Nonfundamental Asset Price Fluctuations and the Distributional Origins of Asset Premia

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Abstract

This paper studies how nonfundamental asset price fluctuations affect macroeconomic aggregates, inequality, household portfolios, and asset premia. To address this question, I estimate a heterogeneous-agent model with incomplete markets, portfolio choice, and nonfundamental asset price shocks using a Bayesian approach in the sequence space. Although nonfundamental asset price shocks have limited effects on aggregate variables and standard inequality measures, they affect households heterogeneously across the wealth distribution. As a result, up to 47 percent of the observed equity premium is explained by the compensation demanded by households exposed to nonfundamental asset price risk. Together with standard business cycle shocks, this mechanism helps reconcile consumption-based asset pricing theory with the empirical magnitude of equity and term premia.

Keywords— Incomplete Markets, Household Savings, Asset Pricing, Equity Premium
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1 Introduction

Asset prices display substantial volatility, reflecting not only fluctuations in economic fundamentals but also variation in expected returns.[1] A prominent strand of the finance literature attributes this excess volatility to nonfundamental asset price movements driven by shifts in investor beliefs. These shifts may arise because traders form expectations under bounded rationality, because of misperceptions,[2] or because they operate with incomplete or dispersed information.[3] While recent research has begun to explore the macroeconomic implications of such nonfundamental fluctuations[4], their role in shaping household portfolios and driving risk premia is insufficiently understood.

This paper contributes to filling this gap by developing a quantitative heterogeneous-agent New Keynesian model with portfolio choice and nonfundamental asset price shocks in a segmented equity market. The model features a standard New Keynesian supply side, where firms and unions set prices and wages under monopolistic competition and nominal rigidities. Households face idiosyncratic income risk and have access only to incomplete markets. To insure against aggregate shocks, they invest in bonds of varying maturities, physical capital, and an equity fund, and demand risk premia as compensation for holding this portfolio. Individual equities, by contrast, are traded in a segmented financial market, where noise traders induce asset price volatility through demand based on fluctuating expectations about future returns or price changes. I estimate the model using Bayesian methods on macroeconomic and financial time-series data, allowing me to quantify how nonfundamental shocks affect aggregate variables, the wealth and income distribution, and households along the distribution.

I find that nonfundamental asset price shocks are empirically important not only for explaining asset price volatility as by construction, but also for shaping the *level* of asset premia. At the same time, their effects on macroeconomic aggregates and inequality are

[1] I follow Cochrane (2011) and use ”expected returns”, ”discount rates”, and ”risk premia” as synonyms. See Cochrane (2011) and Shiller (2014) for literature reviews on the importance of expected returns for explaining asset price fluctuations.

[2] Seminal papers that feature non-rational noise-traders are Kyle (1985), De Long et al. (1990), and Campbell and Kyle (1993). More recent applications of noise traders in exchange rate markets are by Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021), Fukui, Nakamura and Steinsson (2023), and Itskhoki and Mukhin (2025), among others.

[3] Seminal contributions are Futia (1981) and Singleton (1986), but the idea of goes back to the statement of Keynes (1936) that asset markets behave like beauty contests. More recent applications to financial markets and exchange rates include Allen, Morris and Shin (2006), Bacchetta and Wincoop (2006), Rondina and Walker (2021), Caines and Winkler (2021), Angeletos and Huo (2021), and Angeletos, Lorenzoni and Pavan (2023), among others. See Angeletos and Lian (2016) for a review.

[4] Martin and Ventura (2012) and Miao and Wang (2018) study the impact of nonfundamental asset price fluctuations on investment, Gali (2014), Caballero and Simsek (2020), and Gali (2021) on consumption.

significant but modest. Among aggregate variables, the shock contributes most to the forecast error variance of consumption and investment, accounting for 9 and 12 percent of their total variance, respectively. Technology shocks still explain the bulk of investment, consumption, and output fluctuations. In terms of distributional consequences, the shock raises the value of wealth for households at the top of the distribution in response to positive fluctuations, thereby increasing inequality. Quantitatively, however, this effect is small: nonfundamental asset price shocks account for only 7 percent of the variance in the Gini coefficient of wealth.

The model not only replicates the volatility of aggregate variables but also generates sizable financial market premia, both for holding long-term bonds and for equity. Nonfundamental fluctuations in equity prices play a key role in this result. By construction, nonfundamental shocks account for all excess volatility in empirical returns, to which the model is estimated. Through the lens of the model, roughly two thirds of return volatility is attributed to nonfundamental sources. Given incomplete markets and heterogeneous household exposures, even moderately risk-averse households demand substantial compensation for bearing both fundamental and nonfundamental risks. The model produces an equity premium of 3.9 percent, which is somewhat below empirical estimates but remains broadly consistent with them. Approximately 47 percent of this premium arises from exposure to nonfundamental asset price risk. Although equity holders are typically well-insured due to high savings, they demand compensation for the substantial fluctuations induced by nonfundamental asset price risk.

I also show that the model accounts for roughly half of the observed term premia on bonds. Importantly, the model's ability to generate both equity and term premia disappears when the assumption of incomplete markets is removed. This highlights that a largely standard heterogeneous-agent business cycle framework is not only well suited to capturing aggregate dynamics and inequality, but also capable of reproducing key features of financial markets, such as risk premia. Nonfundamental asset price fluctuations play a crucial role in achieving this result.

The model builds on the one-asset framework of Auclert, Rognlie and Straub (2025), which I extend to incorporate household portfolio choice following Auclert et al. (2024). I embed this household block into the quantitative macroeconomic environment of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007), and introduce a segmented equity market featuring nonfundamental asset price shocks in the spirit of De Long et al. (1990), where noise traders drive deviations from fundamental values.[5]

[5] In addition to microfounding nonfundamental asset price fluctuations through noise traders, I derive al-

Financial markets are segmented because households do not directly trade individual stocks. Instead, they invest only in a stock market index. Asset prices arise from the interaction of three agents: rational traders, noise traders, and an index fund. Rational traders price equities based on fundamentals, while noise traders generate unpredictable shifts in demand, leading to deviations from the present discounted value of expected future dividends. The index fund aggregates stocks into a diversified portfolio and sells shares of it to households, thereby exposing them indirectly to aggregate asset price fluctuations driven by noise. Households face uninsurable idiosyncratic income risk and self-insure by choosing from a portfolio of eight assets: equity, physical capital, and government bonds of six different maturities. For each structural shock, I compute the premia households require for holding each asset, allowing me to quantify average asset premia and decompose them by risk source. On the production side, the model features standard New Keynesian frictions. Retail firms differentiate wholesale goods and set prices subject to nominal rigidities and partial inflation indexation. Wholesale firms accumulate capital under adjustment costs, while unions set wages under Calvo-style stickiness, also with partial indexation. The government finances its expenditures through taxes and by issuing bonds of varying maturities. Monetary policy follows a standard Taylor rule aimed at stabilizing inflation and output.

To clarify the intuition behind the key findings, consider first the role of the nonfundamental asset price shock. By construction, this shock captures all fluctuations in equity prices that are not driven by economic fundamentals. It is therefore unsurprising that it accounts for a substantial share of equity price volatility in the model. Why, then, do such shocks generate only limited aggregate fluctuations? Three mechanisms are central. First, equity comprises only about one fifth of total household assets. As a result, even large movements in equity prices induce relatively modest changes in aggregate household wealth. Second, the aggregate marginal propensity to consume (MPC) out of total wealth is low, as high-wealth low-MPC households hold most of the wealth. As a result, fluctuations in asset values translate only weakly into changes in aggregate consumption. Third, the variance of the nonfundamental asset price shock is significantly smaller than that of the main business cycle shocks, such as total factor productivity (TFP) and investment-specific technology. Together, these factors imply that although nonfundamental shocks drive asset price volatility, they have muted effects on macroeconomic aggregates such as consumption and output.

A similar logic applies to their limited role in explaining wealth inequality. While

ternative formulations based on incomplete information, which yield an identical reduced-form pricing equation.

nonfundamental asset price shocks primarily affect wealthier households—who hold disproportionate shares of risky assets—their overall impact on the wealth distribution remains small. This is because the primary sources of redistribution in the model stem from shocks with broader macroeconomic consequences, such as TFP, investment-specific technology, and government expenditure shocks. As a result, nonfundamental shocks have limited power in shaping inequality dynamics relative to other sources of economic risk.

While nonfundamental asset price shocks have limited effects on aggregate variables, they play a central role in shaping asset risk premia. The computation of these premia follows the approach of Auclert et al. (2024), which adapts standard consumption-based asset pricing theory to linearized heterogeneous-agent economies. In this framework, assets that comove positively with the intertemporal marginal rate of substitution must offer higher expected returns to compensate for the risk they impose. This mechanism is particularly relevant for equity holders exposed to nonfundamental asset price risk. Such shocks generate large and persistent fluctuations in asset returns that disproportionately affect wealthy households, who hold the majority of equity. Although these households have low marginal propensities to consume out of wealth, the magnitude of return fluctuations leads to elevated consumption volatility—substantially higher than that of the aggregate. As a result, equity holders demand a premium for bearing this risk.

This paper contributes to four strands of the literature. First, it relates to the work that decomposes asset premia through the lens of estimated macroeconomic models. The seminal contribution by Bansal and Yaron (2004) introduces long-run risk into asset pricing and shows that it helps replicate observed equity premia. Hansen, Heaton and Li (2008) extend this idea by estimating a structural model with long-run risk and evaluating its ability to match asset price behavior. Rudebusch and Swanson (2012) similarly use an estimated macro-finance model to highlight the role of long-run risk in explaining term premia. Closely related, Schorfheide, Song and Yaron (2018) estimate a DSGE model with Epstein-Zin preferences and decompose the equity premium into short- and long-run risk components, as well as a time-varying risk premium arising from changes in volatility. While these studies provide valuable decompositions, they primarily distinguish risk by its persistence—short-run versus long-run—rather than by its structural economic source. This paper goes one step further by decomposing the equity premium into contributions from specific macroeconomic shocks, such as productivity, fiscal policy, and nonfundamental asset price fluctuations. In doing so, it provides a more granular understanding of the economic drivers behind risk premia. Moreover, the model offers a microfoundation for time-varying risk premia, as households' exposure to macroeconomic shocks endogenously determines the compensation they require in equilibrium.

Second, it contributes to the large body of work studying how heterogeneity shapes our understanding of business cycle drivers. Numerous papers have examined the responses of heterogeneous-agent models to monetary[6] and fiscal[7] policy.

By contrast, relatively few contributions study the role of asset price fluctuations in heterogeneous-agent settings. J. Fernández-Villaverde and Levintal (2024) examine the response of a heterogeneous-agent economy to the disaster shock introduced by Barro (2006), but they do not incorporate demand-side determinacy, so the response reflects only the partial-equilibrium reaction of households to changes in returns. Angeletos and Calvet (2006) analytically derive an equilibrium with heterogeneous agents and fluctuating future returns under CARA preferences, but they abstract from Keynesian frictions and do not study heterogeneity in household responses. The present paper differs from these approaches by examining how asset price fluctuations affect a heterogeneous-agent economy with Keynesian frictions, allowing general equilibrium forces, particularly shifts in wages and interest rates, to shape the household-level response. Closest to my approach is Auclert et al. (2024), whose methodology I adopt. However, I extend their framework by allowing for nine distinct assets instead of two and embedding the household block into a fully-specified quantitative macroeconomic environment. My estimation results show that, when applied in this richer setting, their methodology can generate sizable equity premia, contrary to the findings in their tractable HANK model.

Third, this paper relates to the large literature on asset price bubbles. While early work focuses on the theoretical possibility of bubbles,[8] more recent research explores the implications of bubbles for investment dynamics[9] as well as fiscal and monetary policy.[10] The present paper differs by focusing on how asset price fluctuations affect consumption rather than investment. Moreover, while most of the existing literature emphasizes the supply-side effects of bubbles, particularly their role in relaxing binding financial

[6] Among others, see McKay, Nakamura and Steinsson (2016), Kaplan, Moll and Violante (2018), Auclert (2019), Bayer et al. (2019), Acharya and Dogra (2020), Bilbiie (2020), McKay and Wieland (2021), Luetticke (2021), Kekre and Lenel (2022), Acharya, Challe and Dogra (2023), Bayer, Born and Luetticke (2024), and Auclert, Rognlie and Straub (2024a). See McKay and Wolf (2023) for a summary.

[7] Among others, see Kaplan and Violante (2014), McKay and Reis (2016), Bayer, Born and Luetticke (2022), Auclert, Bardóczy and Rognlie (2023), and Angeletos, Lian and Wolf (2024).

[8] See Samuelson (1958), Tirole (1985), Abel et al. (1989), and Santos and Woodford (1997).

[9] See Farhi and Tirole (2011), Martin and Ventura (2012), Miao, Wang and Xu (2015), Miao and Wang (2018), Larin (2020), and Guerron-Quintana, Hirano and Jinnai (2023).

[10] See Diamond (1965), Domeij and Ellingsen (2018), and Angeletos, Collard and Dellas (2023) for fiscal policy, and Kiyotaki and Moore (2019), Asriyan et al. (2020), and Angeletos, Lorenzoni and Pavan (2023) for monetary policy.

frictions, this paper emphasizes demand-side transmission channels. Within the bubble literature, Gali (2014) and Gali (2021) are closest in terms of transmission mechanisms. In their overlapping-generations model, asset price bubbles affect household wealth and thus consumption, which in turn drives aggregate demand. However, Gali (2021) features only stylized heterogeneity via the perpetual youth structure of Yaari (1965) and Blanchard (1985). In contrast, the present paper allows for rich household heterogeneity and therefore provides a more granular assessment of distributional and aggregate effects. Loosely related is also the contribution of Caballero and Simsek (2020), who introduce volatility shocks to asset returns in a New Keynesian model with households holding heterogeneous beliefs. While their model includes New Keynesian frictions, it features only limited heterogeneity, distinguishing just two household types.

Finally, this paper relates to the empirical literature on how asset price fluctuations affect the macroeconomy. Typically, the impact of economic shocks is analyzed empirically. However, because asset prices and real activity are jointly determined, and only few empirical studies examine how asset price changes affect the distribution of households,[11] we adopt a model-based approach instead. Chodorow-Reich, Nenov and Simsek (2021) provides some of the most recent evidence on the aggregate effects of asset price fluctuations. In response to a 20% national increase in stock market valuations, they document a 1.7% increase in local labor supply after two years. Our model-based approach also allows us to study the distributional and welfare consequences of asset price fluctuations. This links our work to studies such as Kuhn, Schularick and Steins (2020) and Cioffi (2021), which examine how asset prices affect the income and wealth distribution, as well as Fagereng et al. (2025), who investigates the welfare implications of asset price changes.

The remainder of the paper is organized as follows: Section 2 presents the model, Section 3 discusses the calibration and the Bayesian estimation of the model. Section 4 illustrates the quantitative effect of an asset price shock in the economy and decomposes the equity premium. Finally, section 5 concludes.

2 HANK Model with Nonfundamental Asset Price Shocks

This section presents a heterogeneous-agent New Keynesian (HANK) model that incorporates household portfolio choice, segmented financial markets, and nonfundamental asset price fluctuations. Households choose between equity, capital, and government bonds of varying maturities to self-insure against idiosyncratic income risk and to hedge

[11] Chodorow-Reich, Nenov and Simsek (2021) reviews some of these contributions.

exposure to aggregate shocks. Risk premia arise endogenously as households demand premia in compensation for utility fluctuations induced by macroeconomic risk. The financial sector features a segmented equity market for individual equities as households can only trade in an equity fund. Fundamental traders interact with noise traders such that equilibrium asset prices feature fluctuations unrelated to fundamentals. These nonfundamental fluctuations are transmitted to households indirectly through an index fund that intermediates between traders and household investors. The model embeds this financial structure into a quantitative general equilibrium framework with standard New Keynesian frictions in pricing, capital adjustment, and wage setting, following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). The government issues bonds of different maturities and conducts fiscal policy through taxes and spending, while monetary policy follows a standard Taylor rule.

2.1 Nonfundamental Asset Price Shocks in the Equity Market

The equity market model[12] builds on De Long et al. (1990) and Gabaix and Maggiori (2015). The market is segmented in the sense that households do not trade individual equities directly, but instead invest exclusively in an equity index fund. Three types of agents operate in the equity market: fundamental traders, noise traders, and an equity fund that intermediates between them and households. I assume a unit continuum of traders indexed by $l \in [0, 1]$, of which a measure ν are fundamental traders ($l \in [0, \nu]$) and a measure $1 - \nu$ are noise traders ($l \in (\nu, 1]$). All traders live for two periods: they purchase a portfolio of assets in the first period and earn returns in the second. Since traders are owned by the equity fund, they finance their purchases with revenues collected from household investments and return profits to the fund, which are ultimately passed on to households. Both types of traders trade a continuum of individual equities indexed by $j \in [0, 1]$, each issued by a retail firm. Individual equity prices are determined in equilibrium through market clearing. The index fund aggregates these equities into a diversified portfolio and sells shares of the fund to households.

Fundamental Traders: Each fundamental trader is risk-neutral[13], derives utility from the profits of their equity portfolio, discounts the future at the risk-free rate $1 + r_{t+1}$, and incurs quadratic disutility from monitoring firm-specific fundamentals, which increases

[12] In Appendix I, I derive an alternative formulation based on incomplete information that yields the same reduced-form equilibrium asset price.

