Precision galaxy measurements

Gary Bernstein (UPenn)

DSFP July 2020

(nearly) All Rubin Obervatory science in 1 table

Moment	Point sources	Resolved sources
0: Flux $f = \int dx\, dy\, I(x,y)$	Stellar evolution & populations, transients, distance scale, minor planet properties, Galactic structure	Galaxy evolution & populations Photometric redshifts
1: Centroid $\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \frac{1}{f} \int dx\, dy \begin{pmatrix} x \\ y \end{pmatrix} I(x,y)$	Galactic structure & dynamics Distance scale Minor planet orbits	*Nobody cares**
2: Size/Shape $\begin{pmatrix} M_r \\ M_+ \\ M_\times \end{pmatrix} = \int dx\, dy \begin{pmatrix} x^2+y^2 \\ x^2-y^2 \\ 2xy \end{pmatrix} I(x,y)$	*Nobody cares**	Weak gravitational lensing

**except as needed to facilitate galaxy shapes*

Part 1: Galaxy Photometry

- Unlike stellar photometry, there is no known best-possible way to do galaxy photometry.
- It is extremely rare to find galaxy photometry with claimed accuracy better than 1% (10 mmag).

Are extragalactic observers dumber than stellar observers? 😏

There are good reasons why galaxy photometry is less accurate:

- There are no edges to galaxies. Deciding what the "total" magnitude is involves some judgment call and/or extrapolation
- Galaxies can overlap on the sky. **Deblending** is a topic for another lecture (or many of them!).
- ***All stars look the same (delta function!) but galaxies take an infinite variety of appearances, so model-fitting cannot be an exact measurement: at some level, no galaxy fits any simple model!***
- OTOH, I know of no scientific tests that would be weakened if we added 1% errors to the brightnesses

Photometer this!

Hickson Compact Group 7
Dark Energy Survey
R. Gruendl

Things I won’t talk about

- Deblending, a.k.a. crowding
- Calibrating the images (zeropoint) - same as for stellar photometry
- Sky subtraction (harder than for stars - galaxies cover more area and generally have less flux than stars of interest, so a sky brightness error is bigger fractional flux error)
- Color terms (also harder than stellar since galaxies have a larger variety of spectra - also different parts of the galaxy are different colors!!)

The galaxy photometry zoo

- Fixed-radius aperture
- Isophotal
- Adaptive apertures: Kron, Petrosian, matched elliptical Gaussian
- Model fitting
- Pre-seeing aperture fluxes / moments

Desirable characteristics of galaxy flux measurements

1. Includes the flux from all the stars in the galaxy (“total” magnitude)
2. Maximized signal-to-noise ratio for flux measurement
3. Returns same mean flux for images taken in different seeing
4. Returns same mean flux for images taken at different noise levels
5. Yields a true color: different bands’ fluxes apply the same weight to the same part of the galaxy, i.e. the measured color is the true color of some (weighted) collection of stars.
6. (stable under deblending)

Your choice of galaxy flux will depend on which of these properties are most important for your scientific needs.

Fixed-aperture fluxes

Recipe

1. Choose fixed diameter (e.g. 3”) tophat aperture
2. Choose aperture center to null the first moments
3. Add up flux within centered aperture.

UGC 342
Dark Energy Survey
R. Gruendl

Pros

- Simple to describe and execute reproducibly
- contribution of background noise to flux estimate is well known:

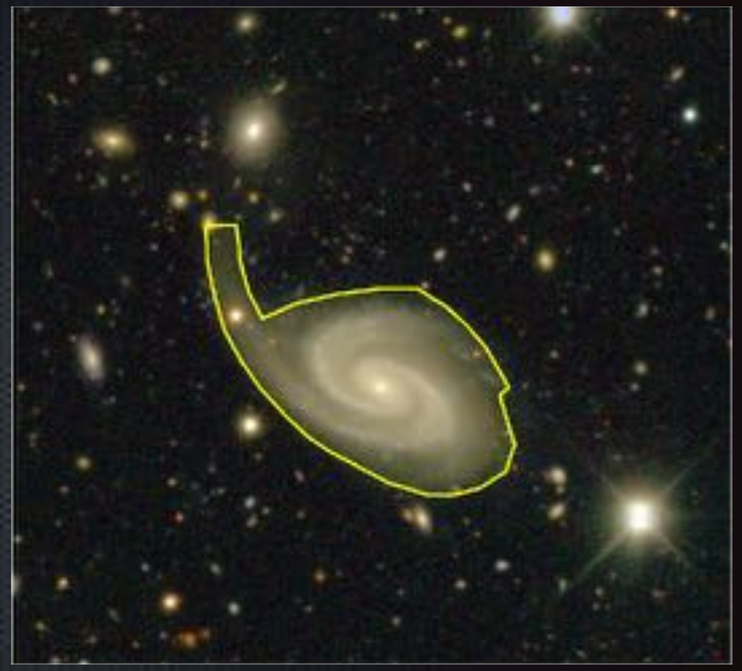
$$\mathrm{Var}(f) = nA_{\mathrm{aper}} = n\pi R^2$$

