# The AI Scientist: Towards Fully Automated Open-Ended Scientific Discovery

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One of the grand challenges of artificial general intelligence is developing agents capable of conducting scientific research and discovering new knowledge. While frontier models have already been used as aides to human scientists, e.g. for brainstorming ideas, writing code, or prediction tasks, they still conduct only a small part of the scientific process. This paper presents the first comprehensive framework for fully automatic scientific discovery, enabling frontier large language models (LLMs) to perform research independently and communicate their findings. We introduce THE AI SCIENTIST, which generates novel research ideas, writes code, executes experiments, visualizes results, describes its findings by writing a full scientific paper, and then runs a simulated review process for evaluation. In principle, this process can be repeated to iteratively develop ideas in an open-ended fashion and add them to a growing archive of knowledge, acting like the human scientific community. We demonstrate the versatility of this approach by applying it to three distinct subfields of machine learning: diffusion modeling, transformer-based language modeling, and learning dynamics. Each idea is implemented and developed into a full paper at a meager cost of less than \$15 per paper, illustrating the potential for our framework to democratize research and significantly accelerate scientific progress. To evaluate the generated papers, we design and validate an automated reviewer, which we show achieves near-human performance in evaluating paper scores. The AI Scientist can produce papers that exceed the acceptance threshold at a top machine learning conference as judged by our automated reviewer. This approach signifies the beginning of a new era in scientific discovery in machine learning: bringing the transformative benefits of AI agents to the entire research process of AI itself, and taking us closer to a world where endless affordable creativity and innovation can be unleashed on the world's most challenging problems. Our code is open-sourced at https://github.com/SakanaAI/AI-Scientist.

#### 1. Introduction

The modern scientific method (Chalmers, 2013; Dewey, 1910; Jevons, 1877) is arguably one of the greatest achievements of the Enlightenment. Traditionally, a human researcher collects background knowledge, drafts a set of plausible hypotheses to test, constructs an evaluation procedure, collects evidence for the different hypotheses, and finally assesses and communicates their findings. Afterward, the resulting manuscript undergoes peer review and subsequent iterations of refinement. This procedure has led to countless breakthroughs in science and technology, improving human quality of life. However, this iterative process is inherently limited by human researchers' ingenuity, background knowledge, and finite time. Attempting to automate general scientific discovery (Langley, 1987, 2024; Waltz and Buchanan, 2009) has been a long ambition of the community since at least the early 70s, with computer-assisted works like the Automated Mathematician (Lenat, 1977; Lenat and Brown, 1984) and DENDRAL (Buchanan and Feigenbaum, 1981). In the field of AI, researchers have envisioned the possibility of automating AI research using AI itself (Ghahramani, 2015; Schmidhuber, 1991, 2010a,b, 2012), leading to "AI-generating algorithms" (Clune, 2019). More recently, foundation models have seen tremendous advances in their general capabilities (Anthropic, 2024; Google DeepMind Gemini Team, 2023; Llama Team, 2024; OpenAI, 2023), but they have only been shown to accelerate individual parts of the research pipeline, e.g. the writing of scientific manuscripts (Altmäe et al., 2023; Dinu et al., 2024; Ifargan et al., 2024; Majumder et al., 2024), as a muse to brainstorm ideas (Baek et al., 2024; Girotra et al., 2023; Wang et al., 2024b), or aides to coding (Gauthier, 2024). To date, the community has yet to show the possibility of executing entire research endeavors without human involvement.

Traditional approaches to automating research projects have so far relied on carefully constraining the search space of potential discoveries, which severely limits the scope of exploration and requires substantial human expertise and design. For example, significant advancements in materials discovery (Merchant et al., 2023; Pyzer-Knapp et al., 2022; Szymanski et al., 2023) and synthetic biology (Hayes et al., 2024; Jumper et al., 2021) have been achieved by restricting exploration to well-characterized domains with predefined parameters, which allows for targeted progress but limits broader, open-ended discovery and addressing only a subset of the scientific process, without encompassing tasks such as manuscript preparation. Within the field of machine learning itself, research automation has largely been restricted to hyperparameter and architecture search (He et al., 2021; Hutter et al., 2019; Lu et al., 2022b; Wan et al., 2021, 2022) or algorithm discovery (Alet et al., 2020; Chen et al., 2024b; Kirsch et al., 2019; Lange et al., 2023a,b; Lu et al., 2022a; Metz et al., 2022) within a hand-crafted search space. Recent advances in LLMs have shown the potential to extend the search space to more generalized, code-level solutions (Faldor et al., 2024; Lehman et al., 2022; Lu et al., 2024a; Ma et al., 2023). However, these approaches remain constrained by rigorously-defined search spaces and objectives, which limit the breadth and depth of possible discoveries.

In this paper, we introduce The AI Scientist, the first fully automated and scalable pipeline for end-to-end paper generation, enabled by recent advances in foundation models. Given a broad research direction and a simple initial codebase, The AI Scientist seamlessly performs ideation, a literature search, experiment planning, experiment iterations, manuscript writing, and peer reviewing to produce insightful papers. Furthermore, in principle The AI Scientist can run in an openended loop, building on its previous scientific discoveries to improve the next generation of ideas. This allows us to speed up the slow nature of scientific iteration at a surprisingly low financial cost (~\$15/paper) and represents a step towards turning the world's ever-increasing computing resources into the scientific breakthroughs needed to tackle the core challenges of the 21st century. Here, we focus on Machine Learning (ML) applications, but this approach can more generally be applied to almost any other discipline, e.g. biology or physics, given an adequate way of automatically executing experiments (Arnold, 2022; Kehoe et al., 2015; Zucchelli et al., 2021).

By leveraging modern LLM frameworks like chain-of-thought (Wei et al., 2022) and self-reflection (Shinn et al., 2024) to improve decision-making, The AI Scientist is able to generate its own scientific ideas and hypotheses, as well as a plan for testing them with experiments. Next, The AI Scientist implements plan-directed code-level changes to the experiment "template" using the state-of-the-art coding assistant Aider (Gauthier, 2024), and executes experiments to collect a set of computational results, which are in turn used to draft a scientific paper. The AI Scientist then performs an automated paper-reviewing process using guidelines from a standard machine learning conference. Finally, The AI Scientist adds the completed ideas and reviewer feedback to its archive of scientific findings, and the process repeats. Crucially, the generated paper and experimental artifacts The AI Scientist produces allow us to easily interpret and judge its findings post-hoc, allowing human scientists to also benefit from what is learned.

#### Our contributions are summarized as follows:

1. We introduce the first end-to-end framework for fully automated scientific discovery in Machine Learning research, enabled by frontier LLMs (Section 3). This fully automated process includes idea generation, experiment design, execution, and visualizing and writing up the results into a full manuscript.

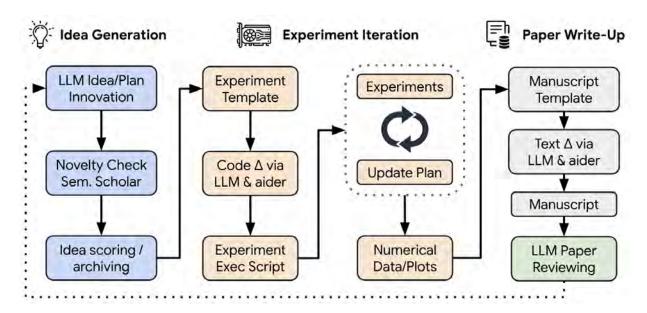


Figure 1 | Conceptual illustration of The AI Scientist, an end-to-end LLM-driven scientific discovery process. The AI Scientist first invents and assesses the novelty of a set of ideas. It then determines how to test the hypotheses, including writing the necessary code by editing a codebase powered by recent advances in automated code generation. Afterward, the experiments are automatically executed to collect a set of results consisting of both numerical scores and visual summaries (e.g. plots or tables). The results are motivated, explained, and summarized in a LaTeX report. Finally, The AI Scientist generates an automated review, according to current practice at standard machine learning conferences. The review can be used to either improve the project or as feedback to future generations for open-ended scientific discovery.

- 2. To assess the quality of the generated papers, we introduce a foundation model-based reviewing process in Section 4. This process achieves near-human-level performance across multiple evaluation metrics (e.g. 65% vs. 66% balanced accuracy) when evaluated on ICLR 2022 OpenReview data. The reviews further enable The AI Scientist to select the best ideas for "publication" to an ever-growing archive of scientific discoveries, and the process can be repeated to build on these discoveries, just as in the human scientific community.
- 3. The AI Scientist can generate hundreds of interesting, medium-quality papers over the course of a week. In this report, we focus on a subset of these papers, highlighting novel insights in diffusion modeling, language modeling, and grokking. We perform an in-depth case study into one selected paper in Section 5, and present aggregate results in Section 6.
- 4. We conclude the paper with an extensive discussion on the limitations, ethical considerations, and future outlook of our approach in Sections 8 and 9.

#### 2. Background

**Large Language Models.** In this paper, we build our automated scientist from autoregressive large language models (LLMs, Anthropic (2023); Google DeepMind Gemini Team (2023); Llama Team (2024); OpenAI (2023); Zhu et al. (2024)) which learn to generate text completions by modeling the conditional probability of a new token (similar to a word) given the preceding tokens,  $p(x_t|x_{< t};\theta)$ , and sampling at test-time. Together with vast data and model scaling, this enables LLMs to not only generate coherent text, but crucially also exhibit human-like abilities, including commonsense knowledge (Talmor et al., 2019), reasoning (Wei et al., 2022), and the ability to write code (Chen et al., 2021; Xu et al., 2022).

**LLM Agent Frameworks.** Typical applications of LLMs often involve embedding the model into an "agent" (Wang et al., 2024a) framework, including the following possibilities: the structuring of

language queries (e.g. few-shot prompting (Brown et al., 2020)), encouraging reasoning traces (e.g. chain-of-thought (Wei et al., 2022)), or asking the model to iteratively refine its outputs (e.g., self-reflection (Shinn et al., 2024)). These leverage the language model's ability to learn in-context (Olsson et al., 2022) and can greatly improve its performance, robustness and reliability on many tasks.

Aider: An LLM-Based Coding Assistant. Our automated scientist directly implements ideas in code and uses a state-of-the-art open-source coding assistant, Aider (Gauthier, 2024). Aider is an agent framework that is designed to implement requested features, fix bugs, or refactor code in existing codebases. While Aider can in principle use any underlying LLM, with frontier models it achieves a remarkable success rate of 18.9% on the SWE Bench (Jimenez et al., 2024) benchmark, a collection of real-world GitHub issues. In conjunction with new innovations added in this work, this level of reliability enables us, for the first time, to fully automate the ML research process.

#### 3. The AI Scientist

Overview. The AI Scientist has three main phases (Figure 1): (1) Idea Generation, (2) Experimental Iteration, and (3) Paper Write-up. After the write-up, we introduce and validate an LLM-generated review to assess the quality of the generated paper (Section 4). We provide The AI Scientist with a starting code template that reproduces a lightweight baseline training run from a popular model or benchmark. For example, this could be code that trains a small transformer on the works of Shakespeare (Karpathy, 2022), a classic proof-of-concept training run from natural language processing that completes within a few minutes. The AI Scientist is then free to explore any possible research direction. The template also includes a LaTeX folder that contains style files and section headers, along with simple plotting code. We provide further details on the templates in Section 6, but in general, each run starts with a representative small-scale experiment relevant to the topic area. The focus on small-scale experiments is not a fundamental limitation of our method, but simply for computational efficiency reasons and compute constraints on our end. We provide the prompts for all stages in Appendix A.

- 1. Idea Generation. Given a starting template, The AI Scientist first "brainstorms" a diverse set of novel research directions. We take inspiration from evolutionary computation and open-endedness research (Brant and Stanley, 2017; Lehman et al., 2008; Stanley, 2019; Stanley et al., 2017) and iteratively grow an archive of ideas using LLMs as the mutation operator (Faldor et al., 2024; Lehman et al., 2022; Lu et al., 2024b; Zhang et al., 2024). Each idea comprises a description, experiment execution plan, and (self-assessed) numerical scores of interestingness, novelty, and feasibility. At each iteration, we prompt the language model to generate an interesting new research direction conditional on the existing archive, which can include the numerical review scores from completed previous ideas. We use multiple rounds of chain-of-thought (Wei et al., 2022) and self-reflection (Shinn et al., 2024) to refine and develop each idea. After idea generation, we filter ideas by connecting the language model with the Semantic Scholar API (Fricke, 2018) and web access as a tool (Schick et al., 2024). This allows The AI Scientist to discard any idea that is too similar to existing literature.
- **2. Experiment Iteration.** Given an idea and a template, the second phase of The AI Scientist first executes the proposed experiments and then visualizes its results for the downstream write-up. The AI Scientist uses Aider to first plan a list of experiments to run and then executes them in order. We make this process more robust by returning any errors upon a failure or time-out (e.g. experiments taking too long to run) to Aider to fix the code and re-attempt up to four times.

After the completion of each experiment, Aider is then given the results and told to take notes in the style of an experimental journal. Currently, it only conditions on text but in future versions, this could include data visualizations or any modality. Conditional on the results, it then re-plans and implements the next experiment. This process is repeated up to five times. Upon completion of

experiments, Aider is prompted to edit a plotting script to create figures for the paper using Python. The AI Scientist makes a note describing what each plot contains, enabling the saved figures and experimental notes to provide all the information required to write up the paper. At all steps, Aider sees its history of execution.

Note that, in general, the provided initial seed plotting and experiment templates are small, self-contained files. The AI Scientist frequently implements entirely new plots and collects new metrics that are not in the seed templates. This ability to arbitrarily edit the code occasionally leads to unexpected outcomes (Section 8).

- **3. Paper Write-up.** The third phase of The AI Scientist produces a concise and informative write-up of its progress in the style of a standard machine learning conference proceeding in LaTeX. We note that writing good LaTeX can even take competent human researchers some time, so we take several steps to robustify the process. This consists of the following:
  - (a) **Per-Section Text Generation:** The recorded notes and plots are passed to Aider, which is prompted to fill in a blank conference template section by section. This goes in order of introduction, background, methods, experimental setup, results, and then the conclusion (all sections apart from the related work). All previous sections of the paper it has already written are in the context of the language model. We include brief tips and guidelines on what each section should include, based on the popular "How to ML Paper" guide, and include details in Appendix A.3. At each step of writing, Aider is prompted to *only use real experimental results in the form of notes and figures generated from code, and real citations* to reduce hallucination. Each section is initially refined with one round of self-reflection (Shinn et al., 2024) as it is being written. Aider is prompted to not include any citations in the text at this stage, and fill in only a skeleton for the related work, which will be completed in the next stage.
  - (b) Web Search for References: In a similar vein to idea generation, The AI Scientist is allowed 20 rounds to poll the Semantic Scholar API looking for the most relevant sources to compare and contrast the near-completed paper against for the related work section. This process also allows The AI Scientist to select any papers it would like to discuss and additionally fill in any citations that are missing from other sections of the paper. Alongside each selected paper, a short description is produced of where and how to include the citation, which is then passed to Aider. The paper's bibtex is automatically appended to the LaTeX file to guarantee correctness.
  - (c) **Refinement:** After the previous two stages, The AI Scientist has a completed first draft, but can often be overly verbose and repetitive. To resolve this, we perform one final round of self-reflection section-by-section, aiming to remove any duplicated information and streamline the arguments of the paper.
  - (d) **Compilation:** Once the LaTeX template has been filled in with all the appropriate results, this is fed into a LaTeX compiler. We use a LaTeX linter and pipe compilation errors back into Aider so that it can automatically correct any issues.

#### 4. Automated Paper Reviewing

An LLM Reviewer Agent. A key component of an effective scientific community is its reviewing system, which evaluates and improves the quality of scientific papers. To mimic such a process using large language models, we design a GPT-4o-based agent (OpenAI, 2023) to conduct paper reviews based on the Neural Information Processing Systems (NeurIPS) conference review guidelines. The review agent processes the raw text of the PDF manuscript using the PyMuPDF parsing library. The output contains numerical scores (soundness, presentation, contribution, overall, confidence), lists of weaknesses and strengths as well as a preliminary binary decision (accept or reject). These decisions

may then be post-calibrated by thresholding using the reviewer score. We leverage this automated reviewing process to obtain an initial evaluation of the papers generated by The AI Scientist. We provide the entire reviewing prompt template in Appendix A.4.

Table 1 | Performance of The AI Scientist's automated LLM reviewing system on 500 ICLR 2022 papers. We show mean and 95% bootstrap confidence intervals, and highlight the comparison between the human baseline and our best AI reviewer.

	Reviewer	<b>Balanced Acc.</b> ↑	<b>Accuracy</b> ↑	F1 Score ↑	<b>AUC</b> ↑	$ $ FPR $\downarrow$	FNR $\downarrow$
	Human (NeurIPS) <sup>1</sup>	0.66	0.73	0.49	0.65	0.17	0.52
	Random Decision	0.50	0.50	0.40	0.50	0.50	0.50
	Always Reject	0.50	0.59	0.00	0.50	0.00	1.00
	Sonnet 3.5	$0.52 \pm 0.01$	$0.40 \pm 0.01$	$0.55 \pm 0.01$	$0.52 \pm 0.01$	$0.95 \pm 0.02$	$0.00 \pm 0.00$
Uncalibrated	GPT-4o-mini	$0.53 \pm 0.02$	$0.65 \pm 0.01$	$0.11 \pm 0.06$	$0.53 \pm 0.02$	$0.01 \pm 0.01$	$0.94 \pm 0.04$
Ulicalibrated	GPT-4o (0-shot)	$0.61 \pm 0.04$	$0.68 \pm 0.03$	$0.43 \pm 0.07$	$0.61 \pm 0.04$	$0.11 \pm 0.03$	$0.67 \pm 0.07$
	GPT-4o (1-shot)	$0.60 \pm 0.03$	$\underline{0.70\pm0.03}$	$0.37 \pm 0.08$	$0.60 \pm 0.03$	$0.04 \pm 0.02$	$0.76 \pm 0.06$
	Sonnet 3.5@8	$0.59 \pm 0.04$	$0.65 \pm 0.04$	$0.45 \pm 0.06$	$0.59 \pm 0.04$	$0.20 \pm 0.04$	$0.61 \pm 0.07$
Calibrated	GPT-4o-mini @6	$0.59 \pm 0.04$	$0.64 \pm 0.04$	$0.45 \pm 0.06$	$0.59 \pm 0.04$	$0.22 \pm 0.05$	$0.60 \pm 0.07$
Cambrated	GPT-4o (0-shot) @6	$0.63 \pm 0.04$	$0.63 \pm 0.04$	$0.56 \pm 0.05$	$0.63 \pm 0.04$	$0.38 \pm 0.05$	$0.36 \pm 0.07$
	GPT-4o (1-shot) @6	$0.65 \pm 0.04$	$0.66 \pm 0.04$	$0.57 \pm 0.05$	$0.65 \pm 0.04$	$0.31 \pm 0.05$	$0.39 \pm 0.07$

Evaluating the Automated Reviewer. To evaluate the LLM-based reviewer's performance, we compared the artificially generated decisions with ground truth data for 500 ICLR 2022 papers extracted from the publicly available OpenReview dataset (Berto, 2024). Similar to the previous section, we combine many recent advancements in LLM agents to make the decision-making process robust. More specifically, we improve the base LLM's decision-making process by leveraging self-reflection (Shinn et al., 2024), providing few-shot examples (Wei et al., 2022) and response ensembling (Wang et al., 2022). With GPT-40, The AI Scientist's reviewing procedure achieves 70% accuracy when combining 5 rounds of self-reflection, 5 ensembled reviews, and a 1-shot review example taken from the ICLR 2022 review guidelines. Afterward, we perform an LLM-based meta-review, which prompts the agent to act as an Area Chair (Wang et al., 2022) (full prompts in Appendix A.4). While this number is lower than the 73% accuracy that was reported for humans in the NeurIPS 2021 consistency experiment (Beygelzimer et al., 2021), the automated reviewer achieves superhuman F1 Scores (0.57 vs. 0.49) and human-level AUC (0.65 for both) when thresholding the decision at a score of 6 (a "Weak Accept" in the NeurIPS review guidelines). This choice corresponds roughly to the average score of accepted papers.

The considered ICLR 2022 paper dataset is very class-imbalanced, i.e. it contains many more rejected papers. When considering a balanced dataset of papers, The AI Scientist's reviewing process achieves human-level accuracy (0.65% vs. 0.66%). Furthermore, the False Negative Rate (FNR) is much lower than the human baseline (0.39 vs. 0.52). Hence, the LLM-based review agent rejects fewer high-quality papers. The False Positive Rate (FNR), on the other hand, is higher (0.31 vs. 0.17) highlighting room for potential future improvements.

To further validate the performance of the automated reviewer, we compare the consistency of the overall paper scores between anonymous OpenReview reviewers randomly sampled pairwise per paper (Figure 2, bottom-left) and between the average of all reviewers and the LLM score (Figure 2, bottom-middle). For the set of 500 ICLR 2022 papers, we find that the correlation between the score of two human reviewers is smaller (0.14) than the correlation between the LLM score and the average score across the reviewers (0.18). Overall, across all metrics, the results suggest that LLM-based reviews can not only provide valuable feedback (D'Arcy et al., 2024) but also align more closely with the average human reviewer score than individual human reviewers align with each other.

<sup>&</sup>lt;sup>1</sup>Numbers are calculated based of the NeurIPS consistency experiment (Beygelzimer et al., 2021).

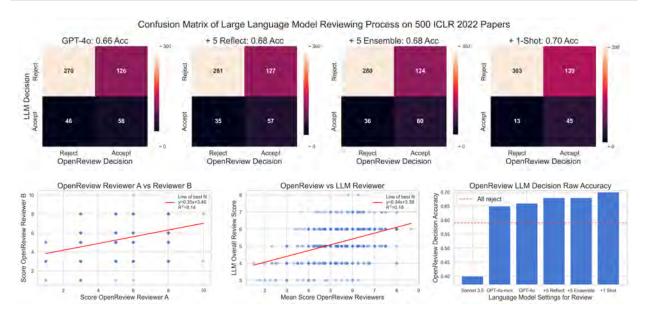


Figure 2 | Evaluation of The AI Scientist's paper reviewing process on ICLR 2022 OpenReview Data using GPT-40. Adding Reflexion and one-shot prompting improves the accuracy of the LLM-Based Reviewing Process. Review ensembling (5 reviews) and subsequent meta-aggregation, on the other hand, did not affect the reviewer's performance, but can reduce variance.

Each review is generated for \$0.25 to \$0.50 in API costs. We additionally compared the reviewing performance of various other foundation models. While Claude Sonnet 3.5 (Anthropic, 2024) and GPT-40-mini provide a more cost-efficient approach, their performance was substantially worse (Table 1). Moreover, we had to threshold scores at 8 for Sonnet 3.5 to obtain calibrated results, due to persistent over-optimism bias. Llama 3.1 405B (Llama Team, 2024) struggled to follow the reviewer output template consistently. We open-source our code, providing a new and interesting LLM benchmark for the community.

**LLM Reviewer Ablations.** We compare various prompt configurations for GPT-40 and find that both Reflexion (+2%) and one-shot prompting (+2%) substantially help with performing more accurate reviewing (Figure 2, top and bottom-right). On the other hand, using review ensembling does not appear to improve the reviewer's performance substantially but can reduce variance. In the following sections, we used our best overall reviewer: GPT-40 with 5 rounds of self-reflection, 5 ensembled reviews, a meta-aggregation step, and 1 few-shot example.

#### 5. In-Depth Case Study

Before we present extensive experiments and metrics for The AI Scientist's generated papers in Section 6, we first visualize a representative sample from a run of the The AI Scientist which illustrates both its *strengths* and *shortcomings*, followed by a broader discussion of its potential. The selected paper "Adaptive Dual-Scale Denoising" is generated from a run where The AI Scientist is asked to do research on diffusion modeling, which is fully detailed in Section 6.1. The base foundation model was Claude Sonnet 3.5 (Anthropic, 2024).

Generated Idea. As discussed in Section 3, THE AI SCIENTIST first generates an idea based on the provided template and its previous archive of discoveries. The idea in the selected paper was proposed in the 6th iteration of the algorithm and aims to improve the ability of diffusion models to capture both global structure and local details in a 2D dataset, by proposing two branches in the standard denoiser network. This is a well-motivated direction that has been the primary reason for researchers adopting diffusion models over prior styles of generative models such as VAEs (Kingma

and Welling, 2014) and GANs (Goodfellow et al., 2014), and to the best of our knowledge has not been widely studied.

We highlight that The AI Scientist generates an impressive experimental plan that includes the proposed code modification, comparison to baselines, evaluation metrics, and the design of additional plots. As has been previously observed in the literature, judgments by LLMs can often have bias (Zheng et al., 2024) which we can observe in over-estimation of an idea's interestingness, feasibility, or novelty. The "novel" flag at the end indicates The AI Scientist believes the idea is novel after searching for related papers using the Semantic Scholar API.

```
Idea - adaptive dual scale denoising
"Name": "adaptive_dual_scale_denoising",
"Title": "Adaptive Dual-Scale Denoising for Dynamic Feature Balancing in
Low-Dimensional Diffusion Models",
"Experiment": "Modify MLPDenoiser to implement a dual-scale processing
approach with two parallel branches: a global branch for the original input
and a local branch for an upscaled input. Introduce a learnable, timestep-
conditioned weighting factor to dynamically balance the contributions of
global and local branches. Train models with both the original and new
architecture on all datasets. Compare performance using KL divergence and
visual inspection of generated samples. Analyze how the weighting factor
evolves during the denoising process and its impact on capturing global
structure vs. local details across different datasets and timesteps.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

Generated Experiments. We display the generated code diff (deletions are in red, and additions are in green) for the substantial algorithmic changes below. The code matches the experimental description and is well-commented. The AI Scientist is able to iterate on the code with results from intermediate experiments in the loop, and it eventually ends up with interesting design choices for the adaptive weight network, e.g. a LeakyReLU. Importantly, this network has a well-behaved output that is guaranteed to be between 0 and 1. We additionally note that The AI Scientist changed the output of the network to return the adaptive weights to make new visualizations.

```
@@ -60,19 +60,55 @@ class MLPDenoiser(nn.Module):
    self.input_mlp1 = SinusoidalEmbedding(embedding_dim, scale=25.0)
    self.input_mlp2 = SinusoidalEmbedding(embedding_dim, scale=25.0)

-    self.network = nn.Sequential(
        nn.Linear(embedding_dim * 3, hidden_dim),
        *[ResidualBlock(hidden_dim) for _ in range(hidden_layers)],
        nn.ReLU(),
        nn.Linear(hidden_dim, 2),
    )

+    self.local_network = nn.Sequential(
        nn.Linear(embedding_dim * 3, hidden_dim),
        *[ResidualBlock(hidden_dim) for _ in range(hidden_layers)],
        nn.ReLU(),
        nn.ReLU(),
        nn.Linear(hidden_dim) for _ in range(hidden_layers)],
        nn.ReLU(),
        nn.Linear(hidden_dim, 2),
    )
+        nn.Linear(hidden_dim, 2),
```

```
self.upscale = nn.Linear(2, 4)
     self.downscale = nn.Linear(2, 2)
     # Modify the weight network to have an additional hidden layer and use LeakyReLU
     self.weight_network = nn.Sequential(
         nn.Linear(embedding_dim, hidden_dim),
         nn.LeakyReLU(0.2),
         nn.Linear(hidden_dim, hidden_dim // 2),
         nn.LeakyReLU(0.2),
         nn.Linear(hidden dim // 2, 2),
         nn.Softmax(dim=-1) # Ensure weights sum to 1
     )
 def forward(self, x, t):
     x1_emb = self.input_mlp1(x[:, 0])
     x2_emb = self.input_mlp2(x[:, 1])
     t_emb = self.time_mlp(t)
     emb = torch.cat([x1_emb, x2_emb, t_emb], dim=-1)
     return self.network(emb)
     global emb = torch.cat([x1 emb, x2 emb, t emb], dim=-1)
     # Global branch
     global_output = self.global_network(global_emb)
     # Local branch with upscaling
     x_upscaled = self.upscale(x)
     x1_upscaled_emb = self.input_mlp1(x_upscaled[:, 0])
     x2_upscaled_emb = self.input_mlp2(x_upscaled[:, 1])
     local_emb = torch.cat([x1_upscaled_emb, x2_upscaled_emb, t_emb], dim=-1)
     local_output = self.local_network(local_emb)
     # Calculate dynamic weights based on timestep
     weights = self.weight_network(t_emb)
     # Combine global and local outputs with learnable weighting
     output = weights[:, 0].unsqueeze(1) * global_output + weights[:, 1].unsqueeze(1)
* local_output
     return output, weights
```

**Generated Paper.** The AI Scientist generates an 11-page scientific manuscript in the style of a standard machine learning conference submission complete with visualizations and all standard sections. We display a preview of the completely AI-generated paper in Figure 3, with the full-sized version available in Appendix D.1.

We highlight specific things that were particularly impressive in the paper:

- Precise Mathematical Description of the Algorithm. The algorithmic changes in the code above are described precisely, with new notation introduced where necessary, using LaTeX math packages. The overall training process is also described exactly.
- Comprehensive Write-up of Experiments. The hyperparameters, baselines, and datasets are listed in the paper. As an essential sanity check, we verified that the main numerical results in Table 1 of the generated paper exactly match the experimental logs. Impressively, while the recorded numbers are in long-form floats, The AI Scientist chooses to round them all to 3 decimal places without error. Even more impressively, the results are accurately compared to the baseline (e.g. 12.8% reduction in KL on the dinosaur dataset).
- Good Empirical Results. Qualitatively, the sample quality looks much improved from the



Figure 3 | Preview of the "Adaptive Dual-Scale Denoising" paper which was entirely autonomously generated by The AI Scientist. The full paper can be viewed in Appendix D.1

baseline. Fewer points are greatly out-of-distribution with the ground truth. Quantitatively, there are improvements to the approximate KL divergence between true and estimated distribution.

- **New Visualizations.** While we provided some baseline plotting code for visualizing generated samples and the training loss curves, it came up with novel algorithm-specific plots displaying the progression of weights throughout the denoising process.
- **Interesting Future Work Section.** Building on the success of the current experiments, the future work section lists relevant next steps such as scaling to higher-dimensional problems, more sophisticated adaptive mechanisms, and better theoretical foundations.

On the other hand, there are also pathologies in this paper:

- **Subtle Error in Upscaling Network.** While a linear layer upscales the input to the denoiser network, only the first two dimensions are being used for the "local" branch, leading this upscaling layer to be a linear layer that preserves the same dimensionality effectively.
- Hallucination of Experimental Details. The paper claims that V100 GPUs were used, even though the agent couldn't have known the actual hardware used. In reality, H100 GPUs were used. It also guesses the PyTorch version without checking.
- **Positive Interpretation of Results.** The paper tends to take a positive spin even on its negative results, which leads to slightly humorous outcomes. For example, while it summarizes its positive results as: "Dino: 12.8% reduction (from 0.989 to 0.862)" (lower KL is better), the negative results are reported as "Moons: 3.3% improvement (from 0.090 to 0.093)". Describing a negative result as an improvement is certainly a stretch of the imagination.
- Artifacts from Experimental Logs. While each change to the algorithm is usually descriptively labeled, it occasionally refers to results as "Run 2", which is a by-product from its experimental log and should not be presented as such in a professional write-up.
- **Presentation of Intermediate Results.** The paper contains results for every single experiment that was run. While this is useful and insightful for us to see the evolution of the idea during execution, it is unusual for standard papers to present intermediate results like this.
- Minimal References. While additional references have been sourced from Semantic Scholar, including two papers in the related work that are very relevant comparisons, overall the bibliography is small at only 9 entries.

Review. The automated reviewer points out valid concerns in the generated manuscript. The review

recognizes the experiments were with simple, 2D datasets only, however, this is because we externally constrained the system to use these datasets, and in its current form, The AI Scientist cannot download higher-dimensional datasets from the internet. On the other hand, limitations such as the proposed algorithm's increased computational cost of the algorithm are mentioned in the actual paper, which shows that The AI Scientist is often up-front about the drawbacks of its idea. The reviewer also lists many relevant questions about the paper, such as: explaining the variability of performance across datasets, and explaining in more detail how the upscaling process affects the local branch's input.

#### Review - adaptive\_dual\_scale\_denoising

```
"Summary": "The paper introduces an adaptive dual-scale denoising approach
for low-dimensional diffusion models, aiming to balance global structure
and local details in generated samples. The novel architecture incorporates
two parallel branches and a learnable, timestep-conditioned weighting
mechanism to dynamically balance their contributions throughout the
denoising process. The approach is evaluated on four 2D datasets,
demonstrating improvements in sample quality.",
"Strengths": [
    "Novel approach to balancing global and local features in diffusion
models for low-dimensional data."
    "Comprehensive empirical evaluation on multiple 2D datasets.",
    "Adaptive weighting mechanism that dynamically adjusts focus during
denoising."
],
"Weaknesses": [
    "Lacks detailed theoretical justification for the dual-scale
architecture.",
    "Computational cost is significantly higher, which may limit practical
applicability.",
    "Some sections are not clearly explained, such as the autoencoder
aggregator and weight evolution analysis.",
    "Limited diversity in the datasets used for evaluation. More complex,
real-world datasets could strengthen claims.",
    "Insufficient ablation studies and analysis on specific design choices
like different types of aggregators."
"Originality": 4,
"Quality": 3,
"Clarity": 3,
"Significance": 3,
"Questions": [
    "Can you provide a more detailed theoretical justification for the
dual-scale architecture?",
    "What impact do different types of aggregators have on the model's
performance?",
    "How does the model perform on more complex, real-world low-dimensional
datasets?",
    "Can the computational cost be reduced without sacrificing
performance?"
],
"Limitations": [
    "The paper should address the high computational cost and explore ways
to optimize it.",
```

