

# A dynamic lithium-ion battery model considering the effects of temperature and capacity fading

O. Erdinc, B. Vural and M. Uzunoglu, *Member, IEEE*

**Abstract**— Battery models capture the characteristics of real-life batteries, and can be used to predict their behavior under various operating conditions. In this paper, a dynamic model of lithium-ion battery has been developed with MATLAB/Simulink® in order to investigate the output characteristics of lithium-ion batteries. Dynamic simulations are carried out, including the observation of the changes in battery terminal output voltage under different charging/discharging, temperature and cycling conditions, and the simulation results are compared with the results obtained from several recent studies. The simulation studies are presented for manifesting that the model is effective and operational.

**Index Terms**—Lithium-ion battery, dynamic model, temperature effect, capacity fading.

## I. INTRODUCTION

BATTERIES directly contribute to the advancement of technologies ranging from portable electronics to the fuel-efficient vehicles. Among the various existing rechargeable batteries, lithium-based batteries appear to occupy a prime position in various aspects [1]. Lithium-ion batteries, with high energy density, high operating voltage levels and long cycle life, have been extensively used in the portable electronic devices, cellular phones and camcorders etc. [2]. In order to evaluate the battery performances under different conditions without resorting to time consuming (and expensive) prototyping and measurement for each alternative, there is a strong need for accurate battery models. Battery models capture the characteristics of real-life batteries, and can be used to predict their behavior under various conditions of charge/discharge. They can also be used to make suggestions towards the improvement and the optimization in battery system [3].

For better understanding of the performance of lithium-ion batteries, numerous authors have developed various models [4-6]. Despite of being prosperous in many aspects, most of them still have several drawbacks. Some of them ignore the transient behaviors [7], while some only work for a fixed state of charge [8] or unable to predict battery runtime [9] which is an important factor for judging the battery performance.

---

This work was supported in part by TUBITAK under Grant 107M355.

O. Erdinc, B. Vural and M. Uzunoglu are with the Department of Electrical Engineering, Yildiz Technical University, Besiktas, Istanbul, 34349, Turkey, (e-mail: oerdinc@yildiz.edu.tr; bvural@yildiz.edu.tr; uzunoglu@yildiz.edu.tr)

Chen and Mora [10] proposed a battery model that can predict both battery runtime and  $I$ - $V$  performance accurately by combining the transient behaviors of battery with a variable open circuit voltage and internal resistance. Besides, the effect of temperature and capacity fading on battery dynamics is neglected in [10]. However, thermal behavior of lithium batteries is important for both the battery operating life requirements and safety considerations [11,12]. Furthermore, the effect of capacity fading is also important for both predicting the battery cycle life exactly and determining the remaining usable battery capacity [6,13,14]. A more accurate battery model can be obtained by considering the impacts of these significant parameters.

In this study, we established an effective dynamic lithium-ion battery simulation model in MATLAB/Simulink environment. The battery model is developed experimentally through the [10], by adding the significant temperature and capacity fading effects on battery dynamics. Dynamic simulations are used to verify the performance of the developed lithium-ion battery model, and the model is validated by comparing with data obtained by several studies realized by different authors.

This paper is organized as follows. Section II describes the mathematical equations used in the developed lithium-ion battery model. Section III represents the results of simulations carried out for observing the changes in battery output characteristics under different charging/discharging, temperature and cycling conditions. Finally, conclusions are given in Section IV.

## II. DESIGN AND MODELING OF LITHIUM-ION BATTERY

In this section, the dynamic model of the lithium-ion battery is introduced. The model used in [10] has been modified by adding the effects of temperature and capacity fading on battery output characteristics. The lithium-ion battery model parameters used in proposed model are as follows:

$V_{bat}$	Battery output voltage [V]
$V_{oc}$	Battery open-circuit voltage [V]
$Z_{eq}$	Battery equivalent internal impedance [ $\Omega$ ]
$I_{bat}$	Battery current [A]
$\Delta E(T)$	Temperature correction of the potential [V]
$SOC$	State of charge
$SOC_{init}$	Initial state of charge
$C_{usable}$	Usable battery capacity [Ah]