- Stable under different noise levels
- The only way to actually measure something approaching “total” is by using large R
- Colors are true if aperture radius >> size of PSF

Cons

- Large fixed apertures have very poor S/N ratio, small apertures miss a large fraction of flux
  - ☞ A single choice of aperture radius can’t be the “best” size for all galaxies!
- Answer depends on seeing unless aperture radius >> size of PSF
- Large apertures inevitably contaminated by other sources

Isophotal fluxes



Recipe

1. Find all the pixels whose brightness is above a certain level (“isophotes” are contours of surface brightness)
2. (optionally: expand this isophotal aperture by some multiplicative factor)
3. Add up flux of these pixels

Pros

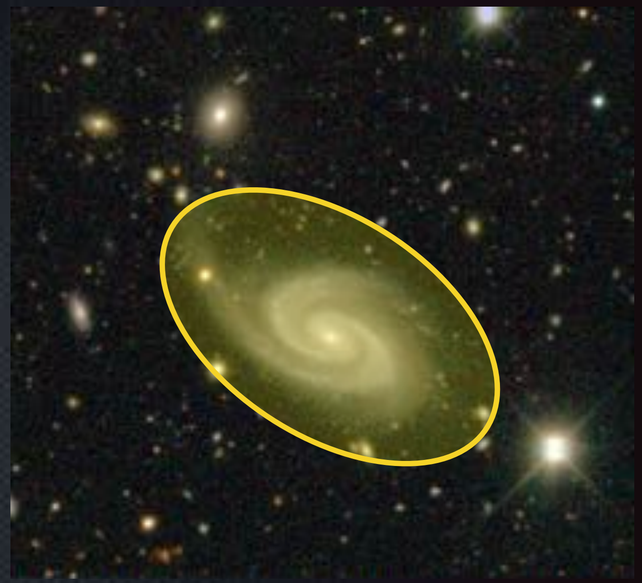
- Simple to code
- Keeps the S/N ratio high by only using pixels with significant signal

Cons

- Noise makes border pixels pop in and out of the apeture, increaseing flux noise and biasing fluxes high.
- Not robust to noise or seeing.
- Not true colors

Not recommended, IMO

Adaptive apertures



Recipe

1. Choose aperture radius to satisfy some criterion for the target galaxy:
   - "(Richard) Kron" magnitudes, SExtractor `MAGAUTO`: choose radius to be a fixed multiple of the measures 1st radial moment, $\sum rI_{xy} \Big/ \sum I_{xy}$
   - "(Vahe) Petrosian" magnitudes: adjust R until the mean surface brightness *within* the boundary is some multiple of the surface brightness *on* the boundary
   - Matched gaussian aperture magnitudes: use a Gaussian aperture, adjust sigma of the weight until it matches the 2nd moment of the pixels inside it
2. Often the aperture is elliptical, with axis ratio & position angle also adjusted to match the 2nd moments of the galaxy inside them (see shape slides later)
3. Add up flux within centered aperture.

Pros

- Apertures are a good compromise between getting near-total flux and getting good S/N
- Relatively (but not precisely) stable under different noise levels
- Recipes can be tweaked to provide "aperture corrections" that are small/stable for common mean galaxy brightness functions (exponential, deVaucouleurs)

Cons

- Answer depends on seeing unless aperture radius >> size of PSF
- Iterative process means it's tough to write a real probability function for errors.

Common "general purpose" use, can be good for "totalness" and accurate colors for well-resolved, bright galaxies.

Galaxy models

Usually some form of an elliptical Sersic model:

$$I(x,y) \propto e^{-\left(\frac{r}{r_0}\right)^{1/n_s}} = e^{-b(n)\left(\frac{r}{r_e}\right)^{1/n_s}}$$

where n_s is the "Sersic index," r_e is the "effective radius" that holds half of the light, and we make the model elliptical by setting

$$r^2 = \frac{x^2}{(1+e)^2} + \frac{y^2}{(1-e)^2}$$

and rotating the coordinates by some position angle θ. This model has 7 free parameters (which means it's hard to fit stably at S/N<10 or so):