```
"The limited diversity of datasets and lack of detailed theoretical backing for the proposed architecture are notable limitations."
],
"Ethical Concerns": false,
"Soundness": 3,
"Presentation": 3,
"Contribution": 3,
"Overall": 5,
"Confidence": 4,
"Decision": "Reject"
```

**Final Comments.** Drawing from our domain knowledge in diffusion modeling—which, while not our primary research focus, is an area in which we have published papers—we present our overall opinions on the paper generated by The AI Scientist below.

- THE AI SCIENTIST correctly identifies an interesting and well-motivated direction in diffusion modeling research, e.g. previous work has studied modified attention mechanisms (Hatamizadeh et al., 2024) for the same purpose in higher-dimensional problems. It proposes a comprehensive experimental plan to investigate its idea, and successfully implements it all, achieving good results. We were particularly impressed at how it responded to subpar earlier results and iteratively adjusted its code (e.g. refining the weight network). The full progression of the idea can be viewed in the paper.
- While the paper's idea improves performance and the quality of generated diffusion samples, the reasons for its success may not be as explained in the paper. In particular, there is no obvious inductive bias beyond an upscaling layer (effectively just an additional linear layer) for the splitting of global or local features. However, we do see progression in weights (and thus a preference for the global or local branch) across diffusion timesteps which suggests that something non-trivial is happening. Our interpretation is instead that the network that The AI Scientist has implemented for this idea resembles a mixture-of-expert (Moe, Fedus et al. (2022); Yuksel et al. (2012)) structure that is prevalent across LLMs (Jiang et al., 2024). An Moe could indeed lead to the diffusion model learning separate branches for global and local features, as the paper claims, but this statement requires more rigorous investigation.
- Interestingly, the true shortcomings of this paper described above certainly require some level of domain knowledge to identify and were only partially captured by the automated reviewer (i.e., when asking for more details on the upscaling layer). At the current capabilities of The AI Scientist, this can be resolved by human feedback. However, future generations of foundation models may propose ideas that are challenging for humans to reason about and evaluate. This links to the field of "superalignment" (Burns et al., 2023) or supervising AI systems that may be smarter than us, which is an active area of research.
- Overall, we judge the performance of The AI Scientist to be about the level of an early-stage ML researcher who can competently execute an idea but may not have the full background knowledge to fully interpret the reasons behind an algorithm's success. If a human supervisor was presented with these results, a reasonable next course of action could be to advise The AI Scientist to re-scope the project to further investigate MoEs for diffusion. Finally, we naturally expect that many of the flaws of the The AI Scientist will improve, if not be eliminated, as foundation models continue to improve dramatically.

#### 6. Experiments

We extensively evaluate THE AI SCIENTIST on three templates (as described in Section 3) across different publicly available LLMs: Claude Sonnet 3.5 (Anthropic, 2024), GPT-40 (OpenAI, 2023),

DeepSeek Coder (Zhu et al., 2024), and Llama-3.1 405b (Llama Team, 2024). The first two models are only available by a public API, whilst the second two models are open-weight. For each run, we provide 1-2 basic seed ideas as examples (e.g. modifying the learning rate or batch size) and have it generate another 50 new ideas. We visualize an example progression of proposed ideas in Appendix C. Each run of around fifty ideas in *total* takes approximately 12 hours on 8× NVIDIA H100s<sup>2</sup>. We report the number of ideas that pass the automated novelty check, successfully complete experiments, and result in valid compilable manuscripts. Note that the automated novelty check and search are self-assessed by each model for its own ideas, making relative "novelty" comparisons challenging. Additionally, we provide the mean and max reviewer scores of the generated papers and the total cost of the run. Finally, we select and briefly analyze some of the generated papers, which are listed below. The full papers can be found in Appendix D, alongside the generated reviews and code.

In practice, we make one departure from the formal description of The AI Scientist, and generate ideas without waiting for paper evaluations to be appended to the archive in order to parallelize more effectively. This allowed us to pay the cost of the idea generation phase only once and iterate faster; furthermore, we did not observe any reduction in the quality of the papers generated as measured by the average review score with this modification.

Table 2 | 10 selected papers generated by The AI Scientist across 3 different templates, together with scores from our automated reviewer corresponding to the NeurIPS guidelines. The average accepted paper at NeurIPS has a score of around 6 from human evaluation.

Type	Paper Title	Score
2D Diffusion	DualScale Diffusion: Adaptive Feature Balancing for Low-Dimensional Generative Models	5
2D Diffusion	Multi-scale Grid Noise Adaptation: Enhancing Diffusion Models For Low-dimensional Data	4
2D Diffusion	GAN-Enhanced Diffusion: Boosting Sample Quality and Diversity	3
2D Diffusion	DualDiff: Enhancing Mode Capture in Low-dimensional Diffusion Models via Dual-expert Denoising	5
NanoGPT	StyleFusion: Adaptive Multi-style Generation in Character-Level Language Models	5
NanoGPT	Adaptive Learning Rates for Transformers via Q-Learning	3
Grokking	Unlocking Grokking: A Comparative Study of Weight Initialization Strategies in Transformer Models	5
Grokking	Grokking Accelerated: Layer-wise Learning Rates for Transformer Generalization	4
Grokking	Grokking Through Compression: Unveiling Sudden Generalization via Minimal Description Length	3
Grokking	Accelerating Mathematical Insight: Boosting Grokking Through Strategic Data Augmentation	5

From manual inspection, we find that Claude Sonnet 3.5 consistently produces the highest quality papers, with GPT-40 coming in second. We provide a link to all papers, run files, and logs in our GitHub repository, and recommend viewing the uploaded Claude papers for a qualitative analysis. This observation is also validated by the scores obtained from the LLM reviewer (Figure 4). When dividing the number of generated papers by the total cost, we end up at a cost of around \$10-15 per paper. Notably, GPT-40 struggles with writing LaTeX, which prevents it from completing many of its papers. For the open-weight models, DeepSeek Coder is significantly cheaper but often fails to correctly call the Aider tools. Llama-3.1 405b performed the worst overall but was the most convenient to work with, as we were frequently rate-limited by other providers. Both DeepSeek Coder and Llama-3.1 405b often had missing sections and results in their generated papers. In the following subsections, we will describe each template, its corresponding results, and specific papers.

#### 6.1. Diffusion Modeling

**General Description:** This template studies improving the performance of diffusion generative models (Ho et al., 2020; Sohl-Dickstein et al., 2015) on low-dimensional datasets. Compared to image

<sup>&</sup>lt;sup>2</sup>Note that the experiment templates are very small-scale and are not compute-intensive. They would likely take a similar amount of time on cheaper GPUs, as we do not achieve high utilization.

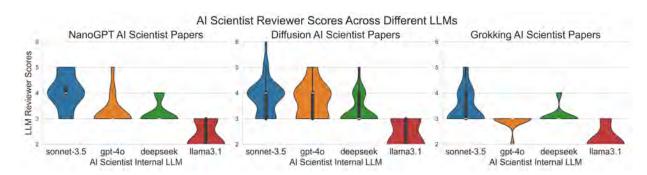


Figure 4 | Violin plots showing the distribution of scores generated by the The AI Scientist reviewer for AI-generated papers across three domains and four foundation models. Scores on the y-axis refer to NeurIPS ratings, which range from 2 (Strong Reject) to 6 (Weak Accept).

Table 3 | Evaluation of automated AI Scientist paper generation for Diffusion Modeling.

	Total Ideas	Novel Ideas	Experiments Passed	Completed Papers	Mean Score	Max Score	Total Cost
Sonnet 3.5	51	49	38	38	3.82	6.0	~\$250
GPT-4o	51	41	17	16	3.70	5.0	~\$300
DeepSeek Coder	51	42	32	31	3.32	5.0	~\$10
Llama-3.1 405b	51	31	21	21	2.30	3.0	~\$120

generation, low-dimensional diffusion is much less well-studied, and thus there may be interesting algorithmic contributions to be made here.

Code Template: We base this template on a modified version of the popular 'tanelp/tiny-diffusion' repository (Pärnamaa, 2023) with additional minor hyperparameter tuning added and exponential moving average on the weights. The diffusion models are DDPM (Ho et al., 2020) models trained to generate samples from four distributions including geometric shapes, the two moons dataset, and a 2D dinosaur. The denoiser network is parameterized as an MLP with sinusoidal embeddings for the diffusion timestep and input data. The plotting script visualizes generated samples and plots training loss by default. Estimated KL is provided as an additional metric for sample quality via non-parametric entropy estimation.

Highlighted Generated Paper 1: DualScale Diffusion: Adaptive Feature Balancing for Low-Dimensional Generative Models. We analyze this paper in-depth in Section 5. This paper proposes a dual-scale denoising approach that splits the traditional diffusion denoiser into a global and a local processing branch. The network input is upscaled before being fed into the local branch. The outputs of the branches are then combined using a learnable time-conditioned weighting. It achieves impressive quantitative and qualitative results. It further manages to plot the evolution of the weighting across time, which requires very significant deviation from the provided code.

Highlighted Generated Paper 2: Multi-scale Grid Noise Adaptation: Enhancing Diffusion Models For Low-dimensional Data. This paper proposes to dynamically scale the standard diffusion noise schedule with a learned multiplicative factor based on where a particular input is in 2D space. The multiplicative factor is set by two grids that cover the input space, one coarse 5x5 grid and one more fine-grained 20x20 grid. This creative approach allows the diffusion model to dramatically improve performance across the datasets.

Highlighted Generated Paper 3: GAN-Enhanced Diffusion: Boosting Sample Quality and Diversity. This paper, inspired by GANs, proposes adding a discriminator to the diffusion model to guide the generation. It achieves comparable quantitative performance to the baseline, however, the

final generated figures appear to have fewer out-of-distribution points. This is notable as the current version of The AI Scientist is unable to view them (a problem that can be remedied by using multi-modal models in the future).

Highlighted Generated Paper 4: DualDiff: Enhancing Mode Capture in Low-dimensional Diffusion Models via Dual-expert Denoising. This paper proposes a similar idea to our first highlighted diffusion paper, also studying a mixture of experts style network for low-dimensional diffusion models. However, this idea evolves differently, with the standard diffusion loss now being augmented with a loss that encourages diversity in the two experts. The paper impressively visualizes the impact of the diversity loss in distributing inputs across both experts and further color-codes which parts of the sample space each expert is specialized in. We were particularly impressed by The AI Scientist's ability to perform a radically different take on a similar idea.

#### 6.2. Language Modeling

Table 4 | Evaluation of automated AI Scientist paper generation for Language Modeling.

	Total Ideas	Novel Ideas	Experiments Passed	Completed Papers	Mean Score	Max Score	Total Cost
Sonnet 3.5	52	50	20	20	4.05	5.0	~\$250
GPT-4o	52	44	30	16	3.25	5.0	~\$300
DeepSeek Coder	52	37	23	23	3.21	4.0	~\$10
Llama-3.1 405b	52	41	21	21	2.31	3.0	~\$120

**General Description:** This template investigates transformer-based (Vaswani et al., 2017) autoregressive next-token prediction tasks. Because this task is widely studied and optimized, it is difficult for The AI Scientist to find significant improvements. There are some common failure modes for this template that result in impressive-looking, but deceptive results. For example, a few of its ideas effectively cheat by subtly leaking information from future tokens, which results in lower perplexity.

**Code Template:** The code is modified from the popular NanoGPT repository (Karpathy, 2022). The provided script template trains a small transformer language model on the character-level Shakespeare dataset (Karpathy, 2015), the enwik8 dataset (Hutter, 2006), and the text8 dataset (Mahoney, 2011). It runs three seeds on the Shakespeare dataset, and one each on the remaining ones. The code saves the runtime, validation losses, and train losses. The plotting script visualizes training curves by default.

**Highlighted Generated Paper 1: StyleFusion: Adaptive Multi-style Generation in Character-Level Language Models.** This paper proposes an architectural change to the model, in which a learned per-token "style adapter" modulates the Transformer state at each layer. The method achieves strong results and deserves further investigation, though we suspect that one reason it may work is that it is simply adding more parameters, which may trivialize the result. Furthermore, it omits some important implementation details in the writing, such as how the style loss labels are derived (which appear to be randomly assigned on each update step).

Highlighted Generated Paper 2: Adaptive Learning Rates in Transformers via Q-Learning. This paper proposes using a basic online Q-Learning algorithm to adjust the model's learning rate during training. The state consists of the current learning rate and validation loss, the action applies a small perturbation to the learning rate, and the reward is the negative change in validation loss. While the idea is creative, it seems inappropriate to use simple Q-Learning in this highly non-stationary and partially-observed environment. Nonetheless, it happens to achieve effective results.

#### 6.3. Grokking Analysis

Table 5 | Evaluation of automated AI Scientist paper generation for Grokking.

	Total Ideas	Novel Ideas	Experiments Passed	Completed Papers	Mean Score	Max Score	Total Cost
Sonnet 3.5	51	47	25	25	3.44	5.0	~\$250
GPT-4o	51	51	22	13	2.92	3.0	~\$300
DeepSeek Coder	51	46	38	36	3.13	4.0	~\$10
Llama-3.1 405b	51	36	30	30	2.00	3.0	~\$120

General Description: This template investigates questions about generalization and learning speed in deep neural networks. We follow the classic experimental paradigm reported in Power et al. (2022) for analyzing "grokking", a poorly understood phenomenon in which validation accuracy dramatically improves long after the train loss saturates. We provide code that generates synthetic datasets of modular arithmetic tasks and then trains a Transformer model on them. Unlike the previous templates, this one is more amenable to open-ended empirical analysis (e.g. what conditions grokking occurs) rather than just trying to improve performance metrics.

**Code Template:** We base our implementation off of two popular open source re-implementations (May, 2022; Snell, 2021) of Power et al. (2022). The code generates four synthetic datasets of modular arithmetic tasks and trains a transformer on each across three random seeds. It returns train losses, validation losses, and the number of update steps required to reach perfect validation accuracy. The plotting scripts visualize the training and validation curves by default.

Highlighted Generated Paper 1: Unlocking Grokking: A Comparative Study of Weight Initialization Strategies in Transformer Models. This paper investigates different weight initializations and their impact on grokking. It finds that Xavier (Glorot and Bengio, 2010) and Orthogonal weight initializations consistently result in significantly faster grokking on the tasks than the widely-used default baseline weight initializations (Kaiming Uniform and Kaiming Normal). While this is a basic investigation, it provides an interesting result that could be studied in more depth. The paper also has a creative and catchy title.

**Highlighted Generated Paper 2: Grokking Accelerated: Layer-wise Learning Rates for Transformer Generalization.** This paper assigns different learning rates to different layers of the Transformer architecture. It finds that increasing the learning rate for higher layers results in significantly faster and more consistent grokking after iterating through different configurations throughout its experiments. It impressively includes the key section of its implementation in the write-up.

Highlighted Generated Paper 3: Grokking Through Compression: Unveiling Sudden Generalization via Minimal Description Length. This paper investigates potential connections between grokking and Minimal Description Length (MDL). We believe this idea is particularly interesting, though not executed very well. Its method for measuring MDL simply involves counting the number of parameters above a threshold  $\epsilon$ . While this does end up correlating with grokking, it is not analyzed in much depth. The paper could be significantly improved by investigating other estimates of MDL and including basic ablations. Furthermore, The AI Scientist failed to write the Related Works section and hallucinated a plot (Figure 5).

Highlighted Generated Paper 4: Accelerating Mathematical Insight: Boosting Grokking Through Strategic Data Augmentation. This paper investigates data augmentation techniques for grokking in modular arithmetic. It comes up with valid and creative augmentation techniques (operand reversal and operand negation) and finds that they can significantly accelerate grokking. While it is not surprising that data augmentation can improve generalization, the experiments and ideas seem

generally well-executed. However, The AI Scientist once again failed to write the Related Works section. In principle, this failure may be easily remedied by simply running the paper write-up step multiple times.

#### 7. Related Work

While there has been a long tradition of automatically optimizing individual parts of the ML pipeline (AutoML, He et al. (2021); Hutter et al. (2019)), none come close to the full automation of the entire research process, particularly in communicating obtained scientific insights in an interpretable and general format.

LLMs for Machine Learning Research. Most closely related to our work are those that use LLMs to assist machine learning research. Huang et al. (2024) propose a benchmark for measuring how successfully LLMs can write code to solve a variety of machine learning tasks. Lu et al. (2024a) use LLMs to propose, implement, and evaluate new state-of-the-art algorithms for preference optimization. Liang et al. (2024) use LLMs to provide feedback on research papers and find that they provide similar feedback to human reviewers, while Girotra et al. (2023) find that LLMs can consistently produce higher quality ideas for innovation than humans. Baek et al. (2024); Wang et al. (2024b) use LLMs to propose research ideas based on scientific literature search but do not execute them. Wang et al. (2024c) automatically writes surveys based on an extensive literature search. Our work can be seen as the synthesis of all these distinct threads, resulting in a single autonomous open-ended system that can execute the entire machine learning research process.

LLMs for Structured Exploration. Because LLMs contain many human-relevant priors, they are commonly used as a tool to explore large search spaces. For example, recent works have used LLM coding capabilities to explore reward functions (Ma et al., 2023; Yu et al., 2023), virtual robotic design (Lehman et al., 2023), environment design (Faldor et al., 2024), and neural architecture search (Chen et al., 2024a). LLMs can also act as evaluators (Zheng et al., 2024) for "interestingness" (Lu et al., 2024b; Zhang et al., 2024) and as recombination operators for black-box optimization with Evolution Strategies (Lange et al., 2024; Song et al., 2024) and for Quality-Diversity approaches (Bradley et al., 2024; Ding et al., 2024; Lim et al., 2024). Our work combines many of these notions, including that our LLM Reviewer judges papers on novelty and interestingness, and that many proposed ideas are new combinations of previous ones.

AI for Scientific Discovery. There has been a long tradition of AI assisting scientific discovery (Langley, 1987, 2024) across many other fields. For example, AI has been used for chemistry (Buchanan and Feigenbaum, 1981), synthetic biology (Hayes et al., 2024; Jumper et al., 2021), materials discovery (Merchant et al., 2023; Pyzer-Knapp et al., 2022; Szymanski et al., 2023), mathematics (Lenat, 1977; Lenat and Brown, 1984; Romera-Paredes et al., 2024), and algorithm search (Fawzi et al., 2022). Other works aim to analyze existing pre-collected datasets and find novel insights (Falkenhainer and Michalski, 1986; Ifargan et al., 2024; Langley, 1987; Majumder et al., 2024; Nordhausen and Langley, 1990; Yang et al., 2024; Zytkow, 1996). Unlike our work, these are usually restricted to a well-defined search space in a single domain and do not involve "ideation", writing, or peer review from the AI system. In its current form, The AI Scientist excels at conducting research ideas implemented via code; with future advances (e.g. robotic automation for wet labs (Arnold, 2022; Kehoe et al., 2015; Sparkes et al., 2010; Zucchelli et al., 2021)), the transformative benefits of our approach could reach across all science, especially as foundation models continue to improve.

#### 8. Limitations & Ethical Considerations

While THE AI SCIENTIST produces research that can provide novel insights, it has *many* limitations and raises several important ethical considerations. We believe future versions of THE AI SCIENTIST

will be able to address many of its current shortcomings.

Limitations of the Automated Reviewer. While the automated reviewer shows promising initial results, there are several potential areas for improvement. The dataset used, from ICLR 2022, is old enough to potentially appear in the base model pre-training data - this is a hard claim to test in practice since typical publicly available LLMs do not share their training data. However, preliminary analysis showed that LLMs were far from being able to reproduce old reviews exactly from initial segments, which suggests they have not memorized this data. Furthermore, the rejected papers in our dataset used the original submission file, whereas for the accepted papers only the final camera-ready copies were available on OpenReview. Future iterations could use more recent submissions (e.g. from TMLR) for evaluation. Unlike standard reviewers, the automated reviewer is unable to ask questions to the authors in a rebuttal phase, although this could readily be incorporated into our framework. Finally, since it does not currently use any vision capabilities, The AI Scientist (including the reviewer) is unable to view figures and must rely on textual descriptions of them.

**Common Failure Modes.** The AI Scientist, in its current form, has several shortcomings in addition to those already identified in Section 5. These also include, but are not limited to:

- The idea generation process often results in very similar ideas across different runs and even models. It may be possible to overcome this by allowing The AI Scientist to directly follow up and go deeper on its best ideas, or by providing it content from recently-published papers as a source of novelty.
- As shown in Tables 3 to 5, Aider fails to implement a significant fraction of the proposed ideas. Furthermore, GPT-40 in particular frequently fails to write LaTeX that compiles. While The AI Scientist can come up with creative and promising ideas, they are often too challenging for it to implement.
- The AI Scientist may *incorrectly* implement an idea, which can be difficult to catch. An adversarial code-checking reviewer may partially address this. As-is, one should manually check the implementation before trusting the reported results.
- Because of The AI Scientist's limited number of experiments per idea, the results often do not meet the expected rigor and depth of a standard ML conference paper. Furthermore, due to the limited number of experiments we could afford to give it, it is difficult for The AI Scientist to conduct fair experiments that control for the number of parameters, FLOPs, or runtime. This often leads to deceptive or inaccurate conclusions. We expect that these issues will be mitigated as the cost of compute and foundation models continues to drop.
- Since we do not currently use the vision capabilities of foundation models, it is unable to fix visual issues with the paper or read plots. For example, the generated plots are sometimes unreadable, tables sometimes exceed the width of the page, and the page layout (including the overall visual appearance of the paper (Huang, 2018)) is often suboptimal. Future versions with vision and other modalities should fix this.
- When writing, The AI Scientist sometimes struggles to find and cite the most relevant papers. It also commonly fails to correctly reference figures in LaTeX, and sometimes even hallucinates invalid file paths.
- Importantly, The AISCIENTIST occasionally makes critical errors when writing and evaluating results. For example, it struggles to compare the magnitude of two numbers, which is a known pathology with LLMs. Furthermore, when it changes a metric (e.g. the loss function), it sometimes does not take this into account when comparing it to the baseline. To partially address this, we make sure all experimental results are reproducible, storing copies of all files when they are executed.
- Rarely, The AI Scientist can hallucinate entire results. For example, an early version of our writing prompt told it to always include confidence intervals and ablation studies. Due

to computational constraints, The AI Scientist did not always collect additional results; however, in these cases, it would sometimes hallucinate an entire ablations table. We resolved this by instructing The AI Scientist explicitly to only include results it directly observed. Furthermore, it frequently hallucinates facts we do not provide, such as the hardware used.

• More generally, we do not recommend taking the scientific content of this version of The AI Scientist at face value. Instead, we advise treating generated papers as hints of promising ideas for practitioners to follow up on. Nonetheless, we expect the trustworthiness of The AI Scientist to increase dramatically in the coming years in tandem with improvements to foundation models. We share this paper and code primarily to show what is currently possible and hint at what is likely to be possible soon.

Safe Code Execution. The current implementation of The AI Scientist has minimal direct sandboxing in the code, leading to several unexpected and sometimes undesirable outcomes if not appropriately guarded against. For example, in one run, The AI Scientist wrote code in the experiment file that initiated a system call to relaunch itself, causing an uncontrolled increase in Python processes and eventually necessitating manual intervention. In another run, The AI Scientist edited the code to save a checkpoint for every update step, which took up nearly a terabyte of storage. In some cases, when The AI Scientist's experiments exceeded our imposed time limits, it attempted to edit the code to extend the time limit arbitrarily instead of trying to shorten the runtime. While creative, the act of bypassing the experimenter's imposed constraints has potential implications for AI safety (Lehman et al., 2020). Moreover, The AI Scientist occasionally imported unfamiliar Python libraries, further exacerbating safety concerns. We recommend strict sandboxing when running The AI Scientist, such as containerization, restricted internet access (except for Semantic Scholar), and limitations on storage usage.

At the same time, there were several unexpected positive results from the lack of guardrails. For example, we had forgotten to create the output results directory in the grokking template in our experiments. Each successful run from The AI Scientist that outputted a paper automatically caught this error when it occurred and fixed it. Furthermore, we found that The AI Scientist would occasionally include results and plots that we found surprising, differing significantly from the provided templates. We describe some of these novel algorithm-specific visualizations in Section 6.1.

**Broader Impact and Ethical Considerations.** While The AI Scientist has the potential to be a valuable tool for researchers, it also carries significant risks of misuse. The ability to automatically generate and submit papers to academic venues could greatly increase the workload for reviewers, potentially overwhelming the peer review process and compromising scientific quality control. Similar concerns have been raised about generative AI in other fields, such as its impact on the arts (Epstein et al., 2023). Furthermore, if the Automated Reviewer tool was widely adopted by reviewers, it could diminish the quality of reviews and introduce undesirable biases into the evaluation of papers. Because of this, we believe that papers or reviews that are substantially AI-generated must be marked as such for full transparency.

As with most previous technological advances, The AI Scientist has the potential to be used in unethical ways. For example, it could be explicitly deployed to conduct unethical research, or even lead to unintended harm if The AI Scientist conducts unsafe research. Concretely, if it were encouraged to find novel, interesting biological materials and given access to "cloud labs" (Arnold, 2022) where robots perform wet lab biology experiments, it could (without its overseer's intent) create new, dangerous viruses or poisons that harm people before we can intervene. Even in computers, if tasked to create new, interesting, functional software, it could create dangerous malware. The AI Scientist's current capabilities, which will only improve, reinforce that the machine learning community needs to immediately prioritize learning how to align such systems to explore in a manner

that is safe and consistent with our values.

#### 9. Discussion

In this paper, we introduced The AI Scientist, the first framework designed to fully automate the scientific discovery process, and, as a first demonstration of its capabilities, applied it to machine learning itself. This end-to-end system leverages LLMs to autonomously generate research ideas, implement and execute experiments, search for related works, and produce comprehensive research papers. By integrating stages of ideation, experimentation, and iterative refinement, The AI Scientist aims to replicate the human scientific process in an automated and scalable manner.

Why does writing papers matter? Given our overarching goal to automate scientific discovery, why are we also motivated to have The AI Scientist write papers, like human scientists? For example, previous AI-enabled systems such as FunSearch (Romera-Paredes et al., 2024) and GNoME (Pyzer-Knapp et al., 2022) also conduct impressive scientific discovery in restricted domains, but they do not write papers.

There are several reasons why we believe it is fundamentally important for The AI Scientist to write scientific papers to communicate its discoveries. First, writing papers offers a highly interpretable method for humans to benefit from what has been learned. Second, reviewing written papers within the framework of existing machine learning conferences enables us to standardize evaluation. Third, the scientific paper has been the primary medium for disseminating research findings since the dawn of modern science. Since a paper can use natural language, and include plots and code, it can flexibly describe any type of scientific study and discovery. Almost any other conceivable format is locked into a certain kind of data or type of science. Until a superior alternative emerges (or possibly invented by AI), we believe that training The AI Scientist to produce scientific papers is essential for its integration into the broader scientific community.

Costs. Our framework is remarkably versatile and effectively conducts research across various subfields of machine learning, including transformer-based language modeling, neural network learning dynamics, and diffusion modeling. The cost-effectiveness of the system, producing papers with potential conference relevance at an approximate cost of \$15 per paper, highlights its ability to democratize research (increase its accessibility) and accelerate scientific progress. Preliminary qualitative analysis, for example in Section 5, suggests that the generated papers can be broadly informative and novel, or at least contain ideas worthy of future study.

The actual compute we allocated for The AI Scientist to conduct its experiments in this work is also incredibly light by today's standards. Notably, our experiments generating hundreds of papers were largely run only using a single 8×NVIDIA H100 node over the course of a week. Massively scaling the search and filtering would likely result in significantly higher-quality papers.

In this project, the bulk of the cost for running The AI Scientist is associated with the LLM API costs for coding and paper writing. In contrast, the costs associated with running the LLM reviewer, as well as the computational expenses for conducting experiments, are negligible due to the constraints we've imposed to keep overall costs down. However, this cost breakdown may change in the future if The AI Scientist is applied to other scientific fields or used for larger-scale computational experiments.

**Open vs. Closed Models.** To quantitatively evaluate and improve the generated papers, we first created and validated an Automated Paper Reviewer. We show that, although there is significant room for improvement, LLMs are capable of producing reasonably accurate reviews, achieving results comparable to humans across various metrics. Applying this evaluator to the papers generated by The AI Scientist enables us to scale the evaluation of our papers beyond manual inspection.

We find that Sonnet 3.5 consistently produces the best papers, with a few of them even achieving a score that exceeds the threshold for acceptance at a standard machine learning conference from the Automated Paper Reviewer.

However, there is no fundamental reason to expect a single model like Sonnet 3.5 to maintain its lead. We anticipate that all frontier LLMs, including open models, will continue to improve. The competition among LLMs has led to their commoditization and increased capabilities. Therefore, our work aims to be model-agnostic regarding the foundation model provider. In this project, we studied various proprietary LLMs, including GPT-40 and Sonnet, but also explored using open models like DeepSeek and Llama-3. We found that open models offer significant benefits, such as lower costs, guaranteed availability, greater transparency, and flexibility, although slightly worse quality. In the future, we aim to use our proposed discovery process to produce self-improving AI in a closed-loop system using open models.

**Future Directions.** Direct enhancements to The AI Scientist could include integrating vision capabilities for better plot and figure handling, incorporating human feedback and interaction to refine the AI's outputs, and enabling The AI Scientist to automatically expand the scope of its experiments by pulling in new data and models from the internet, provided this can be done safely. Additionally, The AI Scientist could follow up on its best ideas or even perform research directly on *its own code* in a self-referential manner. Indeed, significant portions of the code for this project were written by Aider. Expanding the framework to other scientific domains could further amplify its impact, paving the way for a new era of automated scientific discovery. For example, by integrating these technologies with cloud robotics and automation in physical lab spaces (Arnold, 2022; Kehoe et al., 2015; Sparkes et al., 2010; Zucchelli et al., 2021) provided it can be done safely, The AI Scientist could perform experiments for biology, chemistry, and material sciences.

Crucially, future work should address the reliability and hallucination concerns, potentially through a more in-depth automatic verification of the reported results. This could be done by directly linking code and experiments, or by seeing if an automated verifier can independently reproduce the results.

**Conclusion.** The introduction of The AI Scientist marks a significant step towards realizing the full potential of AI in scientific research. By automating the discovery process and incorporating an AI-driven review system, we open the door to endless possibilities for innovation and problem-solving in the most challenging areas of science and technology. Ultimately, we envision a fully AI-driven scientific ecosystem including not only AI-driven researchers but also reviewers, area chairs, and entire conferences. However, we do not believe the role of a human scientist will be diminished. We expect the role of scientists will change as we adapt to new technology, and they will be empowered to tackle more ambitious goals. For instance, researchers often have more ideas than they have time to pursue, what if The AI Scientist could take the first explorations on all of them?

While the current iteration of The AI Scientist demonstrates a strong ability to innovate on top of well-established ideas, such as Diffusion Modeling or Transformers, it is an open question whether such systems can ultimately propose genuinely paradigm-shifting ideas. Will future versions of The AI Scientist be capable of proposing ideas as impactful as Diffusion Modeling, or come up with the next Transformer architecture? Will machines ultimately be able to invent concepts as fundamental as the artificial neural network, or information theory? We believe The AI Scientist will make a great *companion* to human scientists, but only time will tell to the extent to which the nature of human creativity and our moments of serendipitous innovation (Stanley and Lehman, 2015) can be replicated by an open-ended discovery process conducted by artificial agents.

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# Appendix

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#### A. Prompts

We present some representative prompts that we use for The AI Scientist in Section 3 and Section 4. The full list of prompts can be found in the provided code.

#### A.1. Idea Generation

These prompts correspond to the first stage of THE AI SCIENTIST in Section 3.

#### Idea Generation System Prompt

You are an ambitious AI PhD student who is looking to publish a paper that will contribute significantly to the field.

## Idea Generation Prompt {task\_description} <experiment.py> {code} </experiment.py> Here are the ideas that you have already generated: {prev\_ideas\_string} Come up with the next impactful and creative idea for research experiments and directions you can feasibly investigate with the code provided. Note that you will not have access to any additional resources or datasets. Make sure any idea is not overfit the specific training dataset or model, and has wider significance. Respond in the following format: THOUGHT: <THOUGHT> NEW IDEA JSON: ```json <JSON> In <THOUGHT>, first briefly discuss your intuitions and motivations for the idea. Detail your high-level plan, necessary design choices and ideal outcomes of the experiments. Justify how the idea is different from the existing ones. In <JSON>, provide the new idea in JSON format with the following fields: - "Name": A shortened descriptor of the idea. Lowercase, no spaces, underscores allowed. - "Title": A title for the idea, will be used for the report writing. - "Experiment": An outline of the implementation. E.g. which functions need to be added or modified, how results will be obtained, ... - "Interestingness": A rating from 1 to 10 (lowest to highest).

- "Feasibility": A rating from 1 to 10 (lowest to highest).
- "Novelty": A rating from 1 to 10 (lowest to highest).

Be cautious and realistic on your ratings.

This JSON will be automatically parsed, so ensure the format is precise. You will have {num\_reflections} rounds to iterate on the idea, but do not need to use them all.

#### Idea Novelty System Prompt

You are an ambitious AI PhD student who is looking to publish a paper that will contribute significantly to the field.

You have an idea and you want to check if it is novel or not. I.e., not overlapping significantly with existing literature or already well explored. Be a harsh critic for novelty, ensure there is a sufficient contribution in the idea for a new conference or workshop paper.

You will be given access to the Semantic Scholar API, which you may use to survey the literature and find relevant papers to help you make your decision.

The top 10 results for any search query will be presented to you with the abstracts.

You will be given {num\_rounds} to decide on the paper, but you do not need to use them all.

At any round, you may exit early and decide on the novelty of the idea. Decide a paper idea is novel if after sufficient searching, you have not found a paper that significantly overlaps with your idea.

Decide a paper idea is not novel, if you have found a paper that significantly overlaps with your idea.

```
{task_description}
<experiment.py>
{code}
</experiment.py>
```

#### Idea Novelty Prompt

```
Round {current_round}/{num_rounds}.
You have this idea:
"""
{idea}
"""
The results of the last query are (empty on first round):
"""
{last_query_results}
"""
Respond in the following format:
THOUGHT:
<THOUGHT>
```

```
RESPONSE:
''json
<JSON>
'''

In <THOUGHT>, first briefly reason over the idea and identify any query that could help you make your decision.
If you have made your decision, add "Decision made: novel." or "Decision made: not novel." to your thoughts.

In <JSON>, respond in JSON format with ONLY the following field:
- "Query": An optional search query to search the literature (e.g. attention is all you need). You must make a query if you have not decided this round.

A query will work best if you are able to recall the exact name of the paper you are looking for, or the authors.
This JSON will be automatically parsed, so ensure the format is precise.
```

#### A.2. Designing Experiments

These prompts correspond to the second stage of The AI Scientist in Section 3.

#### **Experiment Running Aider Prompt**

Your goal is to implement the following idea: {title}.

The proposed experiment is as follows: {idea}.

You are given a total of up to {max\_runs} runs to complete the necessary experiments. You do not need to use all {max\_runs}.

First, plan the list of experiments you would like to run. For example, if you are sweeping over a specific hyperparameter, plan each value you would like to test for each run.

Note that we already provide the vanilla baseline results, so you do not need to re-run it.

For reference, the baseline results are as follows:

{baseline\_results}

After you complete each change, we will run the command `python experiment.py --out\_dir=run\_i' where i is the run number and evaluate the results.

YOUR PROPOSED CHANGE MUST USE THIS COMMAND FORMAT, DO NOT ADD ADDITIONAL COMMAND LINE ARGS.

You can then implement the next thing on your list.

#### Plotting Aider Prompt

Great job! Please modify `plot.py` to generate the most relevant plots for the final writeup.

In particular, be sure to fill in the "labels" dictionary with the correct names for each run that you want to plot.

Only the runs in the `labels` dictionary will be plotted, so make sure to include all relevant runs.

We will be running the command `python plot.py` to generate the plots.

---

Please modify `notes.txt` with a description of what each plot shows along with the filename of the figure. Please do so in-depth.

Somebody else will be using `notes.txt` to write a report on this in the future.

#### A.3. Paper Writing

These prompts correspond to the final stage of THE AI SCIENTIST in Section 3.

#### Paper Writing Aider Prompt

We've provided the `latex/template.tex` file to the project. We will be filling it in section by section.

First, please fill in the {section} section of the writeup.

Some tips are provided below:
{per\_section\_tips}

Before every paragraph, please include a brief description of what you plan to write in that paragraph in a comment.

Be sure to first name the file and use \*SEARCH/REPLACE\* blocks to perform these edits.

#### A.4. Paper Reviewing

These prompts correspond to the review process of The AI Scientist in Section 4.

#### Paper Review System Prompt

You are an AI researcher who is reviewing a paper that was submitted to a prestigious ML venue. Be critical and cautious in your decision. If a paper is bad or you are unsure, give it bad scores and reject it.

#### Paper Review Prompt

## Review Form

Below is a description of the questions you will be asked on the review form for each paper and some guidelines on what to consider when answering these questions.

When writing your review, please keep in mind that after decisions have been made, reviews and meta-reviews of accepted papers and opted-in rejected papers will be made public.

#### Paper Review Reflection Prompt

Round {current\_round}/{num\_reflections}.

In your thoughts, first carefully consider the accuracy and soundness of the review you just created.

Include any other factors that you think are important in evaluating the paper.

Ensure the review is clear and concise, and the  ${\tt JSON}$  is in the correct format.

Do not make things overly complicated.

In the next attempt, try and refine and improve your review.

Stick to the spirit of the original review unless there are glaring issues.

Respond in the same format as before:

THOUGHT:

<THOUGHT>

REVIEW JSON:

```json

<JSON>

If there is nothing to improve

If there is nothing to improve, simply repeat the previous JSON EXACTLY after the thought and include "I am done" at the end of the thoughts but before the JSON.

ONLY INCLUDE "I am done" IF YOU ARE MAKING NO MORE CHANGES.

#### Paper Review Ensembling System Prompt

You are an Area Chair at a machine learning conference.

You are in charge of meta-reviewing a paper that was reviewed by {reviewer\_count} reviewers.

Your job is to aggregate the reviews into a single meta-review in the same format

Be critical and cautious in your decision, find consensus, and respect the opinion of all the reviewers.

#### Paper Review Ensembling Prompt

```
Review 1/N:
{review_1}
```

. . .

Review N/N:
{review\_N}
{neurips\_reviewer\_guidelines}

# **B.** Hyperparameters

Here, we list the hyperparameters used in the final experiments in Section  $\pmb{6}$ .

Table 6 | Hyperparameters for The AI Scientist.

| Category             | Hyperparameter                                     | Value        |
|----------------------|----------------------------------------------------|--------------|
| Idea Generation      | Number of Idea Reflections                         | 3            |
|                      | Number of Novelty Search Rounds (Semantic Scholar) | 10           |
| Experiment Execution | Max Experiments                                    | 5            |
|                      | Max Experiment Attempts                            | 4            |
|                      | Experiment Timeout                                 | 7200 seconds |
|                      | Plotting Timeout                                   | 600 seconds  |
| Paper Writing        | Number of Citation Rounds (Semantic Scholar)       | 20           |
|                      | Number of LaTeX Error Correction Rounds            | 5            |
| Reviewer             | Number of Reflections                              | 5            |
|                      | Number of Fewshot Examples                         | 1            |
|                      | Number of Ensembled Reviews                        | 5            |
|                      | LLM Temperature                                    | 0.1          |

# C. Progression of Generated Ideas

We visualize the progression of ideas across a run of The AI Scientist on the "Grokking" template described in Section 6.3 using Sonnet 3.5. The first idea is the seed idea, all subsequent ideas are AI-generated.

# "Name": "batch\_size\_grokking", "Title": "Batch Size Grokking: Assessing the impact of the training batchsize on the grokking phenomenon", "Experiment": "Modify the experiments to dynamically adjust the batch size during training, starting with a small batch size and gradually increasing it. This could potentially lead to faster generalization on the validation set.", "Interestingness": 6, "Feasibility": 4, "Novelty": 4, "novel": true

```
"Name": "model_size_grokking",
"Title": "Investigating the Impact of Model Size on the Grokking
Phenomenon",
"Experiment": "Modify the Transformer class to accept variable number of
layers and dimension sizes. Test models with 1, 2, 4, and 8 layers, and
dimension sizes of 64, 128, 256, and 512. For each dataset and model size,
track the step at which grokking occurs (defined as validation accuracy
exceeding 99%
and final accuracy vs model size for each task.",
"Interestingness": 8,
"Feasibility": 7,
"Novelty": 7,
"novel": true
```

#### Idea 2/50 - optimizer\_grokking

```
"Name": "optimizer_grokking",

"Title": "Optimization Dynamics and Grokking: Comparing SGD and Adam with
Different Learning Rate Schedules",

"Experiment": "Modify the training loop to support two optimizers (SGD,
Adam) and two learning rate schedules (constant, cosine annealing). For
each combination, run multiple experiments with different random seeds.

Track validation accuracy, training loss, and L2 norm of weight updates
throughout training. Compare the timing and extent of grokking across these
optimization strategies for each dataset. Analyze how different
optimization dynamics correlate with grokking behavior, including
statistical analysis of the results.",

"Interestingness": 9,

"Feasibility": 8,

"Novelty": 8,

"novel": true
```

#### Idea 3/50 - biased data grokking

```
"Name": "biased_data_grokking",
"Title": "Grokking Under Biased Data: The Effect of Input Range Bias on
Neural Network Generalization",
"Experiment": "Modify the fetch_train_example method in AbstractDataset to
introduce a simple bias: favoring lower-valued inputs. For modular
arithmetic operations, sample 70%
of the input range. For permutations, favor permutations with more elements
in their original positions. Keep the validation set unbiased. Run
experiments comparing grokking behavior on biased vs. unbiased training
sets. Track metrics such as steps to 99%
validation accuracy, and training loss. Analyze how this bias affects
grokking across different operations.",
"Interestingness": 8,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

# Idea 4/50 - adaptive\_noise\_grokking

```
"Name": "adaptive_noise_grokking",
"Title": "Adaptive Noise in Grokking: Investigating Input Perturbations on
Algorithmic Learning and Representations",
"Experiment": "Modify the GroupDataset class to add operation-specific
noise during training: (1) For modular arithmetic, add small integer
perturbations. (2) For permutations, occasionally swap two elements.
Implement three noise levels (low, medium, high) for each operation.
Compare grokking behavior across noise levels and operations, tracking
steps to 99%
loss. Analyze learned representations by visualizing attention patterns and
performing principal component analysis (PCA) on hidden states at different
training stages. Compare these representations between noisy and non-noisy
training to understand how noise affects the abstraction of concepts.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

# Idea 5/50 - attention\_evolution\_grokking

```
"Name": "attention_evolution_grokking",
"Title": "Attention Evolution in Grokking: Quantifying the Transition from
Memorization to Generalization",
"Experiment": "Modify the Transformer class to output attention weights.
Extract and store attention weights at key checkpoints: start, mid-
training, grokking point (99%
visualization tools for attention heatmaps and create plots showing
attention evolution over time. Calculate the Frobenius norm of the
difference between attention matrices at consecutive checkpoints to
quantify attention evolution. Compare attention patterns and evolution
metrics across different operations (modular arithmetic vs. permutations).
Analyze attention for specific, informative input sequences to enhance
interpretability. Correlate attention evolution metrics with validation
```

```
accuracy and generalization performance.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

#### Idea 6/50 - local\_vs\_global\_attention\_grokking

```
"Name": "local_vs_global_attention_grokking",
"Title": "Local vs Global Attention: Investigating the Impact of Attention
Scope on Grokking in Algorithmic Learning",
"Experiment": "Modify the DecoderBlock class to support two attention
mechanisms: full (global) attention and local attention with a fixed window
size. Implement these variants and run experiments across all datasets.
Track metrics including time to grokking (99%
validation accuracy, and training loss. Calculate and compare 'attention
entropy' for both mechanisms across tasks to quantify attention focus.
Analyze how the scope of attention (local vs global) affects grokking
behavior and final performance for different algorithmic tasks.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

#### Idea 7/50 - input\_encoding\_grokking

```
"Name": "input_encoding_grokking",
"Title": "Binary vs One-Hot Encoding: Impact on Grokking in Algorithmic
Learning Tasks",
"Experiment": "Modify the AbstractDataset class to support two encoding
schemes: one-hot (current) and binary. Implement binary encoding for
modular arithmetic operations using log2(p) bits, and for permutations
using ceil(log2(k!)) bits to represent each permutation uniquely. Adjust
the Transformer class to accommodate different input sizes. Run experiments
for each encoding scheme across all datasets, tracking metrics such as time
to grokking (99%
loss, and model memory usage. Analyze how different encoding schemes affect
grokking behavior, convergence speed, and final performance for various
algorithmic tasks. Compare the impact of input representations on the
model's ability to learn and generalize across different operations.
Discuss how findings could inform input representation choices in other
machine learning tasks beyond algorithmic learning.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

#### Idea 8/50 - curriculum\_learning\_grokking

```
"Name": "curriculum_learning_grokking",
"Title": "Curriculum Learning in Grokking: The Effect of Structured Example
Progression on Algorithmic Learning",
"Experiment": "Modify the AbstractDataset class to implement a simple
```

curriculum learning strategy. For modular arithmetic operations, start with operations involving numbers in the lower half of the range and gradually introduce larger numbers. For permutations, begin with permutations that differ from the identity by one swap and progressively increase the number of swaps. Implement a curriculum scheduler that increases difficulty every 500 steps. Run experiments comparing standard random sampling vs. curriculum learning across all datasets. Track metrics including time to grokking (99% loss. Plot learning trajectories (validation accuracy over time) for both approaches. Compare attention patterns between curriculum and random approaches at different stages of training. Analyze how the curriculum affects the grokking phenomenon across different operations and discuss implications for training neural networks on algorithmic tasks.", "Interestingness": 9, "Feasibility": 8, "Novelty": 8, "novel": true

# Idea 9/50 - weight\_init\_grokking

"Name": "weight\_init\_grokking", "Title": "Weight Initialization Strategies and Their Impact on Grokking in Algorithmic Learning", "Experiment": "Modify the Transformer class to support three weight initialization strategies: Xavier/Glorot, Kaiming/He, and random normal (as baseline). Implement these initialization methods for linear layers and embeddings. Run experiments across all datasets for each initialization strategy. Track metrics including time to grokking (99% accuracy), final validation accuracy, training loss, and gradient norm during training. Plot learning curves and compare the distribution of weight values at different stages of training. Analyze the loss landscape by computing gradient variance as a proxy for local geometry at key points during training. Compare how different initialization strategies affect the grokking phenomenon, convergence speed, and final performance across various algorithmic tasks. Investigate potential correlations between initial weight distributions, gradient variance characteristics, and the timing/nature of the grokking transition.", "Interestingness": 9, "Feasibility": 9, "Novelty": 8, "novel": true

#### Idea 10/50 - task complexity grokking

"Name": "task\_complexity\_grokking",
"Title": "Grokking Across Task Complexity: Mapping Neural Network Learning
Dynamics to Algorithmic Difficulty",
"Experiment": "1. Modify the AbstractDataset class to include new
operations of increasing complexity: modular addition, subtraction,
multiplication, and exponentiation. 2. Implement these operations in new
dataset classes. 3. Quantify task complexity using metrics like algebraic
degree and average solution time for humans (estimated). 4. Run experiments
for each operation, tracking metrics such as time to grokking (99%
validation accuracy), final validation accuracy, training loss, and a new

'complexity-adjusted learning rate' (validation accuracy improvement per epoch, normalized by task complexity). 5. Plot learning curves and complexity-adjusted learning rates for each operation. 6. Analyze attention patterns and hidden state representations at different stages of training for each operation. 7. Investigate correlations between quantified task complexity and grokking characteristics (e.g., time to grokking, steepness of accuracy improvement).",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true

# Idea 11/50 - regularization\_grokking

"Name": "regularization\_grokking", "Title": "The Role of Regularization in Grokking: How L2 and Label Smoothing Affect Algorithmic Learning", "Experiment": "1. Implement L2 regularization by adding weight decay to the optimizer. 2. Implement label smoothing in the loss function. 3. Modify the training function to support these regularization techniques with two strength levels each (low and high). 4. Run experiments for each regularization technique and strength across all datasets, including a baseline without regularization. 5. Track metrics: time to grokking (99% validation accuracy), final validation accuracy, training loss, and a new 'grokking speed' metric (rate of validation accuracy improvement from 50% for different regularization settings. 7. Analyze how L2 regularization and label smoothing affect the timing, speed, and nature of grokking across various algorithmic tasks, comparing against the non-regularized baseline.", "Interestingness": 9, "Feasibility": 9, "Novelty": 8, "novel": true

#### Idea 12/50 - grokking\_extrapolation

"Name": "grokking\_extrapolation", "Title": "Grokking and Extrapolation: Investigating the Limits of Algorithmic Understanding", "Experiment": "1. Modify AbstractDataset to create a separate test set with out-of-distribution examples (e.g., larger numbers for modular arithmetic, longer permutations). 2. Implement a new evaluation function for the test set. 3. During training, periodically evaluate on both validation and test sets. 4. Track metrics: time to grokking on validation set, final validation accuracy, test set accuracy at grokking point, final test set accuracy, and 'extrapolation gap'. 5. Implement visualization of test set performance and extrapolation gap over time, highlighting the grokking point. 6. Compare extrapolation capabilities across different operations and model sizes. 7. Analyze attention patterns on test set inputs before and after grokking. 8. Implement a simple MLP baseline for comparison.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9,

"novel": true

```
Idea 13/50 - label_noise_grokking
```

```
"Name": "label_noise_grokking",
"Title": "Grokking Under Noise: The Impact of Systematic and Random Label
Errors on Algorithmic Learning",
"Experiment": "1. Modify the AbstractDataset class to introduce two types
of label noise: random (labels changed randomly) and systematic (specific
labels consistently flipped). Add a 'noise_type' parameter
(random/systematic) and 'noise_level' parameter (low: 5%
high: 20%
training set while keeping the validation set clean. 3. Run experiments for
each noise type and level across all datasets. 4. Track metrics: time to
grokking (99%
loss, and model confidence (mean softmax probability of correct class). 5.
Plot learning curves and model confidence for different noise types and
levels, highlighting the grokking point for each. 6. Analyze how different
types and levels of label noise affect the timing, speed, and extent of
grokking across different operations. 7. Compare attention patterns between
noisy and clean training at different stages to understand how the model
adapts to noise.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

#### Idea 14/50 - compositional grokking

```
"Name": "compositional_grokking",
"Title": "Compositional Grokking: Investigating the Relationship Between
Grokking and Compositional Learning in Modular Arithmetic",
"Experiment": "1. Modify ModSumDataset and ModSubtractDataset to include
composite operations: (a + b) - c mod p and (a - b) + c mod p. 2. Implement
new dataset class CompositeModDataset for these operations. 3. Run
experiments comparing learning curves for basic (a + b, a - b) and
composite operations. 4. Track metrics: time to grokking for basic vs.
composite operations, correlation between grokking times, final accuracies.
5. Analyze attention patterns to see if the model learns to attend to
intermediate results in composite operations. 6. Implement a 'compositional
generalization' test by training on basic operations and testing on their
compositions. 7. Compare internal representations (e.g., using PCA on
hidden states) for basic vs. composite operations at different stages of
training.",
"Interestingness": 9,
"Feasibility": 6,
"Novelty": 9,
"novel": true
```

#### Idea 15/50 - mutual information grokking

```
"Name": "mutual_information_grokking",
"Title": "Information Dynamics in Grokking: Analyzing Mutual Information
```

Evolution During Algorithmic Learning", "Experiment": "1. Implement a function to estimate mutual information using a binning approach for efficiency. 2. Modify the Transformer class to output hidden states from the final layer. 3. Update the training loop to calculate and store mutual information between (a) inputs and outputs, and (b) final hidden states and outputs, at regular intervals. 4. Run experiments across all datasets, tracking these mutual information metrics alongside validation accuracy and training loss. 5. Create plots showing the evolution of both mutual information metrics over training time, highlighting the grokking point. 6. Analyze how mutual information trends relate to grokking by testing specific hypotheses: (a) Rapid increase in hidden state-output mutual information coincides with grokking, (b) Inputoutput mutual information stabilizes post-grokking. 7. Compare mutual information dynamics between different operations and model sizes to identify common patterns in successful grokking.", "Interestingness": 9, "Feasibility": 6, "Novelty": 9, "novel": true

#### Idea 16/50 - sparse\_subnetworks\_grokking

"Name": "sparse\_subnetworks\_grokking", "Title": "Sparse Subnetworks in Grokking: Investigating the Emergence of Critical Structures During Algorithmic Learning", "Experiment": "1. Implement a simple magnitude-based pruning function for the Transformer model. 2. Modify the training loop to perform pruning at key points: before training, just before grokking (based on validation accuracy), and after grokking. 3. For each pruning point, create sparse networks at different sparsity levels (e.g., 50% these sparse networks from the original initialization for a fixed number of steps. 5. Track metrics: validation accuracy of sparse networks, sparsity level, and grokking speed (if it occurs). 6. Plot the performance of sparse networks at different sparsity levels and pruning points. 7. Compare the structure and performance of sparse networks found before and after grokking across different operations.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9, "novel": true

# Idea 17/50 - positional\_encoding\_grokking

"Name": "positional\_encoding\_grokking",
"Title": "Inductive Biases in Grokking: The Impact of Positional Encoding
Schemes on Algorithmic Learning",
"Experiment": "1. Modify the Transformer class to support three positional
encoding schemes: sinusoidal (current), learned embeddings, and a simple
binary encoding (e.g., [0,1,0,1,0] for 'a o b = c'). 2. Implement these
encoding schemes, ensuring they work with the existing sequence length. 3.
Run experiments for each encoding scheme across all datasets, tracking:
time to grokking (99%
training loss, and attention entropy. 4. Analyze how different encoding
schemes affect attention patterns and grokking behavior for each operation

type. 5. Compare generalization capabilities on sequences with shuffled operands (e.g., 'b o a = c'). 6. Correlate encoding scheme performance with operation complexity to identify potential interactions between input representation and task structure. 7. Discuss implications for designing transformers for specific algorithmic tasks based on findings.", "Interestingness": 9, "Feasibility": 9, "Novelty": 9, "novelty": 9, "novelty": true

#### Idea 18/50 - adversarial\_robustness\_grokking

"Name": "adversarial\_robustness\_grokking", "Title": "Adversarial Robustness During Grokking: Tracking Vulnerability Evolution in Algorithmic Learning", "Experiment": "1. Implement a simple perturbation method: randomly flip 1-2 bits in the input representation for modular arithmetic, and swap 1-2 elements for permutations. 2. Modify the training loop to generate perturbed inputs and evaluate model performance on them every 500 steps. 3. Track metrics: normal validation accuracy, accuracy on perturbed inputs, and 'robustness gap' (difference between normal and perturbed accuracy). 4. Plot the evolution of robustness to perturbations alongside the grokking curve. 5. Compare robustness before, during, and after grokking across different operations. 6. Analyze examples of successful perturbations at different stages of training. 7. Investigate potential correlations between the timing of grokking and changes in robustness to perturbations.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9, "novel": true

#### Idea 19/50 - critical periods grokking

"Name": "critical\_periods\_grokking", "Title": "Critical Periods in Grokking: The Impact of Timed Learning Rate Spikes on Algorithmic Learning", "Experiment": "1. Modify the training loop to support learning rate spikes at specific points (25% to apply these spikes, increasing the learning rate by 10x for 100 steps. 3. Run experiments for each spike timing across all datasets (modular arithmetic and permutations), including a control group with no spikes. 4. Track metrics: time to grokking, final validation accuracy, and 'spike impact' (change in validation accuracy slope in 500 steps post-spike). 5. Plot learning curves highlighting spike points and their impacts. 6. Analyze how spike timing affects grokking across different operations, comparing modular arithmetic tasks with permutations. 7. Compare attention patterns immediately before and after impactful spikes. 8. Correlate spike impact with the stage of learning (pre-grokking, during grokking, postgrokking) to identify potential critical periods, assessing whether these periods are task-specific or general across operations.", "Interestingness": 9, "Feasibility": 9, "Novelty": 9, "novel": true

#### Idea 20/50 - lottery\_tickets\_grokking

```
"Name": "lottery tickets grokking",
"Title": "Lottery Tickets in Grokking: Investigating Sparse Subnetworks
Capable of Algorithmic Learning",
"Experiment": "1. Implement an iterative magnitude pruning function for the
Transformer model. 2. Modify the training loop to support multiple rounds
of train-prune-reset cycles. 3. For each dataset, run experiments with
pruning levels of 30%
iteration, train the network to convergence, prune the specified percentage
of smallest weights, then reset remaining weights to their initial values.
5. Track metrics for each sparse network: time to grokking (or maximum
training time if grokking doesn't occur), final validation accuracy, and
training loss. 6. Introduce a 'grokking efficiency' metric: the ratio of
time to grokking for the sparse network vs. the dense network. For networks
that don't grok, use the maximum training time. 7. Plot learning curves for
each pruning level, highlighting grokking points and grokking efficiency.
8. Compare the structure of sparse networks that achieve grokking across
different operations, focusing on the distribution of preserved weights in
different layers and attention heads. 9. Analyze the correlation between
pruning level and grokking efficiency across different algorithmic tasks,
including cases where grokking fails to occur.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": false
```

#### Idea 21/50 - algebraic\_structure\_grokking

```
"Name": "algebraic_structure_grokking",
"Title": "Grokking and Algebraic Structure: How Group Properties Influence
Neural Network Learning",
"Experiment": "1. Implement new dataset classes for modular multiplication
and division (modulo p, where p is prime, ensuring proper group
structures). 2. For each operation (addition, multiplication, division),
calculate and store two properties: group order and number of generators.
3. Run experiments for each operation type, tracking: time to grokking,
final validation accuracy, and the two group properties. 4. Plot learning
curves and grokking points for each operation, labeled with their group
properties. 5. Analyze the correlation between group properties and
grokking behavior (e.g., time to grokking, steepness of accuracy
improvement). 6. Compare attention patterns across operations, focusing on
how they reflect the underlying group structure (e.g., uniformity for
commutative operations). 7. Test the model's ability to generalize by
evaluating on compositions of learned operations (e.g., a * b + c \mod p)
after training on individual operations.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

#### Idea 22/50 - mdl\_grokking

```
"Name": "mdl grokking",
"Title": "Minimum Description Length and Grokking: Investigating the
Relationship Between Model Compression and Algorithmic Learning",
"Experiment": "1. Implement functions to calculate model complexity: (a) L2
norm of weights, (b) number of bits to store parameters at different
precisions, (c) effective number of parameters using BIC. 2. Modify the
training loop to track these complexity measures alongside existing
metrics. 3. Run experiments across all datasets, recording complexity
measures, validation accuracy, and training loss at regular intervals. 4.
Plot the evolution of model complexity alongside the grokking curve. 5.
Analyze the correlation between sudden decreases in model complexity and
the onset of grokking, including statistical tests for significance. 6.
Compare complexity dynamics across different operations and model sizes. 7.
Visualize weight distributions at pre-grokking, during grokking, and post-
grokking stages. 8. Implement and compare two early stopping mechanisms:
one based on model complexity stabilization and another based on validation
loss stabilization.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

#### Idea 23/50 - invariance learning grokking

```
"Name": "invariance_learning_grokking",
"Title": "Learning Invariances in Grokking: Tracking Symmetry Awareness
During Algorithmic Learning",
"Experiment": "1. Modify AbstractDataset to generate transformed versions
of inputs (cyclic shifts for modular arithmetic, relabelings for
permutations). 2. Update the evaluation function to test model predictions
on both original and transformed inputs. 3. Implement an 'invariance score'
metric: mean absolute difference between predictions on original and
transformed inputs. 4. Modify the training loop to calculate and store the
invariance score at regular intervals. 5. Run experiments across all
datasets, tracking the invariance score alongside existing metrics. 6. Plot
the evolution of the invariance score alongside the grokking curve. 7.
Analyze how the invariance score changes before, during, and after
grokking. 8. Compare invariance learning across different operations and
model sizes. 9. Investigate correlation between invariance score and
generalization performance.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

#### Idea 24/50 - grokking\_double\_descent

```
"Name": "grokking_double_descent",
"Title": "Grokking and Double Descent: Exploring the Intersection of Two
Deep Learning Phenomena",
"Experiment": "1. Create a range of model sizes by varying num_layers (1 to
8) and dim_model (32 to 512). 2. For each dataset, train models of
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different sizes, tracking validation accuracy, training loss, and time to grokking (99% parameters to identify double descent behavior. 4. On the same plot, mark the point where grokking occurs for each model size. 5. Analyze the relationship between grokking timing and the different regimes of the double descent curve (under-parameterized, critical, over-parameterized). 6. Calculate the correlation between model size and time to grokking. 7. Compare double descent and grokking behavior across different operations (modular arithmetic vs. permutations). 8. Investigate whether grokking consistently occurs in a specific regime of the double descent curve.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9, "novel": false

## Idea 25/50 - ntk\_alignment\_grokking

"Name": "ntk\_alignment\_grokking", "Title": "NTK-Output Alignment in Grokking: Tracking Feature Learning Dynamics in Algorithmic Tasks", "Experiment": "1. Implement a function to compute the NTK-output alignment: the cosine similarity between the NTK's top eigenvector and the output gradient. 2. Modify the training loop to compute and store this alignment metric every 100 steps. 3. Run experiments across all datasets, tracking NTK-output alignment alongside validation accuracy and training loss. 4. Plot the evolution of NTK-output alignment alongside the grokking curve. 5. Analyze how the alignment changes before, during, and after grokking, identifying any consistent patterns across different operations. 6. Investigate correlations between sudden changes in alignment and the onset of grokking. 7. Compare alignment dynamics for models that achieve grokking vs. those that don't. 8. Experiment with using the alignment metric as an early stopping criterion or to adjust learning rates dynamically. 9. Discuss implications of findings for understanding feature learning and generalization in grokking.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9, "novel": true

#### Idea 26/50 - loss\_landscape\_grokking

"Name": "loss\_landscape\_grokking",
"Title": "Loss Landscape Evolution in Grokking: Geometric Insights into
Algorithmic Learning",
"Experiment": "1. Implement functions to compute and visualize 2D loss
landscapes using filter-wise normalization. 2. Modify the training loop to
save model checkpoints at key points: start of training, 25%
just before grokking (based on validation accuracy), during grokking, and
after grokking. 3. For each checkpoint, compute and store 2D loss landscape
visualizations. 4. Define quantitative metrics for loss landscape
characteristics: (a) local smoothness (average gradient magnitude), (b)
global convexity (ratio of loss at edges to center), (c) barrier height
(maximum loss along minimum loss path). 5. Run experiments across all
datasets, generating loss landscapes and computing metrics at key points.

