Appendix A.1: Literature-based model

The internal cell impedance equations shown here are only for discharge obtained from literature. Charging will be modelled a similar set of equation with different parameters. So the equations shown here are only half of the required equations for the practical circuit-based model

C_s

Short time transient capacitance

Dependent on T, SoC, C-rate, current direction, Ah processed.

$$C_s(SoC,T,I,Ah) = \big(C_s(SoC,I) + C_{s,n}(Ah,T)\big)\cdot Te^{\left(\frac{E_{a,cs}}{R}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right)}$$

$$C_s(SoC,I) = (d_1 + d_5 I + d_6 I^2)e^{-d_2 SoC} + (d_3 + d_7 I + d_8 I^2) + d_4 SoC$$

$$C_{s,n}(Ah,T) = \sum_{i=1}^{E}\left(\sqrt{{C_{s,ni}}^2 + \left(d_{n0}e^{\left(-\frac{E_{a,csn}}{R}\left(\frac{1}{T_i}-\frac{1}{T_{ref}}\right)\right)}\right)^2}\,\Delta Ah_i - C_{s,ni}\right)$$

C_l

Long time transient capacitance

Dependent on T, SoC, C-rate, current direction.

$$C_l(SoC,I,T) = C_l(SoC,I)\cdot e^{\left(\frac{E_{a,cl}}{R}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right)}$$

$$C_l(SoC,I) = h_1 SoC^6 + h_2 SoC^5 + h_3 SoC^4 + h_4 SoC^3 + h_5 SoC^2 + h_6 SoC + h_7 + h_8 I + h_9 I^2$$

R_o

Ohmic resistance

Dependent on T, SoC, C-rate current direction, Ah processed.

$$R_o(SoC,T,I,Ah) = \big(R_o(SoC,I) + R_{o,n}(Ah,T,I)\big)\cdot e^{\left(\frac{E_{a,o}}{R}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right)}$$

$$R_o(SoC,I) = (b_1 + b_4 I + b_5 I^2)e^{-b_2 SoC} + b_3 + b_6 I + b_7 I^2$$

$$R_{o,n}(Ah,T,I) = \sum_{i=1}^{E}\left((b_{n1} + b_{n2}I_i + b_{n3}{I_i}^2)\cdot b_n \Delta Ah_i \cdot e^{\left(-\frac{E_{a,on}}{R}\left(\frac{1}{T_i}-\frac{1}{T_{ref}}\right)\right)}\right)$$

C_{use}

Capacity of the cell

Dependent on T, C-rate, Ah processed.

$$C_{use}(\xi,T) = (Q_{nom} - \xi)\cdot e^{\left(-\frac{E_{a,q}}{R}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right)}$$

$$\xi(T,I,Ah) = \sum_{i}^{E}\left(\frac{1}{2}k_{\sigma,i}(T,I)\Delta {Ah_i}^2 + \Delta Ah_i\sqrt{{k_1}^2 + 2k_{\sigma,i}(T,I)\xi_{i-1}}\right)$$

