Lexical Analysis

DFA Minimization & Equivalence to Regular Expressions

DFA State Minimization

- How to Reduce the Number of States of a DFA?
  - Find unique minimum-state DFA (up to state names)
  - Need to recognize the same language
- Normalization
  - Assume every state has a transition on every symbol
  - If not, just add missing transitions to a dead state
- Key Idea
  - Find string *w* that distinguishes states *s* and *t*
- Algorithm
  - Start with accepting *vs.* non-accepting states partition of states
  - Refine state groups on all input sequences, i.e. by tracing all transitions
  - Until no refinement is possible

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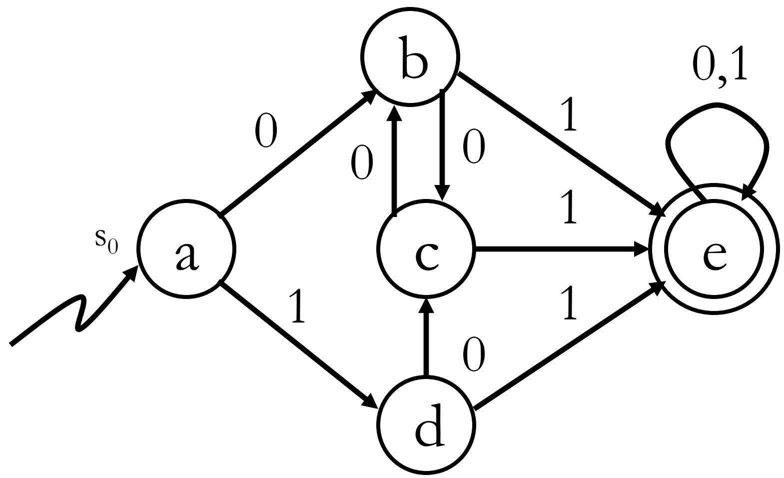
  - Until no refinement is possible

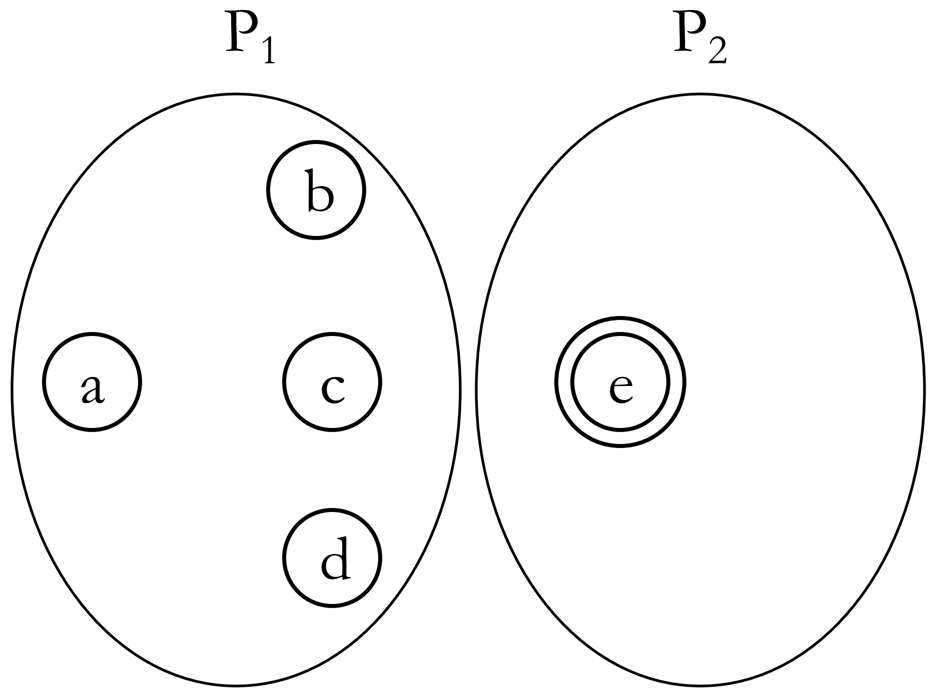
- Does this Terminate?
  - Refinement will end; in the limit 1 partition is 1 state

- What to do When Refinement Terminates?
  - Elect representative state for each partition
  - Merge edges
  - Remove unneeded states in each partition