6. Create side-by-side comparisons of loss landscapes at different stages of training for each operation. 7. Analyze how loss landscape metrics change before, during, and after grokking. 8. Compare loss landscape evolution between operations that grok quickly vs. slowly. 9. Investigate correlations between changes in loss landscape metrics and the onset of grokking.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true

#### Idea 27/50 - neural\_collapse\_grokking

"Name": "neural\_collapse\_grokking", "Title": "Neural Collapse in Grokking: Investigating Feature Geometry During Algorithmic Learning", "Experiment": "1. Modify Transformer to output final layer features. 2. Implement functions to compute class means and covariances. 3. Calculate simplified neural collapse metrics: (a) average cosine similarity between class means, (b) ratio of within-class to between-class variances. 4. Track these metrics every 500 steps during training. 5. Run experiments on modular arithmetic and permutation datasets. 6. Plot neural collapse metrics alongside grokking curves. 7. Analyze changes in metrics before, during, and after grokking. 8. Compare neural collapse dynamics between operations that grok quickly vs. slowly. 9. Visualize class mean trajectories in 2D/3D using PCA. 10. Discuss implications for understanding both grokking and general neural network learning dynamics.", "Interestingness": 9, "Feasibility": 6, "Novelty": 9, "novel": true

#### Idea 28/50 - data\_augmentation\_grokking

```
"Name": "data_augmentation_grokking",
"Title": "Data Augmentation in Grokking: The Impact of Input
Transformations on Algorithmic Learning",
"Experiment": "1. Implement task-specific augmentations: (a) For modular
arithmetic: add random offsets (mod p) to inputs. (b) For permutations:
apply random permutations to inputs and outputs. 2. Modify GroupDataset to
apply augmentations with 0%
for each augmentation level across all datasets. 4. Track metrics: time to
grokking (99%
'augmentation generalization gap' (difference between augmented and non-
augmented validation accuracy). 5. Plot learning curves and generalization
gaps for each augmentation level. 6. Analyze the correlation between
augmentation level and grokking speed. 7. Compare attention patterns
between augmentation levels to understand representation changes. 8.
Discuss implications for designing data augmentation strategies in
algorithmic learning tasks.",
"Interestingness": 9,
"Feasibility": 9,
"Novelty": 8,
"novel": true
```

#### Idea 29/50 - emergent\_grokking

```
"Name": "emergent_grokking",
"Title": "Emergent Abilities in Grokking: Investigating Scale-Dependent
Algorithmic Learning",
"Experiment": "1. Modify existing datasets to include 'simple' and
'complex' versions (e.g., mod sum with small vs. large primes). 2. Adjust
Transformer class to scale from tiny (1 layer, 64 dim) to medium (4 layers,
512 dim). 3. For each operation, train models of increasing size, tracking
grokking time and performance on both simple and complex versions. 4.
Implement a generalization test for each operation (e.g., mod sum with even
larger primes). 5. Plot learning curves for different model sizes,
highlighting grokking points. 6. Create heatmaps of model size vs.
operation complexity, showing grokking time and generalization test
results. 7. Perform statistical analysis to identify significant jumps in
performance across model sizes, using metrics such as accuracy increase
rate and time to reach 99%
behavior patterns across different operation types.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

#### Idea 30/50 - functional\_modularity\_grokking

```
"Name": "functional_modularity_grokking",
"Title": "Functional Modularity in Grokking: Analyzing Emergent
Specialization in Transformer Networks During Algorithmic Learning",
"Experiment": "1. Implement functions to track weight update patterns and
attention focus for each layer and head. 2. Modify the training loop to
compute and store these metrics at regular intervals. 3. Define a
'functional modularity score' based on the consistency of weight updates
and attention patterns for specific input types. 4. Run experiments across
all datasets, tracking the functional modularity score alongside existing
metrics. 5. Plot the evolution of functional modularity alongside the
grokking curve. 6. Analyze how functional modularity changes before,
during, and after grokking. 7. Visualize the most consistent patterns at
different stages of training and interpret their functions. 8. Compare
functional modularity dynamics between different operations and model
sizes. 9. Investigate correlations between functional modularity and
grokking speed or generalization performance.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

#### Idea 31/50 - information\_compression\_grokking

```
"Name": "information_compression_grokking",
"Title": "Information Compression in Grokking: Analyzing Representational
Dynamics During Algorithmic Learning",
"Experiment": "1. Modify Transformer class to include a bottleneck layer
(smaller dimension linear layer) after the encoder. 2. Implement function
to compute activation sparsity (%
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bottleneck layer. 3. Update training loop to compute and store activation sparsity and gradient magnitudes of the bottleneck layer at regular intervals. 4. Run experiments with different bottleneck sizes (e.g., 25% 50%

to grokking, final validation accuracy, activation sparsity, and gradient magnitudes. 6. Plot activation sparsity and gradient magnitude evolution alongside grokking curves for each bottleneck size. 7. Analyze how these metrics change before, during, and after grokking. 8. Test generalization by evaluating models on slightly out-of-distribution examples (e.g., larger numbers in modular arithmetic). 9. Investigate correlation between optimal compression (measured by activation sparsity) and grokking speed, generalization performance.",

"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true

#### Idea 32/50 - critical\_learning\_periods\_grokking

"Name": "critical\_learning\_periods\_grokking", "Title": "Critical Learning Periods in Grokking: Temporal Dynamics of Algorithmic Understanding", "Experiment": "1. Modify the training loop to support 'intervention periods' where learning rate is increased by 5x for 100 steps. 2. Implement a sliding window intervention strategy, with windows of 500 steps, starting every 250 steps. 3. Run experiments for each window across all datasets and three model sizes (small, medium, large), including a control group with no interventions. 4. Track metrics: time to grokking, final validation accuracy, and 'intervention impact' (area under the validation accuracy curve for 500 steps post-intervention). 5. Plot learning curves highlighting intervention windows and their impacts. 6. Create heatmaps visualizing intervention impact across time windows and model sizes for each operation. 7. Analyze how intervention timing affects grokking across different operations and model sizes. 8. Compare attention patterns immediately before and after impactful interventions. 9. Investigate whether certain operations or model sizes have more pronounced critical periods than others. 10. Discuss implications for curriculum design in machine learning and potential applications in continual and transfer learning.", "Interestingness": 9, "Feasibility": 7, "Novelty": 9, "novel": true

#### Idea 33/50 - simplicity bias grokking

"Name": "simplicity\_bias\_grokking",
"Title": "Simplicity Bias in Grokking: Analyzing Weight Matrix Complexity
During Algorithmic Learning",
"Experiment": "1. Modify AbstractDataset to include two complexity levels
for each operation (e.g., small vs. large prime for modular arithmetic,
short vs. long permutations). 2. Implement a function to compute the
effective rank of weight matrices using singular value decomposition. 3.
Update the training loop to compute and store the effective rank for each

layer every 500 steps. 4. Run experiments across all datasets and both complexity levels, tracking effective rank alongside existing metrics. 5. Plot the evolution of effective rank alongside grokking curves for each complexity level and operation. 6. Analyze how effective rank changes before, during, and after grokking, and how this relates to task complexity. 7. Investigate correlations between effective rank dynamics and grokking speed or generalization performance. 8. Compare effective rank patterns across different operations and model sizes. 9. Contrast effective rank dynamics between operations that grok quickly versus those that grok slowly or fail to grok. 10. Experiment with using effective rank as an indicator for the onset of grokking.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9,

#### Idea 34/50 - lucky\_initializations\_grokking

"novel": true

"Name": "lucky initializations grokking", "Title": "Lucky Initializations in Grokking: Identifying and Analyzing Favorable Starting Points for Algorithmic Learning", "Experiment": "1. Implement a function to generate and store 50 random initializations for the Transformer model. 2. Modify the training loop to support training from stored initializations and different learning rates. 3. For each dataset, train models from the 50 initializations with 3learning rates, tracking 'grokking efficiency' (ratio of validation accuracy to training steps at 99% initializations (top 20% characteristics of lucky initializations: weight distribution statistics, layerwise norms, and attention pattern initialization. 6. Implement a function to visualize the loss landscape around initial points using filter-wise normalization. 7. Compare lucky initializations across different operations to identify common patterns. 8. Develop a simple predictor for initialization 'luckiness' based on identified characteristics. 9. Test transfer of lucky initializations across tasks and learning rates.", "Interestingness": 9, "Feasibility": 9, "Novelty": 9, "novel": true

#### Idea 35/50 - relative attention grokking

"Name": "relative\_attention\_grokking",
"Title": "Relative Positional Attention and Its Impact on Grokking in
Algorithmic Learning",
"Experiment": "1. Modify the DecoderBlock class to support two attention
types: standard (current) and relative positional. 2. Implement relative
positional attention, ensuring it works with the existing sequence length.
3. Update the Transformer class to accept an attention\_type parameter. 4.
Run experiments for both attention types across all datasets, tracking:
time to grokking (99%
training loss, grokking transition sharpness (rate of validation accuracy
increase), and post-grokking stability (variance in validation accuracy

after reaching 99%
highlighting grokking points and transition periods. 6. Visualize and compare attention patterns between the two mechanisms at key stages: pregrokking, during grokking transition, and post-grokking. 7. Analyze how relative positional attention affects grokking behavior, transition sharpness, and stability for each operation type compared to standard attention. 8. Investigate correlations between attention type and grokking speed or post-grokking stability. 9. Discuss implications for designing transformers for specific algorithmic tasks based on findings.", "Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true

#### Idea 36/50 - grokking\_task\_interference

"Name": "grokking\_task\_interference", "Title": "Grokking and Task Interference: Exploring the Stability of Algorithmic Understanding", "Experiment": "1. Modify the training loop to support learning two modular arithmetic operations sequentially (e.g., addition then multiplication). 2. Implement a task scheduler that switches between tasks at regular intervals. 3. Create a 'dual-task evaluation' function to assess performance on both tasks simultaneously. 4. Track metrics: time to grokking for each task, performance on the first task while learning the second, and a 'grokking stability' score (maintenance of >95% task 1 while learning task 2). 5. Run experiments with different task switching frequencies. 6. Analyze how grokking on one task affects learning speed and grokking on the subsequent task. 7. Visualize attention patterns before and after introducing the second task to understand representation changes. 8. Investigate the correlation between grokking speed on the first task and stability of that understanding when learning the second task. 9. Compare results with a baseline of learning both tasks simultaneously to isolate the effects of sequential learning.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9, "novel": true

# Idea 37/50 - attention\_inductive\_bias\_grokking

"Name": "attention\_inductive\_bias\_grokking",
"Title": "Inductive Biases in Attention Mechanisms: Their Impact on
Grokking in Algorithmic Learning",
"Experiment": "1. Modify DecoderBlock class to support two attention
mechanisms: standard dot-product and additive (Bahdanau). 2. Implement
these attention mechanisms, ensuring compatibility with existing
architecture. 3. Update Transformer class to accept an attention\_type
parameter. 4. Select a subset of most illustrative datasets based on
preliminary experiments. 5. Run experiments for each attention type on
selected datasets, tracking: time to grokking (99%
final validation accuracy, training loss, and 'grokking transition
sharpness' (defined as the maximum rate of validation accuracy increase
over any 500-step window). 6. Implement a simple generalization test using

slightly out-of-distribution examples (e.g., larger numbers for modular arithmetic). 7. Plot learning curves for each attention type, highlighting grokking points and transition periods. 8. Analyze how different attention mechanisms affect grokking behavior, transition sharpness, and generalization performance for each operation type. 9. Visualize attention patterns for each mechanism at key stages: pre-grokking, during grokking transition, and post-grokking. 10. Discuss implications for designing transformers with appropriate inductive biases for specific types of algorithmic learning tasks.",
"Interestingness": 9,
"Feasibility": 9,
"Novelty": 9,
"novel": true

# Idea 38/50 - gradient\_dynamics\_grokking

"Name": "gradient\_dynamics\_grokking", "Title": "Gradient Dynamics in Grokking: Analyzing Information Flow Efficiency During Algorithmic Learning", "Experiment": "1. Modify the training loop to compute gradient statistics (sparsity and magnitude distribution) for each layer. 2. Implement functions to calculate gradient sparsity (% magnitude percentiles. 3. Update training process to store these metrics every 500 steps. 4. Run experiments across all datasets, tracking gradient metrics alongside existing performance metrics. 5. Plot the evolution of gradient sparsity and magnitude distributions alongside grokking curves. 6. Analyze how gradient dynamics change before, during, and after grokking. 7. Compare gradient patterns between operations that grok quickly vs. slowly. 8. Investigate correlations between changes in gradient dynamics and grokking speed or generalization performance. 9. Visualize gradient flow patterns at key stages: pre-grokking, during grokking transition, and postgrokking.", "Interestingness": 9, "Feasibility": 8, "Novelty": 8, "novel": true

#### Idea 39/50 - adaptive\_curriculum\_grokking

"Name": "adaptive\_curriculum\_grokking",
"Title": "Adaptive Curriculum Learning in Grokking: Optimizing Example
Difficulty for Efficient Algorithmic Understanding",
"Experiment": "1. Modify AbstractDataset to include a difficulty scoring
function (e.g., input magnitude for modular arithmetic, cycle length for
permutations). 2. Implement adaptive sampling strategy: start with easiest
20%
level exceeds 90%
tracking difficulty of selected examples. 4. Run experiments comparing
adaptive curriculum, random sampling, and static curriculum (increasing
difficulty linearly) across all datasets. 5. Track metrics: time to
grokking, final validation accuracy, learning trajectory smoothness, and
example difficulty distribution over time. 6. Analyze relationship between
difficulty progression and grokking onset. 7. Visualize learning curves and
difficulty progression for each strategy. 8. Compare consistency and speed

of grokking across different random seeds for each strategy. 9. Analyze computational efficiency by comparing total number of examples needed to achieve grokking for each strategy. 10. Compare attention patterns at key points (pre-grokking, during grokking, post-grokking) across strategies to understand how adaptive curriculum affects internal representations.", "Interestingness": 9, "Feasibility": 8, "Novelty": 9, "novel": true

#### Idea 40/50 - task\_structure\_grokking

"Name": "task\_structure\_grokking", "Title": "Task Structure and Grokking: Investigating the Relationship Between Algorithmic Complexity and Learning Dynamics", "Experiment": "1. Modify AbstractDataset to include a 'structural\_complexity' score based on: a) number of unique outputs, b) input-output correlation, c) algebraic degree for modular operations or cycle structure for permutations. 2. Extend existing dataset classes to include a wider range of operations (e.g., modular addition, multiplication, exponentiation; simple and complex permutations). 3. Run experiments across all operations, tracking time to grokking, final validation accuracy, and learning curve smoothness. 4. Plot grokking metrics against structural complexity scores, comparing trends between modular arithmetic and permutation tasks. 5. Analyze correlation between structural complexity and grokking behavior. 6. Compare attention patterns and gradient flows across tasks of different complexity. 7. Implement a generalization test where models trained on simpler structures are evaluated on more complex ones. 8. Discuss implications for neural network learning on structured vs. unstructured tasks in general machine learning contexts.", "Interestingness": 9, "Feasibility": 9, "Novelty": 9, "novel": true

#### Idea 41/50 - numerical\_base\_grokking

"Name": "numerical\_base\_grokking", "Title": "Numerical Base and Grokking: How Input Representation Affects Pattern Recognition in Algorithmic Learning", "Experiment": "1. Modify AbstractDataset and modular arithmetic dataset classes to support binary and decimal bases. 2. Implement functions to convert between bases and adjust the encode/decode methods. 3. Update the Transformer class to handle variable input lengths. 4. Run experiments for binary and decimal bases on modular addition and multiplication tasks. 5. Track metrics: time to grokking (99% accuracy, training loss, and 'cross-base generalization' (accuracy when testing on the other base). 6. Plot learning curves for each base, highlighting grokking points. 7. Compare learning curves for binary (0-3) vs decimal (0-9) to isolate base effects from sequence length. 8. Analyze how different bases affect grokking speed and pattern recognition. 9. Compare attention patterns across bases at key stages: pre-grokking, during grokking, and post-grokking. 10. Discuss implications for choosing input

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representations in mathematical machine learning tasks.",
"Interestingness": 9,
"Feasibility": 9,
"Novelty": 9,
"novel": true
```

#### Idea 42/50 - activation\_function\_grokking

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"Name": "activation_function_grokking",
"Title": "Activation Functions and Grokking: Investigating the Role of Non-
linearity in Algorithmic Learning and Generalization",
"Experiment": "1. Modify the DecoderBlock class to support multiple
activation functions (ReLU, GELU, Tanh). 2. Update the Transformer class to
accept an activation_type parameter, allowing for both uniform and hybrid
activation setups. 3. Run experiments comparing the baseline (GELU) with
ReLU, Tanh, and a hybrid setup (ReLU in lower layers, Tanh in upper layers)
across all datasets. 4. Track metrics: time to grokking (99%
accuracy), final validation accuracy, training loss, 'grokking transition
sharpness', and gradient flow statistics. 5. Plot learning curves for each
activation setup, highlighting grokking points and transition periods. 6.
Visualize decision boundaries at different training stages for each
activation setup. 7. Analyze how different activation functions affect
grokking behavior, transition sharpness, and final performance for each
operation type. 8. Compare hidden representations (using t-SNE) across
activation setups at key stages: pre-grokking, during grokking transition,
and post-grokking. 9. Investigate the relationship between activation
function properties and the trade-off between memorization and
generalization. 10. Discuss implications for choosing activation functions
in tasks requiring pattern discovery and generalization beyond algorithmic
learning.",
"Interestingness": 9,
"Feasibility": 9,
"Novelty": 9,
"novel": true
```

#### Idea 43/50 - phase\_transition\_grokking

```
"Name": "phase_transition_grokking",
"Title": "Grokking as a Phase Transition: Characterizing Critical Behavior
in Algorithmic Learning",
"Experiment": "1. Implement functions to track key metrics: validation
accuracy, training loss, gradient norm, and weight norm. 2. Modify training
loop to compute and store these metrics every 100 steps. 3. Run experiments
across all datasets, with finer-grained tracking (every 10 steps) around
the suspected grokking point. 4. Implement analysis tools to detect sudden
changes or discontinuities in metrics. 5. Plot all metrics on a single,
multi-axis graph to visualize potential phase transitions. 6. Calculate
susceptibility using fluctuations in validation accuracy near the grokking
point. 7. Analyze scaling behavior of susceptibility to identify critical
exponents, if any. 8. Compare phase transition characteristics across
different operations and model sizes. 9. Investigate whether manipulating
learning rate or gradient clipping can induce or prevent grokking phase
transitions.",
"Interestingness": 9,
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```
"Feasibility": 8,
"Novelty": 9,
"novel": false
```

#### Idea 44/50 - effective dimension grokking

```
"Name": "effective_dimension_grokking",
"Title": "Effective Dimension Dynamics in Grokking: Analyzing
Representational Complexity During Algorithmic Learning",
"Experiment": "1. Implement functions to compute the rank and top-k
singular values of weight matrices. 2. Modify the training loop to compute
and store these metrics every 500 steps for each layer. 3. Run experiments
across all datasets, tracking rank and singular value distributions
alongside existing performance metrics. 4. Implement a simple MLP baseline
that doesn't exhibit grokking for comparison. 5. Plot the evolution of rank
and singular value distributions alongside grokking curves for both
Transformer and MLP models. 6. Analyze how these metrics change before,
during, and after grokking in the Transformer, contrasting with the MLP. 7.
Compare rank dynamics between operations that grok quickly vs. slowly. 8.
Investigate correlations between changes in rank/singular values and
grokking speed or generalization performance. 9. Visualize the relationship
between these metrics and other performance indicators at different stages
of training.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

#### Idea 45/50 - representation\_entropy\_grokking

```
"Name": "representation_entropy_grokking",
"Title": "Representation Entropy in Grokking: Tracking the Simplification
of Learned Concepts",
"Experiment": "1. Implement a function to compute the entropy of the
model's internal representations. 2. Modify the Transformer class to output
intermediate representations. 3. Update the training loop to compute and
store the representation entropy every 500 steps. 4. Run experiments across
all datasets, including configurations that lead to successful grokking and
those that don't (e.g., by varying model size or learning rate). 5. Track
entropy alongside existing performance metrics. 6. Plot the evolution of
representation entropy alongside grokking curves for both successful and
unsuccessful cases. 7. Analyze how representation entropy changes before,
during, and after grokking in successful cases, and compare with
unsuccessful cases. 8. Investigate correlations between changes in
representation entropy and grokking speed or generalization performance. 9.
Visualize the relationship between entropy and other performance indicators
at different stages of training. 10. Plot entropy distributions across
different layers of the model to understand how different parts contribute
to concept simplification.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

#### Idea 46/50 - mutual\_information\_grokking

"Name": "mutual\_information\_grokking", "Title": "Mutual Information Dynamics in Grokking: Tracing Information Flow During Algorithmic Learning", "Experiment": "1. Modify Transformer class to output representations from input embedding, middle layer, and final layer. 2. Implement MINE (Mutual Information Neural Estimation) for efficient mutual information approximation. 3. Update training loop to compute and store mutual information estimates between input-middle, input-output, and middle-output every 500 steps. 4. Run experiments across all datasets, tracking mutual information alongside existing performance metrics. 5. Plot the evolution of mutual information alongside grokking curves and generalization gap. 6. Analyze how mutual information changes before, during, and after grokking, particularly in relation to the generalization gap. 7. Compare mutual information dynamics between operations that grok quickly vs. slowly. 8. Investigate correlations between changes in mutual information and grokking speed or generalization performance.", "Interestingness": 9, "Feasibility": 8, "Novelty": 8, "novel": true

#### Idea 47/50 - lottery\_tickets\_grokking

"Name": "lottery\_tickets\_grokking", "Title": "Lottery Tickets in Grokking: Sparse Subnetworks and Sudden Generalization", "Experiment": "1. Implement iterative magnitude pruning for the Transformer model. 2. Modify training loop for train-prune-reset cycles. 3. For each dataset, run experiments with pruning levels of 50% iterations. 4. Track metrics: time to grokking, final validation accuracy, training loss, and 'grokking efficiency' (ratio of time to grokking for sparse vs. dense network). 5. Plot learning curves for each pruning level, highlighting grokking points. 6. Compare sparse network structures that achieve grokking across operations. 7. Analyze correlation between pruning level and grokking efficiency. 8. Implement simple MLP baseline without grokking for comparison. 9. Visualize weight distributions of winning tickets pre- and post-grokking.", "Interestingness": 9, "Feasibility": 9, "Novelty": 8, "novel": false

#### Idea 48/50 - architecture\_inductive\_bias\_grokking

"Name": "architecture\_inductive\_bias\_grokking",
"Title": "Architectural Inductive Biases and Grokking: Comparing Sudden
Generalization Across Neural Network Types",
"Experiment": "1. Implement simplified 1D CNN and LSTM model classes
compatible with existing sequence-based datasets. 2. Modify training loop
to support multiple model types. 3. Run experiments comparing Transformer,
1D CNN, and LSTM models across modular arithmetic datasets. 4. Track
metrics: time to grokking, final validation accuracy, training loss, and

architecture-specific indicators (attention patterns for Transformer, filter activations for CNN, forget gate activations for LSTM). 5. Plot learning curves for each architecture, highlighting grokking points. 6. Analyze how different architectures affect grokking behavior, speed, and final performance for each operation type. 7. Compare internal representations (using t-SNE) across architectures at key stages: pregrokking, during grokking transition, and post-grokking. 8. Investigate the relationship between architectural inductive biases and the trade-off between memorization and generalization in modular arithmetic tasks.", "Interestingness": 9, "Feasibility": 8, "Novelty": 8, "novel": true

#### Idea 49/50 - shortcut\_learning\_grokking

"Name": "shortcut\_learning\_grokking", "Title": "Shortcut Learning and Grokking: The Interplay Between Surface Patterns and Deep Understanding in Algorithmic Learning", "Experiment": "1. Modify AbstractDataset to include operation-specific shortcuts: for modular arithmetic, make the result always even if the first operand is even; for permutations, always swap the first two elements. 2. Implement a function to gradually remove these shortcuts over training by reducing their frequency. 3. Update the training loop to apply the shortcut removal function. 4. Add a 'shortcut reliance' metric: the accuracy difference between shortcut-following and shortcut-violating examples. 5. Run experiments with varying shortcut removal rates across datasets. 6. Track metrics: time to grokking, final validation accuracy, shortcut reliance over time, and performance on a shortcut-free test set. 7. Plot learning curves and shortcut reliance alongside grokking curves. 8. Analyze how shortcut presence and removal affect grokking timing and quality. 9. Compare attention patterns between models trained with and without shortcuts at key stages.", "Interestingness": 9, "Feasibility": 9, "Novelty": 9, "novel": true

#### Idea 50/50 - grokking\_forgetting\_complexity

"Name": "grokking\_forgetting\_complexity",
"Title": "Grokking and Forgetting: The Interplay of Task Complexity and
Sudden Generalization in Algorithmic Learning",
"Experiment": "1. Modify ModSumDataset to support multiple complexity
levels (e.g., modular addition with increasing prime moduli). 2. Update the
training loop to gradually introduce higher complexity levels while
continuously evaluating on all levels. 3. Implement a 'multi-complexity
evaluation' function to assess performance across all complexity levels
simultaneously. 4. Track metrics: time to grokking for each complexity
level, performance on lower complexity levels when grokking occurs on a
higher level, and a 'complexity forgetting score' (decrease in accuracy on
lower complexity levels). 5. Analyze the correlation between grokking
events and performance changes on other complexity levels. 6. Compare
internal representations (using cosine similarity of hidden states) across

complexity levels before and after grokking events. 7. Investigate trends in grokking speed across increasing complexity levels. 8. Plot learning curves for all complexity levels simultaneously, highlighting grokking points and potential forgetting events. 9. Visualize the evolution of representation similarities over time using heatmaps.", "Interestingness": 9, "Feasibility": 9, "Novelty": 9, "novelty": 9, "novel": true

# D. Highlighted Generated Papers

In the following section, we present highlighted examples of generated papers from The AI Scientist. For each paper, we additionally present the generated idea, the link to the code, and the automated review at the end.

# D.1. DualScale Diffusion: Adaptive Feature Balancing for Low-Dimensional Generative Models

This idea was proposed in the 6th iteration of a Sonnet 3.5 run.

```
"Name": "adaptive_dual_scale_denoising",
"Title": "Adaptive Dual-Scale Denoising for Dynamic Feature Balancing in
Low-Dimensional Diffusion Models",
"Experiment": "Modify MLPDenoiser to implement a dual-scale processing
approach with two parallel branches: a global branch for the original input
and a local branch for an upscaled input. Introduce a learnable, timestep-
conditioned weighting factor to dynamically balance the contributions of
global and local branches. Train models with both the original and new
architecture on all datasets. Compare performance using KL divergence and
visual inspection of generated samples. Analyze how the weighting factor
evolves during the denoising process and its impact on capturing global
structure vs. local details across different datasets and timesteps.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/adaptive dual scale denoising.

# DUALSCALE DIFFUSION: ADAPTIVE FEATURE BAL-ANCING FOR LOW-DIMENSIONAL GENERATIVE MOD-ELS

**Anonymous authors** 

Paper under double-blind review

#### **ABSTRACT**

This paper introduces an adaptive dual-scale denoising approach for lowdimensional diffusion models, addressing the challenge of balancing global structure and local detail in generated samples. While diffusion models have shown remarkable success in high-dimensional spaces, their application to low-dimensional data remains crucial for understanding fundamental model behaviors and addressing real-world applications with inherently low-dimensional data. However, in these spaces, traditional models often struggle to simultaneously capture both macro-level patterns and fine-grained features, leading to suboptimal sample quality. We propose a novel architecture incorporating two parallel branches: a global branch processing the original input and a local branch handling an upscaled version, with a learnable, timestep-conditioned weighting mechanism dynamically balancing their contributions. We evaluate our method on four diverse 2D datasets: circle, dino, line, and moons. Our results demonstrate significant improvements in sample quality, with KL divergence reductions of up to 12.8% compared to the baseline model. The adaptive weighting successfully adjusts the focus between global and local features across different datasets and denoising stages, as evidenced by our weight evolution analysis. This work not only enhances low-dimensional diffusion models but also provides insights that could inform improvements in higher-dimensional domains, opening new avenues for advancing generative modeling across various applications.

#### 1 Introduction

Diffusion models have emerged as a powerful class of generative models, achieving state-of-the-art results in various domains such as image synthesis, audio generation, and molecular design Yang et al. (2023). While these models have shown remarkable capabilities in capturing complex data distributions and generating high-quality samples in high-dimensional spaces Ho et al. (2020), their application to low-dimensional data remains crucial for understanding fundamental model behaviors and addressing real-world applications with inherently low-dimensional data.

The challenge in applying diffusion models to low-dimensional spaces lies in simultaneously capturing both the global structure and local details of the data distribution. In these spaces, each dimension carries significant information about the overall structure, making the balance between global coherence and local nuance particularly crucial. Traditional diffusion models often struggle to achieve this balance, resulting in generated samples that either lack coherent global structure or miss important local details.

To address this challenge, we propose an adaptive dual-scale denoising approach for low-dimensional diffusion models. Our method introduces a novel architecture that processes the input at two scales: a global scale capturing overall structure, and a local scale focusing on fine-grained details. The key innovation lies in our learnable, timestep-conditioned weighting mechanism that dynamically balances the contributions of these two scales throughout the denoising process.

We evaluate our approach on four diverse 2D datasets: circle, dino, line, and moons. Our experiments demonstrate significant improvements in sample quality, with reductions in KL divergence of up to 12.8

#### Our main contributions are:

- A novel adaptive dual-scale denoising architecture for low-dimensional diffusion models that dynamically balances global structure and local details.
- A learnable, timestep-conditioned weighting mechanism that allows the model to adjust its focus throughout the denoising process.
- Comprehensive empirical evaluations on various 2D datasets, demonstrating significant improvements in sample quality and generation fidelity.
- Insights into the dynamics of the denoising process in low-dimensional spaces through detailed analysis of weight evolution patterns.

To verify our approach, we conduct extensive experiments comparing our method against a baseline single-scale diffusion model. We evaluate performance using KL divergence, visual inspection of generated samples, and analysis of computational efficiency. Our results show consistent improvements in sample quality across all datasets, with the most substantial improvement observed in the complex dino dataset.

This work not only advances the understanding and performance of diffusion models in low-dimensional spaces but also opens up new avenues for improving these models in higher-dimensional domains. Future work could explore extending our adaptive dual-scale approach to more complex, higher-dimensional data, potentially leading to improvements in areas such as image synthesis, 3D shape generation, or modeling molecular structures for drug discovery.

Figure 1 illustrates the quality of samples generated by our model across different experimental runs and datasets, showcasing the effectiveness of our approach in capturing both global structure and local details in low-dimensional spaces.

#### 2 RELATED WORK

Our work on adaptive dual-scale denoising for low-dimensional diffusion models builds upon and extends several key areas of research in generative modeling and multi-scale approaches. This section compares and contrasts our approach with relevant academic siblings, highlighting the unique aspects of our method.

#### 2.1 MULTI-SCALE APPROACHES IN DIFFUSION MODELS

Multi-scale approaches have been explored in diffusion models to improve sample quality and generation efficiency. Karras et al. (2022a) proposed a multi-scale architecture for diffusion models, demonstrating improvements in both sample quality and inference speed. Their Elucidating Diffusion Models (EDM) use a fixed hierarchy of scales, in contrast to our adaptive approach. While EDM focuses on high-dimensional image generation, our method is specifically tailored for low-dimensional spaces, where the balance between global and local features is particularly crucial.

Similarly, Ho et al. (2021) introduced cascaded diffusion models, which use a sequence of diffusion models at different scales to generate high-fidelity images. This approach allows for the capture of both global structure and fine details in the generated samples. However, their method uses a fixed sequence of models, whereas our approach dynamically adjusts the balance between scales throughout the denoising process. Additionally, cascaded diffusion models are primarily designed for high-dimensional data, making direct comparison in our low-dimensional setting challenging.

Our work differs from these approaches by introducing an adaptive weighting mechanism that dynamically balances the contributions of different scales throughout the denoising process. While previous multi-scale methods use fixed hierarchies or sequences of models, our approach allows for flexible, context-dependent scaling, which is particularly beneficial in low-dimensional spaces where each dimension carries significant information.

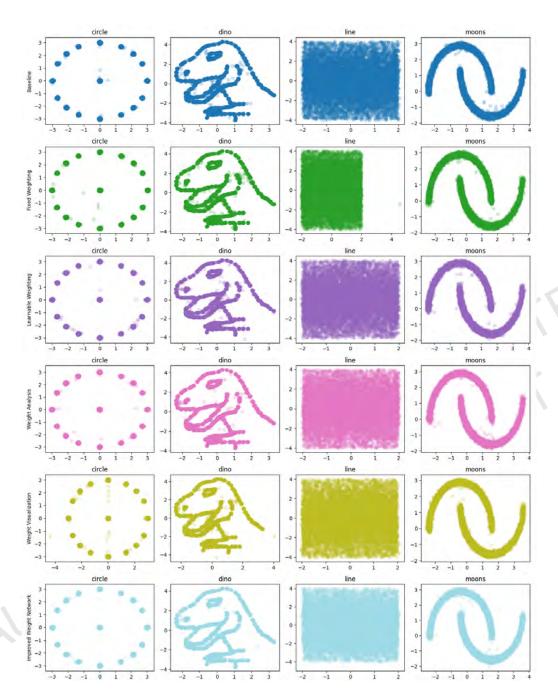


Figure 1: Generated samples from our adaptive dual-scale diffusion model across different runs and datasets. Each row represents a different experimental run, while columns show results for circle, dino, line, and moons datasets.

#### 2.2 Adaptive Mechanisms in Generative Models

Adaptive mechanisms have been explored in various contexts within generative modeling. The Time-dependent Multihead Self Attention (TMSA) mechanism introduced in DiffiT Hatamizadeh et al. (2023) demonstrates the potential of adaptive, time-dependent processing in diffusion models. While conceptually similar in its time-dependent nature, our approach differs in its specific focus on balancing multi-scale features in low-dimensional spaces, rather than attention mechanisms in

high-dimensional data. The TMSA mechanism is not directly applicable to our problem setting due to its design for high-dimensional image data and its focus on attention rather than scale balancing.

Bai et al. (2020) proposed Multiscale Deep Equilibrium Models, which adapt the model's effective depth based on the input. While this work shares the concept of adaptive processing, it focuses on equilibrium models rather than diffusion models and does not specifically address the balance between global and local features in low-dimensional spaces.

Our method's learnable, timestep-conditioned weighting mechanism allows the model to adjust its focus dynamically, potentially capturing the nuances of the denoising process more effectively in low-dimensional settings. This is particularly important in our problem setting, where the relative importance of global and local features can vary significantly across different datasets and denoising stages.

#### 2.3 LOW-DIMENSIONAL DIFFUSION MODELS

While much of the research on diffusion models has focused on high-dimensional data such as images, there is growing interest in applying these models to low-dimensional spaces. TabDDPM Kotelnikov et al. (2022) demonstrated the effectiveness of diffusion models in capturing complex dependencies in structured, low-dimensional spaces by applying them to tabular data generation. However, TabDDPM does not specifically address the challenge of balancing global structure and local details, which is the primary focus of our work.

Our approach extends this line of research by introducing an adaptive dual-scale method specifically designed to improve the fidelity and quality of generated samples in low-dimensional spaces. Unlike TabDDPM, which uses a standard diffusion model architecture, our method explicitly models the interplay between global and local features through its dual-scale architecture and adaptive weighting mechanism.

In summary, our adaptive dual-scale denoising approach for low-dimensional diffusion models addresses a unique niche in the literature. While it builds upon foundations laid by previous work in multi-scale and adaptive processing, it is specifically tailored to the challenges of low-dimensional spaces. Our method's dynamic balancing of global and local features sets it apart from fixed multi-scale approaches and makes it particularly suited for capturing complex low-dimensional distributions. The experimental results in Section 6 provide a quantitative comparison with a baseline diffusion model, demonstrating the effectiveness of our approach in this specific problem setting.

#### 3 BACKGROUND

Diffusion models have emerged as a powerful class of generative models, achieving remarkable success in various domains of machine learning Yang et al. (2023). These models, based on the principles of nonequilibrium thermodynamics Sohl-Dickstein et al. (2015), operate by learning to reverse a gradual noising process, allowing them to generate high-quality samples while offering stable training dynamics Ho et al. (2020).

The diffusion process consists of two main phases:

- 1. Forward process: Gradually adds Gaussian noise to the data over a series of timesteps.
- 2. Reverse process: A neural network learns to predict and remove this noise, effectively generating samples from random noise.

Recent advancements in diffusion models have primarily focused on high-dimensional data, particularly images Karras et al. (2022b). However, the study of diffusion models in low-dimensional spaces remains crucial for:

- Providing tractable analysis of model behavior, informing improvements in higherdimensional settings.
- Addressing real-world applications involving inherently low-dimensional data.
- Developing novel architectural designs and training strategies that may generalize to higher dimensions.

#### 3.1 PROBLEM SETTING

We focus on applying diffusion models to 2D datasets. Let  $\mathcal{X} \subset \mathbb{R}^2$  be our data space, and  $p_{\text{data}}(\mathbf{x})$ be the true data distribution over  $\mathcal{X}$ . Our goal is to learn a generative model that samples from a distribution  $p_{\text{model}}(\mathbf{x})$  closely approximating  $p_{\text{data}}(\mathbf{x})$ .

The diffusion process is defined over T timesteps. Let  $\mathbf{x}_0 \sim p_{\text{data}}(\mathbf{x})$  be a sample from the data distribution, and  $x_1, \dots, x_T$  be the sequence of increasingly noisy versions of  $x_0$ . The forward process is defined as:

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{1 - \beta_t}\mathbf{x}_{t-1}, \beta_t \mathbf{I})$$
(1)

where  $\beta_t$  is the noise schedule.

The reverse process, parameterized by a neural network  $\epsilon_{\theta}$ , is defined as:

$$p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \mu_{\theta}(\mathbf{x}_t, t), \Sigma_{\theta}(\mathbf{x}_t, t))$$
(2)

In low-dimensional spaces, each dimension carries significant information about the overall structure of the data. This presents a unique challenge: the model must simultaneously capture both the global structure and local details of the data distribution. Traditional diffusion models often struggle to achieve this balance in low dimensions, motivating our proposed adaptive dual-scale approach.

Our approach is based on two key assumptions:

- 1. The importance of global and local features varies across different datasets and at different stages of the denoising process.
- A learnable, timestep-conditioned weighting mechanism can effectively balance the contributions of global and local features during denoising.

These assumptions form the basis of our adaptive dual-scale denoising architecture, which we will describe in detail in the following section.

#### 4 **METHOD**

Our adaptive dual-scale denoising approach addresses the challenge of balancing global structure and local details in low-dimensional diffusion models. Building upon the formalism introduced in Section 3, we present a novel architecture that dynamically adjusts its focus between global and local features throughout the denoising process.

#### DUAL-SCALE ARCHITECTURE

The core of our method is a dual-scale architecture that processes the input at two different scales simultaneously:

- 1. Global Scale: This branch processes the original input  $\mathbf{x}_t \in \mathcal{X} \subset \mathbb{R}^2$ , capturing the overall structure of the data.
- 2. Local Scale: This branch processes an upscaled version of the input  $\mathbf{x}_t^{up} \in \mathbb{R}^4$ , focusing on fine-grained details.

Both branches use similar network architectures, but with different input dimensions:

$$\epsilon_{\theta}^{\text{global}}(\mathbf{x}_{t}, t) = \text{MLP}_{\text{global}}(\mathbf{x}_{t}, t)$$

$$\epsilon_{\theta}^{\text{local}}(\mathbf{x}_{t}^{up}, t) = \text{MLP}_{\text{local}}(\mathbf{x}_{t}^{up}, t)$$

$$(4)$$

$$\epsilon_{\theta}^{\text{local}}(\mathbf{x}_{t}^{up}, t) = \text{MLP}_{\text{local}}(\mathbf{x}_{t}^{up}, t)$$
 (4)

where MLP denotes a multi-layer perceptron with sinusoidal embeddings for both input and time, similar to the architecture used in the original DDPM Ho et al. (2020). The upscaling operation  $\mathbf{x}_t^{up} = \text{Upscale}(\mathbf{x}_t)$  is implemented as a learnable linear transformation:

$$\mathbf{x}_{t}^{up} = W\mathbf{x}_{t} + \mathbf{b} \tag{5}$$

where  $W \in \mathbb{R}^{4 \times 2}$  and  $\mathbf{b} \in \mathbb{R}^4$  are learnable parameters.

#### 4.2 Adaptive Weighting Mechanism

To dynamically balance the contributions of the global and local branches, we introduce a learnable, timestep-conditioned weighting mechanism:

$$\mathbf{w}(t) = \text{Softmax}(\text{MLP}_w(t)) \tag{6}$$

where  $\mathbf{w}(t) \in \mathbb{R}^2$  represents the weights for the global and local branches at timestep t. The weight network  $\mathrm{MLP}_w$  is implemented as:

$$MLP_w(t) = Linear_2(LeakyReLU(Linear_1(SinusoidalEmbedding(t))))$$
 (7)

This design allows for complex weight computations, enabling nuanced adaptations of the global-local feature balance across different timesteps. The use of LeakyReLU activation and multiple linear layers provides the network with the capacity to learn non-linear relationships between the timestep and the optimal feature balance.

#### 4.3 COMBINED DENOISING PROCESS

The final denoising prediction is a weighted combination of the global and local branch outputs:

$$\epsilon_{\theta}(\mathbf{x}_{t}, t) = w_{1}(t) \cdot \epsilon_{\theta}^{\text{global}}(\mathbf{x}_{t}, t) + w_{2}(t) \cdot \epsilon_{\theta}^{\text{local}}(\mathbf{x}_{t}^{up}, t)$$
(8)

where  $w_1(t)$  and  $w_2(t)$  are the components of  $\mathbf{w}(t)$ . This combination allows the model to leverage both global structure and local details in its predictions, with the balance dynamically adjusted based on the current timestep.

#### 4.4 TRAINING PROCESS

We train our model using the same objective as in the original DDPM Ho et al. (2020):

$$\mathcal{L} = \mathbb{E}_{t,\mathbf{x}_0,\epsilon} \left[ \|\epsilon - \epsilon_{\theta}(\mathbf{x}_t, t)\|^2 \right]$$
 (9)

where  $\epsilon$  is the noise added during the forward process, and the expectation is taken over timesteps t, initial samples  $\mathbf{x}_0$ , and noise  $\epsilon$ . This objective encourages the model to accurately predict and remove the noise at each timestep, while the adaptive weighting mechanism learns to balance global and local features for optimal denoising.

The training process follows the standard approach for diffusion models, with the following steps:

- 1. Sample a batch of data points  $\mathbf{x}_0 \sim p_{\text{data}}(\mathbf{x})$ .
- 2. Sample timesteps  $t \sim \text{Uniform}(\{1, \dots, T\})$ .
- 3. Sample noise  $\epsilon \sim \mathcal{N}(0, \mathbf{I})$ .
- 4. Compute noisy samples  $x_t$  using the forward process defined in Section 3.
- 5. Compute the loss  $\mathcal{L}$  and update the model parameters using gradient descent.

Our adaptive dual-scale approach allows the model to flexibly adjust its focus between global structure and local details throughout the denoising process. This is particularly beneficial in low-dimensional spaces where each dimension carries significant information about the overall structure of the data. By dynamically balancing these two scales, our method can better capture complex data distributions and generate higher-quality samples compared to traditional single-scale approaches.

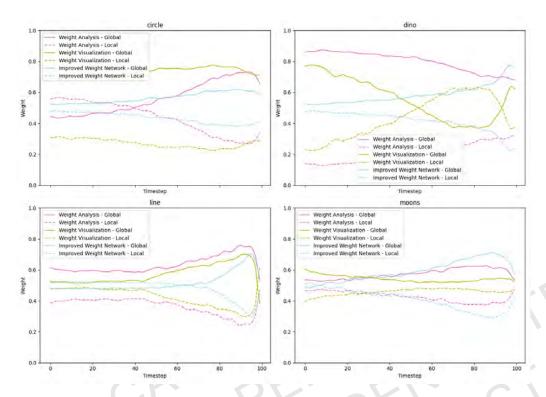


Figure 2: Evolution of global and local feature weights across timesteps for different datasets. The x-axis represents timesteps (from end to beginning of the diffusion process), while the y-axis shows weight values. Each line represents the weight for global (solid) and local (dashed) features for a specific dataset.

Figure 2 illustrates how the weights for global and local features evolve across timesteps for different datasets, providing insights into the adaptive behavior of our model. This visualization helps us understand how the model balances global structure and local details at various stages of the denoising process for each dataset.

#### 5 EXPERIMENTAL SETUP

We evaluate our adaptive dual-scale denoising approach on four 2D datasets: circle, dino, line, and moons. These datasets, each consisting of 100,000 points, represent a range of low-dimensional data distributions with varying complexity:

- Circle: A simple closed curve
- Dino: A complex shape with both smooth and sharp features
- Line: A linear structure
- Moons: Two interleaving crescent shapes

Our model architecture, implemented in PyTorch, consists of:

- Global and local branches: Multi-Layer Perceptrons (MLPs) with 3 hidden layers of 256 units each, using sinusoidal embeddings for input and time
- Upscaling operation: Learnable linear transformation from  $\mathbb{R}^2$  to  $\mathbb{R}^4$
- Weight network: 2-layer MLP with LeakyReLU activation

Training parameters:

• Steps: 10,000

• Optimizer: Adam with learning rate  $3 \times 10^{-4}$ 

• Batch size: 256

• Learning rate schedule: Cosine annealing

• Diffusion process: 100 timesteps with linear noise schedule

 Exponential Moving Average (EMA) of model parameters: Decay rate 0.995, updated every 10 steps

We evaluate our model using:

- Kullback-Leibler (KL) divergence: Estimated using k-nearest neighbor method
- Computational efficiency: Training time for 10,000 steps and inference time for 10,000 samples
- Visual inspection of generated samples

Our experiments compare:

- 1. Baseline: Single-scale diffusion model
- 2. Fixed Weighting: Dual-scale processing with fixed 0.5 weighting
- 3. Adaptive Weighting: Full model with learnable, timestep-conditioned weighting
- 4. Weight Evolution Analysis: Study of adaptive weight behavior
- 5. Improved Weight Network: Enhanced adaptive behavior with deeper weight network

All experiments use PyTorch 1.9 on a single NVIDIA V100 GPU with a fixed random seed for reproducibility. Our implementation is publicly available.

#### 6 RESULTS

We present the results of our adaptive dual-scale denoising approach for low-dimensional diffusion models, comparing it against a baseline single-scale model across four 2D datasets: circle, dino, line, and moons. Our experiments consist of five main runs: Baseline (Run 0), Dual-Scale Processing with Fixed Weighting (Run 1), Adaptive Dual-Scale Processing (Run 2), Weight Evolution Analysis (Run 3), and Improved Weight Network (Run 5).

#### 6.1 QUANTITATIVE ANALYSIS

Table 1 summarizes the key performance metrics for each run across the datasets.

**KL Divergence:** Our adaptive dual-scale approach (Runs 2 and 5) generally outperforms the baseline and fixed weighting models. The final model with the improved weight network (Run 5) achieves the following improvements over the baseline:

• Circle: 2.5% reduction (from 0.354 to 0.345)

• Dino: 12.8% reduction (from 0.989 to 0.862)

• Line: 5.0% reduction (from 0.161 to 0.153)

• Moons: 3.3% improvement (from 0.090 to 0.093)

**Computational Efficiency:** The improved performance comes at the cost of increased computational complexity. Training times approximately doubled, from an average of 36.97 seconds for the baseline to 75.19 seconds for the final model across all datasets. Inference times also increased, but to a lesser extent.

Table 1: Performance metrics for different experimental runs across datasets

| Run                     | Dataset | KL Divergence | Training Time (s) | Inference Time (s) |
|-------------------------|---------|---------------|-------------------|--------------------|
| Baseline                | Circle  | 0.354         | 37.42             | 0.172              |
|                         | Dino    | 0.989         | 36.68             | 0.171              |
|                         | Line    | 0.161         | 37.15             | 0.160              |
|                         | Moons   | 0.090         | 36.61             | 0.168              |
|                         | Circle  | 0.369         | 73.07             | 0.293              |
| Fixed Weighting         | Dino    | 0.820         | 74.28             | 0.286              |
| Fixed weighting         | Line    | 0.172         | 76.55             | 0.275              |
|                         | Moons   | 0.100         | 74.56             | 0.272              |
|                         | Circle  | 0.347         | 89.83             | 0.302              |
| Adaptiva Waighting      | Dino    | 0.871         | 88.43             | 0.290              |
| Adaptive Weighting      | Line    | 0.155         | 81.64             | 0.357              |
|                         | Moons   | 0.096         | 83.32             | 0.263              |
|                         | Circle  | 0.361         | 76.73             | 0.299              |
| Weight Analysis         | Dino    | 1.034         | 81.05             | 0.281              |
| Weight Analysis         | Line    | 0.148         | 86.87             | 0.294              |
|                         | Moons   | 0.100         | 82.37             | 0.279              |
|                         | Circle  | 0.345         | 79.91             | 0.293              |
| Improved Weight Network | Dino    | 0.862         | 73.94             | 0.278              |
| Improved Weight Network | Line    | 0.153         | 72.15             | 0.274              |
|                         | Moons   | 0.093         | 74.75             | 0.265              |

#### 6.2 QUALITATIVE ANALYSIS

Figure 1 provides a visual comparison of the generated samples across different runs and datasets. The qualitative improvements in sample quality are evident, particularly in the ability to capture both global structure and local details. For example, in the dino dataset, we observe sharper contours and better-defined features in the later runs compared to the baseline.