[13] I can also integrate limits to arbitrage by assuming that fundamental traders are risk averse according to a CARA utility function as in De Long et al. (1990), or Bacchetta and Wincoop (2006).

with the size of the trader's net position. Each fundamental trader $l \in [0, \nu]$ chooses a portfolio allocation $\{\theta_{ljt}\}_{j\in[0,1]}$ to maximize utility U_{lt}:

$$U_{lt} = \max_{\{\theta_{ljt}\}} \int_0^1 \left[-q_{jt}\theta_{ljt} + \mathbb{E}_t \left(\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right) \theta_{ljt} - \frac{1}{2}\theta_{ljt}^2 \right] dj,$$

where d_{jt} and q_{jt} denote the dividend and price of equity j, respectively. The functional form yields a linear demand schedule for each equity:

$$\theta_{ljt} = -q_{jt} + \mathbb{E}_t \left[\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right] \quad \forall\, l \in [0, \nu]. \tag{1}$$

Noise Traders: Noise traders represent the second group of market participants. Unlike fundamental traders, their investment behavior is unrelated to economic fundamentals. This may reflect behavioral motives or non-rational stock-picking strategies. Specifically, the demand of each noise trader for stock j is given by:

$$\theta_{ljt} = \tilde{\xi}_t + \epsilon^{\theta}_{ljt},$$

where $\tilde{\xi}_t$ is an aggregate noise-trader demand component and ϵ^{θ}_{ljt} is an idiosyncratic, iid shock to the noise trader–stock j demand.

Equity Fund: The equity fund intermediates between households and traders. It finances trader purchases using household contributions, collects dividend and capital gains from traders' equity holdings, and distributes the resulting returns back to households. The fund aggregates all equities into a single index, which households can invest in. The price of the index fund equals the average price of the underlying equities and pays the average of the underlying dividends:

$$q_t = \int_0^1 q_{jt} dj, \qquad d_t = \int_0^1 d_{jt} dj.$$

Equilibrium Asset Prices: In equilibrium, the aggregate demand for each equity must equal its supply (normalized to one), implying: $\int_0^1 \theta_{ljt} dl = 1$. In Appendix I I illustrate that this market-clearing condition implies that the price of equity j is:

$$q_{jt} = \mathbb{E}_t \left[\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right] + \xi_t, \tag{2}$$

where the effective asset price shock is defined as $\xi_t \equiv \frac{(1-\nu)\tilde{\xi}_t - 1}{\nu}$, and follows an AR(1) process $\xi_t = \rho_q \xi_{t-1} + \epsilon_t^q$ where $\epsilon_t^q \sim \mathcal{N}(0, \sigma_q^2)$. This formulation makes the role of non-fundamental fluctuations explicit and facilitates their estimation. In a symmetric equilibrium where all equities are identical,[14] the aggregate index fund price is:

$$q_t = \mathbb{E}_t \left[\frac{d_{t+1} + q_{t+1}}{1 + r_{t+1}} \right] + \xi_t. \tag{3}$$

Hence, noise trader demand shifts the entire equity price level upward, increasing the valuation of the market even when fundamentals remain unchanged.

2.2 Household Sector with Portfolio Choice and Asset Pricing

The household side of the model combines a standard consumption–savings problem under idiosyncratic income risk with portfolio choice to hedge against aggregate fluctuations. Households earn net labor income, accumulate assets to self-insure against idiosyncratic shocks, and allocate their portfolios across available assets to mitigate exposure to aggregate risk.

Idiosyncratic Risk: There is a continuum of households indexed by $i \in [0, 1]$, which are ex-ante identical, but differ ex-post due to uninsurable idiosyncratic risk in their labor efficiency e_{it} and their discount factor $\tilde{\beta}_{it}$. A Markov Chain describes the transitions between a state $(e, \tilde{\beta})$ and any other state $(e', \tilde{\beta}')$, and the mass of agents in each state is assumed always to equal the mass in the stationary distribution. We assume that the labor productivity and discount factor processes are independent, and normalize the cross-sectional mean of labor productivity to unity.

Household problem: Households can save in $K+1$ assets, subject to a zero-borrowing constraint on their total portfolio wealth, and earn labor income, which is taxed at a rate τ_t. Households have Epstein-Zin preferences over their felicity from consumption c_{it} and labor n_{it}. $1/\rho$ denotes the intertemporal elasticity of substition and γ denote the risk-aversion parameter of households. Households have King, Plosser and Rebelo (1988) utility and obtain utility from consumption, but dislike supplying labor n_{it}, where $v(\cdot)$ quantifies their disutility. The problem of household i in period t, with idiosyncratic income productivity e_{it}, idiosyncratic discount factor β_{it}, and with portfolio holdings

[14] I assume that all retail firms are symmetric. As a result, their equities have identical payoffs, which implies by equation (2) that equity prices are also identical. Thus, all equities are identical.

$\{a_{it}^k\}_{k=0}^K$, where a_{it}^k denotes their portfolio holding of asset $k \in [0, 1, ..., K]$ is given by:

$$V_{it} = \max_{\{c_{it}, n_{it}, \{a_{it}^k\}_{k=0}^K\}_{t=0}^\infty} \left(\beta_{it} \left(c_{it} e^{-v(n_{it})} \right)^{1-\rho} + (1 - \beta_{it}) \left(\mathbb{E}_t \left[V_{it+1}^{1-\gamma} \right] \right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}} \tag{4}$$

$$\text{s.t.} \quad c_{it} + \sum_{k=0}^K q_t^k a_{it}^k \leq \sum_{k=0}^K \left(q_t^k + x_t^k \right) a_{it-1}^k + e_{it}(1 - \tau_t) w_t n_{it}, \tag{5}$$

$$\text{and} \quad \sum_{k=0}^K q_t^k a_{it}^k \geq 0. \tag{6}$$

The household's time-varying discount factor is defined as $\beta_{it} \equiv \tilde{\beta}_{it} \zeta_t$, where $\tilde{\beta}_{it}$ is the idiosyncratic component, and ζ_t is an aggregate discount factor shock. The aggregate component ζ_t evolves according to a log-linear AR(1) process with persistence ρ_ζ and innovation $\epsilon_\zeta \sim \mathcal{N}(0, \sigma_\zeta^2)$. In the household budget constraint (5), q_t^k and x_t^k denote the price and payoff of asset k, respectively. Households allocate wealth across a menu of nine assets: equity, capital, and government bonds with seven different maturities. The pre-tax real wage per unit of efficient labor is denoted by w_t; however, individual households do not choose their own hours worked; instead, labor supply is determined collectively by unions in response to current labor demand.

Solving for Optimal Portfolios and Risk Premia: I solve the model using the sequence-space approach of Auclert et al. (2021), modeling aggregate shocks as first-order "MIT shocks": the economy is perturbed by unanticipated shocks at date $t = 0$, after which all future periods evolve under perfect foresight. In this deterministic environment, all assets yield equal expected returns from period $t > 0$ onward, rendering households locally indifferent across assets and leaving portfolio choice indeterminate. However, at $t = 0$, realized shocks induce variation in ex-post returns, making the portfolio problem well-defined. Auclert et al. (2024) develop a method to recover optimal portfolio allocations and associated risk premia in this setting. A full derivation of the implementation in my model is provided in Appendix II; here, I offer a brief overview of the intuition and key equations.

The core idea is that households, anticipating the economy's response to aggregate shocks, can compute how their marginal utility will adjust—i.e., their exposure to each shock. They also foresee how asset returns will change in period 0. Given this information, households choose portfolios in period -1 to hedge against undesirable utility fluctuations. By imposing market clearing across all assets, the method jointly determines the equilibrium exposure to aggregate risk and the corresponding risk premia that compensate households for bearing this risk.

Formally, let $\boldsymbol{\epsilon}$ denote a vector of Z aggregate shocks, and let households choose asset holdings a_i^k across $K+1$ assets with prices p^k and state-dependent payoffs $x^k(\boldsymbol{\epsilon})$. Given an expected[15] value function $W_i = \mathbb{E}_t \left[V_{it}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, and assuming no binding portfolio constraints, the optimal portfolio satisfies the perturbed Euler equation:

$$\mathbb{E}\left[\frac{x^k(\boldsymbol{\epsilon})}{p^k}\frac{W_i'(\boldsymbol{\epsilon})}{\gamma_i}\right] = 1, \tag{7}$$

where γ_i is the Lagrange multiplier associated with household i's budget constraint.

Portfolio Choice: To derive implications for portfolio choice, Auclert et al. (2024) take a second-order derivative of equation (7) with respect to the volatility of aggregate shocks. This leads to a second-order Euler condition that relates the *exposure of marginal utility* to shocks to the *relative return sensitivity* of assets. Define household i's marginal utility exposure to shock z as $\lambda_{i,z} \equiv \frac{d\log W_i'}{d\epsilon_z}$, and define the relative sensitivity of asset k (in excess of a numéraire asset 0) as $X_{zk} \equiv \frac{\partial \log x^k}{\partial \epsilon_z} - \frac{\partial \log x^0}{\partial \epsilon_z}$. Then the key optimality condition becomes:

$$\mathbf{X}^\top \boldsymbol{\Sigma} \boldsymbol{\lambda}_i = \mathbf{b}, \tag{8}$$

where $\boldsymbol{\lambda}_i$ is a $Z \times 1$ vector of household i's marginal utility exposures, $\mathbf{X}$ is a $Z \times K$ matrix of relative return sensitivities, $\boldsymbol{\Sigma}$ is the diagonal matrix of shock variances, and $\mathbf{b}$ is a $K \times 1$ vector of relative risk premia. Equation (8) states that households optimally choose portfolios to hedge their marginal utility exposure, equating it to the price of risk encoded in $\mathbf{b}$.

If there are sufficiently many linearly independent assets ($K \geq Z$ and $\mathbf{X}^\top\boldsymbol{\Sigma}$ is invertible) then markets are complete with respect to aggregate risk. In this case, equation (8) uniquely determines a vector $\boldsymbol{\lambda}$ such that all households share the same marginal utility exposure to each shock:

$$\frac{d\log W_i'}{d\epsilon_z} = \lambda_z \quad \forall i. \tag{9}$$

Under complete markets, equation (9) characterizes the portfolio holdings required for optimal risk-sharing. By imposing market clearing on individual portfolio demands as given in equation (8), I solve for the common marginal utility exposure λ_z, which in turn determines the optimal portfolio allocations for each household. In my baseline specification, I do not impose portfolio constraints, allowing households to fully insure

[15] Expectations are taken over idiosyncratic fluctuations, as well as over aggregate dynamics conditional on an aggregate shock realization.

against aggregate risk. As a result, this setup may overstate the degree of insurance and understate the required compensation for risk. This implies that the model's implied risk premia represent a lower bound.[16]

Risk-Premia: This framework also provides a closed-form approximation for average risk premia in the economy.[17] For any asset k, its premium over the reference asset 0 is approximately:

$$\frac{R^k(\sigma) - R^0(\sigma)}{R} \approx - \sum_{z=1}^{Z} X_{zk} \lambda_z \overline{\sigma}_z^2 \sigma^2, \tag{10}$$

where X_{zk} captures how asset k responds to shock z, and λ_z captures the average exposure to that shock. This decomposition highlights the standard consumption-based asset pricing logic: *risk premia arise when assets co-move with the marginal value function*. Assets that pay off in states where the marginal value is high must offer higher expected returns as compensation. As the risk premium is additive seperable in shocks, we can calculate the contribution of an individual shock $\mathcal{Z}$ to the total premium of an asset k, as well:

$$\Omega_{k,\mathcal{Z}} = \frac{X_{\mathcal{Z}k} \lambda_z \overline{\sigma}_{\mathcal{Z}}^2}{\sum_{z=1}^{Z} X_{zk} \lambda_z \overline{\sigma}_z^2} \tag{11}$$

The method thus provides a tractable and powerful way to compute endogenous portfolio allocations and risk premia in heterogeneous-agent models using only first-order impulse responses and static model primitives, without the need to solve the full second-order model.

2.3 New Keynesian Firm Sector

We assume a three-tier production structure with a representative wholesale producer, a continuum of retailers, and a final goods producer. The wholesale producer creates a homogeneous wholesale good that is differentiated by retailers into a specific variety. The final goods producer bundles differentiated varieties into the final good. The whole-

[16] Auclert et al. (2024) show how to compute constrained-optimal portfolios in the case of two assets. Extending this to models with more assets, such as the nine-asset setup considered here, is more challenging. They note, however, that imposing portfolio constraints tends to push results toward those implied by exogenous portfolio rules (e.g., constant shares), typically increasing the level of risk premia while reducing the extent of endogenous insurance. Importantly, they find that portfolio constraints have limited implications for aggregate dynamics. Consequently, my estimates for risk premia are likely lower bounds, as imposing portfolio restrictions would increase them.

[17] Asset premia depend on the stochastic structure of the economy, which is time-invariant under a first-order solution. This implies that I can only evaluate the average premia over the estimated periods. In a counterfactual exercises, I reestimate the model for three different periods to study how changes in the volatility and persistence of shocks alter premia.

sale firm accumulates capital subject to investment adjustment costs, while retailers set the prices of their product subject to a Calvo (1983) adjustment friction.

Final goods firm: The final goods firm bundles all j varieties using a Dixit-Stiglitz aggregator

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{1}{\mu_t^p}} \, dj \right)^{\mu_t^p} \tag{12}$$

with elasticity of substitution between varieties of $\mu_t^p/(\mu_t^p - 1) > 1$. We assume that μ_t^p follows a log-AR(1) process with persistence ρ_p and shocks $\epsilon_t^p \sim N(0, \sigma_p^2)$ around the mean of the steady state price markup μ_p. Cost minimization of the final goods producer yields demand Y_{jt} for the individual variety j as

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\mu_t^p/(\mu_t^p - 1)} Y_t, \tag{13}$$

where P_{jt} is the price of the individual variety j is offered at and $P_t = \int_0^1 \left(P_{jt}^{\frac{1}{1-\mu_t^p}} \, dj \right)^{1-\mu_t^p}$ denotes the aggregate price level.

Retail firms: There exists a unit interval of j monopolistically competitive retail firms. Each retail firm buys a homogeneous wholesale good from the wholesale firm at the price mc_t and costlessly differentiates the good into a variety j, for which the producer is a monopolist. As a result of monopolistic competition, each retailer generates a profit which it distributes to equity holders. Each retail firm sets the price for the variety P_{jt} subject to a Calvo (1983) adjustment friction with indexation of prices. Retailers that are unable to re-optimize during the period adjust their price according to the following indexation rule:

$$P_{jt} = P_{jt-1} \Pi_{t-1}^{\iota_p} \Pi^{1-\iota_p}, \tag{14}$$

where Π is the steady state inflation rate, and ι_p reflects the degree of indexation to lagged aggregate inflation Π_{t-1}. For retail firms able to re-optimize, the optimization is to choose a new reset price P_{jt}^* to maximize expected discounted profits until the next re-optimization, given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \lambda_p^s \bar{\beta}^s \left(\frac{P_{jt}^* \Gamma_{t,t+s}}{P_{t+s}} - mc_{t+s} \right) Y_{jt+s} \tag{15}$$

subject to demand by the final goods producer (13) and $\Gamma_{t,t+s} = \prod_{k=1}^{s} \Pi_{t+s-1}^{\iota_p} \Pi^{1-\iota_p}$. λ_p denotes the probability not to adjust the price in a given period and $\bar{\beta}$ denotes the average discount factor of households.[18] The corresponding first-order condition for price setting implies a Phillips curve,

$$\log(\Pi_t) = \frac{\bar{\beta}}{1+\bar{\beta}\iota_p} \mathbb{E}_t \log(\Pi_{t+1}) + \frac{\iota_p}{1+\bar{\beta}\iota_p} \log(\Pi_{t-1}) + \kappa_p \left(mc_t - \frac{1}{\mu^p} \right) + \mu_t^p, \tag{16}$$

where the slope of the Phillips curve is given by $\kappa_p = \frac{1-\lambda_p \bar{\beta}}{1+\iota_p \bar{\beta}} \frac{1-\lambda_p}{\lambda_p}$. I assume a symmetric equilibrium in which aggregate profits in the economy are $d_t = (1 - \frac{1}{\mu_t})Y_t$. These profits are distributed to the owners of shares in the retail firms that are traded at the price q_t.

Wholesale firm: Wholesale goods are produced by a representative wholesale firm using labor and capital:

$$Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha}, \tag{17}$$

where α is the capital share in production, Z_t is total factor productivity that follows a log AR(1) process with persistence ρ_Z and shocks $\epsilon_t^Z \sim N(0, \sigma_Z^2)$, N_t is the labor hired, and K_{t-1} is the capital stock owned by the wholesale firm. Capital accumulates within the firm subject to investment adjustment costs, so that for each unit invested, a firm has to pay the adjustment cost.

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{1}{2\chi}\left(\frac{I_t}{I_{t-1}} - 1\right)^2, \tag{18}$$

where $1/\chi$ is the curvature of the function. Moreover, I allow for shocks to the marginal productivity of investment Ψ_t, such that the capital accumulation equation for the wholesale firm is

$$K_t = (1-\delta)K_{t-1} + \Psi_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{19}$$

where δ is the depreciation rate of capital. I assume that Ψ_t follows a log AR(1) process with persistence ρ_i and shocks $\epsilon_t^i \sim N(0, \sigma_i^2)$. The wholesale firm is perfectly competitive and takes the real wholesale price mc_t and real wage w_t as given, selling all output. In this setting, the wholesale firm entering period t with capital K_{t-1} and past investment

[18] I need to make an assumption about the discount rate with which firms discount future events. Here, I follow Auclert, Rognlie and Straub (2025) and choose the average discount factor in the economy. The average discount factor is the values of the discount factors of households multiplied by the stationary distribution of the Markov Chain that determines the idiosyncratic fluctuations in β_{it}.