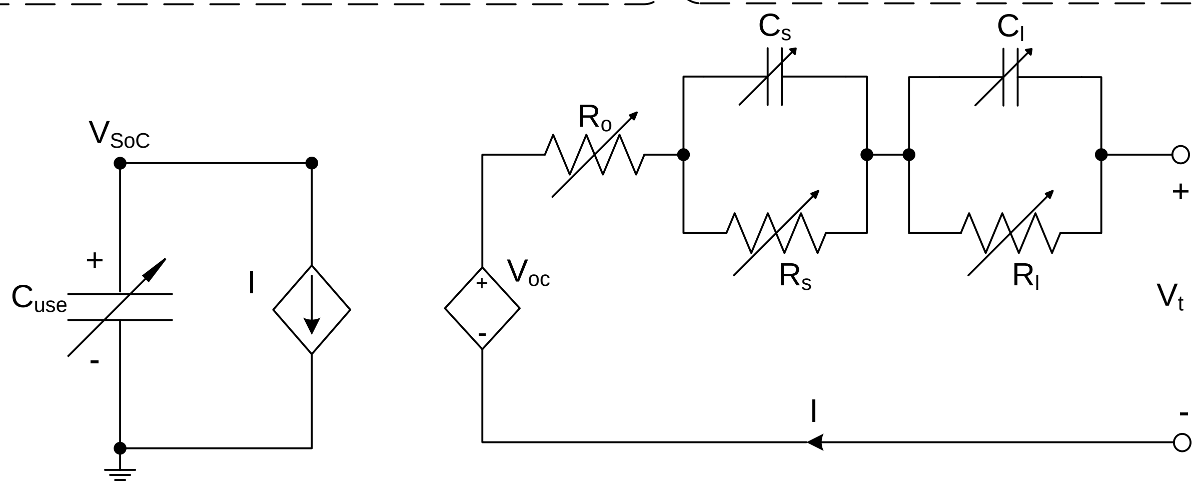
$$k_{\sigma,i} = k_2(k_3 + k_4 I + k_5 I^2)e^{\left(-\frac{E_a}{R}\left(\frac{1}{T,i}-\frac{1}{T_{ref}}\right)\right)}$$

V_{oc}

Open circuit voltage

Dependent on SoC.

$$V_{oc}(SoC) = a_1 e^{-a_2 SoC} + a_3 + a_4 SoC + a_5\, e^{-\frac{a_6}{1-SoC}}$$



R_s

Short time transient resistance

Dependent on T, SoC, C-rate, current direction, Ah processed.

$$R_s(SoC,T,I,Ah) = \big(R_s(SoC,I) + R_{s,n}(Ah,T)\big)\cdot Te^{\left(\frac{E_{a,rs}}{R}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right)}$$

$$R_s(SoC,I) = (c_1 + c_5 I + c_6 I^2)e^{-c_2 SoC} + (c_3 + c_7 I + c_8 I^2) + c_4 SoC$$

$$R_{s,n}(Ah,T) = \sum_{i=1}^{E}\left(\sqrt{{R_{s,ni}}^2 + \left(c_{n0}e^{\left(-\frac{E_{a,rsn}}{R}\left(\frac{1}{T_i}-\frac{1}{T_{ref}}\right)\right)}\right)^2}\,\Delta Ah_i - R_{s,ni}\right)$$

R_l

Long time transient resistance

Dependent on T, SoC, C-rate, current direction.

$$R_l(SoC,I,T) = R_l(SoC,I)\cdot e^{\left(\frac{E_{a,rl}}{R}\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right)}$$

$$R_l(SoC,I) = (g_1 + g_4 I + g_5 I^2)e^{-g_2 SoC} + g_3 + g_6 I + g_7 I^2$$

Appendix A.2: Proposed model

R_o — Series resistance

Dependent on T, SoC, current direction, Ah processed.

$$R_{od,T_{high}}(SoC,T) = (b_{1d}SoC^4 + b_{2d}SoC^3 + b_{3d}SoC^2 + b_{4d}SoC + b_{5d})b_{t1d}e^{\frac{b_{t2d}}{T-b_{t3d}}}$$

$$R_{oc,T_{high}}(SoC,T) = (b_{1c}SoC^4 + b_{2c}SoC^3 + b_{3c}SoC^2 + b_{4c}SoC + b_{5c})b_{t1c}e^{\frac{b_{t2c}}{T-b_{t3c}}}$$

$$R_{od,T_{low}}(SoC,T) = (b_{t1d}T^3 + b_{t2d}T^2 + b_{t3d}T + b_{t4d})SoC^4 + b_{2d}SoC^3 + b_{3d}SoC^2 + b_{4d}SoC + b_{t5d}e^{\frac{b_{t6d}}{T-b_{t7d}}}$$

$$R_{oc,T_{low}}(SoC,T) = (b_{1c}SoC^4 + b_{2c}SoC^3)(b_{t1c}T + b_{t2c}) + (b_{t3c}T^2 + b_{t4c}T + b_{t5c})SoC^2 + (b_{t6c}T^2 + b_{t7c}T + b_{t8c})SoC + b_{t9c}e^{\frac{b_{t10c}}{T-b_{t11c}}}$$

Ageing of R_o, cycled with regenerative braking, varied with Ah processed, but did not show a trend.

