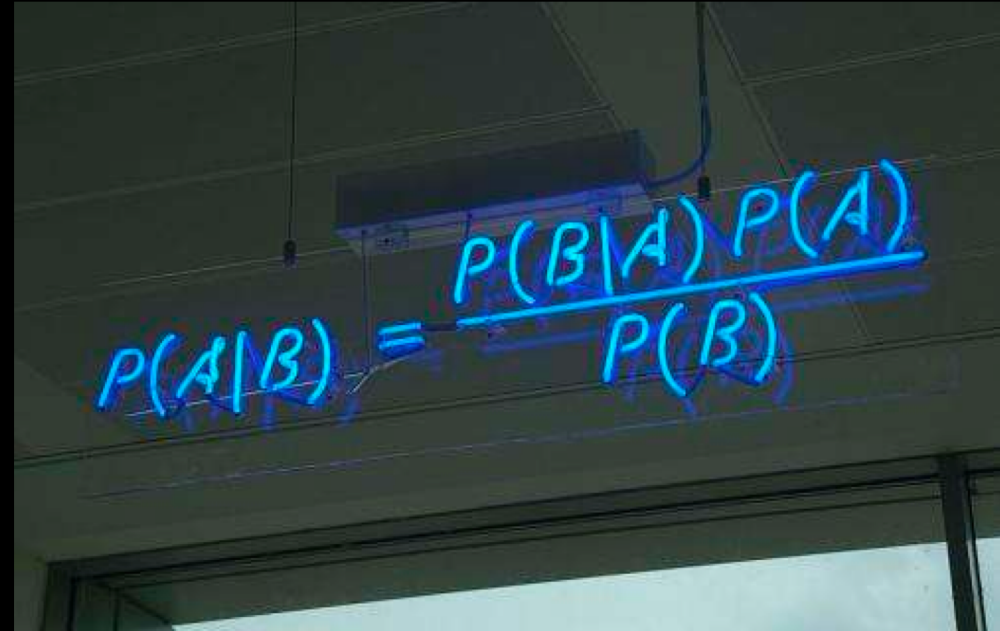


ARTIFICIAL INTELLIGENCE

SUPERVISED LEARNING – WEEK 6

BAYES' THEOREM



A photograph of a presentation screen displaying the formula for Bayes' Theorem in blue ink. The formula is
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

**DESCRIBES THE PROBABILITY OF AN EVENT,
BASED ON PRIOR KNOWLEDGE OF CONDITIONS THAT MIGHT BE RELATED TO THE EVENT.**

PROBABILITY

P(DICE) = DESIRED RESULT / ALL POSSIBILITIES



P(FACE COIN) = $1/2 = 0.5 = 50\%$ (CHANCE)

P(4 IN THE DICE) = $1/6 = 16,67\%$ (CHANCE)



PROBABILITY

P(DICE) = DESIRED RESULT / ALL POSSIBILITIES



P(FACE COIN) = $1/2 = 0.5 = 50\%$ (CHANCE)

P(4 IN THE DICE) = $1/6 = 16,67\%$ (CHANCE)



PROBABILITY

A RARE RARE DISEASE AFFECTS ONLY 0.1% OF THE POPULATION.

THE BLOOD TEST ACCURATELY PREDICTS 99% OF THE CASES OF THE DISEASE.