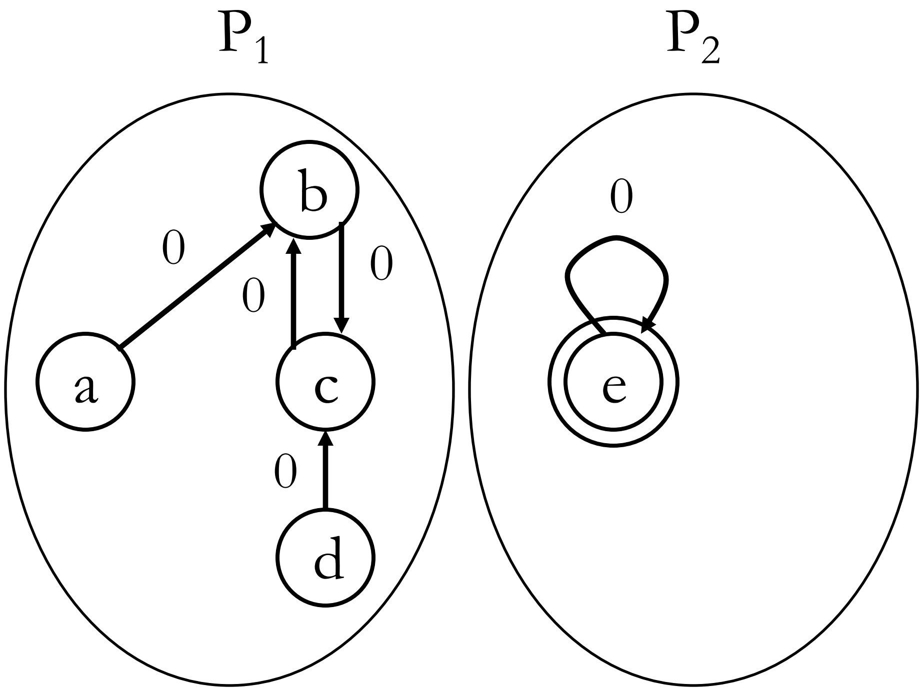
Minimization Example



Minimization Example

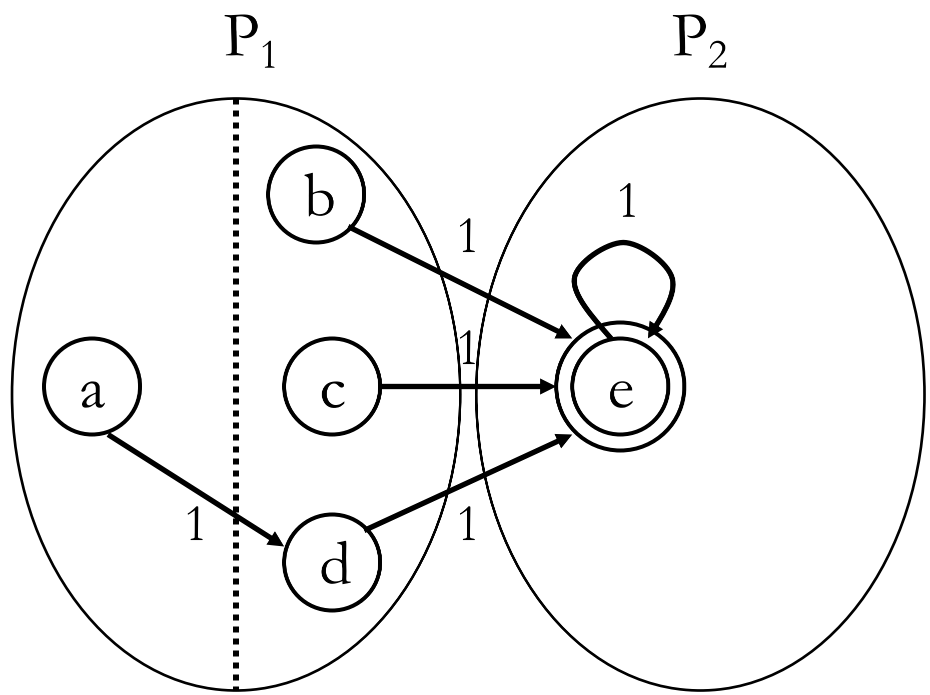


Minimization Example



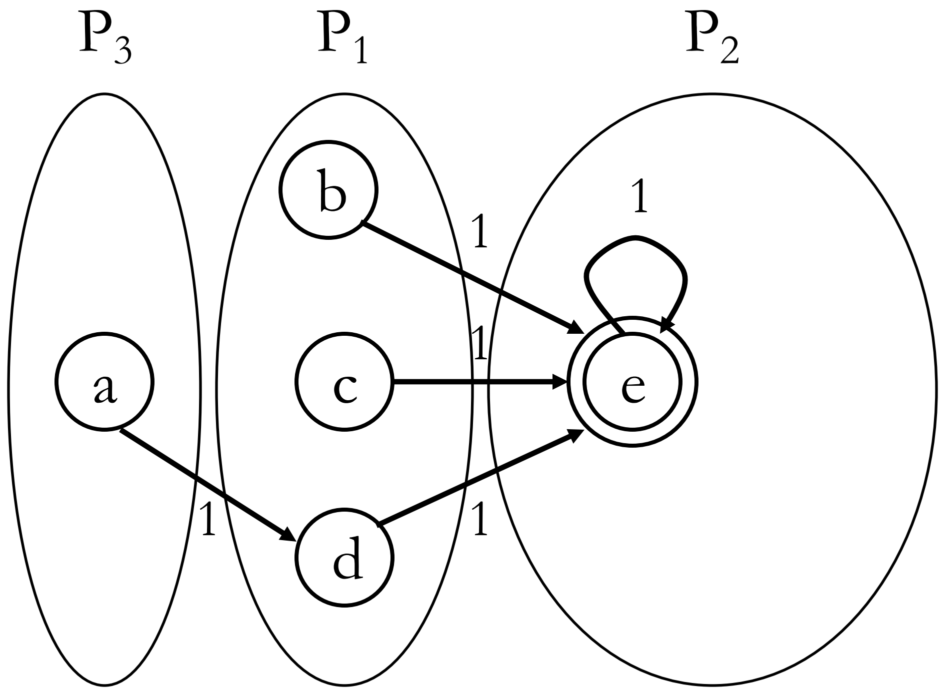
- Label 0 does not split any partition!

