1 Some Theory

Theory Exercise 1 *Proof the following*
If T is a contraction then $T^n x$ is a Cauchy sequence.

Theory Exercise 2 *Consider the growth model with only capital where productivity z is stochastic and $\ln z$ follows an AR-1 process such that $E(\log z'|z) = \mu(1-\rho) + \rho z$. Assume $\delta = 1$. Show that the value function can be written as $V(k, z) = A + B \ln k + C \ln z$.*

2 Basics

Exercise 1 (a.Value Function Iteration) *Assume household income can be either 10/9 or 1/9. The probability to move from high income to low income is 4/90 and the probability to move from low income to high income is 2/5. The household can borrow and lend at a 4/90 interest rate and has log utility in consumption. The borrowing constraint is -9/4. Its discount factor is 0.95. Solve the consumption savings model by value-function iteration. Consider first only on-grid choices and then off-grid choices. Plot value and policy functions Use a 30-point log-grid for asset holdings between the borrowing limit and* 6*.*

Exercise 1 (b. Policy Function Iteration) *Same as Exercise 1, but solve using policy function iteration.*

3 Endogenous grid method

Exercise 2 (Endogenous Grid Method) *Assume household income can be either 10/9 or 1/9. The probability to move from high income to low income is 4/90 and the probability to move from low income to high income is 2/5. The household can borrow and lend at a 4/90 interest rate and has log utility in consumption. The borrowing constraint is -9/4. Its discount factor is 0.95. Solve the consumption savings model using the endogenous grid method and plot value and policy functions and compare time to compute to off-grid VFI! Use a 100-point log-grid for asset holdings between the borrowing limit and* 6*.*

4 Young's method & Solving an Aiyagari economy

Exercise 3 (Bewley Model) *Re-use the codes/model from exercise 2, but now assume that $r = 0$, the asset they save in bears no interest (money) and that households have a CRRA utility function with risk aversion 2. Further assume the household cannot borrow. Set the maximum asset holdings to 3. Solve the model with EGM, then*

1. *Simulate an agent over T=100,000 periods of time and calculate the average asset holding of the agent in periods* $t \geq 10,000$.

2. *Create a transition matrix from the policy functions. Use this to calculate the ergodic distribution of the model. Calculate the expected asset holdings from that ergodic distribution.*

3. *Compare the histograms of the two distributions and aggregate money demand.*

Exercise 4 (Hugget model) *Extend Exercise 2 to calculate the equilibrium interest rate. Use Young's method and MATLAB's* $fzero$ *function to find a root in the excess demand for bonds function. Assume that households have a CRRA utility function with risk aversion 2, set the maximum asset holdings to 10. Plot the demand and supply for funds.*

Exercise 5 (Aiyagari model) *Extend Exercise 4 to calculate the equilibrium interest rate (same as before), but now assume that the asset is physical capital and households have a CRRA utility function with risk aversion 4, set the maximum asset holdings to 20. Plot the demand and supply for funds. The production function is*

$$F(K, N) = K^{\alpha} N^{1-\alpha}. \tag{1}$$

Let $\delta = .1$, $\beta = 0.95, \alpha = 0.36$.

5 A HANC model

Exercise 6 ***Take an Aiyagari setup:*** *Aggregate output is given by*

$$F(Z, K, N) = Z K^{\alpha} N^{1-\alpha}. \tag{2}$$

Aggregate productivity, Z follows a log AR-1 process with autocorrelation $\rho_Z = 0.75$ *and standard deviation* $\sigma = 0.01$. *Other parameters are: depreciation,* $\delta = 1/10$; *discount factor,* $\beta = 0.95$; *capital share,* $\alpha = 0.36$; *risk aversion,* $\gamma = 1$.

Agents can be employed (productivity 10/9) or unemployed. If unemployed they move with 2/5 probability to employment and if employed they loose their job with prob. 4/90. When unemployed, they receive 50% of the employed wage financed by a corresponding labor tax.

1. *Solve for the steady state without aggregate risk.*

2. *Solve using Reiters's method.*