I_{t-1} chooses the amount of labor N_t, capital K_t, and investment I_t to maximize its value:

$$J_t(K_{t-1}, I_{t-1}) = \max_{K_t, I_t, N_t} mc_t F(K_{t-1}, N_t) - w_t N_t - I_t + \mathbb{E}_t \left[\frac{1}{1 + r_{t+1}} J_{t+1}(K_t, I_t) \right] \tag{20}$$

subject to the capital accumulation equation (19). $1 + r_t$ is the gross real interest rate on assets. The optimization problem implies the standard first-order condition for labor demand $w_t = (1-\alpha) mc_t Z_t \left(\frac{K_{t-1}}{N_t} \right)^{\alpha}$, as well as the expression for Tobin's Q and the firm's investment decision:

$$Q_t = \mathbb{E}_t \left[\frac{1}{1 + r_{t+1}} \left((1-\delta) Q_{t+1} + \alpha MC_{t+1} Z_{t+1} \left(\frac{K_t}{N_{t+1}} \right)^{\alpha - 1} \right) \right] \tag{21}$$

$$1 = \Psi_t Q_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \left[\frac{\Psi_{t+1} Q_{t+1}}{1 + r_{t+1}} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \tag{22}$$

Unions: Nominal wages are assumed to be sticky. As in Erceg, Henderson and Levin (2000), unions set nominal wages to maximize agent utility subject to adjustment costs. I adopt the microfoundations for nominal wage rigidities of staggered pricing as in Calvo (1983). I assume that the unions that are not able to adjust their price optimally adjust it following an indexation rule. Finally, I specify the disutility of labor as $v(n_{it}) = \gamma \frac{n_{it}^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}$. I assume that unions allocate all labor hours uniformly across agents, so that $n_{it} = N_t$. This leads to the wage Phillips curve:

$$\begin{aligned} \log(\Pi_t^w) = {} & \frac{\bar{\beta}}{1 + \bar{\beta}\iota_w} \mathbb{E}_t \log(\Pi_{t+1}^w) + \frac{\iota_w}{1 + \bar{\beta}\iota_w} \log(\Pi_{t-1}^w) \\ & + \kappa_w \left(\gamma N_t^{\frac{1}{\phi}} - \frac{(1-\tau) w_t \int_0^1 e_{it} c_{it}^{-1/\sigma} di}{\mu^w} \right) + \mu_t^w, \end{aligned} \tag{23}$$

describing the dynamics of log-wage inflation Π_t^w as a function of aggregate hours N_t, aggregate posttax labor income $(1-\tau)w_t$, and the effective consumption aggregator $\int_0^1 e_{it} c_{it}^{-1/\sigma} di$ that measures how the consumption distribution affects the wealth effect on labor supply. μ_t^w follows a log-AR(1) process with coefficient ρ_w and shocks $\epsilon_t^w \sim N(0, \sigma_w^2)$.

2.4 Government Sector

The government sector consists out of a fiscal authority and a monetary authority.

Fiscal Authority: Fiscal policy sets the tax rate τ_t on dividends and labor, spends G_t on goods, and issues non-contingent debt B_t, with an average return R^F_{t-1}. Since the overall tax revenue is $\tau_t w_t N_t$, the government budget constraint is given by

$$B_t = R^F_{t-1} B_{t-1} + G_t - \tau_t w_t N_t \tag{24}$$

We assume that fiscal policy is specified in terms of plans for government spending G_t which follows a log-AR(1) process with persistence ρ_G and shocks $\epsilon^G_t \sim N(0, \sigma^2_G)$ and a tax rule:

$$\frac{\tau_t}{\tau^{ss}} = \left(\frac{\tau_{t-1}}{\tau^{ss}}\right)^{\rho_\tau} \left(\frac{B_t}{B_{t-1}}\right)^{(1-\rho_\tau)\gamma^B_\tau} \left(\frac{Y_t}{Y_{t-1}}\right)^{(1-\rho_\tau)\gamma^Y_\tau}, \tag{25}$$

where ρ_τ denotes the persistence of the tax rate, γ^B_τ denotes the elasticity of the tax rate to debt growth, and γ^Y_τ denotes the elasticity of the tax rate to output growth. Given a real interest rate, the tax rule and the government budget constraint imply a process for bonds B_t.

Bond Maturity Structure: I model government debt instruments with a range of maturities. To incorporate different maturities in a tractable fashion, I follow Bayer, Born and Luetticke (2022) and assume that along all maturities the bonds are zero-coupon bonds with geometrical decay.[19] The bonds are priced recursively, and their ex-post returns contribute to the weighted average fiscal interest rate R^F_{t-1} the government has to pay to households.

Let $q^{(n)}_t$ denote the price of a government bond at time t with maturity n and $R^{(n)}_{t-1}$ denote the ex-post return of a bond. The price of each bond is set by a no-arbitrage condition and the ex-post return by definition:

$$q^{(n)}_t = \frac{(1-\delta^{(n)})q^{(n)}_{t+1} + 1}{1 + r_{t+1}}, \quad \text{and} \quad R^{(n)}_{t-1} = \frac{(1-\delta^{(n)})q^{(n)}_t + 1}{q^{(n)}_{t-1}} \quad \forall n \tag{26}$$

$\delta^{(n)}$ denotes the maturity-specific retirement rate of the bond and r_t is the risk-free real interest rate. The government pays the weighted average of the ex-post returns R^F_{t-1} across maturities:

$$R^F_{t-1} = \sum_n \omega^{(n)}_t \cdot R^{(n)}_t \quad \text{with} \quad \sum_n \omega^{(n)}_t = 1 \tag{27}$$

[19] This assumption makes the price and the ex-post return of long-term bonds more exposed to more persistent shocks. This feature helps to match the empirical fact that the price of long-run bonds fluctuate more in response to shocks as either their cash-flows or their discount rates are affected by the shock.

where $\omega_t^{(n)}$ denotes the share of government debt issued in maturity n at time t. This composite rate captures the average cost of servicing outstanding government debt, taking into account the maturity composition of the debt portfolio.

Monetary Policy: Monetary policy sets the nominal interest rate i_t, using the following Taylor rule:

$$1 + i_t = (1 + i_{t-1})^{\rho_r} (1 + \pi_t)^{(1-\rho_r)\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{(1-\rho_r)\phi_Y} \exp(\epsilon_t^r) \tag{28}$$

where ρ_r denotes the persistence of the monetary policy rule, ϕ_π and ϕ_Y denote the elasticities of the nominal interest rate to inflation and output growth, and $\epsilon_t^r \sim N(0, \sigma_r^2)$ is an iid monetary policy innovation. Finally, I define the ex-ante real interest rate as $1 + r_t = (1 + i_t)/(1 + \pi_{t+1})$ according to a Fisher equation.

2.5 Market clearing

In equilibrium, the goods market, the labor market, as well as the asset market, have to clear:

$$Y_t = \int c_{it} di + I_t + G_t \qquad \int_0^1 n_{it} dj = N_t \qquad \int a_{it} di = B_t + q_t + J_t. \tag{29}$$

We assume that all firms are symmetric such that $Y_{jt} = Y_t$, $d_{jt} = d_t$, $w_{jt} = w_t$, and $d_{jt} = d_t$.

3 Calibration and Estimation of the Model

This section illustrates the calibration of the steady state and the estimation of the model on U.S. time-series data. First, I calibrate the model to replicate key dimensions of household heterogeneity, and match time-series averages of aggregate variables. Thereafter, I illustrate the Bayesian estimation on U.S. time-series data, illustrate the estimation results and their validity.

3.1 Calibration of the Steady State

Table 1 portrays the parameters used in the calibration. The parameter choices of the household side largely follows Auclert, Rognlie and Straub (2025). To start with, the exogenous income process is the discretized permanent-transitory income process of

Table 1 Calibration Details (Quarterly Frequency)

Parameter	Value	Description	Source / Target
Preferences			
σ	1.000	Elasticity of intertemporal sub.	Standard value
γ	6.000	Risk aversion	Guvenen (2009)
γ	0.787	Disutility from labor	Labor normalization
ϕ	1.000	Frisch elasticity of labor	Standard value
Idiosyncratic risk			
β^H	0.996	Discount factor patience	Total Assets: 18.0
Δ^β	0.010	Difference discount factors	Aggregate MPC: 0.1
ω	0.635	Fraction of patient households	U.S. Lorenz Curve of wealth
ϖ	0.010	Prob. to become impatient	Krusell and Smith (1997)
Production			
Z	0.501	Steady state productivity	Output Normalization
α	0.286	Capital income share	Capital-to-Output Ratio: 11.2
δ	0.020	Capital depreciation	Standard value
μ^p	1.020	Price Markup	Equity-Price-to-Output Ratio: 4.0
μ^w	1.000	Wage Markup	No transfers in steady state
Government			
G	0.200	Government expenditure	Expenditure-to-Output-Ratio: 0.2
B	2.800	Government debt	Debt-to-Output-Ratio: 2.8
r	0.500	Real interest rate in $(\%)$	Postwar annual average
τ	0.305	Tax rate	Finances debt and expenditure

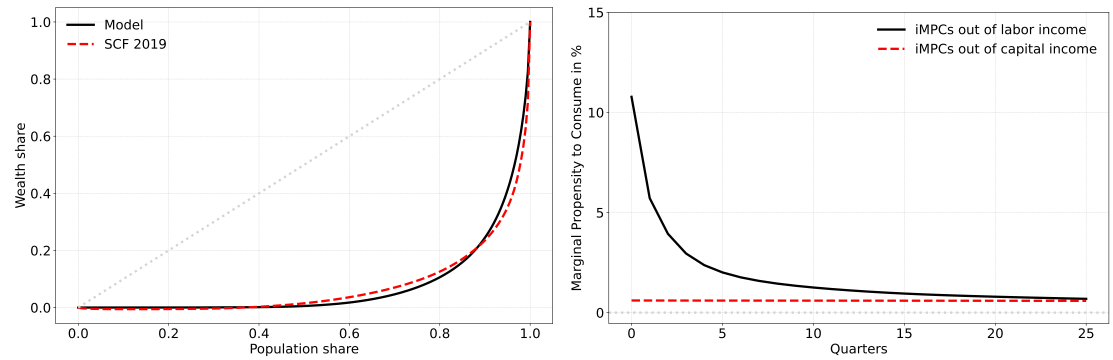
Notes: All parameters in the table are calibrated to a quarterly frequency. Probabilities represent the likelihood within a single quarterly period. Interest rates are reported quarterly.

Kaplan, Moll and Violante (2018), based on their estimates from the Social Security Administration data. I assume standard intertemporal elasticities of substitution and labor supply equal to one and calibrate the disutility from labor γ to normalize labor supply $N_t = 1$. Moreover, I set the risk-aversion parameter equal to 6 as in Guvenen (2009). This parameter value is between the commonly used value of 10 in the finance literature and the commonly used value of 1 when using CRRA preferences in macro models. I then calibrate remaining household parameters to ensure asset market clearing, generate an aggregate marginal propensity to consume (MPC) out of labor income of 0.1, and match the empirical Lorenz curve of wealth inequality in the U.S.[20] To achieve this, I jointly calibrate β^H, the discount factor of patient households; Δ^β, the gap between the discount factors of patient and impatient households; and ω, the stationary share

[20] I choose a lower quarterly target MPC than Auclert, Rognlie and Straub (2024b), as recent evidence of Orchard, Ramey and Wieland (2025) shows that MPC estimates for nondurable consumption are lower than 0.2 per quarter.

Figure 1 Fit of wealth inequality and aggregate marginal propensities to consume

(a) Fit of the Lorenz Curve **(b)** Aggregate MPCs



Notes: Panel 1a) illustrates the fit of the model-implied Lorenz for household wealth compared to the Lorenz curve estimated from the SCF in 2019 by Auclert, Rognlie and Straub (2025). Panel 1b) illustrates the intertemporal marginal propensities to consume (iMPCs) of the model out of labor and capital income.

of patient households in the population. The probability to change patience ϖ follows Krusell and Smith (1997) in matching the entry of a new generation to the economy.

Figure 1 (1a) shows the model's fit to the empirical Lorenz curve from the 2019 Survey of Consumer Finances (SCF). The model matches the Lorenz curve closely, though it features less top wealth inequality than the data. Figure 1 (1b) shows the models aggregate intertemporal marginal propensities to consume (MPCs). While I explicitly target an impact MPC out of labor income of 0.1, all other MPCs are untargeted. The model's intertemporal MPCs align well with empirical estimates from microdata, having a large inpact and then declining quickly. In addition, the untargetted, model-implied MPC out of capital income is 0.01. This calibration is at the lower end of the empirical range of 0 to 0.05 estimated by Chodorow-Reich, Nenov and Simsek (2021).

I set TFP Z_t so that output is normalized to one and α so as to target a quarterly capital-to-output ratio of 11.2 with a standard depreciation rate of $\delta = 0.02$. The markup μ^p is chosen to generate a quarterly stock market-to-GDP ratio of $q_t/Y_t = 4.0$, consistent with historical averages.[21] I assume that the wage markup equals $\mu^w = 1$, such that there are no union transfers in steady state.

The calibration of the government sector follows standard values, as well as historical averages for the debt structure. I set government expenditure equal to 20% of GDP and total government debt equal to 2.8 times quarterly GDP, corresponding to an annual

[21] Bayer, Born and Luetticke (2024) estimate a stockmarket to wealth ratio of 1.14.

Table 2 Calibration of the public debt structure

Bond Label	Maturity Tranche n	Share $\omega^{(n)}$ in Total Debt	Av. Duration $1/\delta^{(n)}$ in Quarters
3M	$\leq 3M$	0.154	1.00
6M	$6M - 9M$	0.129	2.31
1Y	$1Y$	0.043	4.00
2Y	$2Y$	0.138	8.00
5Y	$3Y - 7Y$	0.296	18.32
10Y	$8Y - 12Y$	0.095	37.36
20Y	$15Y \leq$	0.145	95.74

Notes: Maturities show the maturity of the zero coupon bonds in months (M) or years (Y). Share in total debt $\omega^{(n)}$ represents the fraction of total government debt with the respective maturity. The duration $\delta^{(n)}$ illustrates the average duration in quarters of the neighboring maturities I clustered together to create the seven subgroups.

debt-to-GDP ratio of 70%, both reflecting historical averages. The risk-free interest rate is set to 0.5%, in line with the U.S. postwar average. The labor tax rate τ is set to 0.305 such as to balance the government's budget constraint.

To calibrate the maturity structure of government debt, I use historical averages from the database of De Graeve and Mazzolini (2023). Appendix III provides a detailed illustration. Below, I give a brief overview. I take the averages over the market value of public debt at each maturity available in the dataset. I then combine neighboring maturities together in order to obtain seven groups that approximately represent the maturities of three and six months, as well as one, two, five, ten, and fifteen years. I choose these tranches to achieve almost equally weighted tranches and to feature important maturities on the yield curve. Table 2 illustrates the resulting calibration parameters.

Roughly 30% of government debt has a maturity equal to or below one year, approximately 44% has a maturity between one year and five years, and the rest have a maturity longer than five years. I assume that all debt within the dataset are zero-coupon bonds and then calculate the weighted maturity of the empirical groups to arrive at the duration measure δ^m for each group. I then label each of these groups according to their closest full month or year.

3.2 Bayesian Estimation of the Model

To estimate the model, I follow the methodology of Auclert et al. (2021), representing the model in its vector moving average (VMA(∞)) form. For the empirical implementation, I use the same macroeconomic time series as in Bayer, Born and Luetticke (2024), covering the period from the third quarter of 1954 to the fourth quarter of 2019. I augment this dataset with updated series from Shiller (1989) on equity prices, dividends, and returns. The combined dataset includes growth rates of real GDP, consumption, investment, wages, dividends, and stock prices. Hours worked, the (shadow) federal funds rate, the inflation rate, and real equity returns are expressed in logarithmic levels.[22] A detailed description of the data sources and the transformations applied is provided in Appendix III.

To estimate the model, I use the DIME sampler of Boehl (2024). The DIME sampler has two important advantages. First, by combining parallel local chains with a global sampler, it enables the approximation of even multinomal posterior distributions. Hence, it is very robust to explore the unknown posterior distribution of the estimated HANK model. Second, the global sampler also does not require mode optimization before initializing sampling. The sampler learns the shape of the posterior distribution just from sampling and therefore is robost to misspecification of priors. Table 3 illustrates the priors used for estimation, as well as the posterior estimates.

Priors For all exogenous shock processes I impose an identical prior structure. Innovation standard deviations are given Inverse Gamma priors, while the autoregressive coefficients receive Beta priors. This combination is weakly informative, puts mass away from the boundaries, and treats the shocks symmetrically. For the policy parameters, the priors follow Bayer, Born and Luetticke (2024), which aligns the Taylor rule coefficients and interest rate inertia with common benchmarks in the New Keynesian literature. For the indexation parameters ι_p and ι_w I adopt the priors from Smets and Wouters (2007), anchoring beliefs about backward indexation at empirically plausible levels. Overall, the prior block is comparable to the main references and sufficiently diffuse to let the data discipline the posterior. Next, I illustrate the estimation results evaluated at the average of the posterior distribution.

Shock processes Shock dynamics display high persistence for real and fiscal disturbances, with ρ_q, ρ_z, ρ_g near 0.96–0.99 and $\rho_p \approx 0.87$. Innovation scales are heterogeneous: government, wage, and investment shocks have the largest variances in our normalization ($\sigma_g \cdot 100 \approx 2.96,\ \sigma_w \cdot 100 \approx 2.08,\ \sigma_i \cdot 100 \approx 2.21$), whereas markup-type and asset-price shocks are comparatively small ($\sigma_\zeta \cdot 100 \approx 0.18,\ \sigma_q \cdot 100 \approx 0.36$). These magnitudes are broadly consistent with medium-scale NK estimates that attribute persistent yet relatively low-volatility movements to price-setting disturbances and larger innovations to fiscal or cost-side shocks.

[22] Following Bayer, Born and Luetticke (2024), I use the shadow federal funds rate constructed by Wu and Xia (2016) during periods when the federal funds rate is constrained by the zero lower bound.