Minimization Example



- Label 1 splits P_1 and P_2 partitions!

Minimization Example



- Label 1 splits P_1 and P_2 partitions!

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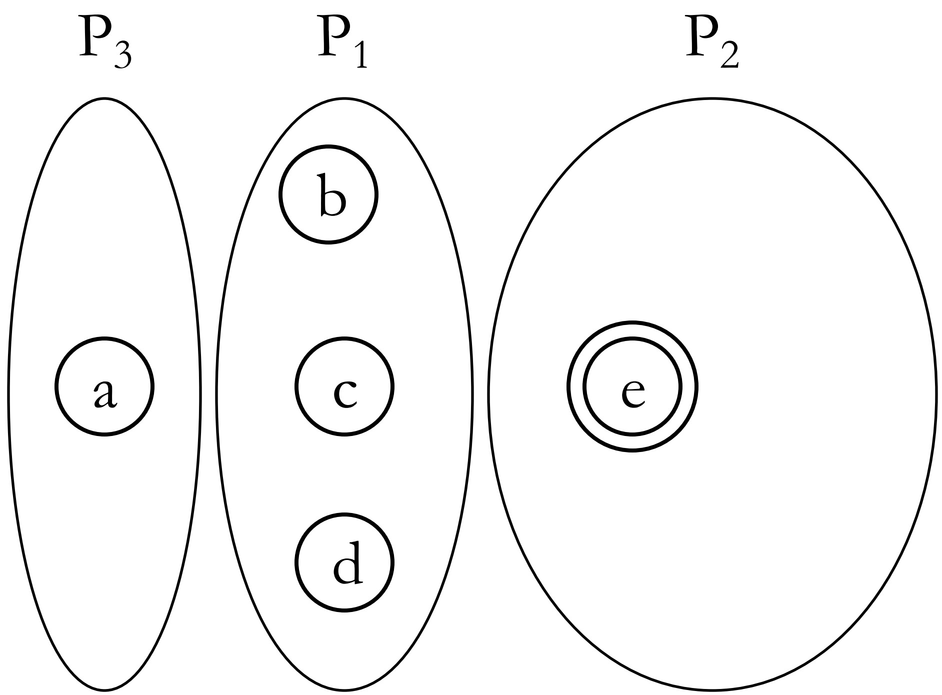

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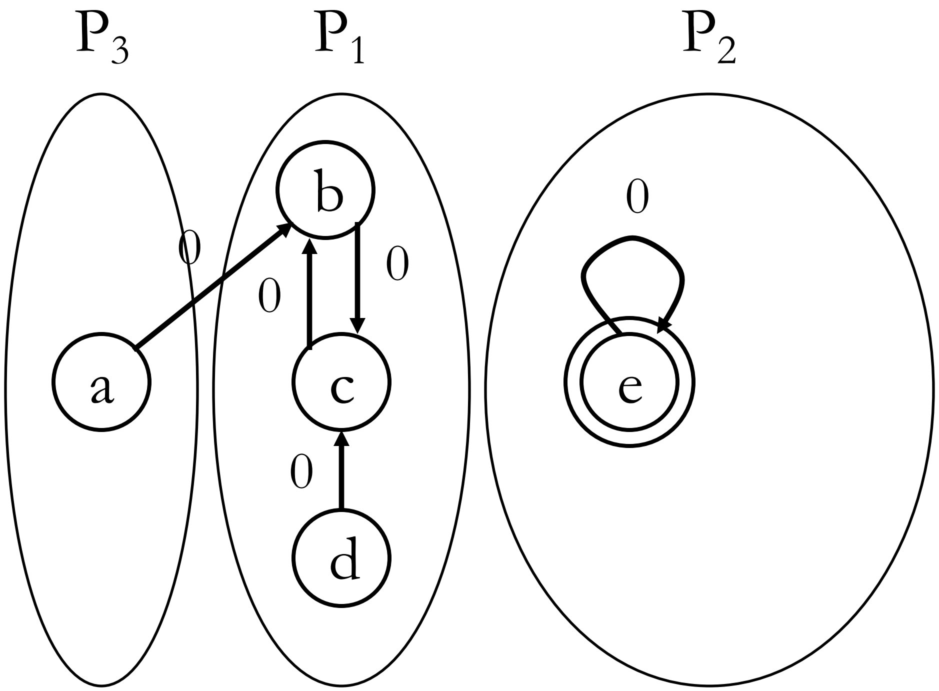