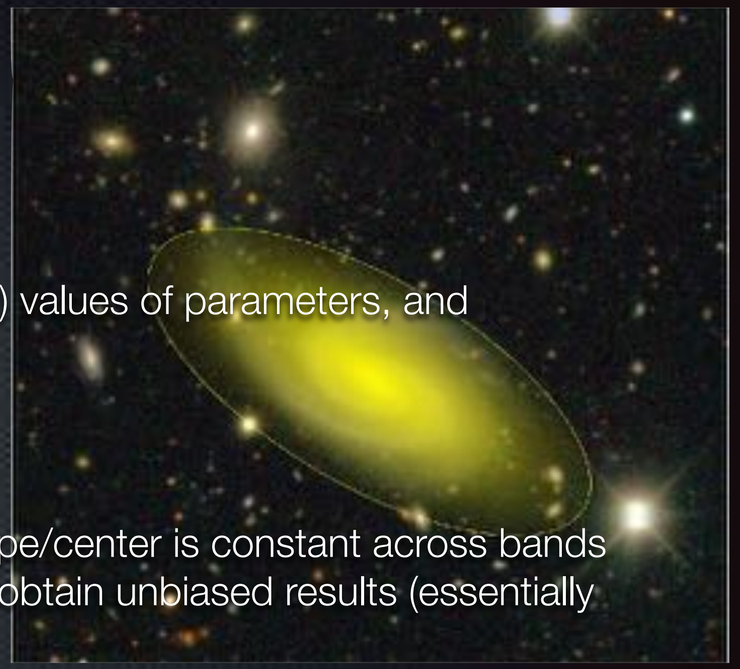
$$\{f, x_0, y_0, r_e, e, \theta, n_s\}$$

Two famous cases of the Sersic are the **exponential disk** $(n_s = 1)$ and the **deVaucouleurs profile** $(n_s = 4)$**,** which are long known as empirical good fits to the disks of spiral galaxies, and to elliptical galaxies, respectively. Model-fitting codes use several common variants:

- Single Sersic (7 params)
- Disk-or-bulge: fit both exponential & deVauc, choose best one (6.x parameters)
- Disk+bulge: fit to sum of both, force same center & shape and size (7 parameters)
- ….or allow disk & bulge to have distinct radii (+1 param) and/or ellipticities (+2 params)

and in all good modern implementations, the galaxy's "intrinisic model" is convolved with the PSF before being compared to the pixel data!

Galaxy model-fitting photometry



Recipe

1. Choose a model for the galaxy, and any **priors** that you wish to place on their parameters.
2. Conduct a non-linear least-squares fit or MCMC to obtain maximum-likelihood (or max-posterior) values of parameters, and confidence regions.
3. Flux is one of the fitted parameters.

Pros

- One can execute the fit using data from multiple exposures and multiple filters, assuming the shape/center is constant across bands
- One can exclude defective pixels (or regions contaminated by other sources) from the fit and still obtain unbiased results (essentially using the model to fill in the missing pixels)
- ***If the galaxy truly looks like the model***, then
  - This process yields maximal signal-to-noise, saturating the Cramer-Rao bound on flux
  - Yields total magnitudes from finite image (assumes that the wings fit the model!)
  - Is (almost) unbiased under varying noise level and seeing
  - Yields true colors

Good choice for marginally-resolved galaxies or where S/N is at a premium, but beware of biases and fitting failures.

Cons

- Because models are nonlinear in parameters other than flux, the max-likelihood solutions are biased by a fractional amount $\propto (S/N)^{-2}$
- The many-parameter fits can become degenerate or fail to converge, esp for faint galaxies.
- The colors will not be “true” if the color varies across the face of the galaxy (“color gradients”)
- ***The galaxy usually departs from the model (possibly at a non-detectable level for low-S/N galaxies)***, so
- the magnitude is not total (it’s guessing about the wings), the answer can depend on seeing and noise level of the image, and there can be color biases.

Pre-seeing apertures

- All the methods of estimating “total” flux of galaxies require tradeoffs between their “totalness,” their noise level, and the amount of assuming being done about the galaxy’s true appearance.
- For photo-z and galaxy-evolution purposes, getting an accurate **color** of some subset of the galaxy is often far more important that measuring the total magnitude. There are methods for insuring that your flux measurement contains the exact same set of stars for all bands. Briefly:
  - ***PSF matching***: convolve all bands’ observed images to make them have the same PSF. Then apply aperture mag of your choice. Downside: you have to use the worst seeing of any of your images.
  - ***Fourier-domain moments***: Express your chosen “pre-seeing” aperture as a sum in Fourier space instead of in real space. In Fourier space, it’s easy to remove the PSF - just divide by the Fourier version of PSF. Easier to assign noise levels than in PSF matching. More on this later.
  - ***GaaP: (Kuijken 2008):*** Clever real-domain image manipulation to obtain the flux that would be obtained by a pre-seeing Gaussian aperture flux measurement. Again, pre-seeing aperture cannot be much smaller than PSF.