C_{use} — Capacity of the cell

Dependent on T, recharge C-rate, SoC_{avg}, SoC_{dev}, Ah processed.

$$C_{use}(T,\xi) = (Q_{nom} - \xi)\cdot e^{k_1\left(\frac{1}{T-k_2} - \frac{1}{T_{ref}-k_2}\right)}$$

$$\xi(T, SoC_{avg}, SoC_{dev}, Ah) = \sum_i^E \left(\left(k_{s1}SoC_{dev,i}\cdot e^{(k_{s2}\cdot SoC_{avg,i})} + k_{s3}e^{k_{s4}SoC_{dev,i}}\right) e^{\left(-\frac{E_a}{R}\left(\frac{1}{T_i} - \frac{1}{T_{ref}}\right)\right)} \right) Ah_i$$

$$SoC_{avg} = \frac{1}{\Delta Ah_m}\int_{Ah_{m-1}}^{Ah_m} SoC(Ah)\,dAh$$

$$SoC_{dev} = \sqrt{\frac{3}{\Delta Ah_m}\int_{Ah_{m-1}}^{Ah_m} \left(SoC(Ah) - SoC_{avg}\right)^2 dAh}$$

Not enough data was available to model the influence of recharge C-rate at low temperatures.

V_{oc} — Open circuit voltage

Dependent on SoC.

$$V_{oc}(SoC) = a_1e^{-a_2SoC} + a_3 + a_4SoC + a_5\,e^{-\frac{a_6}{1-SoC}}$$

C_s — Short time transient capacitance

Dependent on T, SoC, current direction.

$$C_{sd,T_{high}}(SoC,T) = d_{1d}SoC^3 + d_{2d}SoC^2 + (d_{3d} + d_{t1d}\Delta T)SoC + d_{4d} + d_{t2d}\Delta T$$

$$C_{sc,T_{high}}(SoC,T) = d_{1c}SoC^4 + d_{2c}SoC^3 + d_{3c}SoC^2 + d_{4c}SoC + d_{5c} + d_{t1c}\Delta T$$

$$C_{sd,T_{low}}(SoC,T) = d_{1d}(SoC + (d_{t1d}T - d_{t2d}))^3 + d_{2d}(SoC + (d_{t1d}T - d_{t2d}))^2 + d_{4d} + (d_{t3d}T^2 + d_{t4d}T + d_{t5d})(SoC + (d_{t1d}T - d_{t2d}))$$

$$C_{sc,T_{low}}(SoC,T) = (d_{t1c}T^2 + d_{t2c}T + d_{t3c})SoC^4 + (d_{t4c}T^2 + d_{t5c}T + d_{t6c})SoC^3 + d_{t7c}e^{d_{t8c}T}SoC^2 + (d_{t9c}T^2 + d_{t10c}T + d_{t11c})SoC + d_{t12c}T + d_{t13c}$$

C_l — Long time transient capacitance

Dependent on T, SoC, current direction.

