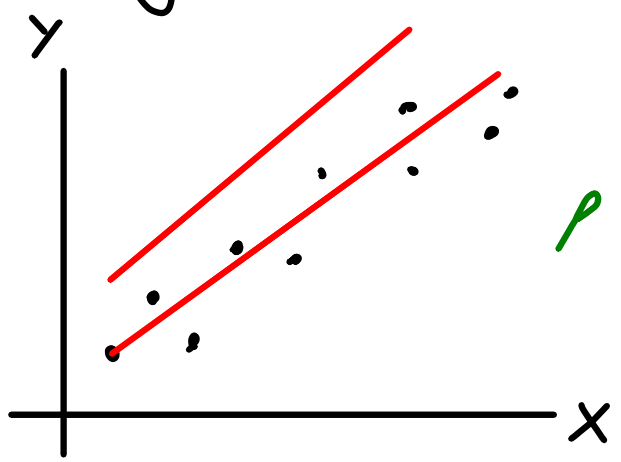
Modelo de Regresión Simple

X	Y
2	5
7	9
⋮	⋮



$\rho \approx 1$

(1) ¿Cómo se que los datos tienen asociación lineal?

$\rho = cor(x,y)$; $-1<\rho<1$

si • $\rho \approx 1$ entonces hay relación lineal directamente proporcional.

• $\rho \approx -1$ entonces hay relación lineal inversamente proporcional.

• $\rho \approx 0$ NO hay relación Lineal

(2) Afirmación

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

$\varepsilon_i \sim N(0,\sigma^2)$; $\forall i = 1,\dots,n$

$$\varepsilon_i = Y_i - \beta_0 - \beta_1 X_i$$

Objetivo: La mejor recta cumple que $\hat{\beta}_0, \hat{\beta}_1$ resuelven:

$$\min_{(\beta_0,\beta_1)\in\mathbb{R}^2} \|(\varepsilon_1,\dots,\varepsilon_n)\|^2$$

$\|(a,b)\|^2 = a^2+b^2$

$$= \min_{(\beta_0,\beta_1)\in\mathbb{R}^2} \sum_{i=1}^{n} \varepsilon_i^2$$

$$= \boxed{\min_{(\beta_0,\beta_1)\in\mathbb{R}^2} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2}$$

Tarea:

- $\dfrac{\partial}{\partial \beta_0}\left(\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2\right) = 0$
- $\dfrac{\partial}{\partial \beta_1}\left(\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2\right) = 0$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \quad ; \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \quad ; \quad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

Recta de Regresión es:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

¿Si $x = 7$ ¿cuanto debe valer aproximadamente Y?

$Y = \beta_0 + \beta_1 \cdot 7 + \varepsilon_i$

$\Rightarrow Y \approx \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 7$

$\Rightarrow Y(x=7) \approx \hat{\beta}_0 + \hat{\beta}_1 \cdot 7$

•—•

$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$; $\varepsilon_i \sim N(0,\sigma^2)$

$\Rightarrow Y_i \sim N(\beta_0 + \beta_1 X_i, \sigma^2)$

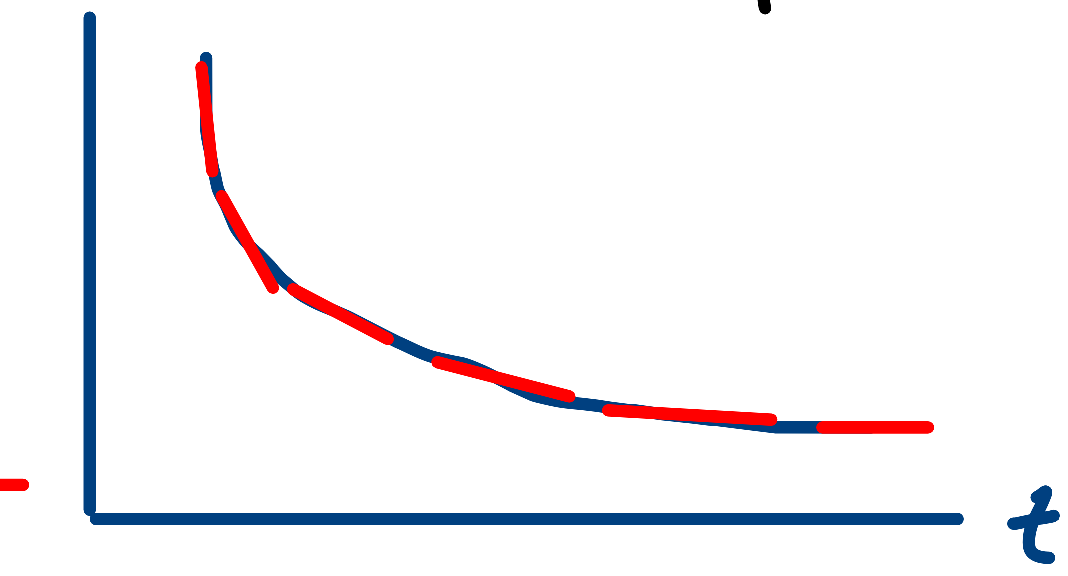
Inferencia: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$

$$\begin{cases} H_0: \beta_1 = 0 \\ H_1: \beta_1 \neq 0 \end{cases}$$

$Log = Ln$

$$\boxed{T(t) = A e^{-Kt}}$$

$Y_i = \beta_0 + \beta_1 X_i$



t	T
t_i	T_i

Ajusta un modelo de Regresión lineal

$$Log(T(t)) = Log(A) - Kt$$

$$Y_t = \beta_0 + \beta_1 t \quad ; \quad Lm(log(T) \sim t)$$

$$\Rightarrow \hat{\beta}_0 = \widehat{Log(A)} \Rightarrow \hat{\beta}_0 = Log(\hat{A})$$

$$\Rightarrow \hat{A} = e^{\widehat{\beta_0}}$$

$$\Rightarrow \hat{\beta}_1 = \widehat{-K} \qquad \Rightarrow \hat{K} = -\hat{\beta}_1$$

Dol Pesos

$\hat{A}, \hat{B}$

$$\frac{\text{Dolar} - A}{\text{Peso} - B} = C$$

$$\Rightarrow \text{Dolar} - A = (\text{Peso} - B)\, C$$

$$\Rightarrow \text{Dolar} = C \cdot \text{Peso} - CB + A$$

$$\Rightarrow Y_i = \beta_1 \cdot X_i + \beta_0$$

Pet	Lit

; A y B

$$Pet = A \underbrace{Sin\left(5 \cdot litros\right)}\ - Cos\left(B\right)$$

$$Y_i = \beta_1 X_i + \beta_0 \; ; Lm(Y \sim x)$$

$$Y_i = Pet$$

$$X_i = Sin(5 \cdot Lit) \; ; Lm(Pet \sim Sin(5 Lit))$$