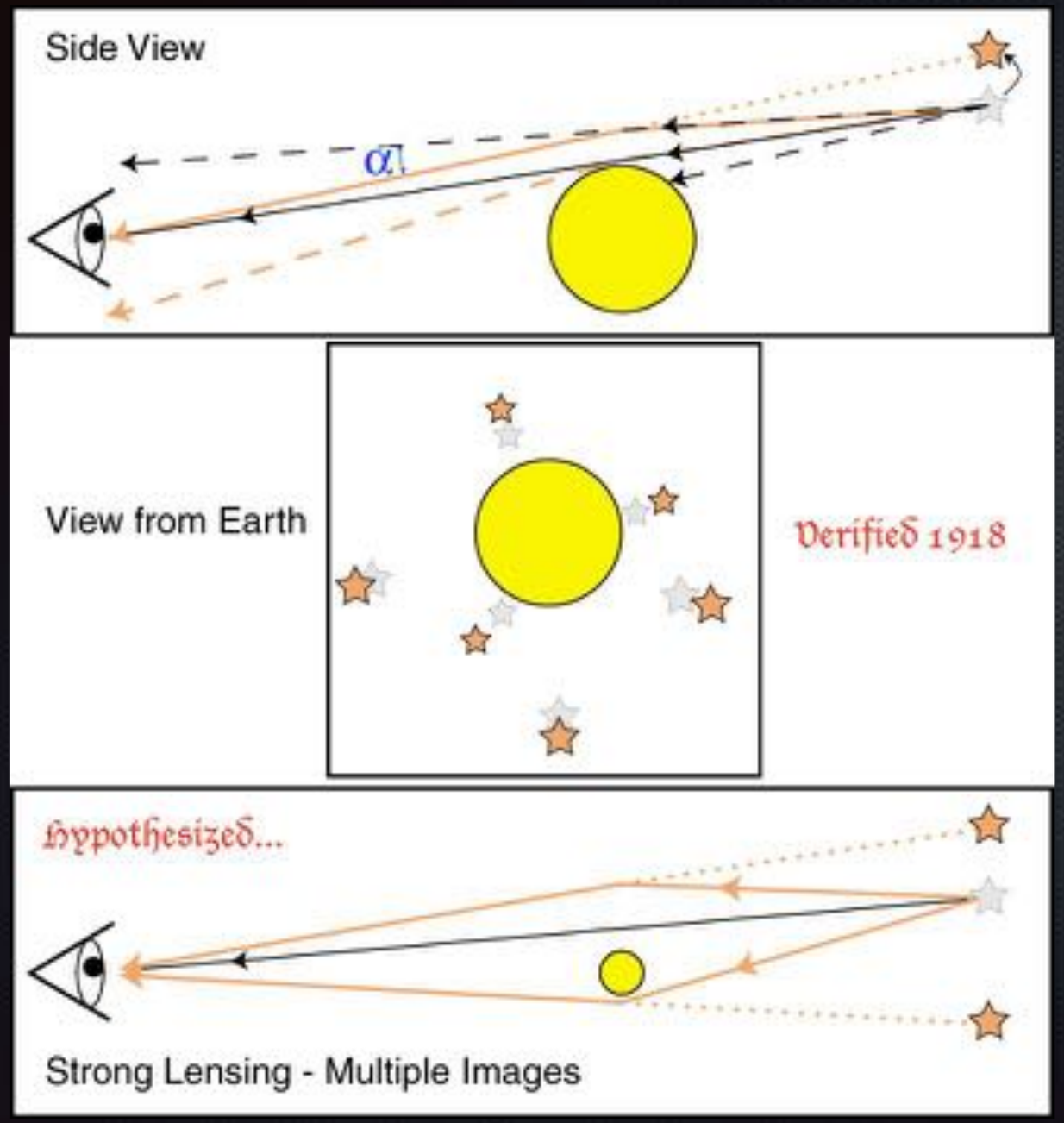
These are the methods of choice if you want precise and accurate colors that have physical meaning, e.g. for photo-z’s, and not-well-resolved galaxies.