$T$	Temperature [ $^{\circ}\text{C}$ - $^{\circ}\text{K}$ ]
$t$	Storage time [months]
$Q_n$	Change in state of charge of battery negative electrode
$N$	Cycle number
$k_1$	Coefficient for the change in SOC of battery negative electrode [ $\text{cycle}^{-2}$ ]
$k_2$	Coefficient for the change in SOC of battery negative electrode [ $\text{cycle}^{-1}$ ]
$k_3$	Coefficient for the change in $R_{\text{cycle}}$ [ $\Omega/\text{cycle}^{1/2}$ ]
$CCF$	Capacity correction factor
$C_{\text{init}}$	Initial battery capacity [Ah]

The battery output voltage can be calculated due to the battery open circuit voltage, voltage drop resulting from the battery equivalent internal impedance and the temperature correction of the battery potential. Accordingly, the battery output voltage may be expressed as [15]

$$V_{\text{bat}} = V_{\text{OC}} - i_{\text{bat}} \times Z_{\text{eq}} + \Delta E(T) \quad (1)$$

#### A. The battery open circuit voltage

The battery open circuit voltage is the difference of the electrical potential between the two terminals of a battery, when there is no external load connected [16]. As the value of battery open circuit voltage is strongly dependent on battery SOC, it can be calculated as [10]

$$V_{\text{oc}}(\text{SOC}) = -1.031 \times \exp(-35 \times \text{SOC}) + 3.685 + 0.2156 \times \text{SOC} - 0.1178 \times \text{SOC}^2 + 0.321 \times \text{SOC}^3 \quad (2)$$

The battery SOC can be expressed as

$$\text{SOC} = \text{SOC}_{\text{init}} - \int (i_{\text{bat}} / C_{\text{usable}}) dt. \quad (3)$$

where usable battery capacity changes depending on capacity fading.

#### B. The effect of capacity fading

Capacity fading refers to the irreversible loss in the usable capacity of a battery due to time, temperature and cycle number. Generally, a battery is considered to be usable until reaching the 80% of its initial capacity [1,14]. So that, modeling the capacity fading is important for predicting the remaining life of the battery. The irreversible loss causing capacity fading is associated with degradation of the battery, and the loss occurs whether the battery is inactive (so-called “calendar life” losses) or exercised (“cycle life” losses) [13]. Both calendar and cycle life losses of a battery appear to be linear with time and dramatically increase with increasing temperature [14]. Therefore, the effect of temperature must be considered while modeling the capacity fading for a battery. The calendar and cycle life losses lead to a capacity correction factor to determine the remaining usable battery capacity. The capacity correction factor can be calculated as

$$CCF = 1 - (\text{Calendar life losses} + \text{Cycle life losses}) \quad (4)$$

Then the remaining usable battery capacity can be defined as

$$C_{\text{usable}} = C_{\text{initial}} \times CCF \quad (5)$$

The calendar life losses of a battery consist of storage losses occurring when the battery is not used. The percentage of storage losses can be expressed as [13,14]

$$\% \text{ storage loss} = 1.544 \times 10^7 \times \exp(40498 / (8.3143 \times T)) \times t \quad (6)$$

where  $T$  is the temperature in  $^{\circ}\text{K}$ .

It is a valid assumption to consider that the only variable related with the other component of capacity fading, cycle life losses, is the negative electrode SOC. The rate of change in negative electrode SOC dependent on cycle number and temperature can be represented as [14]

$$(d\theta_n / dN) = k_1 N + k_2 \quad (7)$$

where the coefficient  $k_1$  accounts for capacity losses that increase rapidly during adverse conditions such as cycling at high temperature, and  $k_2$  is a factor to account for capacity losses under usual conditions of cycling. The values of the coefficients,  $k_1$  and  $k_2$ , change depending on cycling temperature (Table 1). The variations of negative electrode SOC can be considered for simulating the cycle life losses [14].