Table 3 Bayesian estimation results: shock and policy parameters

Shock Parameter	Distribution	Prior Mean	Prior SD	Posterior Mean	Posterior Median	Posterior 5%	Posterior 95%
$\sigma_q \cdot 100$	Inv. Gamma	10.0	25.0	0.357	0.342	0.274	0.443
ρ_q	Beta	0.5	0.2	0.985	0.986	0.980	0.990
$\sigma_\zeta \cdot 100$	Inv. Gamma	10.0	25.0	0.184	0.179	0.151	0.218
ρ_ζ	Beta	0.5	0.2	0.802	0.804	0.767	0.834
$\sigma_z \cdot 100$	Inv. Gamma	10.0	25.0	0.672	0.661	0.601	0.737
ρ_z	Beta	0.5	0.2	0.972	0.972	0.956	0.983
$\sigma_r \cdot 100$	Inv. Gamma	10.0	25.0	0.316	0.310	0.275	0.356
$\sigma_i \cdot 100$	Inv. Gamma	10.0	25.0	2.208	2.187	1.993	2.452
ρ_i	Beta	0.5	0.2	0.665	0.665	0.575	0.748
$\sigma_p \cdot 100$	Inv. Gamma	10.0	25.0	0.301	0.297	0.268	0.330
ρ_p	Beta	0.5	0.2	0.869	0.895	0.299	0.927
$\sigma_w \cdot 100$	Inv. Gamma	10.0	25.0	2.080	2.063	1.624	2.708
ρ_w	Beta	0.5	0.2	0.938	0.940	0.914	0.958
$\sigma_g \cdot 100$	Inv. Gamma	10.0	25.0	2.956	2.958	2.711	3.222
ρ_g	Beta	0.5	0.2	0.962	0.962	0.949	0.972

Policy Parameter	Distribution	Prior Mean	Prior SD	Posterior Mean	Posterior Median	Posterior 5%	Posterior 95%
ρ_r	Beta	0.5	0.2	0.697	0.703	0.654	0.742
ϕ_π	Gamma	1.5	0.3	2.190	2.191	2.007	2.399
ϕ_Y	Normal	0.1	0.1	0.230	0.231	0.161	0.299
ρ_τ	Beta	0.5	0.2	0.373	0.387	0.159	0.580
γ_τ^B	Normal	0.0	1.0	5.850	5.900	4.664	7.228
γ_τ^Y	Normal	0.0	1.0	0.601	0.573	−0.485	1.642
λ_p	Beta	0.5	0.1	0.496	0.498	0.318	0.672
λ_w	Beta	0.5	0.1	0.279	0.279	0.222	0.339
ι_p	Beta	0.5	0.2	0.504	0.501	0.225	0.785
ι_w	Beta	0.5	0.2	0.115	0.112	0.040	0.279
χ	Gamma	4.0	2.0	2.552	2.689	0.826	6.490

Notes: Posterior estimates are based on Bayesian inference using the DIME sampler by Boehl (2024). The sampler was run with 128 parallel chains for 7,500 iterations each. I discard the first 2,500 iterations as burn-in. Reported values are posterior means, medians, as well as 90 percent credible intervals. Reported values for shock standard deviations are scaled by 100 to enhance readability.

Policy Parameters The monetary policy block is well behaved. The posterior mean for interest rate smoothing is $\rho_r \approx 0.70$, consistent with the 0.6–0.8 range reported by Smets and Wouters (2007) and Bayer, Born and Luetticke (2024). The inflation coefficient is $\phi_\pi \approx 2.19$, comfortably above unity and within the 1.5–2.5 interval typically found for post-1990 U.S. data, while the output coefficient $\phi_Y \approx 0.23$ lies in the standard 0.1–0.3 range. The fiscal rule parameters indicate moderate persistence in the fiscal stance, $\rho_\tau \approx 0.37$, a sizable feedback on debt, $\gamma_\tau^B \approx 5.85$, and a smaller feedback on activity,

$\gamma_\tau^Y \approx 0.60$.

Nominal frictions and indexation The posterior for price and wage stickiness, $\lambda_p \approx 0.50$ and $\lambda_w \approx 0.28$, suggests appreciable real rigidity with relatively easier wage adjustment than price adjustment in this specification. These values fall within the empirical range reported in medium-scale NK estimations, though they point to somewhat lower wage stickiness than in some earlier studies. Indexation is asymmetric: price indexation is moderate, $\iota_p \approx 0.50$, whereas wage indexation is low, $\iota_w \approx 0.12$. Relative to Smets and Wouters (2007), which typically finds modest price indexation and non-negligible wage indexation, our estimates imply stronger backward-looking behavior in prices and weaker in wages, consistent with studies emphasizing improved nominal anchoring of wage setting in more recent samples.

Diagnostics and additional evidence To assess convergence and mixing, Appendix IV reports trace plots, posterior densities and convergence checks for all parameters. The traces exhibit stationary, well-overlapped chains for the vast majority of parameters, and the corresponding posterior distributions are unimodal and well behaved. Together with the large number of chains and draws used in estimation, these diagnostics support the reliability of the posterior summaries reported in the table.

4 Decomposing U.S. Business Cycles and Asset Premia

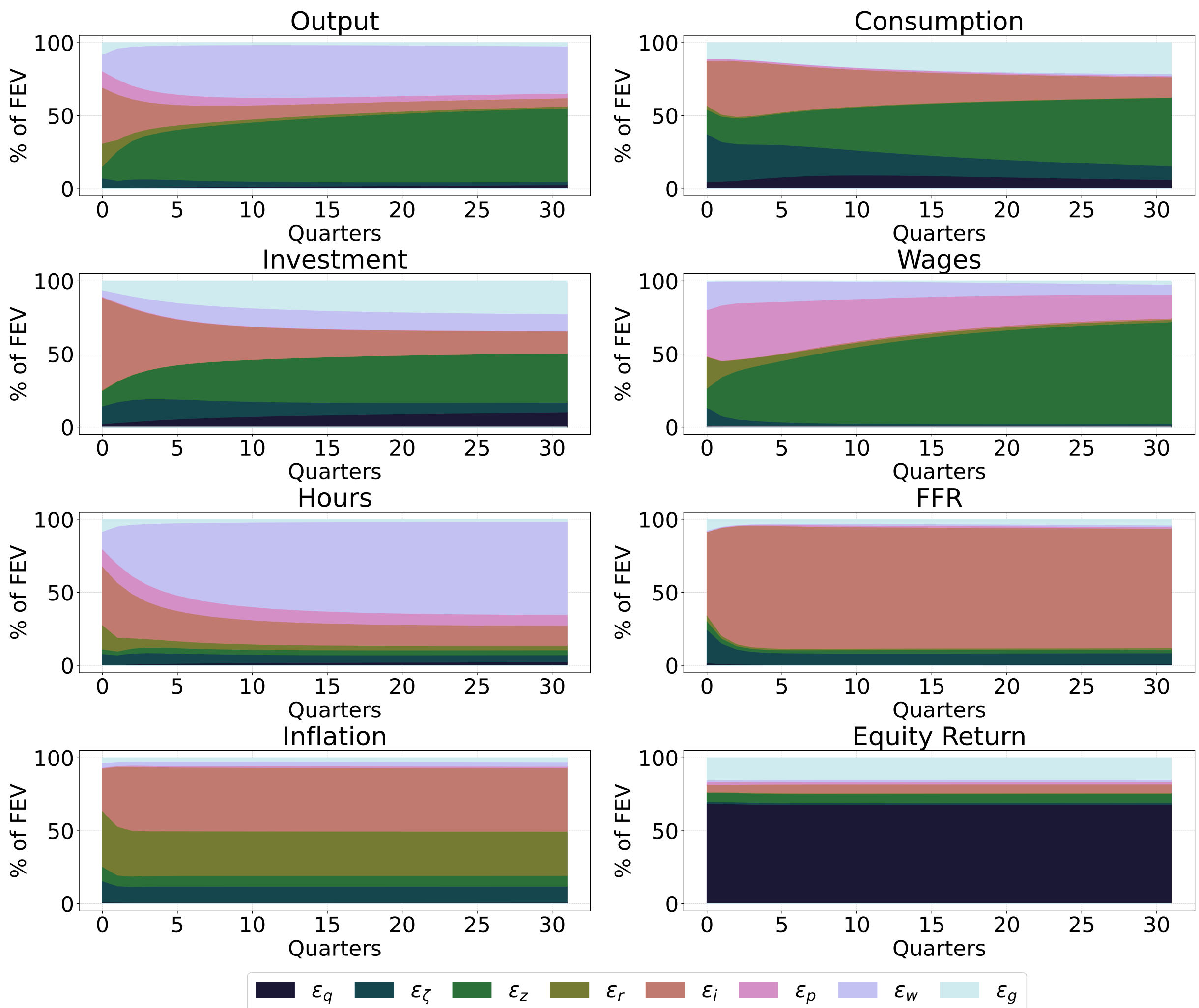
The estimated model enables me to revisit the question on key drivers of the business cycle and household inequality. Thereafter, I can use the model to decompose asset premia into their risk components.

4.1 Decomposing Aggregates and Inequality

Figure 2 and figure 3 show the forecast error variance decomposition (FEVD) for key aggregate variables and measures of inequality over a 32-quarter horizon. The decomposition attributes the variance in each variable to eight structural shocks: the asset price shock (ϵ_q), the discount factor shock (ϵ_ζ), the TFP shock (ϵ_Z), the monetary policy shock (ϵ_r), the investment-specific technology shock (ϵ_i), the price markup shock (ϵ_p), the wage markup shock (ϵ_w), and the government spending shock (ϵ_g).

Decomposing Aggregates: Figure 2 shows the FEVD for aggregates. For output, short-run fluctuations are mostly driven by monetary and fiscal shocks. Over time, supply-side

Figure 2 Forecast Error Variance Decomposition of Aggregate Variables

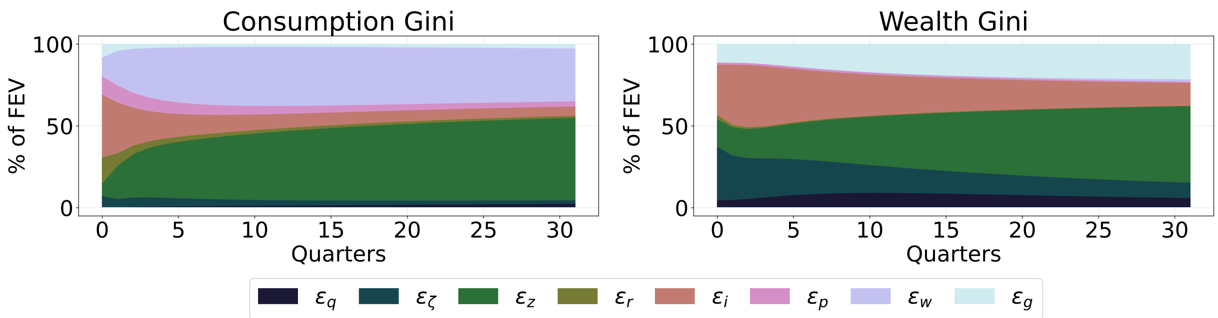


Notes: Forecast error variance decomposition of aggregate variables over 32 quarters. The coloured areas indicate the share of the variance in the illustrated variable due to an individual shock. Decomposition in the asset price shock ϵ_q, discount factor shock ϵ_ζ, TFP shock ϵ_Z, monetary policy shock ϵ_r, investment specific technology shock ϵ_i, price markup shock ϵ_p, wage markup shock ϵ_w, and government expenditure shock ϵ_g.

forces like TFP and investment-specific technology become more important. Consumption follows a similar pattern. In the short run, demand shocks matter most. In the long run, productivity-related shocks take over. Investment is strongly driven by the investment-specific technology shock at all horizons. TFP plays a smaller role, while discount factor and monetary shocks influence short-run movements. Wages are shaped mainly by wage markup and TFP shocks, with smaller effects from price markup and monetary shocks.

Hours worked respond mostly to wage markup and demand shocks in the short run.

Figure 3 Forecast Error Variance Decomposition of Inequality Measures



Notes: Forecast error variance decomposition of Consumption Gini and Wealth Gini over 32 quarters. The coloured areas indicate the share of the variance in the illustrated variable due to an individual shock. Decomposition in the asset price shock ϵ_q, discount factor shock ϵ_ζ, TFP shock ϵ_Z, monetary policy shock ϵ_r, investment specific technology shock ϵ_i, price markup shock ϵ_p, wage markup shock ϵ_w, and government expenditure shock ϵ_g.

Over longer horizons, TFP becomes more relevant. The federal funds rate is largely explained by monetary policy shocks, but it also reacts to inflation and real activity, which bring in price markup and TFP shocks. Inflation is driven by price markup shocks at short horizons. Monetary and demand shocks also play a role. In the long run, technology shocks gradually gain importance. Equity return volatility is mainly driven by the discount factor and asset price shocks. These capture variation in the pricing of risk. TFP and investment-specific technology matter through their effect on cash flows, and markup shocks influence both cash flows and the pricing kernel.

Compared to Smets and Wouters (2007) and Bayer, Born and Luetticke (2024), my model produces notable differences in the importance of shared shocks. Like those papers, TFP and investment shocks are key for long-run output and investment. But my model assigns a larger share of short-run variation in consumption and hours to discount factor and monetary policy shocks. This reflects stronger intertemporal effects due to portfolio choice. Price and wage markup shocks remain central for inflation and wages, as in both benchmark models. However, in my model, their effects last longer. This is likely due to how household heterogeneity and segmented markets affect transmission. Although I use similar shocks, their impact changes because of richer microeconomic structure. These features are not present in the representative-agent model of Smets and Wouters (2007) or the HANK model with limited asset structure in Bayer, Born and Luetticke (2024).

Decomposing Inequality: Figure 3 illustrates the FEVD for the consumption Gini and the Wealth Gini as measures of inequality. For the consumption Gini, the forecast error variance decomposition indicates that TFP shocks (ϵ_Z), investment-specific technology shocks (ϵ_i), and wage markup shocks (ϵ_w) are the primary drivers of short- and medium-run fluctuations. These shocks affect households asymmetrically through differences in labor income exposure and the amplification of nominal rigidities, leading to heterogeneous consumption responses across the distribution. In the short run, nearly all shocks contribute to consumption inequality, but investment-specific technology shocks alone account for roughly one-third of the total variance. Over longer horizons, the importance of TFP and wage markup shocks rises, as persistent changes in productivity and wage-setting behavior reshape income trajectories and alter households' ability to smooth consumption. This shift highlights a transition from transitory, investment-driven dispersion to more structural, long-lasting sources of consumption inequality.

In contrast, the dynamics of the wealth Gini exhibit a distinct shift in the importance of structural shocks over time. In the short run, wealth inequality is primarily driven by the discount factor shock (ϵ_ζ), along with substantial contributions from TFP shocks (ϵ_Z) and investment-specific technology shocks (ϵ_i). These shocks influence asset accumulation through heterogeneous saving behavior and differential exposure to capital income risk. As the forecast horizon increases, however, the role of investment-specific technology shocks diminishes, while the importance of government expenditure shocks (ϵ_g) rises, reflecting the long-term redistributive effects of fiscal policy. At the same time, the influence of the discount factor shock gradually declines, whereas the contribution of TFP shocks grows, consistent with their persistent impact on wealth accumulation through productivity and long-run return dynamics. Notably, the asset price shock (ϵ_q) contributes only marginally at all horizons, suggesting that while it may generate short-term asset price volatility, it does not play a central role in shaping long-run wealth inequality in the model.

Asset Price Shock: The asset price shock (ϵ_q) plays a limited role for most macroeconomic aggregates but becomes more relevant for financial variables. While it contributes only marginally to the variance of output, consumption, or inflation, it has a noticeable impact on the volatility of equity returns. This suggests that the shock captures variation in risk pricing that is not directly linked to fundamental forces. Its effect is concentrated in the short run and does not drive long-term fluctuations in real variables or inequality.

Table 4 Annualized Assets Premia in Excess of the 3-month Government Bond Return

Asset	**HA** (%)	**RA** (%)	**Data** (%)
Equity	3.90	3.0×10^{-4}	5.01
Capital	0.66	1.2×10^{-4}	-
Bond 6m	0.10	1.0×10^{-6}	0.19
Bond 1y	0.25	2.0×10^{-5}	0.36
Bond 2y	0.45	5.0×10^{-5}	0.71
Bond 5y	0.63	8.0×10^{-5}	1.28
Bond 10y	0.77	9.0×10^{-5}	1.76
Bond 20y	1.01	1.1×10^{-4}	1.94

Notes: Annualized premia of the estimated heterogeneous agent (HA) model, for a representative agent (RA) model, and estimates from the data. The premia are calculated using the methodology of Auclert et al. (2024): $\frac{R_1 - R_0}{R} \approx -X\,\bar{\lambda}\,\sigma^2$, where X is the ex-post variation of the excess return of an asset over the 3m-bonds return, and $\bar{\lambda}$ is the pricing kernel of the representative asset buyer. The estimate for the equity premium is calculated as the mean of the annualized excess returns of stock returns over the return of a zero-coupon bond with a ten year maturity. The estimates for the term premia are calculated as the excess returns of zero-coupon bonds with constant maturity over the return of a zero-coupon bond with a maturity of three months. The data on equity returns is the identical dataseries that I use for my estimation. The data on the zero-coupon

4.2 Decomposing the Equity Premium

Table 4 reports annualized asset return premia, expressed in percentage points above the 3-month government bond return, for both a heterogeneous agent (HA) model and a representative agent (RA) version of the model. The premia are computed using the methodology of Auclert, Rognlie and Straub (2024a), which ties asset premia to the product of the pricing kernel and the variance of excess returns.

In the HA model, I obtain a sizable equity premium of 3.9%, which reflects meaningful compensation for holding risky equity. This premium stands in stark contrast to the negligible value generated by the RA model, which lacks sufficient heterogeneity and pricing kernel curvature to explain observed asset return differentials. The HA model thus provides a more realistic account of equity risk compensation.

For capital, the HA model yields a much smaller premium of 0.66%, which is relatively modest compared to equity. This may reflect the absence of nonfundamental fluctuations in housing prices, which are shown to be important in explaining capital returns in models such as Kaplan, Mitman, and Violante (KMV). Incorporating such housing-related asset price dynamics could potentially amplify the premium on capital in future extensions.