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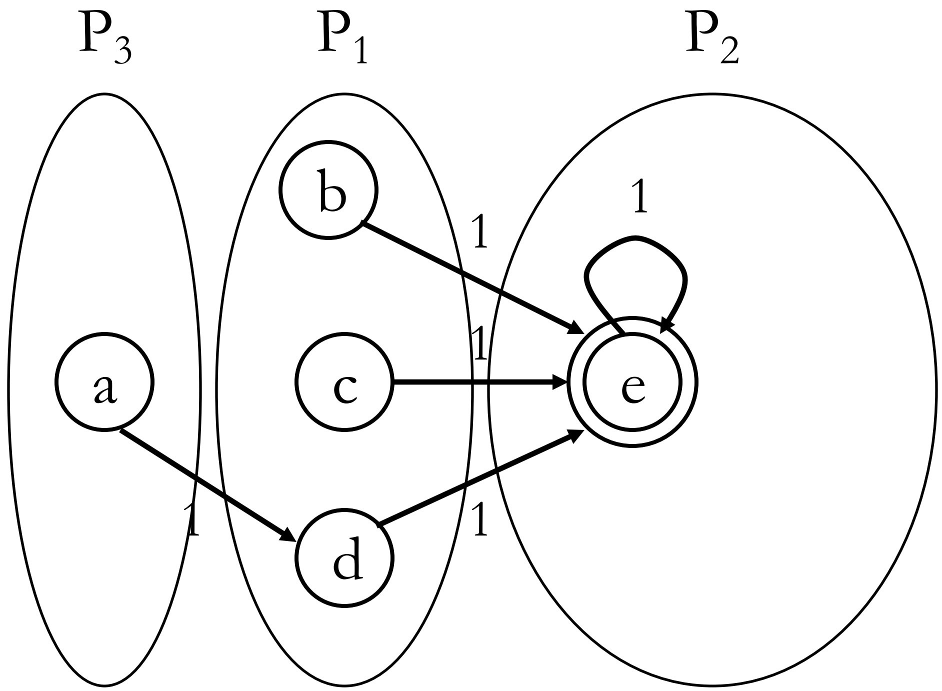

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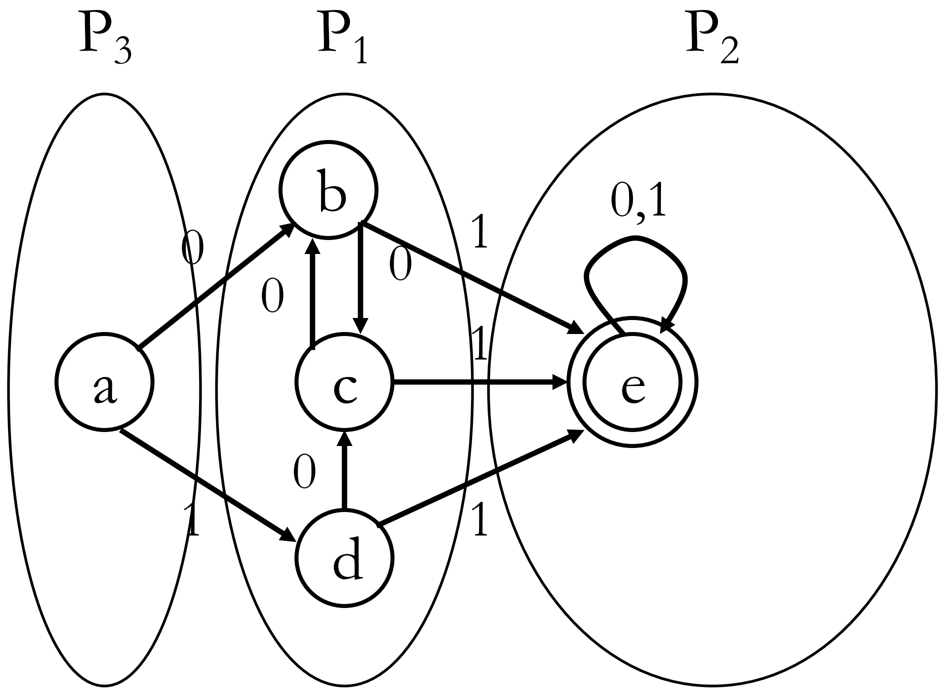
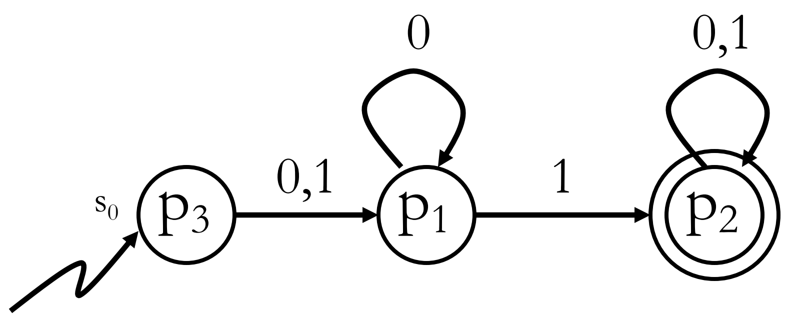