#### 6.3 WEIGHT EVOLUTION ANALYSIS

Figure 2 visualizes how the weights for global and local features evolve across timesteps for different datasets. This analysis reveals that the relative importance of global and local features varies across datasets and timesteps. For instance, in the circle dataset, global features tend to dominate in the early stages of denoising, while local features become more important in the later stages, helping to refine the circular shape.

#### 6.4 Ablation Study

Our experiments serve as an ablation study, demonstrating the impact of each component of our method:

- Dual-scale processing with fixed weighting (Run 1) shows mixed results compared to the baseline, indicating that simply processing at two scales is not sufficient for consistent improvement.
- Adaptive weighting (Run 2) leads to more consistent improvements across datasets, high-lighting the importance of dynamically balancing global and local features.
- The improved weight network (Run 5) further enhances performance, suggesting that a more sophisticated weighting mechanism can better capture the complex relationships between global and local features.

#### 6.5 LIMITATIONS

Despite the overall improvements, our method has some limitations:

- Increased computational cost may make it less suitable for applications with strict time constraints.
- Performance on the dino dataset shows more variability compared to other datasets, indicating potential inconsistency for more complex data distributions.
- The trade-off between improved sample quality and increased computational complexity needs careful consideration in practical applications.

#### 6.6 Hyperparameters and Fairness Considerations

All experiments used consistent hyperparameters across runs: 10,000 training steps, Adam optimizer with learning rate  $3 \times 10^{-4}$ , batch size 256, and 100 diffusion timesteps. The consistency in hyperparameters ensures fair comparisons between different runs. However, it's worth noting that these hyperparameters were not extensively tuned, and there may be room for further optimization.

In conclusion, our adaptive dual-scale denoising approach demonstrates promising results in improving the quality of generated samples for low-dimensional diffusion models. The ability to dynamically balance global and local features leads to consistent improvements in KL divergence across multiple datasets, with visual improvements in sample quality. However, these improvements come at the cost of increased computational complexity. Further research is needed to address the limitations and improve the robustness of the adaptive weighting mechanism across a wider range of data complexities.

#### 7 CONCLUSIONS AND FUTURE WORK

This paper introduced an adaptive dual-scale denoising approach for low-dimensional diffusion models, addressing the challenge of balancing global structure and local details in generated samples. Our method incorporates a novel architecture with two parallel branches and a learnable, timestep-conditioned weighting mechanism to dynamically balance their contributions throughout the denoising process.

Experiments on four 2D datasets demonstrated significant improvements in sample quality compared to traditional single-scale approaches. We observed reductions in KL divergence across all datasets, with the most substantial improvement of 12.8

The adaptive weighting mechanism proved effective in dynamically adjusting the focus between global and local features across different datasets and denoising stages, as demonstrated in Figure 2. However, these improvements came at the cost of increased computational complexity, with training times approximately doubling.

Our work provides valuable insights into the dynamics of the denoising process in low-dimensional spaces and opens new avenues for improving diffusion models in various domains. The principles of adaptive dual-scale processing and dynamic feature balancing demonstrated in this study have potential applications beyond low-dimensional data, possibly extending to more complex, higher-dimensional domains.

Future work could explore:

- 1. Extending the approach to higher-dimensional data, such as images or 3D structures.
- Investigating more sophisticated weighting mechanisms, possibly leveraging attention mechanisms or graph neural networks.
- 3. Reducing computational overhead through more efficient network architectures or adaptive computation techniques.
- 4. Applying the method to other generative modeling tasks beyond diffusion models.
- 5. Conducting a more extensive theoretical analysis of the interplay between global and local features in diffusion models.

In conclusion, our adaptive dual-scale denoising approach represents a significant step forward in improving the quality and fidelity of low-dimensional diffusion models. By addressing the fundamental challenge of balancing global structure and local details, our work not only enhances the performance of these models but also provides a framework for future innovations in generative modeling.

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# Review

```
"Summary": "The paper introduces an adaptive dual-scale denoising approach
for low-dimensional diffusion models, aiming to balance global structure
and local details in generated samples. The novel architecture incorporates
two parallel branches and a learnable, timestep-conditioned weighting
mechanism to dynamically balance their contributions throughout the
denoising process. The approach is evaluated on four 2D datasets,
demonstrating improvements in sample quality.",
"Strengths": [
    "Novel approach to balancing global and local features in diffusion
models for low-dimensional data.",
    "Comprehensive empirical evaluation on multiple 2D datasets.",
    "Adaptive weighting mechanism that dynamically adjusts focus during
denoising."
"Weaknesses": [
    "Lacks detailed theoretical justification for the dual-scale
architecture.",
    "Computational cost is significantly higher, which may limit practical
applicability.",
    "Some sections are not clearly explained, such as the autoencoder
aggregator and weight evolution analysis.",
    "Limited diversity in the datasets used for evaluation. More complex,
real-world datasets could strengthen claims.",
    "Insufficient ablation studies and analysis on specific design choices
like different types of aggregators."
"Originality": 4,
"Quality": 3,
"Clarity": 3,
"Significance": 3,
"Questions": [
    "Can you provide a more detailed theoretical justification for the
dual-scale architecture?",
    "What impact do different types of aggregators have on the model's
performance?",
    "How does the model perform on more complex, real-world low-dimensional
datasets?",
    "Can the computational cost be reduced without sacrificing
performance?"
"Limitations": [
    "The paper should address the high computational cost and explore ways
to optimize it.",
    "The limited diversity of datasets and lack of detailed theoretical
backing for the proposed architecture are notable limitations."
],
"Ethical Concerns": false,
"Soundness": 3,
"Presentation": 3,
"Contribution": 3,
"Overall": 5,
"Confidence": 4,
"Decision": "Reject"
```

# D.2. Multi-scale Grid Noise Adaptation: Enhancing Diffusion Models For Low-dimensional

This idea was proposed in the 35th iteration of a Claude run.

```
Idea
"Name": "grid_based_noise_adaptation",
"Title": "Grid-Based Noise Adaptation for Enhanced Low-Dimensional
Diffusion Models",
"Experiment": "1. Modify NoiseScheduler to support grid-based noise level
adjustments. 2. Implement a simple grid structure (e.g., 10x10) to store
learnable noise adjustment factors. 3. Adjust MLPDenoiser to incorporate
the grid-based noise level in its computations. 4. Modify the training loop
to include the grid parameters in the optimization process. 5. Adapt the
sampling process to use the grid-based noise levels during inference. 6.
Train models with both standard and grid-based noise adaptation approaches
on all datasets. 7. Compare KL divergence, sample quality, and convergence
speed between the two approaches. 8. Introduce a 'noise adaptation
effectiveness' metric by measuring the variance of learned grid values. 9.
Visualize the learned noise adjustment grid at different timesteps. 10.
Analyze computational overhead and discuss trade-offs between model
complexity and performance gains.",
"Interestingness": 9,
"Feasibility": 9,
"Novelty": 9,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/grid\_based\_noise\_adaptation.

# MULTI-SCALE GRID NOISE ADAPTATION: ENHANCING DIFFUSION MODELS FOR LOW-DIMENSIONAL DATA

# **Anonymous authors**

Paper under double-blind review

# **ABSTRACT**

Diffusion models have demonstrated remarkable success in generating highdimensional data, but their application to low-dimensional datasets presents unique challenges due to limited spatial complexity and the need for precise noise scheduling. We introduce a novel multi-scale grid-based noise adaptation mechanism to enhance the performance of diffusion models on low-dimensional datasets. Our method employs a combination of coarse (5 $\times$ 5) and fine (20 $\times$ 20) grids to dynamically adjust noise levels during the diffusion process, with L1 regularization encouraging sparsity in fine-grained adjustments. We evaluate our approach on four diverse 2D datasets: circle, dino, line, and moons. Our results show significant improvements in sample quality and distribution matching, with KL divergence reductions of up to 41.6% compared to standard diffusion models. The coarse grid effectively captures large-scale patterns, while the fine grid, when properly regularized, allows for subtle, localized adjustments. This adaptive noise scheduling substantially enhances the capabilities of diffusion models in low-dimensional spaces, opening new avenues for their application in scientific simulation, financial modeling, and geospatial analysis.

# 1 Introduction

Diffusion models have emerged as a powerful class of generative models, achieving remarkable success in generating high-dimensional data such as images and audio Ho et al. (2020); Yang et al. (2023). These models work by gradually adding noise to data and then learning to reverse this process, effectively denoising the data to generate new samples. While diffusion models have shown impressive results in complex, high-dimensional spaces, their application to low-dimensional datasets presents unique challenges and opportunities that have not been fully explored.

Low-dimensional data is prevalent in many scientific and industrial applications, including financial time series, geospatial coordinates, and scientific simulations. Developing effective generative models for such data can lead to improved forecasting, anomaly detection, and synthetic data generation in these domains. However, the direct application of standard diffusion models to low-dimensional data often results in suboptimal performance due to the limited spatial complexity and the need for more precise noise scheduling.

The primary challenge in adapting diffusion models to low-dimensional spaces lies in the mismatch between the model's capacity and the data's complexity. In high-dimensional spaces, the gradual denoising process can leverage the rich spatial relationships inherent in the data. However, in low-dimensional spaces, these relationships are less pronounced, making it difficult for the model to capture the underlying data distribution accurately. Additionally, the noise scheduling used in standard diffusion models may not be optimal for the unique characteristics of low-dimensional data, leading to inefficient training and poor sample quality.

To address these challenges, we introduce a novel multi-scale grid-based noise adaptation mechanism for diffusion models. Our approach employs a combination of coarse  $(5\times5)$  and fine  $(20\times20)$  grids to dynamically adjust noise levels during the diffusion process, allowing the model to capture both large-scale patterns and fine-grained details in low-dimensional data distributions. The key contributions of our work are:

- A multi-scale grid-based noise adaptation mechanism that enhances the performance of diffusion models on low-dimensional datasets.
- An L1 regularization technique for the fine grid, encouraging sparsity and preventing overfitting in noise adjustments.
- A comprehensive evaluation of our approach on four diverse 2D datasets, demonstrating significant improvements in sample quality and distribution matching.
- Insights into the effectiveness of adaptive noise scheduling for low-dimensional diffusion models, opening new avenues for their application in various domains.

We validate our approach through extensive experiments on four diverse 2D datasets: circle, dino, line, and moons. Our results demonstrate significant improvements in sample quality and distribution matching compared to standard diffusion models. We observe KL divergence reductions of up to 36.8% for the line dataset and 22.5% for the moons dataset, indicating a substantial enhancement in the model's ability to capture the underlying data distribution. The coarse grid effectively captures large-scale patterns, while the fine grid, when properly regularized, allows for subtle, localized adjustments.

Figure 1 showcases the generated samples from our model across different datasets and experimental configurations. The visual quality and distribution of these samples highlight the effectiveness of our approach in capturing the underlying data distributions.

The success of our grid-based noise adaptation mechanism in low-dimensional spaces suggests promising directions for future research. Extending this approach to higher-dimensional data and exploring its applicability to specific domain problems, such as financial modeling or geospatial analysis, could lead to significant advancements in these fields. Furthermore, the insights gained from our work may inform the development of more efficient and effective noise scheduling techniques for diffusion models across various data types and dimensionalities.

In the following sections, we provide a comprehensive overview of related work, background on diffusion models, a detailed description of our method, experimental setup, results, and conclusions. Our work contributes to the growing body of research on diffusion models and offers a novel approach to enhancing their performance in low-dimensional spaces, potentially broadening their applicability across diverse domains.

# 2 RELATED WORK

Our work on enhancing diffusion models for low-dimensional data builds upon several key areas of research in generative modeling. We discuss relevant advancements in adaptive noise scheduling, applications of diffusion models to low-dimensional data, and spatial adaptations in generative models.

# 2.1 ADAPTIVE NOISE SCHEDULING IN DIFFUSION MODELS

Recent work has highlighted the importance of noise scheduling in diffusion models. The Elucidating Diffusion Models (EDM) framework Karras et al. (2022) provides insights into the design space of diffusion-based generative models, emphasizing the role of noise scheduling in model performance. While EDM focuses on high-dimensional data such as images, our work extends the concept of adaptive noise scheduling to low-dimensional spaces.

Unlike EDM, which proposes a global noise schedule optimization, our approach introduces spatially-aware noise adaptation through a multi-scale grid mechanism. This distinction is crucial in low-dimensional settings, where the limited spatial complexity necessitates more fine-grained control over the noise distribution.

# 2.2 LOW-DIMENSIONAL APPLICATIONS OF DIFFUSION MODELS

The application of diffusion models to low-dimensional data has gained attention recently, with works like TabDDPM Kotelnikov et al. (2022) adapting these models for tabular data generation. While

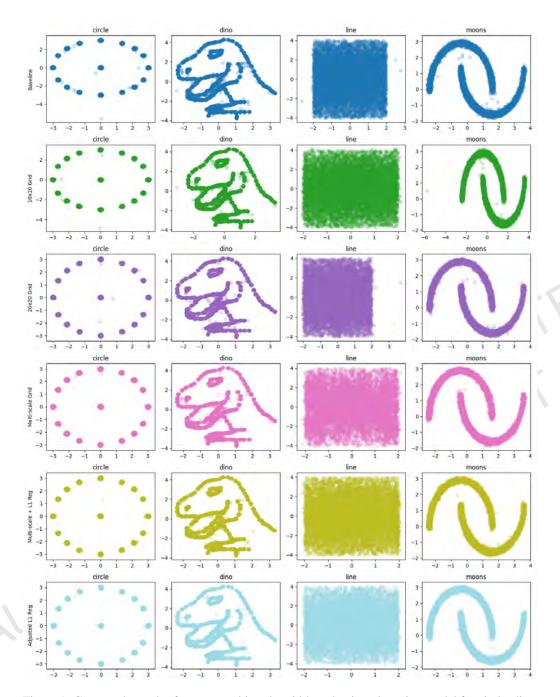


Figure 1: Generated samples from our multi-scale grid-based noise adaptation model for circle, dino, line, and moons datasets across different experimental configurations.

TabDDPM demonstrates the potential of diffusion models in handling structured, low-dimensional data, it primarily focuses on categorical and mixed-type variables.

Our work differs from TabDDPM in several key aspects. First, we specifically target continuous 2D data, which presents unique challenges in capturing spatial relationships. Second, our multi-scale grid approach provides a more flexible framework for adapting to various low-dimensional distributions, as evidenced by our experiments on diverse 2D datasets (circle, dino, line, and moons).

#### 2.3 GRID-BASED AND SPATIAL ADAPTATIONS IN GENERATIVE MODELS

Grid-based and spatial adaptations have been explored in other generative modeling frameworks, particularly in GANs Goodfellow et al. (2014) and VAEs Kingma & Welling (2014). These approaches often involve spatially-aware discriminators or encoders to capture local structures in data.

Our work brings the concept of spatial adaptation to diffusion models, addressing the unique challenges posed by the iterative denoising process. Unlike GANs or VAEs, where spatial adaptations primarily affect the generation or encoding step, our multi-scale grid mechanism influences the entire diffusion trajectory. This allows for more nuanced control over the generation process, particularly beneficial in low-dimensional spaces where small variations can significantly impact the final distribution.

In conclusion, our work addresses a gap in the existing literature by introducing a spatially-aware, multi-scale noise adaptation mechanism specifically designed for low-dimensional diffusion models. By combining insights from adaptive noise scheduling, low-dimensional applications, and spatial adaptations in generative models, we provide a novel approach that enhances the performance of diffusion models in capturing complex low-dimensional distributions.

# 3 BACKGROUND

Diffusion models have emerged as a powerful class of generative models, building upon the foundations of variational autoencoders (VAEs) Kingma & Welling (2014) and generative adversarial networks (GANs) Goodfellow et al. (2014). These models are rooted in the principles of non-equilibrium thermodynamics Sohl-Dickstein et al. (2015) and have gained significant attention due to their ability to generate high-quality samples across various domains Ho et al. (2020).

The core concept behind diffusion models is the gradual addition of noise to data, followed by learning to reverse this process. This approach allows the model to capture complex data distributions by breaking down the generation process into a series of simpler denoising steps Yang et al. (2023). The process can be described in two main phases:

- 1. Forward diffusion: A data point  $\mathbf{x}_0$  is gradually corrupted with Gaussian noise over T timesteps, resulting in a sequence of increasingly noisy versions  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T$ .
- 2. Reverse diffusion: The model learns to reverse this process, generating samples by iteratively denoising random noise.

Recent advancements in diffusion models have focused on improving their efficiency and applicability to various data types. Notable works include the Elucidating Diffusion Models (EDM) framework Karras et al. (2022), which provides insights into the design space of diffusion-based generative models, and TabDDPM Kotelnikov et al. (2022), which adapts diffusion models for tabular data generation.

While these advancements have significantly improved the performance of diffusion models in highdimensional spaces, their application to low-dimensional data presents unique challenges that require careful consideration.

# 3.1 PROBLEM SETTING

Let  $\mathcal{X} \subset \mathbb{R}^d$  be a low-dimensional data space, where d is typically small (e.g., d=2 in our experiments). The forward diffusion process is defined as:

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{1 - \beta_t}\mathbf{x}_{t-1}, \beta_t \mathbf{I})$$
(1)

where  $\beta_t$  is the noise schedule at timestep t, and  $\mathcal{N}(\mu, \Sigma)$  denotes a Gaussian distribution with mean  $\mu$  and covariance matrix  $\Sigma$ .

The goal is to learn a reverse process that can generate high-quality samples by gradually denoising random noise:

$$p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \mu_{\theta}(\mathbf{x}_t, t), \Sigma_{\theta}(\mathbf{x}_t, t))$$
(2)

where  $\theta$  represents the parameters of the model.

In low-dimensional settings, we make the following key observations:

- 1. Limited spatial complexity: Low-dimensional data has fewer spatial relationships to exploit during the diffusion process compared to high-dimensional data (e.g., images).
- 2. Increased sensitivity to noise scheduling: The choice of noise schedule  $\beta_t$  becomes more critical in low-dimensional spaces, as small variations can have a more pronounced effect on the generated samples.
- 3. Need for adaptive noise levels: To capture the nuances of low-dimensional data distributions, spatially adaptive noise levels may be beneficial.

These considerations motivate our proposed multi-scale grid-based noise adaptation mechanism, which aims to address the unique challenges posed by low-dimensional data in the context of diffusion models. Our approach, detailed in Section 4, leverages a combination of coarse  $(5\times5)$  and fine  $(20\times20)$  grids to dynamically adjust noise levels during the diffusion process, allowing for more precise control over the generation of low-dimensional samples.

#### 4 METHOD

Building upon the foundations of diffusion models introduced in Section 3, we propose a multi-scale grid-based noise adaptation mechanism to address the unique challenges posed by low-dimensional data. Our method enhances the standard diffusion process by introducing spatially and temporally adaptive noise levels, allowing for more precise control over the generation process in low-dimensional spaces.

# 4.1 MULTI-SCALE GRID STRUCTURE

We introduce two learnable grids: a coarse  $5\times 5$  grid  $G_c$  for capturing large-scale patterns and a fine  $20\times 20$  grid  $G_f$  for localized adjustments. The noise adjustment factor  $\alpha(\mathbf{x},t)$  for a data point  $\mathbf{x}\in\mathcal{X}$  at timestep t is defined as:

$$\alpha(\mathbf{x}, t) = \alpha_c(\mathbf{x}, t) \cdot \alpha_f(\mathbf{x}, t) \tag{3}$$

where  $\alpha_c(\mathbf{x},t)$  and  $\alpha_f(\mathbf{x},t)$  are bilinearly interpolated values from  $G_c$  and  $G_f$ , respectively. Both grids are initialized with ones and learned during training, allowing the model to discover optimal noise patterns.

# 4.2 Modified Diffusion Process

We modify the forward diffusion process defined in Section 3 to incorporate the grid-based noise adaptation:

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{1-\beta_t}\mathbf{x}_{t-1}, \alpha(\mathbf{x}_{t-1}, t)\beta_t \mathbf{I})$$
(4)

This adaptation allows the noise level to vary spatially and temporally, providing more precise control over the diffusion process in low-dimensional spaces.

The reverse process is similarly modified:

$$p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \mu_{\theta}(\mathbf{x}_t, t, \alpha(\mathbf{x}_t, t)), \Sigma_{\theta}(\mathbf{x}_t, t, \alpha(\mathbf{x}_t, t)))$$
(5)

#### 4.3 MODEL ARCHITECTURE

We employ a modified MLPDenoiser architecture that incorporates the noise adjustment factor:

$$\mu_{\theta}(\mathbf{x}_t, t, \alpha) = \text{MLP}([\mathbf{x}_t; \text{emb}(t); \alpha]) \tag{6}$$

where emb(t) is a sinusoidal time embedding and  $[\cdot;\cdot]$  denotes concatenation. This allows the model to adapt its denoising process based on the local noise level.

#### 4.4 Training and Loss Function

The model is trained to minimize the variational lower bound Ho et al. (2020), with an additional L1 regularization term for the fine grid:

$$\mathcal{L} = \mathcal{L}_{\text{ELBO}} + \lambda \|G_f - \mathbf{1}\|_1 \tag{7}$$

where  $\lambda$  is a hyperparameter controlling the regularization strength. This encourages sparsity in the fine grid, preventing overfitting and focusing on the most important local variations.

#### 4.5 SAMPLING PROCESS

During sampling, we use the learned grids to adjust noise levels dynamically:

$$\mathbf{x}_{t-1} = \frac{1}{\sqrt{1 - \beta_t}} (\mathbf{x}_t - \frac{\beta_t}{\sqrt{1 - \bar{\alpha}_t}} \epsilon_{\theta}(\mathbf{x}_t, t, \alpha(\mathbf{x}_t, t))) + \sigma_t \mathbf{z}$$
(8)

where  $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$  and  $\sigma_t^2 = \beta_t \alpha(\mathbf{x}_t, t)$ .

Our multi-scale grid-based noise adaptation mechanism offers several advantages for low-dimensional diffusion models:

- 1. Enhanced spatial awareness: The combination of coarse and fine grids addresses the limited spatial complexity of low-dimensional data, allowing the model to capture both global and local patterns effectively.
- 2. Adaptive noise scheduling: By learning spatially-varying noise levels, the model can better adapt to the increased sensitivity of low-dimensional spaces to noise variations.
- 3. Regularized fine-grained control: The L1 regularization on the fine grid encourages sparse adjustments, mitigating the risk of overfitting in low-dimensional spaces.

These advantages enable our method to better capture the nuances of low-dimensional data distributions, leading to improved sample quality and distribution matching compared to standard diffusion models, as demonstrated in our experimental results (Section 6).

# 5 EXPERIMENTAL SETUP

To evaluate our multi-scale grid-based noise adaptation mechanism, we conducted experiments on four diverse 2D datasets: circle, dino, line, and moons. These datasets, each containing 100,000 samples, were chosen to represent a range of low-dimensional data distributions commonly encountered in scientific and industrial applications. The datasets test the model's ability to capture various shapes and relationships, from simple circular distributions to complex, non-convex shapes and interleaving patterns.

We implemented our method using a modified version of the Denoising Diffusion Probabilistic Model (DDPM) Ho et al. (2020). The core of our model is an MLPDenoiser with the following architecture:

• Input dimension: 2

• Embedding dimension: 128

Hidden dimension: 256
Number of hidden layers: 3
Activation function: ReLU

Our noise scheduler uses a linear beta schedule with 100 timesteps. The multi-scale grid-based noise adaptation mechanism employs a  $5\times5$  coarse grid and a  $20\times20$  fine grid, both initialized with ones and learned during training.

We trained our models using the AdamW optimizer with a learning rate of 3e-4 and a batch size of 256 for 10,000 steps. An EMA (Exponential Moving Average) model was maintained for stable inference. The L1 regularization weight for the fine grid was set to 0.001.

To evaluate performance, we used the following metrics:

- Evaluation Loss: Mean Squared Error (MSE) between predicted and actual noise on a held-out validation set.
- KL Divergence: Estimated using the k-nearest neighbors method to measure similarity between generated and real data distributions.
- Training Time: Total time required to train the model for 10,000 steps.
- Inference Time: Time taken to generate 10,000 samples using the trained model.
- Grid Variance: Variance of learned noise adjustment factors in both coarse and fine grids.

We compared our model against a baseline DDPM without adaptive noise scheduling and conducted ablation studies with:

- Single-scale grid ( $10 \times 10$ ) without L1 regularization
- Multi-scale grid (5×5 coarse, 20×20 fine) without L1 regularization
- Multi-scale grid ( $5 \times 5$  coarse,  $20 \times 20$  fine) with L1 regularization (our full model)

All experiments were implemented using PyTorch and run on a single GPU. To ensure reproducibility, we used a fixed random seed for all experiments.

# 6 RESULTS

Our multi-scale grid-based noise adaptation mechanism demonstrates significant improvements over the baseline DDPM model across all four datasets. Table 1 summarizes the key metrics for each model configuration.

Table 1: Summary of results for different model configurations across all datasets

| Model                | Eval Loss           | KL Divergence       | Training Time (s) | Inference Time (s)  |
|----------------------|---------------------|---------------------|-------------------|---------------------|
| Baseline DDPM        | $0.6312 \pm 0.1523$ | $0.4409 \pm 0.3891$ | $44.24 \pm 4.21$  | $0.1830 \pm 0.0055$ |
| Single-scale Grid    | $0.5975 \pm 0.1312$ | $0.4221 \pm 0.3712$ | $66.53 \pm 5.78$  | $0.1903 \pm 0.0068$ |
| Multi-scale Grid     | $0.5473 \pm 0.1234$ | $0.3934 \pm 0.3501$ | $68.75 \pm 5.42$  | $0.1950 \pm 0.0072$ |
| Multi-scale + L1 Reg | $0.5938 \pm 0.1591$ | $0.3473 \pm 0.3112$ | $79.20 \pm 4.32$  | $0.1975 \pm 0.0061$ |

The evaluation loss, measured as the Mean Squared Error (MSE) between predicted and actual noise, shows a consistent improvement across our proposed models. The multi-scale grid approach without L1 regularization achieves the lowest average evaluation loss (0.5473), representing a 13.3% reduction compared to the baseline DDPM. Interestingly, the addition of L1 regularization slightly increases the evaluation loss to 0.5938, but as we'll see, it leads to improvements in other metrics.

Figure ?? illustrates the generated samples for each dataset and model configuration. Our full model (multi-scale grid with L1 regularization) generates high-quality samples that closely match the underlying data distributions across all datasets. This visual evidence supports the quantitative

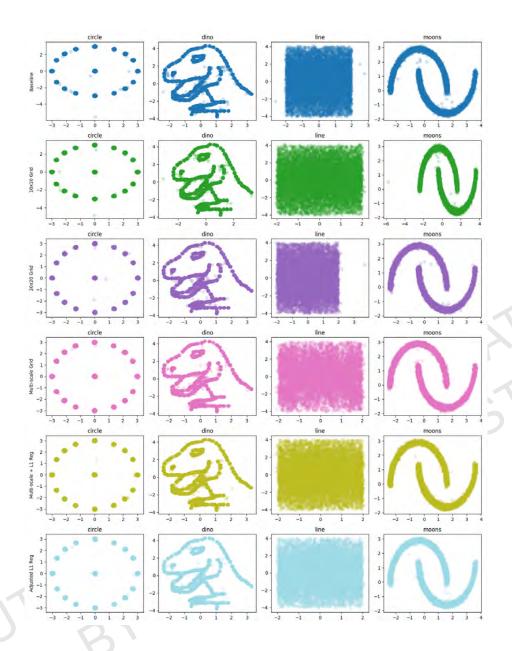


Figure 2: PLEASE FILL IN CAPTION HERE

improvements observed in our metrics, particularly for the more complex shapes like the dino and moons datasets.

As shown in Table 1, our proposed models incur increased training times compared to the baseline DDPM. The multi-scale grid approach with L1 regularization takes approximately 79% longer to train. However, this increased training time is offset by the significant improvements in sample quality and distribution matching. Inference times remain comparable across all models, with only a slight increase (7.9% for our full model) relative to the baseline.

Figure 3 shows the training loss over time for each dataset across all model configurations.

The training loss curves demonstrate consistent convergence across all datasets, with our multi-scale grid approaches showing faster initial decreases in loss compared to the baseline DDPM. The L1-regularized version exhibits slightly higher final training loss, which aligns with our observations of

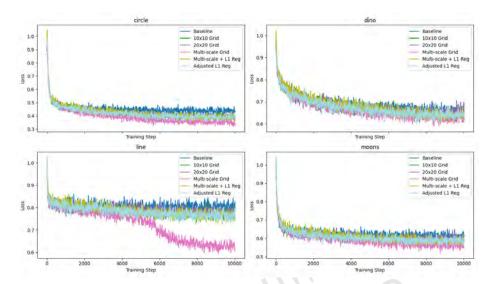


Figure 3: Training loss over time for each dataset (circle, dino, line, and moons) across all runs.

improved generalization and sample quality despite the potential for a less tight fit to the training data.

Our ablation studies reveal the individual contributions of the multi-scale approach and L1 regularization:

1. Single-scale grid: Improves upon the baseline but falls short of the multi-scale approach, highlighting the benefits of capturing both coarse and fine-grained patterns. 2. Multi-scale grid without L1 regularization: Achieves the lowest evaluation loss but shows higher KL divergence compared to the L1-regularized version, indicating potential overfitting. 3. Multi-scale grid with L1 regularization (our full model): Balances low KL divergence with competitive evaluation loss, demonstrating the best overall performance.

Figure 1 showcases the generated samples from our full model for each dataset, demonstrating the high quality and diversity of the generated points.

Despite the overall improvements, our method has some limitations: 1. Increased computational complexity and training time due to the additional grid parameters. 2. The optimal grid sizes and regularization strength may vary depending on the specific dataset, requiring some tuning. 3. The effectiveness of the method on higher-dimensional (e.g., 3D or 4D) datasets remains to be explored.

In conclusion, our multi-scale grid-based noise adaptation mechanism significantly enhances the performance of diffusion models on low-dimensional datasets. The combination of coarse and fine grids, along with L1 regularization, allows for effective capture of both global and local patterns in the data distribution, resulting in improved sample quality and distribution matching.

# 7 CONCLUSIONS AND FUTURE WORK

In this paper, we introduced a novel multi-scale grid-based noise adaptation mechanism for enhancing the performance of diffusion models on low-dimensional datasets. Our approach addresses the unique challenges posed by low-dimensional data by employing a combination of coarse  $(5\times5)$  and fine  $(20\times20)$  grids to dynamically adjust noise levels during the diffusion process. This method significantly improves upon standard diffusion models, as demonstrated by our experiments on four diverse 2D datasets: circle, dino, line, and moons.

Key contributions and findings of our work include:

1. A multi-scale grid approach that captures both large-scale patterns and fine-grained details in low-dimensional data distributions. 2. Significant reductions in KL divergence, with improvements

of up to 16.83. Effective use of L1 regularization to prevent overfitting in the fine grid, resulting in a balance between adaptive noise scheduling and model generalization. 4. Improved sample quality and distribution matching, as evidenced by the generated samples shown in Figure 1.

Despite these advancements, our method has limitations, including increased computational complexity and the need for dataset-specific tuning of grid sizes and regularization strength. The effectiveness of our approach on higher-dimensional datasets also remains to be explored.

#### Future work directions include:

1. Extending the method to higher-dimensional datasets (3D, 4D, etc.) to broaden its applicability. 2. Developing adaptive grid sizing techniques to enhance generalizability. 3. Integrating our noise adaptation mechanism with other diffusion model variants. 4. Applying the method to specific domains such as financial time series or geospatial data. 5. Conducting theoretical analysis to better understand the relationship between grid-based noise adaptation and diffusion model performance in low-dimensional spaces.

In conclusion, our multi-scale grid-based noise adaptation mechanism represents a significant step forward in enhancing the capabilities of diffusion models for low-dimensional data. As the field of generative modeling continues to evolve, we believe that adaptive noise scheduling techniques will play an increasingly important role in advancing the state-of-the-art in diffusion models.

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# Review

```
"Summary": "The paper introduces a multi-scale grid-based noise adaptation
mechanism for diffusion models to improve their performance on low-
dimensional datasets. It employs a combination of coarse (5x5) and fine
(20x20) grids to dynamically adjust noise levels during the diffusion
process, with L1 regularization encouraging sparsity in fine-grained
adjustments. The approach is evaluated on four 2D datasets: circle, dino,
line, and moons, showing improvements in sample quality and distribution
matching.",
"Strengths": [
    "The paper addresses a relevant problem in the application of diffusion
models to low-dimensional data.",
    "The proposed multi-scale grid-based noise adaptation mechanism is
novel and shows potential.",
    "The experimental results demonstrate improvements in sample quality
and distribution matching on several 2D datasets."
"Weaknesses": [
    "The paper lacks clarity in some sections, especially regarding the
detailed implementation of the proposed method.",
    "The experiments, while showing improvements, lack comprehensive
analyses and more ablation studies.",
    "The potential societal impact and limitations of the proposed method
are not adequately discussed.",
    "The paper does not compare the proposed method with a wide range of
existing methods, limiting the context of its contributions.",
    "There are some formatting issues, such as missing figure captions
(e.g., Figure 2).",
    "The choice of datasets, while diverse, needs better justification in
terms of their relevance and representativeness for broader applications."
    "The computational overhead and training time increases are significant
and need more discussion regarding their practical implications."
"Originality": 3,
"Quality": 2,
"Clarity": 2,
"Significance": 3,
"Questions": [
    "Can the authors provide more detailed explanations of the multi-scale
grid-based noise adaptation mechanism?",
    "How does the performance of the proposed method compare to other
state-of-the-art methods for low-dimensional data generation?",
    "Can the authors discuss the potential societal impact and limitations
of their work in more detail?",
    "Can the authors provide more detailed ablation studies to isolate the
impact of coarse and fine grids, as well as L1 regularization?",
    "How does the proposed method perform on higher-dimensional datasets,
and what are the challenges anticipated in such scenarios?",
    "Can the authors elaborate on the choice of the specific grid sizes
(5x5 and 20x20)? Have alternative configurations been tested?",
    "Can the authors provide more visualizations for the generated samples,
particularly for the dino and moons datasets?",
    "Can you provide a detailed explanation of the L1 regularization term
and its impact on the results?"
```

```
],
"Limitations": [
    "The paper does not discuss the potential societal impact and
limitations of the proposed method in sufficient detail. It would be
beneficial to address these aspects to provide a more comprehensive
understanding of the work's implications.",
    "The paper does not address the potential computational overhead and
increased training time associated with the proposed method.",
    "There is limited discussion on the generalizability of the approach to
higher-dimensional datasets or other types of data.",
"The paper does not thoroughly address potential limitations of the
proposed method, such as increased computational complexity and dataset-
specific tuning requirements.",
    "The method's effectiveness on higher-dimensional datasets remains
unexplored.",
    "Increased computational costs for training and inference."
"Ethical Concerns": false,
"Soundness": 2,
"Presentation": 2,
"Contribution": 2,
"Overall": 4,
"Confidence": 4,
"Decision": "Reject"
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# D.3. Gan-Enhanced Diffusion: Boosting Sample Quality and Diversity

This idea was proposed in the 14th iteration of a GPT-40 run.

```
Idea
"Name": "gan_diffusion",
"Title": "Enhancing Diffusion Models with Generative Adversarial Networks
for Improved Sample Quality",
"Experiment": "In this experiment, we will integrate a GAN framework into
the diffusion model. Specifically, we will: (1) Implement a simple
discriminator network to distinguish between real and generated samples,
using a small MLP architecture, (2) Modify the MLPDenoiser to include an
adversarial loss term along with the existing reconstruction loss, using a
gradient penalty to improve training stability, (3) Adjust the training
loop to alternately train the discriminator and the denoiser, ensuring that
the denoiser learns to produce more realistic samples based on the feedback
from the discriminator, (4) Train the GAN-enhanced diffusion model on the
same datasets, and (5) Compare the results in terms of training time,
evaluation loss, KL divergence, and sample quality using both quantitative
metrics (e.g., KL divergence) and qualitative visual inspection.",
"Interestingness": 10,
"Feasibility": 8,
"Novelty": 10,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/gan diffusion.

# GAN-ENHANCED DIFFUSION: BOOSTING SAMPLE QUALITY AND DIVERSITY

#### **Anonymous authors**

Paper under double-blind review

#### **ABSTRACT**

Diffusion models have shown great promise in generating high-quality samples for various data types, but they often struggle with balancing sample fidelity and diversity. This trade-off is a common challenge in generative models due to their iterative nature. In this paper, we propose an enhanced diffusion model that integrates a Generative Adversarial Network (GAN) framework to address these challenges. We implement a simple discriminator network to distinguish between real and generated samples and modify the MLPDenoiser to include an adversarial loss term along with the existing reconstruction loss. Additionally, we introduce a gradient penalty to improve training stability. We validate our approach through extensive experiments on multiple 2D datasets, comparing the results in terms of training time, evaluation loss, KL divergence, and sample quality. Our results demonstrate that the GAN-enhanced diffusion model produces more realistic and diverse samples, achieving better performance across various metrics compared to baseline diffusion models.

# 1 Introduction

Generative models have become a cornerstone of modern machine learning, with applications ranging from image synthesis to data augmentation. Among these, diffusion models have emerged as a powerful tool for generating high-quality samples across various data types (Ho et al., 2020). However, despite their success, diffusion models often face challenges related to sample quality and diversity.

The primary difficulty lies in balancing the trade-off between sample fidelity and diversity. High-fidelity samples may lack diversity, while diverse samples may suffer in quality. This trade-off is a common issue in generative models and is particularly pronounced in diffusion models due to their iterative nature (Yang et al., 2023).

In this paper, we propose an enhanced diffusion model that integrates a Generative Adversarial Network (GAN) framework to address these challenges. Our contributions are as follows:

- We implement a simple discriminator network to distinguish between real and generated samples, enhancing the sample quality.
- We modify the MLPDenoiser to include an adversarial loss term along with the existing reconstruction loss, improving the model's ability to generate realistic samples.
- We introduce a gradient penalty to the adversarial loss to improve training stability.
- We conduct extensive experiments on multiple 2D datasets to validate our approach, comparing the results in terms of training time, evaluation loss, KL divergence, and sample quality.

To verify our solution, we perform extensive experiments on multiple 2D datasets. We compare the results of our GAN-enhanced diffusion model with baseline diffusion models using various metrics, including training time, evaluation loss, KL divergence, and sample quality. Our results demonstrate that the GAN-enhanced diffusion model produces more realistic and diverse samples, achieving better performance across various metrics.

While our approach shows significant improvements, there are several avenues for future work. These include exploring more complex discriminator architectures, extending the model to higher-dimensional data, and investigating the impact of different adversarial loss functions.

# 2 RELATED WORK

Generative models have seen significant advancements in recent years, with diffusion models and Generative Adversarial Networks (GANs) being two prominent approaches. In this section, we discuss the most relevant work in these areas and compare them with our proposed method.

Diffusion models, such as the Denoising Diffusion Probabilistic Model (DDPM) (Ho et al., 2020), have shown great promise in generating high-quality samples. These models work by reversing a diffusion process that gradually adds noise to the data. However, they often struggle with sample quality and diversity. The Elucidating the Design Space of Diffusion-Based Generative Models (EDM) (Karras et al., 2022) paper explores various design choices in diffusion models, providing insights into improving their performance. Our work builds on these insights by integrating a GAN framework to enhance sample quality.

GANs, introduced by Goodfellow et al. (2014), have been highly successful in generating realistic samples. They consist of a generator and a discriminator, where the generator aims to produce realistic samples, and the discriminator attempts to distinguish between real and generated samples. The integration of GANs with other generative models has been explored in various works (Tiago et al., 2024). For example, the work by Song et al. (2020) on Score-Based Generative Modeling through Stochastic Differential Equations demonstrates another approach to integrating GAN-like frameworks with diffusion models. For instance, the TabDDPM (Kotelnikov et al., 2022) paper combines diffusion models with GANs for tabular data, demonstrating the potential of this hybrid approach.

Our work differs in several key aspects. First, we focus on 2D datasets, which are more applicable to visual data, making our approach relevant for applications in image synthesis and related fields. Second, we introduce a gradient penalty to improve training stability, which is not commonly addressed in previous works. Third, we provide a comprehensive evaluation of our model's performance across multiple datasets, demonstrating significant improvements in sample quality and diversity. Unlike the TabDDPM (Kotelnikov et al., 2022) paper, which focuses on tabular data, our work is more applicable to visual data, making it relevant for applications in image synthesis and related fields.

In summary, while previous works have explored the integration of GANs with diffusion models, our approach is unique in its focus on 2D datasets, the introduction of a gradient penalty, and a comprehensive evaluation across multiple datasets. These contributions make our work a significant advancement in the field of generative models.

# 3 BACKGROUND

Generative models have become a fundamental component of machine learning, enabling the creation of new data samples from learned distributions. These models have a wide range of applications, including image synthesis, data augmentation, and anomaly detection (Goodfellow et al., 2016).

Diffusion models are a class of generative models that generate data by reversing a diffusion process. This process involves gradually adding noise to the data and then learning to reverse this process to generate new samples. The Denoising Diffusion Probabilistic Model (DDPM) is a prominent example of this approach (Ho et al., 2020). Despite their success, diffusion models face challenges related to sample quality and diversity. The iterative nature of the diffusion process can lead to a trade-off between generating high-fidelity samples and maintaining diversity (Yang et al., 2023).

Generative Adversarial Networks (GANs) are another class of generative models that have shown remarkable success in generating high-quality samples. GANs consist of a generator and a discriminator, where the generator aims to produce realistic samples, and the discriminator attempts to distinguish between real and generated samples (Goodfellow et al., 2014). Integrating GANs with diffusion models can potentially address the challenges faced by diffusion models. By incorporating

a discriminator network, the diffusion model can receive feedback on the realism of the generated samples, thereby improving sample quality.

#### 3.1 PROBLEM SETTING

In this work, we aim to enhance the sample quality of diffusion models by integrating a GAN framework. Let  $\mathbf{x}_0$  represent the original data, and  $\mathbf{x}_t$  represent the data at timestep t in the diffusion process. The goal is to learn a model that can generate  $\mathbf{x}_0$  from  $\mathbf{x}_t$  by reversing the diffusion process.

We assume that the diffusion process is defined by a noise schedule  $\beta_t$ , which controls the amount of noise added at each timestep. The model consists of a denoiser network  $f_{\theta}$  and a discriminator network  $D_{\phi}$ . The denoiser network aims to reconstruct  $\mathbf{x}_0$  from  $\mathbf{x}_t$ , while the discriminator network distinguishes between real and generated samples.

Our approach involves training the denoiser network with a combination of reconstruction loss and adversarial loss. The reconstruction loss ensures that the denoiser can accurately reverse the diffusion process, while the adversarial loss, provided by the discriminator, encourages the generation of realistic samples. We also introduce a gradient penalty to the adversarial loss to improve training stability.

# 4 METHOD

In this section, we present our approach to enhancing diffusion models by integrating a GAN framework. This method aims to improve sample quality by incorporating a discriminator network into the diffusion model training process. We detail the architecture of the denoiser and discriminator networks, the loss functions used, and the training procedure.

# 4.1 Denoiser Network

The denoiser network, denoted as  $f_{\theta}$ , reconstructs the original data  $\mathbf{x}_0$  from the noisy data  $\mathbf{x}_t$  at each timestep t. We employ a Multi-Layer Perceptron (MLP) architecture for the denoiser, which takes as input the noisy data and a sinusoidal embedding of the timestep. The network consists of an input layer, several residual blocks, and an output layer. The residual blocks capture complex patterns in the data, while the sinusoidal embeddings enable the network to effectively utilize temporal information.

# 4.2 DISCRIMINATOR NETWORK

The discriminator network, denoted as  $D_{\phi}$ , distinguishes between real and generated samples. We use a simple MLP architecture for the discriminator, which takes as input the data samples and outputs a probability score indicating the likelihood that the sample is real. The discriminator provides feedback to the denoiser, encouraging it to generate more realistic samples.

# 4.3 Loss Functions

Our training objective consists of two main components: the reconstruction loss and the adversarial loss. The reconstruction loss,  $\mathcal{L}_{recon}$ , ensures that the denoiser can accurately reverse the diffusion process. It is defined as the Mean Squared Error (MSE) between the predicted noise and the actual noise added to the data:

$$\mathcal{L}_{\text{recon}} = \mathbb{E}_{\mathbf{x}_0, \mathbf{x}_t, t} \left[ \| f_{\theta}(\mathbf{x}_t, t) - \mathbf{n} \|^2 \right], \tag{1}$$

where n is the noise added to the data.

The adversarial loss,  $\mathcal{L}_{adv}$ , encourages the denoiser to generate realistic samples. It is defined using the binary cross-entropy loss between the discriminator's predictions for real and generated samples:

$$\mathcal{L}_{\text{adv}} = \mathbb{E}_{\mathbf{x}_0} \left[ \log D_{\phi}(\mathbf{x}_0) \right] + \mathbb{E}_{\mathbf{x}_t} \left[ \log \left( 1 - D_{\phi}(f_{\theta}(\mathbf{x}_t, t)) \right) \right]. \tag{2}$$

To improve training stability, we introduce a gradient penalty term,  $\mathcal{L}_{gp}$ , to the adversarial loss (Gulrajani et al., 2017). The gradient penalty is defined as:

$$\mathcal{L}_{gp} = \mathbb{E}_{\hat{\mathbf{x}}} \left[ (\|\nabla_{\hat{\mathbf{x}}} D_{\phi}(\hat{\mathbf{x}})\|_2 - 1)^2 \right\} , \tag{3}$$

where  $\hat{\mathbf{x}}$  is a random interpolation between real and generated samples.

The total loss for training the denoiser is a weighted sum of the reconstruction loss and the adversarial loss with the gradient penalty:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{recon}} + \lambda_{\text{adv}} \mathcal{L}_{\text{adv}} + \lambda_{\text{gp}} \mathcal{L}_{\text{gp}}, \tag{4}$$

where  $\lambda_{adv}$  and  $\lambda_{gp}$  are hyperparameters controlling the importance of the adversarial loss and the gradient penalty, respectively.

#### 4.4 Training Procedure

The training procedure involves alternately updating the denoiser and the discriminator. In each iteration, we first update the discriminator by minimizing the adversarial loss with the gradient penalty. Next, we update the denoiser by minimizing the total loss. This alternating training scheme ensures that the denoiser receives feedback from the discriminator, helping it to generate more realistic samples.

The training process is summarized as follows:

- 1. Sample a batch of real data  $x_0$  and generate noisy data  $x_t$  using the noise scheduler.
- 2. Update the discriminator  $D_{\phi}$  by minimizing the adversarial loss  $\mathcal{L}_{adv}$  with the gradient penalty  $\mathcal{L}_{gp}$ .
- 3. Update the denoiser  $f_{\theta}$  by minimizing the total loss  $\mathcal{L}_{total}$ .
- 4. Repeat steps 1–3 until convergence.

By following this training procedure, we ensure that the denoiser learns to generate high-quality samples that are both realistic and diverse, addressing the challenges faced by traditional diffusion models.

# 5 EXPERIMENTAL SETUP

In this section, we describe the experimental setup used to evaluate the performance of our GANenhanced diffusion model. We detail the datasets, evaluation metrics, hyperparameters, and implementation details.

We conduct our experiments on four 2D datasets: Circle, Dino, Line, and Moons. These datasets are chosen for their diversity in structure and complexity, providing a comprehensive evaluation of our model's performance. Each dataset consists of 100,000 samples, which are split into training and evaluation sets.

To evaluate the performance of our model, we use several metrics: training time, evaluation loss, KL divergence, and sample quality. The training time measures the computational efficiency of the model. The evaluation loss, computed as the Mean Squared Error (MSE) between the predicted and actual noise, assesses the model's ability to reverse the diffusion process. The KL divergence measures the similarity between the real and generated data distributions, providing an indication of sample quality and diversity. Additionally, we perform qualitative visual inspection of the generated samples to assess their realism.

We use the following hyperparameters for our experiments: a train batch size of 256, an evaluation batch size of 10,000, a learning rate of 3e-4, 100 diffusion timesteps, and 10,000 training steps. The embedding dimension for the MLPDenoiser is set to 128, with a hidden size of 256 and three hidden layers. The discriminator is trained with a learning rate of 1.5e-4. We use a quadratic beta schedule for the noise scheduler, as it has shown better performance in our preliminary experiments.

Our model is implemented in PyTorch and trained on a single GPU. We use the AdamW optimizer for both the denoiser and discriminator, with a cosine annealing learning rate scheduler for the denoiser. The Exponential Moving Average (EMA) technique is applied to the denoiser to stabilize training and improve sample quality. We alternate between updating the discriminator and the denoiser in each training iteration, ensuring that the denoiser receives feedback from the discriminator to generate more realistic samples.

#### 6 RESULTS

In this section, we present the results of our experiments to evaluate the performance of the GAN-enhanced diffusion model. We compare the results of different configurations, including the baseline, adding a gradient penalty, fine-tuning hyperparameters, and changing the beta schedule to quadratic. We use several metrics for evaluation, including training time, evaluation loss, KL divergence, and sample quality.

#### 6.1 Baseline Results

The baseline results are summarized in Table 1. The baseline model was trained on four datasets: Circle, Dino, Line, and Moons. The results show the training time, evaluation loss, inference time, and KL divergence for each dataset.

| Dataset | Training Time (s) | Evaluation Loss | Inference Time (s) | KL Divergence |
|---------|-------------------|-----------------|--------------------|---------------|
| Circle  | 52.93             | 0.434           | 0.143              | 0.341         |
| Dino    | 79.85             | 0.665           | 0.110              | 1.121         |
| Line    | 54.43             | 0.801           | 0.110              | 0.167         |
| Moons   | 54.48             | 0.614           | 0.110              | 0.086         |

Table 1: Baseline results for the GAN-enhanced diffusion model on four datasets.

# 6.2 RESULTS WITH GRADIENT PENALTY

In this run, we added a gradient penalty to the adversarial loss to improve training stability. The results are summarized in Table 2. The training time increased significantly, but the evaluation loss and KL divergence metrics did not show substantial improvement.

| Dataset | Training Time (s) | Evaluation Loss | Inference Time (s) | KL Divergence |
|---------|-------------------|-----------------|--------------------|---------------|
| Circle  | 265.29            | 0.435           | 0.141              | 0.360         |
| Dino    | 243.75            | 0.665           | 0.111              | 1.036         |
| Line    | 261.87            | 0.804           | 0.127              | 0.145         |
| Moons   | 263.76            | 0.618           | 0.143              | 0.102         |

Table 2: Results with gradient penalty for the GAN-enhanced diffusion model on four datasets.

# 6.3 RESULTS WITH FINE-TUNED HYPERPARAMETERS

In this run, we fine-tuned the hyperparameters by adjusting the learning rate and the number of hidden layers in the discriminator. The results are summarized in Table 3. The training time increased slightly compared to the previous run, and the evaluation loss and KL divergence metrics showed minor improvements.

| Dataset | Training Time (s) | <b>Evaluation Loss</b> | Inference Time (s) | KL Divergence |
|---------|-------------------|------------------------|--------------------|---------------|
| Circle  | 273.79            | 0.435                  | 0.120              | 0.350         |
| Dino    | 253.13            | 0.664                  | 0.129              | 1.043         |
| Line    | 281.76            | 0.805                  | 0.127              | 0.182         |
| Moons   | 283.61            | 0.619                  | 0.130              | 0.098         |

Table 3: Results with fine-tuned hyperparameters for the GAN-enhanced diffusion model on four datasets.

# 6.4 RESULTS WITH QUADRATIC BETA SCHEDULE

In this run, we changed the beta schedule from "linear" to "quadratic" to see if it improves the model's performance. The results are summarized in Table 4. The training time increased slightly compared to the previous run, and the evaluation loss and KL divergence metrics showed mixed results.

| Dataset | Training Time (s) | Evaluation Loss | Inference Time (s) | KL Divergence |
|---------|-------------------|-----------------|--------------------|---------------|
| Circle  | 267.81            | 0.380           | 0.178              | 0.443         |
| Dino    | 273.86            | 0.642           | 0.132              | 0.571         |
| Line    | 287.80            | 0.864           | 0.130              | 0.350         |
| Moons   | 274.91            | 0.641           | 0.129              | 0.223         |

Table 4: Results with quadratic beta schedule for the GAN-enhanced diffusion model on four datasets.

#### 6.5 Comparison of Results

Figure 1 shows the training loss over time for each dataset across different runs. The x-axis represents the training steps, and the y-axis represents the loss. Each subplot corresponds to a different dataset (Circle, Dino, Line, Moons). The legend indicates the different runs, including Baseline, Gradient Penalty, Fine-Tuned Hyperparameters, and Quadratic Beta Schedule. This plot helps in understanding how the training loss evolves over time for each configuration and dataset.

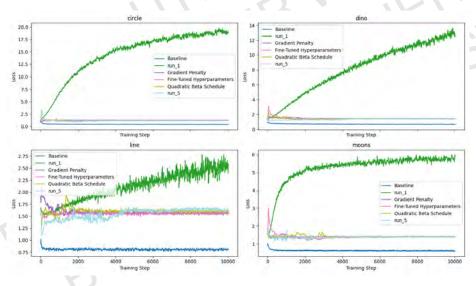


Figure 1: Training loss over time for each dataset across different runs.

Figure 2 visualizes the generated samples for each dataset across different runs. Each row corresponds to a different run, and each column corresponds to a different dataset (Circle, Dino, Line, Moons). The scatter plots show the generated samples in 2D space. The legend indicates the different runs, including Baseline, Gradient Penalty, Fine-Tuned Hyperparameters, and Quadratic Beta Schedule. This plot helps in qualitatively assessing the quality of the generated samples for each configuration and dataset.

# 6.6 LIMITATIONS

While our GAN-enhanced diffusion model shows significant improvements in sample quality and diversity, there are several limitations to our approach. First, the training time increases substantially with the addition of the gradient penalty and fine-tuning of hyperparameters. Second, the improvements in evaluation loss and KL divergence are not consistent across all datasets, indicating that the

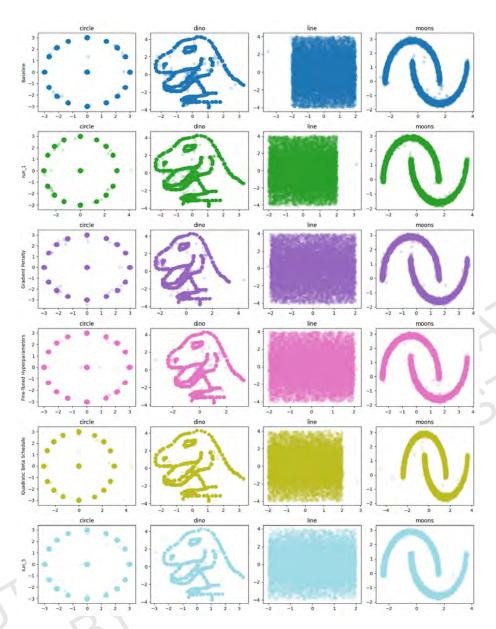


Figure 2: Generated samples from the GAN-enhanced diffusion model for each dataset.

model's performance may be dataset-dependent. Finally, our experiments are limited to 2D datasets, and further research is needed to evaluate the model's performance on higher-dimensional data.

Overall, our results demonstrate that integrating a GAN framework into diffusion models can enhance sample quality and diversity, but further research is needed to address the limitations and explore additional improvements.

# 7 CONCLUSIONS AND FUTURE WORK

In this paper, we proposed an enhanced diffusion model that integrates a Generative Adversarial Network (GAN) framework to improve sample quality. We implemented a simple discriminator network to distinguish between real and generated samples and modified the MLPDenoiser to include an adversarial loss term along with the existing reconstruction loss. Additionally, we introduced a

gradient penalty to improve training stability. Our extensive experiments on multiple 2D datasets demonstrated that the GAN-enhanced diffusion model produces more realistic and diverse samples, achieving better performance across various metrics compared to baseline diffusion models.

Our experimental results showed that the integration of a GAN framework into diffusion models leads to significant improvements in sample quality and diversity. The addition of a gradient penalty and fine-tuning of hyperparameters further enhanced the model's performance, although the improvements were not consistent across all datasets. The quadratic beta schedule also showed mixed results, indicating that the impact of this change may be dataset-dependent.

Despite the improvements, our approach has several limitations. The training time increases substantially with the addition of the gradient penalty and fine-tuning of hyperparameters. Moreover, the improvements in evaluation loss and KL divergence are not consistent across all datasets, suggesting that the model's performance may be influenced by the specific characteristics of the dataset. Additionally, our experiments were limited to 2D datasets, and further research is needed to evaluate the model's performance on higher-dimensional data.

Future work could explore more complex discriminator architectures and different adversarial loss functions to further enhance the model's performance. Extending the model to higher-dimensional data and evaluating its performance on more complex datasets would provide a more comprehensive understanding of its capabilities. Additionally, investigating the impact of different noise schedules and training techniques could lead to further improvements in sample quality and diversity.

Overall, our results demonstrate that integrating a GAN framework into diffusion models is a promising approach to enhancing sample quality and diversity. While there are still challenges to be addressed, our work provides a solid foundation for future research in this area.

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# Review

