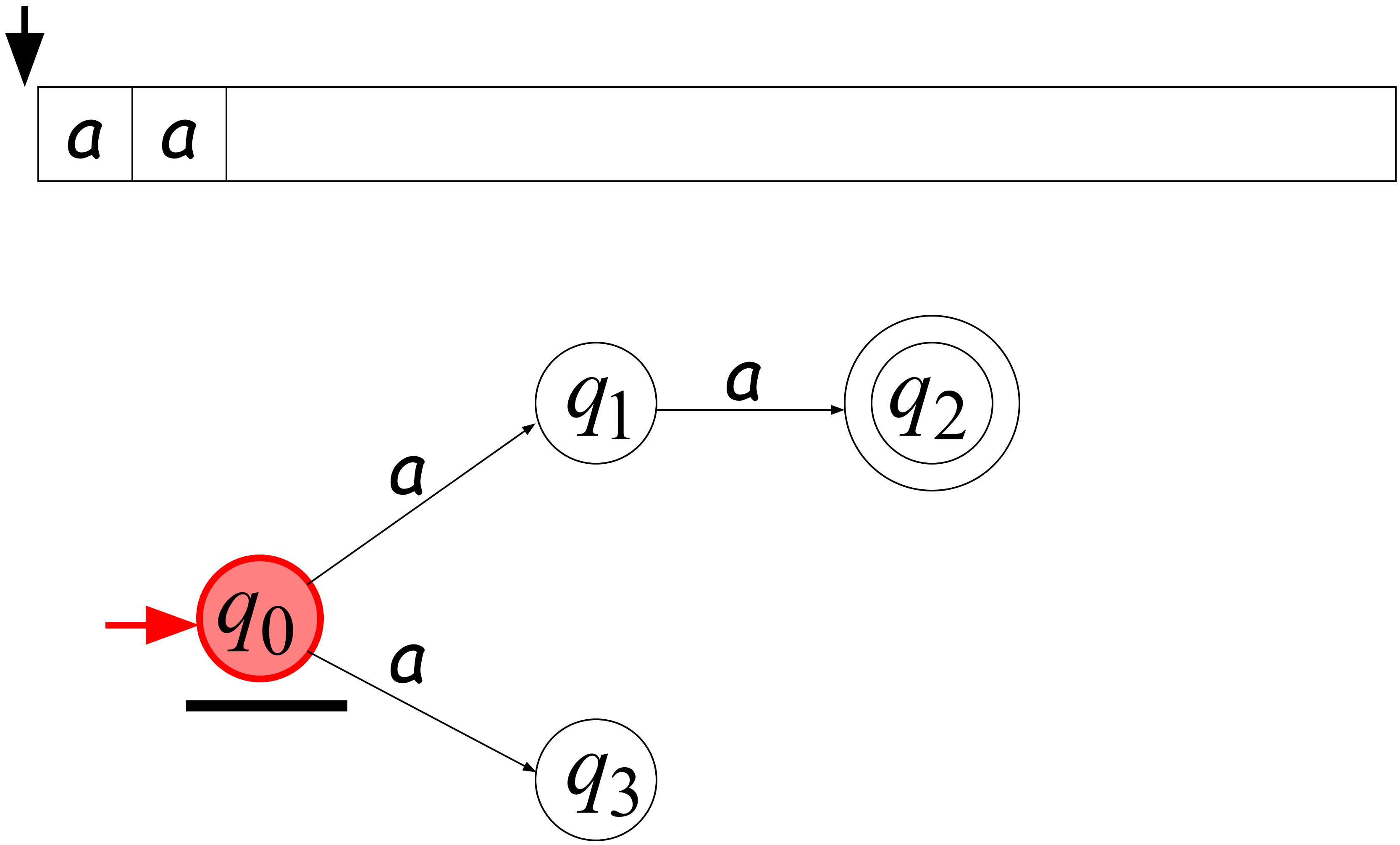
Nondeterministic Finite Accepter (NFA)

Alphabet set = $\{a\}$

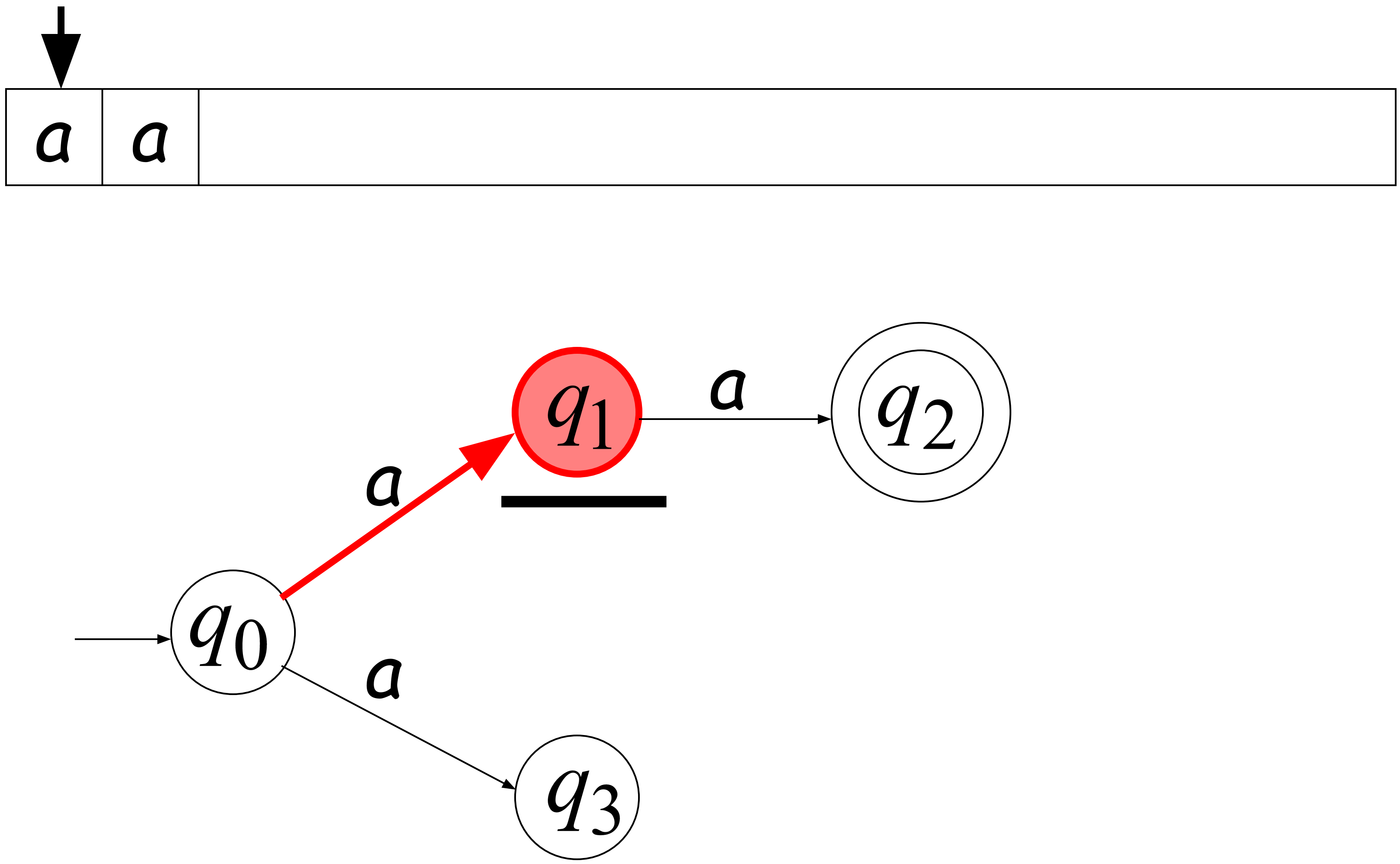
Two choices



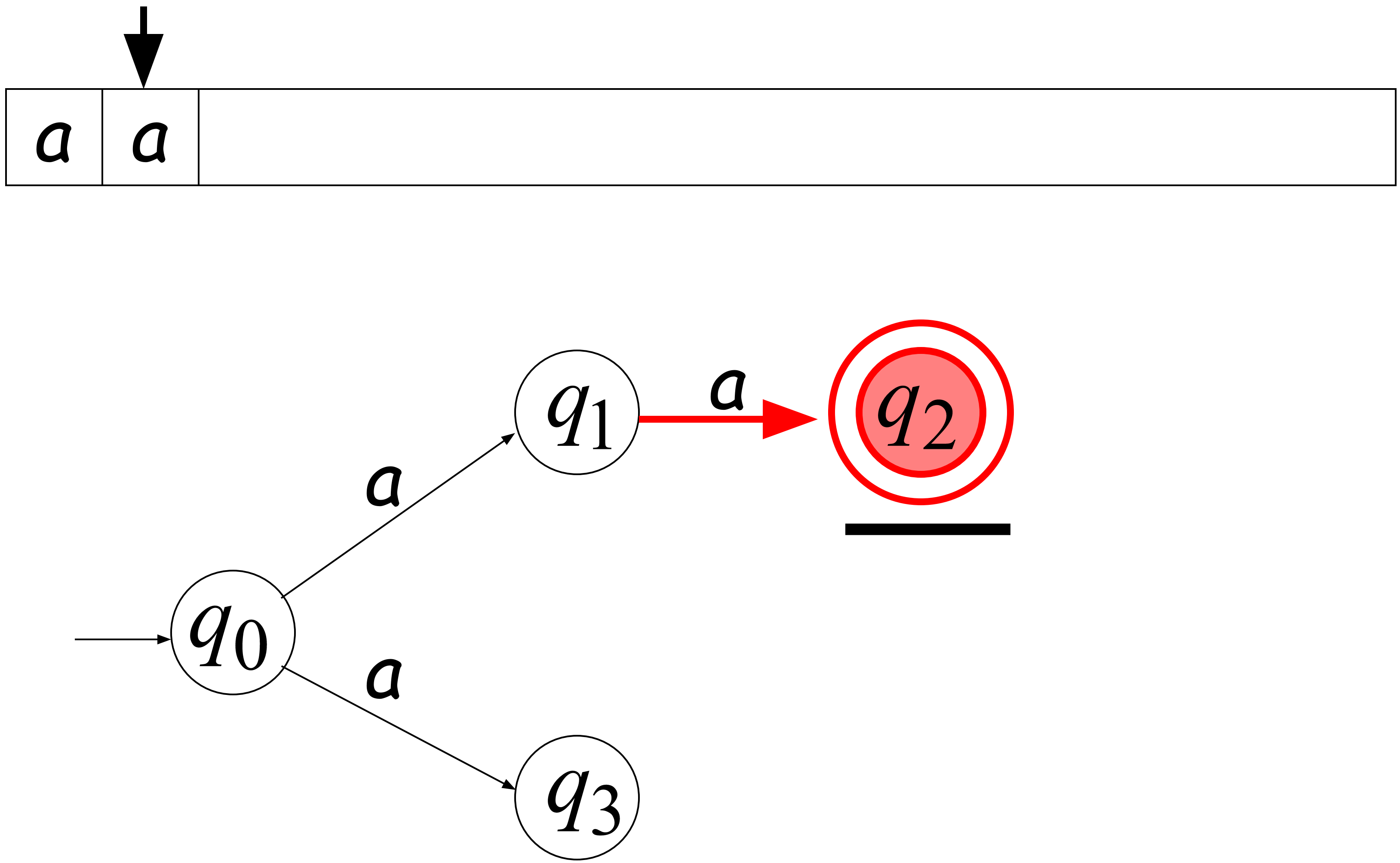
First Choice



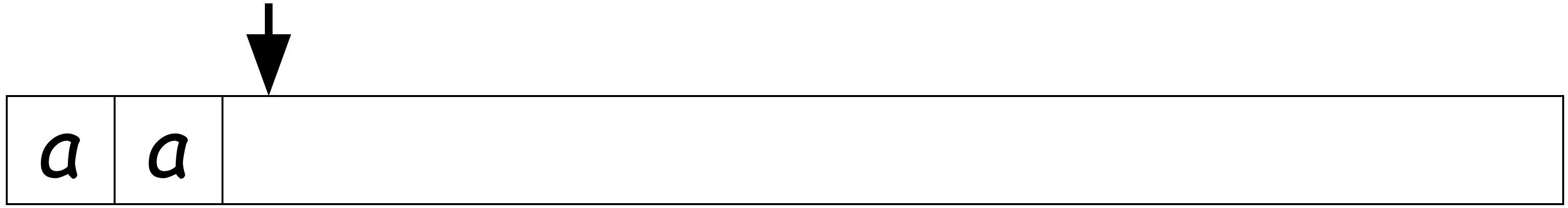
First Choice



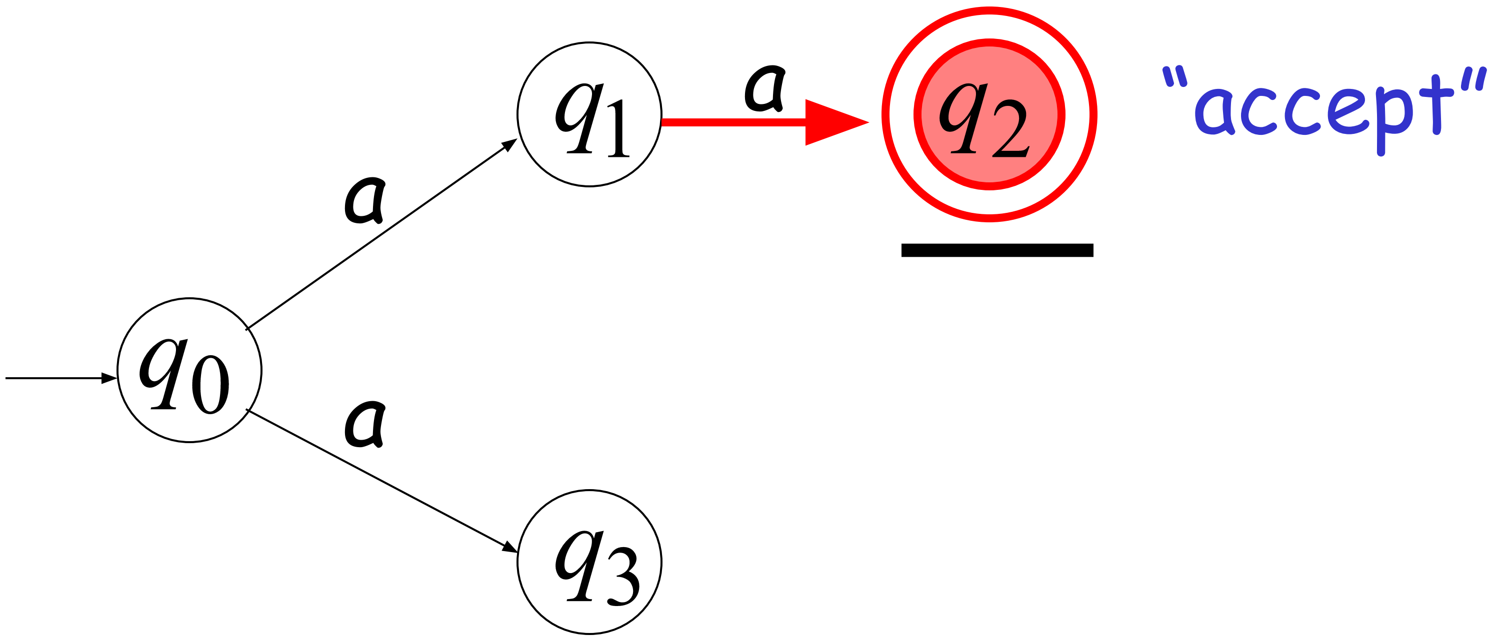
First Choice



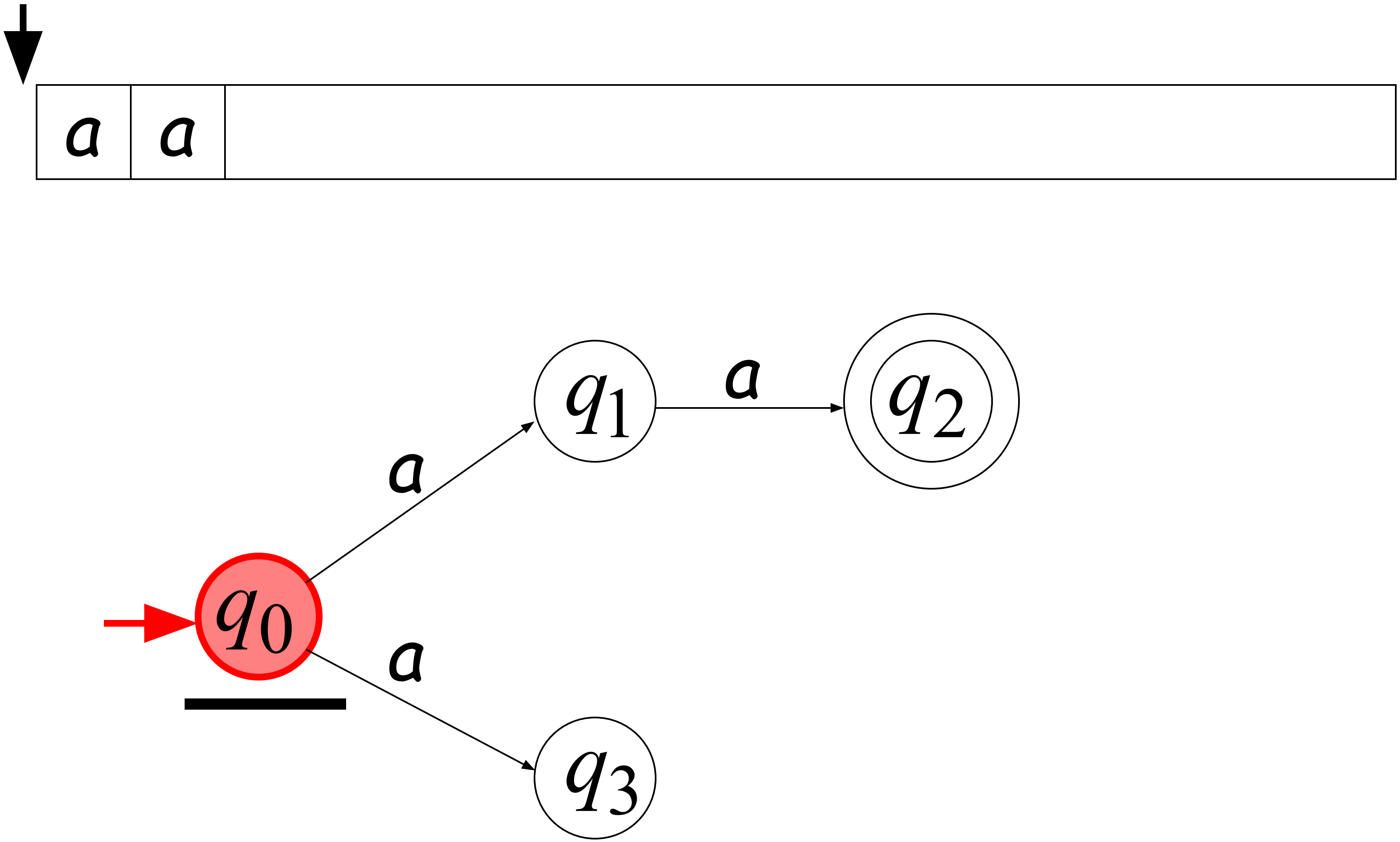
First Choice



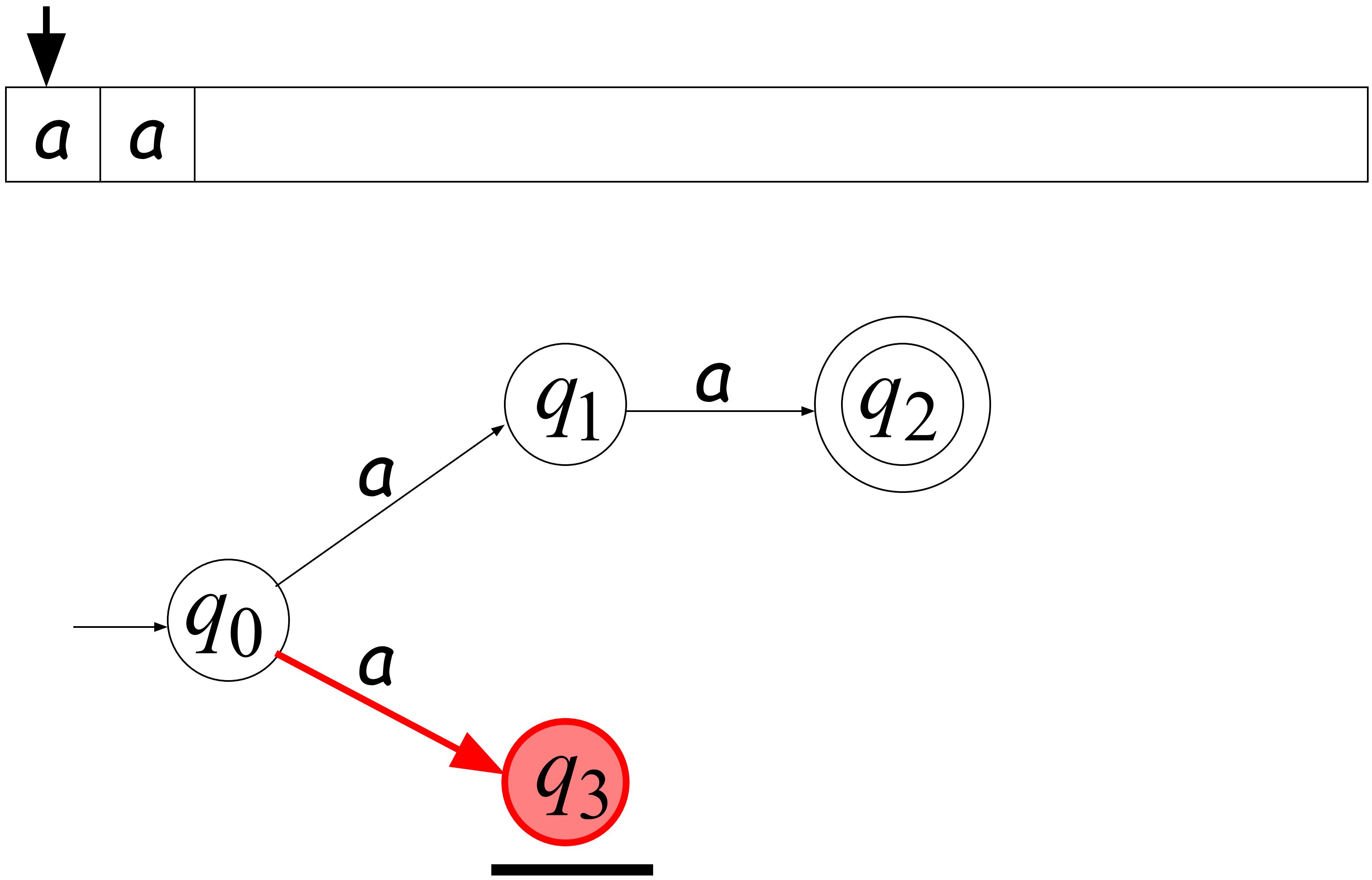
All input is consumed



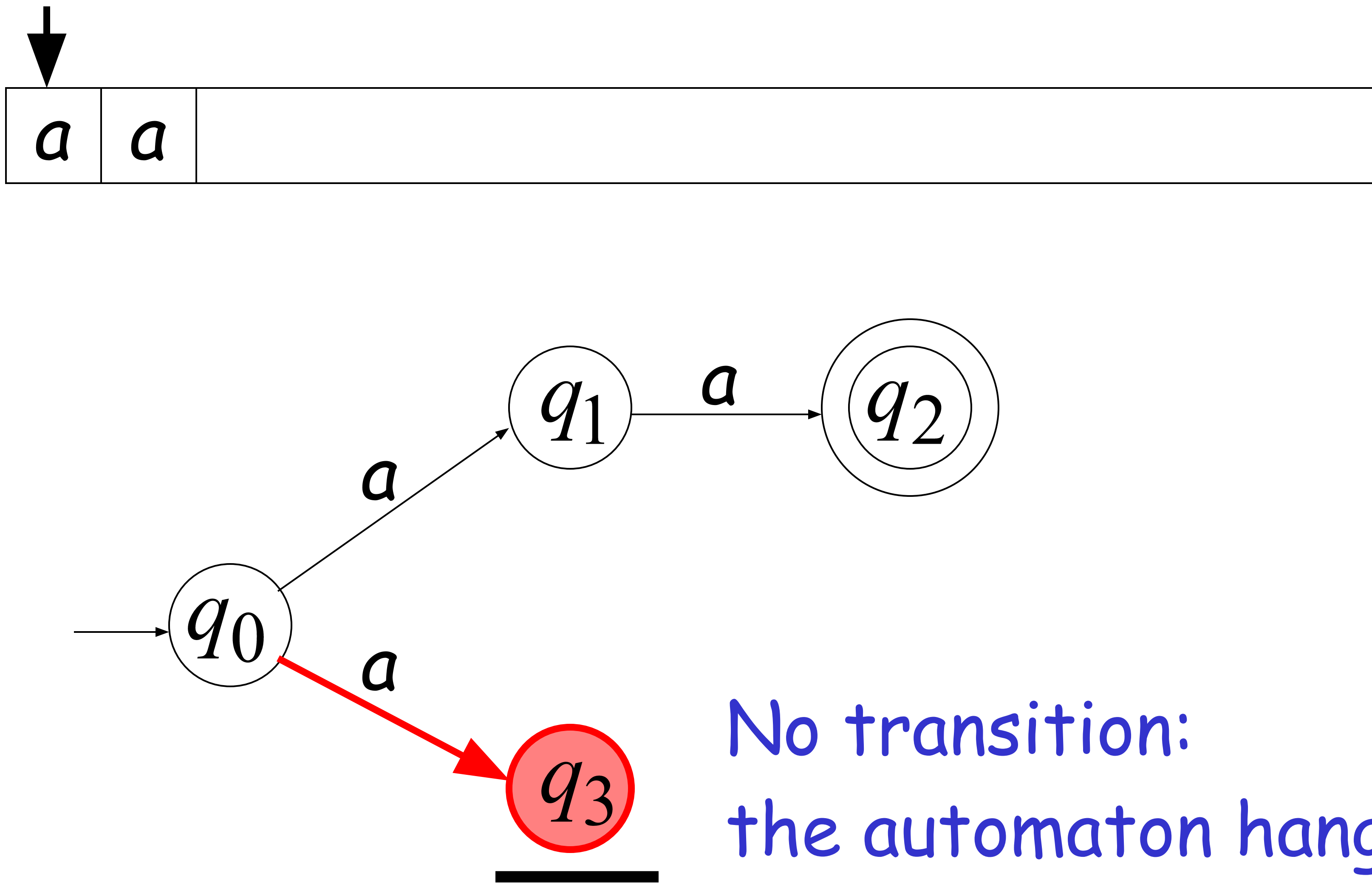
Second Choice



Second Choice

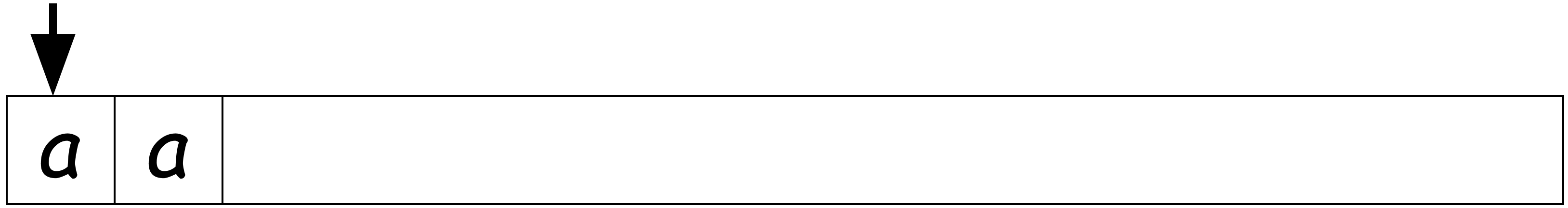


Second Choice

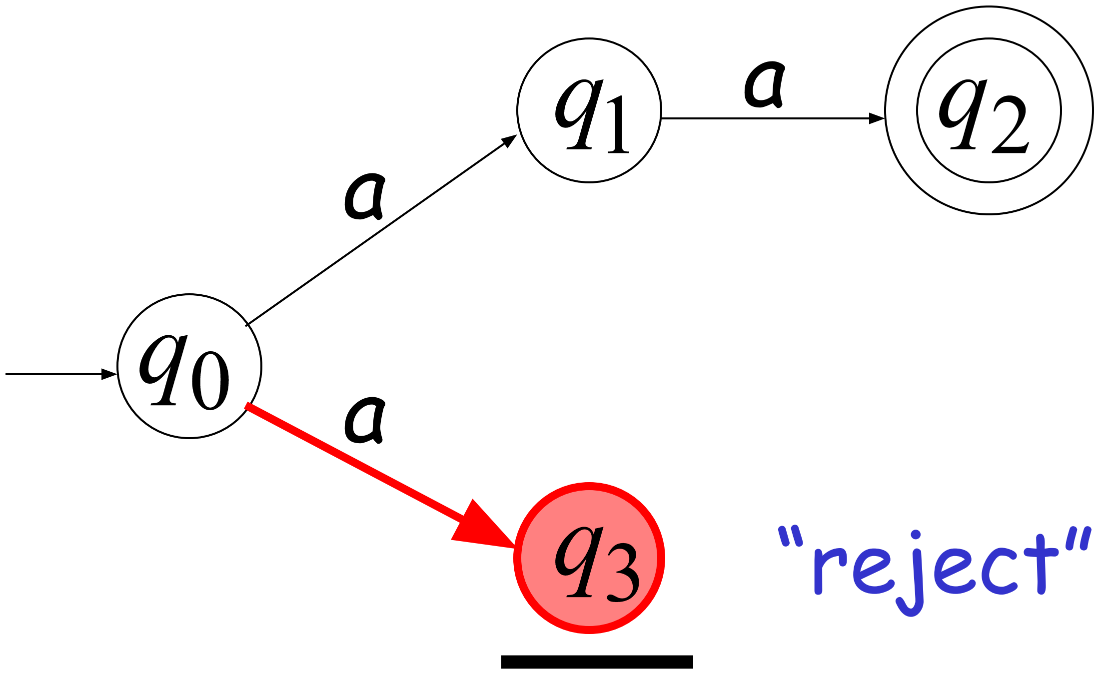


No transition:
the automaton hangs

Second Choice



Input cannot be consumed



"reject"