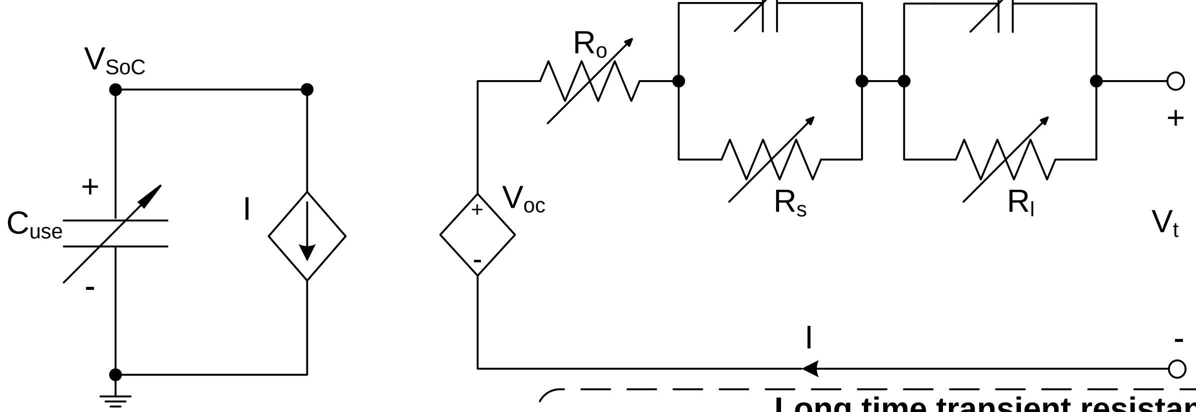
$$C_{ld,T_{high}}(SoC,T) = (h_{1d}SoC^6 + h_{2d}SoC^5 + h_{3d}SoC^4 + h_{4d}SoC^3 + h_{5d}SoC^2 + h_{6d}SoC + h_{7d})h_{t1d}e^{\frac{h_{t2d}}{T}}$$

$$C_{lc,T_{high}}(SoC,T) = (h_{1c}SoC^5 + h_{2c}SoC^4 + h_{3c}SoC^3 + h_{4c}SoC^2 + h_{5c}SoC + h_{6c})h_{t1c}e^{h_{t2c}T}$$

$$C_{ld,T_{low}}(SoC,T) = (h_{1d}SoC^4 + h_{2d}SoC^3 + h_{5d})e^{h_{t1d}T} + h_{3d}SoC^2e^{h_{t2d}T} + h_{4d}SoC\,e^{\frac{h_{t3d}}{T-h_{t4d}}}$$

$$C_{lc,T_{low}}(SoC,T) = (h_{t1c}SoC^5 + h_{t2c}SoC^4 + h_{t3c}SoC^3)e^{h_{t4c}T} + h_{t5c}e^{h_{t6c}T}SoC^2 + h_{t7c}e^{h_{t8c}}SoC + h_{t9c}$$

ΔT is the temperature deviation from reference temperature 25°C



R_s — Short time transient resistance

Dependent on T, SoC, current direction

$$R_{sd,T_{high}}(SoC,T) = c_{1d}e^{-c_{2d}SoC} + c_{3d} + c_{t1d}\Delta T + (c_{4d} + c_{t2d}\Delta T)SoC$$

$$R_{sc,T_{high}}(SoC,T) = c_{t1c}e^{c_{t2c}T + (c_{t3c}T + c_{t4c})SoC} + c_{t5c}T + c_{t6c}$$

$$R_{sd,T_{low}}(SoC,T) = (c_{t1d}T^2 + c_{t2d}T + c_{t3d})e^{-(c_{t4d}T^2 + c_{t5d}T + c_{t6d})SoC} + c_{3d} + c_{4d}SoC$$

$$R_{sc,T_{low}}(SoC,T) = c_{t1c}e^{c_{t2c}T + (c_{t3c}T + c_{t4c})SoC} + c_{t5c}T^2 + c_{t6c}T + c_{t7c}$$

R_l — Long time transient resistance

Dependent on T, C-rate, SoC, current direction.

$$R_{ld,T_{high}}(SoC,T,I) = \left((g_{1d} + g_{t1d}\Delta T)e^{-g_{2d}SoC} + g_{3d} + g_{t2d}\Delta T + g_{4d}SoC\right)\cdot\left(g_{i1d}I_{Crate}{}^{g_{i2d}} + g_{id3}\right)$$

$$R_{lc,T_{high}}(SoC,T,I) = (g_{1c}e^{-g_{2c}SoC} + g_{3c} + g_{4c}SoC)(g_{t1c}T + g_{t2c})\cdot\left(g_{i1c}(I_{Crate}{}^{g_{i2c}} - 1)SoC + I_{Crate}{}^{g_{i3c}}\right)$$