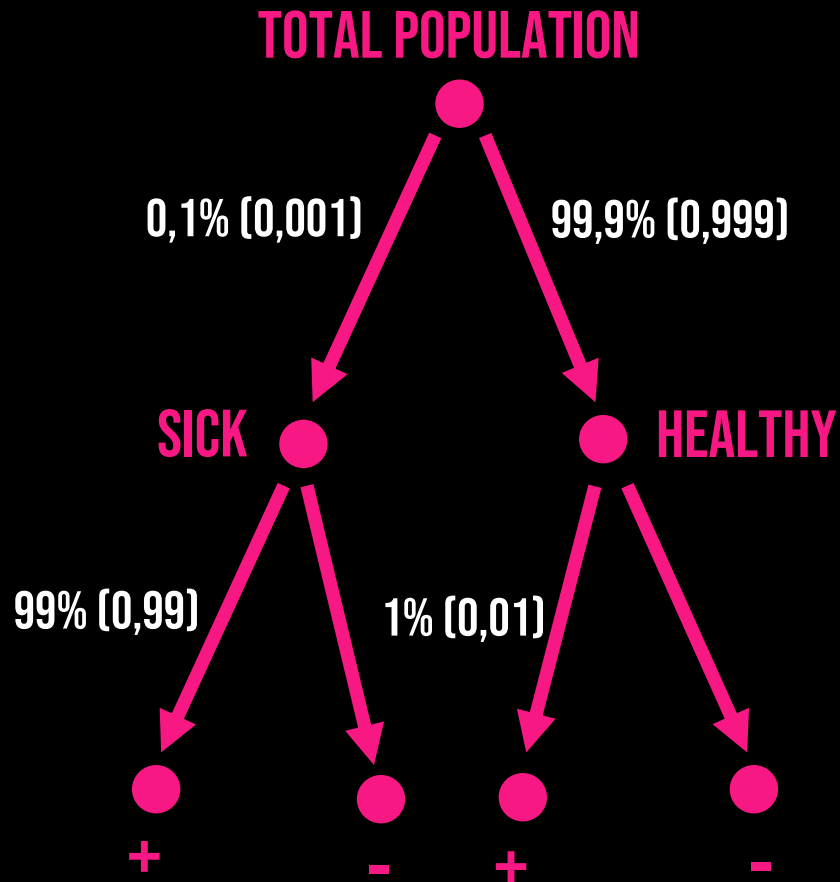
THE SAME BLOOD TEST FAILS 1% OF THE TIME, PREDICTING ILLNESS FOR THOSE WHO DO NOT HAVE IT (FALSE POSITIVE).

IF A PERSON RECEIVES THE POSITIVE TEST, HOW LIKELY IS IT TO ACTUALLY BE SICK?

PROBABILITY

ABOUT 80% OF THE DOCTORS SAY A WRONG RESPONSE

USING BAYES' THEOREM



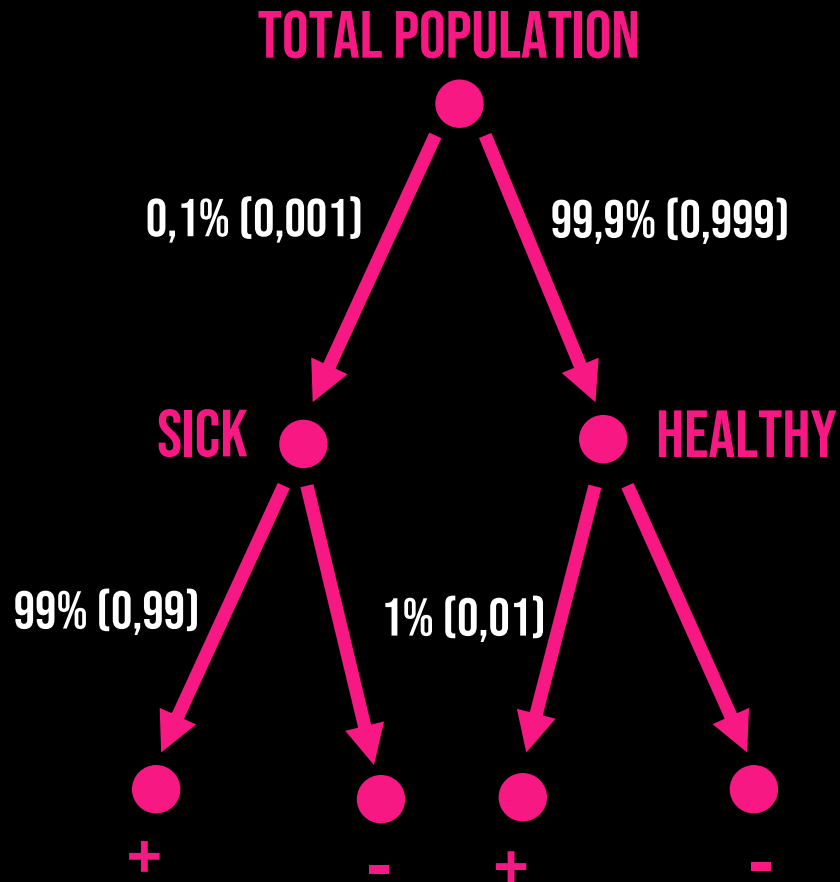
$$P(\text{SICK} | +) = (+ | \text{SICK}) * P(\text{SICK}) / P(+)$$

