Inverting the mass hierarchy of jet quenching effects with prompt b-jet substructure

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The mass of heavy quarks, such as charm and bottom, plays an important role in the formation of parton showers. This effect is apparently not well understood when parton showers evolve in a strongly interacting quark-gluon plasma. We propose a new experimental measurement in relativistic heavy ion collisions, based on a two-prong subjet structure inside a reconstructed heavy flavor jet, which can place stringent constraints on the mass dependence of in-medium splitting functions. We identify the region of jet transverse momenta where parton mass effects are leading and predict a unique reversal of the mass hierarchy of jet quenching effects in heavy ion relative to proton collisions. Namely, the momentum sharing distribution of prompt b-tagged jets is more strongly modified in comparison to the one for light jets. Our work is useful in guiding experimental efforts at the Large Hadron Collider and the Relativistic Heavy Ion Collider in the near future.

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Introduction. Understanding the production and structure of hadronic jets is crucial to test perturbative Quantum Chromodynamics (QCD) and make full use of the data from today's high energy collider experiments. With the ever increasing center-of-mass energies $\sqrt{s_{\rm NN}}$ of hadronic and heavy ion collisions, heavy quarks, such as charm (c) and bottom (b), are copiously produced in parton showers. The fraction of jets initiated by prompt heavy quarks is also becoming sizable, necessitating more precise theoretical control on the effects of parton mass. It was suggested more than a decade ago that these mass effects should be readily observable in heavy ion collisions, leading to reduced radiative energy losses of charm and bottom quarks relative to light quarks [1–3] in hot and dense nuclear matter.

Experimental measurements of open heavy flavor and b-quark jets have not clearly established this "dead cone" effect [1], which was predicted to result in smaller cross section attenuation of D-mesons and, especially, B-mesons relative to light hadrons in ultrarelativistic nuclear collisions. At high transverse momenta the difference in the magnitude of heavy and light quark flavor quenching disappears due to the non-Abelian analog of the Landau-Pomeranchuk-Migdal effect [4] and the different hardness of light parton and heavy quark fragmentation functions. Recently, it was also pointed out that both open heavy flavor and b-jet production receive large contribution from gluon fragmentation into heavy flavor [5, 6]. At low, but perturbatively accessible, transverse momenta at the Relativistic Heavy Ion Collider (RHIC) it was noticed early on that the anticipated mass effect on B-meson quenching is not reflected in the suppression of non-photonic electrons coming from the semileptonic decays of open heavy flavor [7]. This discrepancy has stimulated extensive theoretical work, for a comprehensive review that covers theory and experimental measurements see [8], suggesting that collisional energy loss effects may play a very important role in this kinematic domain. Only very recently has there been an indication at the Large Hadron Collider (LHC) that the suppression of B-mesons, inferred via the $B \to J/\psi$ channel, might be smaller that that of D-mesons and light hadrons [9].

It is, therefore, both timely and critical to identify new experimental observables, which are sensitive to the mass effects in parton branching and shower evolution. Jet substructure [10] is a promising direction to investigate, and a recently proposed study of the two leading subjets inside a reconstructed jet [11] can accurately test the $1 \to 2$ QCD splitting function [12]. The technique itself is based on the "soft drop declustering" [13], which removes soft wide-angle radiation from a jet until hard 2-prong substructure is found. The jet momentum sharing variable is then defined as

$$z_g = \frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} \ , \quad z_g > z_{\rm cut} \left(\frac{\Delta R_{12}}{R} \right)^{\beta} \ , \qquad (1)$$

where p_{T1} and p_{T2} are the transverse momenta for the subjets. Soft bremsstrahlung is eliminated through the minimum z_g requirement, where in Eq. (1) ΔR_{12} is the distance between two subjets and R is the radius of the original jet. In the limit of large jet energies the distribution of z_g maps directly onto the widely used lowest order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting functions.

