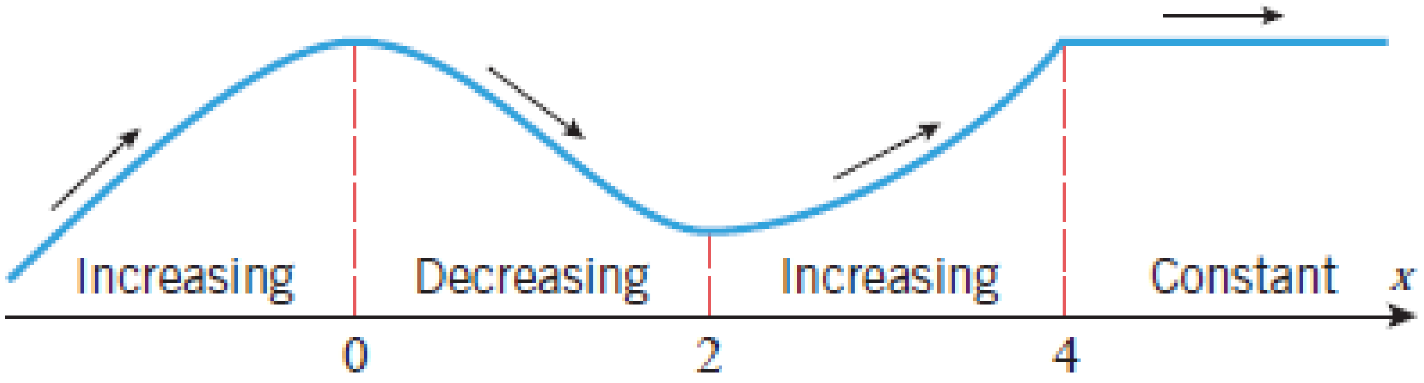
Lecture No 3

4.1 ANALYSIS OF FUNCTIONS I: INCREASE, DECREASE, AND CONCAVITY

INCREASING AND DECREASING FUNCTIONS



▶ **Figure 4.1.1**

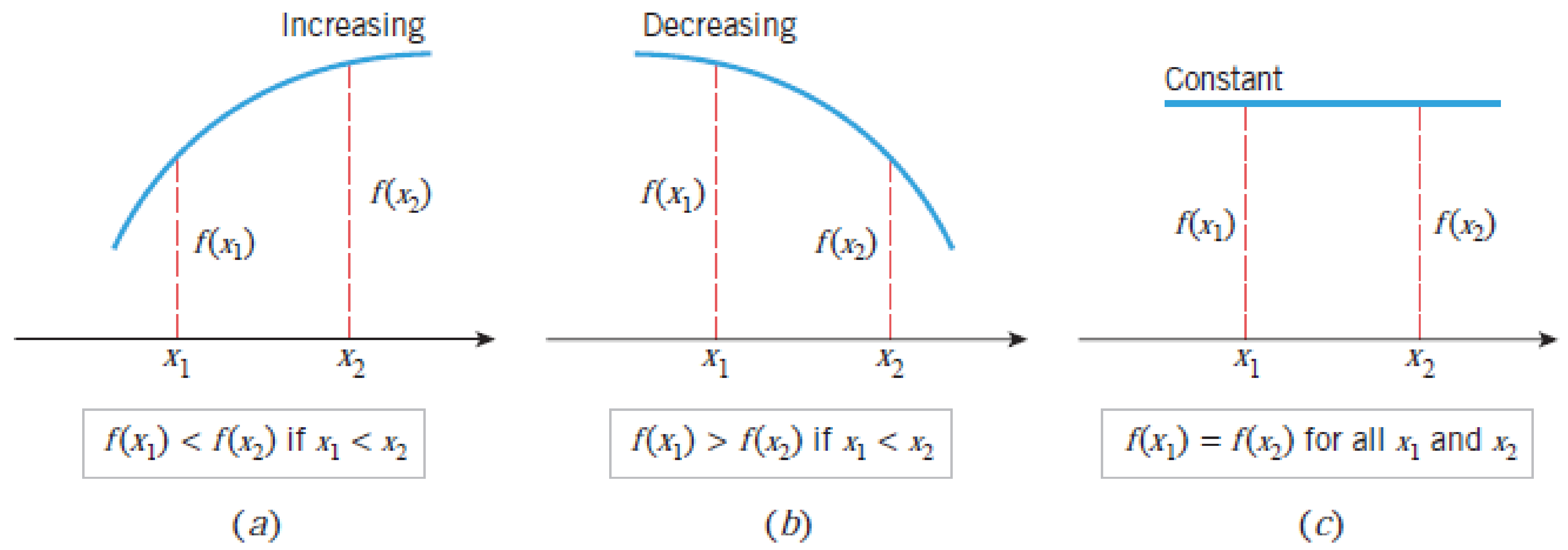
The following definition, which is illustrated in Figure 4.1.2, expresses these intuitive ideas precisely.