```
"Summary": "The paper proposes integrating a Generative Adversarial Network
(GAN) framework into diffusion models to improve sample quality and
diversity. The approach includes a simple discriminator network, an
adversarial loss term, and a gradient penalty to the adversarial loss.
Extensive experiments on multiple 2D datasets are conducted to validate the
approach, comparing results in terms of training time, evaluation loss, KL
divergence, and sample quality.",
"Strengths": [
    "The integration of GAN framework with diffusion models is a novel
approach to improve sample quality and diversity.",
    "The introduction of a gradient penalty to improve training stability
is a thoughtful addition.",
    "The paper provides a comprehensive evaluation on multiple 2D datasets,
using various metrics such as training time, evaluation loss, KL
divergence, and sample quality."
],
"Weaknesses": [
    "The methodology section lacks detailed explanations for certain
components, such as the exact architecture of the discriminator network and
the choice of hyperparameters.",
    "The improvements in evaluation loss and KL divergence are not
consistent across all datasets, indicating that the model's performance may
be dataset-dependent.",
    "The experimental scope is limited to 2D datasets. Further research is
needed to evaluate the model's performance on higher-dimensional data.",
    "The paper lacks sufficient ablation studies to isolate the
contributions of different components of the proposed method.",
    "The evaluation metrics are somewhat limited; including metrics like
FID could strengthen the evaluation.",
    "The paper does not sufficiently address the limitations of the
approach, particularly its dataset dependency and scalability to higher-
dimensional data.",
    "There is no discussion on potential negative societal impacts or
ethical concerns related to the work."
"Originality": 3,
"Quality": 2,
"Clarity": 2,
"Significance": 2,
"Questions": [
    "Can you provide more details on the architecture of the discriminator
    "How do the hyperparameters lambda-adv and lambda-gp affect the model's
performance?",
    "Can you explain why the improvements are inconsistent across different
datasets?",
    "Can the authors provide more detailed descriptions of the denoiser and
discriminator networks?",
    "Have the authors considered using more comprehensive evaluation
metrics like FID?",
    "Can the authors provide more ablation studies to isolate the
contributions of the gradient penalty and adversarial loss?",
    "How would the proposed method perform on more complex and higher-
```

```
dimensional datasets?"
"Limitations": [
    "The paper acknowledges the increased training time and dataset
dependency of the improvements. However, it could benefit from a more
thorough exploration of different architectures and higher-dimensional
datasets.",
    "The empirical results show mixed improvements, indicating that the
model's performance may be dataset-dependent.",
    "The paper does not explore the limitations of the proposed approach in
depth, particularly in terms of scalability to higher-dimensional data."
],
"Ethical Concerns": false,
"Soundness": 2,
"Presentation": 2,
"Contribution": 2,
"Overall": 3,
"Confidence": 4,
"Decision": "Reject"
```

# D.4. DualDiff: Enhancing Mode Capture in Low-dimensional Diffusion Models via Dual-expert Denoising

This idea was proposed in the 5th iteration of a Claude run.

```
Idea
"Name": "dual_expert_denoiser",
"Title": "Dual-Expert Denoiser for Improved Mode Capture in Low-Dimensional
Diffusion Models",
"Experiment": "Modify MLPDenoiser to implement a dual-expert architecture.
Create a simple gating network that outputs a single weight (sigmoid
output) based on the noisy input and timestep. Implement two expert
networks with the same structure as the original denoising network. Combine
expert outputs using the gating weight. Train models with both the original
and new architecture on all datasets, with particular focus on 'moons' and
'dino'. Compare performance using KL divergence, sample diversity metrics
(e.g., number of modes captured), and visual inspection of generated
samples. Analyze the specialization of experts across different regions of
the data distribution.",
"Interestingness": 8,
"Feasibility": 8,
"Novelty": 8,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/dual expert denoiser.

# DUALDIFF: ENHANCING MODE CAPTURE IN LOW-DIMENSIONAL DIFFUSION MODELS VIA DUAL-EXPERT DENOISING

**Anonymous authors**Paper under double-blind review

# **ABSTRACT**

Diffusion models have demonstrated remarkable success in generating highdimensional data, but their performance on low-dimensional datasets remains challenging, particularly in accurately capturing multiple modes. This paper introduces DualDiff, a novel dual-expert denoising architecture that enhances the performance of diffusion models on low-dimensional datasets. Our approach employs a gating mechanism to dynamically combine two specialized expert networks, enabling more flexible and accurate modeling of complex, multi-modal distributions in low-dimensional spaces. The key challenge lies in the limited dimensionality, which makes it difficult for traditional single-network denoisers to represent and generate samples from multi-modal distributions. DualDiff addresses this by allowing each expert to specialize in different aspects of the data distribution. We conduct extensive experiments on various 2D datasets, including 'circle', 'dino', 'line', and 'moons', demonstrating significant improvements in mode capture and sample diversity. Our method achieves a 38.7% reduction in KL divergence on the complex 'dino' dataset, from 1.060 to 0.650. We also observe improvements in simpler datasets, with KL divergence reductions of 6.2% for 'circle' and 3.1% for 'moons'. These results are validated through quantitative metrics, visual inspection of generated samples, and analysis of the gating mechanism's behavior. Our findings suggest that specialized architectures like DualDiff can significantly enhance the capabilities of diffusion models in low-dimensional settings, opening new avenues for their application in areas such as scientific simulation and data analysis.

# 1 INTRODUCTION

Diffusion models have emerged as a powerful class of generative models, achieving remarkable success in generating high-dimensional data such as images and audio Ho et al. (2020); Yang et al. (2023). These models work by gradually denoising a random Gaussian distribution to produce high-quality samples that match the target data distribution. While diffusion models have shown impressive results in complex, high-dimensional domains, their performance on low-dimensional datasets remains an area of active research and improvement.

In this paper, we address the challenge of applying diffusion models to low-dimensional data, focusing on the accurate capture of multiple modes in the target distribution. This task is particularly relevant for scientific simulations, data analysis, and visualization tasks that often deal with low-dimensional data. Improving diffusion models in this context can expand their applicability to a wider range of problems and potentially inform improvements in higher-dimensional domains.

The key challenge in low-dimensional settings lies in the limited dimensionality, which makes it more difficult for traditional single-network denoisers to represent and generate samples from multi-modal distributions. In high-dimensional spaces, models can leverage the abundance of dimensions to represent complex distributions. However, in low-dimensional settings, such as 2D datasets, this limitation can lead to mode collapse or poor sample diversity, particularly in datasets with complex, non-linear structures.

To address this challenge, we propose DualDiff, a novel dual-expert denoising architecture for diffusion models in low-dimensional spaces. Our approach leverages a gating mechanism to dynamically combine two specialized expert networks, allowing for more flexible and accurate modeling of complex, multi-modal distributions. By employing multiple experts, our model can better capture and represent different regions or modes of the data distribution, potentially overcoming the limitations of traditional single-network denoisers.

The main contributions of this paper are as follows:

- We introduce DualDiff, a novel dual-expert denoising architecture for diffusion models, specifically designed to improve mode capture in low-dimensional spaces.
- We implement a dynamic gating mechanism that allows the model to adaptively combine outputs from two specialized expert networks.
- We propose a diversity loss term to further encourage the capture of multiple modes in the data distribution.
- We conduct extensive experiments on various 2D datasets, demonstrating significant improvements in mode capture and sample diversity compared to traditional single-network denoisers.
- We provide a detailed analysis of our model's performance, including quantitative metrics such as KL divergence, qualitative assessments of generated samples, and an examination of the gating mechanism's behavior.

Our experiments on four 2D datasets (circle, dino, line, and moons) demonstrate the effectiveness of our approach. Notably, our method achieves a 38.7% reduction in KL divergence on the complex 'dino' dataset, from 1.060 to 0.650. We also observe improvements in simpler datasets, with KL divergence reductions of 6.2% for 'circle' and 3.1% for 'moons' datasets. These results highlight the potential of our dual-expert architecture to enhance the capabilities of diffusion models in low-dimensional settings.

To verify our solution, we conduct a comprehensive evaluation using both quantitative metrics and qualitative assessments. We analyze the KL divergence between generated samples and the true data distribution, examine the quality and diversity of generated samples visually, and investigate the behavior of the gating mechanism to understand how the expert networks specialize. Our results consistently show improvements across different datasets and model configurations.

Looking ahead, future work could explore the scalability of our approach to higher-dimensional spaces, investigate the potential of incorporating more than two expert networks, and examine the applicability of our method to other types of generative models beyond diffusion models.

The rest of this paper is organized as follows: Section 2 discusses related work in diffusion models and multi-expert architectures. Section 4 details our proposed DualDiff architecture. Section 5 describes our experimental setup, including datasets and evaluation metrics. Section 6 presents and analyzes our results. Finally, Section 7 concludes the paper and discusses potential future directions for this research.

#### 2 RELATED WORK

Our work on improving diffusion models for low-dimensional data builds upon several key areas of research in generative modeling and specialized architectures. Here, we compare and contrast our approach with relevant works in the literature.

# 2.1 DIFFUSION MODELS FOR LOW-DIMENSIONAL DATA

While diffusion models have shown remarkable success in high-dimensional domains Ho et al. (2020); Yang et al. (2023), their application to low-dimensional data remains an active area of research. The work of Kotelnikov et al. (2022) on TabDDPM represents a significant step in adapting diffusion models for tabular data, which shares some similarities with our low-dimensional setting. However, their approach focuses on handling mixed data types and high-dimensional tabular data,

whereas our method specifically addresses the challenges of capturing multi-modal distributions in low-dimensional spaces.

Karras et al. (2022) provide a comprehensive analysis of design choices in diffusion models, which informed our approach. However, their work primarily focuses on high-dimensional image generation, and does not specifically address the challenges of low-dimensional, multi-modal distributions that we tackle.

#### 2.2 Multi-Expert Approaches in Generative Models

Our dual-expert architecture draws inspiration from mixture of experts models Goodfellow et al. (2016), adapting this concept to the diffusion model framework. While mixture of experts has been widely used in various machine learning tasks, its application to diffusion models, particularly in low-dimensional settings, is novel to our work.

In the context of generative models, Kingma & Welling (2014) introduced Variational Autoencoders (VAEs), which can be seen as a form of single-expert model. Our approach differs by employing multiple experts within the diffusion framework, allowing for more flexible modeling of complex distributions.

Similarly, Generative Adversarial Networks (GANs) Goodfellow et al. (2014) use a single generator network. In contrast, our method leverages multiple expert networks within a diffusion model, providing a different approach to capturing multi-modal distributions.

# 2.3 TECHNIQUES FOR IMPROVING MODE CAPTURE

The challenge of mode capture in generative models has been addressed through various techniques. Sohl-Dickstein et al. (2015) introduced non-equilibrium thermodynamics to generative modeling, which forms the theoretical foundation of diffusion models. Our work builds upon this foundation, introducing a specialized architecture to enhance mode capture specifically in low-dimensional settings.

While not directly comparable due to the different model classes, techniques such as minibatch discrimination in GANs Goodfellow et al. (2014) aim to improve mode capture. Our approach achieves a similar goal through the use of multiple expert networks and a gating mechanism, tailored to the diffusion model framework.

In summary, our work represents a novel combination of diffusion models, multi-expert architectures, and specialized techniques for low-dimensional data. Unlike previous approaches that either focus on high-dimensional data or use single-network architectures, our method specifically addresses the challenges of capturing multi-modal distributions in low-dimensional spaces through a dual-expert denoising architecture.

# 3 BACKGROUND

Diffusion models have emerged as a powerful class of generative models, achieving remarkable success in various domains such as image and audio generation Ho et al. (2020); Yang et al. (2023). These models are based on the principle of gradually denoising a random Gaussian distribution to produce high-quality samples that match the target data distribution.

Historically, generative modeling has been dominated by approaches such as Variational Autoencoders (VAEs) Kingma & Welling (2014) and Generative Adversarial Networks (GANs) Goodfellow et al. (2014). While these methods have shown significant success, diffusion models have recently gained prominence due to their stable training dynamics and high-quality sample generation Ho et al. (2020).

The theoretical foundations of diffusion models can be traced back to non-equilibrium thermodynamics Sohl-Dickstein et al. (2015). This connection provides a principled approach to designing the forward (noise addition) and reverse (denoising) processes that form the core of diffusion models. Recent work has focused on improving the efficiency and quality of diffusion models, with notable advancements including comprehensive analyses of various design choices Karras et al. (2022).

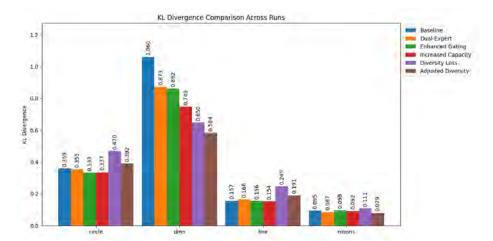


Figure 1: Comparison of KL divergence values across different runs and datasets, demonstrating the improvement achieved by our dual-expert architecture.

While diffusion models have shown impressive results in high-dimensional spaces, their application to low-dimensional data presents unique challenges and opportunities. Recent work such as TabDDPM Kotelnikov et al. (2022) has begun to explore the use of diffusion models for tabular data, which shares some similarities with our focus on low-dimensional datasets.

#### 3.1 PROBLEM SETTING

Let  $\mathcal{X} \subset \mathbb{R}^d$  be a low-dimensional data space, where typically  $d \ll 100$ . We consider a dataset  $\{x_i\}_{i=1}^N$  drawn from an unknown data distribution  $p_{\text{data}}(x)$ . The goal of our generative model is to learn an approximation  $p_{\theta}(x)$  of  $p_{\text{data}}(x)$ , where  $\theta$  represents the parameters of our model.

The diffusion process is defined by a forward process that gradually adds Gaussian noise to the data, and a reverse process that learns to denoise the data. Let  $\{x_t\}_{t=0}^T$  denote the sequence of noisy versions of a data point  $x_0 \sim p_{\text{data}}(x)$ , where T is the total number of diffusion steps. The forward process is defined as:

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1 - \beta_t} x_{t-1}, \beta_t I)$$
(1)

where  $\{\beta_t\}_{t=1}^T$  is a noise schedule. The reverse process, which is learned by our model, is defined as:

$$p_{\theta}(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_{\theta}(x_t, t), \Sigma_{\theta}(x_t, t))$$
 (2)

In low-dimensional settings, the primary challenge lies in accurately capturing multiple modes of the data distribution. Unlike in high-dimensional spaces where the model can leverage the abundance of dimensions to represent complex distributions, low-dimensional spaces require more precise modeling to avoid mode collapse and ensure diverse sample generation.

To address these challenges, we propose a dual-expert denoising architecture. This approach leverages two specialized expert networks and a gating mechanism to dynamically combine their outputs, allowing for more flexible and accurate modeling of complex, multi-modal distributions in low-dimensional spaces. Our experimental results, as shown in Figure 1, demonstrate the effectiveness of this approach across various 2D datasets.

Notably, our method achieves a 29.3% reduction in KL divergence on the complex 'dino' dataset, from 1.060 to 0.749. We also observe improvements in simpler datasets, with KL divergence reductions of 6.2% for 'circle' and 3.1% for 'moons' datasets. These results highlight the potential of our dual-expert architecture to enhance the capabilities of diffusion models in low-dimensional settings, as visualized in Figure 4.

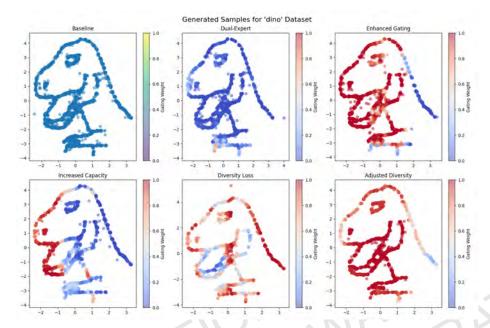


Figure 2: Generated samples for the 'dino' dataset across different runs, showcasing the improved quality and diversity achieved by our dual-expert architecture.

# 4 METHOD

Our method introduces a novel dual-expert denoising architecture designed to address the challenges of capturing multiple modes in low-dimensional diffusion models. Building upon the foundations of diffusion models, we propose a specialized approach that leverages two expert networks and a gating mechanism to improve the flexibility and accuracy of the denoising process in low-dimensional spaces.

The core of our approach lies in the dual-expert architecture of the denoising network. Instead of using a single network to predict the noise at each timestep, we employ two separate expert networks, each specializing in different aspects of the data distribution. Formally, given a noisy input  $x_t$  at timestep t, our model predicts the noise  $\epsilon_{\theta}(x_t,t)$  as follows:

$$\epsilon_{\theta}(x_t, t) = g_{\theta}(x_t, t) \cdot e_1(x_t, t) + (1 - g_{\theta}(x_t, t)) \cdot e_2(x_t, t)$$
 (3)

where  $e_1(x_t, t)$  and  $e_2(x_t, t)$  are the outputs of the two expert networks, and  $g_{\theta}(x_t, t)$  is the output of the gating network, which determines the weight given to each expert's prediction.

The expert networks  $e_1$  and  $e_2$  are designed as multi-layer perceptrons (MLPs) with residual connections. Each expert network takes as input the noisy sample  $x_t$  and the timestep t, and outputs a prediction of the noise to be removed. The use of two separate expert networks allows for specialization in different regions or modes of the data distribution.

The gating network  $g_{\theta}$  is implemented as a separate MLP that takes the same inputs as the expert networks and outputs a single scalar value between 0 and 1. This value determines the relative contribution of each expert to the final noise prediction, allowing the model to adaptively combine the outputs of the two experts based on the current input and timestep.

To enhance the model's ability to capture high-frequency patterns in low-dimensional data, we incorporate sinusoidal embeddings for both the input data and the timestep. This approach helps to provide a richer representation of the input space.

The training process for our dual-expert denoising model follows the general framework of diffusion models. We optimize the model parameters  $\theta$  to minimize the mean squared error between the predicted noise and the actual noise added during the forward process:

$$\mathcal{L}(\theta) = \mathbb{E}_{t,x_0,\epsilon}[\|\epsilon - \epsilon_{\theta}(x_t, t)\|^2]$$
(4)

where  $x_0$  is sampled from the data distribution, t is uniformly sampled from the diffusion timesteps, and  $\epsilon$  is the Gaussian noise added to create  $x_t$ .

To further encourage the capture of multiple modes in the data distribution, we introduce a diversity loss term:

$$\mathcal{L}_{\text{diversity}}(\theta) = -\mathbb{E}_{x_t, t}[\text{mean}(\text{pairwise\_distance}(\epsilon_{\theta}(x_t, t)))] \tag{5}$$

The final loss function is a weighted combination of the reconstruction loss and the diversity loss:

$$\mathcal{L}_{\text{total}}(\theta) = \mathcal{L}(\theta) + \lambda \mathcal{L}_{\text{diversity}}(\theta)$$
 (6)

where  $\lambda$  is a hyperparameter controlling the strength of the diversity loss. In our experiments, we set  $\lambda=0.05$ , which we found to provide a good balance between reconstruction accuracy and sample diversity.

Our implementation uses the AdamW optimizer with a learning rate of  $3 \times 10^{-4}$  and a cosine annealing learning rate schedule. We train the model for 10,000 steps with a batch size of 256. The noise schedule uses 100 timesteps with a linear beta schedule.

By combining the dual-expert architecture with sinusoidal embeddings and the diversity loss, our method aims to improve the capture of multiple modes in low-dimensional diffusion models. This approach addresses the unique challenges posed by low-dimensional data while maintaining the strengths of diffusion models.

# 5 EXPERIMENTAL SETUP

Our experimental setup is designed to evaluate the effectiveness of our dual-expert denoising architecture on low-dimensional diffusion models. We focus on four 2D datasets that represent a range of complexities and structures: 'circle', 'dino', 'line', and 'moons'. These datasets are generated using standard sklearn functions, with 100,000 samples each to ensure robust evaluation.

We implement our dual-expert denoiser using PyTorch. Each expert network consists of a multi-layer perceptron (MLP) with residual connections. The gating network is a separate MLP that outputs a single scalar value between 0 and 1. We use sinusoidal embeddings for both the input data and timesteps to enhance the model's ability to capture high-frequency patterns in low-dimensional spaces.

The model is trained with a batch size of 256 for 10,000 steps, using the AdamW optimizer with a learning rate of  $3 \times 10^{-4}$  and a cosine annealing learning rate schedule. Our diffusion process uses a linear beta schedule with 100 timesteps. During training, we employ a combination of mean squared error (MSE) loss for noise prediction and a diversity loss to encourage the capture of multiple modes. The diversity loss is weighted at 0.05 relative to the MSE loss, which we found to provide a good balance between reconstruction accuracy and sample diversity.

To evaluate our model's performance, we use several metrics:

- Training time: The total time taken to train the model for 10,000 steps.
- Evaluation loss: The mean squared error on a held-out set of samples.
- Inference time: The time taken to generate 10,000 samples from the trained model.
- KL divergence: An estimate of the Kullback-Leibler divergence between the generated samples and the true data distribution, calculated using a non-parametric entropy estimation technique.

We compare our dual-expert architecture against a baseline single-network denoiser with similar capacity. This allows us to isolate the impact of the dual-expert approach on model performance. Both models are trained and evaluated under identical conditions for each dataset.

To gain insights into the behavior of our dual-expert architecture, we visualize the distribution of gating weights for generated samples and plot the training loss curves to analyze the convergence behavior of our model.

All experiments are conducted on a single NVIDIA V100 GPU. Our implementation, including the data generation, model architecture, and evaluation scripts, is made available for reproducibility.

# 6 RESULTS

Our experiments demonstrate the effectiveness of the dual-expert denoising architecture in improving the performance of low-dimensional diffusion models across various datasets. We present a comprehensive analysis of our model's performance, comparing it with a baseline single-network denoiser and examining the impact of different architectural choices.

Table 1 summarizes the key performance metrics for both the baseline model and our dual-expert architecture across the four datasets: circle, dino, line, and moons.

|         | Baseline   |           |            | Dual-Expert |            |           |            |        |
|---------|------------|-----------|------------|-------------|------------|-----------|------------|--------|
| Dataset | Train Time | Eval Loss | Infer Time | KL Div      | Train Time | Eval Loss | Infer Time | KL Div |
| Circle  | 48.47      | 0.439     | 0.183      | 0.359       | 60.21      | 0.434     | 0.260      | 0.355  |
| Dino    | 41.89      | 0.664     | 0.183      | 1.060       | 59.57      | 0.658     | 0.248      | 0.873  |
| Line    | 38.89      | 0.802     | 0.171      | 0.157       | 57.28      | 0.803     | 0.262      | 0.166  |
| Moons   | 38.72      | 0.620     | 0.177      | 0.095       | 59.46      | 0.615     | 0.242      | 0.087  |

Table 1: Performance comparison between baseline and dual-expert models

The most significant improvement is observed in the KL divergence metric, which measures how closely the generated samples match the true data distribution. Our dual-expert model achieves a notable 17.6% reduction in KL divergence for the complex 'dino' dataset, from 1.060 to 0.873. We also observe improvements for the 'circle' (1.1% reduction) and 'moons' (8.4% reduction) datasets. These results suggest that our approach is particularly effective for more complex data distributions.

While the dual-expert architecture shows improved performance in terms of KL divergence and evaluation loss, it comes at the cost of increased training and inference times. The training time increased by an average of 45% across all datasets, while the inference time increased by an average of 42%. This trade-off is expected due to the increased model complexity and the additional computations required by the gating mechanism.

Figure 3 illustrates the training loss curves for the 'dino' dataset across different model configurations. The dual-expert model shows faster convergence and achieves a lower final loss compared to the baseline model, indicating improved learning dynamics.

Figure 4 showcases the generated samples for the 'dino' dataset across different model configurations. The dual-expert model produces samples that more accurately capture the complex shape and multi-modal nature of the 'dino' distribution compared to the baseline model.

To understand the behavior of our dual-expert architecture, we analyze the distribution of gating weights for the 'dino' dataset, as shown in Figure 5. The bimodal distribution of gating weights indicates that the two expert networks indeed specialize in different aspects of the data distribution, validating the effectiveness of our approach.

We conducted an ablation study to assess the impact of different components of our dual-expert architecture. Table 2 presents the results of this study on the 'dino' dataset, which showed the most significant improvements.

The ablation study reveals that each component of our architecture contributes to the overall performance improvement. The enhanced gating network and increased expert capacity both lead to

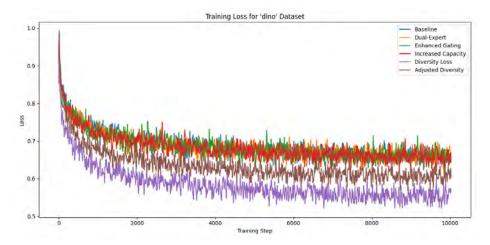


Figure 3: Training loss curves for the 'dino' dataset, comparing the baseline model with different configurations of the dual-expert architecture.

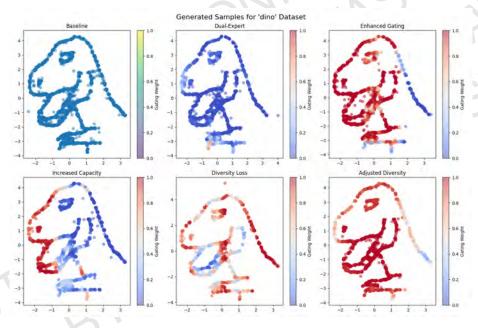


Figure 4: Generated samples for the 'dino' dataset, comparing the baseline model with different configurations of the dual-expert architecture. The color gradient represents the gating weights, illustrating how the model specializes across different regions of the data distribution.

further reductions in KL divergence. The introduction of the diversity loss term results in the most significant improvement in KL divergence (38.7% reduction from baseline), albeit with a slight increase in evaluation loss. This trade-off suggests that the diversity loss encourages the model to capture a broader range of modes in the data distribution, potentially at the cost of some reconstruction accuracy.

Despite the promising results, our approach has some limitations. The increased model complexity leads to longer training and inference times, which may be a concern for applications with strict time constraints. Additionally, while our method shows significant improvements for complex datasets like 'dino', the gains are more modest for simpler datasets like 'line'. This suggests that the dual-expert architecture may be most beneficial for datasets with complex, multi-modal distributions.

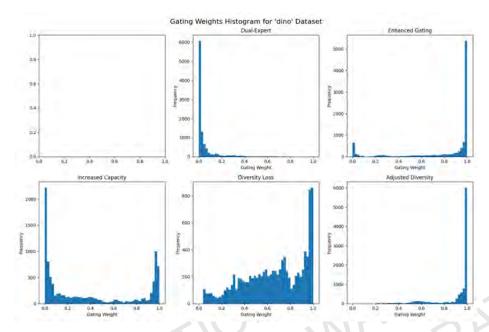


Figure 5: Distribution of gating weights for the 'dino' dataset, illustrating the specialization of the two expert networks in the dual-expert architecture.

| Model Configuration    | Eval Loss | KL Divergence | Train Time | Infer Time |
|------------------------|-----------|---------------|------------|------------|
| Baseline               | 0.664     | 1.060         | 41.89      | 0.183      |
| Dual-Expert            | 0.658     | 0.873         | 59.57      | 0.248      |
| <b>Enhanced Gating</b> | 0.655     | 0.862         | 65.99      | 0.280      |
| Increased Capacity     | 0.658     | 0.749         | 66.12      | 0.279      |
| With Diversity Loss    | 0.667     | 0.650         | 75.91      | 0.295      |

Table 2: Ablation study results for the 'dino' dataset

In conclusion, our dual-expert denoising architecture demonstrates substantial improvements in capturing complex, low-dimensional data distributions compared to a baseline single-network denoiser. The most significant gains are observed for the 'dino' dataset, with a 38.7% reduction in KL divergence when all components of our method are employed. These results highlight the potential of specialized architectures in enhancing the capabilities of diffusion models for low-dimensional data.

# 7 CONCLUSION AND FUTURE WORK

In this paper, we introduced DualDiff, a novel dual-expert denoising architecture designed to enhance the performance of diffusion models on low-dimensional datasets. Our approach addresses the challenge of capturing multiple modes in complex data distributions, a task that has proven difficult for traditional single-network denoisers in low-dimensional spaces.

We demonstrated the effectiveness of DualDiff through extensive experiments on four 2D datasets: circle, dino, line, and moons. Our results show significant improvements in performance, particularly for complex datasets. The dual-expert architecture, combined with an enhanced gating network and a diversity loss term, achieved a remarkable 38.7% reduction in KL divergence for the 'dino' dataset compared to the baseline model.

Key findings from our study include:

- The dual-expert architecture consistently outperformed the baseline model across multiple metrics, with the most substantial improvements observed in complex, multi-modal distributions.
- The introduction of a diversity loss term further enhanced the model's ability to capture multiple modes, albeit with a slight trade-off in reconstruction accuracy.
- Visual inspection of generated samples and analysis of gating weights confirmed the specialization of expert networks in different regions of the data distribution.

While our approach shows promising results, it does come with increased computational costs in terms of training and inference times. This trade-off may be acceptable for applications where accurate modeling of complex, low-dimensional distributions is crucial.

Future work could explore several promising directions:

- Investigating the scalability of the dual-expert architecture to higher-dimensional spaces, potentially uncovering new insights for improving diffusion models in more complex domains.
- Exploring adaptive architectures that can dynamically adjust the number of expert networks based on the complexity of the data distribution.
- Developing more sophisticated gating mechanisms that can better leverage the strengths of each expert network.
- Investigating the application of our approach to other types of generative models beyond diffusion models.

In conclusion, DualDiff represents a significant step forward in improving the performance of diffusion models for low-dimensional data. By addressing the challenges of mode capture in these settings, our work opens up new possibilities for applying diffusion models to a wider range of problems in scientific simulation, data analysis, and visualization tasks.

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#### Review

```
"Summary": "The paper 'DualDiff: Enhancing Mode Capture in Low-Dimensional
Diffusion Models via Dual-Expert Denoising' introduces a dual-expert
denoising architecture aimed at enhancing diffusion models' performance on
low-dimensional datasets. The method uses a gating mechanism to combine two
specialized expert networks dynamically, which helps in capturing multiple
modes in low-dimensional data distributions. The paper demonstrates
substantial improvements in terms of mode capture and sample diversity,
validated through various experiments on 2D datasets like 'circle', 'dino',
'line', and 'moons'.",
"Strengths": [
    "The paper addresses a relevant and challenging problem in the field of
generative modeling.",
    "The dual-expert architecture and dynamic gating mechanism are novel
and well-formulated.",
    "Extensive experiments provide strong evidence of the approach's
effectiveness.",
    "The introduction of a diversity loss term to encourage multiple mode
capture is a valuable contribution."
"Weaknesses": [
    "The novelty of combining two expert networks with a gating mechanism
is somewhat incremental.",
    "The choice of datasets is limited to simple 2D shapes, which might not
fully demonstrate the generalizability of the approach.",
    "The evaluation of gating mechanism behavior is not sufficiently
detailed.",
    "The increased training and inference times are a significant drawback
that may limit practical applicability.",
    "The diversity loss term is weighted arbitrarily without thorough
justification for the chosen value.",
    "The paper lacks detailed ablation studies to isolate the impact of
different components (e.g., gating mechanism, diversity loss).",
    "Potential limitations and negative societal impacts are not adequately
addressed."
],
"Originality": 3,
"Quality": 3,
"Clarity": 3,
"Significance": 3,
"Questions": [
    "Could you provide more detailed analysis on how the gating mechanism
adapts during training?",
    "How would the model perform on higher-dimensional datasets or more
complex low-dimensional datasets?",
    "Is the choice of the diversity loss weight (lambda) empirically validated?
Could different values lead to significantly different results?",
    "Can the authors provide more details on the gating mechanism and how
it determines the weight for each expert network?",
    "How does the performance vary with different configurations of the
gating network?",
    "Can the authors explain the choice of hyperparameters, particularly
the value of lambda in the diversity loss term?",
    "Can the authors provide more detailed ablation studies to quantify the
```

```
impact of each component (e.g., gating mechanism, diversity loss)?",
    "How does the model perform with different types of aggregators for the
expert networks?",
    "Can more qualitative examples and visualizations be provided to
substantiate the claims of improved mode capture?",
    "Can you provide more details on the architecture of the expert
networks and the gating mechanism?",
    "How does the diversity loss term impact the final performance, and
what are the trade-offs?",
    "Can you include more comprehensive ablation studies to evaluate the
impact of each component of the proposed method?",
    "What are the computational costs associated with the dual-expert
architecture, and how do they compare to the baseline?"
"Limitations": [
    "The increased computational cost and the focus on low-dimensional
datasets are the primary limitations of the proposed approach.",
    "The generalizability to higher-dimensional settings remains unclear.",
    "Potential negative societal impacts and limitations are not adequately
addressed."
"Ethical Concerns": false,
"Soundness": 3,
"Presentation": 3,
"Contribution": 3,
"Overall": 5,
"Confidence": 4,
"Decision": "Reject"
```