In heavy ion (A+A) collisions the interactions of the outgoing partons with the hot and dense QCD medium, the quark-gluon plasma (QGP), may change the jet splitting functions relative to the simpler proton-proton (p+p) case. Pioneering experimental studies of this observable have recently been carried out by the CMS collaboration [14] at the LHC and the STAR collaboration [15] at RHIC. Theoretically, it was shown that the nuclear modification of the jet momentum sharing distribution [16] is directly related to the in-medium splitting functions [17], which can be obtained in the framework of soft-collinear effective theory with Glauber gluon interactions ($\mathrm{SCET_G}$) [18, 19]. Similar conclusion was reached in other works focused on the soft gluon emission parton energy loss limit [20, 21] and Monte Carlo studies [22, 23]. Thus, the momentum sharing distribution of jets not only complements the extensive suite of jet quenching measurements in A+A reactions at RHIC [24, 25] and LHC [26–39], but also provides direct access to the fundamental many-body QCD physics probed in heavy ion collisions.

Recent advances in generalizing $\mathrm{SCET_G}$ to include finite heavy quark masses [5] is the stepping stone for the first calculation of the heavy flavor jet splitting function in Au+Au collisions at RHIC and Pb+Pb collisions at the LHC, which we present in this Letter. We are interested in the limit $0 < m \ll p^+$ [40, 41], where m is the heavy quark mass and p^+ is the large lightcone momentum[1]. Consider an off-shell parton of momentum $[p^+, p^-, \mathbf{0}_\perp]$ that splits into two daughter partons $[xp^+, \mathbf{k}_\perp^2/xp^+, \mathbf{k}_\perp]$ and $[(1-x)p^+, \mathbf{k}_\perp^2/(1-x)p^+, -\mathbf{k}_\perp]$. In the absence of a QCD medium the massive vacuum splitting kernels $Q \to Qg$, $Q \to gQ$, and $g \to Q\bar{Q}$ read

$$\left(\frac{dN^{\rm vac}}{dzd^2\mathbf{k}_\perp}\right)_{Q\to Qg} = \frac{\alpha_s}{2\pi^2}\frac{C_F}{\mathbf{k}_\perp^2+z^2m^2} \times\left(\frac{1+(1-z)^2}{z}-\frac{2z(1-z)m^2}{\mathbf{k}_\perp^2+z^2m^2}\right) , \tag{2}$$

$$\left(\frac{dN^{\rm vac}}{dzd^2\mathbf{k}_\perp}\right)_{g\to Q\bar{Q}} = \frac{\alpha_s}{2\pi^2}\frac{T_R}{\mathbf{k}_\perp^2+m^2} \times\left(z^2+(1-z)^2+\frac{2z(1-z)m^2}{\mathbf{k}_\perp^2+m^2}\right) , \tag{3}$$

$$\left(\frac{dN^{\rm vac}}{dzd^2\mathbf{k}_\perp}\right)_{Q\to gQ} = \left(\frac{dN^{\rm vac}}{dzd^2\mathbf{k}_\perp}\right)_{Q\to Qg}(z\to 1-z) . \tag{4}$$

Here, C_F and C_A are Casimir operators of the fundamental and the adjoint representation of SU(3) and $T_R = 1/2$ is the trace normalization of the fundamental representation. The above equations reduce to the massless splitting functions when $m = 0$, and the $g \to gg$ kernel is well known and not shown here. If we denote by $r_g \equiv \Delta R_{12}$ the angular separation between the two final state partons and $E_0 = p^+/2$ is the energy, $\mathbf{k}_\perp = z(1-z)r_gE_0$ when r_g is not too large. To identify the region of phase space where heavy quark mass effects are leading, we consider a typical separation between the two subjets inside a reconstructed jet $r_g = 0.2$ and a momentum sharing fraction $z \sim 1/2$. The essential $\mathbf{k}_\perp^2 < z^2m^2, m^2, (1-z)^2m^2$ condition will be satisfied for prompt b-jets of energy $E_0 \leq 25$ GeV. While those energies are smaller than the typical jet energies that have so far been studied in heavy ion collisions at the LHC, they are within reach of improved experimental jet reconstruction techniques [25]. Such moderate energy jets are also the cornerstone of the jet physics program with the future sPHENIX experiment at RHIC [43].