- Elect Representative and Merge Edges

Minimization Example



- Elect Representative and Merge Edges

DFA State Minimization: Algorithm

DFA = $\{D, \sum, d, s_0, D_F\}$

```
P ← {D_F , {D - D_F}}
while (P is still changing)
  T ← Ø
  for each set p ∈ P
     T ← T ∪ Split(p)
  P ← T
```

```
Split(S)
  for each c ∈ ∑
    if c splits S into s_1 and s_2
      then return {s_1 , s_2}
  return S
```

DFA to RE: Kleene Construction

- Path Problem over the DFA
  - Starting from state s_1 (numbering of states is 1 … N - **important**)
  - Label all edges through all states to an accepting state
  - What to do with cycles in the DFA? as there are infinite length paths?
- Kleene Construction*
  - Iterate and merge path expressions for every pair of nodes i and j not going through any node with label higher then k
  - Increase k up to N
  - At the end, do the union of all path expressions that start at s_1 and end in a final state.

* This is a dynamic programming algorithmic approach

DFA to RE: Kleene Construction

for i = 1 to N

  for j = 1 to N

    $R^0_{ij} = \{a \mid \delta(s_i,a) = s_j\}$

    if (i = j) then

      $R^0_{ij} = R^0_{ij} \mid \{\varepsilon\}$

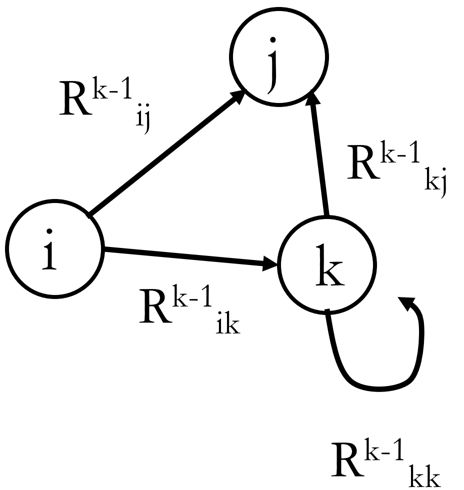
for k = 1 to N

  for i = 1 to N

    for j = 1 to N

      $R^k_{ij} = R^{k-1}_{ik} (R^{k-1}_{kk})^* R^{k-1}_{kj} \mid R^{k-1}_{ij}$

$L = \mid s_j \in S_F \; R^N_{1j}$



DFA to RE: Kleene Construction

for i = 1 to N

  for j = 1 to N

    $R^0_{ij} = \{a \mid \delta(s_i, a) = s_j\}$

    if (i = j) then

      $R^0_{ij} = R^0_{ij} \mid \{\varepsilon\}$

Direct Path

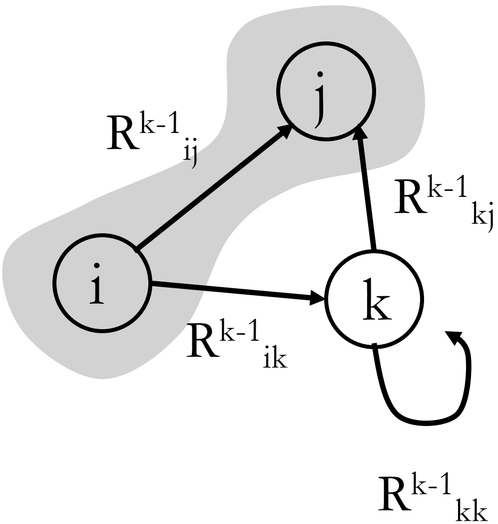
for k = 1 to N

  for i = 1 to N

    for j = 1 to N

      $R^k_{ij} = R^{k-1}_{ik} (R^{k-1}_{kk})^* R^{k-1}_{kj} \mid R^{k-1}_{ij}$

$L = |_{s_j \in S_F} R^N_{1j}$



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DFA to RE: Kleene Construction

Direct Path

```
for i = 1 to N
  for j = 1 to N
    R^0_ij = {a | δ(s_i,a) = s_j}
    if (i = j) then
      R^0_ij = R^0_ij | {ε}
```

$$R^{0}_{ij} = \{a \mid \delta(s_i,a) = s_j\}$$

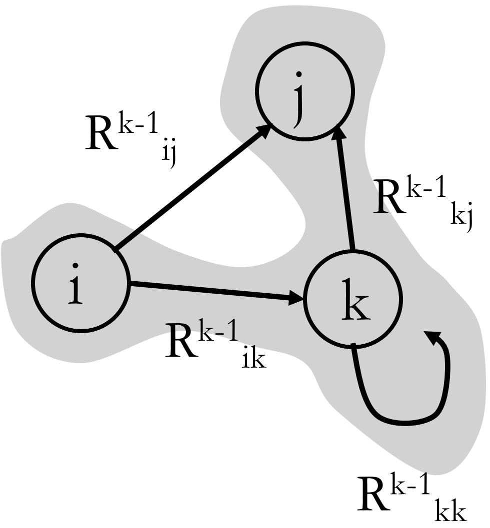
$$R^{0}_{ij} = R^{0}_{ij} \mid \{\varepsilon\}$$