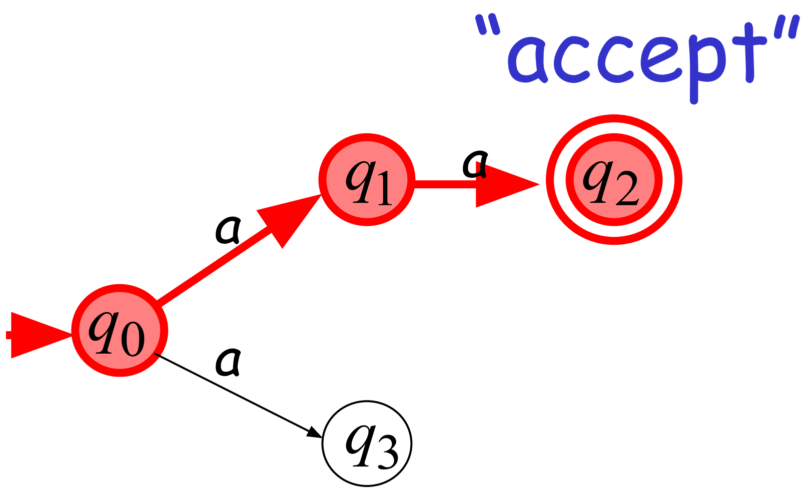
An NFA accepts a string:
when there is a computation of the NFA
that accepts the string

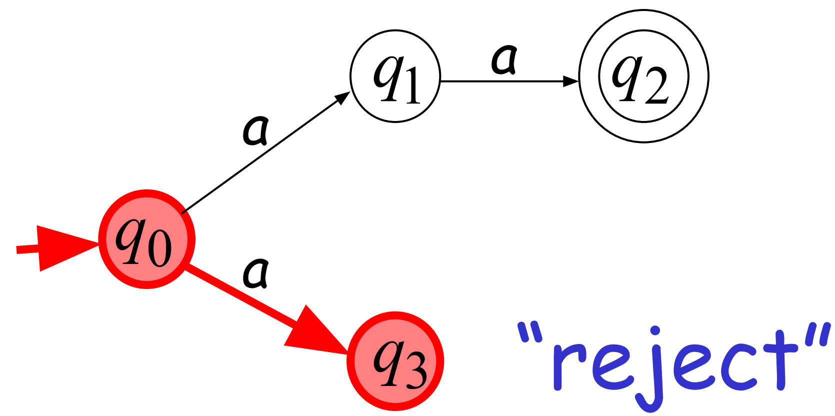
AND

all the input is consumed and the automaton
is in a final state

Example

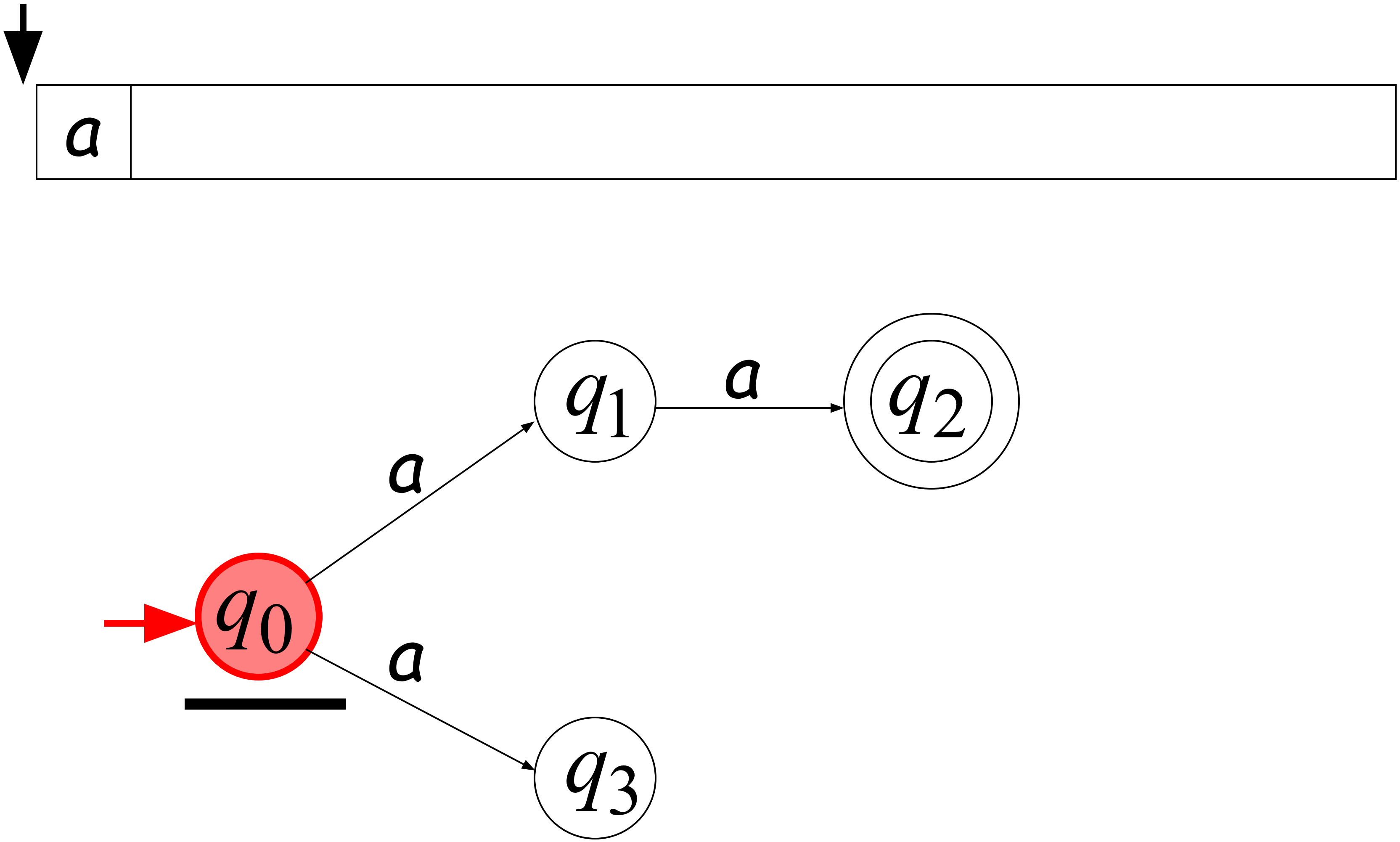
aa is accepted by the NFA:



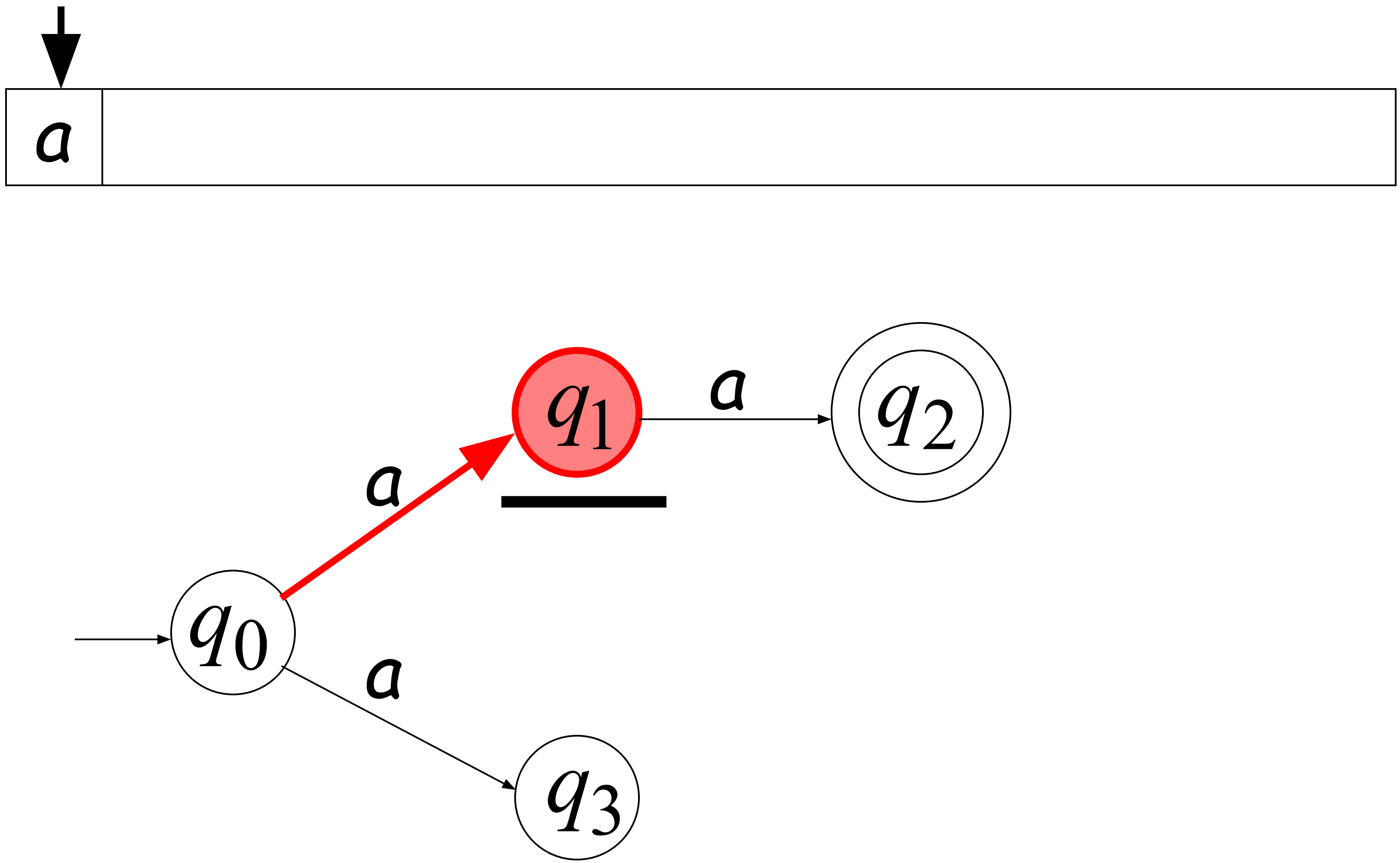


because this computation accepts aa

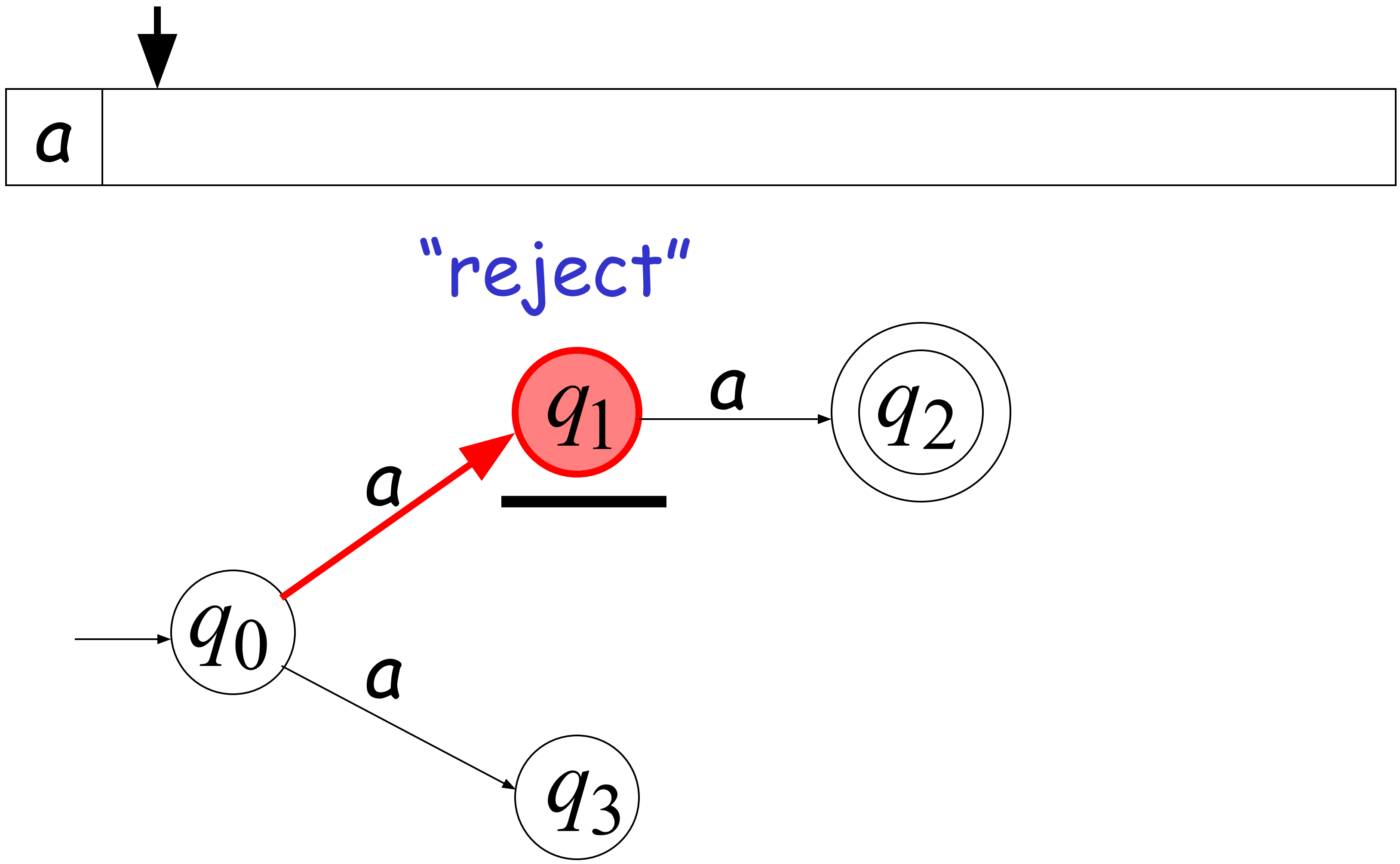
Rejection example



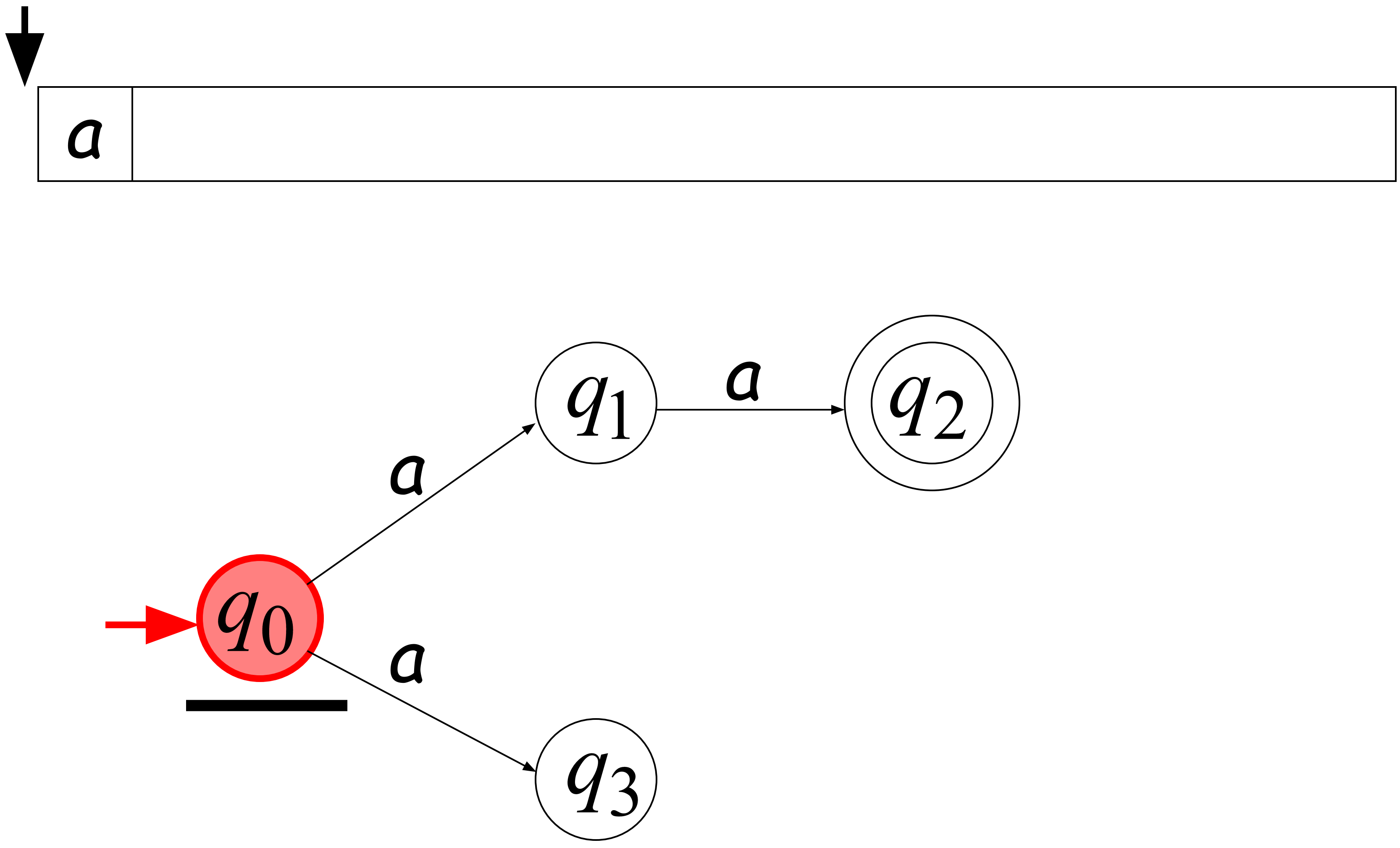
First Choice



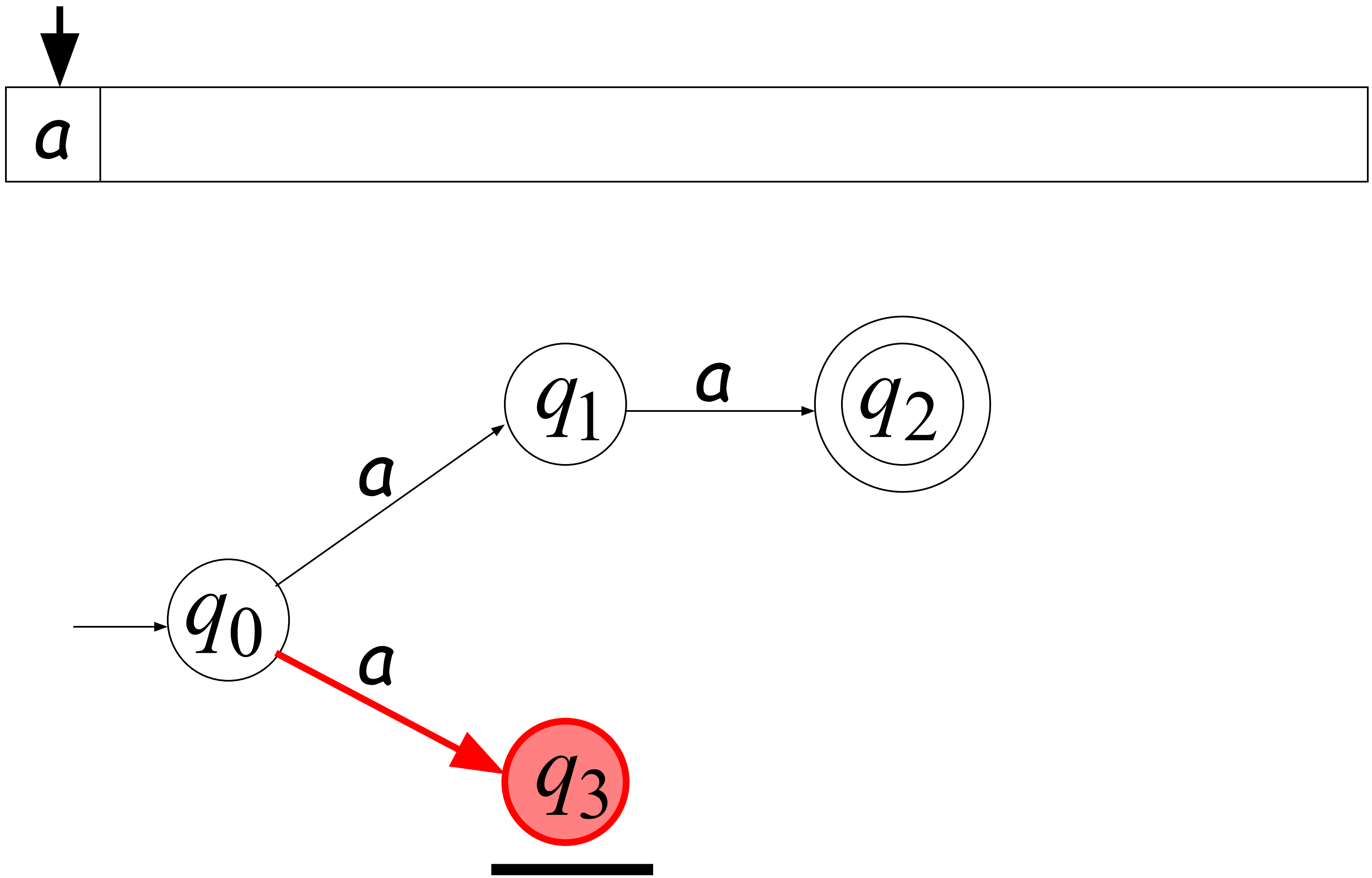
First Choice



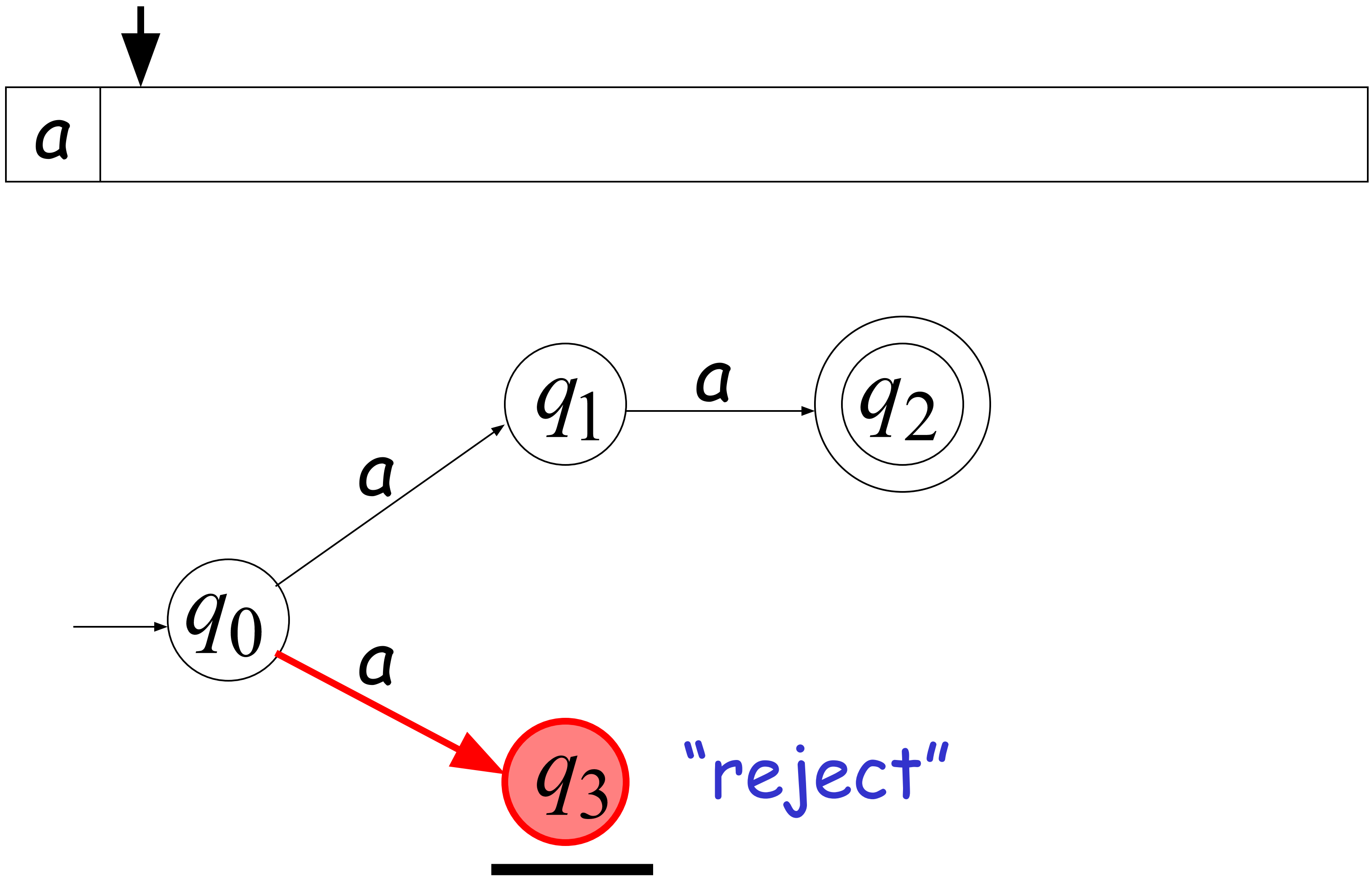
Second Choice



Second Choice



Second Choice



An NFA rejects a string:

when there is no computation of the NFA that accepts the string:

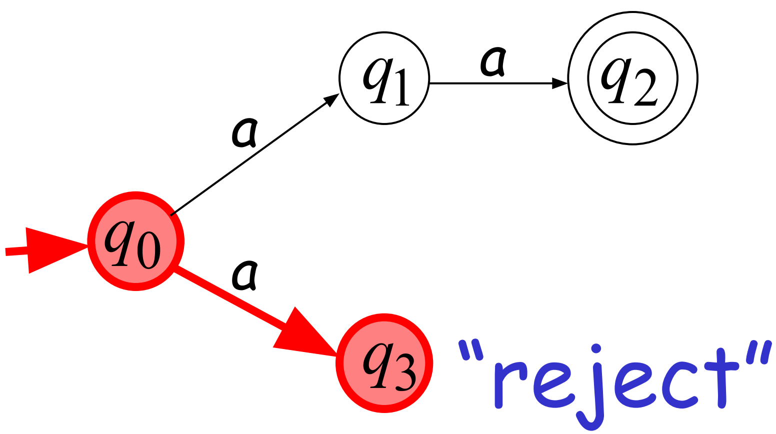
- All the input is consumed and the automaton is in a non final state

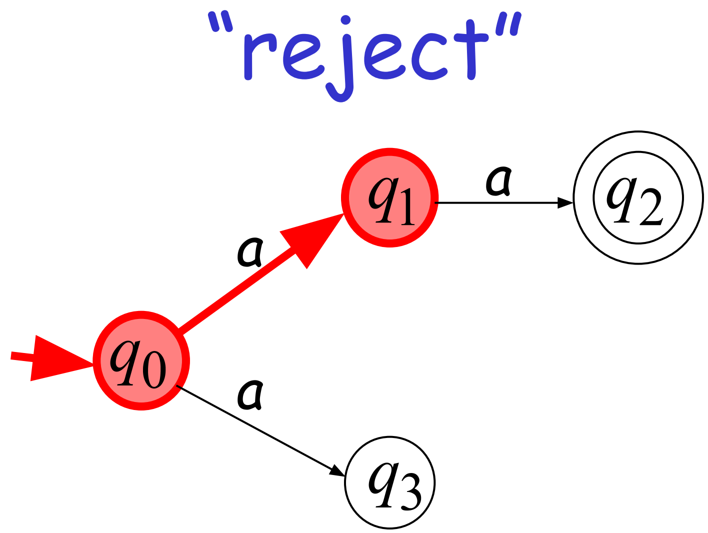
OR

- The input cannot be consumed

Example

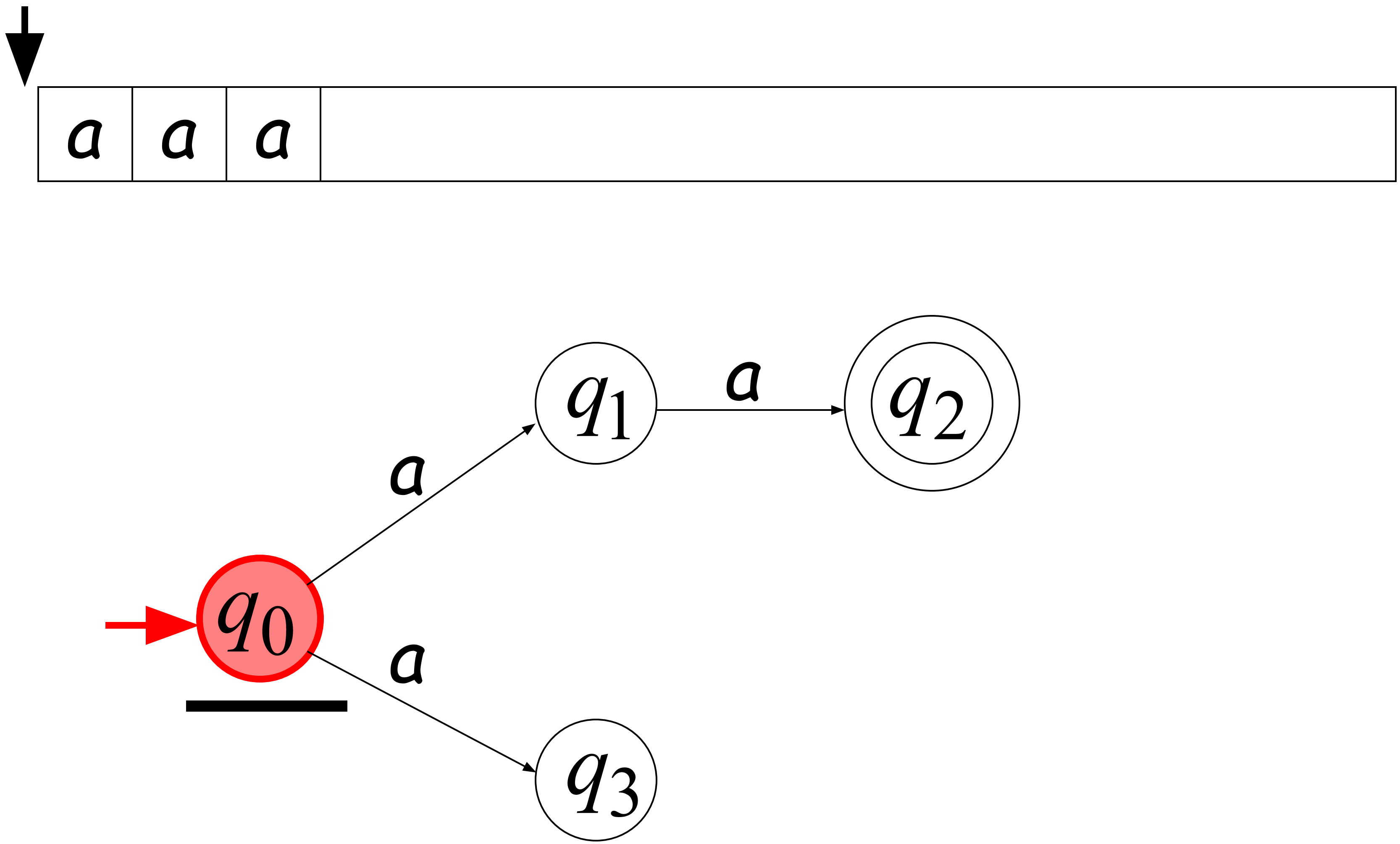
a is rejected by the NFA:



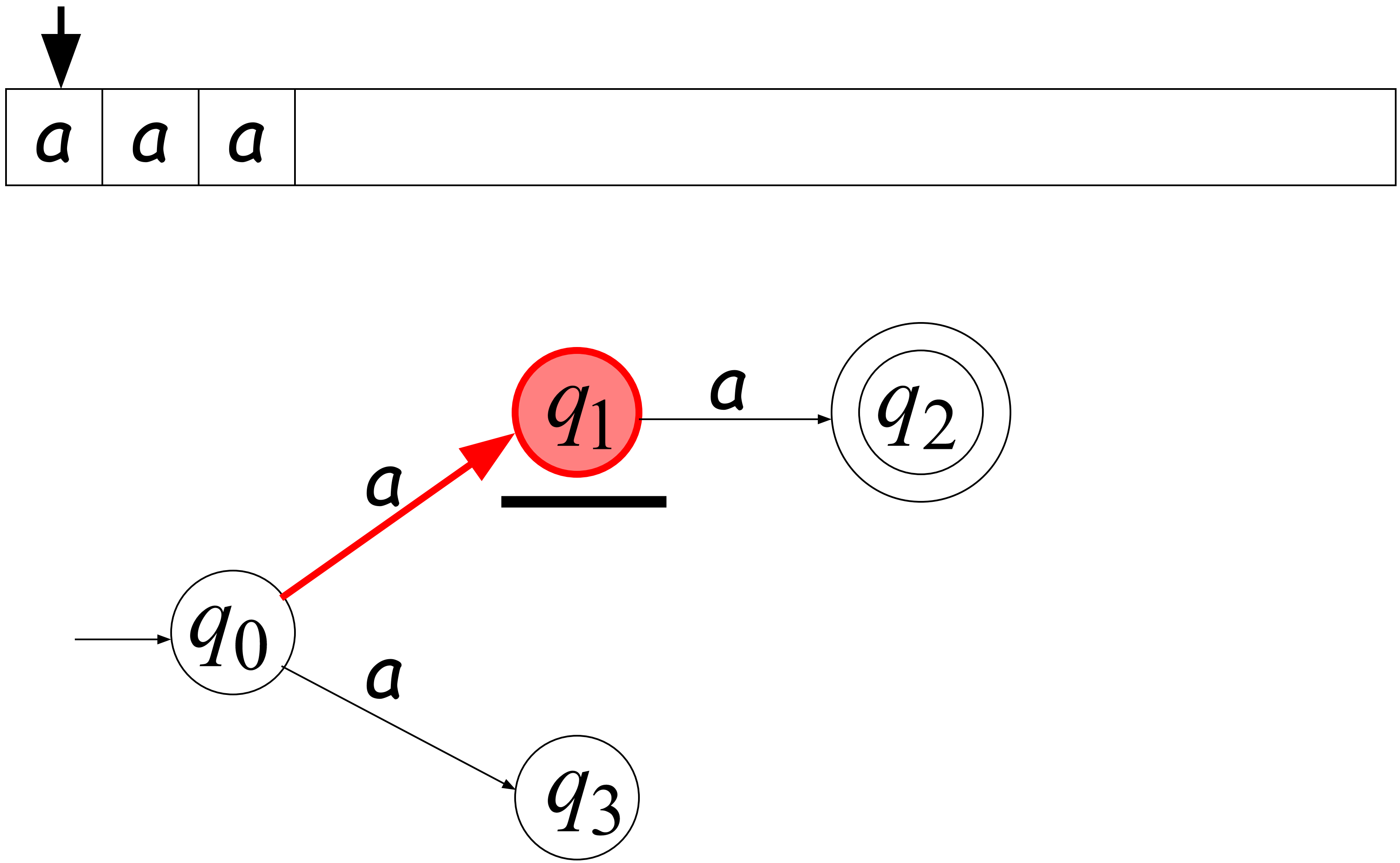


All possible computations lead to rejection

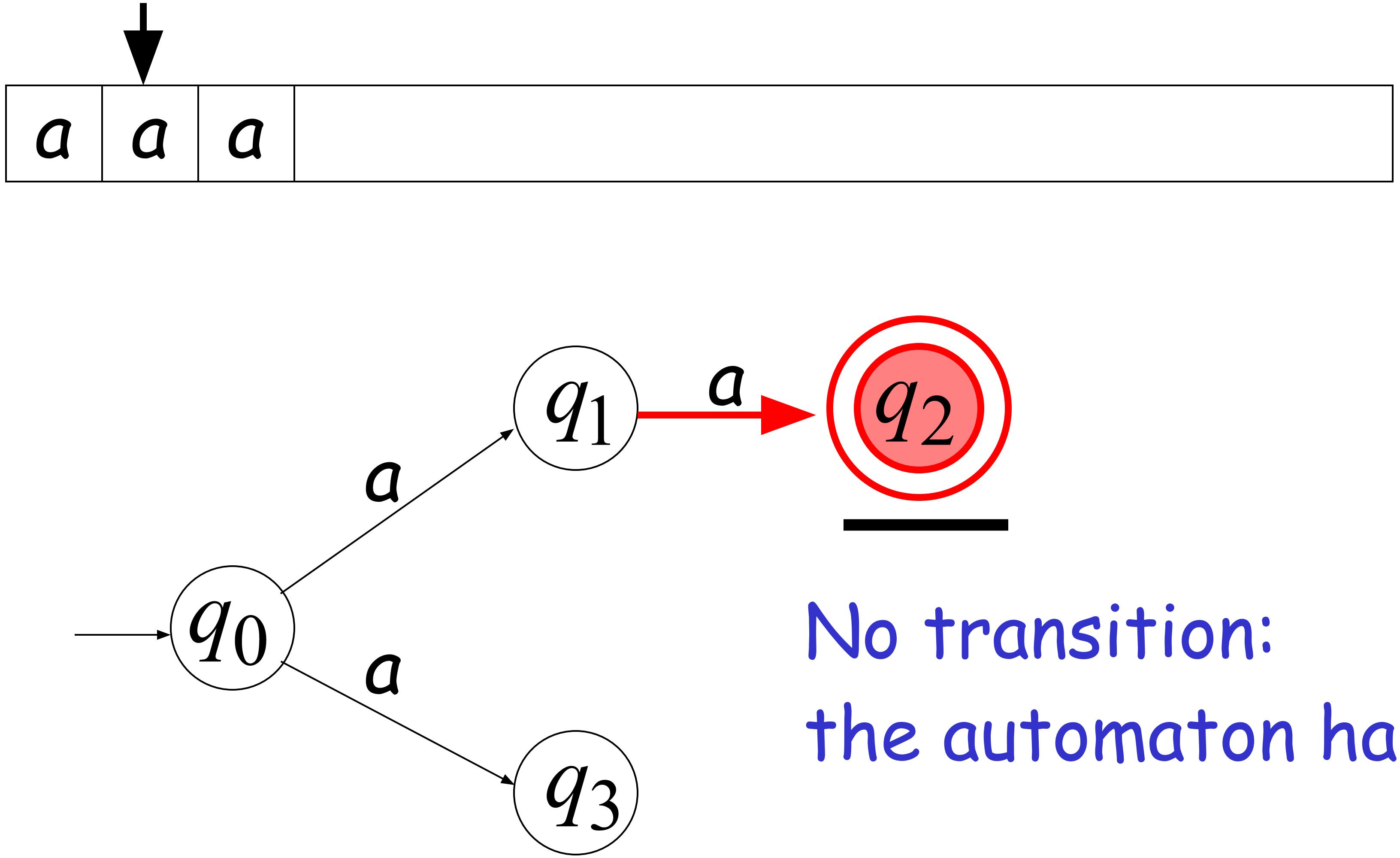
Rejection example



First Choice

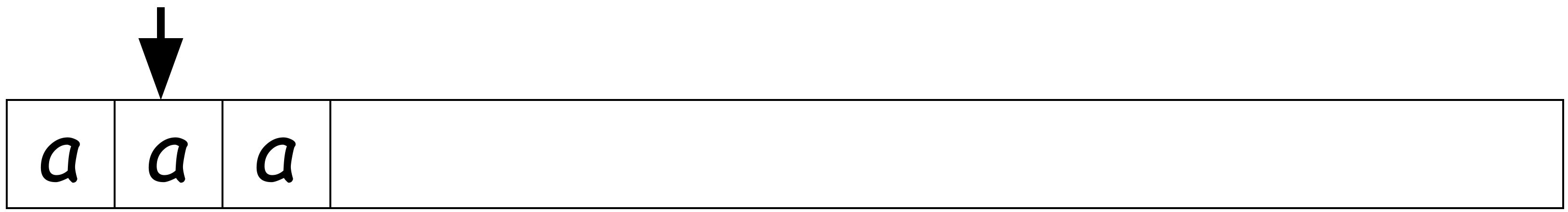


First Choice

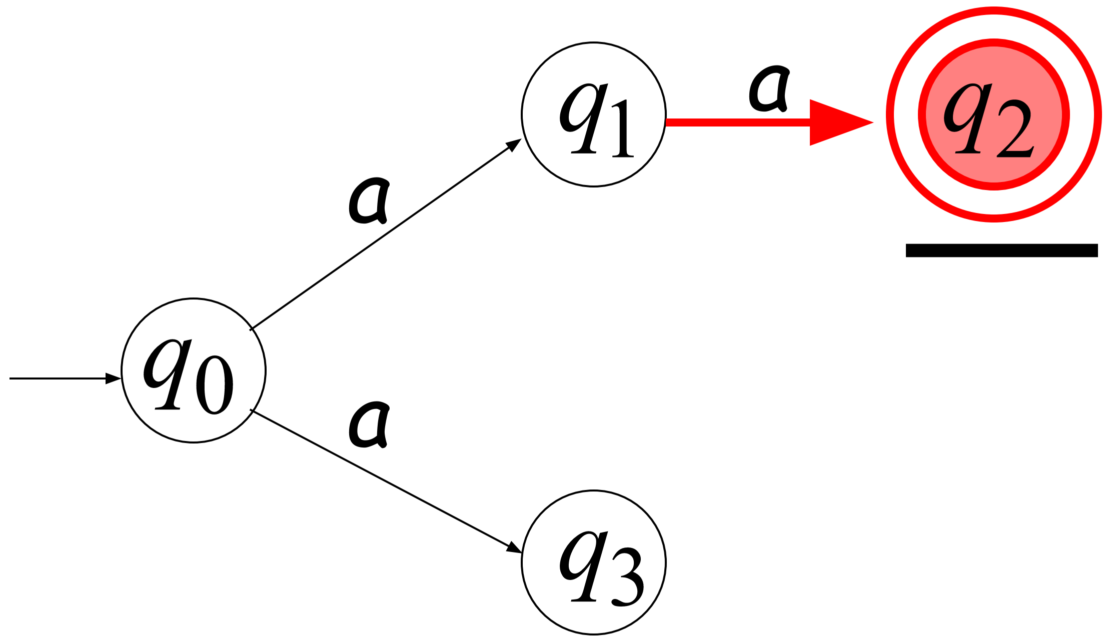


No transition:
the automaton hangs

First Choice

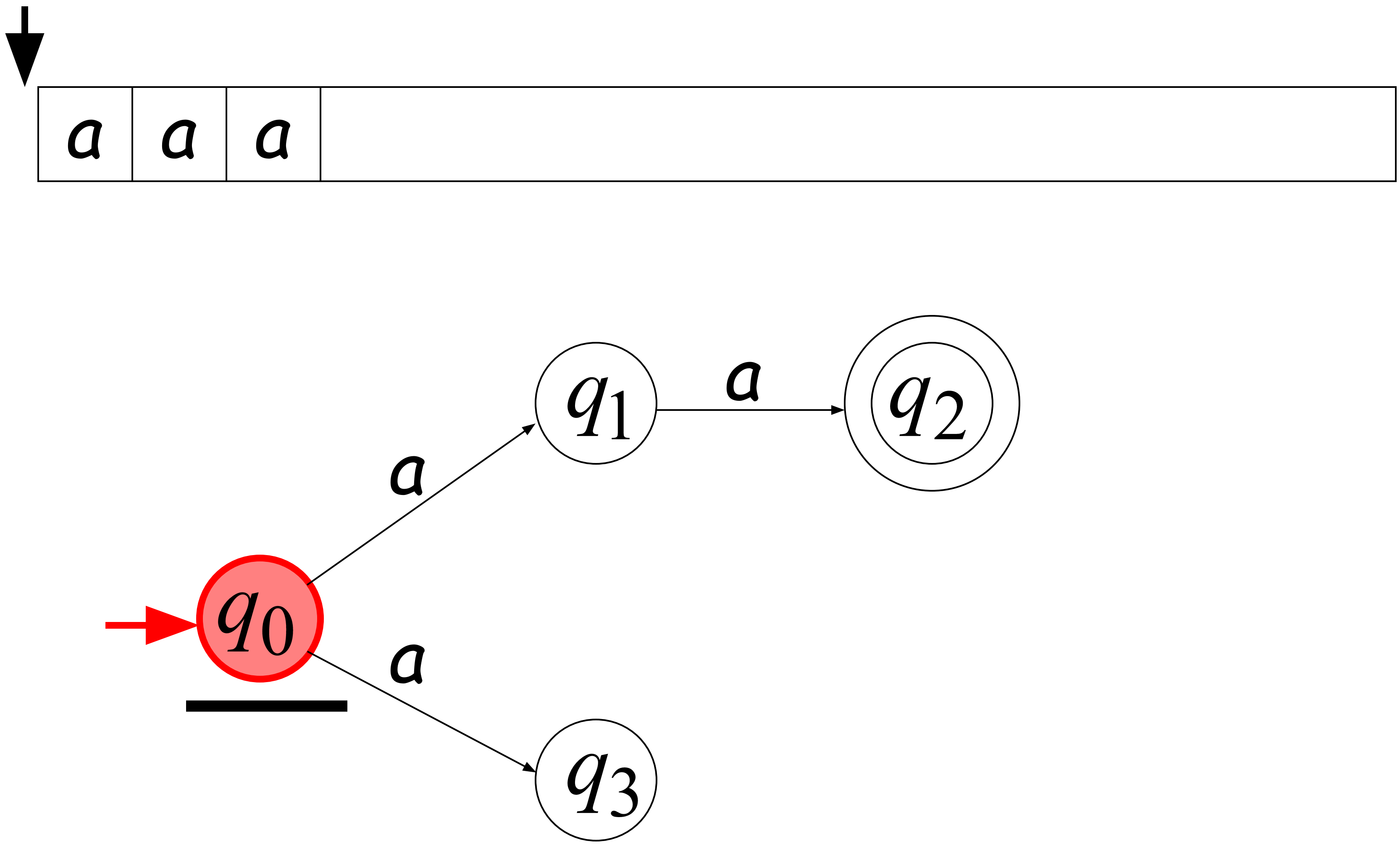


Input cannot be consumed

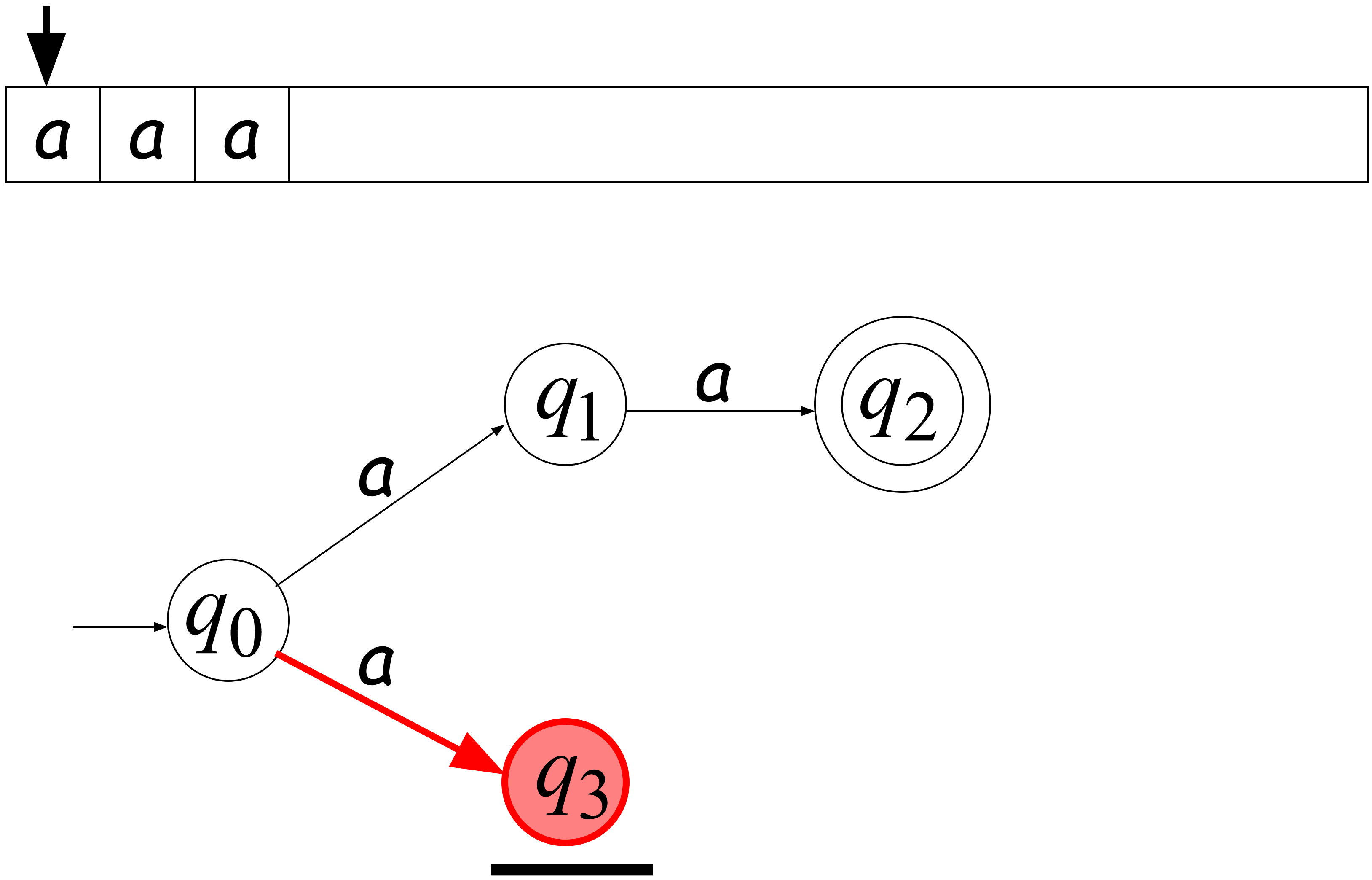


"reject"