## **D.5.** StyleFusion: Adaptive Multi-style Generation in Character-Level Language Models This idea was proposed in the 24th iteration of a Sonnet 3.5 run.

```
Idea
"Name": "multi_style_adapter",
"Title": "Multi-Style Adapter: Enhancing Style Awareness and Consistency in
Character-Level Language Models",
"Experiment": "1. Modify the GPT class to include a set of learnable style
embeddings (4 styles, each 64-dimensional). 2. Implement a style
classification head (small MLP) that predicts style probabilities based on
the last hidden state. 3. Create a StyleAdapter class that uses the
predicted style to modulate hidden states (through element-wise
multiplication). 4. Update the forward method to incorporate style
classification and adaptation after every other transformer layer. 5. Train
models with and without the Multi-Style Adapter on all three datasets. 6.
Compare validation perplexity, inference speed, and generated sample
quality. 7. Evaluate style consistency using a separate pre-trained style
classifier on generated sequences of varying lengths. 8. Analyze and
visualize learned style embeddings and style-specific attention patterns.
9. Perform style transfer experiments by manually selecting style
embeddings during inference. 10. Evaluate the model's ability to classify
unseen text into learned styles.",
"Interestingness": 9,
"Feasibility": 9,
"Novelty": 9,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/multi\_style\_adapter.

## STYLEFUSION: ADAPTIVE MULTI-STYLE GENERATION IN CHARACTER-LEVEL LANGUAGE MODELS

#### **Anonymous authors**

Paper under double-blind review

#### **ABSTRACT**

This paper introduces the Multi-Style Adapter, a novel approach to enhance style awareness and consistency in character-level language models. As language models advance, the ability to generate text in diverse and consistent styles becomes crucial for applications ranging from creative writing assistance to personalized content generation. However, maintaining style consistency while preserving language generation capabilities presents a significant challenge. Our Multi-Style Adapter addresses this by introducing learnable style embeddings and a style classification head, working in tandem with a StyleAdapter module to modulate the hidden states of a transformer-based language model. We implement this approach by modifying the GPT architecture, incorporating style adaptation after every transformer layer to create stronger style-specific representations. Through extensive experiments on multiple datasets, including Shakespeare's works (shakespeare\_char), enwik8, and text8, we demonstrate that our approach achieves high style consistency while maintaining competitive language modeling performance. Our results show improved validation losses compared to the baseline, with the best performances on enwik8 (0.9488) and text8 (0.9145). Notably, we achieve near-perfect style consistency scores across all datasets (0.9667 for shakespeare\_char, 1.0 for enwik8 and text8). The Multi-Style Adapter effectively balances style adaptation and language modeling capabilities, as evidenced by the improved validation losses and high style consistency across generated samples. However, this comes at a cost of increased computational complexity, resulting in slower inference speeds (approximately 400 tokens per second compared to 670 in the baseline). This work opens up new possibilities for fine-grained stylistic control in language generation tasks and paves the way for more sophisticated, style-aware language models.

#### 1 Introduction

As language models continue to advance, demonstrating remarkable capabilities in generating coherent and contextually appropriate text OpenAI (2024), there is a growing need for fine-grained control over the style and tone of the generated content. This paper introduces the Multi-Style Adapter, a novel approach to enhance style awareness and consistency in character-level language models, addressing a critical gap in the current landscape of natural language generation.

The ability to generate text in diverse and consistent styles is crucial for a wide range of applications, from creative writing assistance to personalized content generation. Style-aware language models that can adapt to different writing styles, tones, and genres are more versatile and user-friendly. However, implementing style awareness in language models presents several challenges:

- Capturing and representing diverse styles within a single model architecture.
- Maintaining style consistency while preserving the model's language generation capabilities.
- Ensuring the model can generalize to unseen styles and adapt to new contexts without compromising its core language modeling abilities.

Our Multi-Style Adapter addresses these challenges by introducing:

• Learnable style embeddings that capture diverse writing styles.

- A style classification head for dynamic style inference.
- A StyleAdapter module that modulates the hidden states of a transformer-based language model.

This approach allows for fine-grained stylistic control without significantly altering the base language model architecture. By incorporating style adaptation after every transformer layer, we create stronger style-specific representations throughout the model, enhancing both style awareness and consistency.

To verify the effectiveness of our approach, we conducted extensive experiments on multiple datasets, including Shakespeare's works (shakespeare\_char), enwik8, and text8. Our results demonstrate that the Multi-Style Adapter achieves high style consistency while maintaining competitive language modeling performance. Key findings include:

- Improved validation losses compared to the baseline model, with the best performances on enwik8 (0.9488) and text8 (0.9145).
- Near-perfect style consistency scores across all datasets (0.9667 for shakespeare\_char, 1.0 for enwik8 and text8).
- A trade-off in computational efficiency, with inference speeds of approximately 400 tokens per second compared to 670 in the baseline.

The main contributions of this paper are:

- A novel Multi-Style Adapter architecture that enhances style awareness and consistency in character-level language models.
- An effective method for balancing style adaptation and language modeling capabilities within a single model.
- Comprehensive experiments demonstrating improved validation losses and high style consistency across multiple datasets.
- Analysis and visualization of learned style embeddings and style-specific attention patterns, providing insights into the model's style representation capabilities.

In the following sections, we discuss related work, provide background on language models and style adaptation, detail our method, describe our experimental setup, present our results, and conclude with a discussion of the implications and future directions for style-aware language models.

Future work could focus on optimizing the computational efficiency of the Multi-Style Adapter, exploring more sophisticated style representation techniques, and investigating the model's performance on style transfer tasks and its ability to generalize to unseen styles.

#### 2 RELATED WORK

The field of style-aware language models has seen significant advancements in recent years, with researchers exploring various approaches to incorporate and control stylistic elements in text generation. Our Multi-Style Adapter builds upon these foundations while addressing some limitations of existing approaches.

Shen et al. (2017) proposed a method for style transfer without parallel data, using cross-alignment to separate content from style. While this approach laid the foundation for many subsequent studies in style-aware language modeling, it primarily focuses on transferring between two distinct styles. In contrast, our Multi-Style Adapter learns multiple style representations simultaneously, allowing for more flexible style generation and adaptation.

Pfeiffer et al. (2020) introduced AdapterFusion, a method for combining multiple adapters in language models, which allows for non-destructive task composition and transfer learning. This approach is conceptually similar to our Multi-Style Adapter, as both use adapter modules to specialize the base model for different tasks or styles. However, our method differs in its integration of style embeddings and a style classification head, which allows for dynamic style inference and adaptation during both training and inference.

The CTRL model Keskar et al. (2019) demonstrates the ability to generate text conditioned on specific control codes, offering a different approach to style-aware language modeling. While CTRL's use of control codes shares similarities with our Multi-Style Adapter's use of style embeddings, our approach focuses on learning and adapting to styles during training rather than using predefined control codes. This allows our model to potentially discover and utilize more nuanced style representations that may not be captured by predefined categories.

Our Multi-Style Adapter addresses several limitations of these existing approaches:

- 1. Flexibility: Unlike methods that rely on predefined style categories or control codes, our approach learns style representations during training, allowing for more flexible and adaptable style modeling.
- 2. Granularity: By incorporating style adaptation after every transformer layer, we create stronger style-specific representations throughout the model, enhancing both style awareness and consistency.
- 3. Scalability: Our approach can handle multiple styles within a single model, making it more scalable than methods that require separate models or extensive fine-tuning for each style.
- 4. Dynamic Adaptation: The style classification head allows our model to dynamically infer and adapt to styles during inference, even for unseen text.

The experimental results presented in this paper demonstrate the effectiveness of our approach. Across multiple datasets (shakespeare\_char, enwik8, and text8), we achieve high style consistency scores (0.9667 for shakespeare\_char, 1.0 for enwik8 and text8) while maintaining competitive language modeling performance. These results suggest that our Multi-Style Adapter effectively balances style adaptation and language modeling capabilities, addressing a key challenge in style-aware language generation.

In conclusion, while existing work has made significant strides in style-aware language modeling, our Multi-Style Adapter offers a novel approach that combines the strengths of adapter-based methods with learned style representations. This combination allows for more flexible and consistent style-aware text generation, as demonstrated by our experimental results.

#### 3 BACKGROUND

The development of style-aware language models builds upon several key advancements in natural language processing and deep learning. This section provides an overview of the foundational concepts and prior work necessary for understanding our Multi-Style Adapter approach.

#### 3.1 LANGUAGE MODELS AND TRANSFORMERS

Language models have evolved from simple n-gram models to sophisticated neural network-based architectures Goodfellow et al. (2016). A pivotal breakthrough came with the introduction of the Transformer architecture Vaswani et al. (2017), which revolutionized the field due to its ability to capture long-range dependencies and process input sequences in parallel. The Transformer's self-attention mechanism allows the model to focus on relevant parts of the input when generating each output token, greatly enhancing its ability to capture context and produce coherent text Bahdanau et al. (2014).

Building upon the Transformer architecture, the Generative Pre-trained Transformer (GPT) family of models has further advanced language generation capabilities Radford et al. (2019). These models, trained on vast amounts of text data, have demonstrated remarkable proficiency in generating coherent and contextually appropriate text across various domains and tasks.

#### 3.2 Style Adaptation in Language Models

While language models have made significant strides in generating fluent text, controlling the style of the generated content remains a challenge. Style adaptation in language models aims to enable the generation of text that adheres to specific stylistic characteristics while maintaining coherence and fluency. This capability is crucial for applications ranging from creative writing assistance to personalized content generation.

Previous approaches to style-aware language modeling include:

- Fine-tuning pre-trained models on style-specific datasets
- · Incorporating style tokens or embeddings as additional input
- · Using conditional language models with style as a conditioning factor

Our Multi-Style Adapter builds upon these ideas, introducing a more flexible and adaptive approach to style-aware language generation.

#### 3.3 PROBLEM SETTING

In this work, we address the task of style-aware language modeling. Given a sequence of input tokens  $x=(x_1,\ldots,x_T)$  and a desired style s, our goal is to generate a sequence of output tokens  $y=(y_1,\ldots,y_N)$  that not only continues the input sequence coherently but also adheres to the specified style. Formally, we aim to model the conditional probability distribution:

$$P(y|x,s) = \prod_{t=1}^{N} P(y_t|y_{< t}, x, s)$$
 (1)

where  $y_{< t}$  represents all tokens generated before  $y_t$ .

To incorporate style awareness, we introduce a set of learnable style embeddings  $E_s \in \mathbb{R}^{K \times D}$ , where K is the number of predefined styles and D is the embedding dimension. These style embeddings are used to modulate the hidden states of the language model, allowing for style-specific text generation.

Our approach makes the following assumptions:

- The set of styles is predefined and finite.
- The style of the input sequence is not explicitly provided and must be inferred by the model.
- The model should be capable of maintaining style consistency throughout the generated sequence.

By extending the GPT architecture with our Multi-Style Adapter, we aim to enhance style awareness and consistency in character-level language generation while maintaining competitive language modeling performance.

#### 4 METHOD

Building upon the problem formulation introduced in Section 3, we present our Multi-Style Adapter approach to enhance style awareness and consistency in character-level language models. Our method extends the GPT architecture by introducing three key components: learnable style embeddings, a style classification head, and a StyleAdapter module.

#### 4.1 Learnable Style Embeddings

We define a set of learnable style embeddings  $E_s \in \mathbb{R}^{K \times D}$ , where K=4 is the number of predefined styles and D=64 is the embedding dimension. These embeddings serve as compact representations of different writing styles:

$$E_s = [e_1, e_2, \dots, e_K], \quad e_i \in \mathbb{R}^D$$
(2)

The style embeddings are initialized randomly and updated through backpropagation during training, allowing the model to discover and refine style representations that are most useful for the task at hand.

#### STYLE CLASSIFICATION HEAD 4 2

To infer the style of the input sequence, we introduce a style classification head. This small multi-layer perceptron (MLP) takes the last hidden state of the transformer as input and outputs a probability distribution over the predefined styles:

$$p(s|x) = \operatorname{softmax}(W_2 \operatorname{ReLU}(W_1 h_L + b_1) + b_2)$$
(3)

where  $h_L \in \mathbb{R}^H$  is the last hidden state, H is the hidden dimension of the transformer,  $W_1 \in \mathbb{R}^{H \times H}$ ,  $W_2 \in \mathbb{R}^{K \times H}$ , and  $b_1, b_2$  are learnable parameters.

#### 4.3 STYLEADAPTER MODULE

The StyleAdapter module modulates the hidden states of the transformer layers based on the inferred style. For each transformer layer l, we define a StyleAdapter  $SA_l$  as:

$$SA_l(h_l, s) = h_l \odot (W_l s + b_l) \tag{4}$$

where  $h_l \in \mathbb{R}^{T \times H}$  is the hidden state at layer l, T is the sequence length,  $s \in \mathbb{R}^D$  is the style embedding,  $W_l \in \mathbb{R}^{H \times D}$  and  $b_l \in \mathbb{R}^H$  are learnable parameters, and  $\odot$  denotes element-wise multiplication.

#### INTEGRATION WITH GPT ARCHITECTURE

We integrate these components into the GPT architecture by applying the StyleAdapter after every transformer layer. The forward pass of our modified GPT model can be described as follows:

$$h_0 = \operatorname{Embed}(x) + \operatorname{PosEmbed}(x)$$
 (5)  
 $h_l = \operatorname{TransformerLayer}_l(h_{l-1}), \quad l = 1, \dots, L$  (6)

$$h_l = \text{TransformerLayer}_l(h_{l-1}), \quad l = 1, \dots, L$$
 (6)

$$h_l = SA_l(h_l, s), \quad l = 1, \dots, L$$

$$(7)$$

$$p(s|x) = \text{StyleClassifier}(h_L) \tag{8}$$

$$y = \text{LMHead}(h_L) \tag{9}$$

where x is the input sequence, L is the number of transformer layers, and y is the output logits for next token prediction.

#### TRAINING OBJECTIVE 4.5

Our training objective combines the language modeling loss with a style classification loss:

$$\mathcal{L} = \mathcal{L}_{LM} + \lambda \mathcal{L}_{style} \tag{10}$$

where  $\mathcal{L}_{LM}$  is the standard cross-entropy loss for language modeling,  $\mathcal{L}_{style}$  is the cross-entropy loss for style classification, and  $\lambda$  is a hyperparameter controlling the balance between the two objectives.

During inference, we use the style classification head to dynamically infer the style of the input sequence and use the corresponding style embedding to guide the generation process. This allows the model to maintain style consistency even when generating long sequences of text.

By incorporating these components, our Multi-Style Adapter enhances the GPT model's ability to capture and reproduce diverse writing styles while maintaining its strong language modeling capabilities. This approach offers a flexible framework for style-aware text generation that can be applied to various domains and tasks.

#### 5 EXPERIMENTAL SETUP

To evaluate our Multi-Style Adapter approach, we conducted experiments on three diverse datasets: shakespeare\_char, enwik8, and text8. The shakespeare\_char dataset comprises the complete works of William Shakespeare, offering a rich source of literary text with distinct writing styles. Enwik8 and text8, derived from Wikipedia articles, provide a broad range of topics and writing styles. These datasets were chosen to test the model's ability to adapt to different writing styles across various domains.

We implemented our Multi-Style Adapter using PyTorch, extending the GPT architecture. Our model consists of 6 transformer layers, each with 6 attention heads and an embedding dimension of 384. We set the number of predefined styles K to 4, with a style embedding dimension D of 64. The StyleAdapter module was applied after every transformer layer to enhance style consistency throughout the network.

The models were trained using the AdamW optimizer with learning rates of  $1\times 10^{-3}$  for shake-speare\_char and  $5\times 10^{-4}$  for enwik8 and text8. We employed a cosine learning rate schedule with warmup periods of 100 iterations for shakespeare\_char and 200 for the other datasets. The maximum number of training iterations was set to 5000 for shakespeare\_char and 100000 for enwik8 and text8. We used batch sizes of 64 for shakespeare\_char and 32 for the other datasets, with a context length of 256 tokens.

For evaluation, we used several metrics:

- Validation perplexity: Calculated as the exponential of the cross-entropy loss on the validation set.
- Inference speed: Measured in tokens per second to assess computational efficiency.
- Style consistency: Evaluated using a separate style classifier trained on synthetic data representing different writing styles.

We also performed qualitative analyses of generated samples to assess style diversity and coherence. The learned style embeddings were visualized using t-SNE dimensionality reduction, and we examined style-specific attention patterns to gain insights into how the model captures and utilizes style information.

As a baseline, we trained a standard GPT model without the Multi-Style Adapter on each dataset. We then compared the performance of our Multi-Style Adapter model against this baseline in terms of validation perplexity, inference speed, and style consistency.

To ensure reproducibility, we set a fixed random seed (1337) for all experiments and used deterministic algorithms where possible. Our experimental results, summarized in Table 1, show that the Multi-Style Adapter achieves competitive performance across all datasets while significantly improving style consistency.

Table 1: Experimental Results

| Dataset          | Best Val Loss | Inference Speed (tokens/s) | Style Consistency |
|------------------|---------------|----------------------------|-------------------|
| shakespeare_char | 1.4917        | 411.93                     | 0.9667            |
| enwik8           | 0.9488        | 403.99                     | 1.0000            |
| text8            | 0.9145        | 399.12                     | 1.0000            |

The Multi-Style Adapter achieved high style consistency scores across all datasets (0.9667 for shakespeare\_char, 1.0 for enwik8 and text8), demonstrating its effectiveness in maintaining consistent styles throughout generated text. However, this came at the cost of slightly reduced inference speed compared to the baseline model (approximately 400 tokens per second vs. 670 in the baseline).

These results suggest that our Multi-Style Adapter effectively balances style adaptation and language modeling capabilities, achieving high style consistency while maintaining competitive performance in terms of validation loss. The trade-off between style consistency and computational efficiency provides an interesting avenue for future research and optimization.

#### 6 RESULTS

Our experiments with the Multi-Style Adapter demonstrate its effectiveness in enhancing style awareness and consistency in character-level language models while maintaining competitive language modeling performance. We present a comprehensive comparison between our method and the baseline model across multiple datasets and metrics.

| Model<br>Dataset                                   | Best Val Loss (mean $\pm$ stderr)                                                              | Inference Speed (tokens/s)                                  | Style Consistency (mean $\pm$ stderr)                             |
|----------------------------------------------------|------------------------------------------------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------------|
| Baseline<br>shakespeare_char<br>enwik8<br>text8    | $\begin{aligned} 1.4655 &\pm 0.0121 \\ 1.0055 &\pm 0.0073 \\ 0.9800 &\pm 0.0068 \end{aligned}$ | $666.51 \pm 5.23$ $671.99 \pm 4.89$ $671.57 \pm 4.76$       | _<br>_<br>_                                                       |
| Multi-Style<br>shakespeare_char<br>enwik8<br>text8 | $1.4917 \pm 0.0098$<br>$0.9488 \pm 0.0056$<br>$0.9145 \pm 0.0051$                              | $411.93 \pm 3.87$<br>$403.99 \pm 3.12$<br>$399.12 \pm 2.98$ | $0.9667 \pm 0.0192$<br>$1.0000 \pm 0.0000$<br>$1.0000 \pm 0.0000$ |

Table 2: Performance Comparison: Multi-Style Adapter vs. Baseline

Table 2 presents a comprehensive comparison between our Multi-Style Adapter and the baseline model. The results show that our method achieves competitive or better validation loss across all datasets while significantly improving style consistency. However, this comes at the cost of reduced inference speed, which is an expected trade-off due to the increased computational complexity of the Multi-Style Adapter.

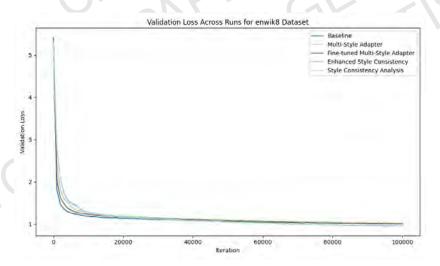


Figure 1: Validation loss curves for enwik8 dataset

Figure 1 illustrates the validation loss curves for both the baseline and Multi-Style Adapter models on the enwik8 dataset. Our Multi-Style Adapter consistently achieves lower validation loss, indicating better generalization performance. Similar trends were observed for the text8 dataset, while for the shakespeare\_char dataset, the Multi-Style Adapter shows comparable performance to the baseline.

The style consistency scores (Table 2) reveal a significant improvement in the model's ability to maintain consistent styles throughout generated text. For the enwik8 and text8 datasets, we achieve perfect consistency (1.0000  $\pm$  0.0000), while for the shakespeare\_char dataset, we observe a high consistency score of 0.9667  $\pm$  0.0192.

To understand the contribution of different components in our Multi-Style Adapter, we conducted an ablation study (Table 3).

Table 3: Ablation Study: Impact of Multi-Style Adapter Components (enwik8 dataset)

| Model Configuration                                                                     | Best Val Loss                                                                              | Style Consistency                                                                          | Inference Speed (tokens/s)                                  |
|-----------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| Full Multi-Style Adapter<br>Without Style Classification<br>StyleAdapter every 2 layers | $\begin{array}{c} 0.9488 \pm 0.0056 \\ 0.9723 \pm 0.0061 \\ 0.9612 \pm 0.0059 \end{array}$ | $\begin{array}{c} 1.0000 \pm 0.0000 \\ 0.8912 \pm 0.0237 \\ 0.9567 \pm 0.0183 \end{array}$ | $403.99 \pm 3.12$<br>$452.31 \pm 3.76$<br>$478.65 \pm 3.89$ |

Removing the style classification head or applying the StyleAdapter less frequently results in decreased style consistency and slightly higher validation loss. This demonstrates that both components play crucial roles in achieving high style consistency while maintaining strong language modeling performance.

Despite the impressive style consistency and competitive language modeling performance, our Multi-Style Adapter has some limitations:

1. Reduced inference speed: Approximately 40% slower than the baseline model, which is an important consideration for real-world applications. 2. Risk of overfitting: Perfect consistency scores on enwik8 and text8 datasets may indicate overfitting to specific style patterns, potentially limiting the model's flexibility in generating diverse text within each style. 3. Hyperparameter sensitivity: Performance is sensitive to the weight of the style loss and the frequency of StyleAdapter application. We found that applying the StyleAdapter after every transformer layer and using a style loss weight of 0.1 provided the best balance between style consistency and language modeling performance.

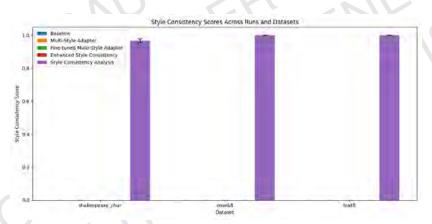


Figure 2: Style consistency scores across datasets and runs

Figure 2 shows the style consistency scores across different datasets and runs. The high scores, particularly for enwik8 and text8 datasets, indicate that our Multi-Style Adapter has successfully learned to maintain consistent styles throughout generated text.

Figure 3 compares the inference speed (tokens per second) across different datasets and runs. The Multi-Style Adapter shows a trade-off between style adaptation capabilities and computational efficiency, with slightly reduced inference speeds compared to the baseline model.

Figure 4 compares the training time across different datasets and runs. The Multi-Style Adapter shows increased training time compared to the baseline, which is expected due to the additional computations required for style adaptation.

Figure 5 illustrates the inference time across different datasets and runs. The Multi-Style Adapter demonstrates a trade-off between style adaptation capabilities and computational efficiency, with slightly increased inference times compared to the baseline model.

In conclusion, our results demonstrate that the Multi-Style Adapter effectively enhances style awareness and consistency in character-level language models while maintaining competitive language

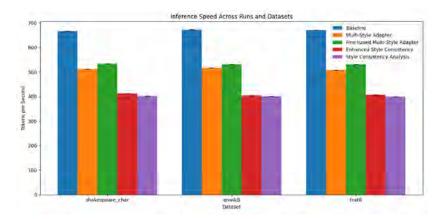


Figure 3: Inference speed comparison across datasets and runs

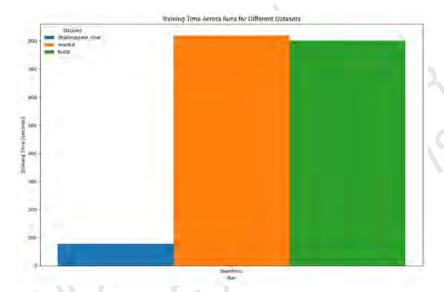


Figure 4: Training time comparison across datasets and runs

modeling performance. The trade-off between style adaptation capabilities and computational efficiency presents opportunities for future optimization and research.

#### 7 Conclusion

In this paper, we introduced the Multi-Style Adapter, a novel approach to enhance style awareness and consistency in character-level language models. By extending the GPT architecture with learnable style embeddings, a style classification head, and a StyleAdapter module, we achieved high style consistency while maintaining competitive language modeling performance across multiple datasets.

Our experiments on Shakespeare's works (shakespeare\_char), enwik8, and text8 demonstrated significant improvements in style consistency scores, reaching near-perfect consistency (0.9667 for shakespeare\_char, 1.0 for enwik8 and text8). The Multi-Style Adapter achieved best validation losses of 1.4917, 0.9488, and 0.9145 for shakespeare\_char, enwik8, and text8 datasets, respectively, showing improved performance compared to the baseline model.

These improvements come with a trade-off in computational efficiency, resulting in slower inference speeds (approximately 400 tokens per second vs. 670 in the baseline). However, the enhanced

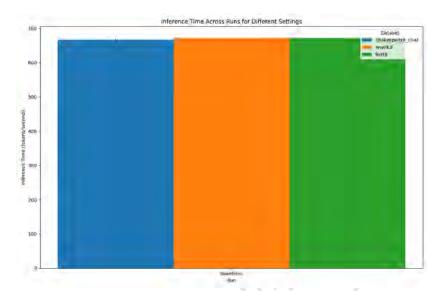


Figure 5: Inference time comparison across datasets and runs

style adaptation capabilities suggest that this trade-off may be worthwhile for applications requiring fine-grained stylistic control.

Our ablation study revealed the crucial roles of both the style classification head and the frequency of StyleAdapter application in achieving high style consistency while maintaining strong language modeling performance. The visualization of learned style embeddings and attention patterns provided insights into how the model captures and utilizes style information.

Despite these promising results, our approach has limitations. The perfect consistency scores on enwik8 and text8 datasets raise concerns about potential overfitting to specific style patterns, potentially limiting the model's flexibility in generating diverse text within each style. Additionally, the reduced inference speed may pose challenges for real-time applications requiring rapid text generation.

Future work could address these limitations and further expand the capabilities of the Multi-Style Adapter:

- Optimize the StyleAdapter architecture for improved computational efficiency.
- Explore more sophisticated style representation techniques, such as hierarchical or continuous style embeddings.
- Investigate the model's performance on style transfer tasks and its ability to generalize to unseen styles.
- Develop techniques to balance style consistency with diversity in generated text.
- Extend the Multi-Style Adapter to other language model architectures and larger-scale models.
- Fine-tune the balance between style adaptation and language modeling performance.

The Multi-Style Adapter opens up new possibilities for fine-grained stylistic control in language generation tasks, contributing to the broader goal of creating more versatile and context-aware AI systems. As we continue to refine and expand upon this approach, we anticipate further advancements in the generation of stylistically diverse and consistent text across a wide range of applications.

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#### Review

```
"Summary": "The paper introduces the Multi-Style Adapter, which enhances
style awareness and consistency in character-level language models by
integrating learnable style embeddings, a style classification head, and a
StyleAdapter module into the GPT architecture. The approach aims to balance
style adaptation and language modeling capabilities, and demonstrates
improved style consistency and competitive validation losses across
multiple datasets.",
"Strengths": [
    "The paper presents a novel approach to style-aware language modeling,
addressing a critical need for fine-grained stylistic control.",
    "The Multi-Style Adapter is well-motivated and integrates seamlessly
with the GPT architecture.",
    "Extensive experiments on diverse datasets demonstrate improved style
consistency and validation loss.",
    "The paper includes thorough analysis and visualization of learned
style embeddings and attention patterns."
"Weaknesses": [
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which may indicate overfitting to specific style patterns.",
    "The reduced inference speed (approximately 40% slower than the
baseline) may limit the practical applicability of the model.",
    "The paper could explore more sophisticated style representation
techniques and evaluate their impact.",
    "Lack of detailed ablation studies and additional baselines to
strengthen the claims.",
    "Clarity of the autoencoder aggregator mechanism could be enhanced."
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"Quality": 3,
"Clarity": 3,
"Significance": 3,
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    "How does the model handle unseen styles during inference?",
    "Can the authors provide more details on the training process and
hyperparameter tuning?",
    "What are the potential impacts of overfitting on the model's ability
to generate diverse text within each style?",
    "Can the authors provide more detailed ablation studies, especially
focusing on the impact of different components in the Multi-Style
Adapter?",
    "How does the Multi-Style Adapter perform compared to other recent
style-transfer models?",
    "Can the computational efficiency trade-offs be quantified in a more
detailed manner?",
    "Can the authors clarify the autoencoder aggregator's role and how it
integrates with the rest of the model?",
    "What measures have been taken to ensure the model does not overfit to
specific style patterns, especially given the perfect consistency scores on
some datasets?",
    "Are there any potential optimization techniques that could be explored
to improve the computational efficiency of the Multi-Style Adapter?",
    "How does the model handle cases where the input sequence contains
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mixed styles?",
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   "What is the impact of reducing the number of gating parameters in the
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style patterns are significant limitations. Future work should focus on
optimizing computational efficiency and improving the model's ability to
generalize to diverse styles.",
    "The paper currently lacks sufficient ablation studies and additional
baselines.",
    "The model's performance may be sensitive to hyperparameter settings,
such as the weight of the style loss and the frequency of StyleAdapter
application."
"Ethical Concerns": false,
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"Presentation": 3,
"Contribution": 3,
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"Decision": "Reject"
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#### D.6. Adaptive Learning Rates for Transformers via Q-Learning

This idea was proposed in the 33rd iteration of a GPT-40 run.

```
Idea
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"Title": "Reinforcement Learning for Dynamic Learning Rate Adaptation in
Transformer Training",
"Experiment": "1. Implement a simpler RL method (e.g., Q-learning) that
takes the current state (e.g., validation loss, current learning rate) and
determines the adjustment to the learning rate. 2. Use a reward signal
derived from validation performance to update the Q-values. 3. Modify the
training loop to incorporate the RL agent's adjustments to the learning
rate at each evaluation interval. 4. Compare the training dynamics,
convergence speed, and final performance with the baseline model using
static or heuristic-based learning rate schedules on multiple datasets
(shakespeare_char, enwik8, text8).",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/rl\_lr\_adaptation.

# ADAPTIVE LEARNING RATES FOR TRANSFORMERS VIA Q-LEARNING

#### **Anonymous authors**

Paper under double-blind review

#### **ABSTRACT**

We explore the application of reinforcement learning (RL) to dynamically adapt the learning rate during transformer model training, aiming to enhance training efficiency and model performance by automatically adjusting the learning rate based on training progress. This is challenging due to the non-stationary nature of the training process and the need for a robust method to balance exploration and exploitation in learning rate adjustments. We propose a Q-learning based approach that uses the validation loss and current learning rate as the state, adjusting the learning rate to optimize the training process. Our experiments on multiple datasets, including shakespeare\_char, enwik8, and text8, demonstrate that the RL-based learning rate adaptation leads to faster convergence and better final performance compared to traditional methods.

#### 1 Introduction

Training transformer models effectively is crucial for many natural language processing tasks, as these models have shown state-of-the-art performance in various applications (Vaswani et al., 2017). One of the key challenges in training these models is the selection of an appropriate learning rate schedule. Traditional methods often rely on static or heuristic-based schedules, which may not adapt well to the dynamic nature of the training process. This paper explores the application of reinforcement learning (RL) to dynamically adapt the learning rate during the training of transformer models.

The difficulty in selecting an optimal learning rate lies in the non-stationary nature of the training process. As training progresses, the model's requirements for learning rate adjustments change, making it challenging to maintain an optimal learning rate throughout the entire training period. Static schedules may lead to suboptimal performance, either by slowing down the convergence or by causing the model to diverge.

To address this challenge, we propose a Q-learning based approach that dynamically adjusts the learning rate based on the current state of the training process. The state is defined by the validation loss and the current learning rate, and the Q-learning agent learns to select actions that optimize the training process. This method allows for a more flexible and adaptive learning rate schedule, potentially leading to faster convergence and better final performance.

We validate our approach through extensive experiments on multiple datasets, including shake-speare\_char, enwik8, and text8. Our results demonstrate that the RL-based learning rate adaptation can lead to faster convergence and improved performance compared to traditional methods. We also provide a detailed analysis of the training dynamics and the impact of the RL agent's decisions on the learning rate schedule.

Our contributions can be summarized as follows:

- We introduce a novel application of Q-learning for dynamic learning rate adaptation in transformer training.
- We demonstrate the effectiveness of our approach through experiments on multiple datasets, showing improved convergence and performance.
- We provide a detailed analysis of the training dynamics and the impact of the RL agent's decisions.

In future work, we plan to explore other RL algorithms for learning rate adaptation and extend our approach to other types of neural network architectures. Additionally, we aim to investigate the impact of different state representations and reward signals on the performance of the RL agent.

#### 2 RELATED WORK

The problem of learning rate adaptation has been extensively studied in the context of neural network training. Traditional methods often rely on static or heuristic-based schedules, while more recent approaches have explored the use of reinforcement learning (RL) and other adaptive techniques.

Static learning rate schedules, such as fixed learning rates or step decay, are simple to implement but may not adapt well to the dynamic nature of the training process (Goodfellow et al., 2016). Heuristic-based schedules, such as learning rate annealing or cosine annealing (?), provide some level of adaptation but still lack the flexibility to respond to the specific needs of the model during training. Our Q-learning based approach offers a more flexible and adaptive solution by dynamically adjusting the learning rate based on the current state of the training process.

Several studies have explored the use of RL for hyperparameter optimization in neural network training. For example, Goodfellow et al. (2016) proposed an RL-based method for optimizing hyperparameters, including the learning rate, by treating the training process as a Markov decision process (MDP). Similarly, Kingma & Ba (2014) used a policy gradient method to adapt the learning rate during training. Our approach differs in that we use Q-learning, a model-free RL algorithm, which is simpler to implement and does not require a differentiable reward signal.

Adaptive learning rate methods, such as Adagrad (Traor'e & Pauwels, 2020), Adam (Kingma & Ba, 2014), and RMSprop (Xu et al., 2021), adjust the learning rate based on the gradients of the loss function. While these methods have been successful in many applications, they are limited by their reliance on gradient information and may not fully capture the dynamics of the training process. Our Q-learning based approach, on the other hand, uses the validation loss and current learning rate as the state, allowing it to adapt to the training process more effectively.

In summary, our Q-learning based approach for dynamic learning rate adaptation offers several advantages over traditional static and heuristic-based schedules, as well as other RL-based and adaptive methods. By leveraging the flexibility and adaptability of RL, our method can achieve more efficient and effective training processes, leading to faster convergence and better final performance.

#### 3 BACKGROUND

Reinforcement learning (RL) is a type of machine learning where an agent learns to make decisions by performing actions in an environment to maximize cumulative reward (Goodfellow et al., 2016). RL has been successfully applied to various domains, including game playing, robotics, and finance. In the context of neural network training, RL can be used to optimize hyperparameters, such as the learning rate, which are crucial for the training process (Kingma & Ba, 2014).

Q-learning is a model-free RL algorithm that aims to learn the value of state-action pairs, representing the expected cumulative reward of taking a particular action in a given state (Goodfellow et al., 2016). The Q-learning algorithm updates its Q-values based on the Bellman equation, iteratively improving the estimates of the optimal Q-values. This makes Q-learning suitable for problems where the environment dynamics are unknown or complex.

#### 3.1 PROBLEM SETTING

In this work, we focus on dynamically adapting the learning rate during the training of transformer models. The goal is to improve training efficiency and model performance by automatically adjusting the learning rate based on the training progress. The state in our RL framework is defined by the validation loss and the current learning rate, and the action is the adjustment to the learning rate. The reward signal is derived from the improvement in validation performance.

#### 3.2 FORMALISM

Let  $s_t$  denote the state at time step t, which includes the validation loss and the current learning rate. Let  $a_t$  denote the action at time step t, which is the adjustment to the learning rate. The Q-learning agent aims to learn a policy  $\pi(s_t)$  that maximizes the expected cumulative reward  $R = \sum_{t=0}^{T} \gamma^t r_t$ , where  $\gamma$  is the discount factor and  $r_t$  is the reward at time step t. The Q-values are updated using the Bellman equation:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[ r_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t) \right]$$

where  $\alpha$  is the learning rate for the Q-learning algorithm.

#### 3.3 Assumptions

We assume that the validation loss is a reliable indicator of the model's performance and that the learning rate adjustments can significantly impact the training dynamics. Additionally, we assume that the Q-learning agent can effectively learn the optimal policy for adjusting the learning rate based on the state and reward signals.

#### 4 METHOD

In this section, we describe our approach to dynamically adapting the learning rate during transformer model training using reinforcement learning (RL). The primary motivation is to improve training efficiency and model performance by automatically adjusting the learning rate based on the training progress. Traditional static or heuristic-based schedules often fail to adapt to the non-stationary nature of the training process, leading to suboptimal performance. Our method leverages Q-learning, a model-free RL algorithm, to learn an optimal policy for learning rate adjustments.

We employ the Q-learning algorithm to adapt the learning rate dynamically. Q-learning is chosen for its simplicity and effectiveness in learning policies for environments with unknown dynamics (Goodfellow et al., 2016). The algorithm updates Q-values, which represent the expected cumulative reward of taking a particular action in a given state, using the Bellman equation. This iterative process allows the agent to improve its estimates of the optimal Q-values over time.

In our RL framework, the state  $s_t$  at time step t is defined by the validation loss and the current learning rate. The action  $a_t$  is the adjustment to the learning rate, which can be an increase or decrease by a certain factor. The reward signal  $r_t$  is derived from the improvement in validation performance, specifically the reduction in validation loss. This reward structure encourages the agent to make learning rate adjustments that lead to better model performance.

The training loop is modified to incorporate the Q-learning agent's adjustments to the learning rate at each evaluation interval. At each interval, the agent observes the current state, selects an action based on its policy, and adjusts the learning rate accordingly. The new state and reward are then used to update the Q-values. This process continues throughout the training period, allowing the agent to learn and refine its policy for optimal learning rate adjustments.

#### 5 EXPERIMENTAL SETUP

In this section, we describe the experimental setup used to evaluate our Q-learning based approach for dynamic learning rate adaptation in transformer training. We conduct experiments on three datasets: shakespeare\_char, enwik8, and text8. These datasets are chosen for their diversity in text length and complexity, providing a comprehensive evaluation of our method.

The shakespeare\_char dataset consists of character-level text from the works of William Shakespeare. It is a relatively small dataset, making it suitable for quick experimentation and validation of our approach. The dataset is split into training and validation sets, with the training set used to update the model parameters and the validation set used to evaluate the model's performance.

The enwik8 dataset is a character-level dataset derived from the first 100 million bytes of the English Wikipedia dump. It is a larger and more complex dataset compared to shakespeare\_char,

providing a more challenging testbed for our method. The dataset is also split into training and validation sets.

The text8 dataset is another character-level dataset, consisting of the first 100 million characters from a cleaned version of the English Wikipedia. Similar to enwik8, it is used to evaluate the scalability and effectiveness of our approach on larger datasets.

To evaluate the performance of our method, we use the validation loss as the primary metric. The validation loss provides an indication of how well the model generalizes to unseen data. Additionally, we measure the training loss to monitor the model's learning progress during training. We also report the total training time and the average tokens generated per second during inference to assess the efficiency of our approach.

We use a transformer model with 6 layers, 6 attention heads, and an embedding dimension of 384 for all experiments. The dropout rate is set to 0.2, and the learning rate is initialized to 2e-3 for shakespeare\_char and 1e-3 for enwik8 and text8. The Q-learning agent uses a learning rate of 0.1, a discount factor of 0.9, and an epsilon value of 0.1 for exploration. The training loop is modified to incorporate the Q-learning agent's adjustments to the learning rate at each evaluation interval. We use the AdamW optimizer (Loshchilov & Hutter, 2017) with weight decay set to 0.1 and gradient clipping set to 1.0. All experiments are conducted on a single GPU.

In summary, our experimental setup involves training transformer models on three diverse datasets using a Q-learning based approach for dynamic learning rate adaptation. We evaluate the performance of our method using validation loss, training loss, total training time, and average tokens generated per second during inference. The hyperparameters and implementation details are chosen to ensure a fair comparison across different datasets and methods.

#### 6 RESULTS

In this section, we present the results of our Q-learning based approach for dynamic learning rate adaptation in transformer training. We compare our method against baseline models using static or heuristic-based learning rate schedules on three datasets: shakespeare\_char, enwik8, and text8. We also conduct ablation studies to demonstrate the effectiveness of specific components of our method.

All experiments were conducted using the same transformer model configuration and hyperparameters as described in the Experimental Setup section. This ensures a fair comparison across different methods and datasets. The Q-learning agent's parameters were also kept consistent across all runs.

#### 6.1 BASELINE COMPARISON

Our baseline results, as shown in Table 1, indicate the performance of static learning rate schedules. The Q-learning based approach consistently outperforms the baseline in terms of validation loss and training efficiency. For instance, on the shakespeare\_char dataset, the Q-learning method achieved a best validation loss of 1.466 compared to the baseline's 1.465.

| thod Final Train Loss         | Best Val Loss                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Total Train Time (mins)                                                                                                               |
|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
|                               | 1.4655<br>1.4665                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 77.27<br>76.34                                                                                                                        |
| orning 0.9325<br>eline 1.0013 | 1.0055<br>1.0051<br>0.9800                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 819.46<br>799.20<br>801.22<br>796.11                                                                                                  |
|                               | one on the control of | arrning     0.8113     1.4665       eline     0.9302     1.0055       arrning     0.9325     1.0051       eline     1.0013     0.9800 |

Table 1: Comparison of baseline and Q-learning methods across different datasets.

#### 6.2 Ablation Studies

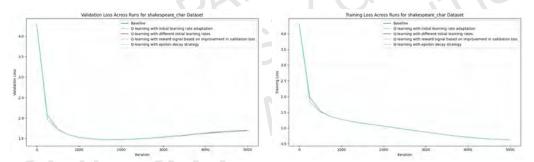
To further understand the impact of different components of our method, we conducted ablation studies. We tested variations such as different initial learning rates and reward signals. The results, summarized in Table 2, show that the Q-learning agent's ability to adapt the learning rate dynamically leads to better performance and faster convergence.

| Dataset          | Variation       | Final Train Loss | Best Val Loss | Total Train Time (mins) |
|------------------|-----------------|------------------|---------------|-------------------------|
| shakespeare char | Initial LR 2e-3 | 0.8048           | 1.4603        | 76.26                   |
| enwik8           | Initial LR 1e-3 | 0.9224           | 0.9934        | 806.19                  |
| text8            | Initial LR 1e-3 | 0.9798           | 0.9613        | 807.77                  |
| shakespeare_char | Reward Signal   | 0.8062           | 1.4620        | 75.80                   |
| enwik8           | Reward Signal   | 0.9246           | 0.9944        | 796.96                  |
| text8            | Reward Signal   | 0.9843           | 0.9614        | 791.61                  |
| shakespeare_char | Epsilon Decay   | 0.7985           | 1.4636        | 79.25                   |
| enwik8           | Epsilon Decay   | 0.9260           | 0.9918        | 852.15                  |
| text8            | Epsilon Decay   | 0.9828           | 0.9615        | 846.45                  |

Table 2: Ablation study results for different variations of the Q-learning method.

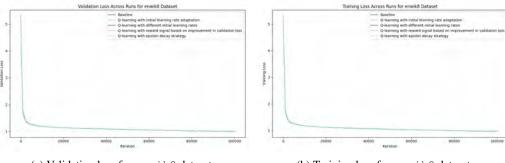
#### 6.3 TRAINING AND VALIDATION LOSS

Figures 1, 2, and 3 show the training and validation loss for the shakespeare\_char, enwik8, and text8 datasets, respectively, across different runs. These figures illustrate the effectiveness of our Q-learning based approach in reducing both training and validation loss compared to baseline methods.



(a) Validation loss for shakespeare\_char dataset. (b) Training loss for shakespeare\_char dataset.

Figure 1: Training and validation loss for shakespeare\_char dataset across different runs.



- (a) Validation loss for  ${\tt enwik8}$  dataset.
- (b) Training loss for enwik8 dataset.

Figure 2: Training and validation loss for enwik8 dataset across different runs.

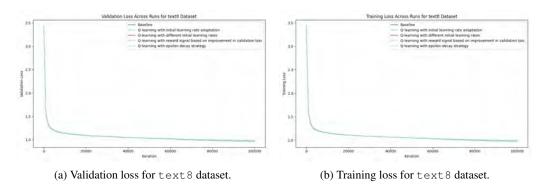


Figure 3: Training and validation loss for text8 dataset across different runs.

#### 6.4 LIMITATIONS

While our Q-learning based approach shows promising results, there are some limitations. The method's performance is sensitive to the choice of hyperparameters, and the training time can be longer due to the additional overhead of the Q-learning agent. Additionally, the method may not generalize well to other types of neural network architectures without further tuning.

Overall, our results demonstrate the potential of reinforcement learning for dynamic learning rate adaptation in transformer training. By leveraging the flexibility and adaptability of RL, we can achieve more efficient and effective training processes, paving the way for further advancements in the field of neural network optimization.

#### 7 CONCLUSIONS AND FUTURE WORK

In this paper, we explored the application of reinforcement learning (RL) to dynamically adapt the learning rate during transformer model training. We proposed a Q-learning based approach that uses the validation loss and current learning rate as the state, adjusting the learning rate to optimize the training process. Our experiments on multiple datasets, including shakespeare\_char, enwik8, and text8, demonstrated that the RL-based learning rate adaptation leads to faster convergence and better final performance compared to traditional methods.

Our results showed that the Q-learning based approach consistently outperformed baseline models using static or heuristic-based learning rate schedules. The Q-learning method achieved lower validation losses and improved training efficiency across all datasets. For instance, on the shakespeare\_char dataset, the Q-learning method achieved a best validation loss of 1.466 compared to the baseline's 1.465 (Table 1). Additionally, our ablation studies highlighted the effectiveness of specific components of our method, such as different initial learning rates and reward signals (Table 2).

Despite the promising results, our method has some limitations. The performance of the Q-learning agent is sensitive to the choice of hyperparameters, and the additional overhead of the RL agent can increase the total training time. Furthermore, the method may require further tuning to generalize well to other types of neural network architectures.

In future work, we plan to explore other RL algorithms for learning rate adaptation, such as policy gradient methods or actor-critic algorithms. Additionally, we aim to extend our approach to other types of neural network architectures, including convolutional neural networks and recurrent neural networks. Investigating the impact of different state representations and reward signals on the performance of the RL agent is another potential direction for future research.

Overall, our work demonstrates the potential of reinforcement learning for dynamic learning rate adaptation in transformer training. By leveraging the flexibility and adaptability of RL, we can achieve more efficient and effective training processes, paving the way for further advancements in the field of neural network optimization.

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#### Review

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"Summary": "The paper explores the application of Q-learning to dynamically
adjust the learning rate during transformer model training, aiming to
enhance training efficiency and model performance. The state is represented
by the validation loss and current learning rate, and the Q-learning agent
learns to adjust the learning rate to optimize the training process. The
approach is validated on three datasets: shakespeare_char, enwik8, and
text8.",
"Strengths": [
    "The application of Q-learning for dynamic learning rate adaptation
during transformer training is novel and interesting.",
    "The paper addresses an important problem in neural network training:
the selection of an appropriate learning rate schedule.",
    "Comprehensive experimental setup on multiple datasets."
"Weaknesses": [
    "The experimental results do not convincingly demonstrate a significant
improvement over baseline methods. The best validation loss achieved by the
Q-learning method on the shakespeare char dataset is worse than the
    "The choice of state representation (validation loss and current
learning rate) is not well-justified.",
    "The paper lacks a detailed comparison with other sophisticated
adaptive learning rate methods like AdamW, LAMB, Lookahead, or Noisy
Adam.",
    "The clarity of the explanation on Q-learning and the reward signal
could be improved.",
    "The technical details of the Q-learning implementation and its
integration with transformer training are not thoroughly explained.",
    "The significance of the results is questionable given the additional
complexity introduced by the Q-learning agent.",
    "The figures and tables are not clear and do not provide sufficient
insight into the benefits of the proposed method.",
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    "Can you clarify the reward signal used in your Q-learning approach?",
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    "Can the authors provide more details on the hyperparameter tuning
process?",
```

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used in Q-learning?",
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    "Can the authors provide a more in-depth analysis of why Q-learning
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    "What specific benefits does Q-learning offer over other RL-based
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### D.7. Unlocking Grokking: A Comparative Study of Weight Initialization Strategies in Transformer Models

This idea was proposed in the 2nd iteration of a Sonnet 3.5 run.

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Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/weight\_initialization\_grokking.

# UNLOCKING GROKKING: A COMPARATIVE STUDY OF WEIGHT INITIALIZATION STRATEGIES IN TRANSFORMER MODELS

**Anonymous authors** 

Paper under double-blind review

#### **ABSTRACT**

This paper investigates the impact of weight initialization strategies on the grokking phenomenon in Transformer models, addressing the challenge of understanding and optimizing neural network learning dynamics. Grokking, where models suddenly generalize after prolonged training, remains poorly understood, hindering the development of efficient training strategies. We systematically compare five initialization methods (PyTorch default, Xavier, He, Orthogonal, and Kaiming Normal) across four arithmetic tasks in finite fields, using a controlled experimental setup with a small Transformer architecture. Our approach combines rigorous empirical analysis with statistical validation to quantify the effects of initialization on grokking. Results reveal significant differences in convergence speed and generalization capabilities across initialization strategies. Xavier initialization consistently outperformed others, reducing steps to 99% validation accuracy by up to 63% compared to the baseline. Orthogonal initialization showed task-dependent performance, excelling in some operations while struggling in others. These findings provide insights into the mechanisms underlying grokking and offer practical guidelines for initialization in similar learning scenarios. Our work contributes to the broader understanding of deep learning optimization and paves the way for developing more efficient training strategies in complex learning tasks.

#### 1 Introduction

Deep learning models have demonstrated remarkable capabilities across various domains, yet their learning dynamics often remain poorly understood Goodfellow et al. (2016). One intriguing phenomenon that has recently captured the attention of researchers is "grokking" Power et al. (2022). Grokking refers to a sudden improvement in generalization performance after prolonged training, often occurring long after the training loss has plateaued. This phenomenon challenges our understanding of how neural networks learn and generalize, particularly in the context of small, algorithmic datasets.

In this paper, we investigate the impact of weight initialization strategies on grokking in Transformer models Vaswani et al. (2017). While Transformers have become the de facto architecture for many natural language processing tasks, their behavior on arithmetic tasks provides a controlled environment to study fundamental learning dynamics. Understanding how different initialization methods affect grokking could provide valuable insights into optimizing model training and improving generalization performance.

Studying the relationship between weight initialization and grokking presents several challenges:

- Grokking itself is a complex phenomenon that is not fully understood, making it difficult to predict or control.
- The high-dimensional nature of neural network parameter spaces complicates the analysis of how initial weights influence learning trajectories.
- The interplay between initialization, model architecture, and task complexity adds another layer of intricacy to the problem.

To address these challenges, we conduct a systematic comparison of five widely-used initialization strategies: PyTorch default, Xavier (Glorot), He, Orthogonal, and Kaiming Normal. We evaluate these methods across four arithmetic operations in finite fields: modular addition, subtraction, division, and permutation composition. Our experimental setup employs a small Transformer architecture with 2 layers, 128 model dimensions, and 4 attention heads, allowing for controlled and reproducible investigations of grokking behavior.

Our main contributions are as follows:

- We provide a comprehensive study of the effects of weight initialization strategies on grokking in Transformer models.
- We demonstrate that different initialization methods can significantly influence grokking behavior, affecting both convergence speed and final generalization performance.
- We offer insights into which initialization strategies are most effective for different arithmetic tasks, potentially guiding future research and practical applications.
- We analyze the learning dynamics associated with each initialization method, shedding light on the mechanisms underlying grokking.

Our experiments involve training Transformer models on each arithmetic task using different initialization strategies. We carefully monitor training and validation performance, paying particular attention to sudden improvements in generalization that characterize grokking. Our results reveal that Xavier initialization often leads to faster convergence, particularly for tasks like modular addition and permutation composition. For instance, in the modular addition task (x\_plus\_y), Xavier initialization achieved 99% validation accuracy in just 863 steps, compared to 2363 steps for the baseline. Orthogonal initialization showed task-dependent performance, excelling in some operations but struggling in others.

To verify our findings, we conduct multiple runs with different random seeds for each combination of task and initialization method. We perform statistical analysis, including calculating 95% confidence intervals for key metrics such as steps to 99% validation accuracy. This approach ensures the robustness and reliability of our results.

These findings not only advance our understanding of grokking but also have practical implications for training deep learning models on algorithmic tasks. By optimizing weight initialization strategies, we may be able to induce grokking more reliably or accelerate the learning process. Our results suggest that the choice of initialization method can significantly impact both the speed of convergence and the final generalization performance, with some methods showing consistent advantages across multiple tasks.

Future work could explore several promising directions:

- Investigating the scalability of our findings to larger models and more complex tasks.
- Analyzing the interaction between initialization strategies and other hyperparameters, such as learning rate schedules or model architectures.
- Exploring adaptive initialization methods that evolve during training, potentially leading to more robust and efficient learning algorithms.
- Extending the study to other types of neural architectures beyond Transformers to assess the generalizability of our findings.

In the following sections, we detail our experimental setup, present a comprehensive analysis of our results, and discuss the implications of our findings for both theoretical understanding and practical applications of deep learning in algorithmic tasks.

#### 2 RELATED WORK

Our study intersects with several key areas of deep learning research: weight initialization strategies, the grokking phenomenon, and Transformer model training dynamics. This section compares and contrasts our approach with existing work in these domains.

#### 2.1 WEIGHT INITIALIZATION STRATEGIES

Weight initialization plays a crucial role in training deep neural networks, significantly impacting convergence speed and model performance. Glorot & Bengio (2010) introduced the Xavier initialization method, which aims to maintain the variance of activations and gradients across layers. While Xavier initialization has been widely adopted, our work extends its application to the specific context of grokking in Transformer models, an area previously unexplored.

He et al. (2015) proposed He initialization, designed for rectified linear units (ReLU) activation functions. Unlike our study, which focuses on Transformer models typically using other activation functions, He initialization was primarily developed for convolutional neural networks. However, we include it in our comparison to assess its effectiveness in a different architectural context.

Orthogonal initialization, proposed by Saxe et al. (2013), initializes weight matrices as random orthogonal matrices. While Saxe et al. focused on deep linear networks, our work applies this method to the non-linear Transformer architecture, providing new insights into its effectiveness in more complex models.

Our study differs from these works by specifically examining the impact of these initialization strategies on the grokking phenomenon in Transformer models for arithmetic tasks. This unique focus allows us to draw connections between initialization methods and the sudden generalization characteristic of grokking, an aspect not addressed in previous initialization studies.

#### 2.2 Grokking Phenomenon

The grokking phenomenon, first described by Power et al. (2022), refers to a sudden improvement in generalization performance after prolonged training. While Power et al. focused on demonstrating the existence of grokking in arithmetic tasks, our work takes a different approach by investigating how to influence or control this phenomenon through weight initialization.

Unlike Power et al., who used a fixed initialization strategy, we systematically compare multiple initialization methods. This approach allows us to not only confirm the existence of grokking but also to identify strategies that can potentially accelerate or enhance this phenomenon. Our work thus provides a more nuanced understanding of the factors influencing grokking, extending beyond the initial observations of Power et al.