4.1.1 DEFINITION Let f be defined on an interval, and let x_1 and x_2 denote points in that interval.

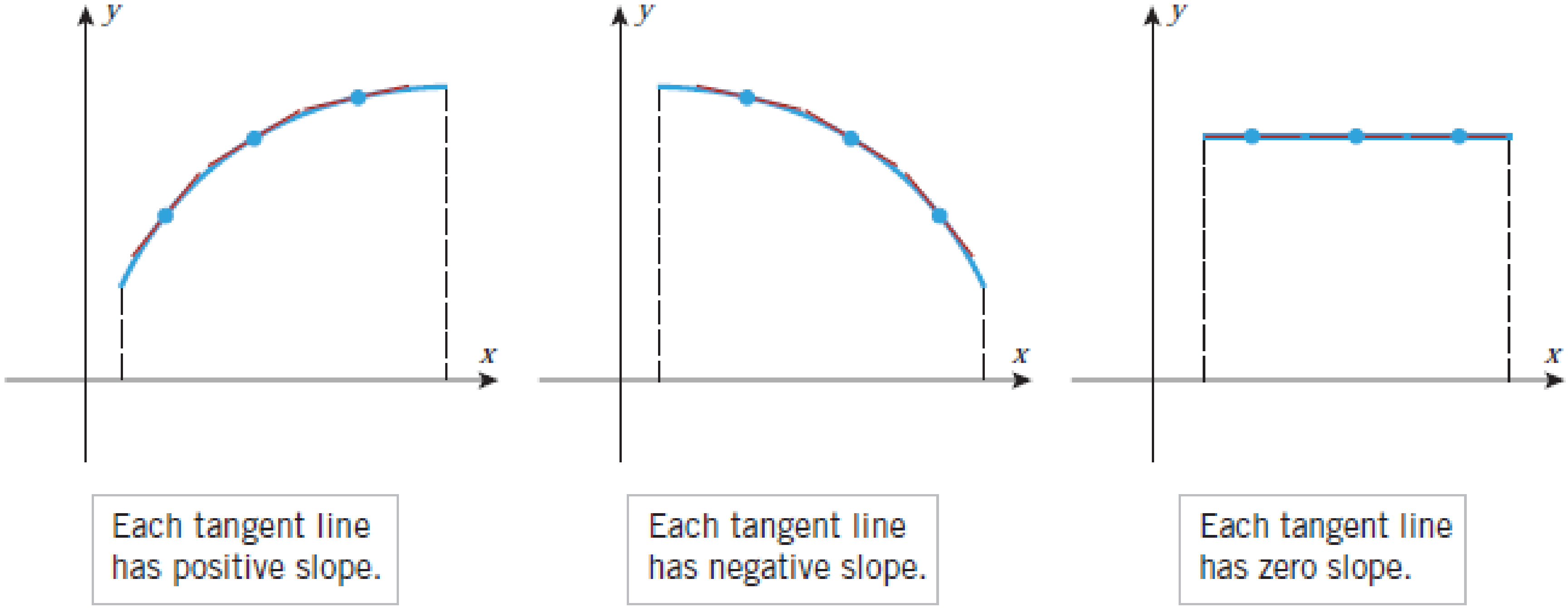
(a) f is ***increasing*** on the interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.