The model successfully captures a sizable share of the term premium on government

Table 5 Decomposition of the Annual Equity Premium in individual risk components

Risk Component $\mathcal{Z}$	Abs. Contribution	Rel. Contribution $\Omega_{k,\mathcal{Z}}$ (in $\%$)
Asset-Price	1.69	47.00
Discount-Factor	-0.43	-11.01
Productivity	0.74	17.22
Monetary Policy	0.12	2.72
Investment specific Prod.	0.82	22.13
Price Markup	-0.12	-1.90
Wage Markup	0.20	1.36
Government Exp.	0.90	22.49
Total	3.90	

Notes: Contribution Ω_ϵ of aggregate shocks to total equity premium of 3.90%. The contribution is calculated as $\Omega_\epsilon = \frac{X_\epsilon \bar{\lambda}_\epsilon \sigma_\epsilon^2}{X \bar{\lambda} \sigma^2}$, hence how much of the total equity premium is explained through one individual shock.

bonds. For example, the 10-year bond yields a premium of 0.77%, rising to 1.01% for the 20-year bond. This upward-sloping pattern reflects the idea that longer-duration assets are riskier and thus command higher compensation. While the model accounts for roughly half of the empirically observed term premia, this already marks a substantial improvement over representative-agent benchmarks. Further amplification via stronger long-horizon risk pricing or deeper bond market segmentation could be help to fully match observed bond return data.

Table 5 presents a decomposition of the model-implied annual equity premium into contributions from eight structural shocks. The asset-price shock, which captures non-fundamental fluctuations, is the most important source of risk compensation. It accounts for 1.69 percentage points, or 47%, of the total equity premium. Including this shock not only helps the model match the volatility of equity prices and returns but also generates a sizeable premium. This result implies that a meaningful share of the equity premium can be attributed to nonfundamental fluctuations, which households perceive as undesirable and therefore require compensation for. Incorporating this shock substantially improves the model's ability to replicate empirically observed equity premia.

The remaining 53% of the premium is explained by classical business cycle shocks. Among these, technology and investment-specific technology shocks jointly account for 1.56 percentage points, or 39%, of the total. This underscores the importance of long-run productivity and investment dynamics in shaping asset prices, consistent with standard macro-finance insights. The government expenditure shock contributes 0.90 percentage points, or 22.49%, to the premium. Its strong effect reflects a key feature of the HANK framework: fiscal shocks trigger endogenous tax adjustments that crowd out private consumption. This magnifies household-level consumption volatility and makes fiscal risk particularly salient and highly priced by households. In contrast, the remaining shocks—discount factor, price markup, wage markup, and monetary policy—contribute only modestly. The discount factor shock has a negative contribution of 11.01%, indicating that equities serve as a hedge against discount rate fluctuations in this model.

Taken together, these results show that nonfundamental fluctuations, alongside technology-driven risks, are central to understanding the equity premium in a setting with household heterogeneity and incomplete markets.

5 Conclusion

This paper develops and estimates a quantitative heterogeneous-agent New Keynesian model with portfolio choice and nonfundamental asset price shocks. By introducing noise traders in a segmented equity market, the model captures fluctuations in asset prices that are disconnected from economic fundamentals. While these nonfundamental shocks generate substantial volatility in equity prices, their aggregate macroeconomic effects are limited. This is due to the relatively small share of equity in total household wealth, low marginal propensities to consume out of wealth, and the comparatively low variance of the shock itself. As a result, nonfundamental fluctuations play only a minor role in explaining movements in aggregate consumption, investment, and inequality.

Despite their limited impact on macroeconomic aggregates, nonfundamental shocks are crucial for understanding asset pricing. They account for nearly 70 percent of the variance in equity returns and explain 40 percent of the model-implied equity premium. This result arises because equity holders, who are disproportionately wealthy, face substantial consumption volatility due to large return fluctuations, even though they are otherwise well-insured. The model thus bridges a gap in the literature by showing that nonfundamental asset price risk can generate sizable premia in a setting with realistic heterogeneity and limited aggregate effects. These findings suggest that the pricing of risk in financial markets is shaped not only by fundamentals but also by how nonfundamental shocks interact with portfolio heterogeneity and risk-sharing frictions.

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Appendix

I Appendix: Derivations for Equity Price

This section illustrates the derivation of the equilibrium equity price in the main text, and illustrates an alternative derivation based on an incomplete information setting.

I.1 Derivation of Equilibrium Equity Price

For each equity j, market clearing requires that aggregate demand equals the (normalized) unit supply:

$$\int_0^1 \theta_{ljt}\, dl \;=\; 1. \tag{30}$$

From the fundamental trader problem, the optimal demand for equity j is

$$\theta^F_{ljt} \;=\; -\,q_{jt} \;+\; \mathbb{E}_t\!\left[\frac{d_{jt+1}+q_{jt+1}}{1+r_{t+1}}\right] \quad \text{for all } l \in [0,\nu], \tag{31}$$

while noise traders follow the rule

$$\theta^N_{ljt} \;=\; \tilde{\xi}_t \;+\; \epsilon^\theta_{ljt} \quad \text{for all } l \in (\nu,1], \tag{32}$$

where ϵ^θ_{ljt} is iid with zero mean across l (and j). Integrating (31) over the mass ν of fundamental traders and (32) over the mass $1-\nu$ of noise traders yields

$$\int_0^\nu \theta^F_{ljt}\, dl = \nu\left(-\,q_{jt} + \mathbb{E}_t\!\left[\frac{d_{jt+1}+q_{jt+1}}{1+r_{t+1}}\right]\right), \tag{33}$$

$$\int_\nu^1 \theta^N_{ljt}\, dl = (1-\nu)\tilde{\xi}_t \;+\; \underbrace{\int_\nu^1 \epsilon^\theta_{ljt}\, dl}_{=0}. \tag{34}$$

The trader-stock specific shock ϵ^θ_{ljt} washes out when averaged over traders due to its iid structure. Substituting (33) and (34) into (30) gives

$$\nu\left(-\,q_{jt} + \mathbb{E}_t\!\left[\frac{d_{jt+1}+q_{jt+1}}{1+r_{t+1}}\right]\right) \;+\; (1-\nu)\tilde{\xi}_t \;=\; 1. \tag{35}$$

Solving (35) for q_{jt} yields

$$q_{jt} \;=\; \mathbb{E}_t\!\left[\frac{d_{jt+1}+q_{jt+1}}{1+r_{t+1}}\right] \;+\; \frac{(1-\nu)\tilde{\xi}_t - 1}{\nu}. \tag{36}$$

Define the effective nonfundamental asset-price term as

$$\xi_t \;\equiv\; \frac{(1-\nu)\tilde{\xi}_t - 1}{\nu}, \tag{37}$$

then (36) becomes

$$q_{jt} \;=\; \mathbb{E}_t\!\left[\frac{d_{jt+1}+q_{jt+1}}{1+r_{t+1}}\right] \;+\; \xi_t, \tag{38}$$

which matches equation (2) in the main text. By assumption, ξ_t follows the AR(1) process $\xi_t = \rho_q \xi_{t-1} + \epsilon_t^q$ with $\epsilon_t^q \sim \mathcal{N}(0, \sigma_q^2)$. In a symmetric equilibrium with identical firms, $q_{jt} = q_t$ and $d_{jt} = d_t$ for all j. Aggregating (38) across j yields the index-fund pricing equation

$$q_t \;=\; \mathbb{E}_t\!\left[\frac{d_{t+1} + q_{t+1}}{1 + r_{t+1}}\right] \;+\; \xi_t, \tag{39}$$

which coincides with equation (3).

I.2 Alternative Microfoundation of Asset Price Shock

This subsection provides an alternative microfoundation of asset price shocks based on incomplete information as in Futia (1981), Singleton (1986), Bacchetta and Wincoop (2006), Angeletos and Lian (2016), and Rondina and Walker (2021). This subsection provides an alternative microfoundation for nonfundamental asset price fluctuations based on incomplete information. Each trader $m \in [0, 1]$ observes a noisy signal about the future payoff of each equity $j \in [0, 1]$, given by:

$$x_{mjt} = d_{jt+1} + u_{mjt}, \quad \text{where } u_{mjt} \sim \mathcal{N}(\tilde{\xi}_t, \sigma_u^2).$$

The noise term u_{mjt} contains a cross-sectionally common distortion $\tilde{\xi}_t$, which biases the beliefs of all traders in the same direction.

Traders with Imperfect Information. Each trader lives for two periods,[23] is risk-neutral, discounts the future at the risk-free rate $1 + r_{t+1}$, and incurs quadratic disutility from monitoring firm-specific signals. Each trader chooses a portfolio allocation $\{\theta_{mjt}\}_{j \in [0,1]}$ to maximize:

$$U_{mt} = \max_{\{\theta_{mjt}\}} \int_0^1 \left[-q_{jt}\theta_{mjt} + \mathbb{E}_{mt}\left(\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right) \theta_{mjt} - \frac{1}{2}\theta_{mjt}^2 \right] dj,$$

where $\mathbb{E}_{mt}[\cdot]$ denotes trader m's subjective expectation, based on the signal x_{mjt}. The optimal portfolio demand satisfies:

$$\theta_{mjt} = \mathbb{E}_{mt}\left[\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right] - q_{jt}.$$

[23] By assuming that traders only live for two periods, higher-order believes beliefs of traders about next periods price become irrelevant. This assumption makes the solution more tractable, but as Bacchetta and Wincoop (2006) show in their paper, does not change the implications.

Equilibrium Asset Prices. Market clearing requires that the average demand equals the unit supply of each equity, that is:

$$\int_0^1 \theta_{mjt} dm = 1.$$

Substituting the demand expression yields the asset pricing equation:

$$q_{jt} = \bar{\mathbb{E}}_t \left[\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right] - 1,$$

where $\bar{\mathbb{E}}_t[\cdot]$ denotes the cross-sectional average of individual expectations.

Belief Distortions and Nonfundamental Prices. Bayesian updating under normally distributed noise implies that all traders share a distorted belief about the average payoff:

$$\bar{\mathbb{E}}_t \left[\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right] = \mathbb{E}_t^{\text{true}} \left[\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right] + \frac{\tilde{\xi}_t}{1 + r_{t+1}}.$$

Substituting this into the pricing equation yields:

$$q_{jt} = \mathbb{E}_t^{\text{true}} \left[\frac{d_{jt+1} + q_{jt+1}}{1 + r_{t+1}} \right] + \xi_t,$$

where the effective asset price shock is defined as:

$$\xi_t \equiv \frac{\tilde{\xi}_t}{1 + r_{t+1}} - 1.$$

As in the main text, we assume ξ_t follows a stationary AR(1) process:

$$\xi_t = \rho_q \xi_{t-1} + \epsilon_t^q, \quad \epsilon_t^q \sim \mathcal{N}(0, \sigma_q^2).$$

Symmetric Equilibrium and Index Fund Pricing. Assuming that all equities are symmetric and deliver identical payoffs, the price of the equity index fund satisfies:

$$q_t = \mathbb{E}_t^{\text{true}} \left[\frac{d_{t+1} + q_{t+1}}{1 + r_{t+1}} \right] + \xi_t,$$

which corresponds exactly to equation (3) in the main text. Hence, distorted beliefs due to incomplete information can rationalize the same reduced-form expression for nonfundamental price movements as in the model with noise traders.

Implications for Returns. As before, asset price fluctuations translate into excess returns through:

$$r^e_t = \frac{q_t - q_{t-1} + d_t}{q_{t-1}},$$

such that nonfundamental shocks affect both prices and returns, even in the absence of changes to dividends or discount factors.

II Appendix: Derivations of the endogenous portfolios

This section derives the results of Auclert et al. (2024) in a unified manner.

Setting and perturbation

There exists a continuum of heterogeneous agents with index i who can allocate their wealth a_i to up to $K+1$ assets. An asset k has supply A^k and stochastic payoff $x^k(\epsilon)$, where $\epsilon \equiv (\epsilon_1, ..., \epsilon_Z)$ denotes the vector of Z exogenous shocks. We suppose that $\epsilon_Z = \sigma\overline{\epsilon}_Z$, with $\overline{\epsilon}_Z \sim N(0, \overline{\sigma}^2_Z)$, such that σ is the common volatility that exists in the economy. Denoting the value function of household i by W_i and given the price of asset k as p^k, the problem of household i is

$$\max_{a_i^k} \mathbb{E}_\epsilon \left[W_i \left(\sum_{k=0}^{K} x^k(\epsilon) a_i^k, \epsilon \right) \right] \tag{40}$$

$$\text{s.t.} \quad \sum_{k=0}^{K} p^k a_i^k = a_i \tag{41}$$

$$\text{and} \quad \sum_{k=0}^{K} \theta_l^k p^k a_i^k \geq 0 \tag{42}$$

with $W_i(a', \epsilon) = \mathbb{E}_{s',s}\left[V(a', s', \epsilon)\right]$ where s' denotes the idiosyncratic risk (without aging transition) and θ_l^k denotes the coefficient for asset k in the constraint l. Denoting the Lagrange multiplier on i's budget constraint by γ_i and the Lagrange multiplier on the l non-negativity constraints with η_{il}, the problem has the first-order conditions

$$\mathbb{E}_\epsilon \left[\frac{x^k(\epsilon)}{p^k} \frac{W_i'(\epsilon)}{\gamma_i} \right] = 1 + \sum_{l=0}^{L} \frac{\eta_{il}}{\gamma_i} \sum_{k=0}^{K} \theta_l^k \tag{43}$$

which must hold for every i and for every k. Writing di for distribution of agents i, market clearing in all asset markets imposes:

$$\int a_i^k di = A^k \quad \forall k. \tag{44}$$

Given primitives a_i and W_i, as well as the parameter σ, an equilibrium is a set of prices for each asset p^k and Lagrange multipliers for each agent γ_i and η_{il}, such that the optimality conditions (43) are satisfied for each (i,k) pair, and all asset markets clear, i.e. (44) holds for all k.

We now work out the implications of these equations for a perturbation in σ up to the second order. We write $p^k(\sigma)$, $\gamma_i(\sigma)$, and $\eta_{il}(\sigma)$ for the solution at a given σ and study their second-order Taylor expansion around $\sigma = 0$. We note that, given that the distribution of ϵ is symmetric, these must be even functions of σ: $p^k(-\sigma) = p^k(\sigma)$, $\gamma_i(-\sigma) = \gamma_i(\sigma)$ and $\eta_{il}(-\sigma) = \eta_{il}(\sigma)$. This implies, in particular, that $\frac{d\gamma_i}{d\sigma} = \frac{dp^k}{d\sigma} = \frac{d\eta_{il}}{d\sigma} = 0$[24], a result that we will use several times below.

Zero-th and first-order perturbation: Applying (43) at $\sigma = 0$, we find $\gamma_i/W_i' = x^k/p^k$ for all i and k, where p^k is short-hand for $p^k(0)$, γ_i for $\gamma_i(0)$, x^k for $x^k(0)$, and W_i' for $W_i'(\sum_{k=0}^K x^k a_i^k, 0)$. Hence, the returns on all assets must equal a common constant R, and this is also the rate entering the Euler equation of all agents:

$$\gamma_i/W_i' = x^k/p^k = R \tag{45}$$

In particular, $\sum_{k=0}^K x^k a_i^k$ is also just $R\sum_{k=0}^K p^k a_i^k = Ra_i$. Equation (45) gives the usual result that, with no aggregate uncertainty, all assets must have equal returns.

Next, differentiating (43) with respect to σ (and around $\sigma = 0$) gives us

$$\mathbb{E}\left[\frac{dx^k}{d\sigma}W_i' + x^k\frac{dW_i'}{d\sigma}\right] = \frac{d\gamma_i}{d\sigma}p^k + \left(\gamma_i + \sum_{l=0}^{L}\eta_{il}\theta_l^k\right)\frac{dp^k}{d\sigma} + \sum_{l=0}^{L}\frac{d\eta_{il}}{d\sigma}\theta_l^k p^k \tag{46}$$

Given the definition $x^k(\epsilon) = x^k(\sigma\bar{\epsilon}_1, ..., \sigma\bar{\epsilon}_Z)$, and $W_i(\sum_{k=0}^K x^k(\sigma\bar{\epsilon})a_i^k, \sigma\bar{\epsilon})$, we have that

$$\frac{dx^k}{d\sigma} = \sum_{z=1}^{Z}\frac{\partial x^k}{\partial \epsilon_z}\bar{\epsilon}_Z \quad \text{and} \quad \frac{dW_i'}{d\sigma} = \sum_{z=1}^{Z}\frac{dW_i'}{d\epsilon_z}\bar{\epsilon}_z \tag{47}$$

[24] Up to first order, the portfolios are not determined such that the portfolio constraints do not bind. The only constraint that might bind is the constraint on total wealth.

where we have defined the total deriviate of W_i' with respect to ϵ_z as

$$\frac{dW_i'}{d\epsilon_z} \equiv W_i'' \sum_{k=0}^{K} \frac{\partial x^k}{\partial \epsilon_z} a_i^k + \frac{\partial W_i'}{\partial \epsilon_z}$$

Since $\mathbb{E}[\overline{\epsilon}_z] = 0$, using equation (47) to substitute into (46), we see that the left-hand side is zero. The right-hand side of (46) is also zero, given our symmetry result above, so equation (46) holds regardless of portfolios.