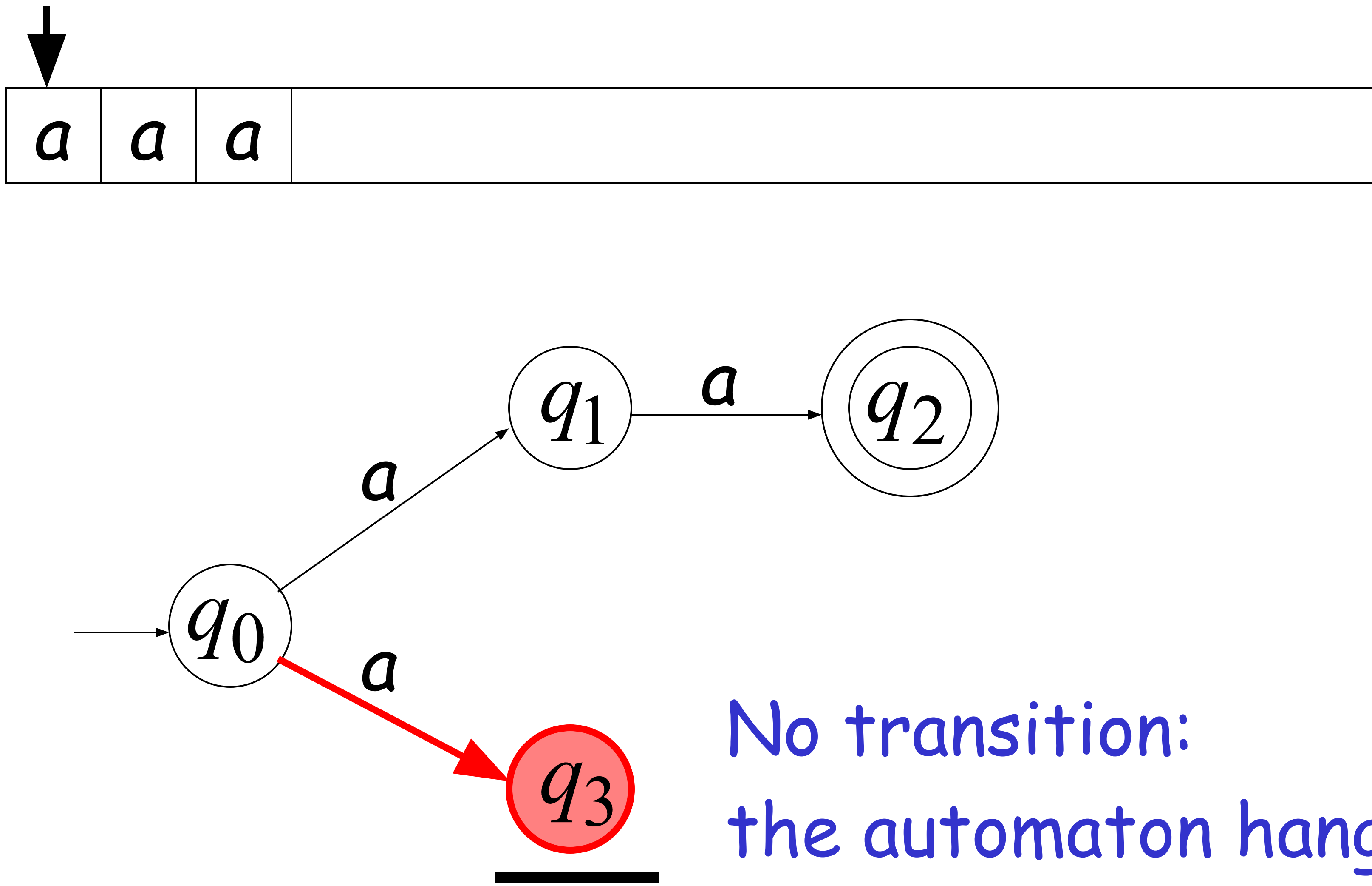
Second Choice



Second Choice

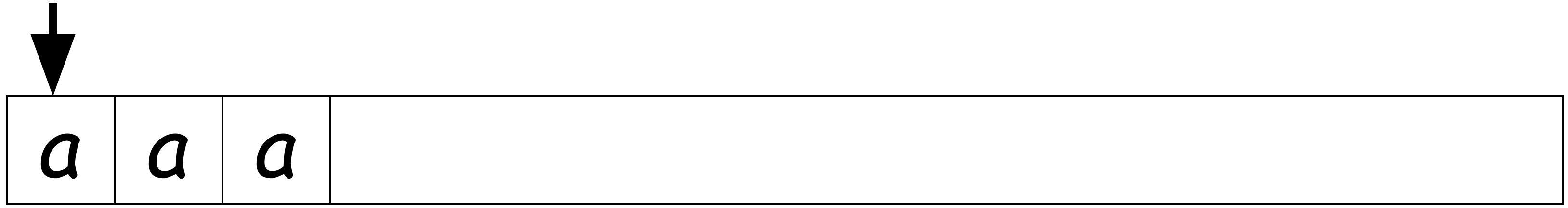


Second Choice

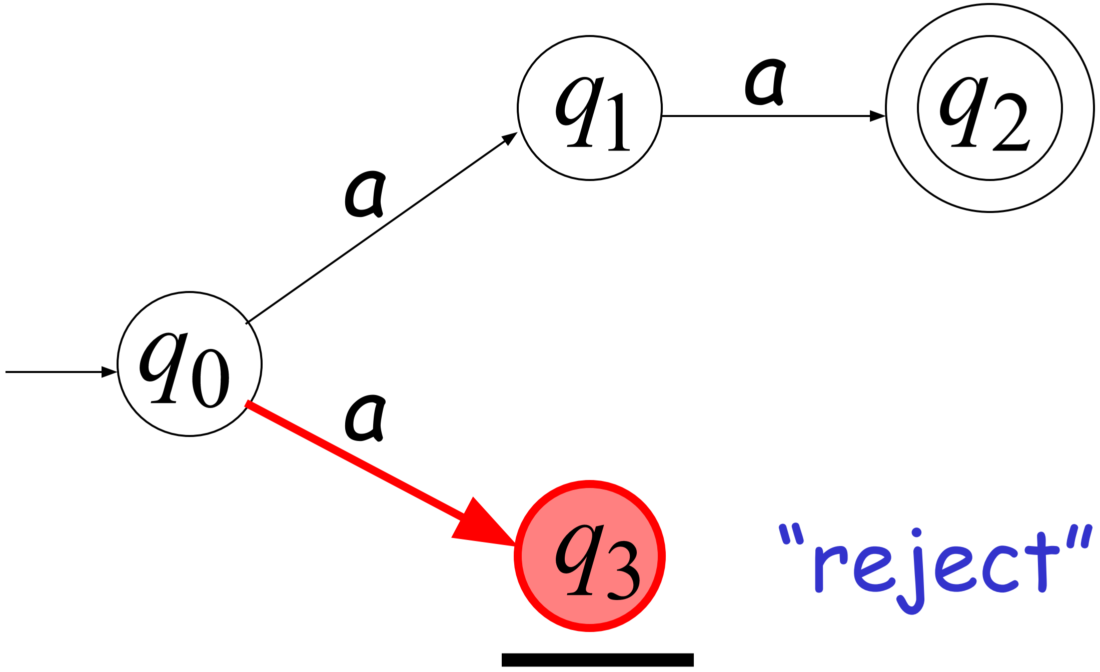


No transition:
the automaton hangs

Second Choice

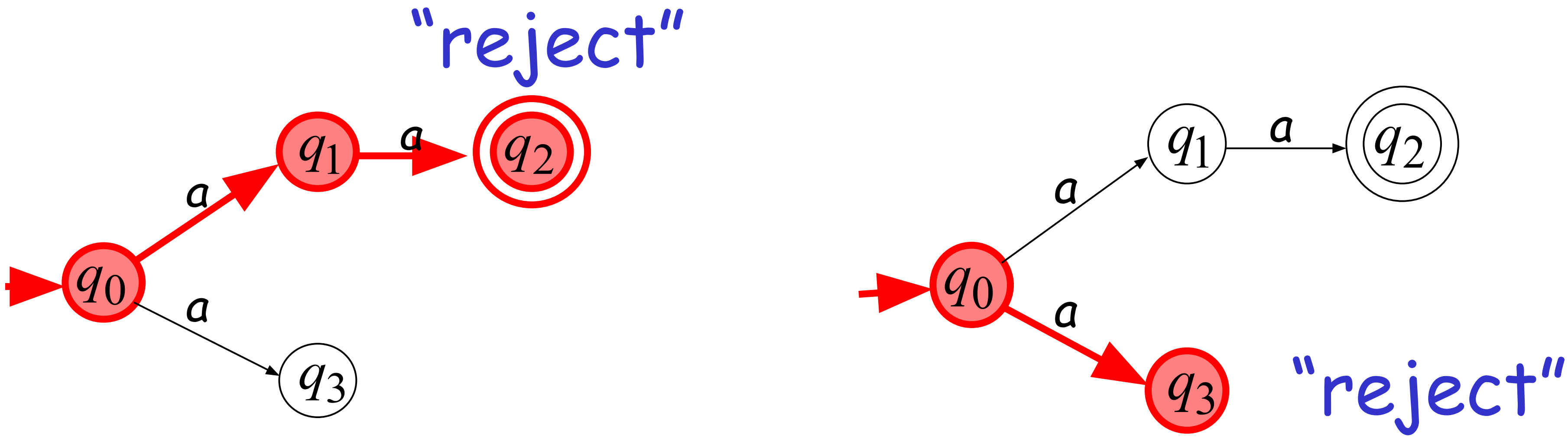


Input cannot be consumed



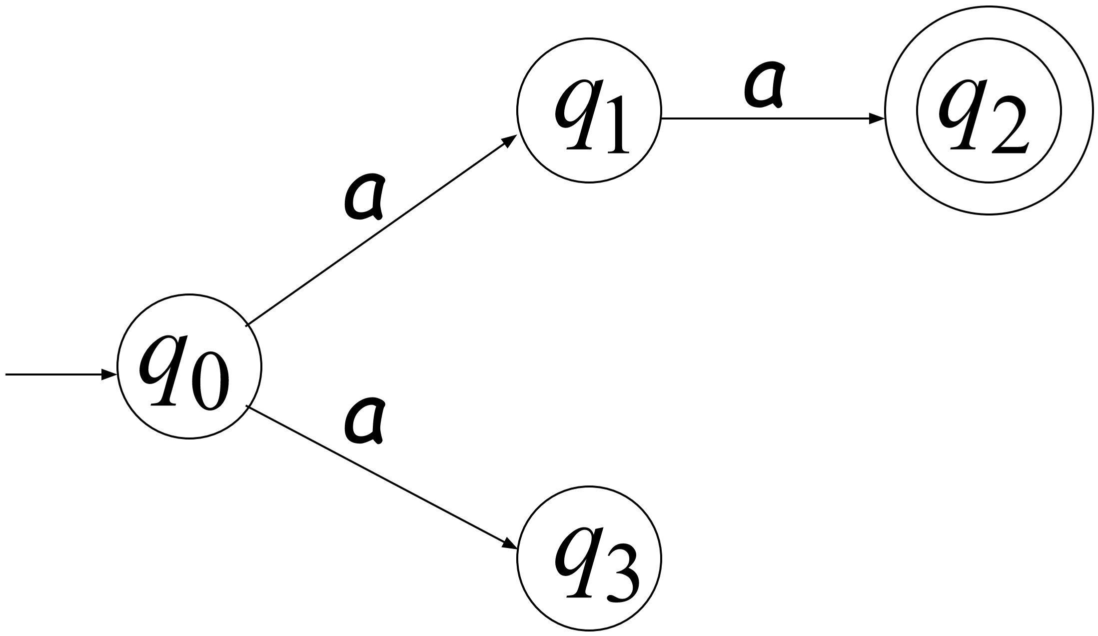
"reject"

aaa is rejected by the NFA:

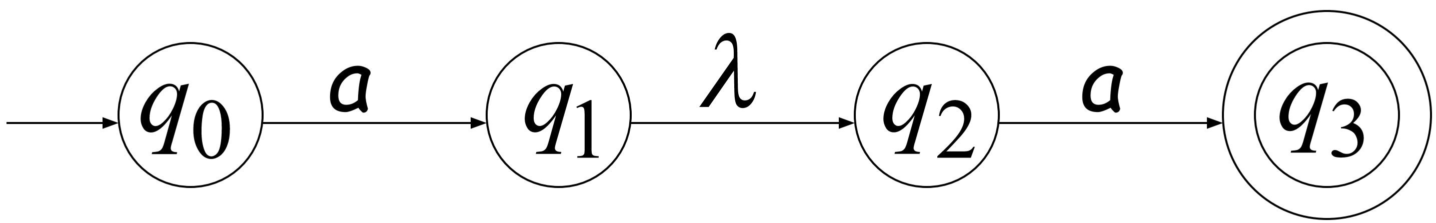


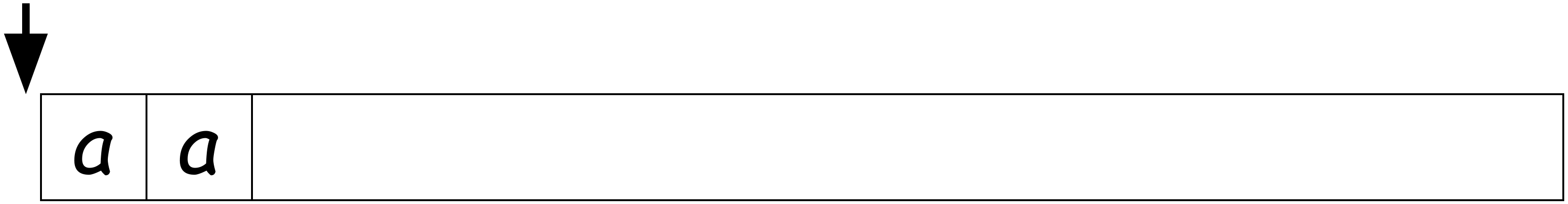
All possible computations lead to rejection

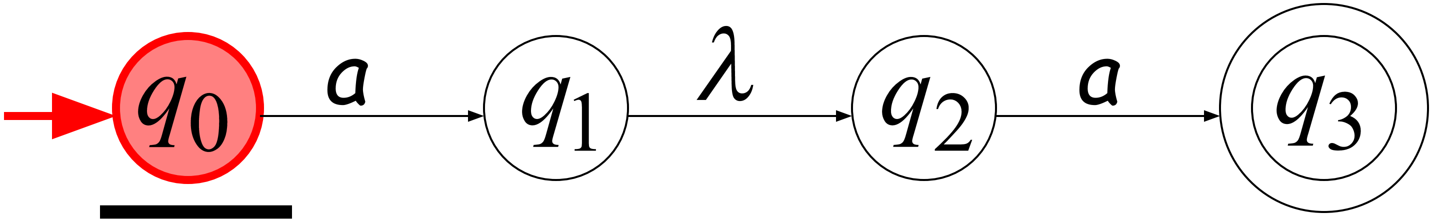
Language accepted: $L = \{aa\}$

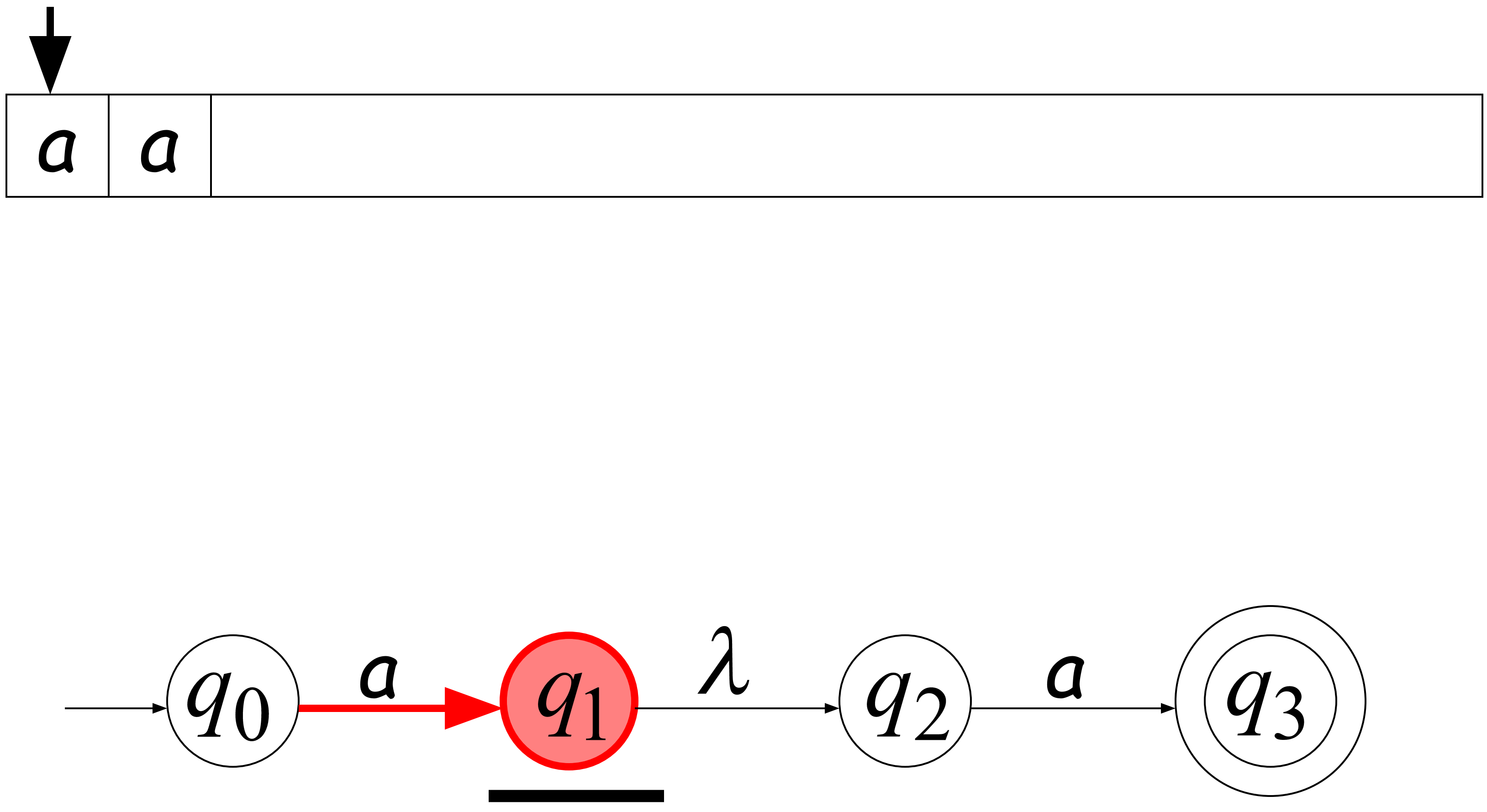


Lambda Transitions



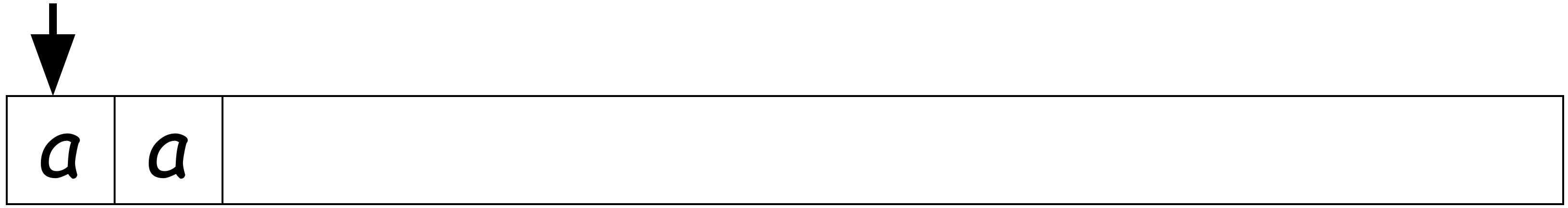

a
a

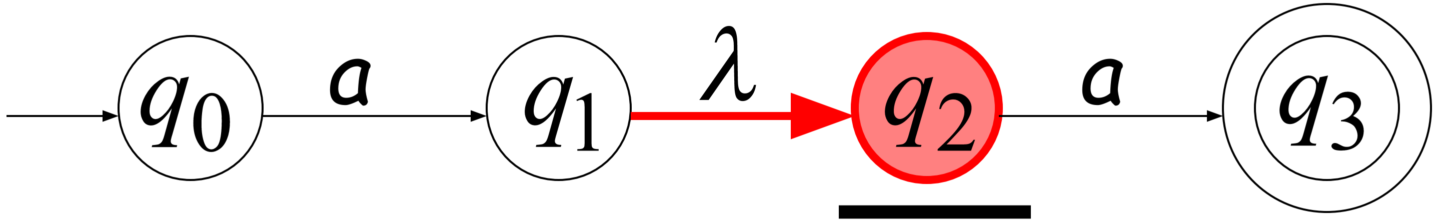

q_0
a
q_1
λ
q_2
a
q_3

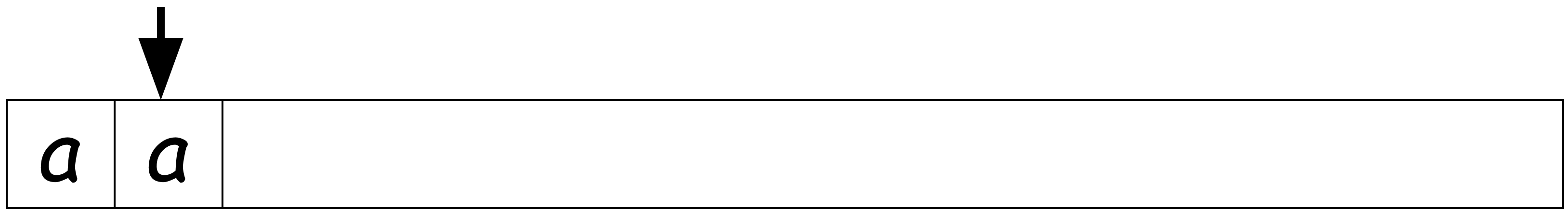


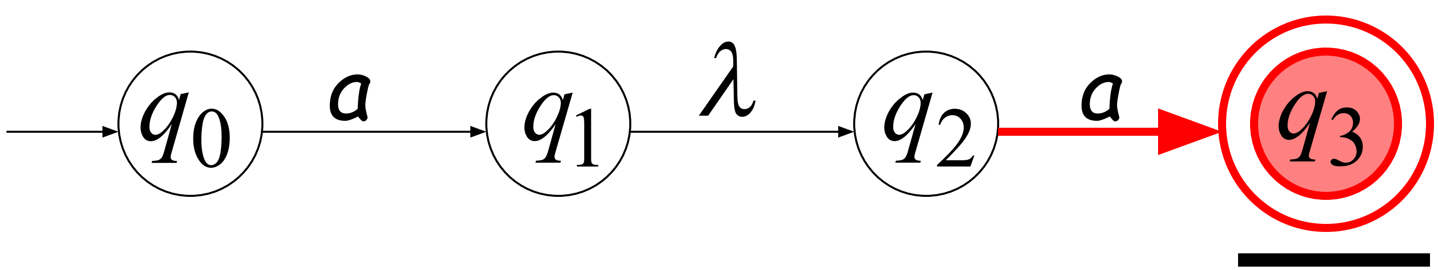
a
a
q_0
a
q_1
λ
q_2
a
q_3

(read head does not move)