Indirect Path

```
for k = 1 to N
  for i = 1 to N
    for j = 1 to N
      R^k_ij = R^(k-1)_ik (R^(k-1)_kk)* R^(k-1)_kj | R^(k-1)_ij
L = | s_j ∈ S_F R^N_1j
```

$$R^{k}_{ij} = R^{k-1}_{ik}\,(R^{k-1}_{kk})^{*}\,R^{k-1}_{kj} \mid R^{k-1}_{ij}$$

$$L = \mid_{s_j \in S_F} R^{N}_{1j}$$



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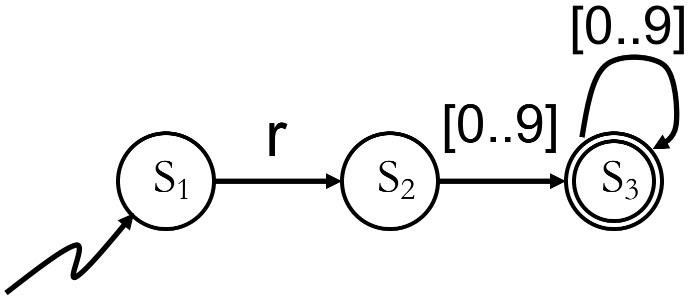
DFA to RE: Example

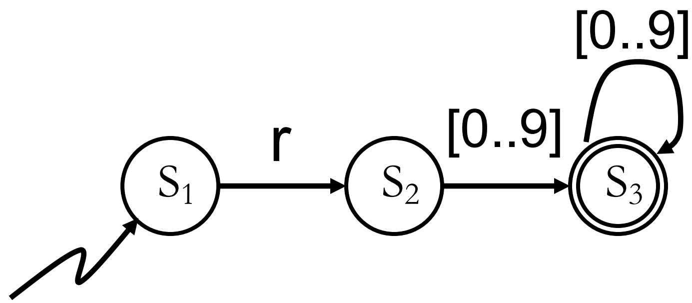


$R^0_{12} = \text{r}$

$R^0_{23} = [0..9]$

$R^0_{33} = [0..9] \mid \varepsilon$

DFA to RE: Example



R^0_{12} = r

R^0_{23} = [0..9]

R^0_{33} = [0..9] | ε

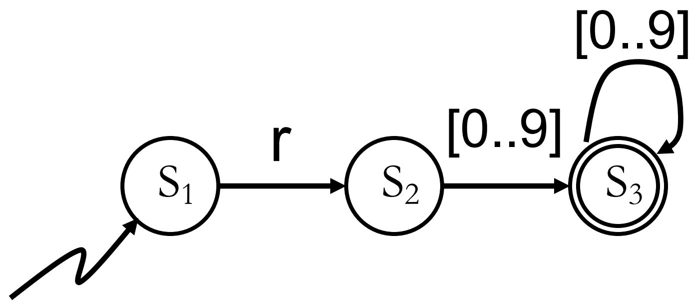
R^0_{kk} = nil otherwise

$R^1_{13} = R^0_{11} (R^0_{11})^* R^0_{13} \mid R^0_{13}$ = nil

$R^1_{23} = R^0_{21} (R^0_{11})^* R^0_{13} \mid R^0_{23}$ = [0..9]

$R^1_{33} = R^0_{31} (R^0_{11})^* R^0_{13} \mid R^0_{13}$ = [0..9] | ε

DFA to RE: Example



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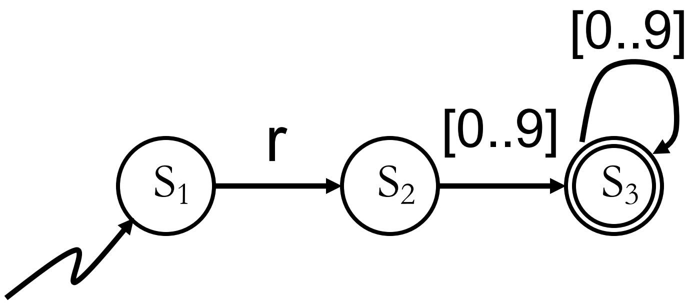
$R^1_{33} = R^0_{31} (R^0_{11})^* R^0_{13} \mid R^0_{13}$ = [0..9] | ε

$R^2_{13} = R^1_{12} (R^1_{22})^* R^1_{23} \mid R^1_{13}$ = r . ε * [0..9]