Second-order perturbation: Now, differentiating (46) with respect to σ gives us:

$$\begin{aligned}\mathbb{E}\left[\frac{d^2x^k}{d\sigma^2}\right] W_i' + 2\mathbb{E}\left[\frac{dx^k}{d\sigma}\frac{dW_i'}{d\sigma}\right] + x^k\mathbb{E}\left[\frac{d^2W_i'}{d\sigma^2}\right] = \\ \frac{d^2\gamma_i}{d\sigma^2}p^k + 2\left(\frac{d\gamma_i}{d\sigma} + \sum_{l=0}^{L}\frac{d\eta_{il}}{d\sigma}\theta_l^k\right)\frac{dp^k}{d\sigma} + \gamma_i\frac{d^2p^k}{d\sigma^2} + \sum_{l=0}^{L}\frac{d^2\eta_{il}}{d\sigma^2}\theta_l^k p^k \end{aligned} \tag{48}$$

Using our symmetry results from above, and dividing all entries by $x^k W_i' = \gamma_i p^k$ from (45), we can write this simply as:

$$\mathbb{E}\left[\frac{dx^k/x^k}{d\sigma}\frac{dW_i'/W_i'}{d\sigma}\right] = \alpha_i + \beta^k + \delta_i^k, \tag{49}$$

where α_i, which only depends on household i, β^k, which only depends on asset k, and δ_i^k, which depends on both are defined as

$$\begin{aligned}\alpha_i &\equiv \frac{1}{2}\left(\frac{d^2\gamma_i/\gamma_i}{d\sigma^2} - \mathbb{E}\left[\frac{d^2W_i'/W_i'}{d\sigma^2}\right]\right) \\ \beta^k &\equiv \frac{1}{2}\left(\frac{d^2p^k/p^k}{d\sigma^2} - \mathbb{E}\left[\frac{d^2x^k/x^k}{d\sigma^2}\right]\right) \\ \delta_i^k &\equiv \sum_{l=0}^{L}\theta_l^k\frac{d^2\eta_{il}/\gamma_i}{d\sigma^2}\end{aligned}$$

Using (47), and the fact that $\mathbb{E}[\overline{\epsilon\epsilon}'] = \Sigma$, we can rewrite (49) as

$$\sum_{z=1}^{Z}\frac{\partial x^k/x^k}{\partial\epsilon_z}\frac{dW_i'/W_i'}{d\epsilon_z}\overline{\sigma}_z^2 = \alpha_i + \beta^k + \delta_i^k \quad \forall\, i,k \tag{50}$$

We note that this applies to the product of two first derivatives, and therefore, intuitively, places restrictions on the relationship between the impulse response of returns and marginal utilities. Finally, using (50) for asset k relative to asset 0 (where we note that 0 could correspond to any reference asset in the economy), we obtain:

$$\sum_{z=1}^{Z}\left(\frac{\partial x^k/x^k}{\partial \epsilon_z}-\frac{\partial x^0/x^0}{\partial \epsilon_z}\right)\frac{dW_i'/W_i'}{d\epsilon_z}\overline{\sigma}_z^2=\underbrace{\beta^k-\beta^0}_{b^k}+\underbrace{\delta_i^k-\delta_i^0}_{d_i^k}\quad \forall\, i,k \tag{51}$$

Equation (51) says that all households with an internal portfolio solution (implying $\delta_i^k=0$) equalize their average sensitivity to shocks z, interacted with the relative returns on asset k, to a k-specific term b^k. We will soon see that this term has the interpretation of a relative risk premium on asset k. Stacking $\mathbf{b}\equiv(b^1,...,b^K)'$ as a $K\times 1$ vector of relative risk premia, $\boldsymbol{\lambda}_i\equiv(\frac{dW_i'/W_i'}{d\epsilon_1},...,\frac{dW_i'/W_i'}{d\epsilon_Z})'$ as a $Z\times 1$ vector of sensitivities of marginal utility to each shock, defining the $Z\times K$ matrix $\mathbf{X}$ with elements equal to the relative returns of each asset to each shock $X_{zk}\equiv\frac{\partial x_k/x_k}{\partial\epsilon_Z}-\frac{\partial x_0/x_0}{\partial\epsilon_Z}$, letting $\boldsymbol{\Sigma}$ denote the $Z\times Z$ matrix with $\overline{\sigma}_z^2$ on its diagonal, and defining the matrix $\boldsymbol{\Theta}$ by $\Theta_{lk}=\theta_l^k-\theta_l^0$ and the vector $\boldsymbol{\eta}_{il}=\frac{d^2\chi_{il}/\gamma_i}{d\sigma^2}$ such that $\boldsymbol{\Theta}'\boldsymbol{\eta}_i\equiv(d_i^1,...,d_i^K)'$ is a $K\times 1$ vector, equation (51) becomes:

$$\mathbf{X}'\boldsymbol{\Sigma}\boldsymbol{\lambda}_i=\boldsymbol{b}+\boldsymbol{\Theta}'\boldsymbol{\eta}_i\quad\forall\, i \tag{52}$$

The term $\boldsymbol{\Theta}'\boldsymbol{\eta}_i$ reflects the shadow value of the constraints.

Complete markets

Suppose that $K=Z$, such that the number of assets equals the number of shocks plus one. This effectively allows households to insure against all aggregate shocks by taking respective portfolio positions. We say that this corresponds to complete markets with respect to aggregate risk. Then $\mathbf{X}$ is a square matrix. Additionally, suppose the following assumptions are fulfilled:

Assumption 1 (Spanning)**.** *The rows of* $\mathbf{X}$ *are linearly independent.*

and

Assumption 2 (Constraints)**.** *There are no portfolio constraints, such that* $\eta_{it}=\delta_i^k=0$ *and* $\boldsymbol{\Theta}'\boldsymbol{\eta}_i=\mathbf{0}_K$.

Assumption 1 says that the relative returns across assets vary sufficiently across shocks, while assumption 2 abstracts from portfolio constraints. Under the first assumption, the $Z\times Z$ matrix $\mathbf{X}'\boldsymbol{\Sigma}$ is invertible, while the second assumption abstracts from idiosyncratic binding constraints. Condition (52) can therefore be rewritten:

$$\boldsymbol{\lambda}_i=(\mathbf{X}')^{-1}\boldsymbol{\Sigma}^{-1}\mathbf{b}\equiv\boldsymbol{\lambda}, \tag{53}$$

which yields the first main result.

Proposition 1. *Suppose that $K = Z$ and assumptions 1 and 2 hold. Then for each shock z, there exists a λ_z such that*

$$\frac{dW_i'/W_i'}{d\epsilon_z} = \lambda_z \quad \forall\, i. \tag{54}$$

Proposition 1 provides us with a simple test of portfolio optimality in a setting where $K = Z$. To understand the test, note that standard first-order methods allow us relatively easily to solve for steady-state x^k, W_i, as well as $\frac{x^k}{\partial \epsilon_z}$ and $\frac{dW_i'}{d\epsilon_z}$ for given shocks z, conditional on given incoming portfolios $\{a_i^k\}$ for all agents. With these objects, one can form the matrix of relative returns $\mathbf{X}$ to test if the spanning assumption 1 is satisfied, and then test whether $\frac{dW_i'/W_i'}{d\epsilon_z}$ are equalized across agents i for all shocks z. If so, proposition 1 tells us that the portfolios are optimal.

Proposition 1 also implies a method for solving for optimal portfolios directly. Suppose that $\overline{a}_i^k$ is an exogenous portfolio and let t_i be the excess payoff from another portfolio a_i^k such that

$$t_i \equiv \sum_{k=0}^{K} x^k(\epsilon)(a_i^k - \overline{a}_i^k). \tag{55}$$

Moreover, let $\overline{W}_i\left(\sum_{k=0}^{K} x^k(\epsilon)\overline{a}_i^k, \boldsymbol{\epsilon}\right)$ denote the value function under the exogenous portfolio, whereas $W_i(t_i, \boldsymbol{\epsilon}) \equiv \overline{W}_i\left(\sum_{k=0}^{K} x^k(\epsilon)\overline{a}_i^k + t_i, \epsilon\right)$ denotes the value function under the portfolio a_i^k. With complete markets and assumptions 1 and 2 in place, households portfolios should satisfy the risk-sharing condition (54). We can find the corresponding excess payoff t_i, by imposing that it satisfies the risk-sharing condition. Given the exogenous portfolio $\overline{a}_i^k$, we can approximate the risk-sharing condition at the optimal portfolio around the utility change in the exogenous portfolio case as

$$\frac{d\overline{W}_i'/\overline{W}_i'}{d\epsilon_z} + \frac{\overline{W}_i''}{\overline{W}_i'}\frac{dt_i}{d\epsilon_z} = \lambda_z. \tag{56}$$

The first term on the right-hand side of (56) refers to the direct exposure to shocks under exogenous portfolios, whereas the second term denotes the "transfer" exposure to shocks under a portfolio that achieves optimal aggregate risk-sharing. Intuitively, equation (56) provides a condition to solve for transfers contingent on shocks $dt_i/d\epsilon_z$:

$$\frac{dt_i}{d\epsilon_z} = \frac{\overline{W}_i'}{\overline{W}_i''}\left(\lambda_z - \frac{d\overline{W}_i'/\overline{W}_i'}{d\epsilon_z}\right) \tag{57}$$

and since transfers have to sum to zero, $\int \frac{dt_i}{d\epsilon_z} di = 0$, we obtain:

$$\lambda_z = \left(\int \frac{\overline{W_i'}}{\overline{W_i''}} di \right)^{-1} \int \frac{\overline{W_i'}}{\overline{W_i''}} \frac{d\overline{W_i'}/\overline{W_i'}}{d\epsilon_z} di. \tag{58}$$

We can use these two equations to derive λ_z via equation (58) and then obtain excess returns via equation (57). From the definition of the excess returns (55), we then obtain the relation between transfers and the endogenous portfolios that ensures optimal insurance against aggregate risk:

$$\frac{dt_i}{d\epsilon_z} = \sum_{k=0}^{K} \frac{dx^k}{d\epsilon_z}(\epsilon)(a_i^k - \overline{a}_i^k) \tag{59}$$

Using the definition of portfolio shares $\omega_i^k = \frac{a_i^k}{a_i}$ and $\overline{\omega}_i^k = \frac{\overline{a}_i^k}{a_i}$ we can rewrite equation (59) to

$$\begin{aligned} \frac{\partial t_i}{\partial \epsilon_z} &= \sum_{k=0}^{K} \frac{\partial x^k}{\partial \epsilon_z}(\epsilon)(a_i^k - \overline{a}_i^k) \\ &= a_i \sum_{k=0}^{K} \frac{\partial x^k}{\partial \epsilon_z}(\epsilon)(\omega_i^k - \overline{\omega}_i^k) \\ &= a_i \sum_{k=1}^{K} \left(\frac{\partial x^k}{\partial \epsilon_z}(\epsilon) - \frac{\partial x^0}{\partial \epsilon_z}(\epsilon) \right) (\omega_i^k - \overline{\omega}_i^k). \end{aligned}$$

Using the definition of $\mathbf{X}$ from above, and defining vectors $\boldsymbol{\omega}_i = (\omega_i^1, ..., \omega_i^K)'$, $\overline{\boldsymbol{\omega}}_i = (\overline{\omega}_i^1, ..., \overline{\omega}_i^K)'$, and $\mathbf{t}_i = (\frac{\partial t_i}{\partial \epsilon_1}, ..., \frac{\partial t_i}{\partial \epsilon_Z})'$, we can write the optimal portfolio weights as

$$\mathbf{t}_i = \mathbf{X}(\boldsymbol{\omega}_i - \overline{\boldsymbol{\omega}}_i) a_i \quad \Leftrightarrow \quad \boldsymbol{\omega}_i = \overline{\boldsymbol{\omega}}_i + \mathbf{X}^{-1} \frac{\mathbf{t}_i}{a_i} \tag{60}$$

For $K = 1$ (two assets) the relation becomes

$$\omega_i^1 = \overline{\omega}_i^1 + \frac{1}{a_i} \left(\frac{\partial x^1}{\partial \epsilon_z}(\epsilon) - \frac{\partial x^0}{\partial \epsilon_z}(\epsilon) \right)^{-1} \frac{dt_i}{d\epsilon_z}.$$

Finally, we can calculate the risk premia associated with the individual assets. We want to approximate risk-premia up to second order around $\sigma = 0$. First, let $R^k(\sigma) = \mathbb{E}\left[x^k(\sigma)\right] / p^k(\sigma)$ define the expected return on asset k. From equation (45), we have

$R^k(0) = R$. The derivative of the expected return with respect to σ is

$$\frac{dR^k(\sigma)}{d\sigma} = \mathbb{E}\left[\frac{dx^k(\sigma)}{d\sigma}\right]\frac{1}{p^k(\sigma)} - \mathbb{E}\left[\frac{x^k(\sigma)}{p^k(\sigma)}\right]\frac{dp^k(\sigma)/p^k(\sigma)}{d\sigma} = 0, \tag{61}$$

which uses equation (47), $\mathbb{E}\left[\frac{dx^k}{d\sigma}\right] = 0$ and $\frac{dp^k}{d\sigma} = 0$ from our symmetry result. Finally, the second-order derivative of the expected return of asset k with respect to σ is

$$\begin{aligned}\frac{d^2R^k(\sigma)}{d\sigma^2} =& \mathbb{E}\left[\frac{d^2x^k(\sigma)}{d\sigma^2}\right]\frac{1}{p^k(\sigma)} - 2\mathbb{E}\left[\frac{dx^k(\sigma)}{d\sigma}\right]\frac{dp^k(\sigma)/p^k(\sigma)}{d\sigma}\frac{1}{p^k(\sigma)} \\ &- \mathbb{E}\left[x^k(\sigma)\right]\left[\frac{\frac{d^2p^k(\sigma)}{d\sigma^2}p^k(\sigma)^2 - 2\frac{dp^k(\sigma)}{d\sigma}p^k(\sigma)}{\left(p^k(\sigma)\right)^4}\right] \\ =& \mathbb{E}\left[\frac{d^2x^k(\sigma)}{d\sigma^2}\right]\frac{1}{p^k(\sigma)} - \mathbb{E}\left[\frac{x^k(\sigma)}{p^k(\sigma)}\right]\left[\frac{d^2p^k(\sigma)/p^k(\sigma)}{d\sigma^2}\right],\end{aligned} \tag{62}$$

where we have used again that the derivatives of the first order of payoffs $x^k(\sigma)$ and prices $p^k(\sigma)$ are zero. Note that $\frac{d^2R^k(\sigma)}{d\sigma^2} = -2R^k(\sigma)\beta^k$. A second-order Taylor expansion of the expected return $R^k(\sigma)$ around $\sigma = 0$ yields

$$R^k(\sigma) \approx R^k(0) + \frac{dR^k(0)}{d\sigma}\sigma + \frac{1}{2}\frac{d^2R^k(0)}{d\sigma^2}\sigma^2 = R - R\beta^k\sigma^2,$$

such that the relative risk premium of asset k against asset 0 has the second order expansion

$$\frac{R^k(\sigma) - R^0(\sigma)}{R} \approx -(\beta^k - \beta^0)\sigma^2 = -b^k\sigma^2. \tag{63}$$

We can use equation (51) and from proposition 1 equation (54) to obtain:

$$b^k = \sum_{z=1}^{Z}\underbrace{\left(\frac{\partial x^k/x^k}{\partial\epsilon_z} - \frac{\partial x^0/x^k}{\partial\epsilon_z}\right)}_{X_{zk}}\underbrace{\frac{dW_i'/W_i'}{d\epsilon_z}}_{\lambda_z}\overline{\sigma}_z^2 = \sum_{z=1}^{Z}X_{zk}\lambda_z\overline{\sigma}_z^2 \tag{64}$$

such that

Proposition 2. *Suppose markets are complete and assumptions (1) and (2) hold. Then, the risk premia of asset k relative to asset 0 satisfies, to second order*

$$\frac{R^k(\sigma) - R^0(\sigma)}{R} \approx -\sum_{z=1}^{Z}X_{zk}\lambda_z\overline{\sigma}_z^2\sigma^2. \tag{65}$$

Proposition 2 allows us to approximate the risk premia on an assets k using only the information from a first-order perturbation.

III Appendix: Data sources and transformations

This section describes the data used for the calibration and in the Bayesian estimation of the model in the main text.

III.1 Estimation on Time-Series Data

The observables used for the estimation can be summarized as

$$obs_t = \begin{bmatrix} \Delta\log(Y_t) \\ \Delta\log(C_t) \\ \Delta\log(I_t) \\ \Delta\log(w_t) \\ \Delta\log(q_t) \\ \Delta\log(d_t) \\ \log(N_t) \\ \log(1+r_t^{eq}) \\ \log(\frac{1}{q_t^{3m}}) \\ \log(1+\pi_t^p) \end{bmatrix} - \begin{bmatrix} \overline{\Delta\log(Y_t)} \\ \overline{\Delta\log(C_t)} \\ \overline{\Delta\log(I_t)} \\ \overline{\Delta\log(w_t)} \\ \overline{\Delta\log(q_t)} \\ \overline{\Delta\log(d_t)} \\ \overline{\log(N_t)} \\ \overline{\log(1+r_t^{eq})} \\ \overline{\log(\frac{1}{q_t^{3m}})} \\ \overline{\log(1+\pi_t^p)} \end{bmatrix}. \tag{66}$$

The Δ denotes the first difference between variables, and bars over variables denote the time-series averages. Except for the stock price and dividend series, all series are obtained from the St. Louise FED - FRED database. All data series from FRED are available at a quarterly frequency. The time series of stock prices, and dividends are obtained from the online database of Robert Shiller. The data was first generated for Shiller (1989), but was updated until today. The up-to-date time series can be accessed here. I extract monthly data on nominal stock prices Q_t^{eq} and dividends D_t from the online dataset and convert them to real series. I illustrate the transformation from monthly to quarterly frequency below and from nominal to real below.

Output Y_t: Sum of gross private domestic investment (GPDI), personal consumption expenditures for nondurable goods (PCND), durable goods (PCDG), and services (PCESV), and government consumption expenditures and gross investment (GCE) divided by the GDP deflator (GDPDEF) and the civilian noninstitutional population (CNP16OV).

Consumption C_t**:** Sum of personal consumption expenditures for nondurable goods (PCND), durable goods (PCDG), and services (PCESV) divided by the GDP deflator (GDPDEF) and the civilian noninstitutional population (CNP16OV).

Investment I_t**:** Gross private domestic investment (GPDI) divided by the GDP deflator (GDPDEF) and the civilian noninstitutional population (CNP16OV).

Real wage w_t**:** Hourly compensation in the nonfarm business sector (COMPNFB) divided by the GDP deflator (GDPDEF).

Hours worked N_t**:** Nonfarm business hours worked (HOANBS) divided by the civilian noninstitutional population (CNP16OV).

Inflation π_t**:** Computed as the log-difference of the GDP deflator (GDPDEF).