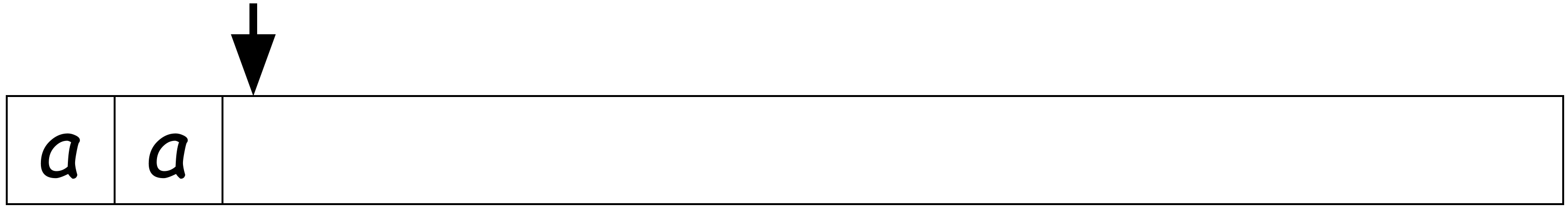




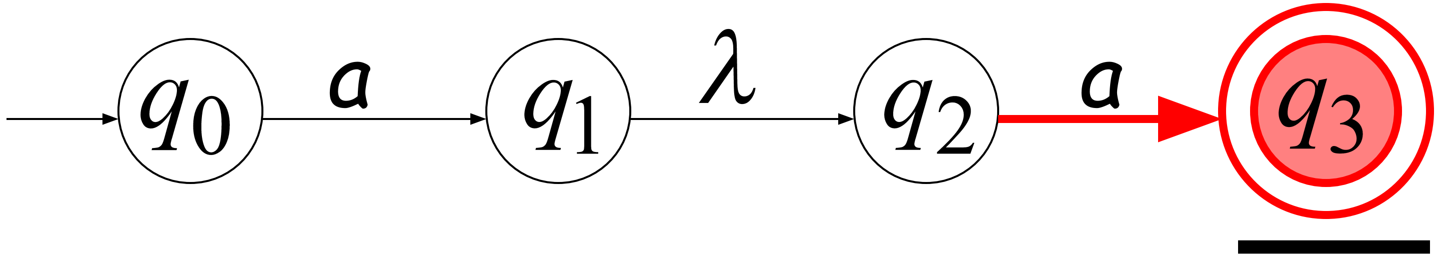

a a


q_0
a
q_1
λ
q_2
a
q_3

all input is consumed

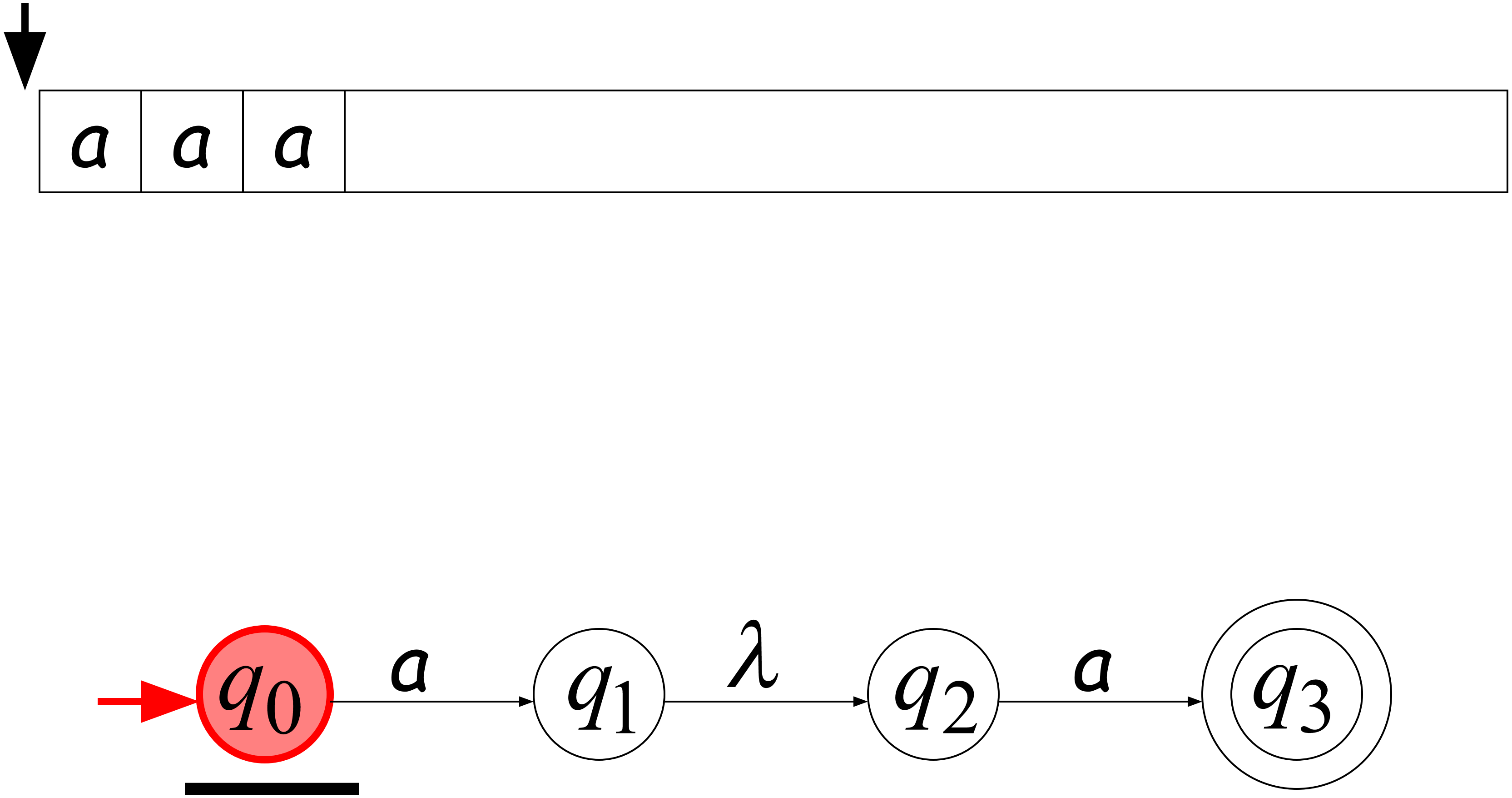


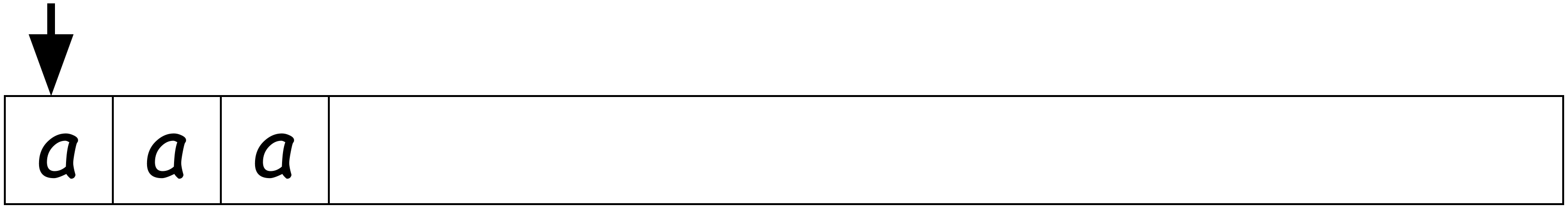
"accept"

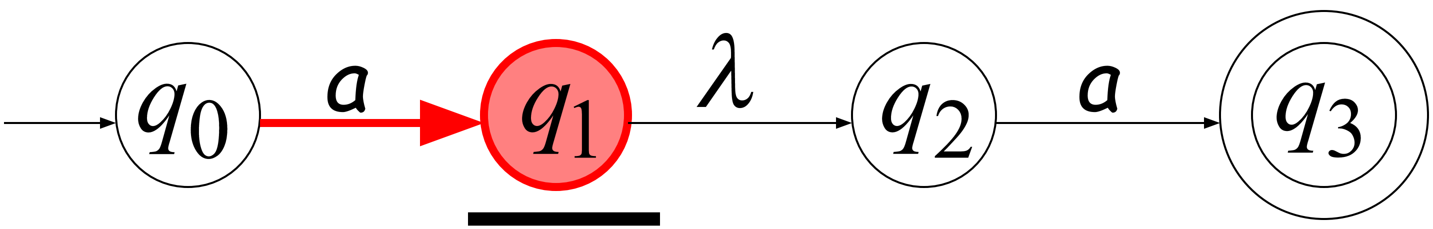


String aa is accepted

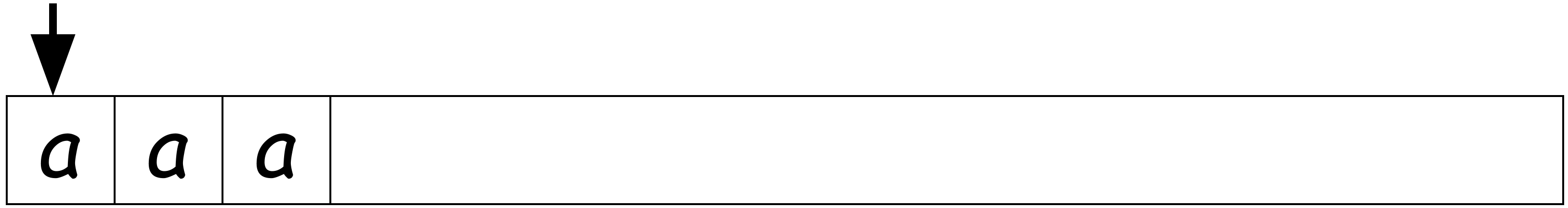
Rejection Example

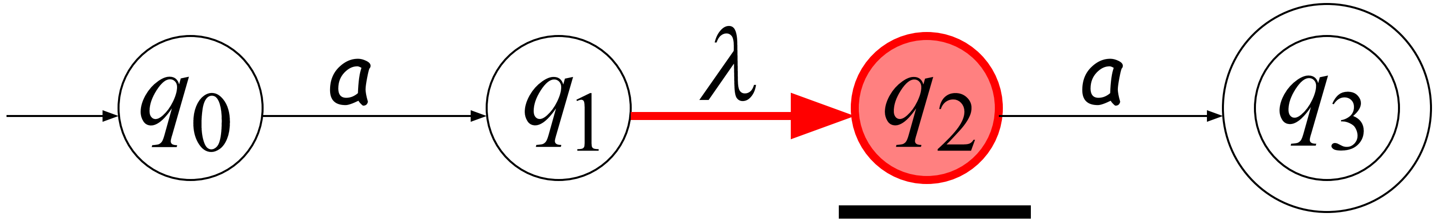


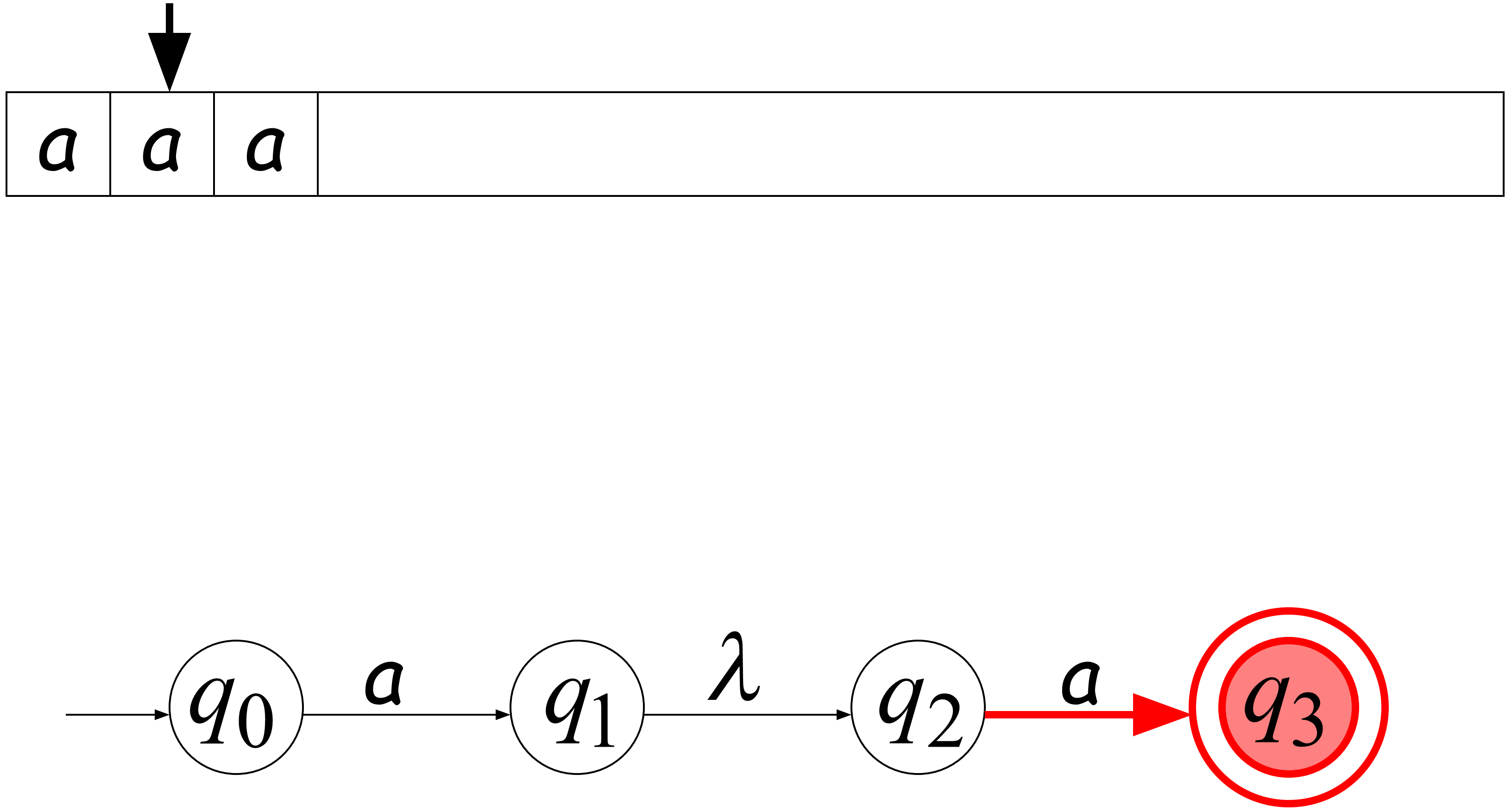

a
a
a


q_0
a
q_1
λ
q_2
a
q_3

(read head doesn't move)