TABLE 1

VALUES OF THE COEFFICIENTS DEPENDENT ON CYCLING TEMPERATURE [14]			
Cycling temperature [ $^{\circ}\text{C}$ ]	$k_1$ [ $\text{cycle}^{-2}$ ]	$k_2$ [ $\text{cycle}^{-1}$ ]	$k_3$ [ $\Omega/\text{cycle}^{1/2}$ ]
25	$8.5 \times 10^{-8}$	$2.5 \times 10^{-4}$	$1.5 \times 10^{-3}$
50	$1.6 \times 10^{-6}$	$2.9 \times 10^{-4}$	$1.7 \times 10^{-3}$

#### C. The variable equivalent internal impedance of battery

The battery equivalent internal impedance consists of a series resistor composed of  $R_{\text{series}}$  and  $R_{\text{cycle}}$ , and two RC networks composed of  $R_{\text{Transient}_S}$ ,  $C_{\text{Transient}_S}$ ,  $R_{\text{Transient}_L}$  and  $C_{\text{Transient}_L}$ , as shown in Fig. 1.

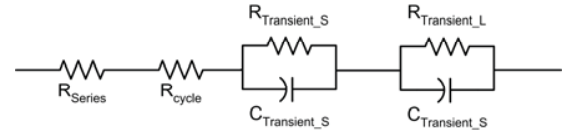


Fig 1. The battery equivalent internal impedance.

$R_{\text{series}}$  is responsible for the instantaneous voltage drop in battery terminal voltage. The other component of series resistor,  $R_{\text{cycle}}$ , is used to explain the increase in the battery resistance with cycling. The components of RC networks are responsible for short and long-time transients in battery internal impedance. The values of  $R_{\text{series}}$ ,  $R_{\text{Transient}_S}$ ,  $C_{\text{Transient}_S}$ ,  $R_{\text{Transient}_L}$  and  $C_{\text{Transient}_L}$  due to battery SOC can be calculated as [10]

$$R_{\text{series}}(\text{SOC}) = 0.1562 \times \exp(-24.37 \times \text{SOC}) + 0.07446 \quad (8)$$

$$R_{\text{Transient}_S}(\text{SOC}) = 0.3208 \times \exp(-29.14 \times \text{SOC}) + 0.04669 \quad (9)$$

$$C_{\text{Transient}_S}(\text{SOC}) = 752.9 \times \exp(-13.51 \times \text{SOC}) + 703.6 \quad (10)$$

$$R_{\text{Transient}_L}(\text{SOC}) = 6.603 \times \exp(-155.2 \times \text{SOC}) + 0.04984 \quad (11)$$

$$C_{\text{Transient}_L}(\text{SOC}) = -6056 \times \exp(-27.12 \times \text{SOC}) + 4475 \quad (12)$$

And  $R_{\text{cycle}}$  can be expressed as [14]

$$R_{\text{cycle}} = k_3 \times N^{1/2} \quad (13)$$

where the coefficient,  $k_3$ , was found to be quite independent of temperature changes by experimental studies (Table 1).

#### D. Temperature correction of the potential

$\Delta E(T)$  is a potential correction term used to compensate for the variation of equilibrium potential that is induced by temperature changes. The figure describing the change of  $\Delta E(T)$  due to temperature can be found in Ref. [15].

### III. TEST AND RESULTS

In this paper, MATLAB/Simulink<sup>®</sup> is used to setup the lithium-ion battery dynamic model. Fig. 2 shows the overall dynamic Simulink model of lithium-ion battery based on Eqs. (1) - (13).

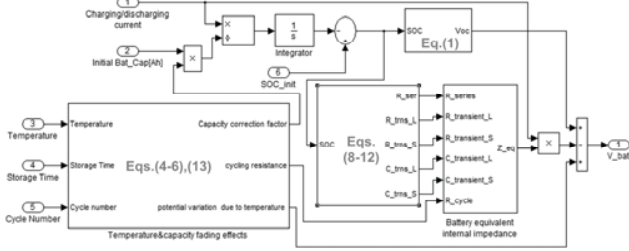


Fig. 2. Overall dynamic model of lithium-ion battery.