(b) f is ***decreasing*** on the interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.

(c) f is ***constant*** on the interval if $f(x_1) = f(x_2)$ for all points x_1 and x_2.



Increasing
f(x1)
f(x2)
x1
x2
f(x1) < f(x2) if x1 < x2
(a)
Decreasing
f(x1)
f(x2)
x1
x2
f(x1) > f(x2) if x1 < x2
(b)
Constant
f(x1)
f(x2)
x1
x2
f(x1) = f(x2) for all x1 and x2
(c)


y
x
Each tangent line has positive slope.
y
x
Each tangent line has negative slope.
y
x
Each tangent line has zero slope.

▶ **Figure 4.1.3**

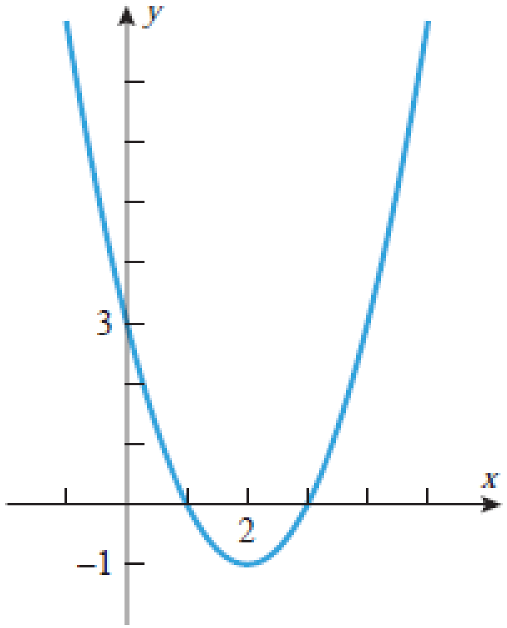
4.1.2 THEOREM *Let f be a function that is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b).*

(a) *If $f'(x) > 0$ for every value of x in (a, b), then f is increasing on $[a, b]$.*

(b) *If $f'(x) < 0$ for every value of x in (a, b), then f is decreasing on $[a, b]$.*

(c) *If $f'(x) = 0$ for every value of x in (a, b), then f is constant on $[a, b]$.*

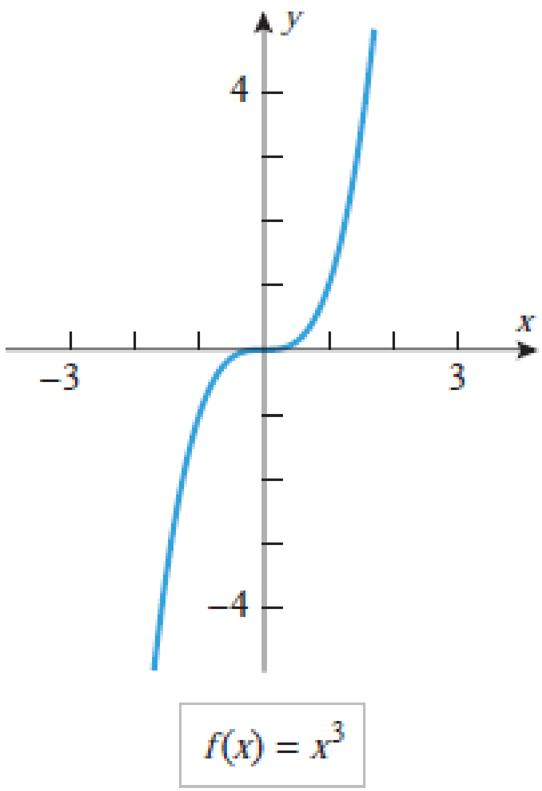
▶ **Example 1** Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing.



$f(x) = x^2 - 4x + 3$

▶ **Figure 4.1.4**

▶ Example 2 Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.