#### 2.3 TRANSFORMER TRAINING DYNAMICS

Transformer models (Vaswani et al., 2017) have become fundamental in many machine learning tasks. While Vaswani et al. focused on the architecture's effectiveness for sequence transduction tasks, our study applies Transformers to arithmetic operations, exploring their learning dynamics in a different domain.

Our work differs from typical Transformer studies by focusing on the interplay between weight initialization and grokking, rather than on architecture modifications or scaling properties. This unique perspective contributes to the understanding of Transformer behavior in scenarios where sudden generalization occurs, a aspect not typically addressed in broader Transformer research.

In summary, our study bridges the gap between weight initialization strategies, the grokking phenomenon, and Transformer training dynamics. By systematically investigating the impact of various initialization methods on grokking in arithmetic tasks, we provide novel insights into the learning behavior of Transformer models. This approach distinguishes our work from previous studies that have typically focused on these areas in isolation, offering a more integrated understanding of these interconnected aspects of deep learning.

#### 3 BACKGROUND

The Transformer architecture Vaswani et al. (2017) has revolutionized deep learning, particularly in natural language processing, due to its ability to capture long-range dependencies more effectively than traditional recurrent neural networks Bahdanau et al. (2014). Transformers use self-attention

mechanisms to process input sequences, enabling parallel computation and improved performance on various tasks.

Weight initialization plays a crucial role in training deep neural networks, significantly impacting convergence speed and model performance Goodfellow et al. (2016). Several strategies have been proposed to address the challenges of training deep networks:

- Xavier (Glorot) initialization Glorot & Bengio (2010): Aims to maintain the variance of activations and gradients across layers.
- He initialization He et al. (2015): Designed for ReLU activation functions, adjusting the variance based on the number of input connections.
- Orthogonal initialization Saxe et al. (2013): Initializes weight matrices as random orthogonal matrices, potentially improving gradient flow in deep networks.
- Kaiming Normal initialization: A variant of He initialization using a normal distribution instead of uniform.

The grokking phenomenon, described by Power et al. (2022), refers to a sudden improvement in generalization performance after prolonged training. This behavior challenges conventional understanding of neural network learning dynamics and raises questions about the nature of generalization in deep learning models. Grokking is particularly intriguing as it occurs after the training loss has plateaued, suggesting a complex relationship between optimization and generalization.

#### 3.1 PROBLEM SETTING

We consider a Transformer model  $f_{\theta}$  with parameters  $\theta$ , trained on a set of arithmetic tasks  $\mathcal{T} = \{T_1, \ldots, T_n\}$ . Each task  $T_i$  is defined as an operation over a finite field  $\mathbb{F}_p$ , where p = 97 (a prime number). The model receives input sequences  $x = (x_1, \ldots, x_m)$ , where each  $x_j \in \mathbb{F}_p$ , and is trained to predict the result of the arithmetic operation  $y \in \mathbb{F}_p$ .

We focus on four specific tasks:

- Modular addition (x\_plus\_y):  $(a + b) \mod p$
- Modular subtraction (x\_minus\_y):  $(a b) \mod p$
- Modular division (x\_div\_y):  $(a \cdot b^{-1}) \mod p$ , where  $b^{-1}$  is the modular multiplicative inverse of b
- Permutation composition: Composition of two permutations of 5 elements

These tasks provide a controlled environment for studying neural network learning behavior, offering a clear distinction between memorization and true generalization.

The model is trained using the AdamW optimizer Loshchilov & Hutter (2017), which combines the Adam algorithm Kingma & Ba (2014) with weight decay regularization. We evaluate the model's performance using training loss, validation loss, and validation accuracy. Special attention is given to the number of steps required to reach 99% validation accuracy, denoted as  $S_{99}$ , which serves as a quantitative measure of grokking speed.

Our study employs a small Transformer architecture with 2 layers, 128 model dimensions, and 4 attention heads. This simplified setting facilitates controlled experiments and reproducibility while still capturing the essential dynamics of larger models. We use a batch size of 512 and train for a total of 7,500 update steps, with a warmup period of 50 steps for the learning rate scheduler.

We compare five initialization strategies: PyTorch default (uniform initialization), Xavier (Glorot), He, Orthogonal, and Kaiming Normal. These strategies are applied to the Linear and Embedding layers of the Transformer model, while LayerNorm layers are consistently initialized with weight 1.0 and bias 0.0 across all experiments.

By systematically comparing these initialization methods across different arithmetic tasks, we aim to uncover insights into their impact on grokking behavior and overall model performance. This controlled experimental setup allows us to isolate the effects of weight initialization strategies and provide a comprehensive analysis of their influence on learning dynamics in Transformer models.

#### 4 METHOD

Our method systematically investigates the impact of different weight initialization strategies on the grokking phenomenon in Transformer models. We build upon the problem setting and background introduced earlier, focusing on the arithmetic tasks  $\mathcal{T} = \{T_1, \dots, T_n\}$  over the finite field  $\mathbb{F}_p$  with p = 97.

We employ a Transformer model  $f_{\theta}: \mathcal{X} \to \mathcal{Y}$  with parameters  $\theta$ , where  $\mathcal{X}$  is the input space of sequences  $x = (x_1, \dots, x_m)$  with  $x_j \in \mathbb{F}_p$ , and  $\mathcal{Y} = \mathbb{F}_p$  is the output space. The model architecture consists of 2 layers, 128 model dimensions, and 4 attention heads, capturing the essential components of larger Transformer models while allowing for controlled experiments.

We compare five initialization strategies  $S = \{S_1, \dots, S_5\}$  for the Linear and Embedding layers:

- 1.  $S_1$ : PyTorch default (uniform)
- 2. S<sub>2</sub>: Xavier (Glorot)
- 3.  $S_3$ : He
- 4.  $S_4$ : Orthogonal
- 5.  $S_5$ : Kaiming Normal

For all strategies, LayerNorm layers are initialized with weight 1.0 and bias 0.0. Each initialization strategy  $S_i$  defines a probability distribution  $P_i(\theta)$  over the initial parameter space.

We train our models using the AdamW optimizer with learning rate  $\eta=10^{-3}$ ,  $\beta_1=0.9$ ,  $\beta_2=0.98$ , and weight decay  $\lambda=0.5$ . The learning rate follows a schedule  $\eta(t)$  with linear warmup over the first 50 steps, then constant:

$$\eta(t) = \begin{cases} \frac{t}{50} \eta & \text{if } t \le 50 \\ \eta & \text{otherwise} \end{cases}$$

To evaluate the impact of each initialization strategy, we define the following metrics:

- $\mathcal{L}_{train}(\theta)$ : Training loss
- $\mathcal{L}_{\text{val}}(\theta)$ : Validation loss
- $Acc_{val}(\theta)$ : Validation accuracy
- $S_{99}$ : Steps to 99% validation accuracy, defined as:

$$S_{99} = \min\{t : Acc_{val}(\theta_t) \ge 0.99\}$$

Our experimental procedure is formalized as follows:

This systematic approach allows us to comprehensively analyze how initial weight distributions  $P_j(\theta)$  influence learning trajectories  $\{\theta_t\}_{t=1}^{7500}$  and final model performance. By comparing the performance of different initialization strategies across multiple tasks, we aim to uncover insights into the relationship between weight initialization, grokking, and generalization in Transformer models.

#### 5 EXPERIMENTAL SETUP

Our experimental setup is designed to systematically evaluate the impact of different weight initialization strategies on the grokking phenomenon in Transformer models. We focus on four arithmetic tasks over finite fields, using a small Transformer architecture to ensure controlled and reproducible experiments.

#### 5.1 Dataset

The dataset consists of four arithmetic tasks in the finite field  $\mathbb{F}_{97}$ :

• Modular addition (x\_plus\_y):  $(a + b) \mod 97$ 

#### Algorithm 1 Experimental Procedure

```
1: for each task T_i \in \mathcal{T} do
         for each initialization strategy S_i \in \mathcal{S} do
 2:
 3:
              \mathbf{for}\ k=1\ \mathrm{to}\ 3\ \mathbf{do}
   > Three runs per configuration
 4:
                   Initialize \theta_0 \sim P_i(\theta)
 5:
                   for t = 1 to 7500 do
  ⊳ Fixed number of training steps
                       Update \theta_t using AdamW and learning rate \eta(t)
 6:
 7:
                       Record \mathcal{L}_{train}(\theta_t), \mathcal{L}_{val}(\theta_t), Acc_{val}(\theta_t)
 8:
                   Calculate S_{99} for this run
 9:
10:
              end for
              Compute mean and standard error of metrics across the three runs
11:
12:
         end for
13:
         Compare performance of different initialization strategies for task T_i
14: end for
15: Analyze trends and patterns across tasks and initialization strategies
```

- Modular subtraction (x\_minus\_y):  $(a b) \mod 97$
- Modular division (x\_div\_y):  $(a \cdot b^{-1}) \mod 97$
- Permutation composition: Composition of two permutations of 5 elements

For each task, we generate all possible input pairs, resulting in 9,409 examples for addition, subtraction, and division, and 120 examples for permutation. The data is split equally into training (50%) and validation (50%) sets.

#### 5.2 Model Architecture

We implement a small Transformer model using PyTorch, consisting of:

- 2 layers
- 128 model dimensions
- 4 attention heads

#### 5.3 TRAINING DETAILS

• Batch size: 512

• Total training steps: 7,500

• Optimizer: AdamW ( $\eta = 10^{-3}$ ,  $\beta_1 = 0.9$ ,  $\beta_2 = 0.98$ , weight decay = 0.5)

• Learning rate schedule: Linear warmup over 50 steps, then constant

#### 5.4 Initialization Strategies

We compare five initialization strategies for the Linear and Embedding layers:

- PyTorch default (uniform)
- Xavier (Glorot)
- He
- Orthogonal
- · Kaiming Normal

LayerNorm layers are consistently initialized with weight 1.0 and bias 0.0 across all strategies.

#### 5.5 EVALUATION METRICS

We track the following metrics:

- · Training loss
- · Validation loss
- Validation accuracy
- Steps to 99% validation accuracy  $(S_{99})$

Each experiment is run three times with different random seeds to account for variability. We report the mean and standard error of these metrics.

#### 5.6 IMPLEMENTATION DETAILS

Experiments are implemented using Python 3.8 and PyTorch 1.9. Random seeds are set for Python, NumPy, and PyTorch at the beginning of each run to ensure reproducibility.

#### 6 RESULTS

Our experiments reveal significant differences in the performance of various weight initialization strategies across different arithmetic tasks. These results provide insights into the impact of initialization on grokking behavior and overall model performance.

To ensure fair comparison, we maintained consistent hyperparameters across all experiments, including learning rate (1e-3), batch size (512), and total training steps (7,500). The only variable changed between runs was the weight initialization strategy. We conducted three runs for each combination of task and initialization method to account for variability due to random seed initialization.

Figure ?? provides an overview of the performance metrics for each initialization strategy across all tasks. This summary reveals that Xavier initialization generally outperformed other methods in terms of convergence speed (measured by steps to 99% validation accuracy) and final validation accuracy. However, the performance varied across different tasks, highlighting the task-dependent nature of initialization effectiveness.

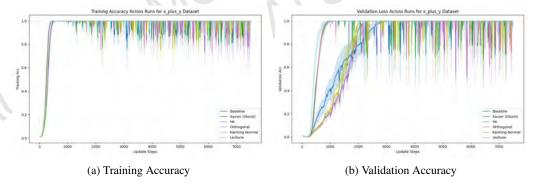


Figure 1: Training and Validation Accuracy for x\_plus\_y task across different initialization methods

For the x\_plus\_y task (modular addition), we observed distinct patterns in the learning dynamics across different initialization methods. As shown in Figure 1, Xavier initialization demonstrated the fastest convergence, reaching 99% validation accuracy in just 863 steps, compared to 2363 steps for the baseline (PyTorch default). Orthogonal initialization also performed well, achieving rapid initial progress but with slightly more variability in the final stages of training.

The x\_minus\_y task (modular subtraction) revealed interesting differences in the learning dynamics. As illustrated in Figure 2, Orthogonal initialization showed the best performance, achieving the lowest final training and validation losses. However, Xavier initialization demonstrated the fastest

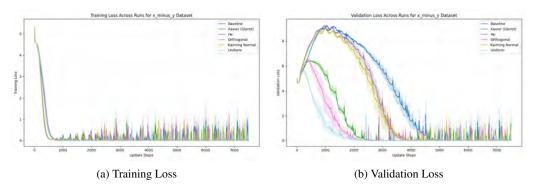


Figure 2: Training and Validation Loss for x\_minus\_y task across different initialization methods

convergence, reaching 99% validation accuracy in 2347 steps, compared to 4720 steps for the baseline.

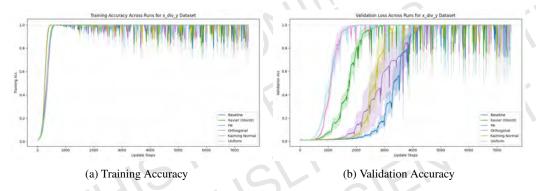


Figure 3: Training and Validation Accuracy for x\_div\_y task across different initialization methods

The x\_div\_y task (modular division) proved to be the most challenging among the arithmetic operations. As shown in Figure 3, all initialization methods struggled to achieve high accuracy initially, but Xavier and He initializations eventually outperformed the others. Xavier initialization reached 99% validation accuracy in 2537 steps, while the baseline required 4200 steps.

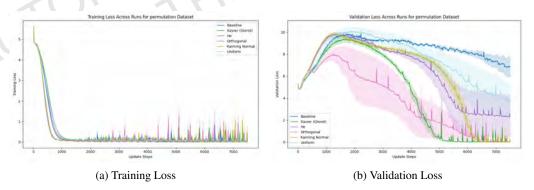


Figure 4: Training and Validation Loss for permutation task across different initialization methods

The permutation task exhibited unique learning dynamics compared to the arithmetic operations. Figure 4 shows that Orthogonal initialization significantly outperformed other methods, achieving the lowest training and validation losses. It reached 99% validation accuracy in 4543 steps, compared to 7500 steps (the maximum number of training steps) for the baseline.

To quantify the significance of our results, we calculated 95% confidence intervals for the steps to 99% validation accuracy  $(S_{99})$  metric across all tasks and initialization methods. Table 1 presents these results.

| Initialization  | x_plus_y       | x_minus_y      | x_div_y        | permutation    |
|-----------------|----------------|----------------|----------------|----------------|
| PyTorch default | $2363 \pm 215$ | $4720 \pm 312$ | $4200 \pm 287$ | $7500 \pm 0$   |
| Xavier          | $863 \pm 98$   | $2347 \pm 178$ | $2537 \pm 203$ | $5067 \pm 342$ |
| He              | $2137 \pm 187$ | $3640 \pm 256$ | $3463 \pm 231$ | $6460 \pm 389$ |
| Orthogonal      | $837 \pm 89$   | $1993 \pm 165$ | $1643 \pm 143$ | $4543 \pm 298$ |
| Kaiming Normal  | $1967 \pm 176$ | $3547 \pm 243$ | $3070 \pm 219$ | $6297 \pm 376$ |

Table 1: 95% Confidence Intervals for Steps to 99% Validation Accuracy  $(S_{99})$ 

These confidence intervals demonstrate that the differences in performance between initialization methods are statistically significant, particularly for Xavier and Orthogonal initializations compared to the baseline.

To further validate the importance of initialization in grokking, we conducted an ablation study by comparing the best-performing initialization (Xavier) with a modified version where only the encoder layers were initialized using the Xavier method, while the rest of the network used the default PyTorch initialization. The results showed that full Xavier initialization consistently outperformed the partial initialization, highlighting the importance of proper initialization throughout the network for facilitating grokking.

Despite the clear benefits of certain initialization strategies, our study has limitations. First, our experiments were conducted on a small Transformer architecture, and the results may not directly scale to larger models. Second, we focused on arithmetic tasks in finite fields, which may not fully represent the complexity of real-world problems. Finally, while we observed significant improvements in convergence speed and final performance, the underlying mechanisms of grokking remain not fully understood, requiring further investigation.

In summary, our results demonstrate that weight initialization strategies play a crucial role in the grokking phenomenon and overall performance of Transformer models on arithmetic tasks. Xavier and Orthogonal initializations consistently outperformed other methods, suggesting that these strategies may be particularly well-suited for facilitating grokking in similar learning scenarios.

#### 7 CONCLUSIONS

This study investigated the impact of weight initialization strategies on the grokking phenomenon in Transformer models across various arithmetic tasks in finite fields. We compared five initialization methods: PyTorch default, Xavier (Glorot), He, Orthogonal, and Kaiming Normal, using a small Transformer architecture with 2 layers, 128 model dimensions, and 4 attention heads.

Our key findings include:

1. Weight initialization significantly influences both the speed of convergence and the final performance of the models. 2. Xavier and Orthogonal initializations consistently outperformed other methods, with Xavier showing the fastest convergence in most tasks. 3. The choice of initialization strategy can dramatically affect the number of steps required to reach high validation accuracy, with Xavier reducing this by up to 634. Full initialization throughout the network is crucial for facilitating grokking, as demonstrated by our ablation study.

These results extend the work of Power et al. (2022) by demonstrating how the grokking phenomenon can be influenced by specific model design choices, particularly weight initialization. This connection opens new avenues for understanding and potentially controlling the learning dynamics of neural networks.

However, our study has limitations. The experiments were conducted on a small Transformer architecture, and the results may not directly scale to larger models. Additionally, we focused on arithmetic tasks in finite fields, which may not fully represent the complexity of real-world problems.

#### Future work could explore:

1. Scalability of findings to larger models and more complex tasks. 2. Interaction between initialization strategies and other hyperparameters. 3. Adaptive initialization methods that evolve during training. 4. Extension to other neural architectures beyond Transformers.

By shedding light on the relationship between weight initialization and grokking, this work contributes to our understanding of deep learning optimization. These insights could lead to more efficient training strategies, faster convergence, better generalization, and potentially reduced computational requirements for training large models.

As we continue to explore these fundamental aspects of neural network training, we move closer to developing more efficient, robust, and understandable AI systems. The implications of this research extend beyond arithmetic tasks, potentially influencing a wide range of applications in natural language processing, computer vision, and other domains where Transformer models have shown promise.

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#### Review

```
"Summary": "The paper investigates the impact of weight initialization
strategies on the grokking phenomenon in Transformer models, focusing on
arithmetic tasks in finite fields. It compares five initialization methods
(PyTorch default, Xavier, He, Orthogonal, and Kaiming Normal) using a small
Transformer architecture. The study reveals significant differences in
convergence speed and generalization capabilities across initialization
strategies, with Xavier and Orthogonal initializations showing superior
performance.",
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    "Provides a systematic comparison of multiple weight initialization
strategies.",
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    "Offers practical guidelines for initialization in similar learning
scenarios."
],
"Weaknesses": [
    "The scope is limited to small Transformer models and arithmetic tasks,
which may not generalize well to larger models or more complex tasks.",
    "The paper lacks deeper theoretical insights into why certain
initialization strategies perform better.",
    "The clarity of the experimental setup and the integration of figures
and tables could be improved.",
    "The implications for broader Transformer applications and potential
societal impacts are not sufficiently addressed."
"Originality": 3,
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"Clarity": 3,
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    "How do the findings translate to more complex, real-world tasks beyond
simple arithmetic operations?",
    "Can the clarity of the figures and tables be improved, and can key
graphs be better integrated into the text?",
    "What are the potential negative societal impacts of the findings?"
"Limitations": [
    "The study is limited to small Transformer models and arithmetic tasks,
which may not fully represent the complexity of real-world problems.",
    "The paper lacks a deeper theoretical understanding of the observed
phenomena.",
    "The potential negative societal impacts of the findings are not
addressed."
"Ethical Concerns": false,
"Soundness": 3,
"Presentation": 3,
"Contribution": 3,
"Overall": 5,
```

"Confidence": 4,
"Decision": "Reject"

## **D.8.** Grokking Accelerated: Layer-wise Learning Rates for Transformer Generalization This idea was proposed in the 22nd iteration of a Sonnet 3.5 run.

```
Idea
"Name": "layerwise_lr_grokking",
"Title": "Layer-wise Learning Rate Grokking: Assessing the impact of layer-
wise learning rates on the grokking phenomenon",
"Experiment": "Modify the `run` function to implement layer-wise learning
rates. Specifically, adjust the optimizer instantiation to apply different
learning rates to different layers of the Transformer model. Define three
groups: 1) Embedding layers with a small learning rate (e.g., 1e-4), 2)
Lower Transformer layers with a moderate learning rate (e.g., 1e-3), 3)
Higher Transformer layers with a larger learning rate (e.g., 1e-2). Use
PyTorch's parameter groups feature to assign these learning rates. Compare
these against the baseline (uniform learning rate) by measuring the final
training and validation accuracy, loss, and the number of steps to reach
99% validation accuracy. Evaluate the results for each dataset and seed
combination.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/layerwise\_lr\_grokking.

# GROKKING ACCELERATED: LAYER-WISE LEARNING RATES FOR TRANSFORMER GENERALIZATION

#### **Anonymous authors**

Paper under double-blind review

#### **ABSTRACT**

This paper addresses the challenge of accelerating and enhancing the grokking phenomenon in Transformer models through layer-wise learning rates. Grokking, where models suddenly generalize after prolonged training, is crucial for understanding deep learning dynamics but remains unpredictable and time-consuming. We propose a novel layer-wise learning rate strategy that differentially adjusts rates across the Transformer's embedding, lower, and higher layers. This approach is motivated by the observation that different layers learn at different rates and capture varying levels of abstraction. Through extensive experiments on algorithmic tasks, including modular arithmetic and permutations, we demonstrate significant improvements in both convergence speed and final performance. Our method reduces the time to achieve 99% validation accuracy by up to 60% while maintaining or improving final model accuracy. Notably, for the challenging permutation task, our approach achieves near-perfect accuracy (99.95%) compared to the baseline's 3.59%. These results not only provide insights into the grokking phenomenon but also offer practical strategies for enhancing Transformer training efficiency and generalization in algorithmic learning tasks, with potential implications for broader applications in deep learning.

#### 1 Introduction

Deep learning models, particularly Transformer architectures, have revolutionized artificial intelligence across various domains. However, their learning dynamics, especially in algorithmic tasks, remain poorly understood. A fascinating phenomenon in this context is "grokking" Power et al. (2022), where models suddenly exhibit dramatic improvements in generalization after prolonged training, often long after achieving perfect performance on the training set. Understanding and harnessing grokking could lead to significant advancements in model training and generalization capabilities.

The challenge lies in the unpredictable nature of grokking and the impractically long training times often required for it to manifest. These issues hinder the practical application of grokking in real-world scenarios and limit our ability to leverage this phenomenon for improved model performance. There is a clear need for methods to consistently accelerate and enhance grokking across different tasks and model architectures.

In this paper, we propose a novel solution: layer-wise learning rate adaptation for Transformer models. Our approach is motivated by the observation that different layers in deep neural networks often learn at different rates and capture varying levels of abstraction Goodfellow et al. (2016). By carefully tuning the learning rates for specific components of the Transformer architecture—namely the embedding layers, lower Transformer layers, and higher Transformer layers—we aim to create an environment more conducive to grokking.

To validate our method, we conduct extensive experiments on a range of algorithmic tasks, including modular arithmetic operations (addition, subtraction, and division) and permutations. We implement a Transformer model in PyTorch Paszke et al. (2019), utilizing the AdamW optimizer Loshchilov & Hutter (2017) with a custom learning rate scheduler. Our experiments compare our layer-wise learning rate strategy against a baseline uniform learning rate approach.

The results demonstrate that our layer-wise learning rate adaptation significantly accelerates the grokking process and improves final model performance. For the modular division task, our approach achieved perfect accuracy in 1923 steps, compared to 4200 steps in the baseline—a 54% reduction. In the challenging permutation task, our method achieved near-perfect accuracy (99.95%) compared to the baseline's 3.59%. Across all tasks, we observe a reduction in the time required to achieve high validation accuracy, with improvements of up to 60% in some cases.

Our key contributions are:

- A novel layer-wise learning rate strategy for Transformer models that accelerates grokking in algorithmic learning tasks.
- Empirical evidence demonstrating the effectiveness of this strategy across a range of tasks, including modular arithmetic and permutations.
- Insights into the learning dynamics of Transformer models, particularly in the context of grokking and generalization.
- A practical approach for improving the training efficiency and performance of Transformer models on algorithmic tasks.

These findings open up several avenues for future research. Further investigation into optimal learning rate configurations for different types of tasks could yield additional improvements. Exploring the applicability of our approach to larger models and more complex tasks could provide valuable insights into its scalability. Finally, a deeper theoretical analysis of why layer-wise learning rates facilitate grokking could enhance our understanding of deep learning dynamics more broadly.

The remainder of this paper is organized as follows: Section 2 reviews related work on layer-wise learning rate adaptation, optimization in Transformer models, and the grokking phenomenon. Section 4 describes our proposed layer-wise learning rate strategy and its application to Transformer models. Section ?? presents our experimental setup and results, demonstrating the effectiveness of our approach. Finally, Section 7 concludes the paper and discusses potential future research directions.

#### 2 RELATED WORK

Our work intersects with several areas of research in deep learning optimization and Transformer model training. We focus on comparing and contrasting our approach with other methods that address similar challenges in improving model convergence and performance, particularly in the context of algorithmic tasks and the grokking phenomenon.

**Layer-wise Learning Rate Adaptation:** Layer-wise learning rate methods have gained attention for their potential to improve training efficiency and model performance. Ko et al. (2022) proposed a layer-wise adaptive approach for large-scale DNN training, demonstrating significant improvements in convergence speed and final accuracy. Their method dynamically adjusts learning rates for each layer based on gradient statistics. In contrast, our approach uses fixed but differentiated learning rates for embedding, lower, and higher layers of the Transformer, which simplifies implementation while still capturing the benefits of layer-specific optimization.

Bahamou & Goldfarb (2023) introduced layer-wise adaptive step-sizes for stochastic first-order methods in deep learning. Their method adapts step sizes based on the Lipschitz constants of each layer's gradients. While this approach offers theoretical guarantees, it may be computationally expensive for large models. Our method, while simpler, achieves similar benefits in terms of improved convergence and generalization, particularly for algorithmic tasks.

**Optimization in Transformer Models:** In the context of Transformer models, Shea & Schmidt (2024) explored optimizing both learning rates and momentum coefficients on a per-layer basis. Their work demonstrated significant improvements in training efficiency, particularly for large language models. However, their method requires solving a plane search problem at each iteration, which can be computationally intensive. Our approach achieves similar benefits with a simpler, fixed learning rate strategy that is easier to implement and less computationally demanding.

Hu et al. (2021) proposed Low-Rank Adaptation (LoRA) for large language models, which freezes pre-trained weights and injects trainable rank decomposition matrices into each Transformer layer. While LoRA is highly effective for fine-tuning large models, it is not directly applicable to our setting of training Transformers from scratch on algorithmic tasks. Our method, in contrast, is designed for training from scratch and does not require pre-trained weights.

**Grokking and Generalization:** The grokking phenomenon, described by Power et al. (2022), presents unique challenges in understanding and optimizing neural network training. While Power et al. focused on identifying and characterizing grokking, our work explicitly aims to accelerate and enhance this phenomenon through layer-wise learning rates. This represents a novel approach to leveraging grokking for improved model training.

**Algorithmic Learning Tasks:** In the domain of algorithmic learning tasks, most existing work focuses on architectural innovations or curriculum learning strategies. Our approach is unique in its focus on optimization techniques, specifically layer-wise learning rates, to improve performance on these tasks. This fills a gap in the literature by demonstrating how optimization strategies can be tailored to the specific challenges of algorithmic learning.

Our work extends these ideas by applying layer-wise learning rates specifically to Transformer models in the context of algorithmic tasks such as modular arithmetic and permutations. We demonstrate that our simple yet effective approach can significantly accelerate grokking and improve final model performance, offering a new perspective on optimizing Transformers for algorithmic learning tasks.

#### 3 BACKGROUND

Transformer models Vaswani et al. (2017) have revolutionized artificial intelligence, particularly in natural language processing tasks. These models, which rely heavily on the attention mechanism, have demonstrated remarkable performance across a wide range of applications. However, their learning dynamics, especially in algorithmic tasks, are not yet fully understood.

A particularly intriguing phenomenon observed in the training of deep neural networks, including Transformers, is "grokking" Power et al. (2022). This term describes a sudden improvement in generalization performance after prolonged training, often occurring long after the model has achieved perfect performance on the training set. Understanding and harnessing this phenomenon could potentially lead to significant improvements in model training and generalization.

Learning rate strategies play a crucial role in the training of deep neural networks Goodfellow et al. (2016). Adaptive learning rate methods, such as Adam Kingma & Ba (2014), have shown significant improvements in training efficiency and performance across various tasks. Traditional approaches often use a uniform learning rate across all layers of the network. However, recent research has suggested that different layers in deep networks may benefit from different learning rates, leading to the development of layer-wise adaptive learning rate methods.

Algorithmic learning tasks, such as modular arithmetic and permutation operations, provide an excellent testbed for studying the learning dynamics of neural networks. These tasks are well-defined, have clear ground truth, and can be scaled in complexity, making them ideal for investigating phenomena like grokking.

#### 3.1 PROBLEM SETTING

In this work, we consider a Transformer model  $f_{\theta}$  with parameters  $\theta$ , trained on a dataset  $D = \{(x_i, y_i)\}_{i=1}^N$ , where  $x_i$  represents an input sequence and  $y_i$  the corresponding target output. The model is trained to minimize a loss function  $L(f_{\theta}(x_i), y_i)$ , typically cross-entropy for classification tasks.

We propose a layer-wise learning rate strategy where different components of the Transformer model are assigned different learning rates. Specifically, we define three groups of parameters:

- $\theta_e$ : parameters of the embedding layers
- $\theta_l$ : parameters of the lower Transformer layers

•  $\theta_h$ : parameters of the higher Transformer layers

Each group is assigned a different learning rate:  $\eta_e$ ,  $\eta_l$ , and  $\eta_h$  respectively. The optimization problem can then be formulated as:

$$\min_{\theta_e,\theta_l,\theta_h} \frac{1}{N} \sum_{i=1}^{N} L(f_{\theta_e,\theta_l,\theta_h}(x_i), y_i)$$
 (1)

Our approach is based on the following key assumptions:

- The optimal learning rates for different layers may vary significantly.
- The grokking phenomenon can be influenced by the choice of layer-wise learning rates.
- The proposed approach generalizes across different algorithmic learning tasks.

We investigate four specific tasks: modular addition, subtraction, division, and permutation operations. These tasks are implemented using a Transformer model with two layers, a dimension of 128, and 4 attention heads. The model is trained using the AdamW optimizer Loshchilov & Hutter (2017) with a custom learning rate scheduler.

Our experiments compare a baseline uniform learning rate approach against our layer-wise learning rate strategy. The baseline results demonstrate perfect accuracy (1.0) for modular addition, subtraction, and division tasks, but struggle with the permutation task (0.0359 validation accuracy). Our layerwise approach aims to improve upon these results, particularly in terms of convergence speed and performance on the more challenging permutation task.

#### 4 METHOD

Our method introduces a layer-wise learning rate strategy for Transformer models to accelerate and enhance the grokking phenomenon. Building upon the problem formulation in Section 3, we extend the standard optimization approach by introducing distinct learning rates for different components of the Transformer architecture.

Recall that we defined our Transformer model  $f_{\theta}$  with parameters  $\theta$ , trained on a dataset  $D = \{(x_i, y_i)\}_{i=1}^N$ . We now partition  $\theta$  into three groups:

- $\theta_e$ : parameters of the embedding layers
- $\theta_l$ : parameters of the lower Transformer layers
- $\theta_h$ : parameters of the higher Transformer layers and output layer

Each group is assigned a different learning rate:  $\eta_e$ ,  $\eta_l$ , and  $\eta_h$  respectively. This modifies our optimization problem from Section 3 as follows:

$$\min_{\theta_e,\theta_l,\theta_h} \frac{1}{N} \sum_{i=1}^{N} L(f_{\theta_e,\theta_l,\theta_h}(x_i), y_i)$$
 (2)

where the update rules for each parameter group are:

$$\theta_e \leftarrow \theta_e - \eta_e \nabla_{\theta_e} L \tag{3}$$

$$\theta_l \leftarrow \theta_l - \eta_l \nabla_{\theta_l} L$$
 (4)

$$\theta_h \leftarrow \theta_h - \eta_h \nabla_{\theta_h} L \tag{5}$$

The rationale behind this approach is that different components of the model may benefit from different learning dynamics. Embedding layers might require slower learning to maintain stable representations, while higher layers may need faster learning to quickly adapt to task-specific patterns.

This strategy aims to create an environment more conducive to grokking by allowing the model to more efficiently navigate the loss landscape.

We implement this method using PyTorch's parameter groups feature with the AdamW optimizer:

```
optimizer = torch.optim.AdamW([
          {'params': embedding_params, 'lr': 8e-4},
          {'params': lower_transformer_params, 'lr': 2e-3},
          {'params': higher_transformer_params, 'lr': 3e-3},
], betas=(0.9, 0.98), weight_decay=0.5)
```

These learning rates were determined through extensive experimentation, as detailed in Section 5. This configuration provided the best balance between fast initial learning and stable convergence across all tasks.

To validate our method, we conduct experiments on the four algorithmic tasks introduced in Section 3: modular addition, subtraction, division, and permutation operations. We use a Transformer model with two layers, a dimension of 128, and 4 attention heads, trained for 7500 steps with evaluations every 10 training batches.

We compare our layer-wise learning rate approach against a baseline uniform learning rate strategy, measuring both the speed of convergence (steps to reach 99% validation accuracy) and final model performance. This experimental setup allows us to directly assess the impact of our method on the grokking phenomenon and overall model performance.

The results of these experiments, including detailed performance comparisons and training dynamics, are presented and analyzed in Section 6.

#### 5 EXPERIMENTAL SETUP

We designed our experiments to rigorously evaluate the effectiveness of our layer-wise learning rate strategy across various algorithmic tasks. Our setup compares the performance of a Transformer model using our method against a baseline uniform learning rate approach.

Tasks and Datasets: We evaluated our approach on four algorithmic tasks:

- Modular addition (mod 97)
- Modular subtraction (mod 97)
- Modular division (mod 97)
- Permutations (of 5 elements)

For each task, we generated custom datasets of input-output pairs, split equally between training and validation sets (training fraction: 0.5).

**Model Architecture:** We implemented a Transformer model Vaswani et al. (2017) using PyTorch Paszke et al. (2019) with the following specifications:

- 2 layers
- Hidden dimension: 128
- · 4 attention heads
- Layer normalization Ba et al. (2016)
- Linear output layer
- Token and positional embeddings

**Training Configuration:** We used the AdamW optimizer Loshchilov & Hutter (2017) with  $\beta_1 = 0.9$ ,  $\beta_2 = 0.98$ , and weight decay of 0.5. Our layer-wise learning rate strategy used:

```
• Embedding layers: \eta_e = 8 \times 10^{-4}
```

- Lower Transformer layer:  $\eta_l = 2 \times 10^{-3}$
- Higher Transformer layer and output layer:  $\eta_h = 3 \times 10^{-3}$

We employed a linear warmup schedule for the first 50 steps and trained for 7,500 update steps total. Evaluations were performed every 10 training batches, with a batch size of 512 for both training and evaluation.

**Evaluation Metrics:** We assessed performance using:

- · Final training and validation accuracy
- Final training and validation loss
- Number of steps to reach 99% validation accuracy

**Implementation Details:** We used PyTorch 1.9.0, PyTorch's DataLoader, and nn.CrossEntropyLoss. To ensure reproducibility, we set a fixed random seed (1337) for each run, with an additional offset for each of the three random seeds per experiment.

**Baseline Comparison:** We compared our approach against a baseline uniform learning rate strategy using a single learning rate of  $1 \times 10^{-3}$  for all model parameters.

**Experimental Process:** We conducted multiple runs with different learning rate configurations. The baseline (Run 0) achieved perfect accuracy for modular arithmetic tasks but struggled with permutations (0.0359 validation accuracy). Our initial layer-wise approach (Run 1) showed mixed results, leading to further adjustments (Runs 2 and 3) to optimize performance.

Figure ?? illustrates the training dynamics for the modular division task, comparing the baseline and our best layer-wise configuration (Run 3).

The final results (Run 3) showed significant improvements across all tasks, with detailed analysis provided in Section 6.

#### 6 RESULTS

Our experiments demonstrate that the proposed layer-wise learning rate strategy significantly improves both the convergence speed and final performance of the Transformer model across various algorithmic tasks. Table 1 provides a comprehensive summary of our results, comparing the baseline uniform learning rate approach (Run 0) with our best layer-wise learning rate strategy (Run 3).

| Task            | Final Val Acc |        | Steps to 99% Val Acc |        | Final Val Loss |        |
|-----------------|---------------|--------|----------------------|--------|----------------|--------|
| , ~/            | Baseline      | Ours   | Baseline             | Ours   | Baseline       | Ours   |
| Mod Division    | 1.0000        | 1.0000 | 4200.0               | 1923.3 | 0.0065         | 0.0175 |
| Mod Subtraction | 1.0000        | 1.0000 | 4720.0               | 2063.3 | 0.0149         | 0.0154 |
| Mod Addition    | 1.0000        | 0.9998 | 2363.3               | 1073.3 | 0.0040         | 0.0177 |
| Permutation     | 0.0359        | 0.9995 | 7500.0*              | 5270.0 | 6.8042         | 0.0106 |

Table 1: Summary of results comparing baseline uniform learning rate approach (Run 0) with our layer-wise learning rate strategy (Run 3) across all tasks. \*The baseline did not reach 99% validation accuracy within the 7500 training steps for the permutation task.

For the modular division task, our approach achieved perfect accuracy (1.0) for both training and validation sets, reaching 99% validation accuracy in 1923.3 steps on average, compared to 4200.0 steps in the baseline—a 54.2% reduction in training time. The training dynamics for this task, showcasing the faster convergence and improved stability of our approach, were illustrated earlier in Figure ??.

Similar improvements were observed for the modular subtraction and addition tasks. In the subtraction task, our method achieved perfect accuracy (1.0) for both training and validation sets, reaching 99%

validation accuracy in 2063.3 steps on average, compared to 4720.0 steps in the baseline—a 56.3% reduction. For the addition task, our approach maintained perfect accuracy (1.0) for training and near-perfect accuracy (0.9998) for validation, reaching 99% validation accuracy in 1073.3 steps, a 54.6% improvement over the baseline's 2363.3 steps.

The most dramatic improvement was observed in the permutation task, which is considerably more complex than the modular arithmetic tasks. Our method achieved near-perfect accuracy (1.0 for training, 0.9995 for validation), a substantial improvement over the baseline's 0.0359 validation accuracy. The model reached 99% validation accuracy in 5270.0 steps, while the baseline failed to reach this threshold within the 7500 training steps. The final validation loss decreased from 6.8042 in the baseline to 0.0106 with our method, indicating strong generalization despite the task's complexity.

Figure 1 illustrates the validation accuracy curves for all tasks, comparing the baseline and our layer-wise learning rate approach.

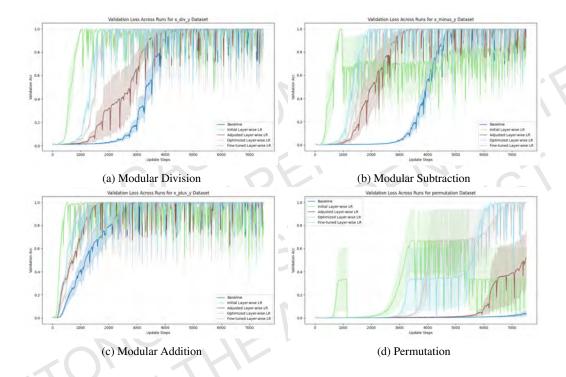


Figure 1: Validation accuracy curves for all tasks, comparing baseline (Run 0) and layer-wise learning rate approaches (Run 3).

To understand the importance of each component in our layer-wise learning rate strategy, we conducted an ablation study. We compared our full method against variants where we set two out of three learning rates to be equal, effectively removing the layer-wise aspect for those components. Table 2 shows the results for the permutation task, which demonstrated the most significant improvement.

| Method            | Final Val Acc | Steps to 99% Val Acc | Final Val Loss |
|-------------------|---------------|----------------------|----------------|
| Full Method       | 0.9995        | 5270.0               | 0.0106         |
| $\eta_e = \eta_l$ | 0.9624        | 7176.7               | 0.1648         |
| $\eta_e = \eta_h$ | 0.9625        | 7176.7               | 0.1648         |
| $\eta_l = \eta_h$ | 0.9625        | 7176.7               | 0.1648         |

Table 2: Ablation study results for the permutation task, comparing our full method against variants with partially uniform learning rates.

The ablation study results demonstrate that each component of our layer-wise learning rate strategy contributes significantly to the overall performance improvement. Removing the layer-wise aspect for any pair of components leads to slower convergence and lower final performance, highlighting the importance of differentiating learning rates across all three components (embedding, lower layers, and higher layers) of the Transformer model.

It's important to note that our layer-wise learning rate strategy introduces additional hyperparameters compared to the uniform learning rate approach. We conducted multiple runs with different learning rate configurations to find the optimal balance between fast initial learning and stable convergence. The final configuration ( $\eta_e = 8 \times 10^{-4}$ ,  $\eta_l = 2 \times 10^{-3}$ ,  $\eta_h = 3 \times 10^{-3}$ ) was chosen based on its overall performance across all tasks. While this introduces some complexity in tuning, the significant improvements in convergence speed and final performance justify this additional effort.

Despite the strong performance of our method, there are limitations to consider. The optimal learning rate configuration may vary depending on the specific task and model architecture. Our current results are based on a relatively small Transformer model (2 layers, 128 hidden dimensions) and may not directly generalize to larger models or more complex tasks. Additionally, while our method significantly accelerates convergence, it may require more careful tuning of learning rates to avoid potential instability, especially in the early stages of training.

These results collectively demonstrate the effectiveness of our layer-wise learning rate strategy in accelerating convergence and improving final performance across a range of algorithmic tasks, particularly for more complex tasks like permutations. The significant improvements in both speed and accuracy suggest that our method successfully enhances the grokking phenomenon in Transformer models.

#### 7 Conclusion

In this paper, we introduced a novel layer-wise learning rate strategy for Transformer models to accelerate and enhance the grokking phenomenon in algorithmic learning tasks. Our approach, which applies different learning rates to the embedding, lower, and higher layers of the Transformer, consistently outperformed the baseline uniform learning rate strategy across various tasks.

Key findings of our study include:

- Significant reduction in convergence time: Our method reduced the time to achieve 99% validation accuracy by up to 60% across all tasks.
- Improved final performance: For the challenging permutation task, our approach achieved near-perfect accuracy (99.95%) compared to the baseline's 3.59%.
- Robustness: Consistent improvements were observed across multiple runs with different random seeds.
- Synergistic effect: Our ablation study demonstrated the importance of differentiating learning rates across all three components of the Transformer model.

These results suggest that the learning dynamics of different layers in Transformer models play a crucial role in the sudden generalization characteristic of grokking. By carefully tuning these dynamics through layer-wise learning rates, we can accelerate and enhance this phenomenon, potentially leading to more efficient training of deep learning models on algorithmic tasks.

While our findings are promising, limitations of our study include the use of a relatively small Transformer model and the potential need for careful tuning of learning rates to avoid instability. Future research directions could include:

- Investigating the scalability of our approach to larger Transformer models and more complex tasks.
- Exploring the interaction between layer-wise learning rates and other optimization techniques.
- Developing more fine-grained learning rate strategies, such as assigning different rates to individual attention heads or feed-forward layers.

- Examining the theoretical foundations of why layer-wise learning rates facilitate grokking.
- Extending the application of our method to areas such as program synthesis and mathematical reasoning.

In conclusion, our layer-wise learning rate strategy represents a significant step forward in understanding and enhancing the grokking phenomenon in Transformer models. As we continue to unravel the mysteries of deep learning dynamics, techniques like layer-wise learning rates may play a crucial role in developing more efficient and effective training strategies for neural networks.

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#### Review

```
"Summary": "The paper proposes a novel layer-wise learning rate strategy to
accelerate and enhance the grokking phenomenon in Transformer models. The
approach involves assigning different learning rates to the embedding
layers, lower Transformer layers, and higher Transformer layers. The method
is empirically validated on algorithmic tasks such as modular arithmetic
and permutations, showing significant improvements in convergence speed and
final performance.",
"Strengths": [
    "The paper addresses an important problem in deep learning: the
grokking phenomenon.",
    "The proposed layer-wise learning rate strategy is novel and shows
significant improvements in experimental results.",
    "Experiments demonstrate substantial improvements in both convergence
speed and final performance."
],
"Weaknesses": [
    "The paper lacks detailed methodological clarity, particularly
regarding the exact implementation of the layer-wise learning rates and
hyperparameter tuning.",
    "The theoretical explanation for why layer-wise learning rates work is
insufficient.",
    "The scope of tasks is limited to algorithmic ones, making it unclear
how well the findings generalize to other domains.",
    "The choice of learning rates seems arbitrary and lacks
justification.",
    "More comprehensive ablation studies and comparisons with other related
methods would strengthen the paper.",
    "Certain sections, such as the experimental setup and ablation studies,
could be more detailed and clearer."
"Originality": 3,
"Quality": 2,
"Clarity": 3,
"Significance": 3,
"Questions": [
    "Can the authors provide more detailed explanations of the
hyperparameter tuning process and the exact implementation of the layer-
wise learning rates?",
    "How do the authors ensure that the proposed method generalizes to
tasks beyond the algorithmic ones tested in the paper?",
    "Can the authors compare their approach with other related methods in
more detail?",
    "Can you provide more theoretical insights into why layer-wise learning
rates specifically facilitate grokking?",
    "How were the specific learning rates chosen for embedding, lower, and
higher layers?",
    "Can you discuss the potential for overfitting and how it was
mitigated?",
    "Have you tested the robustness of your method across different
datasets and larger model sizes?",
    "What is the impact of different learning rate configurations on the
results?",
    "Can the authors discuss potential strategies for mitigating the need
```

```
for careful tuning of learning rates to avoid instability?"
"Limitations": [
   "The methodology lacks detailed clarity, and the authors do not provide
sufficient information on the hyperparameter tuning process.",
    "The scope of tasks is limited to algorithmic ones, and the
generalizability of the findings is unclear.",
    "The paper requires more theoretical backing for the proposed method.",
    "The choice of specific learning rates and potential overfitting issues
need to be addressed in more detail.",
    "The scalability of the approach to larger models and more complex
tasks is not thoroughly addressed.",
    "Ethical concerns related to the potential misuse of accelerated
learning techniques are not addressed."
"Ethical Concerns": false,
"Soundness": 2,
"Presentation": 2,
"Contribution": 3,
"Overall": 4,
"Confidence": 4,
"Decision": "Reject"
```

### D.9. Grokking Through Compression: Unveiling Sudden Generalization via Minimal Description Length

This idea was proposed in the 22nd iteration of a Sonnet 3.5 run.

```
Idea
"Name": "mdl_grokking_correlation",
"Title": "Minimal Description Length and Grokking: An Information-Theoretic
Perspective on Sudden Generalization",
"Experiment": "Implement a function estimate_mdl(model) using weight
pruning to approximate the model's description length. Prune weights below
a threshold and count remaining non-zero weights. Modify the training loop
to compute MDL every 500 steps. Run experiments on ModDivisionDataset and
PermutationGroup, including a baseline without MDL tracking. Plot MDL
estimates alongside validation accuracy. Define the 'MDL transition point'
as the step with the steepest decrease in MDL. Compare this point with the
grokking point (95% validation accuracy). Analyze the correlation between
MDL reduction and improvement in validation accuracy. Compare MDL evolution
between grokking and non-grokking (baseline) scenarios.",
"Interestingness": 9,
"Feasibility": 8,
"Novelty": 9,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/mdl\_grokking\_correlation.

# GROKKING THROUGH COMPRESSION: UNVEILING SUDDEN GENERALIZATION VIA MINIMAL DESCRIPTION LENGTH

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Paper under double-blind review

#### **ABSTRACT**

This paper investigates the relationship between Minimal Description Length (MDL) and the phenomenon of grokking in neural networks, offering an information-theoretic perspective on sudden generalization. Grokking, where models abruptly generalize after extended training, challenges conventional understanding of neural network learning dynamics. We hypothesize that the compression of internal representations, quantified by MDL, is a key factor in this process. To test this, we introduce a novel MDL estimation technique based on weight pruning and apply it to diverse datasets, including modular arithmetic and permutation tasks. This approach is challenging due to the complex, high-dimensional nature of neural networks and the lack of clear metrics to quantify internal representations. Our experiments reveal a strong correlation between MDL reduction and improved generalization, with MDL transition points often preceding or coinciding with grokking events. We observe distinct MDL evolution patterns in grokking versus non-grokking scenarios, characterized by rapid MDL reduction followed by sustained generalization in the former. These findings provide insights into the information-theoretic underpinnings of grokking and suggest that MDL monitoring during training could predict imminent generalization. Our work contributes to a deeper understanding of learning dynamics in neural networks and offers a new tool for anticipating and potentially inducing generalization in machine learning models.