To test the developed dynamic lithium-ion battery model, different temperature, cycling and charging/discharging conditions are applied, and the results obtained by simulating the model are shown in Fig. 3-6.

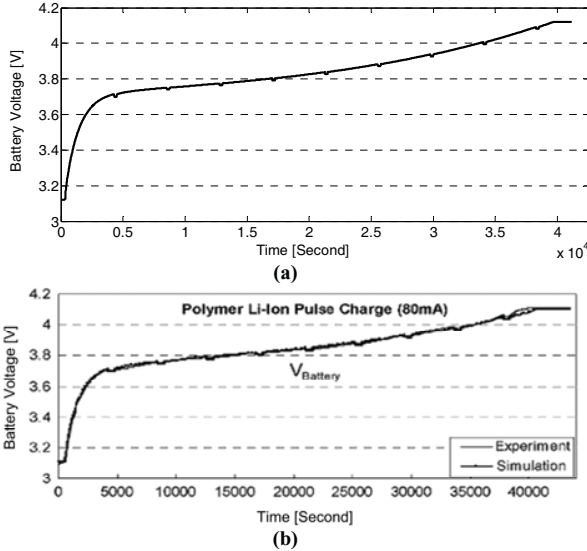
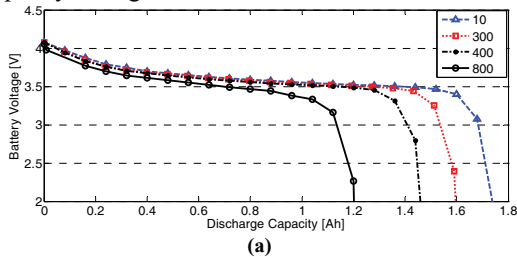


Fig. 3. Battery voltage response curve during 80 mA pulse charging condition (a) Simulation results of the proposed model, (b) Simulation and experimental results given in [10].

Fig. 3 (a)-(b) shows the simulation results of the proposed model and the results obtained by Chen et al [10], for a 80 mA pulse charging condition of a 850 mAh lithium-ion battery at room temperature, 25°C. During the charging process, the battery is charged with a 80 mA current, then with a constant voltage at around 4.1 V. From Fig. 3, it can be clearly seen that the results obtained by the simulation of the proposed model are quite similar with the results given in [10], which ignores the dependency of battery dynamics on temperature and capacity fading.



(a)

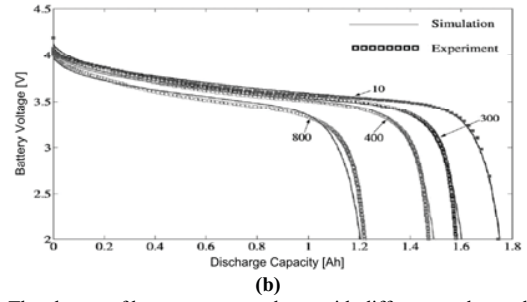
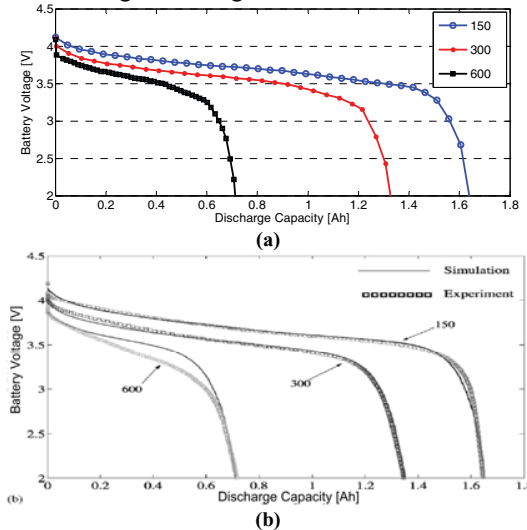


Fig. 4. The change of battery output voltage with different cycle numbers for 25°C (a) Simulation results of the proposed model, (b) Results given in [14].