$$P(\text{SICK} | +) = (0,99) * (0,001) / \\ (0,001 * 0,99) + (0,999 * 0,01)$$

$$P(\text{SICK} | +) = (0,99) * (0,001) / \\ (0,001 * 0,99) + (0,999 * 0,01)$$

$$P(\text{SICK} | +) = 9\%$$

USING BAYES' THEOREM



$$P(\text{SICK} | +) = (+ | \text{SICK}) * P(\text{SICK}) / P(+)$$

$$P(\text{SICK} | +) = (0,99) * (0,001) / \\ (0,001 * 0,99) + (0,999 * 0,01)$$

$$P(\text{SICK} | +) = (0,99) * (0,001) / \\ (0,001 * 0,99) + (0,999 * 0,01)$$

$$P(\text{SICK} | +) = 9\%$$

NAIVE BAYES

OUTLOOK	TEMPERATURE	HUMIDITY	WINDY	PLAY
RAINY	HOT	HIGH	FALSE	NO
RAINY	HOT	HIGH	TRUE	NO
OVERCAST	HOT	HIGH	FALSE	YES
SUNNY	MILD	HIGH	FALSE	YES
SUNNY	COOL	NORMAL	FALSE	YES
SUNNY	COOL	NORMAL	TRUE	NO
OVERCAST	COOL	NORMAL	TRUE	YES
RAINY	MILD	HIGH	FALSE	NO
RAINY	COOL	NORMAL	FALSE	YES
SUNNY	MILD	NORMAL	FALSE	YES
RAINY	MILD	NORMAL	TRUE	YES
OVERCAST	MILD	HIGH	TRUE	YES
OVERCAST	HOT	NORMAL	FALSE	YES
SUNNY	MILD	HIGH	TRUE	NO

$$P(\text{YES}) = 9/14$$

$$P(\text{NO}) = 5/14$$

FREQUENCY AND LIKELIHOOD TABLES

FREQUENCY TABLE		PLAY GOLF	
		YES	NO
OUTLOOK	SUNNY	3	2
	OVECAST	4	0
	RAINY	2	3

$$P(X|C) = P(\text{SUNNY}|\text{YES}) = 3/9 = 0.33$$

LIKELIHOOD TABLE		PLAY GOLF		
		YES	NO	
OUTLOOK	SUNNY	3/9	2/5	5/14
	OVECAST	4/9	0	4/14
	RAINY	2/9	3/5	5/14
		9/14	5/14	

$$P(C) = P(\text{YES}) = 9/14 = 0.64 \quad P(X) = P(\text{SUNNY}) = 5/14 = 0.36$$

POSTERIOR PROBABILITY: $P(C|X) = P(X|C) * P(C) / P(X)$

$$P(\text{YES}|\text{SUNNY}) = P(\text{SUNNY}|\text{YES}) * P(\text{YES}) / P(\text{SUNNY})$$

$$P(\text{YES}|\text{SUNNY}) = 0,33 * 0,64 / 0,36$$

$$P(\text{YES}|\text{SUNNY}) = 0,6$$

POSTERIOR PROBABILITY: $P(C|X) = P(X|C) * P(C) / P(X)$

$$P(\text{NO}|\text{SUNNY}) = P(\text{SUNNY}|\text{NO}) * P(\text{NO}) / P(\text{SUNNY})$$