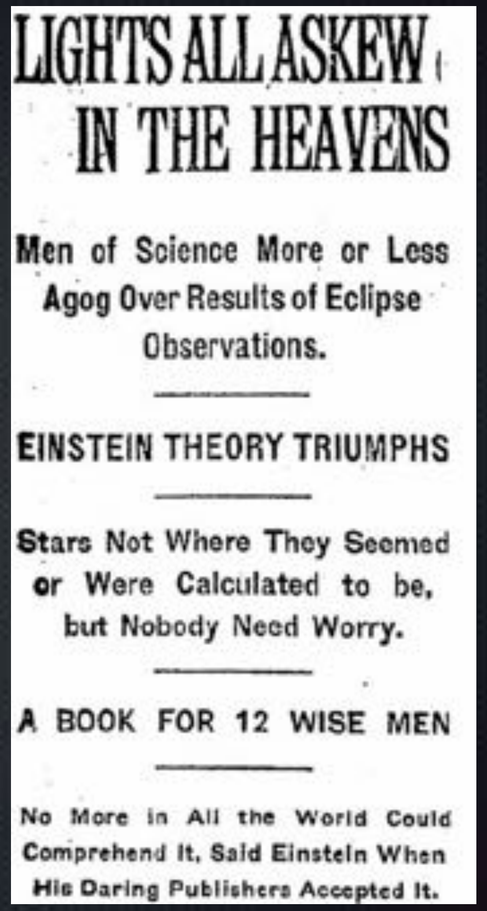
Galaxy photometry summary

- Many different flux definitions and algorithms, which do not even all have the same “truth.” There is no single answer!
- Choose the one that emphasizes the properties your science requires. In particular, do you care about color, or about total flux?
- Always be careful about knowing the shortcomings of your methods (e.g. biases, un-real colors, not total, etc.) and pay attention when you get magnitudes from some source or paper.

Part 2: Galaxy shapes & sizes for weak gravitational lensing

- Intro to weak gravitational lensing (WL)
- Relating precision galaxy shapes, sizes, and fluxes to WL
- Quick description of the two leading approaches to shape/WL measurement:
  - **Metacalibration:** model fitting (Huff & Mandelbaum 2017; Sheldon & Huff 2017)
  - Bayesian Fourier-Domain Moments (**BFD**) (**Bernstein et al 2016**)





LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

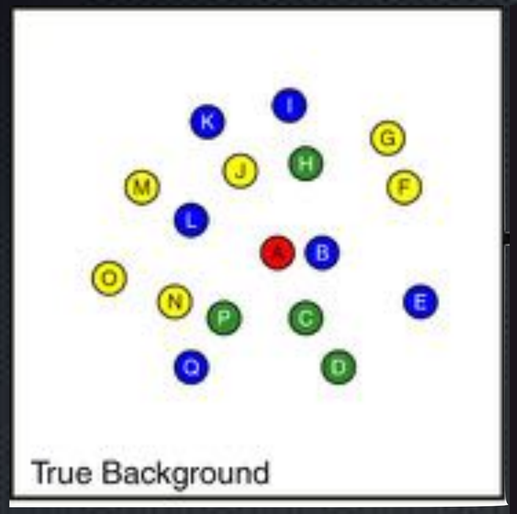
A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

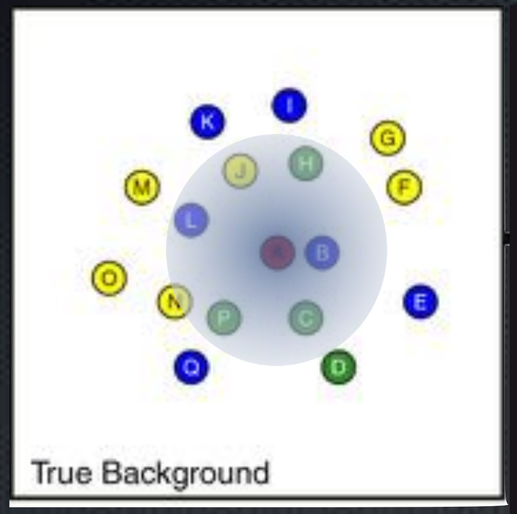
New York Times
Nov 10, 1919

Beautiful strong lensing:

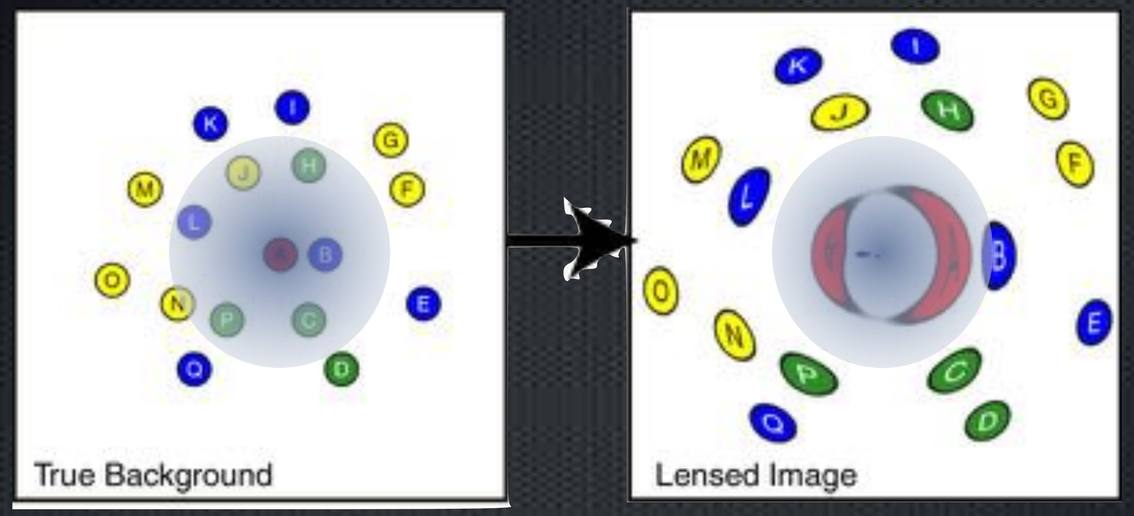
Weak vs strong lensing



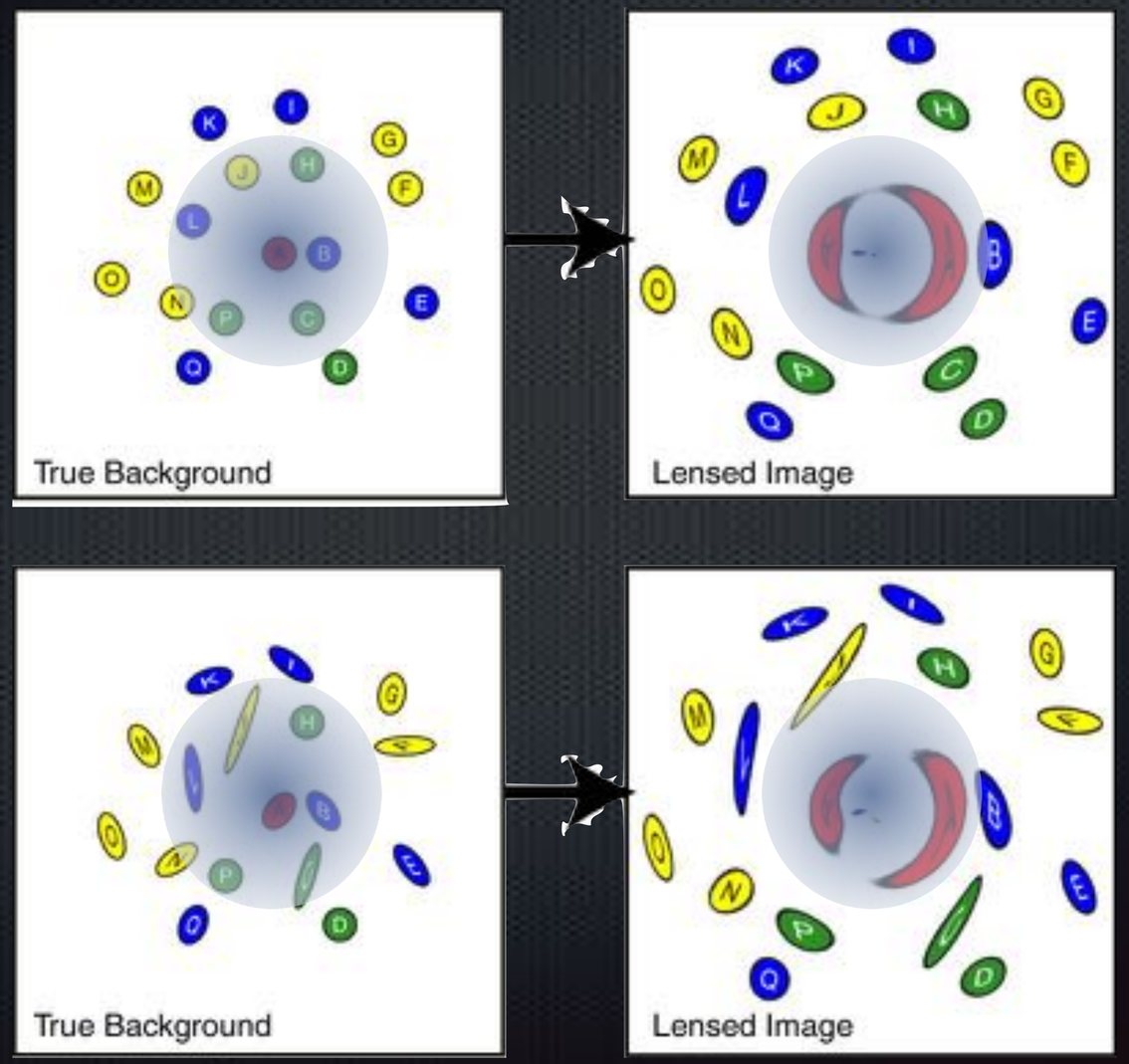
Weak vs strong lensing



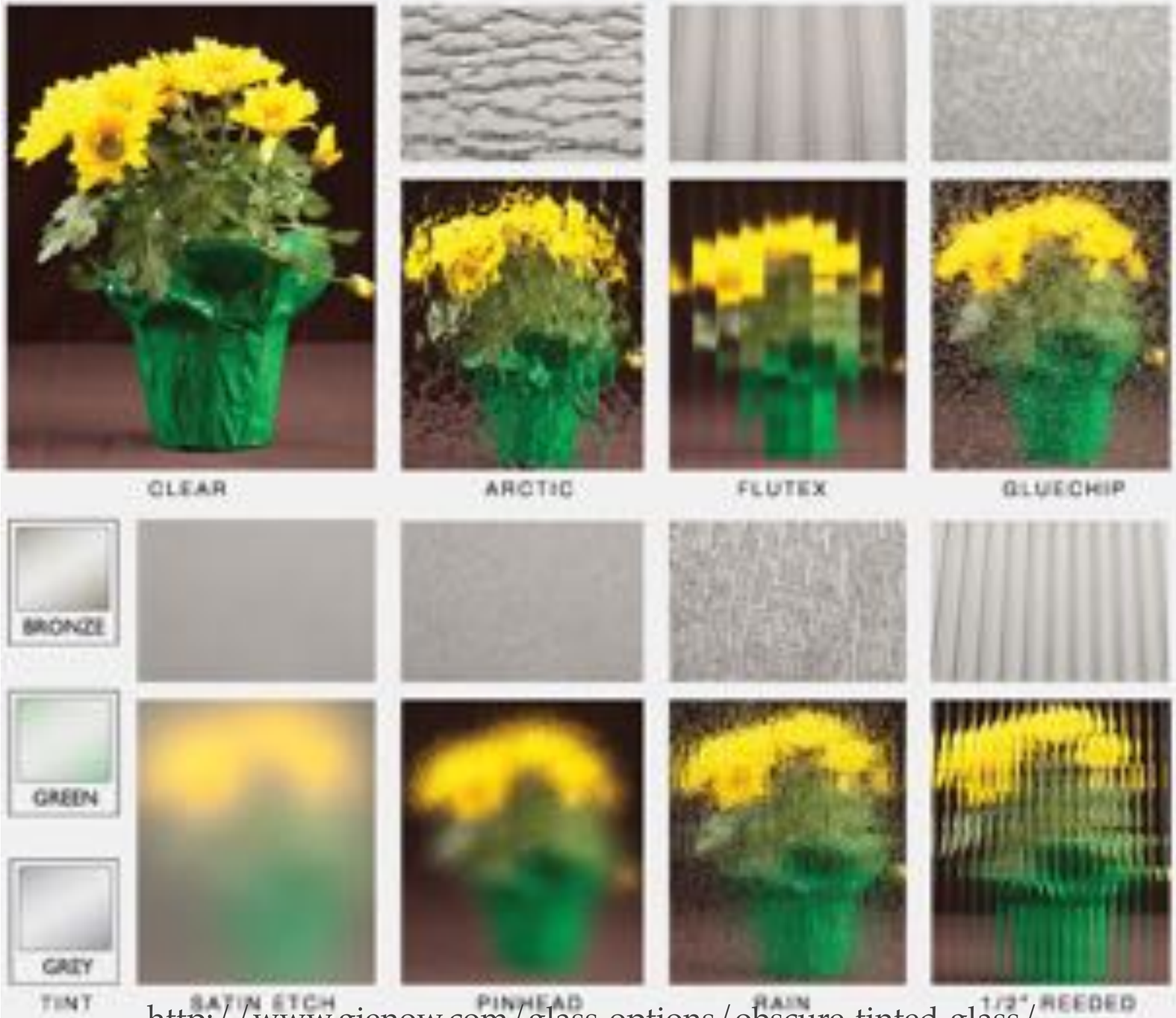
Weak vs strong lensing



Weak vs strong lensing



This is your brain on weak lensing

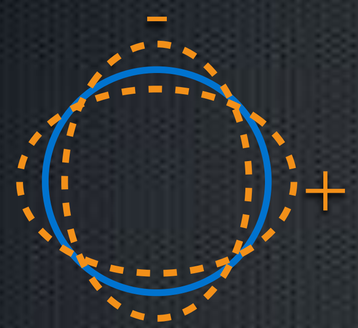


http://www.gienow.com/glass-options/obscure-tinted-glass/

Measuring WL with galaxy images

- A typical line of sight to LSST galaxy is magnified or sheared by just 1-2%!
- While its intrinsic shape is typically non-circular by 20-30%. So *individual* galaxy measurements of WL are very weak (duh!)
- But LSST will measure shapes of ~1B galaxies, meaning that the *average* WL is in principle measurable to $0.2/\sqrt{10^9} \approx 10^{-5}$ which is 0.001 of the typical signal!! Extreme accuracy is required *collectively* even though each measurement is nearly useless!
- We need to find some number(s) we can measure on galaxies that:
  - Are altered by the actions of WL on the images
  - Have values *before* WL that we know to high accuracy
    - This means that we have to be able to remove the effects of PSF on observed shape!
  - Respond to the action of WL in a *precise and accurately known* way even though we are looking at blurry, noisy versions of the galaxy.
  - Extract most of the information about WL that each galaxy image gives.
  - Is feasible to compute on ~1B LSST galaxies!!!

