

Test 10

1) a) $x^2 - 9 \neq 0 \Leftrightarrow x^2 \neq 9 \Leftrightarrow x \neq 3 \cup x \neq -3$

$D_f =]-\infty, -3[\cup]3, +\infty[$

b) $x^2 - 1 \neq 0 \Leftrightarrow x \neq 1 \cup x \neq -1$

$D_g =]-\infty, -1[\cup]1, +\infty[$

2) a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{(x-3)(x+3)}{x-3} = x+3 = 3+3=6$

b) $\lim_{x \rightarrow +\infty} \frac{2x^3 - x - 4}{5x^3 + 3x^2} = \frac{2 - \frac{1}{x} + \frac{4}{x^2}}{5 + \frac{3}{x}} = \frac{2 - 0 + 0}{5 + 0} = \frac{2}{5}$

3) $f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x-2)(x+2)}{x-2} = x+2 = 2+2=4$

4) $f(0) = x^3 - 3x + 1 = (0)^3 - 3(0) + 1 = 0 - 0 + 1 = 1$

$f(1) = x^3 - 3x + 1 = (1)^3 - 3(1) + 1 = 1 - 3 + 1 = -1$

$f(0) = 1 > 0$

$f(1) = -1 < 0$

5) $x-2 \neq 0 \Rightarrow x \neq 2$

$f(2^+) = \frac{2x^2 - x + 1}{x-2} = \frac{2(2^+)^2 - 2^+ + 1}{2^--2} = \frac{8 - 2 + 1}{0^+} = \frac{7}{0^+} = +\infty$

$f(2^-) = \frac{2x^2 - x + 1}{x-2} = \frac{2(2^-)^2 - 2^- + 1}{2^--2} = \frac{7}{0^-} = -\infty$

~~$$m = \frac{2x^2 - x + 1}{x(x-1)} = \frac{2x^2 - x + 1}{x^2 - x} = 2 \cdot \frac{x^2 - x + 1}{x^2 - x} \quad 2 \times 1 = 2$$~~

$$b = \lim_{x \rightarrow 100} \left[\frac{2x^2 - x + 1}{x - 1} - 2x \right]$$

6) $f(2) = 2^3 - 3 \cdot 2^2 - 9 \cdot 2 + 5 = (2)^3 - 3(2)^2 - 9(2) + 5 = 8 - 12 - 18 + 5 = -17$

$f'(2) = 3x^2 - 6x - 9 = 3(2)^2 - 6(2) - 9 = 12 - 12 - 9 = -9$

$y = -9x + (-17 - (-9(2))) = -9x + 1$

7) a) $\int 4x^3 - 2x + 5 \, dx = \frac{4x^4}{4} - \frac{2x^2}{2} + 5x + C, C \in \mathbb{R}$

b) $\int \frac{6x^2}{x^3+1} \, dx = \int 2 \frac{3x^2}{1+x^3} \, dx = 2 \arctan(x^3) + C, C \in \mathbb{R}$

c) $\int x \cdot \cos(x) \, dx = x \cdot \sin(x) - \int \cos(x) \, dx = x \cdot \sin(x) + \sin(x)$

$x \cdot \sin(x) + \sin(x) + C, C \in \mathbb{R}$

8) a) $\int_1^3 2x - 1 \, dx = \left[\frac{2x^2}{2} \right]_1^3 = \frac{2(3)^2}{2} - \frac{2(1)^2}{2} = \frac{18}{2} - \frac{2}{2} = 9 - 1 = 8$

b) $\int_0^1 x \cdot e^x \, dx = \left[x \cdot e^x \right]_0^1 - \int_0^1 e^x \, dx = \left[x \cdot e^x - e^x \right]_0^1 =$

$\cancel{(1 \cdot e^1 - e^1)} - (0 \cdot e^0 - e^0) = 0 - (-1) = 0 + 1 = 1$