$$R_{ld,T_{low}}(SoC,T,I) = \left(g_{t1d}e^{\left(\frac{g_{t2d}}{T-g_{t3d}} - g_{2d}SoC\right)} + g_{t4d}e^{\frac{g_{t5d}}{T}} + g_{t6d}e^{\frac{g_{t7d}}{T}}SoC\right)\cdot\left(g_{i1d}I_{Crate}{}^{g_{i2d}} + g_{id3}\right)$$

$$R_{lc,T_{low}}(SoC,T,I) = \left(g_{t1c}\cdot e^{g_{t2c}T - g_{2c}SoC} + g_{t3c}T^2 + g_{t4c}T + g_{t5c} + g_{4c}SoC\right)\cdot\left(g_{i1c}(I_{Crate}{}^{g_{i2c}} - 1)SoC + I_{Crate}{}^{g_{i3c}}\right)$$

Appendix B: Model parameters

k_1	-5.738	k_{s2}	-2.1665
k_2	2.099E2	k_{s3}	1.408E-5
k_{s1}	-2.046E-4	k_{s4}	3.065

Table B.1: The parameter values of the cell capacity C_{use}.

a_1	-5.863E-1	a_4	1.102E-1
a_2	21.90	a_5	-1.718E-1
a_3	3.414	a_6	8.000E-3

Table B.2: The parameter values of the OCV V_{oc}.

b_{1d}	1.298E-1	c_{2d}	11.03	d_{4d}	3.897E2	h_{2d}	-6.007E6
b_{2d}	-2.892E-1	c_{3d}	1.827E-2	g_{1d}	2.950E-1	h_{3d}	6.271E6
b_{3d}	2.273E-1	c_{4d}	-6.462E-3	g_{2d}	20.00	h_{4d}	-2.958E6
b_{4d}	-7.216E-2	d_{1d}	1.697E2	g_{3d}	4.722E-2	h_{5d}	5.998E5
b_{5d}	8.980E-2	d_{2d}	-1.007E3	g_{4d}	-2.420E-2	h_{6d}	-3.102E4
c_{1d}	1.080E-2	d_{3d}	1.408E3	h_{1d}	2.130E6	h_{7d}	2.232E3

Table B.3: The parameter values of the SoC dependency of internal cell impedance for discharging.

b_{1c}	1.369E-1	c_{2c}	-18.75	d_{5c}	6.849E2	h_{2c}	2.042E5
b_{2c}	-2.518E-1	c_{3c}	1.388E-2	g_{1c}	8.913E-15	h_{3c}	-4.009E3
b_{3c}	1.609E-1	d_{1c}	-1.026E4	g_{2c}	-32.23	h_{4c}	-8.124E4
b_{4c}	-4.100E-2	d_{2c}	1.723E4	g_{3c}	3.100E-2	h_{5c}	2.283E4
b_{5c}	8.210E-2	d_{3c}	-1.013E4	g_{4c}	7.473E-3	h_{6c}	7.144E3
c_{1c}	5.896E-10	d_{4c}	2.340E3	h_{1c}	-1.541E5		

Table B.4: The parameter values of the SoC dependency of internal cell impedance for charging.

g_{i1d}	6.993E-1	g_{i1c}	-4.124E-1
g_{i2d}	-6.919E-1	g_{i2c}	-1.082
g_{i3d}	2.902E-1	g_{i3c}	-8.730E-1

Table B.5: The parameter values of the C-rate dependency of internal cell impedance for both discharging and charging.

b_{t1d}	7.613E-1	c_{t1d}	-3.697E-4	d_{t2d}	12.11	h_{t1d}	3.128E3
b_{t2d}	10.14	c_{t2d}	2.225E-4	g_{t1d}	6.718E-3	h_{t2d}	-2.398E3
b_{t3d}	2.608E2	d_{t1d}	-6.580	g_{t2d}	-5.967E-4		

Table B.6: The parameter values of the high temperature dependency of internal cell impedance for discharging.