$R^2_{33} = R^1_{32} (R^1_{22})^* R^1_{23} \mid R^1_{33}$ = [0..9] | ε

$R^2_{33} = R^1_{32} (R^1_{22})^* R^1_{23} \mid R^1_{33}$ = [0..9] | ε

DFA to RE: Example



R^0_{12} = r

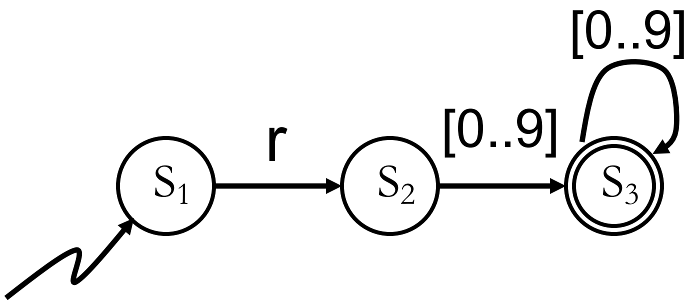
R^0_{23} = [0..9]

R^0_{33} = [0..9] | ε

R^0_{kk} = nil otherwise

$R^3_{13} = R^2_{13}\,(R^2_{33})^*\,R^2_{33} \mid R^2_{13}$

DFA to RE: Example



R^0_{12} = r

R^0_{23} = [0..9]

R^0_{33} = [0..9] | ε

R^0_{kk} = nil otherwise

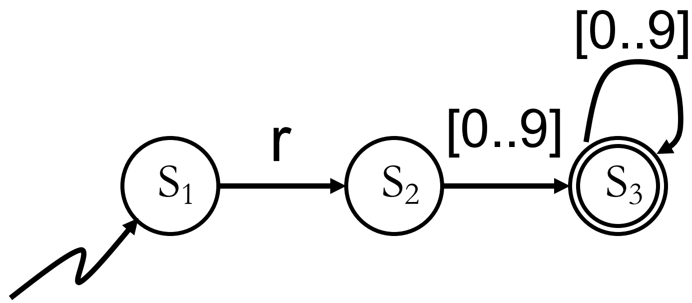
$R^3_{13} = R^2_{13} (R^2_{33})^* R^2_{33} \mid R^2_{13}$

= (r . ε * [0..9])([0..9]*)([0..9]) | r . ε * [0..9]

= (r . [0..9]+) | r . [0..9]

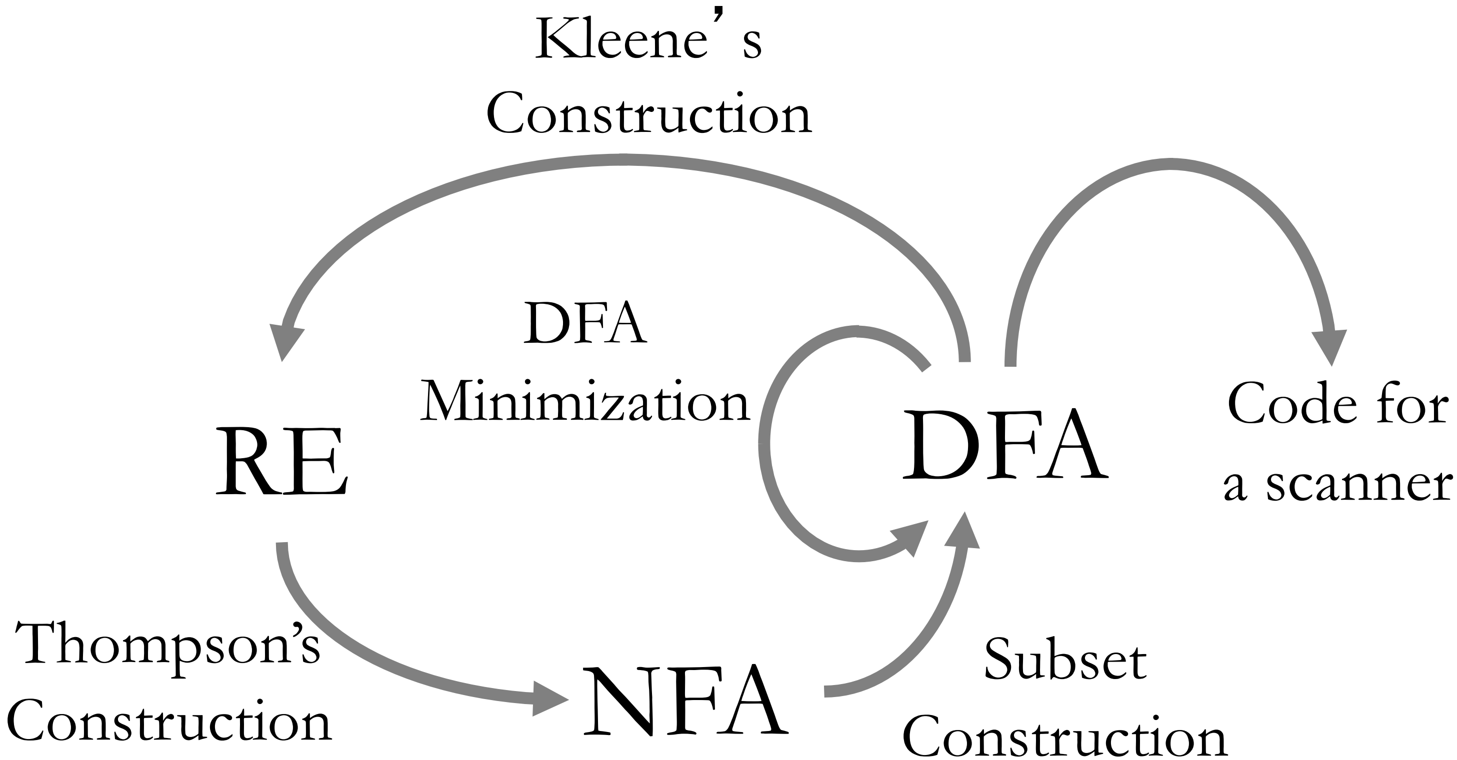
= (r . [0..9]+)