Fig. 4 displays the change of battery output voltage with different cycle numbers for 25°C, while Fig. 5 depicts the variation of the battery output voltage for 50°C. Fig. 4 (a) and Fig. 5 (a) display the results acquired with the simulation of proposed model, as Fig. 4 (b) and Fig. 5 (b) show the results obtained by Ramadass et al [14], for a 0.9 A discharging condition of a 1.8 Ah lithium-ion battery. The strong influence of rising temperature and cycle number on capacity fading can be clearly seen from Fig. 4 and Fig. 5, as the capacity fading significantly increases with the increment of temperature and cycle number. Besides, the simulation results of proposed model are rather alike with the results displayed in [14], as depicted from Fig. 4 and Fig. 5.



(a)

As a new contribution, the effects of temperature and capacity fading on battery runtime for different discharging rates are analyzed in this study. The temperature, cycle number and discharging rate values utilized in this analyze are determined identically with the values applied in above studies. The discharging curves for a 2 Ah fresh cell for a 0.1 A discharging current at -20°C, 0°C, 23°C and 45°C can be seen in Fig. 6. The battery output voltage is affected by the temperature change, although the total battery runtime remains almost unchanged despite of the change in temperature for the discharging condition [17], as depicted from Fig. 6. The same results can be obtained for a charging condition as the charging and discharging processes for a battery have nearly the same kinetics at all temperatures for the same current rates [18]. For different discharging currents (0.1 A, 0.5 A, 1 A), the simulation is run, and the results are given in Table 2.

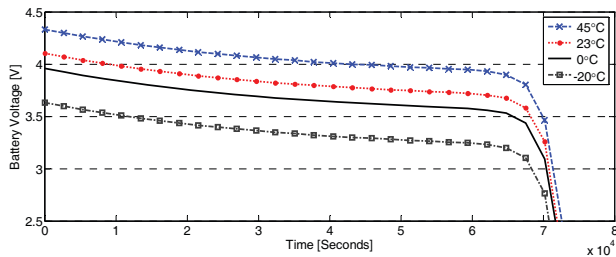


Fig. 6. The change in battery runtime for a 0.1 A discharging condition at different temperatures.

TABLE 2  
THE BATTERY RUNTIME VALUES FOR DIFFERENT DISCHARGING CURRENTS AT DIFFERENT TEMPERATURES

Temperature [°C]	Battery runtime [sec] (0.1 A)	Battery runtime [sec] (0.5 A)	Battery runtime [sec] (1 A)
-20	70858	13504	5392
0	71685	13528	5430
23	72014	13532	5434
45	72671	13547	5448

However, if the capacity fading for different cycle numbers is considered, the battery runtime changes dramatically due to the sharp increment in battery remaining usable capacity. Table 3 shows the battery runtime values for different discharging currents at different cycle numbers (10, 100, 300, 600), for the reference temperature, 23°C. The results obtained by simulating the proposed model for different cycle numbers show the importance of considering the capacity fading effect on battery dynamics, as considering only temperature effect is not sufficient for developing an accurate battery model.

TABLE 3  
THE BATTERY RUNTIME VALUES FOR DIFFERENT DISCHARGING CURRENTS AT DIFFERENT CYCLE NUMBERS, FOR 230C

Cycle Number	Battery runtime [sec] (0.1 A)	Battery runtime [sec] (0.5 A)	Battery runtime [sec] (1 A)
10	70771	12717	5016
100	67254	11232	4395
300	61767	9792	3662
600	51799	7996	2821

As summarizing the above statements, one can see that the battery dynamics are strongly affected by charging/discharging rates, temperature and capacity fading factors, which can not be omitted in a model if the aim is to evaluate the battery dynamics truly for all the operating conditions. All the simulation response curves are agreed with the results obtained by different authors, which means that the built up model can simulate the dynamic response of lithium-ion battery accurately. At the same time, the developed modeling and design methodology has a modular structure that can be implemented, simulated and analyzed with different power and parameters of any desired lithium-ion battery system.