Theoretical formalism. To set up the stage for the jet splitting function calculation in heavy ion collisions, we start with the vacuum case. We denote by $j \to i\bar{i}$ the parton branchings and define $r_g = \theta_g R$. The θ_g and z_g distribution for parton j, after soft-drop grooming is

$$\left(\frac{dN^{\rm vac}}{dz_gd\theta_g}\right)_j = \frac{\alpha_s}{\pi}\frac{1}{\theta_g}\sum_i P^{\rm vac}_{j\to i\bar{i}}(z_g) . \tag{5}$$

When the splitting probability becomes large, resummation is necessary and was performed to modified leading-logarithmic (MLL) accuracy in Ref. [13]. The resummed distribution for a j-type jet, initiated by a massless quark or a gluon, is

$$\frac{dN_j^{\rm vac,MLL}}{dz_gd\theta_g} = \sum_i\left(\frac{dN^{\rm vac}}{dz_gd\theta_g}\right)_{j\to i\bar{i}} \underbrace{\exp\left[-\int_{\theta_g}^1 d\theta\int_{z_{\rm cut}}^{1/2}dz\sum_i\left(\frac{dN^{\rm vac}}{dzd\theta}\right)_{j\to i\bar{i}}\right]}_{\text{Sudakov Factor}} . \tag{6}$$

The normalized joint probability distribution then reads

$$p(\theta_g, z_g)\big|_j = \frac{\frac{dN_j^{\rm vac,MLL}}{dz_gd\theta_g}}{\int_0^1 d\theta\int_{z_{\rm cut}}^{1/2}dz\frac{dN_j^{\rm vac,MLL}}{dzd\theta}} . \tag{7}$$

Suppose that we can distinguish the splitting process involving heavy flavor, for example by tagging jets and subjets with leading charm and beauty mesons (D, B). In the absence of a QCD medium, such study was proposed in Ref. [44] and simulations performed using a Monte Carol event generator framework. Analytically, Eq. (7) can be extended to the case of a heavy flavor jet splitting, such as $b \to bg$ or $c \to cg$, in a straight forward way. For gluon splitting into heavy quark pairs the probability function is defined as

$$p(\theta_g, z_g)\big|_{g\to Q\bar{Q}} = \frac{\left(\frac{dN^{\rm vac}}{dz_gd\theta_g}\right)_{g\to Q\bar{Q}}\Sigma_g(\theta_g)}{\int_0^1 d\theta\int_{z_{\rm cut}}^{1/2}dz\left(\frac{dN^{\rm vac}}{dzd\theta}\right)_{g\to Q\bar{Q}}\Sigma_g(\theta)} , \tag{8}$$

where $\Sigma_g(\theta_g)$ is the Sudakov factor for gluon evolution in Eq. (6) and it expatiates all the possible contributions from gluon splitting, such as $g \to gg$ and $g \to q\bar{q}$. Thus, we find that MLL resummation can change significantly the predictions for the $g \to Q\bar{Q}$ channel relative to the leading order (LO) results. The final probability distribution for z_g is defined as

$$p(z_g)\big|_j = \frac{1}{\sigma_j}\int dp_Td\eta\frac{d\sigma_j}{dp_Td\eta}\int_0^1 d\theta\ p(\theta, z_g)\big|_j , \tag{9}$$

[1] For the case $m \sim p^+$, which is beyond the scope of this Letter, see [42].

