HOMEWORK 2

SUBMIT YOUR SOLUTION BY ~~TUESDAY~~ ~~2nd OF APRIL~~ MONDAY 23rd MARCH 2020

EXERCISE 1 (GRAPH COLOURING)

a) FOR THE GRAPH G IN FIG. 1 FIND THE CLIQUE NUMBER $\omega(G)$ AND USE IT TO PROVIDE AN LOWER BOUND FOR THE CHROMATIC NUMBER $c(G)$

b) FIND THE MAX DEGREE $\Delta(G)$ AND USE IT TO FIND AN UPPER BOUND FOR $c(G)$

c) IS THE THE COLORING IN FIG. 2 A PROPER 3-COLOURING?

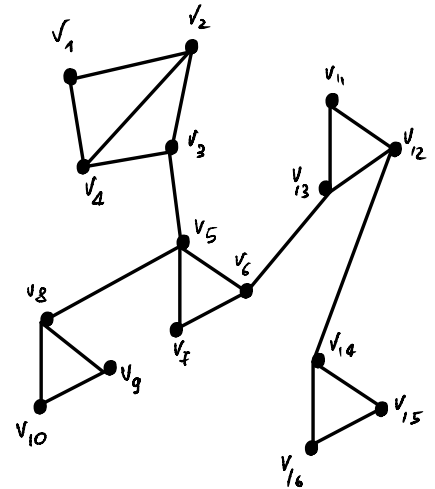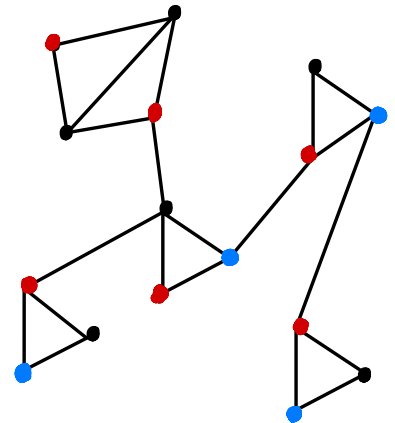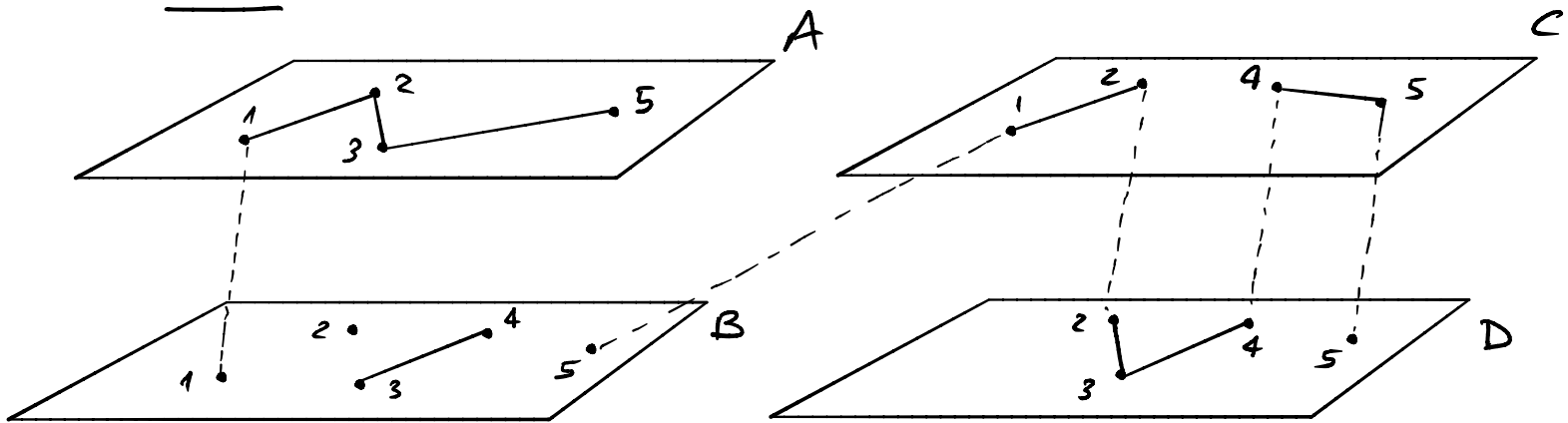d) FIND THE PARTITION OF THE VERTEX SET INDUCED BY THE COLOURING OF FIG. 2

FIG. 1

FIG. 2

EXERCISE 2 (MULTI-LAYER GRAPHS)

FIG. 1

CONSIDER THE MULTI-LAYER NETWORK IN FIG. 1 WITH 4 LAYERS, LABELLED A, B, C, D.

a) FIND:

a.1) THE INTRA-LAYER EDGE SETS E_A

a.2) THE INTER-LAYER EDGE SETS E_C

a.3) THE COUPLING EDGE SET $E_{\tilde{C}}$

b) IS THE NETWORK IN FIG. 1 FULLY INTERCONNECTED?

c) PROVIDE A TENSOR REPRESENTATION OF THE NETWORK. DENOTE WITH $A \in \{0,1\}^{5 \times 5 \times 4 \times 4}$ THE TENSOR ENCODING THE NETWORK CONNECTIVITY

EXERCISE 3 (MULTI-LAYER GRAPHS)

a) FOR THE MULTI-LAYER GRAPH IN FIG. 1, PROVIDE A TENSOR REPRESENTATION

b) LET a_{ij}^{α} BE THE (i,j)-th ELEMENT OF THE ADJACENCY MATRIX FOR THE LAYER α, WITH $i,j = 1,\dots,4$ AND $\alpha \in \{A,B,C,D\}$ FIND:

b.1) THE DEGREE d_i^{α} OF THE NODE i IN THE LAYER α, WITH $i = 1,\dots,4$ AND $\alpha \in \{A,B,C,D\}$.

b.2) THE OVERLAPPING DEGREE O_i WITH $i = 1,\dots,4$

b.3) FIND THE EIGENVECTORS OF EACH NODE BY APPLYING THE UNIFORM-VECTOR-LIKE EIGENVECTOR CENTRALITY

FIG. 1