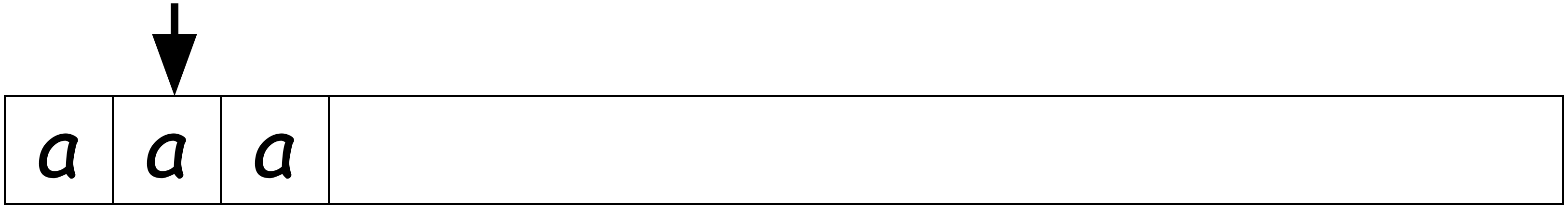


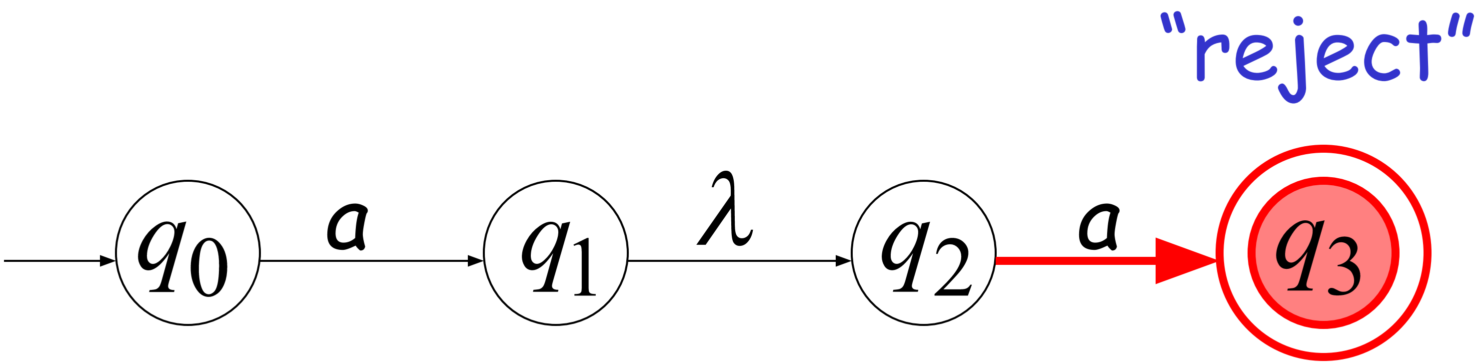




No transition:
the automaton hangs

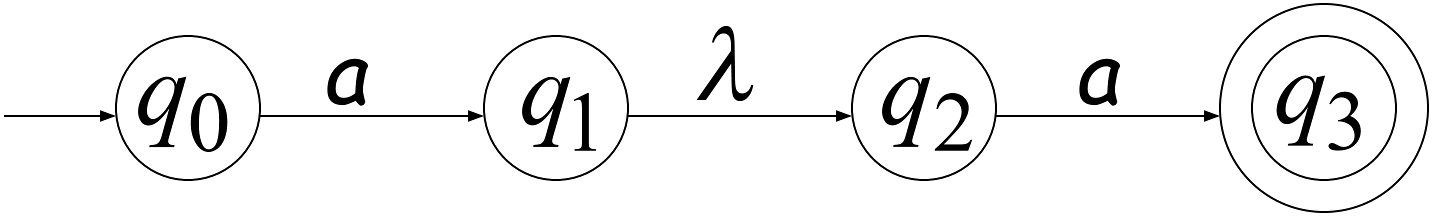
Input cannot be consumed



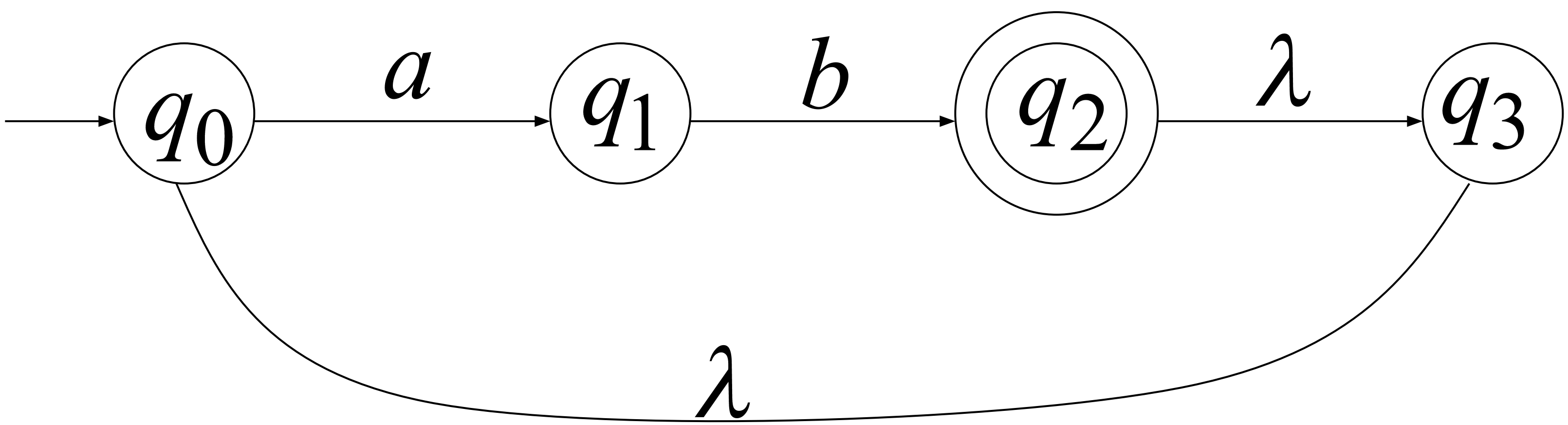


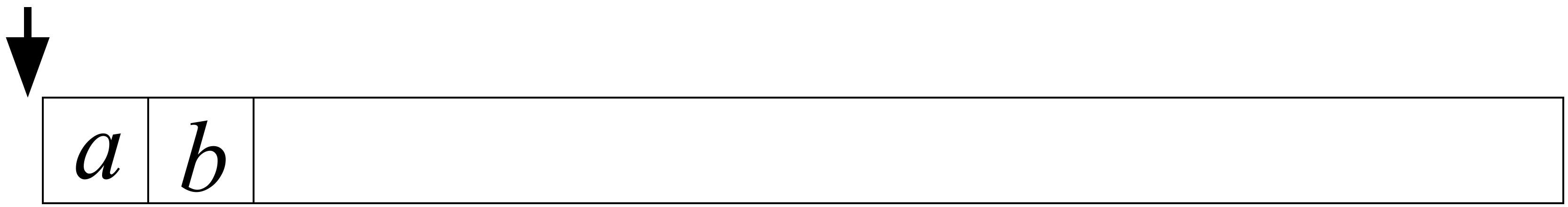
String aaa is rejected

Language accepted: $L = \{aa\}$

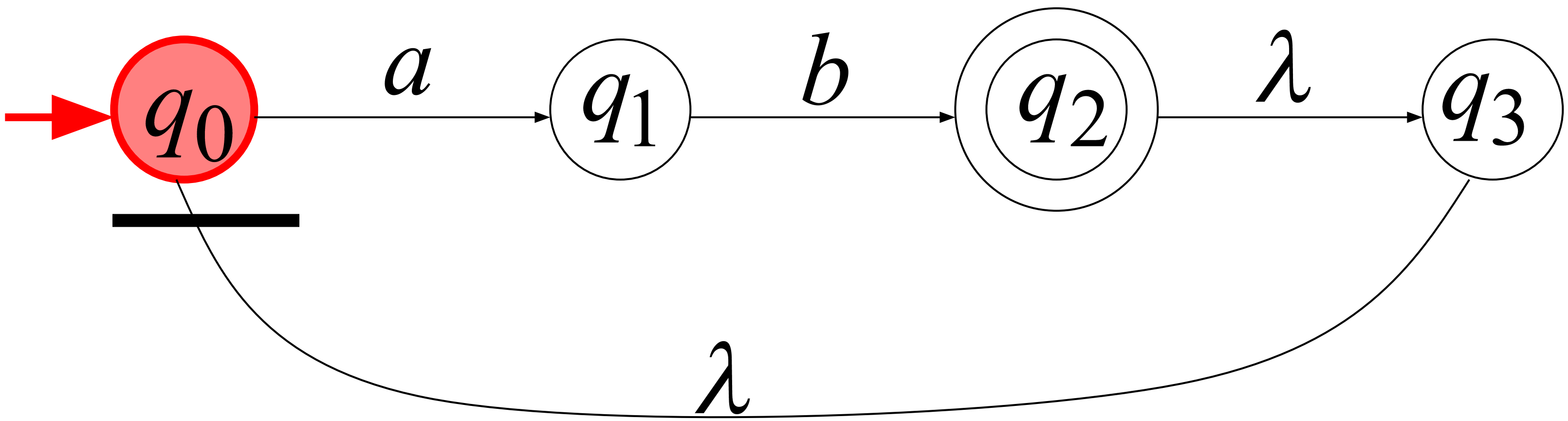


Another NFA Example

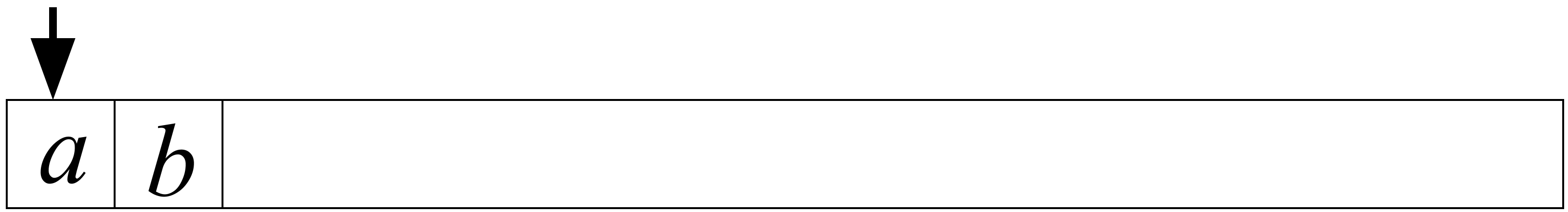




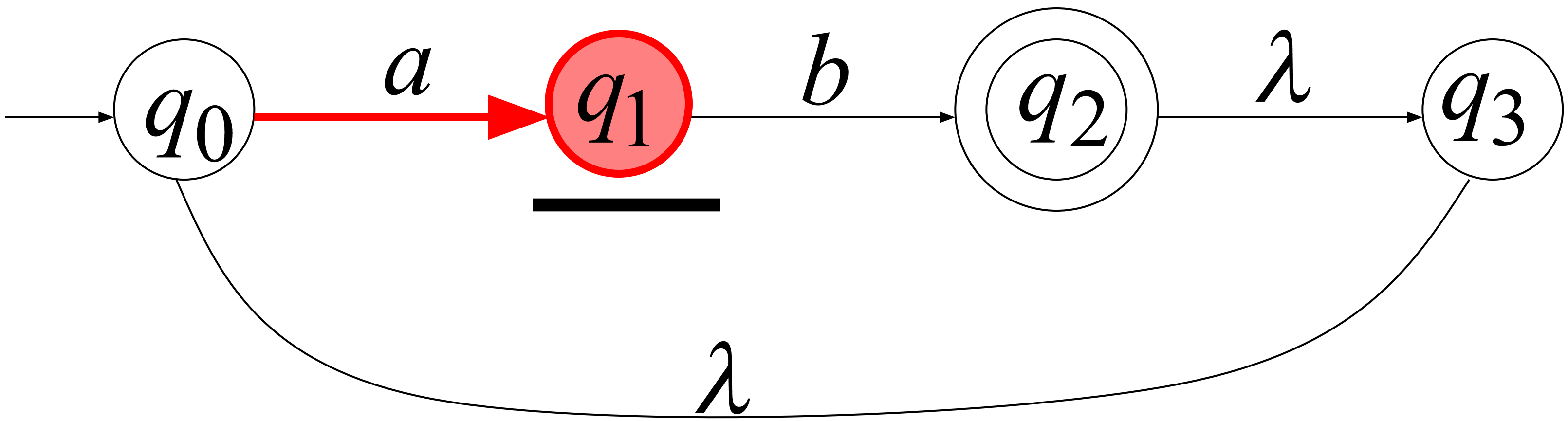
a
b



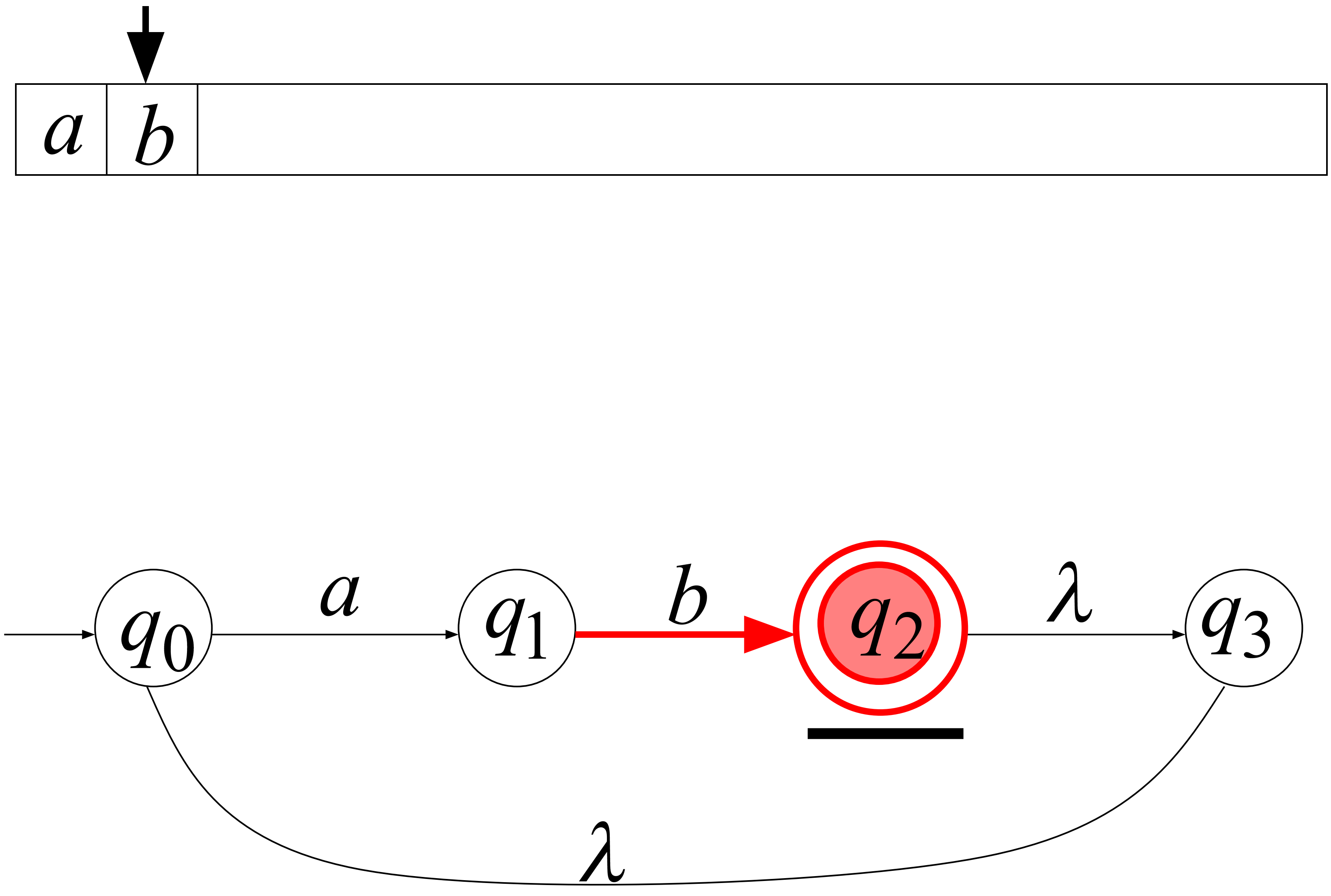
q_0
a
q_1
b
q_2
λ
q_3
λ



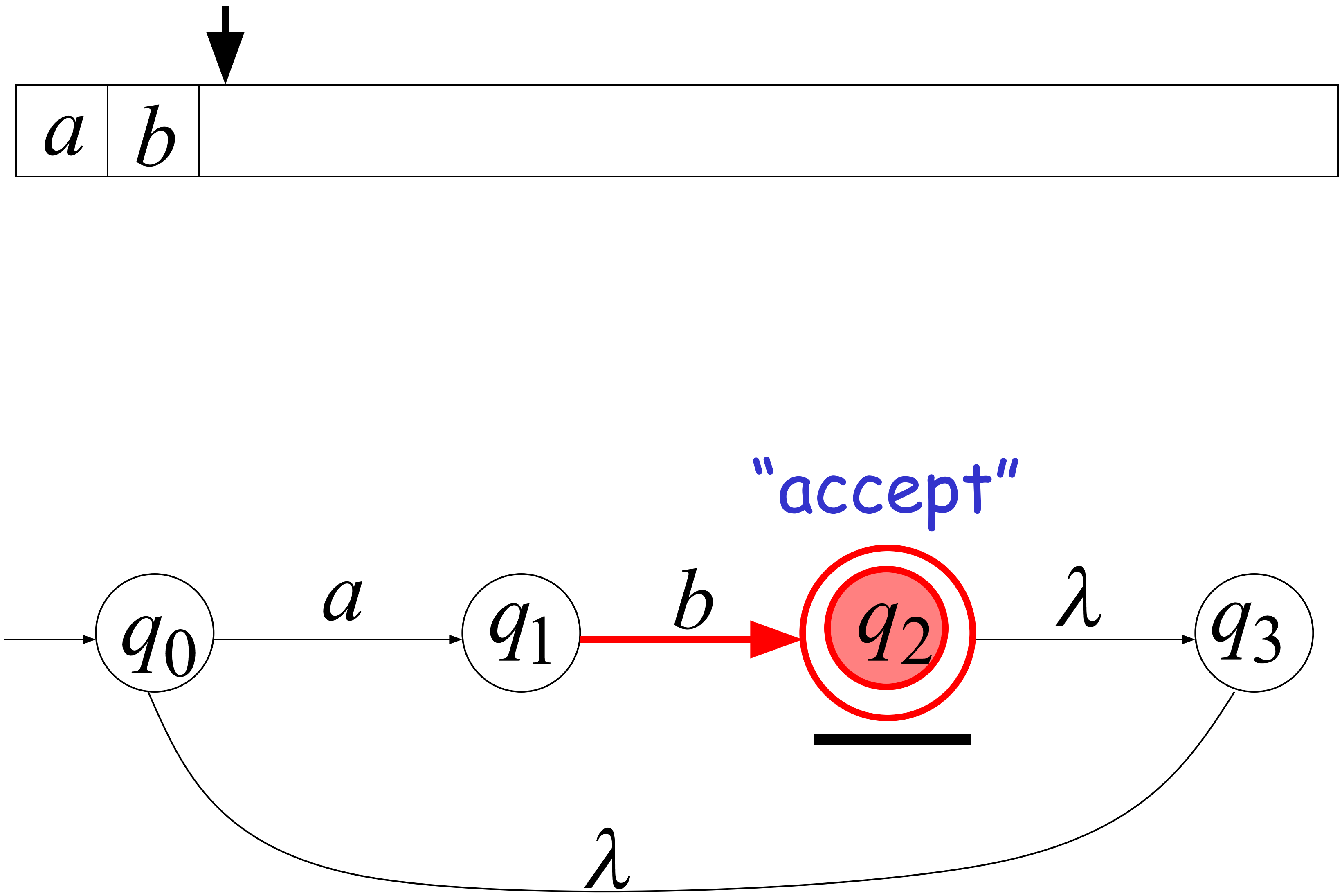
a
b



q_0
a
q_1
b
q_2
λ
q_3
λ

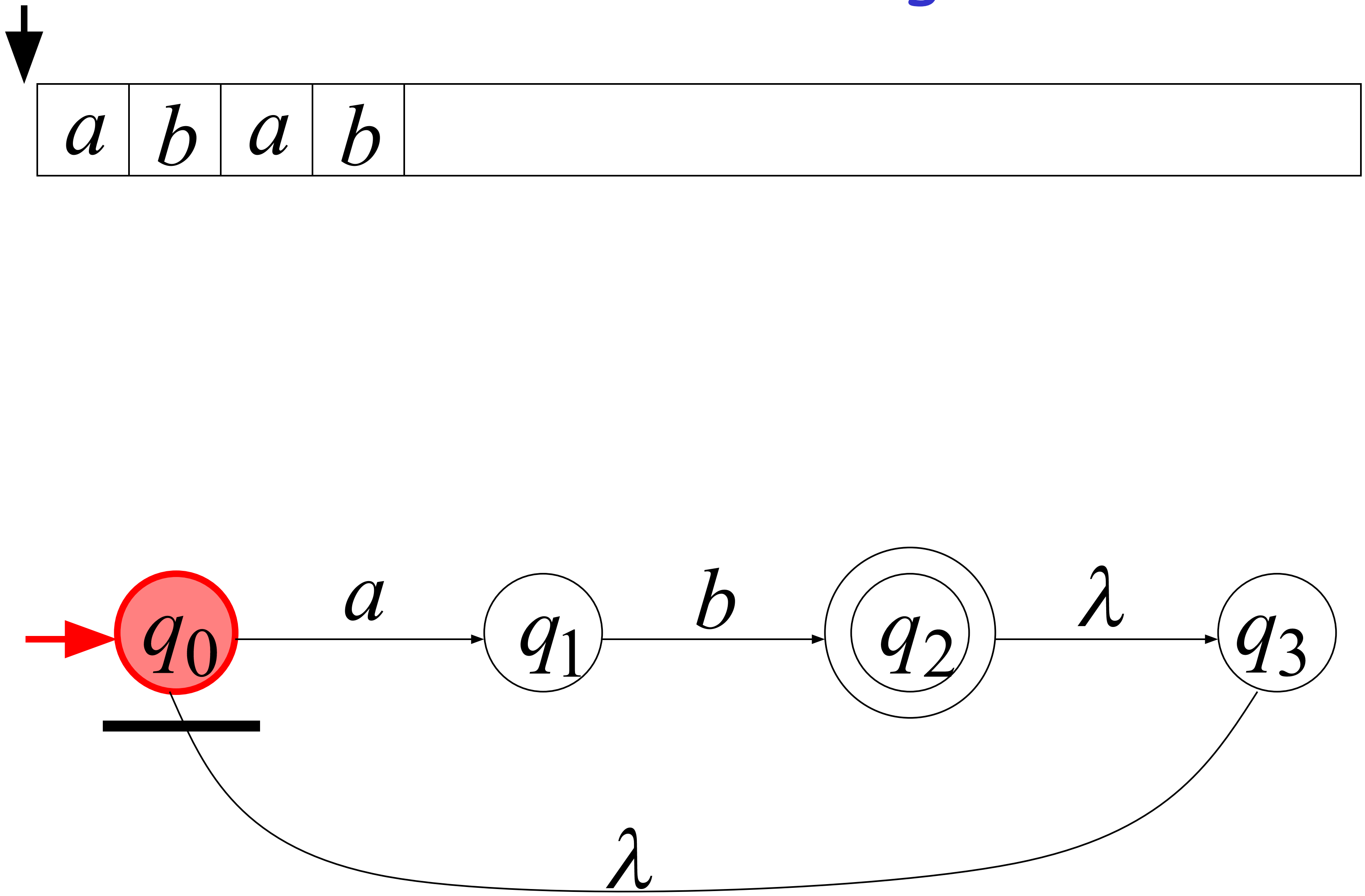


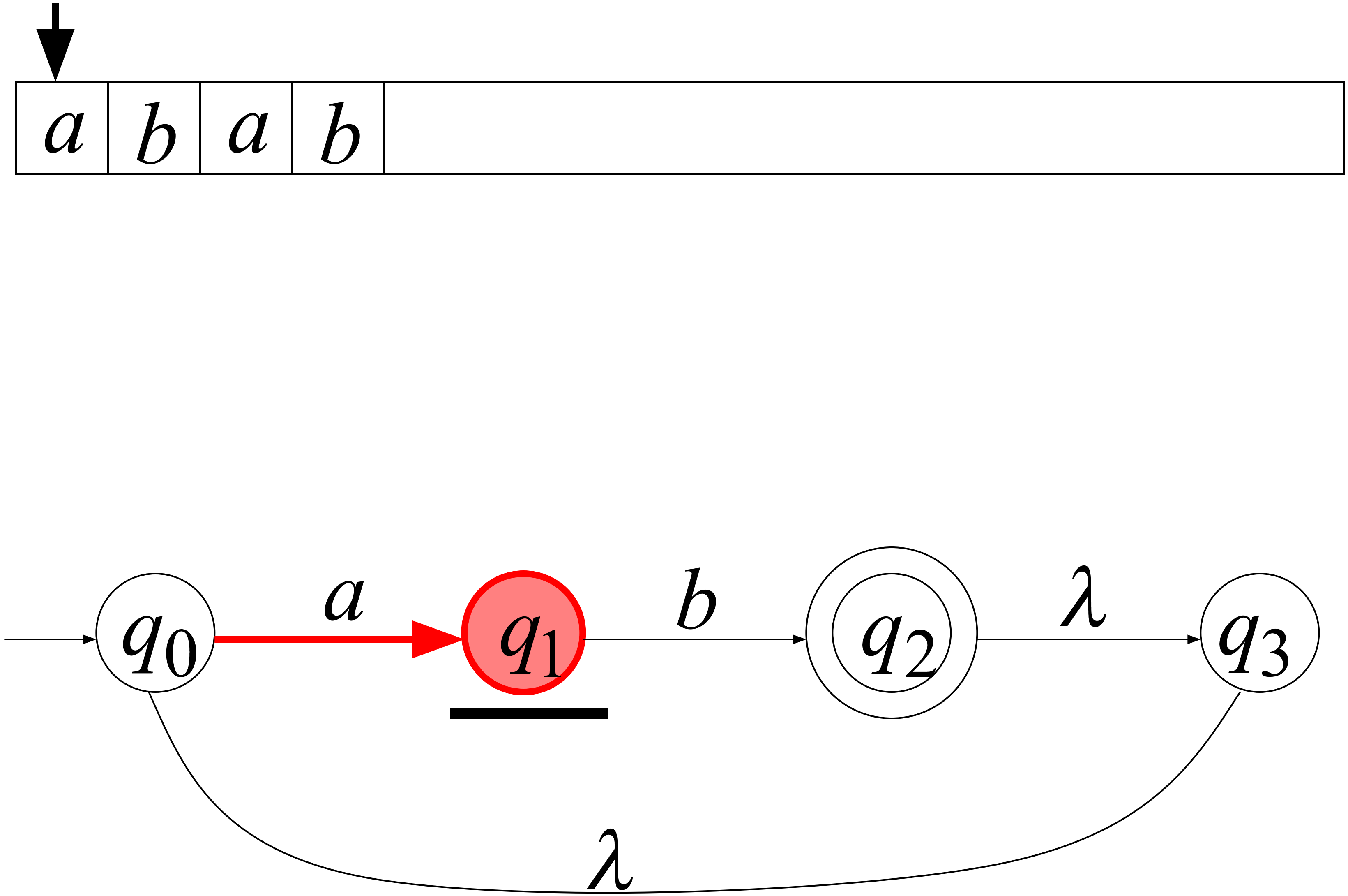
a
b
q_0
a
q_1
b
q_2
λ
q_3
λ

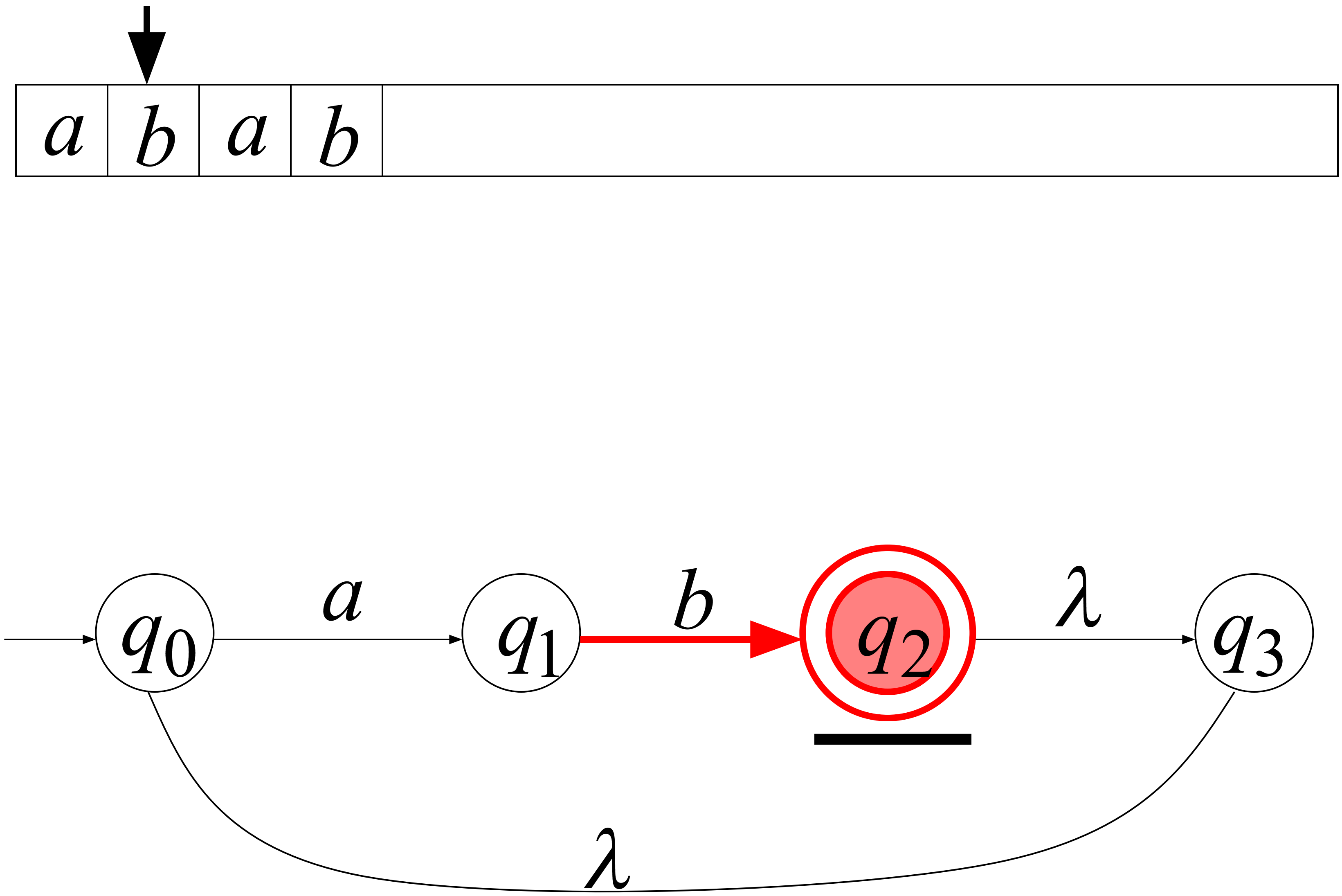


a
b
"accept"
q_0
a
q_1
b
q_2
λ
q_3
λ

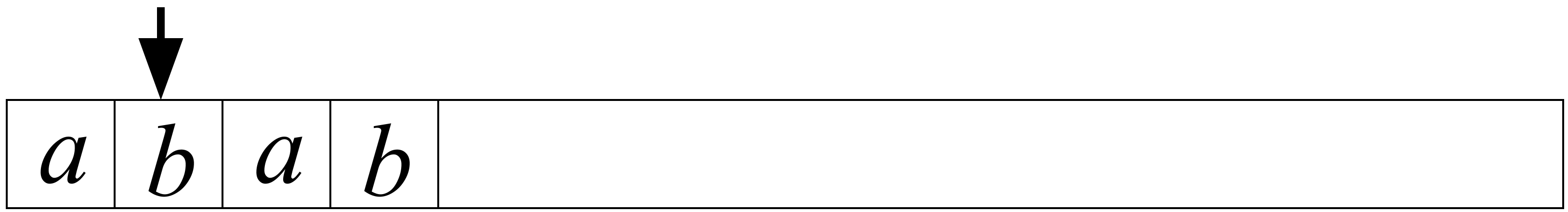
Another String



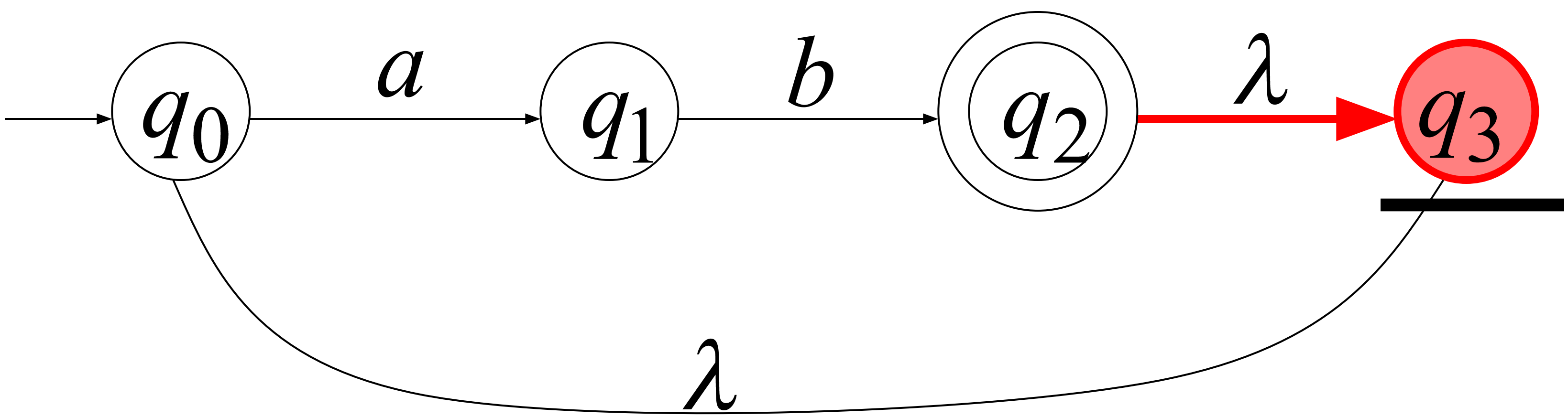

a
b
a
b
q_0
a
q_1
b
q_2
λ
q_3
λ



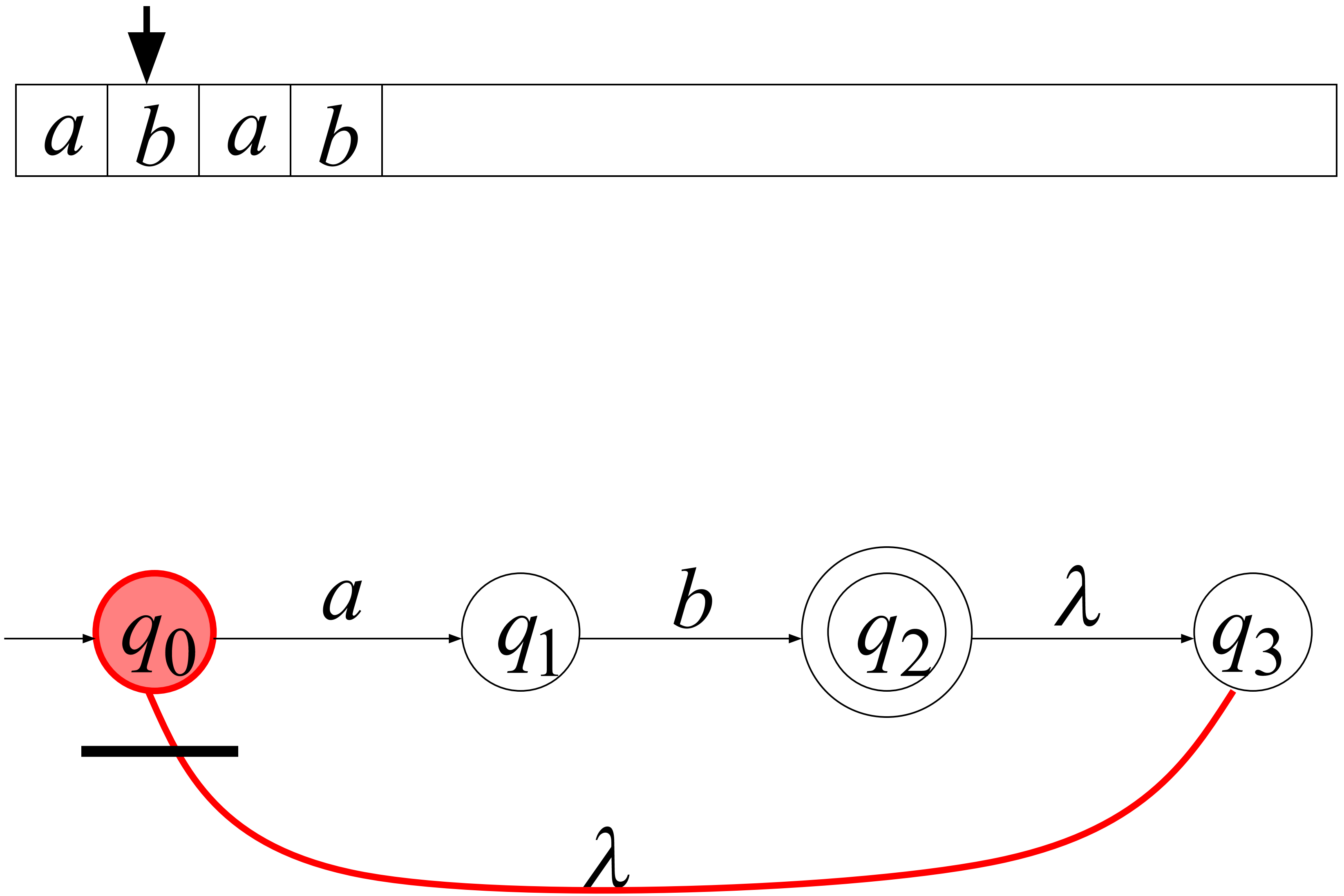
a
b
a
b
a
b
λ
λ
q_0
q_1
q_2
q_3

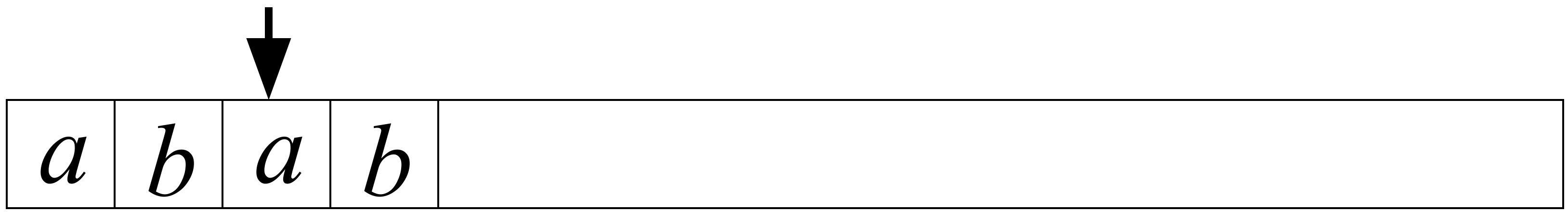


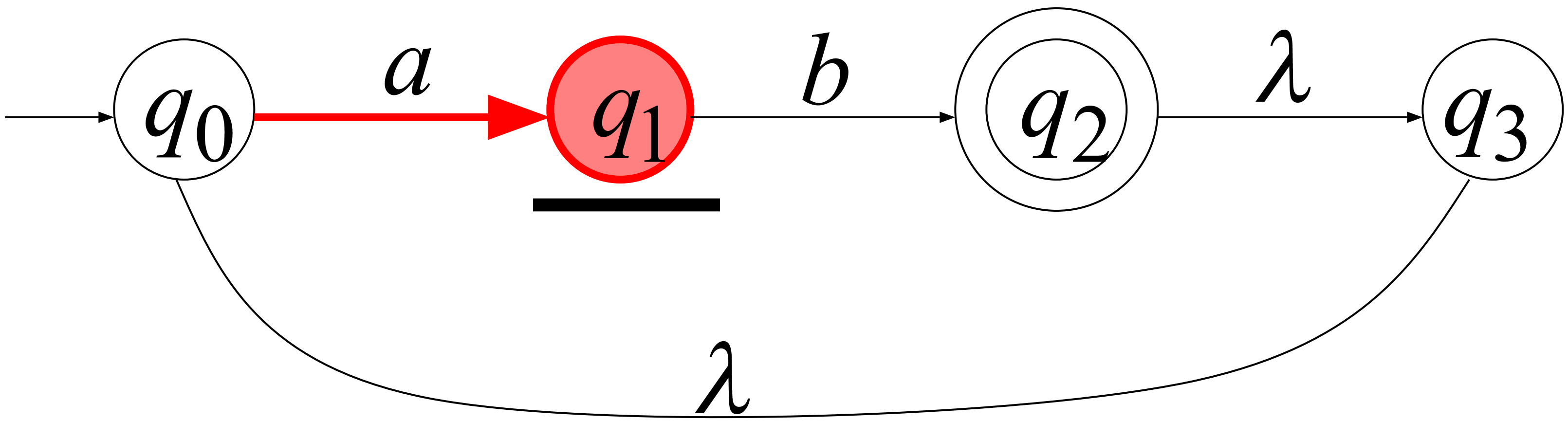
a
b
a
b

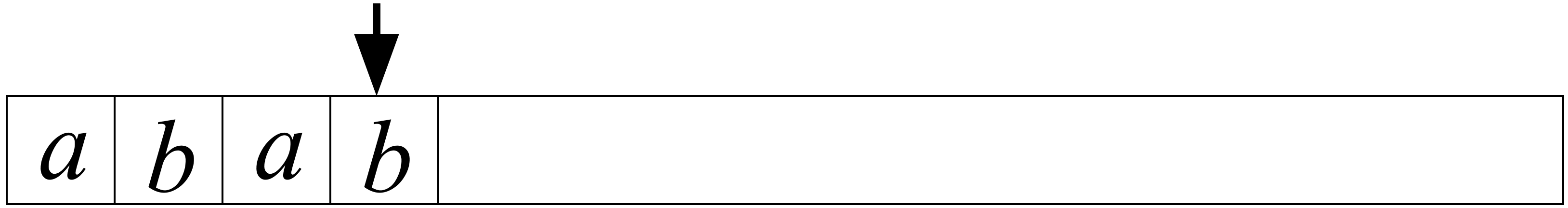


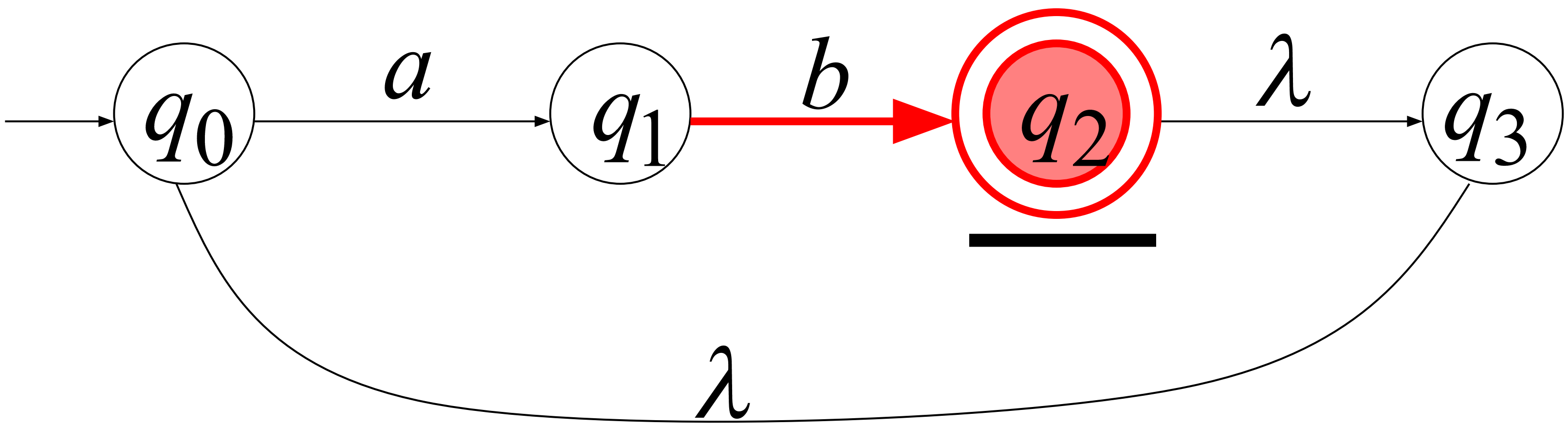
a
b
λ
λ
q0
q1
q2
