# A Model-Driven Optimization Approach to Solving Partial Differential Equations

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*Abstract*—The abstract goes here.

## I. INTRODUCTION

In many fields, like computational fluid dynamics, computational electromagnetics, and structural mechanics, programs are required that model phenomena. A feature common to these scientific computations is that they involve the resolution of partial differential equations (PDEs). Numerical techniques, like Finite Volume Method (FVM) and Finite Element Method (FEM), are widely employed in real-world applications to approximate solutions of PDEs. Unstructured meshes are often used to discretize the PDE domain, since their geometric flexibility allows such PDE solvers to be extrimely effective. However, they are also the main responsible for performance degradation due to the introduction of indirect accesses (e.g. `A[B[i]]`) to access mesh components. A typical problem, therefore, is that implementing a scientific application requires a great effort, other than domain knowledge, and even more time is needed to optimize their execution time.

An unstructured mesh represents a discretization of the equation domain into two- or three-dimensional cells (or "elements"), which define the smallest area within which the solution is sought. As the number of elements can be of the order of millions, a major issue is the time required to perform the numerical method, which can be of the order of hours or days. OP2 [2] and Lizst [1] are Domain Specific Languages that target mesh-based computations by allowing programmers to achieve highly-efficient parallel execution. The successful porting of Hydra, a CFD industrial application devised by Rolls Royce for turbomachinery design (based on FVM, roughly 50000 lines of code and a mesh size that can be over 100 millions edges), to OP2, demonstrates the effectiveness of this framework for implementing PDEs solvers [?]. Both OP2 and Lizst adopt a kernel-oriented programming model: the computation semantics is expressed through self-contained functions (or "kernels"), each function applied in parallel to all items in a specific set of mesh components (e.g. edges, vertices, elements), with an implicit synchronization between the application of two different functions. Together with the indirections problem, having systematic optimizations for the kernels is the major concern in unstructured mesh applications.

In this paper, we describe a model-driven optimization strategy for a class of kernels used in Finite Element computations. The working set of kernels used in PDE solvers is usually small enough to fit the L1 cache, and definitely fits the L2 cache when high-order methods are employed to improve the accuracy of the found solution. However, the latter case is not considered in our work. This implies that main actors of our study will be register allocation and instruction-level parallelism, especially in the form of SIMD vectorisation.

## II. BACKGROUND

## III. A MODEL-DRIVEN APPROACH TO CODE OPTIMIZATION

## IV. THE PYOP2 COMPILER

## V. PERFORMANCE EVALUATION

## VI. RELATED WORK

## VII. CONCLUSIONS

### ACKNOWLEDGMENT

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