Nominal interest rate $\frac{1}{q_t^{3m}}$**:** Quarterly average of the effective federal funds rate (FEDFUNDS). From 2009-Q1 to 2015-Q4, I use the shadow federal funds rate of Wu and Xia (2016) instead of the federal funds rate, which was at the zero-lower bound.

Real stock prices q_t**:** The nominal stock price Q_t^{eq} (S&P Comp. P) is available at a monthly frequency in Robert Shiller's database. I convert the series to a quarterly frequency by taking the average over the realizations of the monthly stock price. Thereafter, the quarterly series is divided by the GDP deflator (GDPDEF) to obtain real stock prices.

Real dividends d_t**:** The nominal dividend D_t (Dividend) is available at a monthly frequency in Robert Shiller's database. I convert the series to a quarterly frequency by taking the average over the realizations of the monthly stock price. Thereafter, the quarterly series is divided by the GDP deflator (GDPDEF) to obtain the real dividend.

Real dividends d_t**:** The nominal dividend (Dividend) is available at a monthly frequency in Robert Shiller's database. I convert the series to a quarterly frequency by taking the average over the realizations of the monthly dividend. Thereafter, the quarterly series is divided by the GDP deflator (GDPDEF) to obtain the real dividend.

Real return r_t^{eq}**:** I use the quarterly nominal equity price Q_t^{eq} and the quarterly nominal dividend D_t to calculate the nominal equity return as $r_t^{eq,nom} = (Q_t^{eq} - Q_{t-1}^{eq})/Q_{t-1}^{eq} + D_t/D_{t-1}^{eq}$ based on the calculation of Jordà et al. (2019). I then convert the return from nominal to real by dividing it by the inflation rate obtained above: $1 + r_t^{eq} \equiv \frac{q_t + d_t}{q_{t-1}} = \frac{1 + r_t^{eq,nom}}{1 + \pi_t^p}$.

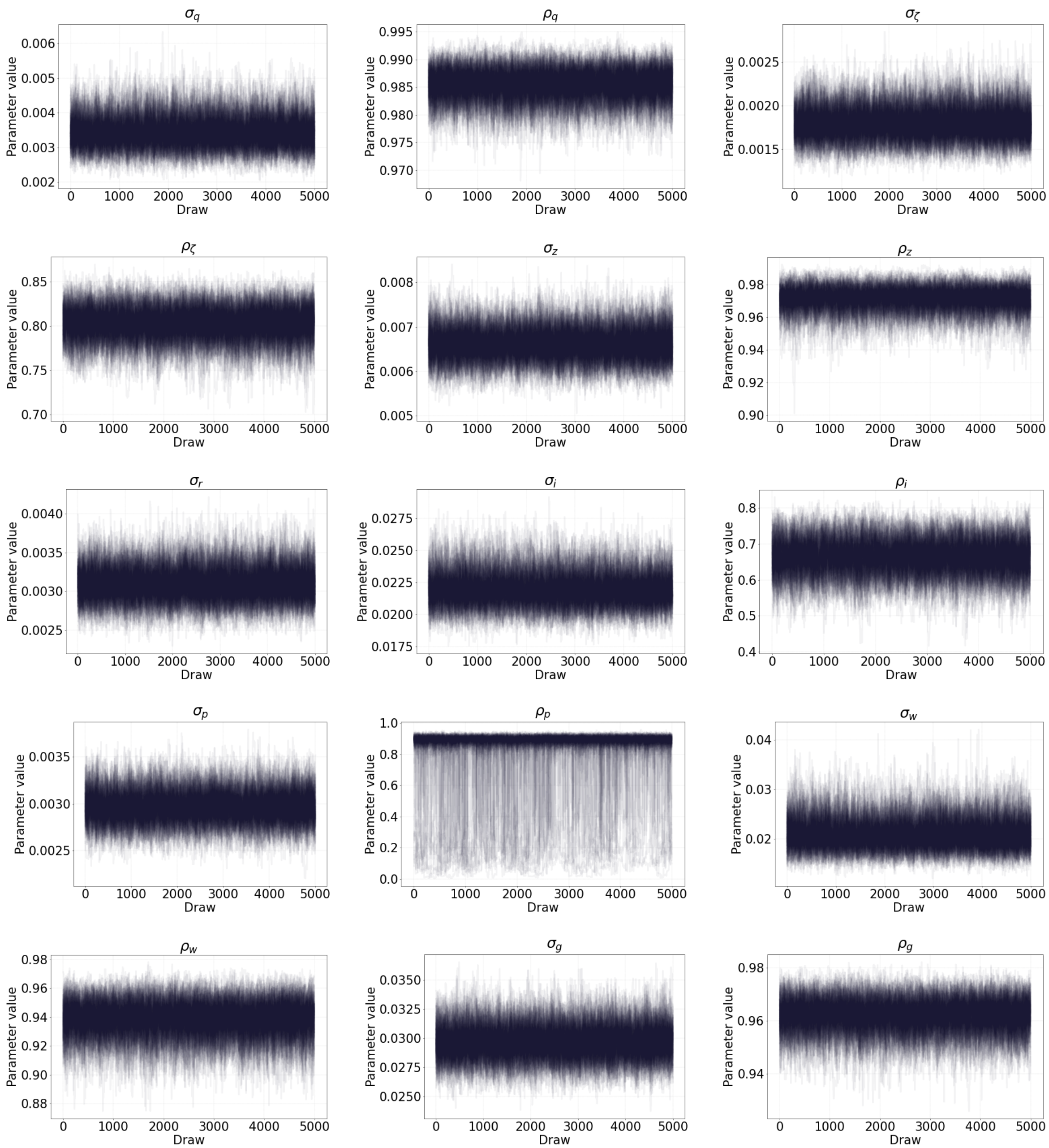
III.2 Calculation of Time-Series AVerages

IV Appendix: Estimation Diagnostics

Figures 4 and 5 illustrate the trace plots of the 128 chains over 5000 draws after a 2500 draw burn-in. Visual inspection of the trace plots indicates satisfactory mixing. For most parameters the chains resemble a classic hairy caterpillar pattern with stationary fluctuations around a stable level and no visible drift or regime shifts, and different chains overlap closely which suggests convergence. Shock standard deviations appear especially well behaved with tight stationary bands. Autoregressive coefficients mix somewhat more slowly as expected when persistence is high; for the autocorrelation of the price markup shock ρ_p, the mass concentrates close to one which introduces higher autocorrelation, yet the chains continue to explore the relevant region of the posterior rather than remaining stuck. In sum the traces support reliable posterior inference with only modest caution warranted for the most persistent coefficient.

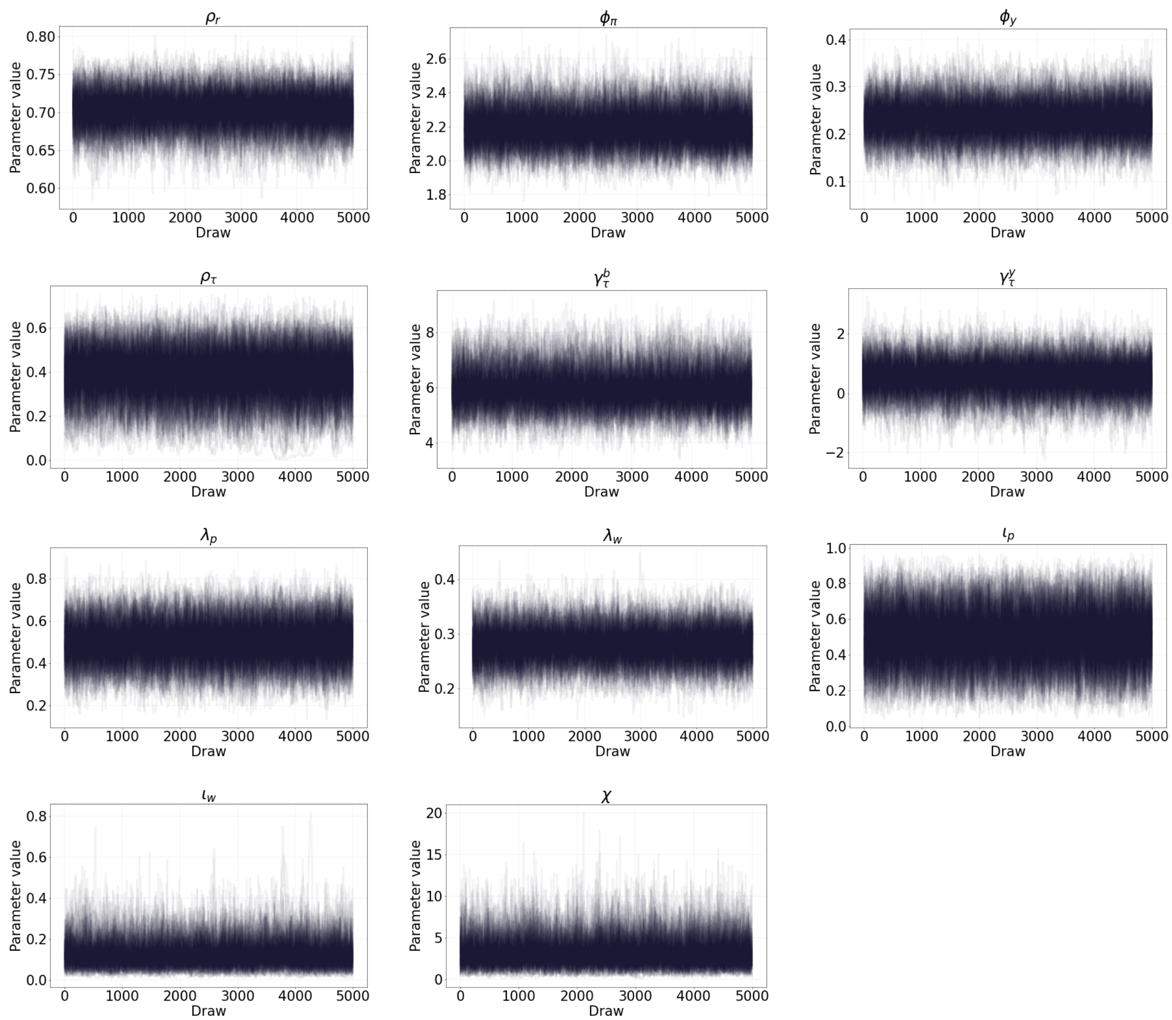
The traces for the policy parameters and frictions indicate generally satisfactory mixing after the burn in period. The chains for ρ_r, ϕ_π, and ϕ_y fluctuate around stable centers with frequent crossovers and no visible drift, which points to convergence. The frictions λ_p, λ_w, and ι_p also show tight stationary bands and good overlap. The fiscal block is somewhat more variable: ρ_τ explores a wider interval and moves more slowly, and γ_τ^y displays heavier tails, though both still travel across the high posterior region. The level parameter χ mixes the most slowly and spans the widest range, implying higher autocorrelation and a lower effective sample size relative to the others. Overall the figure supports reliable inference for most parameters, with mild caution warranted for χ and to a lesser extent for ρ_τ and γ_τ^y.

Figure 4 Traceplots for shock parameters after 2500 burn-in draws



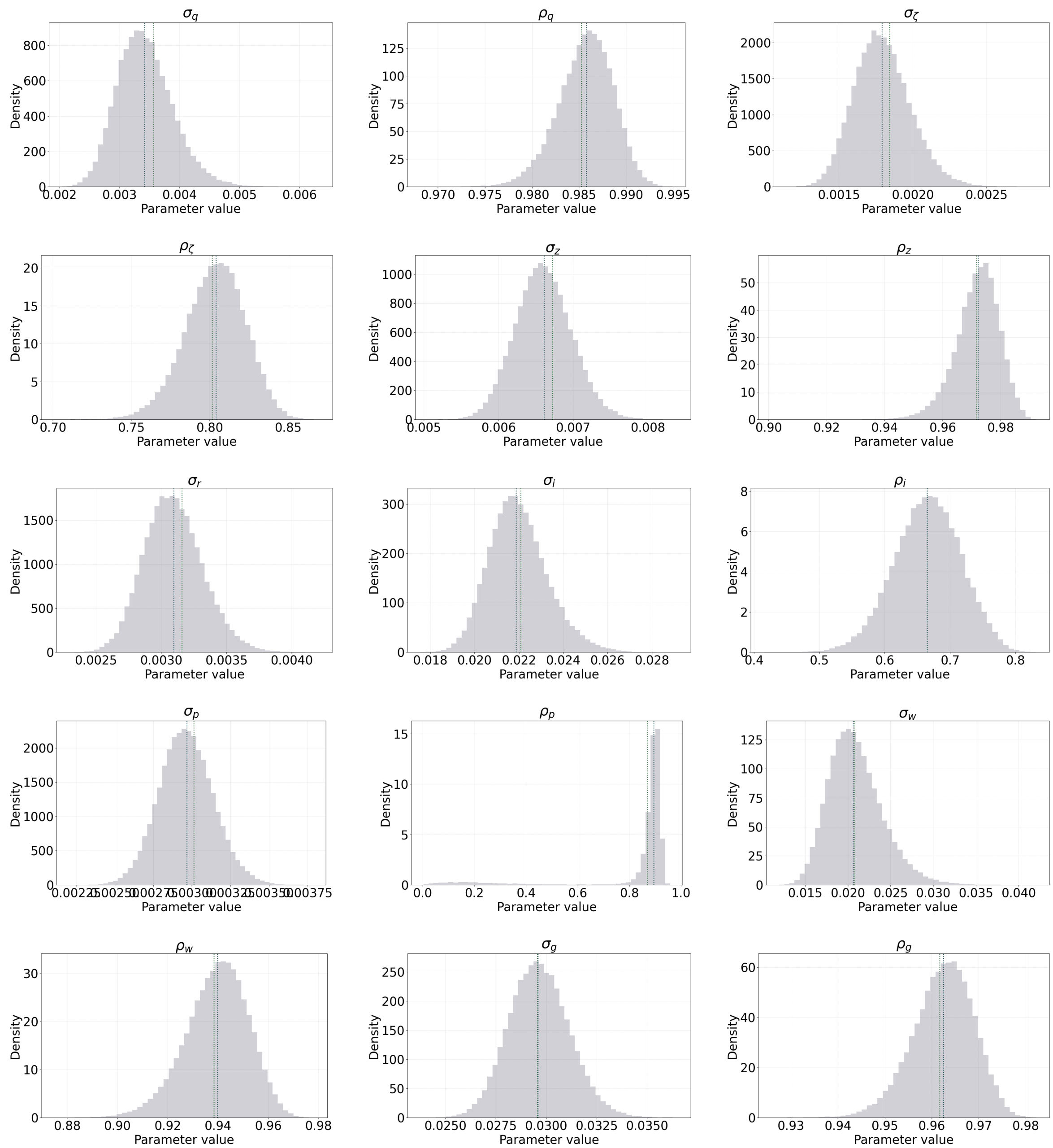
Notes: Traceplots of the 128 chains used in estimation. The traceplots only illustrates the last 5000 draws from all 128 chains. I discarded the first 2500 draws per chain as burn-in.

Figure 5 Traceplots for policy parameters and frictions after 2500 burn-in draws



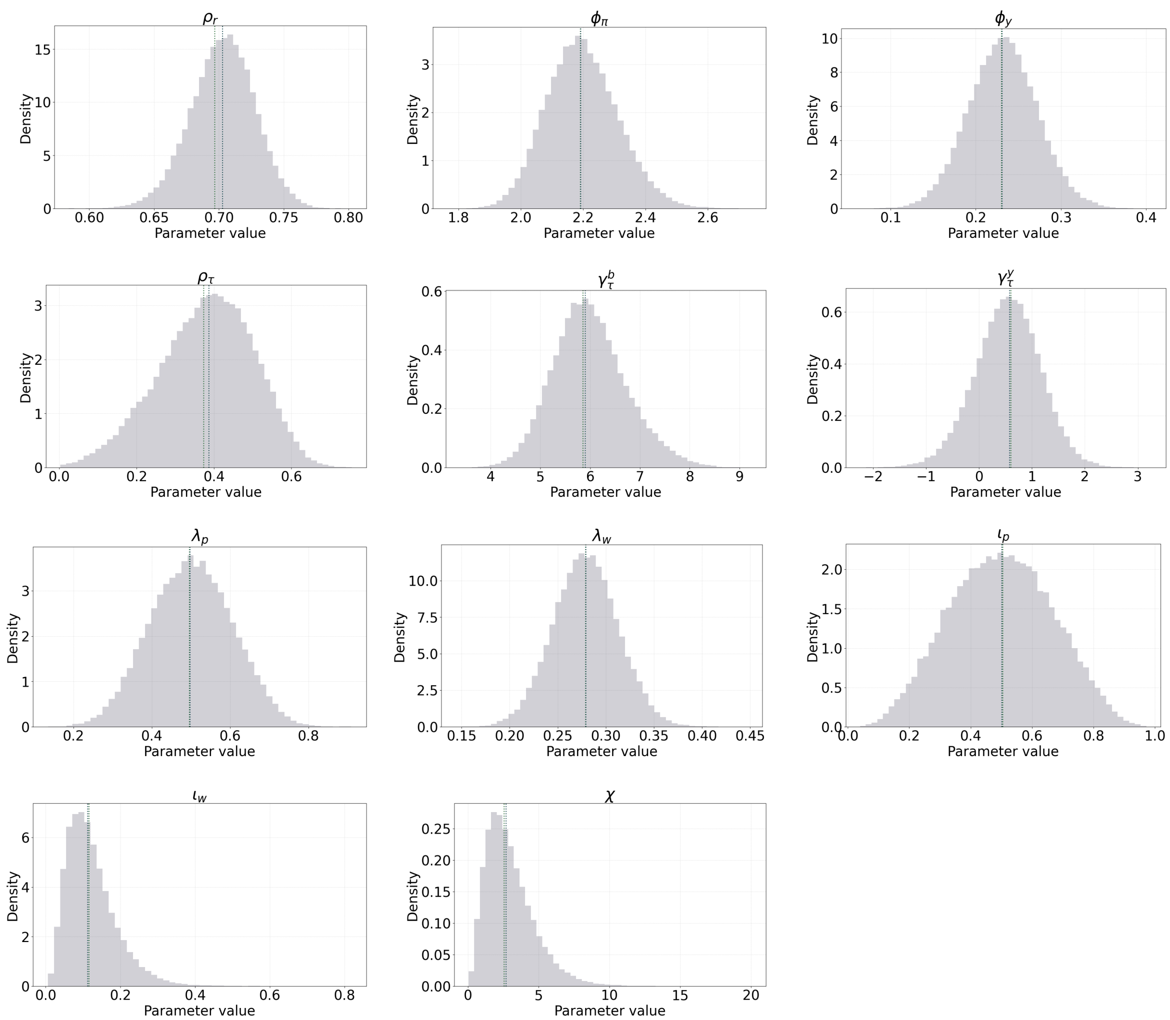
Notes: Traceplots of the 128 chains used in estimation. The traceplots only illustrates the last 5000 draws from all 128 chains. I discarded the first 2500 draws per chain as burn-in.