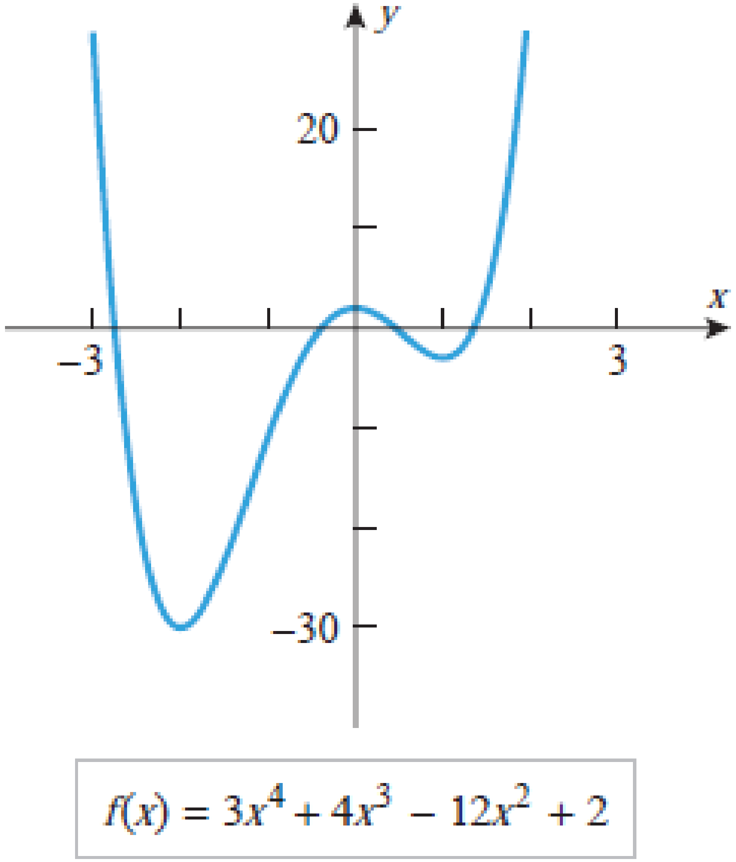


▶ Figure 4.1.5

▶ Example 3

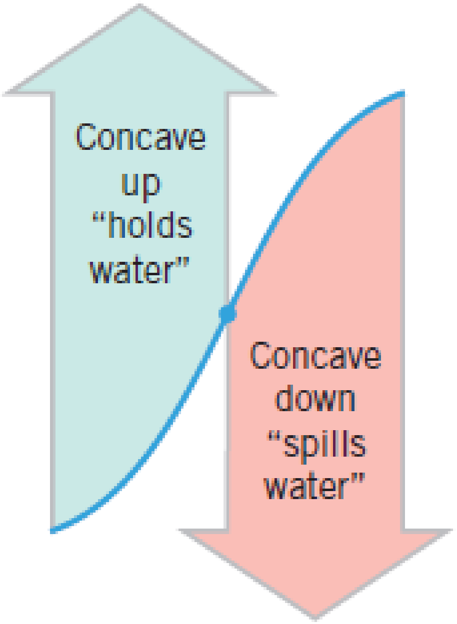
(a) Use the graph of $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ in Figu
about the intervals on which f is increasing or decreasi