q3

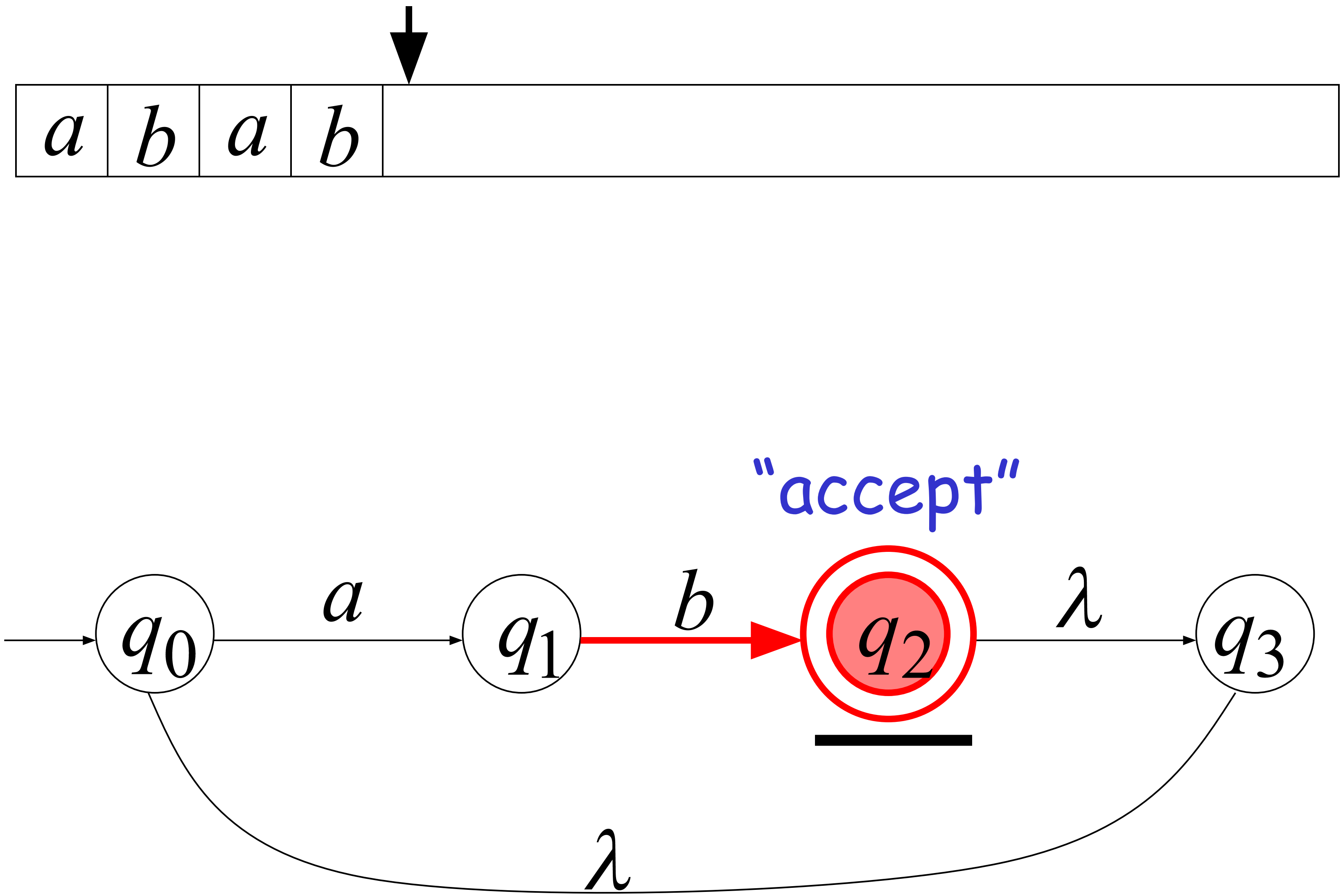

a
b
a
b
a
b
λ
λ
q_0
q_1
q_2
q_3


a
b
a
b


q_0
a
q_1
b
q_2
λ
q_3
λ


a
b
a
b

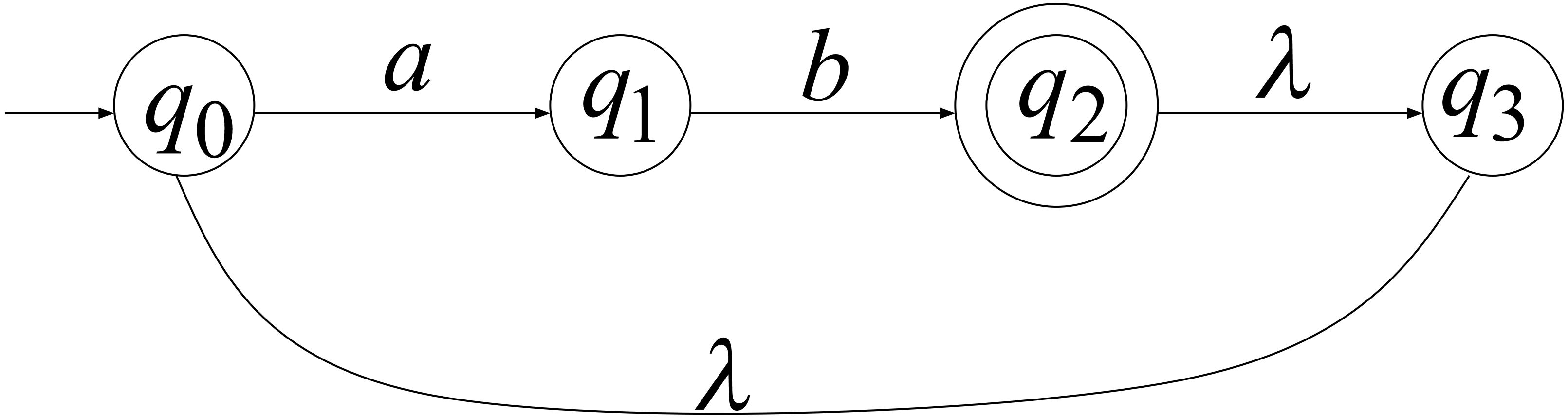

q_0
a
q_1
b
q_2
λ
q_3
λ


a
b
a
b
"accept"
q_0
a
q_1
b
q_2
λ
q_3
λ

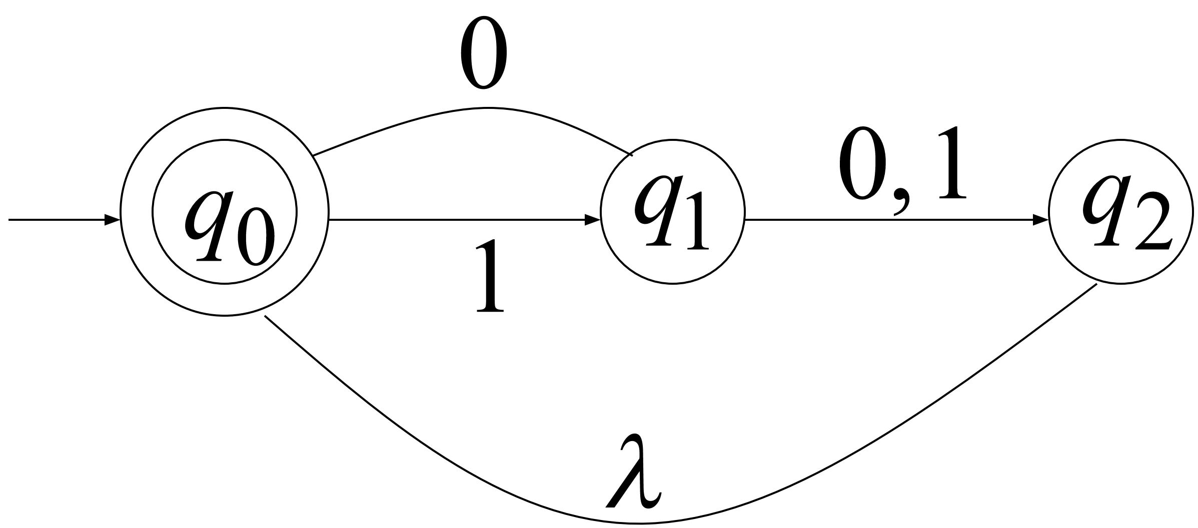
Language accepted

$$L = \{ab,\ abab,\ ababab,\ \ldots\}$$

$$= \{ab\}^+$$

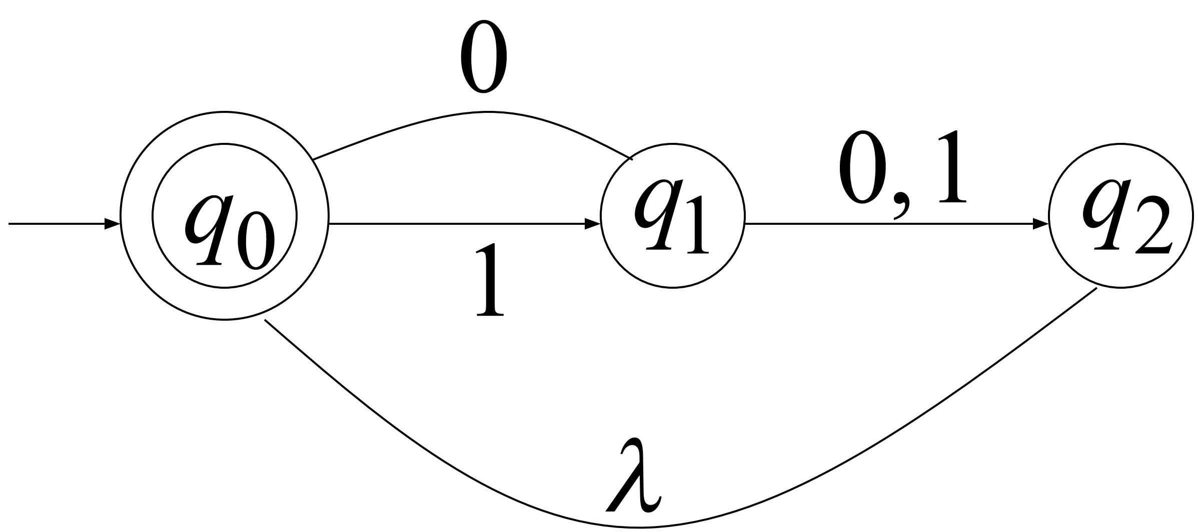


Another NFA Example



Language accepted

$$L(M) = \{\lambda,\ 10,\ 1010,\ 101010,\ \ldots\}$$
$$= \{10\}^*$$



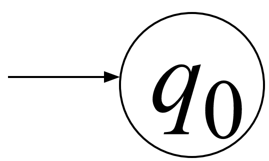
(redundant state)

Remarks:

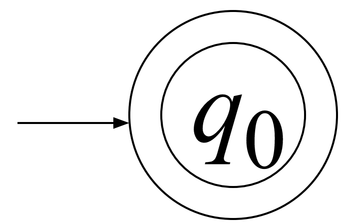
- The λ symbol never appears on the input tape

- Simple automata:

M_1



M_2



$$L(M_1) = \{\}$$

$$L(M_2) = \{\lambda\}$$

Formal Definition of NFAs

$$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

Q : Set of states, i.e. $\{q_0, q_1, q_2\}$

Σ : Input aplhabet, i.e. $\{a, b\}$

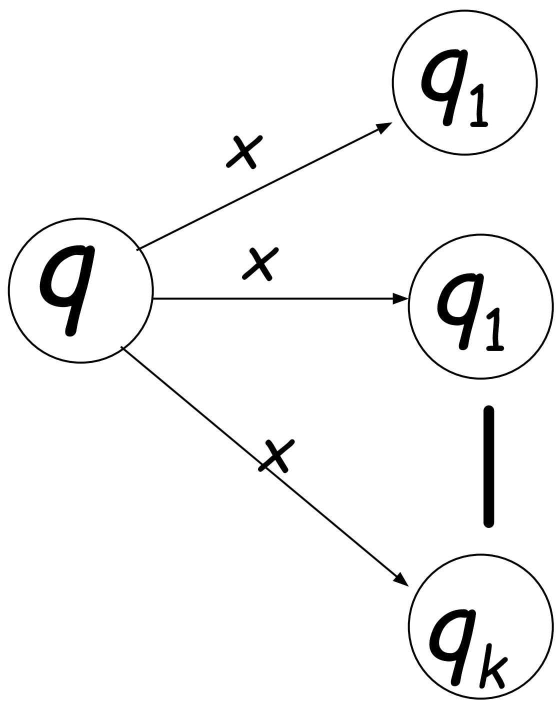
δ : Transition function

q_0 : Initial state

F : Final states

Transition Function δ

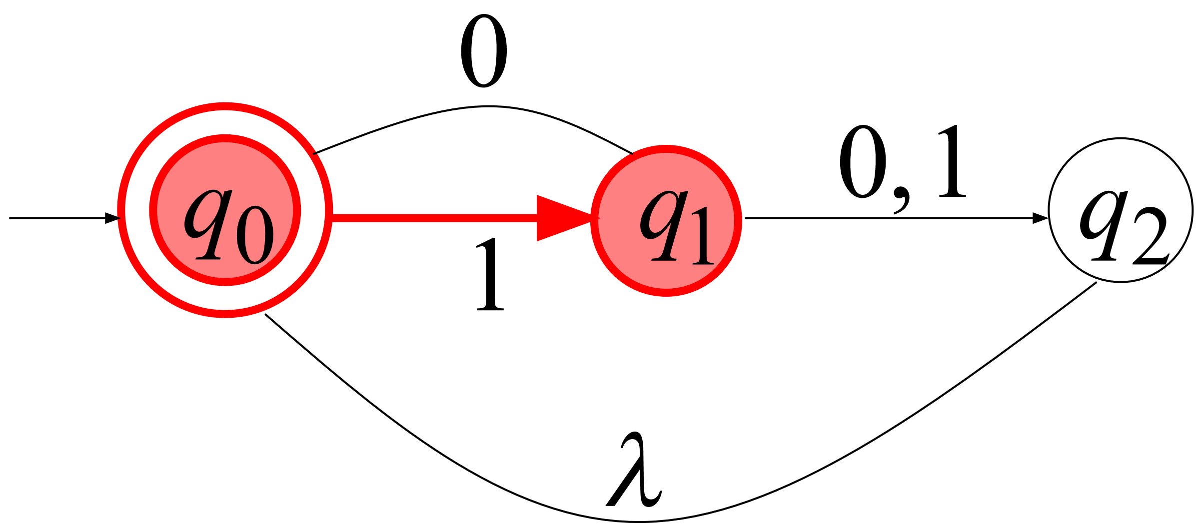
$$\delta(q, x) = \{q_1, q_2, \ldots, q_k\}$$



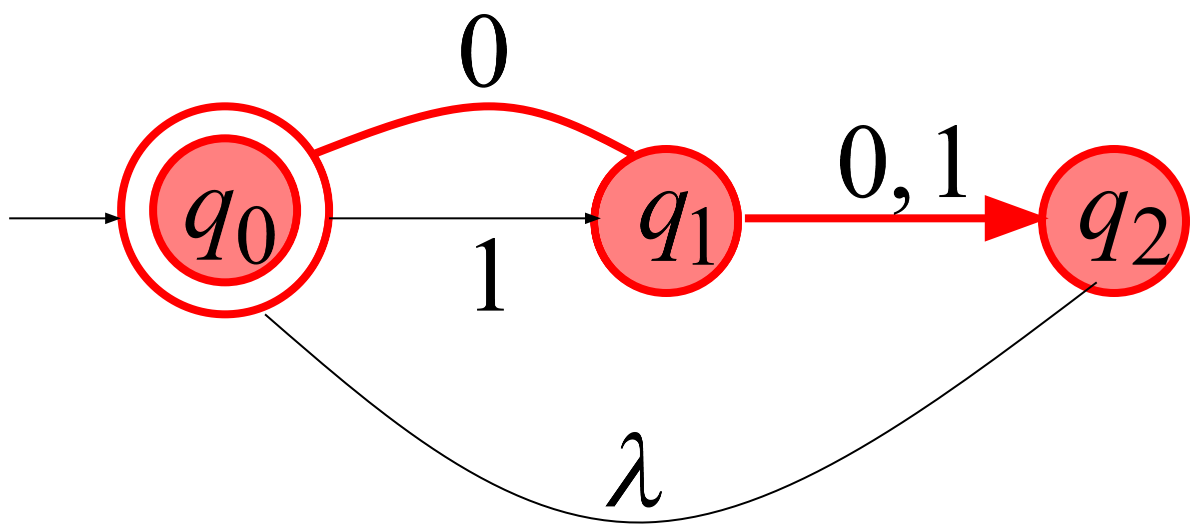
resulting states with following one transition with symbol x