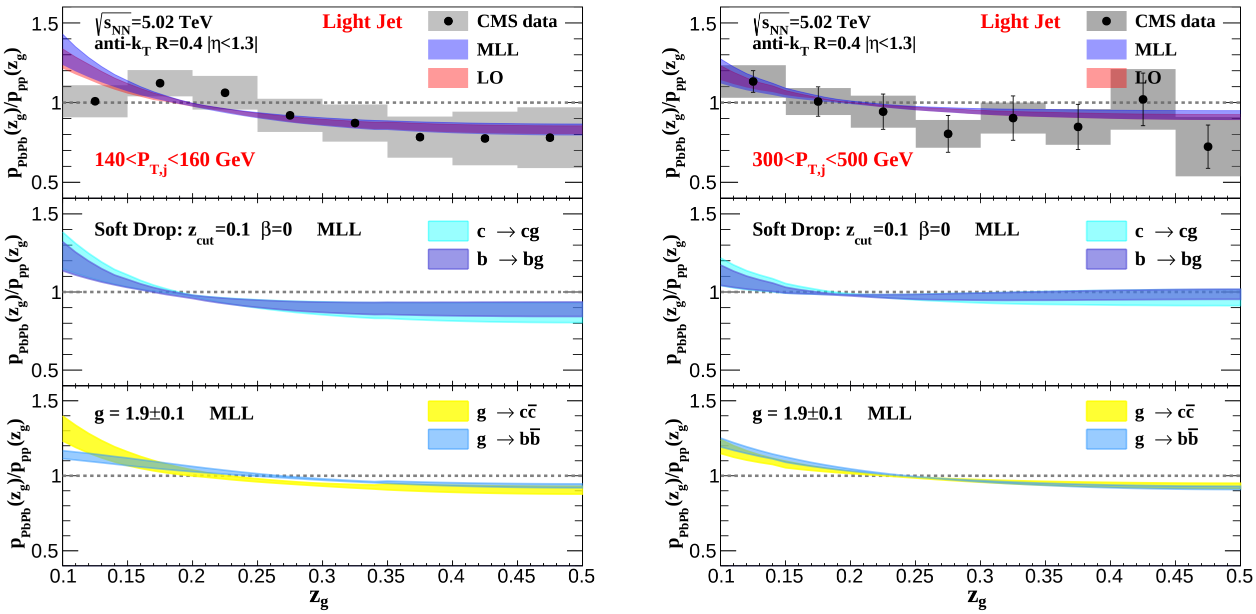

FIG. 1: The modification of the jet splitting functions in 0-10% central Pb+Pb collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV for two p_T bins $140 < p_{T,j} < 160$ GeV (left panel) and $300 < p_{T,j} < 500$ GeV (right panel). The upper panels compare the LO and MLL predictions to CMS light jet substructure measurements. The middle and lower panels present the MLL modifications for heavy flavor tagged jet - the $Q \to Qg$ and $\to Q\bar{Q}$, respectively.

where σ_j is the cross section of the j-parton production. Strictly speaking, the quark or gluon production cross section is not well-defined. In the view of the perturbative nature of higher order corrections, it is sufficient to use the LO predictions for the current calculations [45]. We use MADGRAPH5_AMC@NLO [46] and NNPDF2.3 LO PDF sets [47] to generate the LO events for jet production.

In the presence of a QCD medium the splitting kernels read

$$\left(\frac{dN^{\rm full}}{dzd^2\mathbf{k}_\perp}\right)_j = \left(\frac{dN^{\rm vec}}{dzd^2\mathbf{k}_\perp}\right)_j + \left(\frac{dN^{\rm med}}{dzd^2\mathbf{k}_\perp}\right)_j , \qquad (10)$$

where the complete sets of $dN^{\rm med}/dzd^2\mathbf{k}_\perp$ for zero and finite quark masses on the right hand side can be found in Refs. [5, 17] to first order in opacity. They have been applied to describe and/or predict jet quenching effects for inclusive hadrons and heavy mesons [5, 48, 49], as well as jets and jet substructure [16, 50, 51], in fixed order and resummed calculations. The full in-medium splitting kernels provide a systematic framework to study high transverse momentum observables in the ambiance of nuclear matter beyond the traditional energy loss approaches. We evaluate them in a QGP background simulated by 2+1-dimensional viscous event-by-event hydrodynamics [52], which was recently used to calculate quarkonium suppression at the LHC [53].

Numerical results. For all predictions we use one-loop running coupling α_s and choose the scale as $\max(\mu, \mu_{\rm NP})$, $\mu_{\rm NP}$ being the non-perturbative value where we freeze the coupling. The default scale choices are $\mu = k_\perp$ and $\mu_{\rm NP} = 1$ GeV. The renormalization and factorization scales for the LO jet production are chosen as $\mu_h = p_{T1} + p_{T2}$, where p_{T1} and p_{T2} are the transverse momenta of the jets. The uncertainties are obtained by varying μ, $\mu_{\rm NP}$, μ_h by a factor of two. We also include in all figures the uncertainty from the variation of the coupling g between the jet and the QGP.