(b) Use Theorem 4.1.2 to determine whether your conjectu

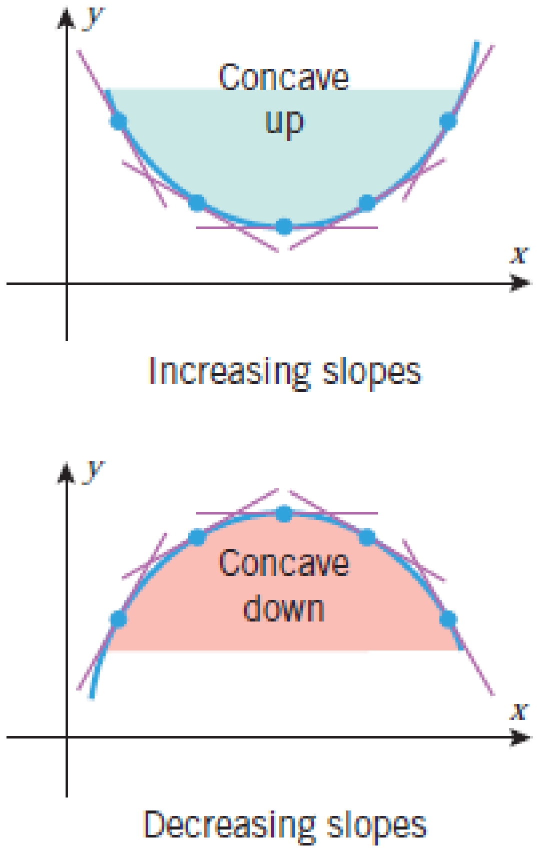


▶ **Figure 4.1.6**

CONCAVITY



▶ Figure 4.1.7



▶ Figure 4.1.8

4.1.3 DEFINITION If f is differentiable on an open interval, then f is said to be ***concave up*** on the open interval if f' is increasing on that interval, and f is said to be ***concave down*** on the open interval if f' is decreasing on that interval.

4.1.4 THEOREM *Let f be twice differentiable on an open interval.*

(a) *If $f''(x) > 0$ for every value of x in the open interval, then f is concave up on that interval.*

(b) *If $f''(x) < 0$ for every value of x in the open interval, then f is concave down on that interval.*

▶ **Example 4** Figure 4.1.4 suggests that the function $f(x) = x^2 - 4x + 3$ is concave up on the interval $(-\infty, +\infty)$. This is consistent with Theorem 4.1.4, since $f'(x) = 2x - 4$ and $f''(x) = 2$, so

$$f''(x) > 0 \quad \text{on the interval } (-\infty, +\infty)$$

INFLECTION POINTS

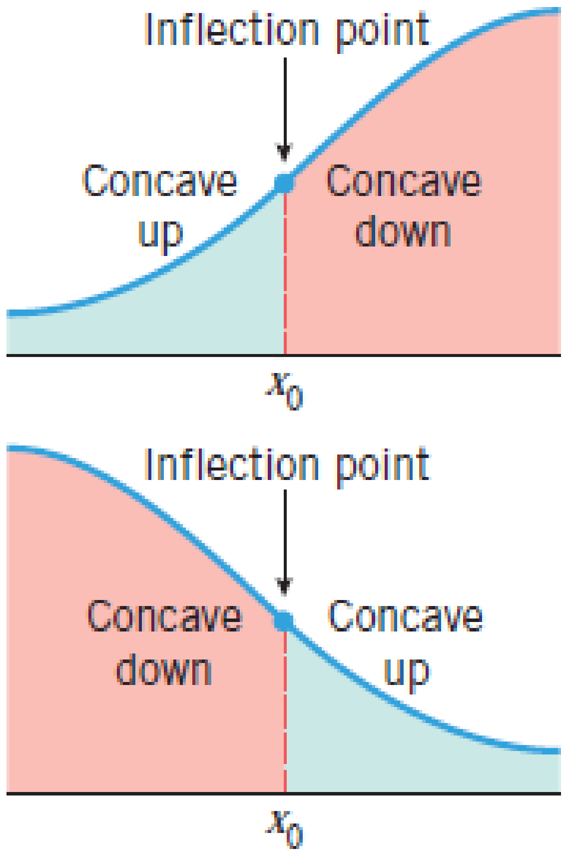


Figure 4.1.9

4.1.5 DEFINITION If f is continuous on an open interval containing a value x_0, and if f changes the direction of its concavity at the point $(x_0, f(x_0))$, then we say that f has an ***inflection point at*** x_0, and we call the point $(x_0, f(x_0))$ on the graph of f an ***inflection point*** of f (Figure 4.1.9).

► **Example 5** Figure 4.1.10 shows the graph of the function $f(x) = x^3 - 3x^2 + 1$. Use the first and second derivatives of f to determine the decreasing, concave up, and concave down. Locate a your conclusions are consistent with the graph.

