Homework 5 - Theory/Laboratory

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1 The Watts and Strogatz's Model

- The local *clustering coefficient* (CC), provided in the *Watts and Strogatz* paper, measures the cliquishness of a neighbourhood (you can see this feature also as how likely 2 nodes that are connected are part of some larger highly connected group of node - i.e. clique).

  This statistic can assume values between 0 and 1, indicating the fraction of possible interconnections between the neighbours of a node (visually you can consider that 0 indicate a start topology graph, meanwhile 1 means a clique).

  To compute the CC of a graph you just need to consider the average value between all the CC of each node belonging to the graph.

- Considering the scenario of having $n = 400$ nodes, $K = 10$ and the rewiring probability $q = 0.01 + 0.01j$, for $j = 1, ..., 99$.

  The average clustering coefficient (CC(q)) and the average path length (APL(q)) for each q has been calculated using the following *MatLab* code:

```
clear all;
clc;

n = 400;
K = 10;

q_cumulative = (0.02:0.01:1)';
q_length = size(q_cumulative,1);
storage = zeros(q_length,2);

for j=1:q_length
    q = q_cumulative(j,1);
    WS = WattsStrogatz(n,K,q);
    cc = clustering_coef_bu(WS.adjacency);

```

```
    storage(j,1) = mean(cc);
    storage(j,2) = mean(distances(WS),'all');
end

figure(1)
plot(q_cumulative,storage(:,1),'k-');
title('CC(q)');
xlabel('Re-wiring probability');
ylabel('Clustering coefficient');

figure(2)
plot(q_cumulative,storage(:,2),'k-');
title('APL(q)');
xlabel('Re-wiring probability');
ylabel('Average path length');
```

Which generates the following two graphs:

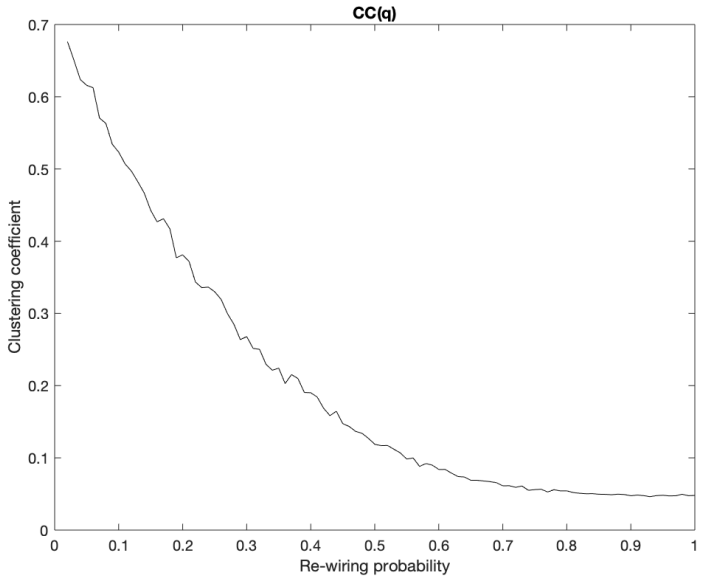

Figure 1: Average clustering coefficient

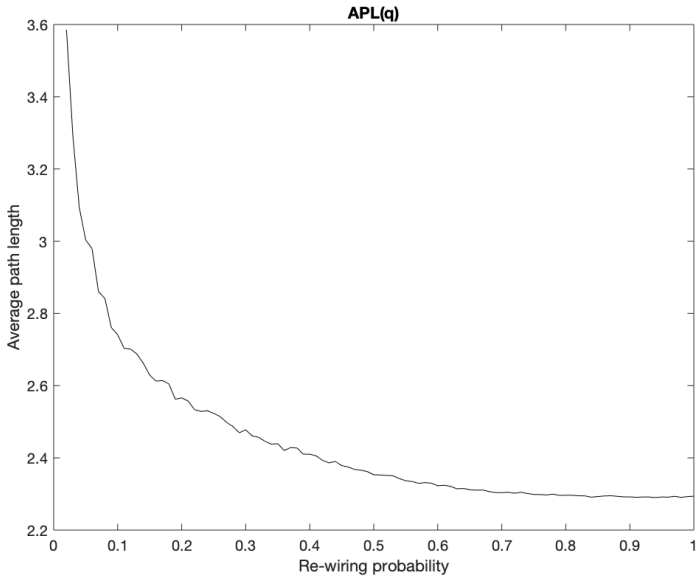

Figure 2: Average path length

- By setting the logarithmic horizontal scale to the previous graphs it has been more clear that the $APL(q)$ drops very quickly (meaning that thanks to the re-wiring process the distance between the nodes in the network decreased), meanwhile the $CC(q)$ at the beginning (regular lattice 'state') decrease very slowly, indicating that the transition to a small world is almost undetectable at the local level.