Transition Function δ

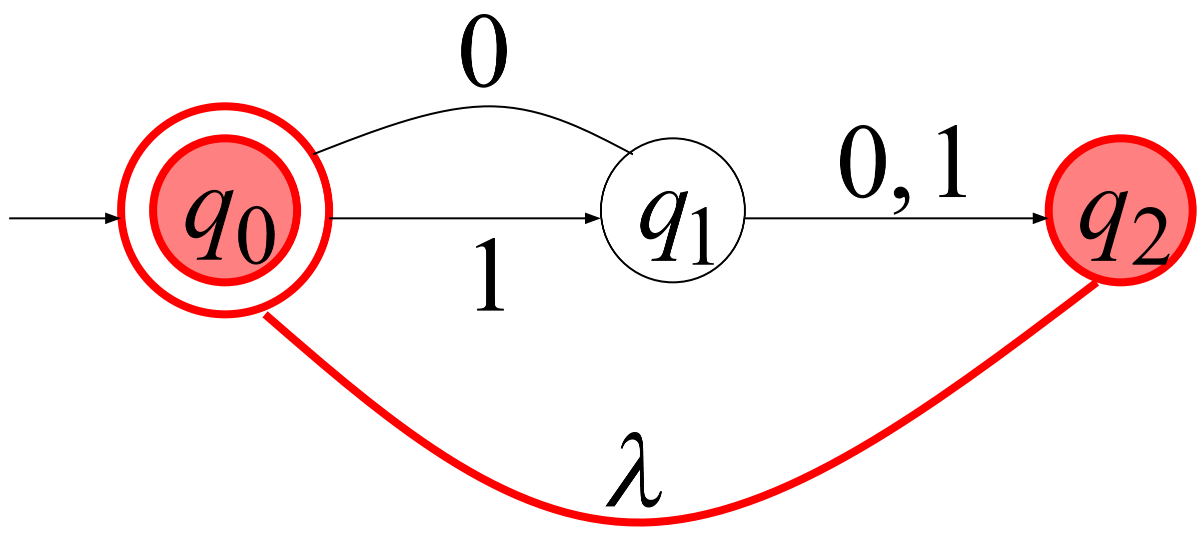
$$\delta(q_0, 1) = \{q_1\}$$



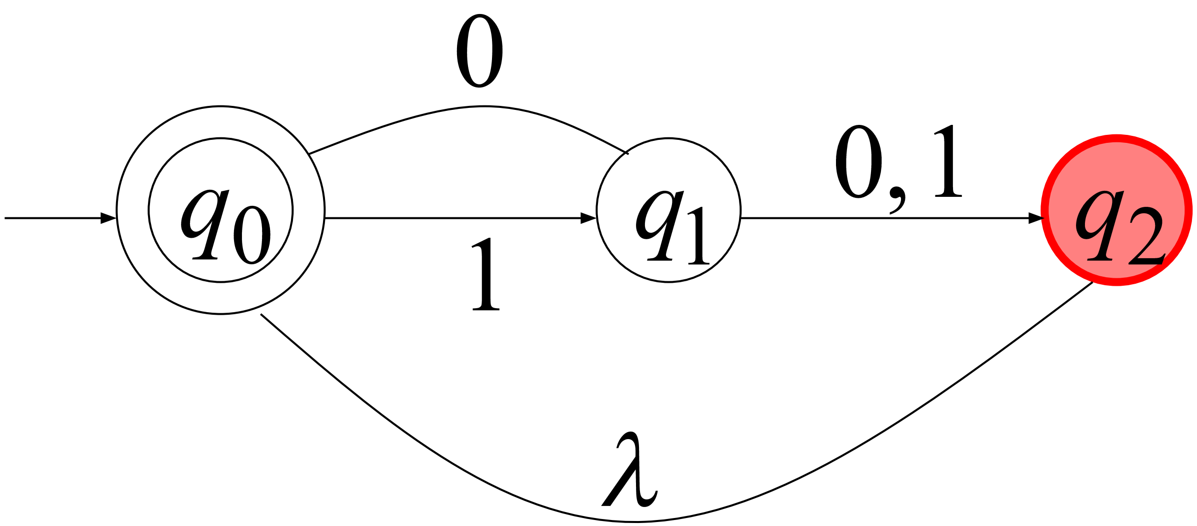
$$\delta(q_1, 0) = \{q_0, q_2\}$$



$$\delta(q_0, \lambda) = \{q_0, q_2\}$$

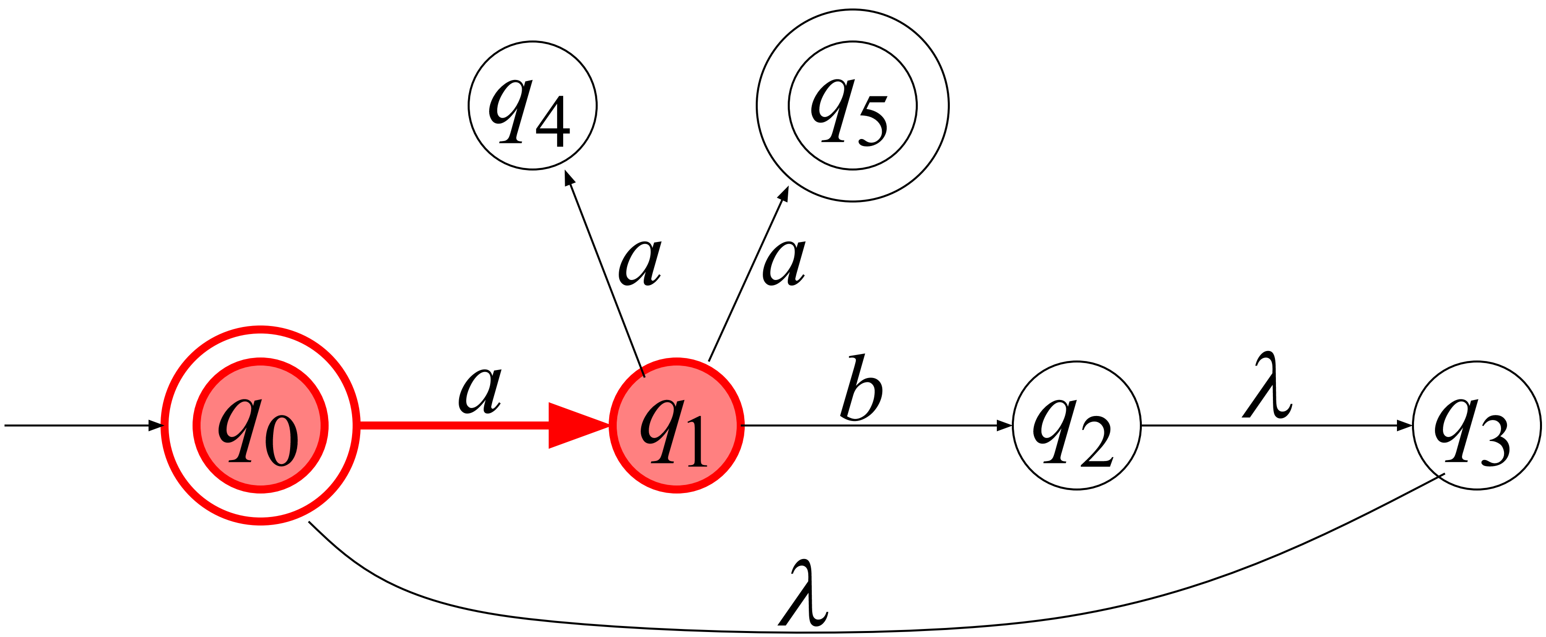


$$\delta(q_2, 1) = \varnothing$$

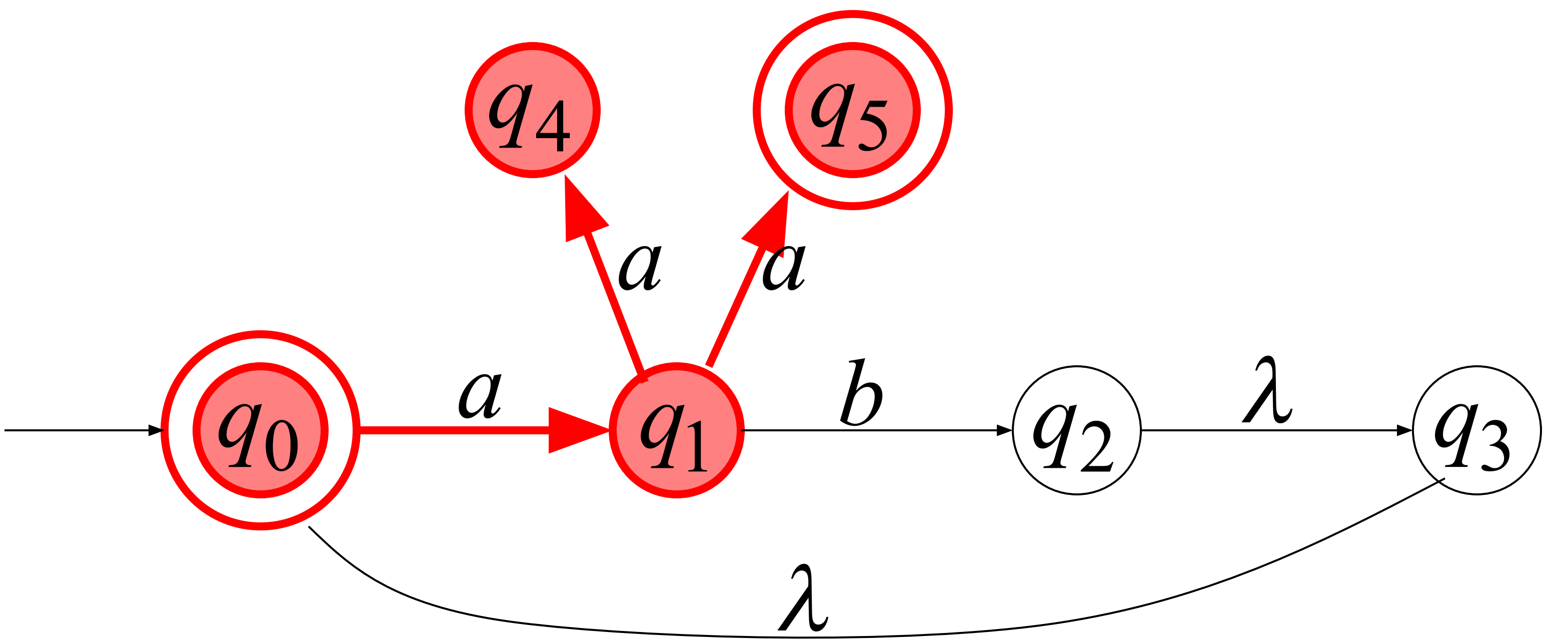


Extended Transition Function δ^*

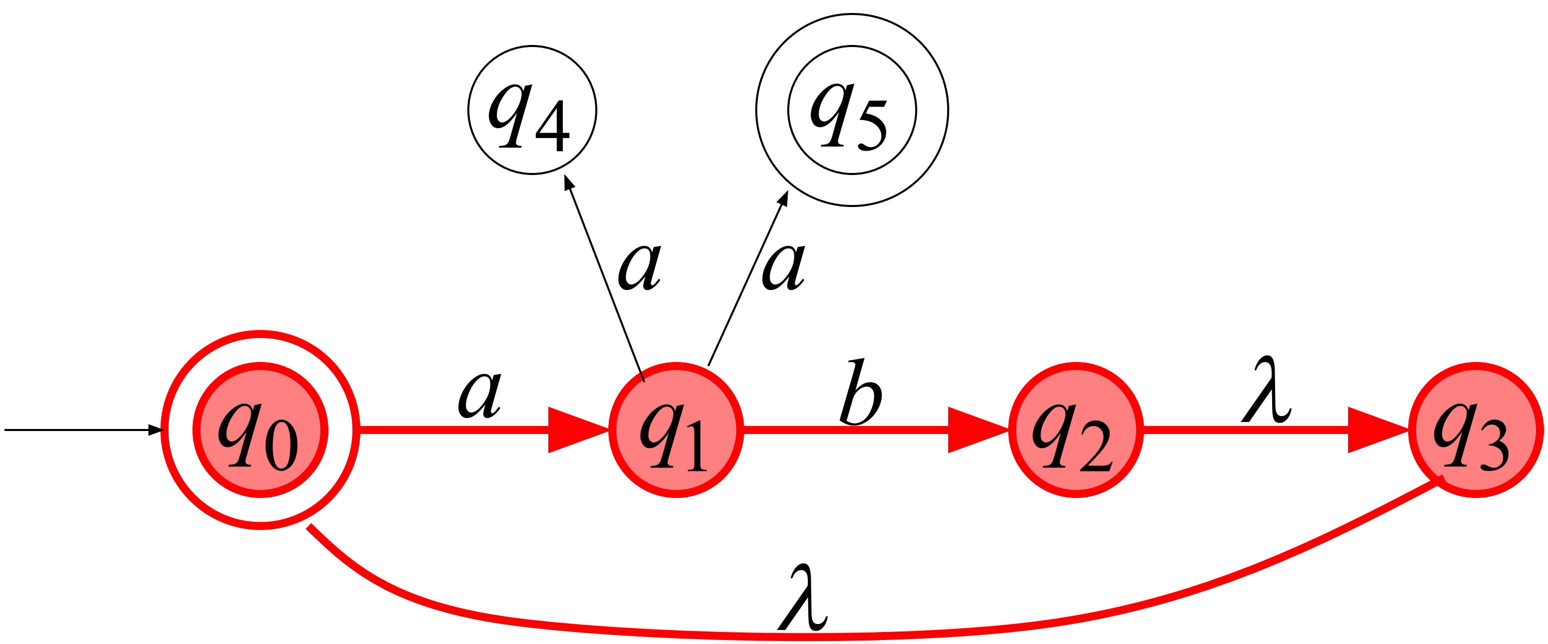
$$\delta^*(q_0, a) = \{q_1\}$$



$$\delta^*(q_0, aa) = \{q_4, q_5\}$$



$$\delta^*(q_0, ab) = \{q_2, q_3, q_0\}$$



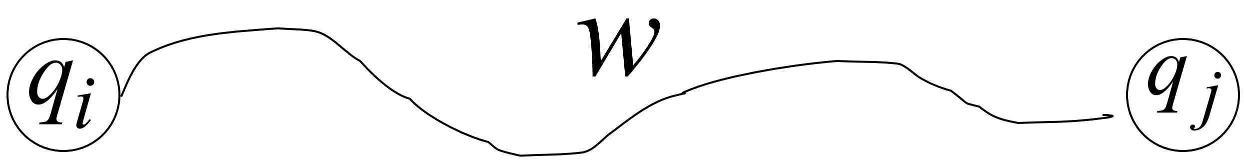
Special case:

for any state q

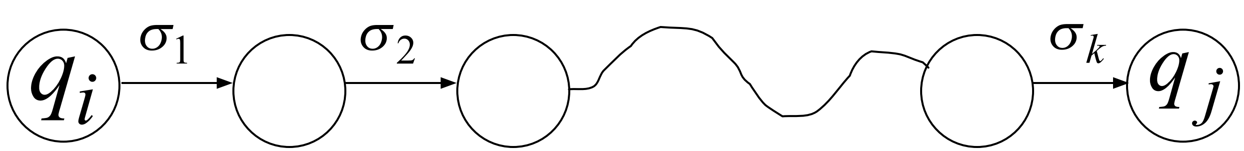
$$q \in \delta^*(q, \lambda)$$

Formally

$q_j \in \delta^*(q_i, w)$: there is a walk from q_i to q_j with label w

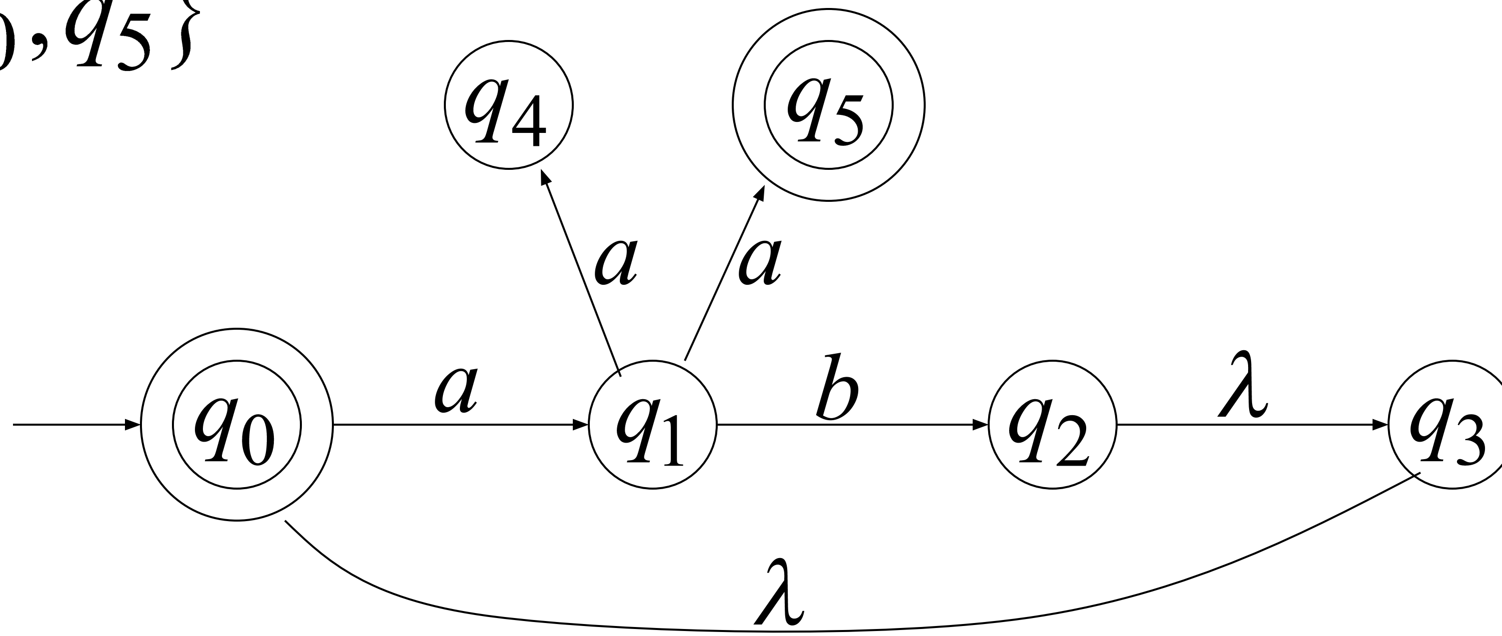


$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



The Language of an NFA M

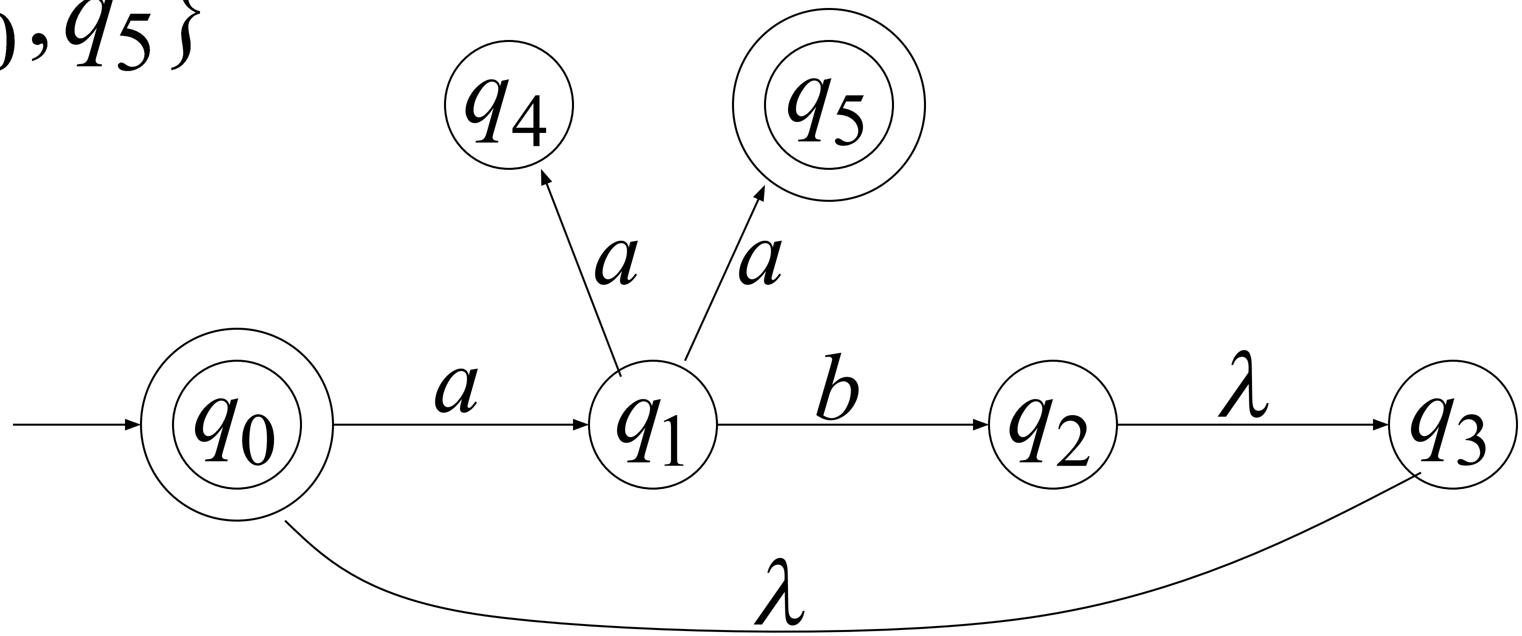
$$F = \{q_0, q_5\}$$



$$\delta^*(q_0, aa) = \{q_4, \underline{q_5}\} \qquad aa \in L(M)$$

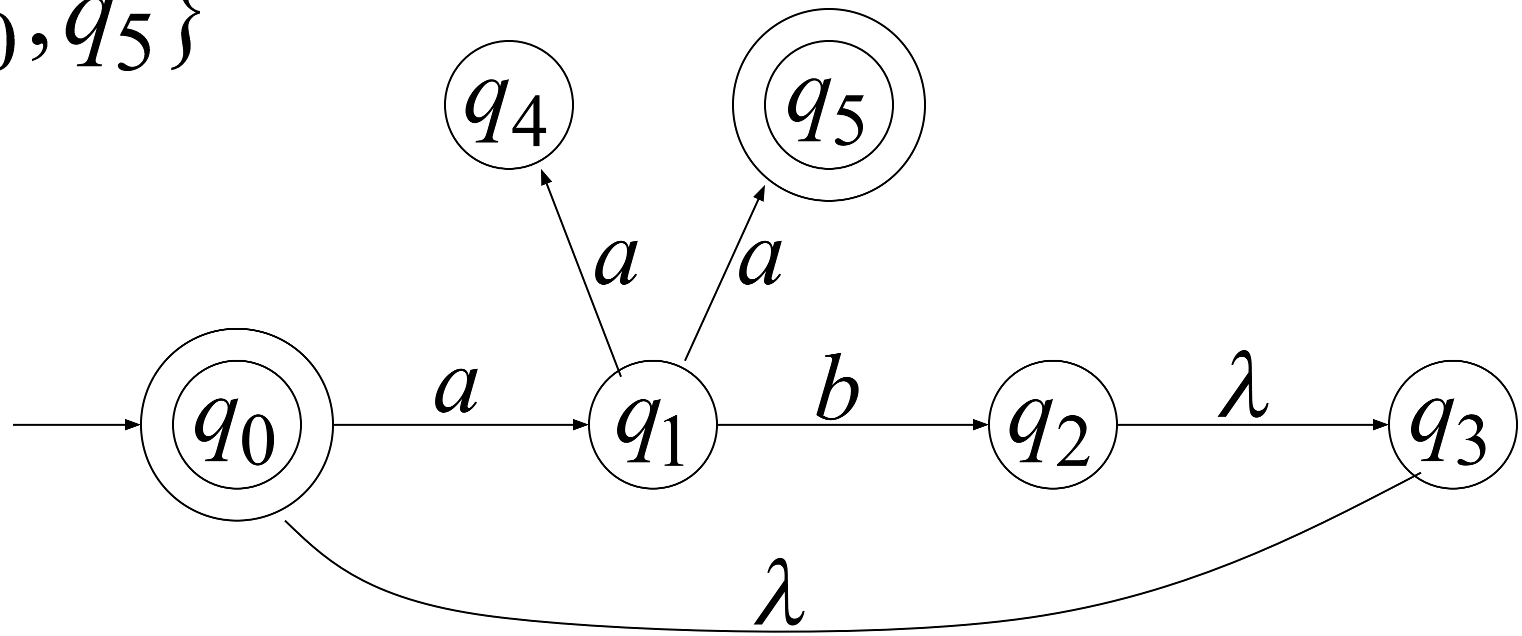
$$\in F$$

$$F = \{q_0, q_5\}$$



$$\delta^*(q_0, ab) = \{q_2, q_3, \underbrace{q_0}_{\in F}\} \qquad ab \in L(M)$$

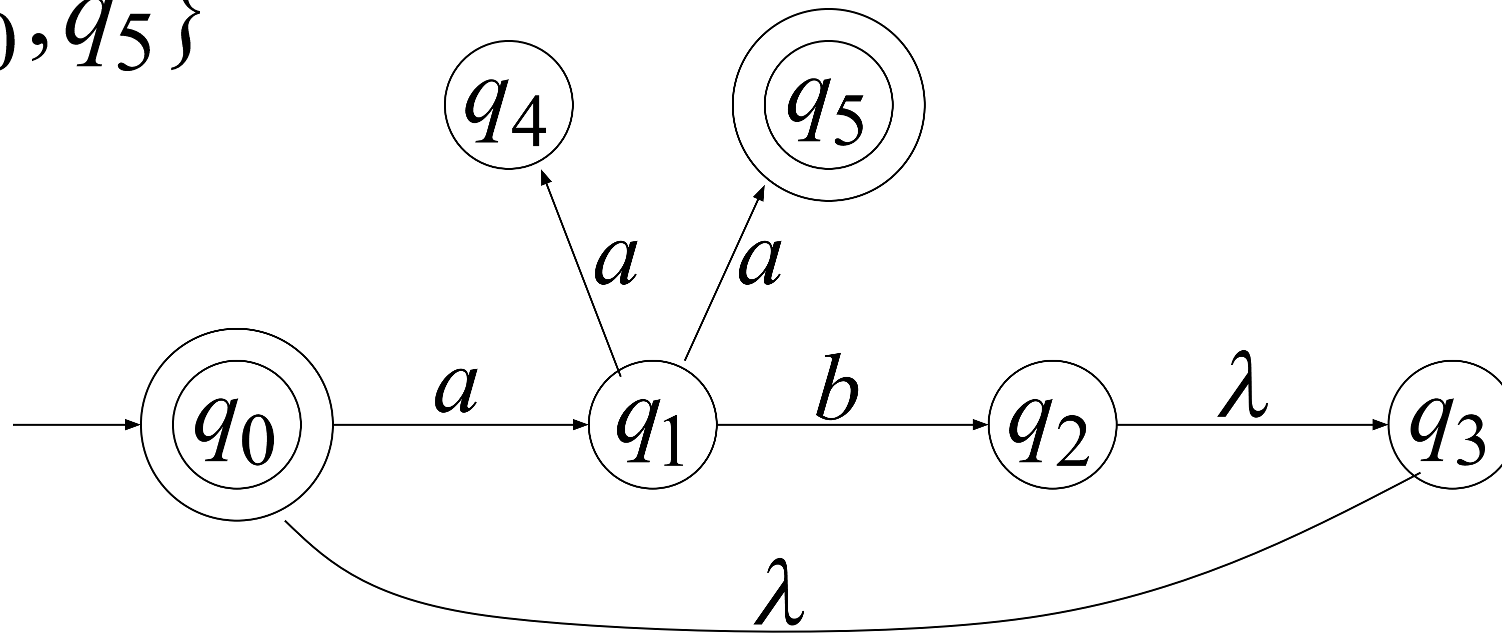
$$F = \{q_0, q_5\}$$



$$\delta^*(q_0, abaa) = \{q_4, \underline{q_5}\} \qquad aaba \in L(M)$$

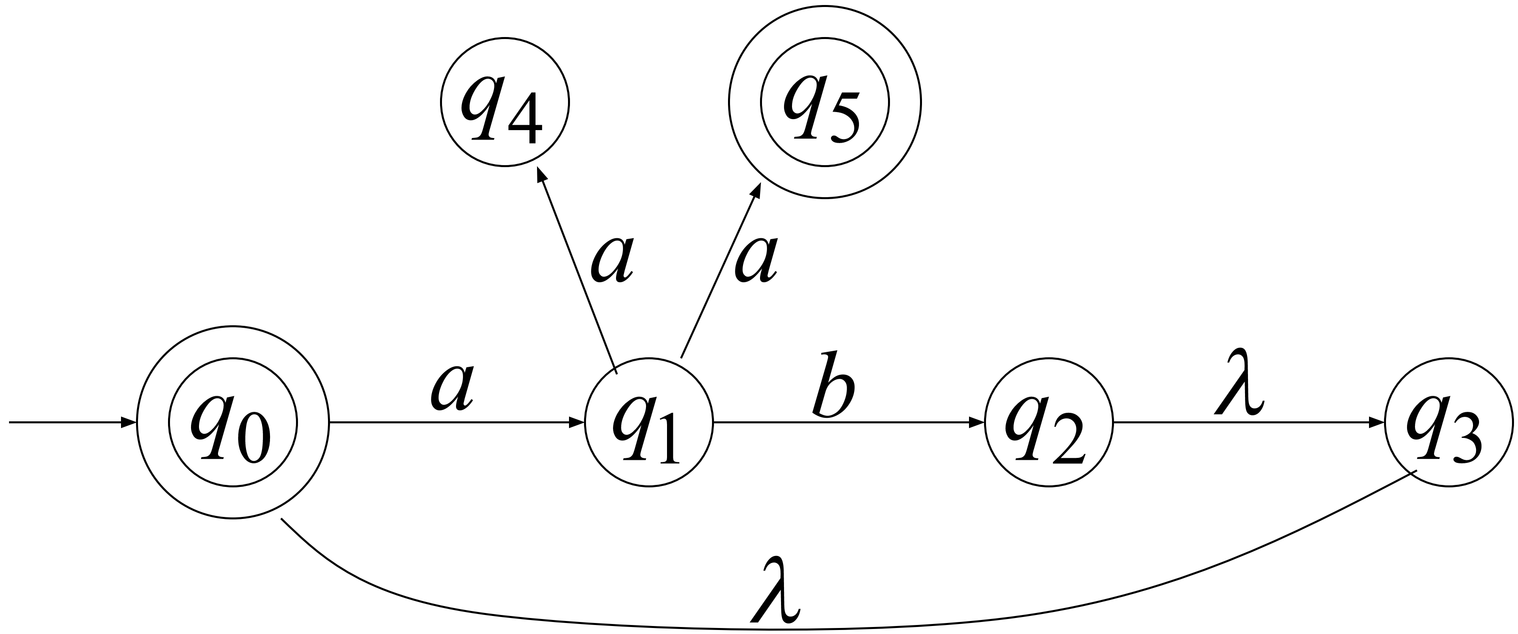
$$\searrow \in F$$

$$F = \{q_0, q_5\}$$



$$\delta^*(q_0, aba) = \{q_1\} \qquad aba \notin L(M)$$

$$\notin F$$



$$L(M) = \{ab\}^* \cup \{ab\}^* \{aa\}$$

Formally

The language accepted by NFA M is:

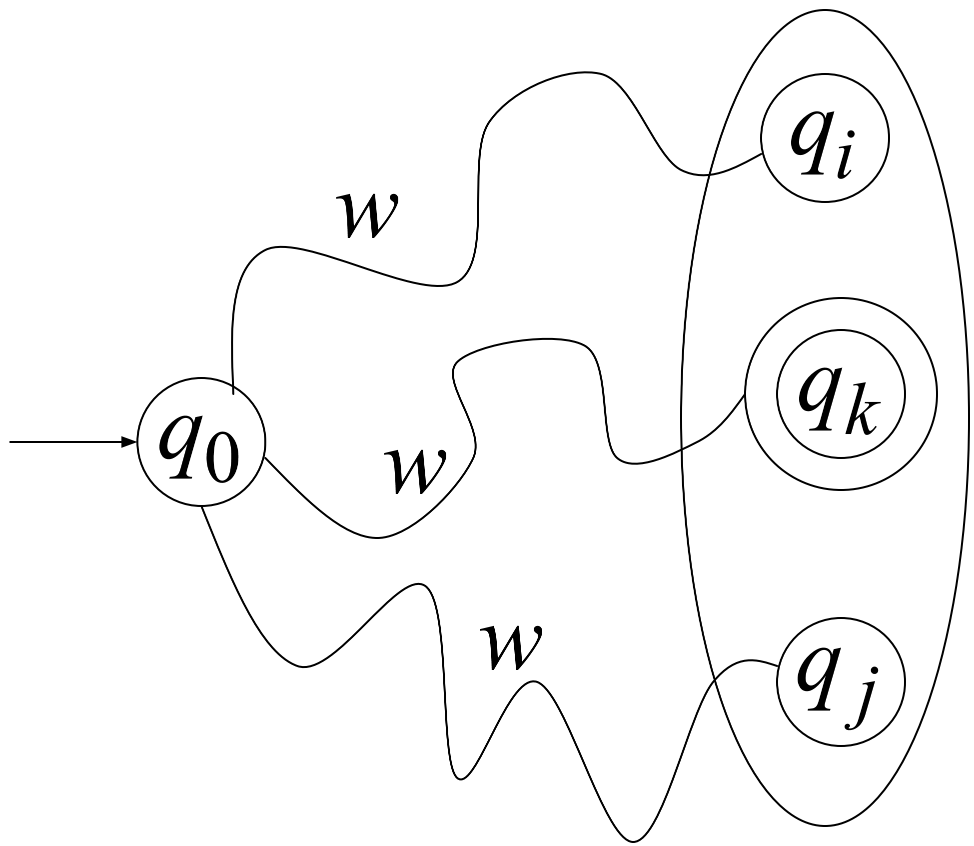
$$L(M) = \{w_1, w_2, w_3, \ldots\}$$

where $\delta^*(q_0, w_m) = \{q_i, q_j, \ldots, q_k, \square\}$

and there is some $q_k \in F$ (final state)

$w \in L(M)$

$\delta^*(q_0, w)$



$q_k \in F$