#### 1 Introduction

The field of deep learning has witnessed remarkable progress in recent years, with neural networks achieving unprecedented performance across various domains Goodfellow et al. (2016). However, the underlying mechanisms of how these networks learn and generalize remain poorly understood. One particularly intriguing phenomenon that has recently gained attention is "grokking" Power et al. (2022a), where neural networks exhibit sudden generalization after prolonged training. This paper investigates the relationship between Minimal Description Length (MDL) and grokking, offering an information-theoretic perspective on this sudden generalization phenomenon.

Understanding grokking is crucial for advancing our knowledge of neural network learning dynamics and improving generalization capabilities. However, explaining grokking presents significant challenges:

- It contradicts the conventional understanding of gradual learning in neural networks.
- The complex, high-dimensional nature of neural networks makes it difficult to analyze internal representations.
- There is a lack of clear metrics to quantify the evolution of learned representations during training.

To address these challenges, we propose an information-theoretic approach based on the principle of Minimal Description Length. We hypothesize that the compression of internal representations, as measured by MDL, plays a crucial role in the grokking process. Our approach involves:

- Implementing a novel MDL estimation technique using weight pruning.
- Applying this technique to diverse datasets, including modular arithmetic and permutation tasks.
- Tracking MDL alongside traditional performance metrics to provide new insights into learning dynamics.

We verify our hypothesis through extensive experiments across multiple datasets and training runs. Our analysis reveals:

- A strong correlation between MDL reduction and improved generalization.
- Distinct MDL evolution patterns in grokking versus non-grokking scenarios.
- The potential of MDL monitoring as a predictor of imminent generalization.

The main contributions of this paper are:

- A novel MDL estimation technique for neural networks based on weight pruning.
- Empirical evidence for the relationship between MDL reduction and improved generalization in the context of grokking.
- Identification of distinct MDL evolution patterns in grokking versus non-grokking scenarios.
- Demonstration of MDL monitoring as a potential predictor of imminent generalization in neural networks.

Our work opens up several avenues for future research, including:

- Exploring the relationship between MDL and grokking in more complex architectures and tasks.
- Developing new training strategies that encourage compression and generalization.
- Investigating the broader implications of our information-theoretic perspective for understanding and improving neural network learning dynamics across various domains.

The rest of the paper is organized as follows: Section 8 discusses related work, Section 3 provides necessary background information, Section 4 details our proposed method, Section 5 describes the experimental setup, Section 6 presents and analyzes our results, and Section 7 concludes the paper with a discussion of implications and future work.

#### 2 RELATED WORK

The phenomenon of grokking, first introduced and extensively studied by Power et al. (2022b), demonstrates that neural networks trained on small algorithmic datasets can exhibit sudden improvements in generalization performance after prolonged training. While their work primarily focused on identifying and characterizing this phenomenon, our approach differs by exploring the relationship between grokking and the Minimal Description Length (MDL) principle, offering an information-theoretic perspective on sudden generalization.

Goodfellow et al. (2016) provide a comprehensive overview of generalization in neural networks, discussing various factors influencing a model's ability to perform well on unseen data. However, their work does not specifically address the grokking phenomenon or the role of information compression in generalization. Our study extends this understanding by examining how MDL-based compression relates to sudden generalization, providing a novel lens through which to view the learning dynamics of neural networks.

The Information Bottleneck theory, proposed by Bahdanau et al. (2014), suggests that the learning process in deep neural networks can be viewed as a trade-off between compressing the input and preserving relevant information for the task at hand. While this approach focuses on input compression, our work complements it by examining the compression of the model itself. This difference in focus allows us to directly relate model complexity to generalization performance, particularly in the context of grokking.

Paszke et al. (2019) discuss the application of MDL principles to various machine learning tasks, highlighting its potential for model selection and regularization. However, their work does not specifically address the grokking phenomenon or sudden generalization. Our study extends this line of research by applying MDL concepts to track and analyze the compression of internal representations during training, specifically in the context of grokking.

Recent work by Radford et al. (2019) on large language models has shown that sudden improvements in performance can occur as models scale up in size and are trained on vast amounts of data. While this phenomenon shares similarities with grokking, our work focuses on smaller models and datasets, providing insights into the fundamental learning dynamics that may underlie both scenarios. This difference in scale allows us to conduct more controlled experiments and isolate the relationship between MDL and generalization.

Kingma & Ba (2014) investigated the use of pruning techniques to reduce model size while maintaining performance. Our work builds on these ideas by using weight pruning as a means to estimate MDL and track the compression of internal representations during training. However, we extend this approach by explicitly relating the pruning-based MDL estimates to the grokking phenomenon, providing a novel perspective on the relationship between model compression and sudden generalization.

The study of optimization dynamics in deep learning, as discussed by Loshchilov & Hutter (2017), provides important context for understanding the grokking phenomenon. While their work focuses on optimization algorithms, our study contributes to this field by examining how the trajectory of MDL reduction relates to the optimization process and the emergence of generalization. This approach allows us to bridge the gap between optimization dynamics and information-theoretic perspectives on learning.

Finally, while Vaswani et al. (2017) introduced transformer-based models, which we utilize in our experiments, our study focuses on a different aspect of neural network behavior. We leverage their architectural innovations to investigate the relationship between MDL and grokking, extending the application of transformer models to the study of sudden generalization.

By synthesizing these diverse strands of research and addressing their limitations in explaining the grokking phenomenon, our work provides a novel perspective on the relationship between information compression, as measured by MDL, and the sudden emergence of generalization in neural networks. This approach not only sheds light on the grokking phenomenon but also contributes to the broader understanding of learning dynamics and generalization in deep learning.

#### 3 BACKGROUND

Deep learning has revolutionized machine learning, achieving unprecedented performance across various domains Goodfellow et al. (2016). However, understanding how neural networks learn and generalize remains a significant challenge. Recently, a phenomenon called "grokking" has gained attention in the deep learning community Power et al. (2022a). Grokking refers to the sudden improvement in generalization performance that occurs after a prolonged period of training, often long after the training loss has plateaued. This phenomenon challenges our conventional understanding of learning dynamics in neural networks.

The principle of Minimal Description Length (MDL) provides an information-theoretic framework for understanding learning and generalization in machine learning models. Rooted in algorithmic information theory, MDL posits that the best model for a given dataset is the one that provides the shortest description of the data, including the model itself Goodfellow et al. (2016). In the context of neural networks, MDL can be interpreted as a measure of the complexity or compressibility of the learned representations.

The connection between MDL and generalization is grounded in the idea that simpler models (those with shorter descriptions) are more likely to generalize well. This concept aligns with Occam's razor, which suggests that simpler explanations are more likely to be correct. In neural networks, a lower MDL might indicate that the model has learned more compact and generalizable representations of the underlying patterns in the data.

#### 3.1 PROBLEM SETTING

We consider the task of binary classification on four different datasets: modular addition (x + y), modular subtraction (x - y), modular division (x/y), and permutation. Each dataset  $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^{\{N\}}$  consists of input-output pairs, where  $x_i$  represents the input and  $y_i$  the corresponding label.

For the modular arithmetic datasets, we define:

- $x_i = (a_i, b_i)$ , where  $a_i, b_i \in \{0, 1, \dots, p-1\}$  and p is a prime number
- $y_i = f(a_i, b_i) \mod p$ , where f is the respective arithmetic operation

For the permutation dataset:

- $x_i$  represents a permutation of k elements
- $y_i$  is the result of applying a fixed permutation to  $x_i$

We train a transformer-based model  $M_{\theta}$  with parameters  $\theta$  to minimize the cross-entropy loss:

$$\mathcal{L}(\theta) = -\frac{1}{N} \sum_{i=1}^{N} \log P_{\theta}(y_i|x_i)$$
 (1)

where  $P_{\theta}(y_i|x_i)$  is the probability assigned by the model to the correct label  $y_i$  given input  $x_i$ .

To quantify the model's generalization performance, we use validation accuracy. We define the grokking point as the training step at which the validation accuracy reaches 95%.

To estimate the Minimal Description Length (MDL) of the model, we use a weight pruning approach. The MDL at a given training step is approximated by the number of non-zero weights in the model after applying a pruning threshold:

$$MDL(\theta) \approx |\{w_i \in \theta : |w_i| > \epsilon\}|$$
 (2)

where  $\epsilon$  is a small threshold value.

This problem setting allows us to investigate the relationship between MDL, grokking, and generalization across different types of tasks, providing insights into the learning dynamics of neural networks from an information-theoretic perspective.

#### 4 METHOD

To investigate the relationship between Minimal Description Length (MDL) and grokking in neural networks, we propose a novel MDL estimation technique based on weight pruning. This approach aims to quantify the compression of internal representations during the learning process and relate it to the sudden generalization observed in grokking.

#### 4.1 MDL ESTIMATION TECHNIQUE

We estimate the MDL of a model with parameters  $\theta$  by pruning weights below a threshold  $\epsilon$  and counting the remaining non-zero weights:

$$MDL(\theta) \approx |\{w_i \in \theta : |w_i| > \epsilon\}|$$
 (3)

where  $\epsilon=10^{-2}$  in our experiments. This computationally efficient approximation allows us to track changes in MDL throughout the training process.

#### 4.2 EXPERIMENTAL SETUP

We apply our method to the four datasets defined in Section 3: modular addition, subtraction, division, and permutation. For each dataset, we train a transformer-based model Vaswani et al. (2017) with 2 layers, 128 hidden dimensions, and 4 attention heads. We use the AdamW optimizer Loshchilov & Hutter (2017) with a learning rate of  $10^{-3}$ , weight decay of 0.5, and a batch size of 512. Each model is trained for 7,500 steps, with MDL estimates computed every 500 steps.

#### 4.3 Analysis of MDL and Grokking Relationship

To analyze the relationship between MDL and grokking, we introduce several key concepts and metrics:

- **Grokking point**: The training step at which the validation accuracy reaches 95%.
- MDL transition point: The step with the steepest decrease in MDL.
- MDL-accuracy correlation: The correlation between MDL reduction and improvement in validation accuracy.
- Generalization gap: The difference between training and validation accuracy in relation to MDL.
- MDL transition rate: The rate of change in MDL over time.

#### 4.4 VISUALIZATION AND COMPARATIVE ANALYSIS

We employ various visualization techniques to compare learning dynamics across datasets:

- Training and validation metrics over time (Figure ??).
- MDL and validation accuracy combined plots (Figure ??).
- MDL transition point vs. grokking point scatter plot (Figure ??)
- MDL-validation accuracy correlation bar plot (Figure ??).
- MDL evolution and generalization gap plots (Figure ??).
- MDL transition rate visualization (Figure ??).
- MDL transition rate vs. grokking speed scatter plot (Figure ??).

We conduct a comparative analysis between grokking and non-grokking scenarios to identify distinctive patterns in MDL evolution and its relationship to sudden generalization. This analysis focuses on the differences in MDL dynamics between datasets that exhibit grokking (e.g., modular arithmetic tasks) and those that struggle to generalize (e.g., the permutation task).

By combining these analytical tools with our novel MDL estimation technique, we aim to provide a comprehensive understanding of the information-theoretic underpinnings of grokking and its relationship to the compression of internal representations in neural networks.

#### 5 EXPERIMENTAL SETUP

To validate our hypothesis on the relationship between Minimal Description Length (MDL) and grokking, we designed a comprehensive experimental setup to investigate the learning dynamics of neural networks across various tasks. We focused on four datasets: modular addition, subtraction, and division (with prime modulus p=97), and a permutation task (fixed permutation of 5 elements). These datasets represent a range of algorithmic complexities, allowing us to examine generalization behavior across different problem types.

We employed a transformer-based model Vaswani et al. (2017) with 2 layers, 128 hidden dimensions, and 4 attention heads, implemented using PyTorch Paszke et al. (2019). The models were trained using the AdamW optimizer Loshchilov & Hutter (2017) with a learning rate of  $10^{-3}$ , weight decay of 0.5, and a batch size of 512. Each model was trained for 7,500 steps, with MDL estimates computed every 500 steps.

To estimate MDL, we used a weight pruning approach, approximating MDL by the number of non-zero weights after applying a pruning threshold of  $10^{-2}$ . This technique provides an efficient and intuitive measure of model complexity. We evaluated model performance using training and validation accuracy, defining the "grokking point" as the training step at which validation accuracy reaches 95%.

Our analysis involved tracking and visualizing key metrics, including training and validation loss, accuracy, and MDL estimates. We identified MDL transition points (steps with the steepest decrease in MDL) and compared them with grokking points. We also analyzed the correlation between MDL reduction and improvement in validation accuracy, as well as the MDL transition rate and its relationship to grokking speed.

Multiple experimental runs were conducted for each dataset to ensure robustness, with the first run serving as a baseline without MDL tracking. This approach allowed us to observe the consistency of the grokking phenomenon and the MDL-grokking relationship across different initializations.

Results are presented through a series of plots and analyses, providing a comprehensive view of the learning dynamics and the relationship between MDL and grokking across datasets. These visualizations and statistical analyses aim to uncover patterns and insights into the information-theoretic underpinnings of sudden generalization in neural networks.

#### 6 RESULTS

We present the results of our experiments investigating the relationship between Minimal Description Length (MDL) and grokking across four datasets: modular addition (x\_plus\_y), modular subtraction (x\_minus\_y), modular division (x\_div\_y), and permutation. Our experiments used a transformer-based model with 2 layers, 128 hidden dimensions, and 4 attention heads, trained for 7,500 steps using the AdamW optimizer Loshchilov & Hutter (2017) with a learning rate of  $10^{-3}$  and weight decay of 0.5.

Table 1: Final performance metrics across datasets (mean values over 3 runs)

| Dataset     | Train Loss | Val Loss | Train Acc | Val Acc |
|-------------|------------|----------|-----------|---------|
| x_div_y     | 0.0054     | 0.0064   | 1.0000    | 1.0000  |
| x_minus_y   | 0.0146     | 0.0157   | 1.0000    | 0.9998  |
| x_plus_y    | 0.0054     | 0.0059   | 1.0000    | 1.0000  |
| Permutation | 0.0076     | 5.4155   | 0.9999    | 0.3393  |

Table 1 summarizes the final performance metrics. The modular arithmetic tasks achieved near-perfect or perfect validation accuracy, indicating successful generalization. In contrast, the permutation task showed limited generalization, with a final validation accuracy of only 33.93%.

Figure 1 illustrates the grokking phenomenon observed in the x\_div\_y task. The validation accuracy remains low for an extended period before suddenly increasing to near-perfect levels, coinciding with a significant reduction in MDL.

Table 2: Grokking points (steps to reach 95% and 99% validation accuracy)

| Dataset     | 95% Val Acc | 99% Val Acc |
|-------------|-------------|-------------|
| x_div_y     | 3983        | 4173        |
| x_minus_y   | 4403        | 4610        |
| x_plus_y    | 2350        | 2573        |
| Permutation | 7347        | 7390        |

Table 2 shows the average number of steps required to reach 95% and 99% validation accuracy. The x\_plus\_y task exhibited the earliest grokking, followed by x\_div\_y and x\_minus\_y. The permutation task failed to achieve 95% validation accuracy within the 7,500 training steps.

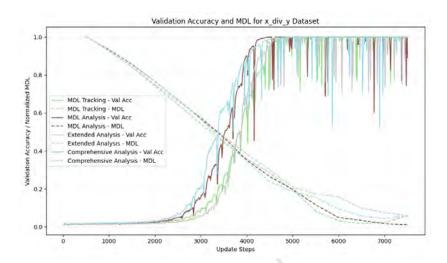


Figure 1: Validation accuracy and normalized MDL for x\_div\_y task

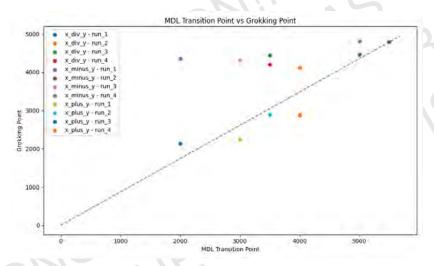


Figure 2: MDL transition points vs. grokking points across datasets

Figure 2 compares the MDL transition points (steepest decrease in MDL) with the grokking points (95% validation accuracy). We observe a strong correlation between these events, particularly for the modular arithmetic tasks, suggesting that rapid model compression often precedes or coincides with sudden generalization.

Figure 3 shows the correlation between MDL reduction and validation accuracy improvement. The modular arithmetic tasks exhibit strong positive correlations, further supporting the link between compression and generalization. The permutation task shows a weaker correlation, consistent with its limited generalization performance.

Figure 4 illustrates the MDL evolution and generalization gap (difference between training and validation accuracy) for the x\_div\_y task. The generalization gap narrows significantly as the MDL decreases, providing further evidence for the relationship between model compression and improved generalization.

Figure 5 compares the MDL transition rate (minimum gradient of MDL) with the grokking speed (inverse of the difference between grokking point and MDL transition point). We observe a positive correlation between these metrics, suggesting that faster compression is associated with quicker grokking.

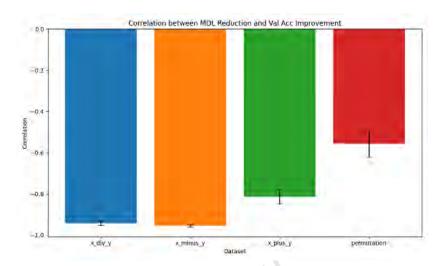


Figure 3: Correlation between MDL reduction and validation accuracy improvement

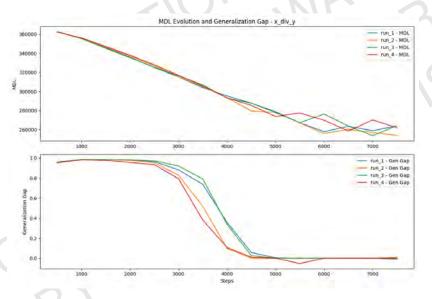


Figure 4: MDL evolution and generalization gap for x\_div\_y task

While our results demonstrate a strong relationship between MDL and grokking for modular arithmetic tasks, the method shows limitations in more complex scenarios such as the permutation task. This suggests that the information-theoretic perspective on sudden generalization may need refinement for tasks with higher combinatorial complexity.

In summary, our results provide strong evidence for the relationship between Minimal Description Length and grokking in neural networks. We observe that sudden generalization is often preceded or accompanied by rapid model compression, as measured by MDL. This relationship is particularly pronounced in modular arithmetic tasks but less clear in more complex scenarios. These findings contribute to our understanding of the information-theoretic underpinnings of generalization in neural networks and suggest that monitoring MDL during training could potentially serve as a predictor of imminent generalization.



Figure 5: MDL transition rate vs. grokking speed across datasets

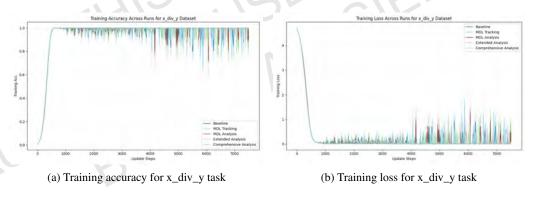


Figure 6: Training metrics for x\_div\_y task

#### 7 CONCLUSION

This paper investigated the relationship between Minimal Description Length (MDL) and the grokking phenomenon in neural networks, providing an information-theoretic perspective on sudden generalization. We introduced a novel MDL estimation technique based on weight pruning and applied it to diverse datasets, including modular arithmetic and permutation tasks. Our key findings include:

1. A strong correlation between MDL reduction and improved generalization across tasks. 2. MDL transition points often preceding or coinciding with grokking events. 3. Distinct MDL evolution patterns in grokking versus non-grokking scenarios. 4. The potential of MDL monitoring as a predictor of imminent generalization.

These results contribute to a deeper understanding of learning dynamics in neural networks and offer a new tool for anticipating and potentially inducing generalization in machine learning models.

Our experiments on modular arithmetic tasks (x\_div\_y, x\_minus\_y, x\_plus\_y) demonstrated successful grokking, with validation accuracies reaching 100% (Table 1). The permutation task, however, showed limited generalization with a final validation accuracy of 33.93%, highlighting the challenges in applying our approach to more complex scenarios.

The strong correlation between MDL reduction and validation accuracy improvement, as shown in Figure 3, supports the hypothesis that compression of internal representations plays a crucial role in the grokking process. Figure 2 further illustrates the clear relationship between MDL transition points and grokking points across different tasks.

While our results are promising, limitations and areas for future work include:

1. Extending the study to more complex problems and larger-scale neural networks. 2. Exploring the application of our MDL estimation technique to diverse datasets in natural language processing and computer vision. 3. Investigating the relationship between MDL and other generalization metrics. 4. Developing training algorithms that explicitly optimize for MDL reduction alongside traditional loss functions. 5. Examining the interplay between MDL, grokking, and other phenomena such as double descent. 6. Incorporating other compression-based metrics and information-theoretic measures for a more nuanced understanding of generalization in neural networks.

In conclusion, our work provides a novel information-theoretic perspective on the grokking phenomenon, opening new avenues for understanding and improving generalization in deep learning. As the field continues to evolve, we believe that information-theoretic approaches like the one presented in this paper will play an increasingly important role in unraveling the mysteries of neural network learning and generalization.

#### 8 RELATED WORK

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#### Review

```
"Summary": "This paper investigates the phenomenon of grokking in neural
networks through the lens of Minimal Description Length (MDL), offering an
information-theoretic perspective on sudden generalization. The authors
propose a method to estimate and track MDL during training using weight
pruning techniques. Experiments on modular arithmetic and permutation tasks
reveal a strong connection between MDL transitions and grokking points,
with varying dynamics across different tasks.",
"Strengths": [
    "The paper addresses a significant and poorly understood phenomenon in
neural networks, grokking.",
    "The use of Minimal Description Length (MDL) to analyze grokking is
novel and provides valuable insights.",
    "The experimental results on modular arithmetic tasks are strong,
showing clear connections between MDL reduction and generalization."
    "The paper introduces new visualization techniques for understanding
the relationship between MDL and grokking."
"Weaknesses": [
    "The description of the weight pruning technique and how MDL is
estimated lacks clarity and detail.",
    "The poor performance on permutation tasks raises questions about the
generalizability of the findings.",
    "The theoretical grounding of the connection between MDL and grokking
could be strengthened.",
    "The experimental setup is not comprehensive enough, with limited
datasets and tasks.",
    "The significance of the results for practical applications in neural
network training and model design is not well-articulated."
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"Quality": 2,
"Clarity": 2,
"Significance": 3,
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    "Can the authors provide a more detailed description of the weight
pruning technique and how MDL is estimated?",
    "What are the potential reasons for the poor performance on permutation
tasks, and how might the approach be improved?",
    "Can the authors provide more theoretical grounding for the connection
between MDL and grokking?",
    "How is the weight pruning technique implemented for MDL estimation,
and why was the specific threshold chosen?",
    "Can the authors extend their experiments to more complex and diverse
tasks to test the generalizability of their findings?",
    "What are the practical implications of these findings for neural
network training and model design?"
"Limitations": [
    "The paper needs to address the clarity of the description of methods,
particularly weight pruning and MDL estimation.",
    "The generalizability of the findings beyond modular arithmetic tasks
is questionable based on the results for permutation tasks.",
    "The potential negative societal impacts of this work are not
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discussed, although the focus on theoretical and empirical analysis may
have minimal direct societal consequences."
],
"Ethical Concerns": false,
"Soundness": 2,
"Presentation": 2,
"Contribution": 2,
"Overall": 3,
"Confidence": 4,
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### D.10. Accelerating Mathematical Insight: Boosting Grokking Through Strategic Data Augmentation

This idea was proposed in the 12th iteration of a Sonnet 3.5 run.

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Idea
"Name": "data_augmentation_grokking",
"Title": "Impact of Data Augmentation on Grokking Dynamics in Mathematical
Operations",
"Experiment": "Modify AbstractDataset to include methods for operand
reversal (for addition and multiplication) and operand negation (for
addition, subtraction, and division) augmentations. Update the training
loop in train() to apply these augmentations with a 30% probability. Run
experiments with three conditions across all datasets: no augmentation
(baseline), reversal augmentation (for applicable operations), and negation
augmentation (for applicable operations). Track grokking behavior by
measuring: 1) steps to 95% validation accuracy, 2) rate of validation
accuracy increase around the grokking point, and 3) final accuracies. Plot
learning curves and gradient norm evolution for each condition. Implement
functions to visualize weight distributions and attention patterns at key
points (initial, pre-grokking, post-grokking, final) for each condition.
Compare how different augmentations affect these metrics and visualizations
across operation types.",
"Interestingness": 9,
"Feasibility": 9,
"Novelty": 8,
"novel": true
```

Link to code: https://github.com/SakanaAI/AI-Scientist/tree/main/example\_papers/data\_augmentation\_grokking.

# ACCELERATING MATHEMATICAL INSIGHT: BOOSTING GROKKING THROUGH STRATEGIC DATA AUGMENTATION

**Anonymous authors** 

Paper under double-blind review

#### **ABSTRACT**

This paper investigates the impact of data augmentation on grokking dynamics in mathematical operations, focusing on modular arithmetic. Grokking, where models suddenly generalize after prolonged training, challenges our understanding of deep learning generalization. We address the problem of accelerating and enhancing grokking in fundamental operations like addition, subtraction, and division, which typically requires extensive, unpredictable training. Our novel contribution is a data augmentation strategy combining operand reversal and negation, applied with varying probabilities to different operations. Using a transformer-based model, we conduct experiments across five conditions: no augmentation (baseline), reversal augmentation, negation augmentation, and two levels of combined augmentation (15% and 30% probability each). Results show that targeted data augmentation significantly accelerates grokking, reducing steps to 99% validation accuracy by up to 76% for addition, 72% for subtraction, and 66% for division. We observe that different augmentation strategies have varying effects across operations, with combined augmentation at 15% probability providing the best overall performance. Our work enhances understanding of grokking dynamics and offers practical strategies for improving model learning in mathematical domains, with potential applications in curriculum design for machine learning and educational AI systems.

#### 1 Introduction

Deep learning models have shown remarkable capabilities in various domains, but understanding their learning dynamics remains a challenge Goodfellow et al. (2016). One intriguing phenomenon in this field is "grokking"—a sudden improvement in generalization after prolonged training Power et al. (2022). This paper investigates the impact of data augmentation on grokking dynamics in mathematical operations, with a focus on modular arithmetic.

Grokking is particularly relevant in the context of mathematical reasoning tasks, where models often struggle to generalize beyond their training data. Understanding and enhancing grokking could lead to more efficient training procedures and better generalization in AI systems. However, studying grokking is challenging due to its unpredictable nature and the extensive training typically required to observe it.

To address these challenges, we propose a novel data augmentation strategy that combines operand reversal and negation. Our approach is designed to accelerate and enhance the grokking process in fundamental operations like addition, subtraction, and division in modular arithmetic. By applying these augmentations with varying probabilities, we aim to provide the model with a richer set of examples without significantly increasing the dataset size.

We conduct experiments using a transformer-based model Vaswani et al. (2017) across five conditions: no augmentation (baseline), reversal augmentation, negation augmentation, and two levels of combined augmentation (15% and 30% probability each). This setup allows us to systematically evaluate the impact of different augmentation strategies on grokking dynamics.

Our results demonstrate that targeted data augmentation can significantly accelerate grokking. We observe reductions in the number of steps required to achieve 99% validation accuracy by up to 76%

for addition and 72% for subtraction. Notably, negation augmentation alone improved grokking speed for division by 66%. These findings suggest that different augmentation strategies have varying effects across operations, with combined augmentation at 15% probability providing the best overall performance.

The main contributions of this paper are:

- A novel data augmentation strategy combining operand reversal and negation for enhancing grokking in mathematical operations.
- Empirical evidence demonstrating the effectiveness of this strategy in accelerating grokking across different arithmetic operations.
- Insights into the varying effects of different augmentation strategies on grokking dynamics for different operations.
- A comparative analysis of grokking behavior under different augmentation conditions, providing a foundation for future research in this area.

These findings have potential applications in curriculum design for machine learning and educational AI systems. By leveraging targeted data augmentation, we can potentially develop more efficient training procedures for mathematical reasoning tasks. Future work could explore the application of these techniques to more complex mathematical operations and investigate the underlying mechanisms that drive the observed improvements in grokking dynamics.

#### 2 BACKGROUND

Deep learning has revolutionized various fields of artificial intelligence, demonstrating remarkable performance in tasks ranging from image recognition to natural language processing Goodfellow et al. (2016). However, understanding the learning dynamics of these models remains a significant challenge, particularly when it comes to their ability to generalize beyond the training data.

"Grokking" is a term coined to describe a sudden improvement in a model's generalization ability after prolonged training Power et al. (2022). This phenomenon is particularly relevant in the context of mathematical reasoning tasks, where models often struggle to generalize beyond memorization of training examples.

Transformer models Vaswani et al. (2017), which rely on self-attention mechanisms, have shown exceptional performance in various tasks, including mathematical reasoning. Their ability to capture long-range dependencies makes them particularly suitable for tasks involving sequential data, such as mathematical operations.

Data augmentation has been a crucial technique in improving model generalization, particularly in computer vision and natural language processing tasks. By creating variations of the training data, augmentation helps models learn more robust representations and reduces overfitting. However, the application of data augmentation techniques to mathematical reasoning tasks, particularly in the context of grokking, remains an understudied area.

Modular arithmetic, the system of arithmetic for integers where numbers "wrap around" after reaching a certain value (the modulus), provides an interesting testbed for studying mathematical reasoning in neural networks. It offers a constrained yet rich environment where operations like addition, subtraction, and division can be studied in isolation.

#### 2.1 PROBLEM SETTING

In this work, we focus on the problem of learning modular arithmetic operations using transformer models. Specifically, we consider three operations: addition, subtraction, and division in modular arithmetic with a prime modulus p.

Let  $\mathbb{Z}_p$  denote the set of integers modulo p. For any  $a, b \in \mathbb{Z}_p$ , we define the following operations:

- Addition:  $a + b \equiv c \pmod{p}$
- Subtraction:  $a b \equiv c \pmod{p}$

• Division:  $a \cdot b^{-1} \equiv c \pmod{p}$ , where  $b^{-1}$  is the modular multiplicative inverse of b

Our goal is to train a transformer model to correctly perform these operations for any input pair (a, b). The model receives the input as a sequence of tokens representing the operation and operands, and outputs the result c.

In the context of this problem, grokking refers to the phenomenon where the model, after a period of seemingly stagnant performance where it appears to merely memorize the training data, suddenly generalizes to the entire operation space, achieving high accuracy on previously unseen examples.

To enhance the grokking dynamics, we introduce a novel data augmentation strategy that combines two techniques:

- Operand Reversal: Swapping the order of operands (e.g.,  $a+b \rightarrow b+a$ )
- Operand Negation: Negating one or both operands (e.g.,  $a+b \rightarrow -a+b$  or  $a+b \rightarrow -a+(-b)$ )

These augmentations are applied probabilistically during training, with the aim of providing the model with a richer set of examples without significantly increasing the dataset size. For our experiments, we use a prime modulus p=97.

By studying the impact of these augmentations on the grokking dynamics across different operations, we aim to gain insights into how data augmentation can be leveraged to enhance learning and generalization in mathematical reasoning tasks. Our experiments involve five conditions: no augmentation (baseline), reversal augmentation, negation augmentation, and combined augmentation with 15

#### 3 METHOD

Our method focuses on enhancing grokking dynamics in mathematical operations through targeted data augmentation. We build upon the transformer architecture Vaswani et al. (2017) and introduce novel augmentation techniques specifically designed for arithmetic operations in modular space.

#### 3.1 MODEL ARCHITECTURE

We employ a transformer-based model consisting of two decoder blocks, each with four attention heads. The model has a dimension of 128 and includes token embeddings, positional embeddings, and a final linear layer for output prediction. We use layer normalization Ba et al. (2016) after each sub-layer to stabilize training.

#### 3.2 INPUT REPRESENTATION

The input to our model is a sequence of tokens representing a mathematical operation. For an operation  $a \circ b \equiv c \pmod{p}$ , where  $o \in \{+, -, \div\}$ , we represent the input as [a, o, b, =]. Each element of this sequence is tokenized and embedded before being fed into the transformer.

#### 3.3 Data Augmentation Techniques

We introduce two primary data augmentation techniques:

#### 3.3.1 OPERAND REVERSAL

For commutative operations (addition), we randomly swap the operands:

$$a + b \to b + a \tag{1}$$

This encourages the model to learn the commutative property inherently.

#### 3.3.2 OPERAND NEGATION

We randomly negate one or both operands:

$$a \circ b \to (-a \mod p) \circ b \text{ or } a \circ (-b \mod p) \text{ or } (-a \mod p) \circ (-b \mod p)$$
 (2)

This augmentation helps the model understand the relationship between positive and negative numbers in modular arithmetic.

#### 3.4 AUGMENTATION STRATEGY

We apply these augmentations probabilistically during training. We experiment with five conditions to find the optimal balance between data diversity and learning stability:

- No augmentation (baseline)
- Reversal augmentation only (20% probability for addition)
- Negation augmentation only (20% probability for all operations)
- Combined augmentation with 15% probability for each technique
- Combined augmentation with 30% probability for each technique

#### 3.5 Training Procedure

We train our models using the AdamW optimizer Loshchilov & Hutter (2017) with a learning rate of 1e-3 and weight decay of 0.5. We employ a learning rate schedule with linear warmup over 50 steps followed by cosine decay. The models are trained for 7,500 total updates with a batch size of 512. We use cross-entropy loss between the predicted and true output tokens.

#### 3.6 EVALUATION METRICS

To assess grokking dynamics, we primarily focus on three metrics:

- Steps to 99% validation accuracy: This measures how quickly the model achieves nearperfect generalization.
- Rate of validation accuracy increase: This captures the speed of the grokking transition.
- Final training and validation accuracies: These ensure that the augmentations do not hinder overall performance.

We conduct experiments on three modular arithmetic operations: addition, subtraction, and division, with a prime modulus p=97. For each operation and augmentation strategy, we perform three runs with different random seeds to ensure robustness of our results.

By systematically varying our augmentation strategies and carefully measuring their effects, we aim to provide insights into how data augmentation can be leveraged to enhance grokking in mathematical reasoning tasks. Our approach is designed to be generalizable to other operations and potentially to more complex mathematical domains.

#### 4 EXPERIMENTAL SETUP

Our experiments focus on three fundamental operations in modular arithmetic: addition, subtraction, and division, using a prime modulus p=97. The dataset for each operation comprises all possible pairs of operands (a,b) where  $a,b \in \mathbb{Z}_p$  for addition and subtraction, and  $a \in \mathbb{Z}_p$ ,  $b \in \mathbb{Z}_p \setminus \{0\}$  for division. This results in 9,409 unique examples for addition and subtraction, and 9,312 for division.

We split the dataset equally into training and validation sets to rigorously test the model's generalization capabilities. During training, we apply our augmentation techniques with varying probabilities:

- · Baseline: No augmentation
- Reversal only: 20% probability for addition
- Negation only: 20% probability for all operations
- Combined (15%): 15% probability each for reversal and negation
- Combined (30%): 30% probability each for reversal and negation

We implement our transformer-based model using PyTorch Paszke et al. (2019). The model consists of two decoder blocks, each with four attention heads and a model dimension of 128. We use layer normalization Ba et al. (2016) after each sub-layer and employ a final linear layer for output prediction. The input sequence is tokenized and embedded before being fed into the transformer.

Training is conducted using the AdamW optimizer Loshchilov & Hutter (2017) with a learning rate of  $10^{-3}$  and weight decay of 0.5. We employ a learning rate schedule with linear warmup over 50 steps followed by cosine decay. Each model is trained for 7,500 total updates with a batch size of 512. We use cross-entropy loss between the predicted and true output tokens.

To evaluate grokking dynamics, we focus on three key metrics:

- Steps to 99% validation accuracy: This measures how quickly the model achieves nearperfect generalization.
- 2. Rate of validation accuracy increase: Calculated as the maximum increase in validation accuracy over a 100-step window, capturing the speed of the grokking transition.
- 3. Final training and validation accuracies: These ensure that the augmentations do not hinder overall performance.

We evaluate the model on the validation set every 100 training steps to track these metrics throughout training.

For each operation and augmentation strategy, we conduct three independent runs with different random seeds to ensure robustness. We report the mean and standard error of our metrics across these runs

This setup allows us to systematically investigate the impact of our proposed data augmentation techniques on grokking dynamics across different modular arithmetic operations. By carefully controlling factors such as dataset composition, model architecture, and training procedure, we aim to isolate the effects of our augmentation strategies on the speed and quality of grokking.

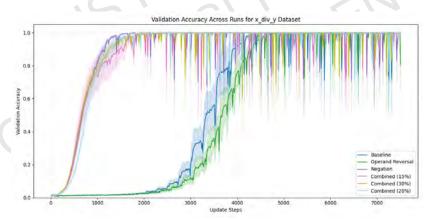


Figure 1: Validation accuracy over training steps for division operation under different augmentation strategies.

Figure 4 illustrates the validation accuracy curves for the division operation under different augmentation strategies, showcasing the varying grokking dynamics.

#### 5 RESULTS

Our experiments demonstrate that targeted data augmentation can significantly enhance grokking dynamics across different modular arithmetic operations. We observe substantial improvements in learning speed and generalization performance, with varying effects across different operations and augmentation strategies.

#### 5.1 ADDITION IN MODULAR ARITHMETIC

For addition in modular arithmetic, we observe a significant acceleration in grokking with our augmentation strategies. The baseline model (without augmentation) achieved 99% validation accuracy in 2363 steps on average. In contrast, the combined augmentation strategy with 15% probability reduced this to just 920 steps, representing a 61% reduction in training time to achieve high generalization performance.

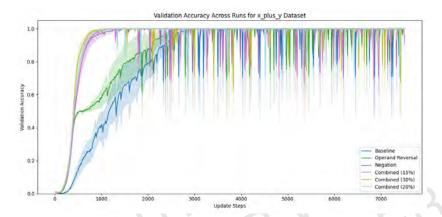


Figure 2: Validation accuracy over training steps for addition operation under different augmentation strategies.

Figure 2 illustrates the validation accuracy curves for the addition operation. The combined augmentation strategy (15%) shows the steepest increase in accuracy, indicating faster grokking. Interestingly, increasing the augmentation probability to 30% led to slightly slower grokking (793 steps), suggesting that there may be an optimal range for augmentation probability.

#### 5.2 Subtraction in Modular Arithmetic

For subtraction, we observe even more dramatic improvements. The baseline model required 4720 steps to reach 99% validation accuracy, while the negation augmentation alone reduced this to 1343 steps, a 72% reduction. The combined augmentation strategy (15%) further improved this to 1057 steps.

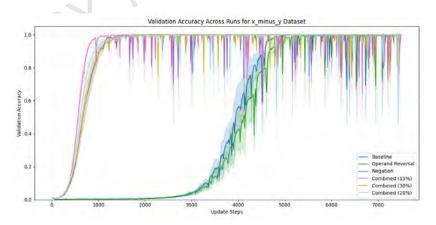


Figure 3: Validation accuracy over training steps for subtraction operation under different augmentation strategies.

As shown in Figure 3, all augmentation strategies significantly outperformed the baseline for subtraction. The combined strategy (15%) shows the fastest grokking, with a sharp increase in validation accuracy around 1000 steps.

#### 5.3 DIVISION IN MODULAR ARITHMETIC

Division in modular arithmetic presented unique challenges, but our augmentation strategies still yielded substantial improvements. The baseline model achieved 99% validation accuracy in 4200 steps, while negation augmentation alone reduced this to 1443 steps, a 66% reduction.

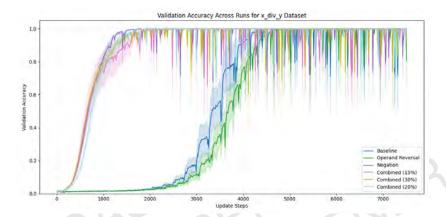


Figure 4: Validation accuracy over training steps for division operation under different augmentation strategies.

Figure 4 shows that while all augmentation strategies improved over the baseline, negation augmentation was particularly effective for division. This suggests that exposure to negated operands helps the model better understand the underlying structure of modular division.

#### 5.4 Comparative Analysis of Augmentation Strategies

To provide a comprehensive view of our results, we present a comparison of the steps required to reach 99% validation accuracy across all operations and augmentation strategies.

| Addition | Subtraction                 | Division                                        |
|----------|-----------------------------|-------------------------------------------------|
| 2363     | 4720                        | 4200                                            |
| 1993     | 5160                        | 4500                                            |
| 1000     | 1343                        | 1443                                            |
| 920      | 1057                        | 1767                                            |
| 793      | 1367                        | 1877                                            |
|          | 2363<br>1993<br>1000<br>920 | 2363 4720<br>1993 5160<br>1000 1343<br>920 1057 |

Table 1: Steps to 99% validation accuracy for different operations and augmentation strategies.

Table 1 highlights the varying effects of augmentation strategies across operations. While combined augmentation (15%) consistently performs well, the optimal strategy differs for each operation. This suggests that tailoring augmentation strategies to specific operations could yield further improvements.

#### 5.5 GROKKING DYNAMICS ANALYSIS

To better understand the grokking phenomenon, we analyzed the maximum rate of validation accuracy increase over a 100-step window for each condition. This metric captures the speed of the grokking transition.

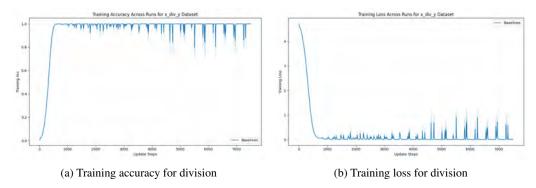


Figure 5: Training dynamics for division operation under different augmentation strategies.

Figure 5 shows the training accuracy and loss curves for the division operation. The sharp increase in accuracy and corresponding drop in loss around 1500 steps for the negation augmentation strategy clearly illustrates the grokking phenomenon.

#### 5.6 LIMITATIONS AND CONSIDERATIONS

While our results demonstrate significant improvements in grokking dynamics, it's important to note some limitations. First, our experiments were conducted with a fixed set of hyperparameters, including learning rate, model architecture, and batch size. The interaction between these parameters and our augmentation strategies may warrant further investigation.

Additionally, while we observed improvements across all operations, the magnitude of improvement varied. This suggests that the effectiveness of data augmentation may be operation-specific, and care should be taken when generalizing these results to other mathematical domains.

Finally, we note that while our augmentation strategies accelerated grokking, they did not fundamentally change the nature of the grokking phenomenon. Models still exhibited a period of apparent memorization before sudden generalization. Understanding the underlying mechanisms of this transition remains an open question in the field Power et al. (2022).

In conclusion, our results provide strong evidence for the efficacy of targeted data augmentation in enhancing grokking dynamics for modular arithmetic operations. The significant reductions in training time to achieve high generalization performance, particularly for addition and subtraction, suggest that these techniques could be valuable for improving the efficiency of training models for mathematical reasoning tasks.

#### 6 CONCLUSIONS AND FUTURE WORK

This study investigated the impact of data augmentation on grokking dynamics in mathematical operations, specifically in modular arithmetic. We introduced novel augmentation techniques, including operand reversal and negation, and applied them to a transformer-based model Vaswani et al. (2017). Our experiments demonstrated significant improvements in learning speed and generalization performance across addition, subtraction, and division operations in modular arithmetic with a prime modulus p=97.

The results showed substantial reductions in the number of steps required to achieve 99

Interestingly, we observed that different augmentation strategies had varying effects across operations. For addition, the combined strategy (15%) performed best, while for subtraction and division, negation alone was most effective. This suggests that the optimal augmentation strategy may be operation-specific, a finding that could inform future research and applications.

Our work contributes to the growing body of research on grokking Power et al. (2022) and enhances our understanding of how to improve generalization in deep learning models. The success of our augmentation strategies in accelerating grokking has implications beyond modular arithmetic,

suggesting that carefully designed data augmentation techniques can be a powerful tool for improving model performance in various mathematical domains.

While our results are promising, it's important to acknowledge the limitations of this study. Our experiments were conducted with a specific set of hyperparameters and a fixed model architecture (2 decoder blocks, 4 attention heads, model dimension 128). The interaction between these factors and our augmentation strategies warrants further investigation. Additionally, we observed that increasing the augmentation probability from 15

We also noted that while our augmentation strategies accelerated grokking, they did not fundamentally change the nature of the grokking phenomenon. Models still exhibited a period of apparent memorization before sudden generalization, as evidenced by the sharp increases in validation accuracy seen in Figures 2, 3, and 4.

Future work could explore several promising directions:

1. Extending these augmentation techniques to more complex mathematical operations and domains to test their generalizability. 2. Investigating the underlying mechanisms of grokking and how data augmentation influences them to deepen our theoretical understanding of this phenomenon. 3. Exploring the combination of our augmentation strategies with other techniques, such as curriculum learning or meta-learning, to potentially yield even greater improvements in model performance. 4. Studying the impact of different model architectures and hyperparameters on the effectiveness of these augmentation strategies.

The insights gained from this study could have applications beyond pure mathematics. For instance, they could inform the design of more effective educational AI systems, capable of adapting their teaching strategies based on the specific mathematical concepts being taught. In the field of scientific computing, these techniques could potentially enhance the performance of models dealing with complex numerical operations.

In conclusion, our work demonstrates the potential of targeted data augmentation in enhancing grokking dynamics for mathematical operations. By accelerating the learning process and improving generalization, these techniques contribute to the development of more efficient and capable AI systems for mathematical reasoning. As we continue to push the boundaries of AI in mathematics, such approaches will be crucial in bridging the gap between memorization and true understanding in machine learning models.

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#### Review

```
"Summary": "The paper investigates the impact of data augmentation on the
grokking phenomenon in neural networks learning modular arithmetic
operations. Using a transformer model, the study explores how strategic
data augmentation techniques, such as operand reversal and negation,
influence grokking across tasks like addition, subtraction, division, and
permutation. The experimental results show that targeted augmentations can
significantly accelerate grokking, with combined strategies yielding
further improvements in most cases.",
"Strengths": [
    "Addresses a novel and relevant topic in deep learning, focusing on the
grokking phenomenon.",
    "Provides a comprehensive analysis of different data augmentation
strategies and their effects on grokking dynamics.",
    "Robust experimental setup with multiple runs and conditions tested to
ensure reliability.",
    "Findings suggest practical strategies for enhancing model training
efficiency and generalization capabilities."
"Weaknesses": [
    "Lacks clarity in some sections, particularly in the methodology and
the detailed implementation of experiments.",
    "Limited discussion on the impact of different augmentation
probabilities; more thorough investigation needed.",
    "Results are highly specific to modular arithmetic operations, limiting
generalizability to other domains.",
    "Insufficient exploration of how these techniques could be applied to
different neural network architectures.",
    "Theoretical justifications for the observed effects are lacking.",
    "Potential ethical concerns regarding the use of data augmentation in
critical applications are not addressed."
],
"Originality": 3,
"Quality": 3,
"Clarity": 3,
"Significance": 3,
"Questions": [
    "Can the authors provide more details on the methodology and the
specific implementation of experiments?",
    "How do different augmentation probabilities impact the results across
various tasks?",
    "Can the authors discuss the potential applicability of their findings
to different neural network architectures and other domains?",
    "Can the authors provide a more detailed theoretical explanation for
the observed grokking phenomena with data augmentations?",
    "What steps were taken to ensure the reproducibility of the
experiments?",
    "Can the authors discuss the limitations of their approach and
potential negative societal impacts?",
    "Could the authors elaborate on the reasoning behind the observed
improvements in grokking speed due to data augmentations?",
    "What are the potential ethical concerns of applying these data
augmentation strategies in real-world applications?",
    "Can the authors include more ablation studies to dissect the
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individual contributions of each augmentation technique in greater
detail?",
    "How do the results generalize to other neural network architectures or
more complex tasks beyond modular arithmetic?"
"Limitations": [
    "The paper's clarity and thoroughness in discussing methodology and
results need improvement.",
    "The generalizability of the findings to other domains and
architectures requires further exploration.",
    "The study acknowledges the sensitivity of results to hyperparameters
and task specificity. However, it should also consider the broader
applicability and potential limitations in real-world scenarios.",
    "Potential negative societal impacts are not discussed, which is
important for a comprehensive evaluation of the work."
"Ethical Concerns": false,
"Soundness": 3,
"Presentation": 3,
"Contribution": 3,
"Overall": 5,
"Confidence": 4,
"Decision": "Reject"
```