#### IV. CONCLUSIONS

In this paper, a dynamic model of lithium-ion battery considering the significant temperature and capacity fading effects is proposed. The simulation results shows that the developed model can truly reflect the dynamic output

characteristic of lithium-ion battery. The developed model is able to evaluate the battery performance under several different operating conditions, and it can be directly used in different simulation models including battery systems.

#### REFERENCES

- [1] B. Kennedy, D. Patterson and S. Camilleri, "Use of lithium-ion batteries in electric vehicles", *Journal of Power Sources*, vol. 90, pp. 156–162, 2000.
- [2] Q. Huang, M. Yan and Z. Jiang, "Thermal study on single electrodes in lithium-ion battery", *Journal of Power Sources*, vol. 156, pp. 541–546, 2006.
- [3] D. Ra and K. S. Han, "Used lithium ion rechargeable battery recycling using Etoile-Rebatt technology", *Journal of Power Sources*, vol. 163, pp. 284–288, 2006.
- [4] K. A. Smith, "Electrochemical Modeling, Estimation and Control of Lithium-ion Batteries", Ph. D. Dissertation, Department of Mechanical Engineering, The Pennsylvania State University, USA, 2006.
- [5] D. W. Dees, V. Battaglia and A. Belanger, "Electrochemical modeling of lithium polymer batteries", *Journal of Power Sources* vol. 110, pp. 310–320, 2002.
- [6] P. Rong and M. Pedram, "Dynamic Lithium-Ion Battery Model for System Simulation", *IEEE Transactions on Very Large Scale Integration Systems*, vol. 14, pp. 441–451, 2006.
- [7] A. Hajizadeh and M. A. Golkar, "Intelligent power management strategy of hybrid distributed generation system", *Int. Journal of Electrical Power and Energy Systems*, vol. 24, pp. 783–795, 2007.
- [8] S. Buller, M. Thele, R. W. D. Doncker and E. Karden, "Impedance based simulation models of supercapacitors and Li-ion batteries for power electronic applications", *IEEE Transactions on Industry Applications*, vol. 41, pp. 742–747, 2005.
- [9] B. Y. Liaw, G. Nagasubramanian, R. G. Jungst and D. H. Doughty, "Modeling of lithium ion cells—A simple equivalent-circuit model approach", *Journal of Solid State Ionics*, vol. 175, pp. 835–839, 2004.
- [10] M. Chen and G. A. R. Mora, "Accurate electrical battery model capable of predicting runtime and I-V performance", *IEEE Transactions on Energy Conversion*, vol. 21, pp. 504–511, 2006.
- [11] S. C. Chen, C.C. Wan and Y.Y. Wang, "Thermal analysis of lithium-ion batteries", *Journal of Power Sources*, vol. 140, pp. 111–124, 2005.
- [12] D. H. Doughty, P. C. Butler, R. G. Jungst and E. P. Roth, "Lithium battery thermal models", *Journal of Power Sources*, vol. 110, pp. 357–363, 2002.
- [13] R. Spotnitz, "Simulation of capacity fade in lithium-ion batteries", *Journal of Power Sources*, vol. 113, pp. 72–80, 2003.
- [14] P. Ramadass, B. Haran, R. White and B. N. Popov, "Mathematical modeling of the capacity fade of Li-ion cells", *Journal of Power Sources*, vol. 123, pp. 230–240, 2003.
- [15] L. Gao, S. Liu and R. A. Dougal, "Dynamic Lithium-Ion Battery Model for System Simulation", *IEEE Transactions on Components and Packaging Technologies*, vol. 25, pp. 495–505, 2002.
- [16] T. R. Crompton, "Battery Reference Book", third ed., Newnes, Oxford, 2000.
- [17] S. S. Zhang, K. Xu and T. R. Jow, "Study of the charging process of a LiCoO<sub>2</sub>-based Li-ion battery", *Journal of Power Sources*, vol. 160, pp. 1349–1354, 2006.
- [18] S. S. Zhang, K. Xu and T. R. Jow, "Charge and discharge characteristics of a commercial LiCoO<sub>2</sub>-based 18650 Li-ion battery", *Journal of Power Sources*, vol. 160, pp. 1403–1409, 2006.