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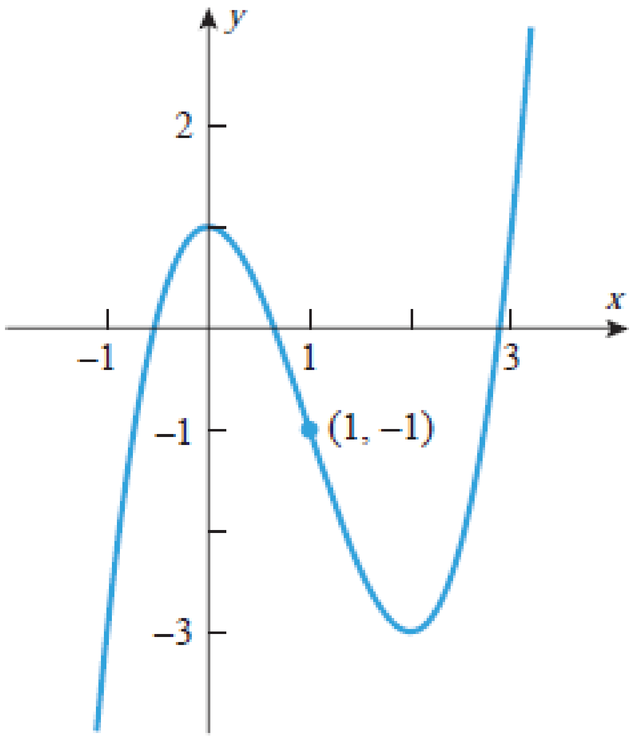


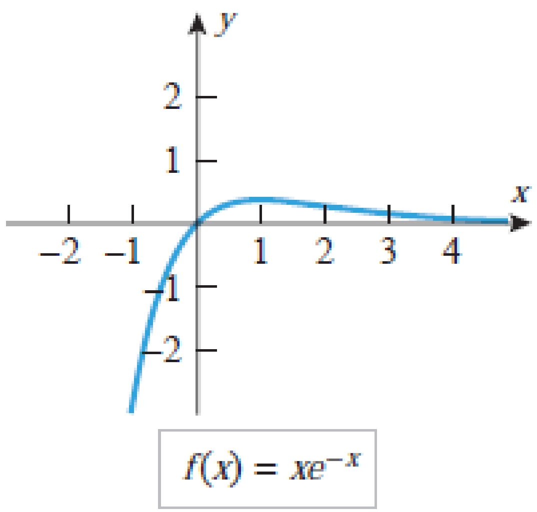
$f(x) = x^3 - 3x^2 + 1$

► **Figure 4.1.10**

INTERVAL	$(3x)(x-2)$	$f'(x)$	CONCLUSION
$x < 0$	$(-)(-)$	$+$	f is increasing on $(-\infty, 0]$
$0 < x < 2$	$(+)(-)$	$-$	f is decreasing on $[0, 2]$
$x > 2$	$(+)(+)$	$+$	f is increasing on $[2, +\infty)$

INTERVAL	$6(x-1)$	$f''(x)$	CONCLUSION
$x < 1$	$(-)$	$-$	f is concave down on $(-\infty, 1)$
$x > 1$	$(+)$	$+$	f is concave up on $(1, +\infty)$

▶ Example 6 Figure 4.1.11 suggests that the function $f(x) = xe^{-x}$ has an inflection point but its exact location is not evident from the graph in this figure. Use the first and second derivatives of f to determine the intervals on which f is increasing, decreasing, concave up, and concave down. Locate all inflection points.



▶ **Figure 4.1.11**

INTERVAL	$(1-x)(e^{-x})$	$f'(x)$	CONCLUSION
$x < 1$	$(+)(+)$	$+$	f is increasing on $(-\infty, 1]$
$x > 1$	$(-)(+)$	$-$	f is decreasing on $[1, +\infty)$

INTERVAL	$(x-2)(e^{-x})$	$f''(x)$	CONCLUSION
$x < 2$	$(-)(+)$	$-$	f is concave down on $(-\infty, 2)$
$x > 2$	$(+)(+)$	$+$	f is concave up on $(2, +\infty)$

Practice Questions: 4.1
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