Figure 6 Posterior histogram of shock parameters



Notes: Posterior histogram from Bayesian estimation. The histogram only illustrates the last 5000 draws from all 128 chains. I discarded the first 2500 draws per chain as burn-in.

Figure 7 Posterior histogram of policy parameters and frictions

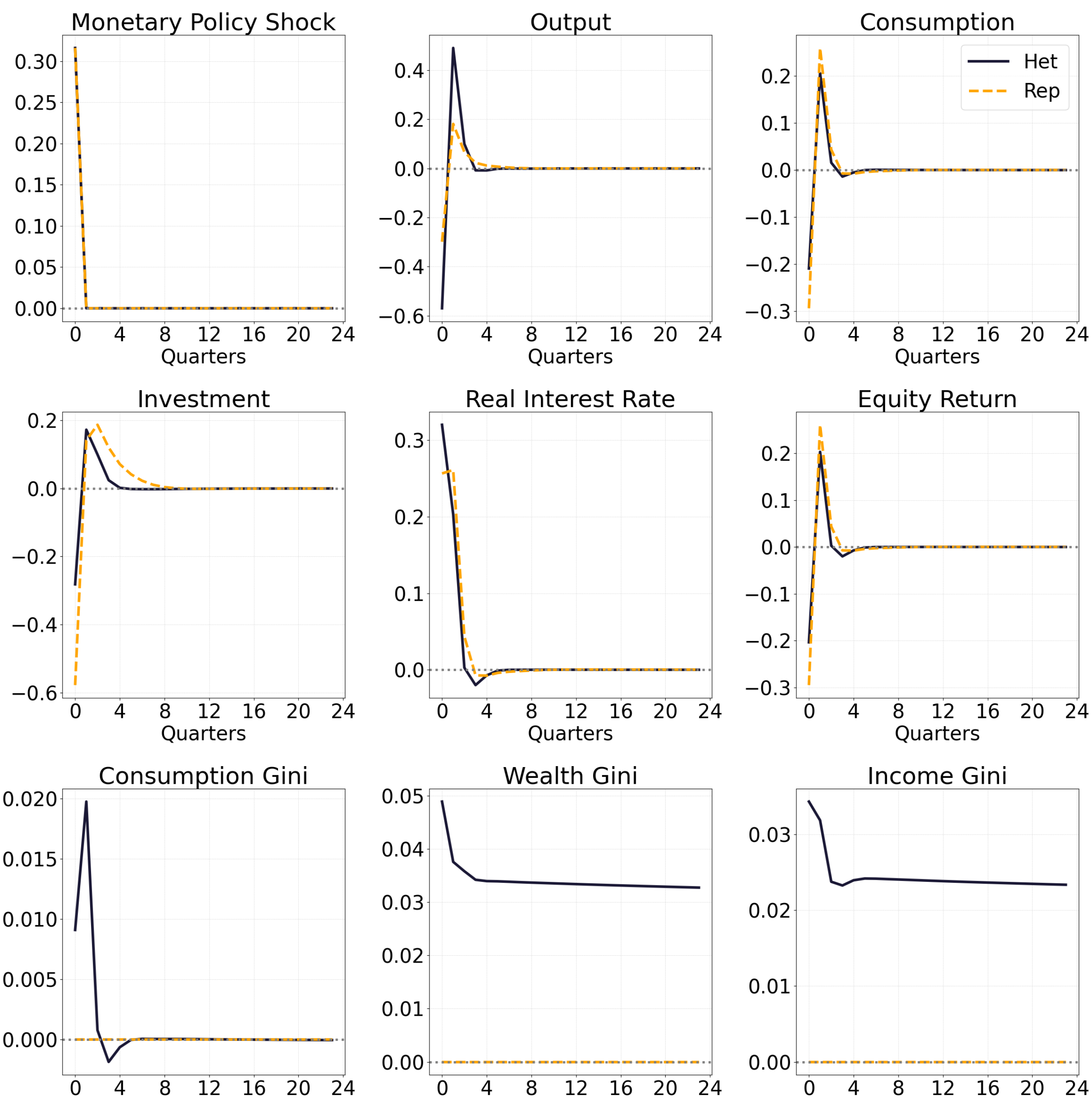


Notes: Posterior histogram from Bayesian estimation. The histogram only illustrates the last 5000 draws from all 128 chains. I discarded the first 2500 draws per chain as burn-in.

V Appendix: Structural Analysis

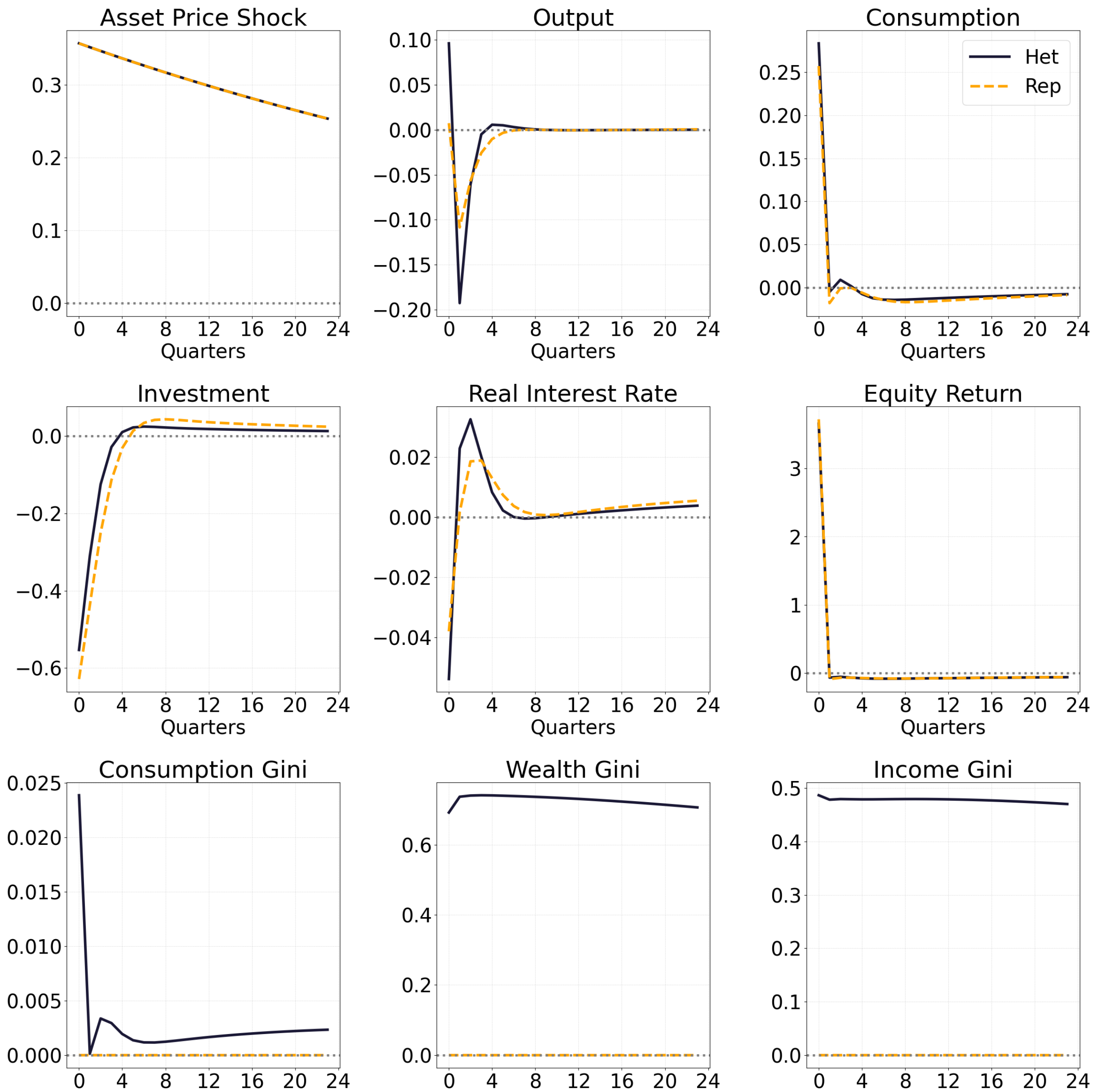
This section illustrates the results of a structural analysis following the estimation. Figures (8) and (9) illustrate the impulse response functions of the estimated heterogeneous agent (Het) and representative agent (Rep) model version to a monetary policy and to a asset price shock. Moreover, figure 11 provides a historical decomposition of the individual variables on which I estimate the model.

Figure 8 IRFs of Heterogeneous and Representative Agent Model to Monetary Policy



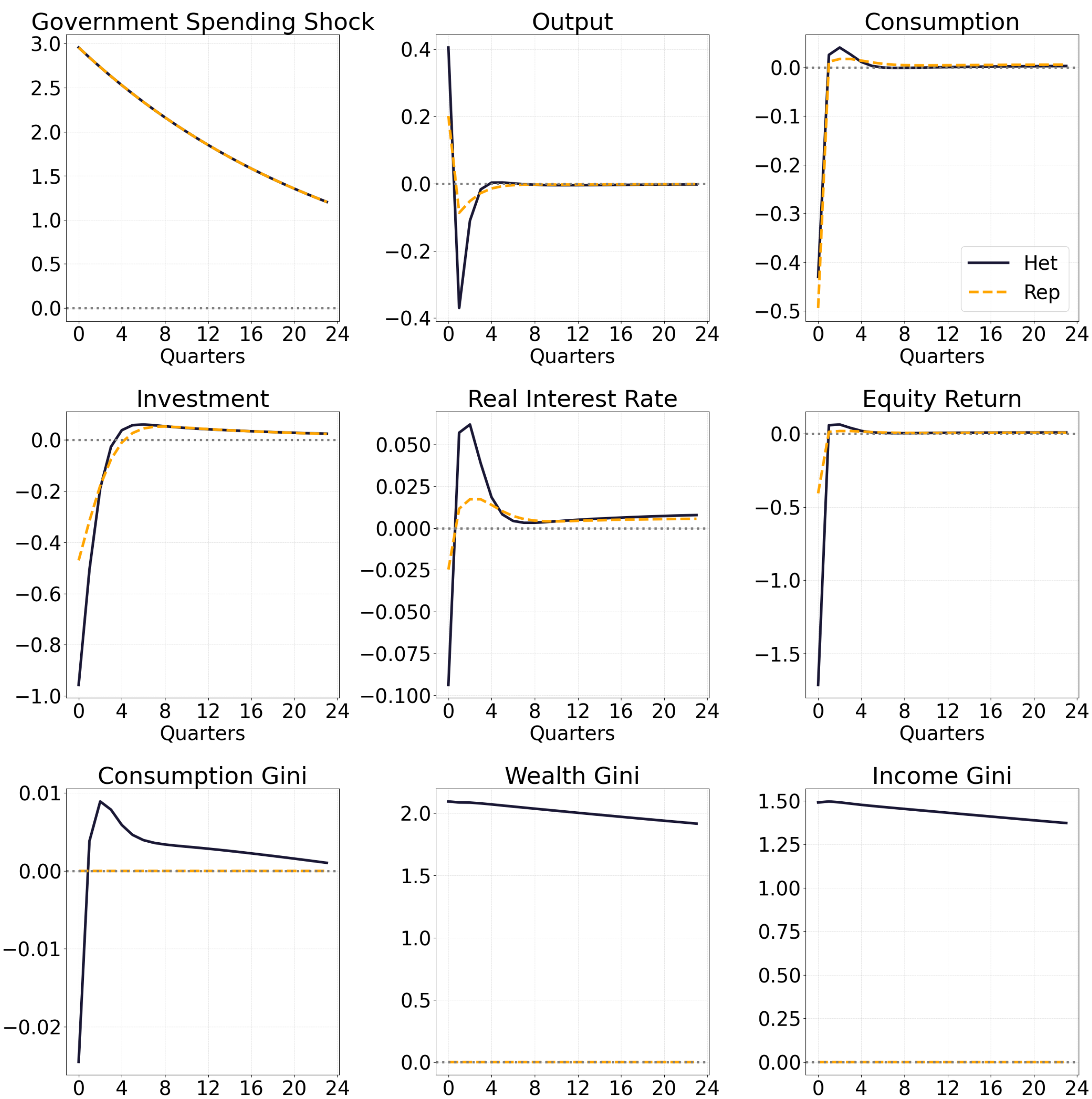
Notes: Impulse response functions (IRFs) of aggregates to monetary policy shock in the heterogeneous agent (Het) model and the representative agent (Rep) model version. The impulse responses shock absolute deviations from the steady state variable in response to the shock.

Figure 9 IRFs of Heterogeneous and Representative Agent Model to Asset Price Shock



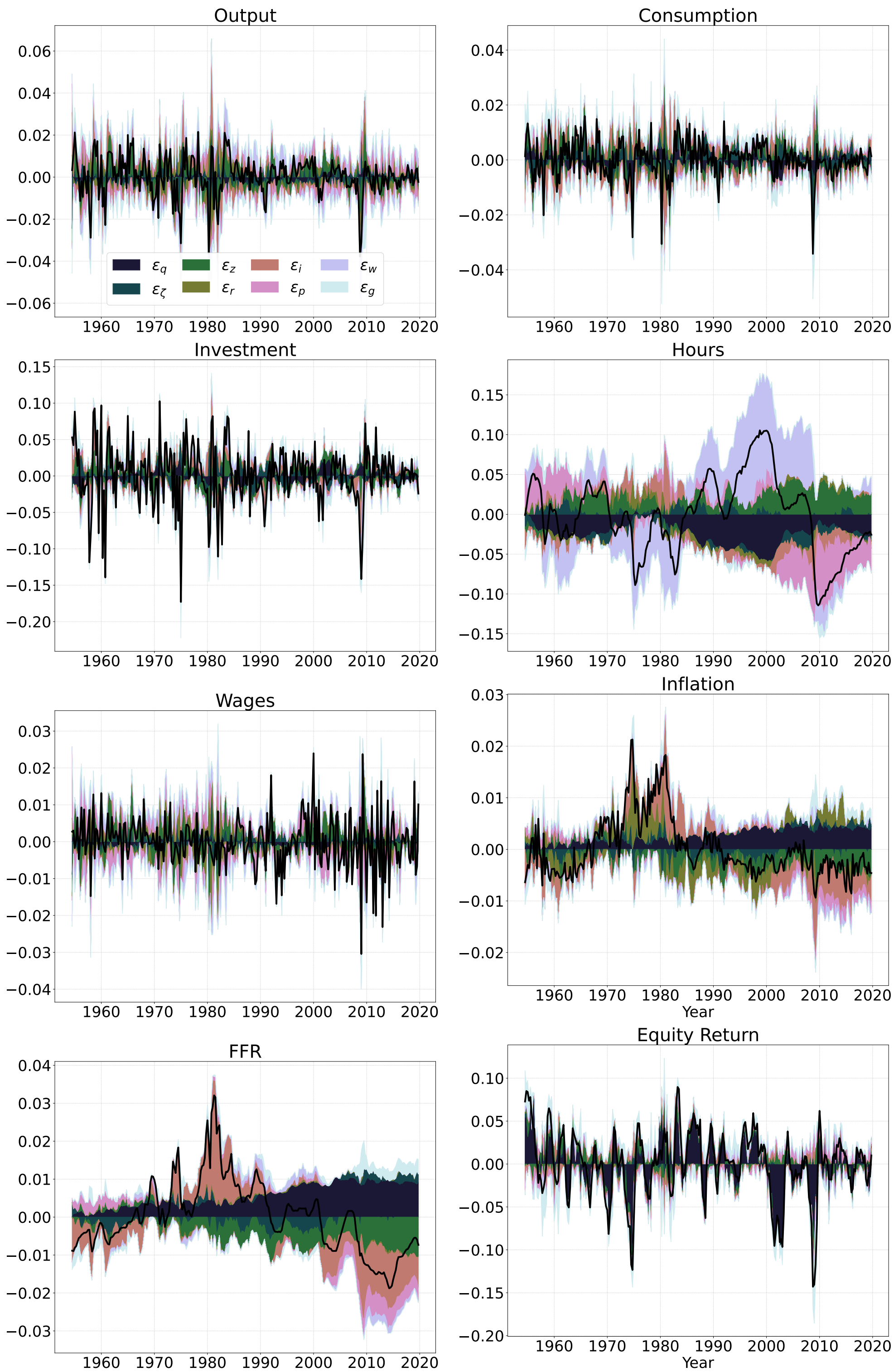
Notes: Impulse response functions (IRFs) of aggregates to monetary policy shock in the heterogeneous agent (Het) model and the representative agent (Rep) model version. The impulse responses shock absolute deviations from the steady state variable in response to the shock.

Figure 10 IRFs of Heterogeneous and Representative Agent Model to Government Expenditure Shock



Notes: Impulse response functions (IRFs) of aggregates to a government expenditure shock in the heterogeneous agent (Het) model and the representative agent (Rep) model version. The impulse responses shock absolute deviations from the steady state variable in response to the shock.

Figure 11 Historical Decomposition of Observed Variables



Notes: Historical decomposition of time-series data into the contribution of individual structural shocks.