In Fig. 1 we show the modifications of the resummed momentum sharing distributions of inclusive jets over the kinematic ranges $140 < p_{T,j} < 160$ GeV and $300 < p_{T,j} < 400$ GeV in 0-10% central Pb+Pb collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV. Results for light jets are compared to measurements by the CMS collaboration at the LHC Run II [14]. Jets are reconstructed using anti-k_T algorithm [54] with $R = 0.4$ and $|\eta| < 1.3$ in both p+p and Pb+Pb collisions. An additional cut on the distance of the two subjets $\Delta R_{12} > 0.1$ is applied due to the detector resolution. Importantly, we notice in the upper panels that there is little difference between the fixed order and resummed calculations. First predictions for the modification of prompt $b-$jet and c-jet substructure are given in the middle panel. We find that jet quenching effects for $p(z_g)$ are comparable to that of of light jets and, at least for $p_T \sim 100$ GeV, we expect it can be measured by the LHC experiments. The mass effect slowly vanishes with increasing jet energy. Finally, the bottom panels of Fig. 1 illustrate that QGP effects for the $g \to Q\bar{Q}$ channel are somewhat smaller in comparison to the other splitting functions.

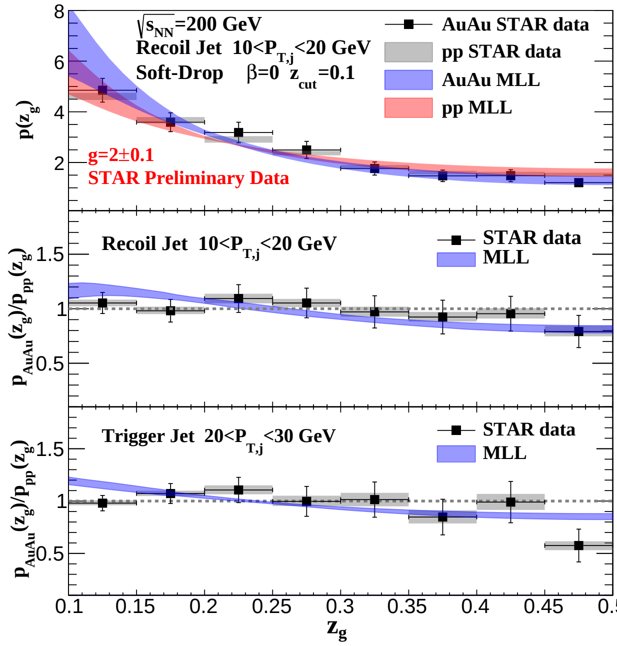

FIG. 2: Distribution of z_g and its modification for recoil and trigger jets at $\sqrt{s_{\rm NN}} = 200$ GeV Au+Au collisions. The upper panel compares the theoretical predictions for the recoil jet to the preliminary STAR data [15]. The middle and bottom panels show the predictions and measurements for the modification of recoil and trigger jets, respectively.

Next, we turn to the jet momentum sharing distribution modification in Au+Au collisions at RHIC. Non-perturbative effects should be more important at lower center-of-mass energies. However, we find that in the normalized $p(z_g)$ distribution and, especially, in the ratio $p_{\rm AuAu}(z_g)/p_{\rm pp}(z_g)$ the sensitivity to non-perturbative physics is reduced. Figure 2 compares the MLL jet splitting functions for trigger the recoil jets in p+p and Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV to measurements from the STAR collaboration [15]. The theoretical calculations in heavy ion collisions take into account the geometric bias due to triggering and the slightly different 0-20% centrality in comparison to the LHC results. Both of the splitting functions and the modification for the recoil jet are in good agreement with data. As we can see, both the MLL results and the measured modifications are smaller that those at the LHC. The bottom panel of Fig. 2 compares our calculation and the measurement for the trigger jet and they are consistent within experimental uncertainties.