WL observables

		Model fitting	Moments
Magnification:			
κ		$f \to (1+2\kappa)f$ $r_e \to (1+\kappa)r_e$	$M_f = \int dx\,dy\,I(x,y) \to M_f(1+2\kappa)$ $M_r = \int dx\,dy\,I(x,y)\left(x^2+y^2\right) \to M_r(1+\kappa)$
Shear:			
γ_+		$e_+ = e\cos 2\theta \to e_+ + 2\gamma_+$	$M_+ = \int dx\,dy\,I(x,y)\left(x^2-y^2\right) \to M_+ + 2M_f\gamma_+$
$\gamma_\times$		$e_\times = e\sin 2\theta \to = e_\times 2\gamma_\times$	$M_\times = \int dx\,dy\,I(x,y)\left(2xy\right) \to M_\times + 2M_f\gamma_\times$

Rules of thumb

- The *observed* ellipticity is reduced from the *intrinsic* one by

$$e_{\rm obs} = e_{\rm true} \frac{R_{\rm gal}^2}{R_{\rm gal}^2 + R_{\rm psf}^2}$$

  which also reduces the effect of shear.

- A Fisher analysis shows that we obtain errors on shape of

$$\sigma_{e_{\rm obs}} \approx \frac{1}{\nu}$$

- but there is an builtin "shape noise" of $\sigma_{e_{\rm true}} \approx 0.15$ that we must add in quadrature to each galaxy.

- The significance of the detection is

$$\nu^2 \approx \frac{f^2}{n\pi \left(R_{\rm gal}^2 + R_{\rm psf}^2 \right)}$$

Metacalibration

- Start with Sersic model-fitting, obtain e's - fitting a pre-seeing model removes most PSF bias.
- *But this is not the right model, there are model biases, plus the maximum-likelihood solution is known to be biased. The level of bias depends on what kinds of galaxies exist on the sky…*
- This bias must be measured to high precision: "metacalibration" does this by:
  - Deconvolving the PSF from the galaxy image (but amplifies the noise!)
  - Shear the image
  - Reconvolve the PSF
  - Remeasure e to determine the response of e to shear
- There are many subtleties… but the idea is: shear the real sky to determine the impact of shear on the real sky.

Bayesian Fourier-domain moments (BFD)

- Step 1: Reduce each galaxy image(s) to a set of pre-seeing moments:

(FT of the image)

Weight function (controls noise)

$$D = \begin{Bmatrix} \hat{M}_f \\ \hat{M}_r \\ \hat{M}_+ \\ \hat{M}_\times \end{Bmatrix} = \int d^2k \, \frac{\tilde{I}(k)}{\tilde{T}(k)} W(k) \begin{Bmatrix} 1 \\ k_x^2 + k_y^2 \\ k_x^2 - k_y^2 \\ 2k_x k_y \end{Bmatrix}$$

(FT of the PSF)

- Moments are invariant under change in PSF or noise level
- Moments are linear in the image, so we know exactly what the likelihood $\mathscr{L}(\hat{M} \,|\, M)$ of the observed moments given the truth.

Bayesian Fourier-domain moments (BFD)

- Step 2: Calculate rigorous Bayesian probability of the data vs WL shear:

Likelihood of the observations

$$P(D|\gamma) = \int dM\, \mathcal{L}(\hat{M}|M) P(M|\gamma)$$

Integrate over all possible true moment vectors

The prior distribution of truth is taken by measuring the galaxy population and the effect of shear from low-noise “deep fields”

Summary

- WL cosmology has motivated the development of a new flavor of precision astronomical measurement, more demanding than flux and centroids!
- Rubin Observatory shear goals demand that galaxy *e* or 2nd-moment measurements be unbiased to about 10^{-5}! At this level there are ***many*** things to worry about. For model fitters: model bias (if model fitting) and the biases of maximum likelihood fits of nonlinear models.
- Two methods have shown ability to do this on isolated galaxies. Both BFD & Metacal:
  - Depend on accurate PSF models to measure “pre-seeing” shapes
  - Use the real sky to determine “priors” of what kinds of galaxies are measured.
  - Can use multiband, multi-exposure data
  - Can correct for selection biases
  - Still need plenty of work to get going on Rubin data, esp. dealing with blending!

Useful references

- Review article on precision WL: Mandelbaum 2018
- Calculating LSST limiting magnitudes and SNR
- "Advanced Exposure Time Calculations" paper by GMB, 2002
- Many details about optimal S/N for shear measurements: Bernstein & Jarvis 2002:
- Metacalibration: Huff & Mandelbaum 2017; Sheldon & Huff 2017
- BFD: Bernstein et al 2016
- GaaP: (Kuijken 2008)
- SDSS guide to different magnitudes
- SExtractor documentation