DFA to RE: Example

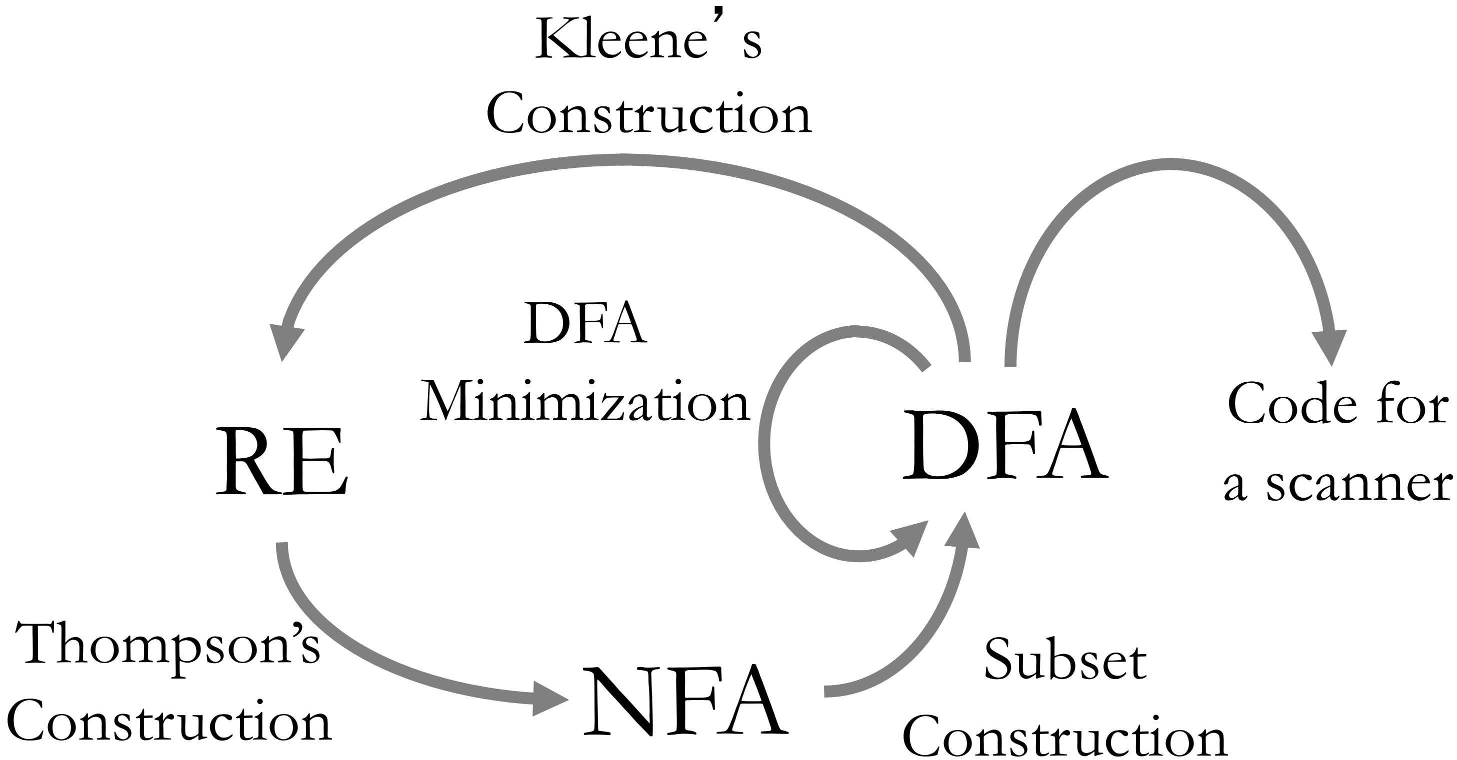


$L(M) = R^3_{13} =$ r . [0..9]+

DFA, NFA and REs



DFA, NFA and REs



Regular Expressions and FA are Equivalent

Summary

- DFA Minimization
  - Find sequence *w* that discriminates states
  - Iterate until no possible refinement

- DFA to RE
  - Kleene construction
  - Combine Path Expression for an increasingly large set of states

- DFA and RE are Equivalent
  - Given one you can derive an equivalent representation in the other