Predictions for the momentum sharing distribution ratios for heavy flavor tagged jets in Au+Au to p+p collisions at $\sqrt{s_{\rm NN}} = 200$ GeV are presented in Fig. 3. We consider $10 < p_{T,j} < 30$ GeV where our analysis suggests that heavy quark mass effects on parton shower formation are the largest, especially for bottom quarks. For $c \to cg$, the $p(z_g)$ modification in the QGP is similar to

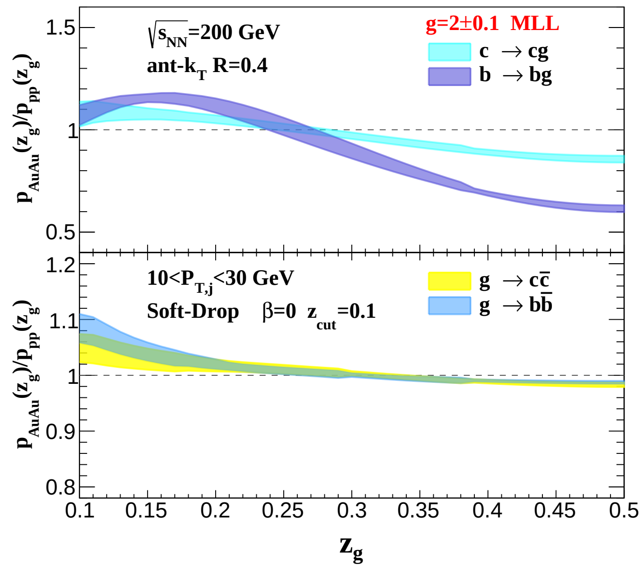

FIG. 3: The modifications of the splitting functions for heavy flavor tagged jet at $\sqrt{s_{\rm NN}} = 200$ GeV Au+Au collisions. Note the strong quenching effects for prompt b-jets contrasted by the lack of QGP-induced modification for the $g \to Q\bar{Q}$ splitting.

the one for light jets, however, the $b \to bg$ channel exhibits much larger in-medium effects. This unique reversal in the mass hierarchy of jet quenching effects[2] stems from the way in which the mass terms in the splitting kernels Eqs. (2)-(4) alter the longitudinal z dependence of parton branching. Namely, if $\mathbf{k}_\perp^2 \ll z_g^2 m^2$ this $Q \to Qg$ distribution is considerably steeper that the one for light partons [5] amplifying the $p_{\rm AA}(z_g)$ versus the $p_{\rm pp}(z_g)$ difference. Conversely, when the $\mathbf{k}_\perp^2 \ll m^2$ the z dependence in the $g \to Q\bar{Q}$ channel is approximately constant, leading to no nuclear modification. This can be clearly seen in the bottom panel of Fig. 3.

Conclusion. In this Letter, we presented the first resummed calculations of the soft-drop groomed momentum sharing distributions in heavy ion collisions in the framework of recently developed effective theories of light parton and heavy quark propagation in the dense QCD matter. For light jets, the modification of this observable in Au+Au and Pb+Pb reactions agrees well with the recent experimental measurements over a wide range of center-of-mass energies, validating the theoretical approach. The most important advances reported in this work, however, relate to heavy flavor tagged jets. We demonstrated that jet splitting functions are especially sensitive to heavy quark mass effects on parton shower

[2] We have checked that in the soft gluon emission limit parton energy loss ordering $\Delta E_b^{\rm rad} < \Delta E_c^{\rm rad} < \Delta E_{u,d}^{\rm rad} < \Delta E_g^{\rm rad}$ holds.

evolution and can be used to constrain the still not well understood dead cone effect in the QGP. In the kinematic domain where those effects are important we predict a unique inversion of the mass hierarchy of jet quenching, with the modification of the momentum sharing distribution for prompt b-jets being the largest. This work opens a new direction of research on heavy flavor jet substructure in ultrarelativistic nuclear collisions and can be extended to different energy correlators in jets [55]. It is also useful in guiding the next generation of jet measurements in heavy ion reactions at RHIC and LHC.

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