$$P(\text{NO}|\text{SUNNY}) = 0,4 * 0,36 / 0,36$$

$$P(\text{NO}|\text{SUNNY}) = 0,4$$

FREQUENCY TABLE		PLAY GOLF	
		YES	NO
OUTLOOK	SUNNY	3	2
	OVECAST	4	0
	RAINY	2	3



LIKELIHOOD TABLE		PLAY GOLF	
		YES	NO
OUTLOOK	SUNNY	3/9	2/5
	OVECAST	4/9	0
	RAINY	2/9	3/5

FREQUENCY TABLE		PLAY GOLF	
		YES	NO
HUMIDITY	HIGH	3	4
	NORMAL	6	1



LIKELIHOOD TABLE		PLAY GOLF	
		YES	NO
HUMIDITY	SUNNY	3/9	2/5
	OVECAST	4/9	0

FREQUENCY TABLE		PLAY GOLF	
		YES	NO
TEMPERATURE	HOT	2	2
	MILD	4	2
	COLD	3	1



LIKELIHOOD TABLE		PLAY GOLF	
		YES	NO
TEMPERATURE	SUNNY	3/9	2/5
	OVECAST	4/9	0
	RAINY	2/9	3/5

FREQUENCY TABLE		PLAY GOLF	
		YES	NO
WINDY	FALSE	6	2
	TRUE	3	3



LIKELIHOOD TABLE		PLAY GOLF	
		YES	NO
OUTLOOK	SUNNY	3/9	2/5
	OVECAST	4/9	0

PREDICTING

OUTLOOK	TEMPERATURE	HUMIDITY	WINDY	PLAY
RAINY	COOL	HIGH	TRUE	?

$$P(\text{YES} | X) = P(\text{RAINY} | \text{YES}) * P(\text{COOL} | \text{YES}) * P(\text{HIGH} | \text{YES}) * P(\text{TRUE} | \text{YES}) * P(\text{YES}) / P(X)$$

$$P(\text{YES} | X) = 2/9 * 3/9 * 3/9 * 3/9 * 9/14 / P(X)$$

$$P(\text{YES} | X) = 0,00529 / P(X)$$

$$P(\text{YES} | X) = 0,00529 / 0,0218$$

$$P(\text{YES} | X) = 0,24$$

$$P(X) = P(\text{RAINY}) * P(\text{COOL}) * P(\text{HIGH}) * P(\text{TRUE})$$

$$P(X) = 5/14 * 4/14 * 7/14 * 6/14$$

$$P(X) = 0,0218$$

$$P(\text{NO} | X) = P(\text{RAINY} | \text{NO}) * P(\text{COOL} | \text{NO}) * P(\text{HIGH} | \text{NO}) * P(\text{TRUE} | \text{NO}) * P(\text{NO}) / P(X)$$

$$P(\text{NO} | X) = 3/5 * 1/5 * 4/5 * 3/5 * 5/14 / P(X)$$

$$P(\text{NO} | X) = 0,02057 / P(X)$$

$$P(\text{NO} | X) = 0,02057 / 0,0218$$

$$P(\text{NO} | X) = 0,942$$

PREDICTING

OUTLOOK	TEMPERATURE	HUMIDITY	WINDY	PLAY
RAINY	COOL	HIGH	TRUE	?

$$P(\text{YES} | X) = P(\text{RAINY} | \text{YES}) * P(\text{COOL} | \text{YES}) * P(\text{HIGH} | \text{YES}) * P(\text{TRUE} | \text{YES}) * P(\text{YES}) / P(X)$$

$$P(\text{YES} | X) = 2/9 * 3/9 * 3/9 * 3/9 * 9/14 / P(X)$$

$$P(\text{YES} | X) = 0,00529 / P(X)$$

$$P(\text{YES} | X) = 0,00529 / 0,0218$$

$$P(\text{YES} | X) = 0,24$$

$$P(X) = P(\text{RAINY}) * P(\text{COOL}) * P(\text{HIGH}) * P(\text{TRUE})$$

$$P(X) = 5/14 * 4/14 * 7/14 * 6/14$$

$$P(X) = 0,0218$$

$$P(\text{NO} | X) = P(\text{RAINY} | \text{NO}) * P(\text{COOL} | \text{NO}) * P(\text{HIGH} | \text{NO}) * P(\text{TRUE} | \text{NO}) * P(\text{NO}) / P(X)$$

$$P(\text{NO} | X) = 3/5 * 1/5 * 4/5 * 3/5 * 5/14 / P(X)$$

$$P(\text{NO} | X) = 0,02057 / P(X)$$

$$P(\text{NO} | X) = 0,02057 / 0,0218$$

$$P(\text{NO} | X) = 0,942$$