b_{t1c}	7.192E-1	c_{t2c}	-1.479E-1	c_{t6c}	7.054E-2	h_{t1c}	2.611E-5
b_{t2c}	33.91	c_{t3c}	-1.178E-1	d_{t1c}	8.814	h_{t2c}	3.541E-2
b_{t3c}	1.999E2	c_{t4c}	13.99	g_{t1c}	-1.344E-2		
c_{t1c}	9.869E8	c_{t5c}	-1.897E-4	g_{t2c}	5.011		

Table B.7: The parameter values of the high temperature dependency of internal cell impedance for charging.

b_{2d}	-2.819E-1	c_{4d}	-6.462E-3	d_{t2d}	4.068	g_{t7d}	4.202E3
b_{3d}	2.448E-1	c_{t1d}	1.923E-4	d_{t3d}	-6.807E-1	h_{1d}	-3.751E-11
b_{4d}	-9.630E-2	c_{t2d}	-1.166E-1	d_{t4d}	4.002E2	h_{2d}	6.764E-11
b_{t1d}	-5.930E-6	c_{t3d}	17.66	d_{t5d}	-5.769E4	h_{3d}	-2.219E-11
b_{t2d}	5.011E-3	c_{t4d}	1.098E-2	g_{2d}	20.00	h_{4d}	10.75
b_{t3d}	-1.411	c_{t5d}	-5.644	g_{t1d}	8.238E-3	h_{5d}	1.534E-13
b_{t4d}	1.325E2	c_{t6d}	7.299E2	g_{t2d}	1.805E2	h_{t1d}	1.220E-1
b_{t5d}	6.297E-2	d_{1d}	-1.173E3	g_{t3d}	2.321E2	h_{t2d}	1.241E-1
b_{t6d}	24.37	d_{2d}	8.278E2	g_{t4d}	1.589E-7	h_{t3d}	-4.492E2
b_{t7d}	2.363E2	d_{4d}	1.005E2	g_{t5d}	3.779E3	h_{t4d}	3.474E2
c_{3d}	1.827E-2	d_{t1d}	1.428E-2	g_{t6d}	-2.208E-8		

Table B.8: The parameter values of the low temperature dependency of internal cell impedance for discharging.

b_{1c}	2.192E-1	c_{t1c}	7.008E29	d_{t7c}	-3.688E-2	g_{t5c}	5.225
b_{2c}	-3.968E-1	c_{t2c}	-3.051E-1	d_{t8c}	4.261E-2	h_{t1c}	6.343E-6
b_{t1c}	-8.297E-2	c_{t3c}	2.782E-1	d_{t9c}	-1.199	h_{t2c}	-1.817E-5
b_{t2c}	24.82	c_{t4c}	-63.04	d_{t10c}	7.103E2	h_{t3c}	1.887E-5
b_{t3c}	2.136E-4	c_{t5c}	2.840E-6	d_{t11c}	-1.027E5	h_{t4c}	8.551E-2
b_{t4c}	-1.422E-1	c_{t6c}	-1.733E-3	d_{t12c}	8.493	h_{t5c}	-1.414E-5
b_{t5c}	23.48	c_{t7c}	2.786E-1	d_{t13c}	-1.902E3	h_{t6c}	8.381E-2
b_{t6c}	-1.795E-4	d_{t1c}	-11.23	g_{2c}	-32.23	h_{t7c}	1.057E-5
b_{t7c}	1.077E-1	d_{t2c}	5.941E3	g_{4c}	1.571E-2	h_{t8c}	7.897E-2
b_{t8c}	-16.20	d_{t3c}	-7.848E5	g_{t1c}	3.726E-8	h_{t9c}	6.962E2
b_{t9c}	4.968E-2	d_{t4c}	16.18	g_{t2c}	-4.951E-2		
b_{t10c}	28.78	d_{t5c}	-8.523E3	g_{t3c}	5.921E-5		
b_{t11c}	2.376E2	d_{t6c}	1.123E6	g_{t4c}	-3.507E-2		

Table B.9: The parameter values of the low temperature dependency of internal cell impedance for charging.