Exercícios (TPC) → Integração por partes

Fórmula: $\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x)\, g'(x)\, dx$

(1) $\int (\underbrace{x}_{f'(x)} \times \underbrace{\arcsin(x^2)}_{g(x)})\, dx =$

Cálculos Auxiliares:
$f(x) = \frac{x^2}{2}$; $f'(x) = x$
$g(x) = \arcsin(x^2)$
$g'(x) = \frac{2x}{\sqrt{1-x^4}}$

$= \frac{x^2}{2} \times \arcsin(x^2) - \int \frac{x^2}{\cancel{2}} \times \frac{\cancel{2}x}{\sqrt{1-x^4}}\, dx =$

$= \frac{x^2}{2} \arcsin(x^2) - \int \frac{x^3}{\sqrt{1-x^4}}\, dx =$

$= \frac{x^2}{2} \arcsin(x^2) - \left(-\frac{1}{4}\right) \int \underbrace{-4x^3}_{g'(x)} \times (\underbrace{1-x^4}_{g(x)})^{-1/2}\, dx$

$= \frac{x^2}{2} \arcsin(x^2) + \frac{1}{4} \frac{[1-x^4]^{-1/2+1}}{-1/2+1} + C$ $\quad -1/2+1 = 1/2$

$= \frac{x^2}{2} \arcsin(x^2) + \frac{1}{4} \times 2\, [1-x^4]^{1/2} + C$

$= \frac{x^2}{2} \arcsin(x^2) + \frac{1}{2} \sqrt{1-x^4} + C, \quad C \in \mathbb{R}$ //

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(2) $\int (\underbrace{1}_{f'(x)} \times \underbrace{\cos(\ln(x))}_{g(x)})\, dx =$

Cálculos Aux.:
$f(x) = x$; $f'(x) = 1$
$g(x) = \cos(\ln(x))$
$g'(x) = (\ln(x))' \times (-\sin(\ln(x)))$
$= -\frac{1}{x} \sin(\ln(x))$

$= x \times \cos(\ln(x)) - \int x \times \left(-\frac{1}{x}\right) \sin(\ln(x))\, dx =$

$= x \cos(\ln(x)) + \int \underbrace{1}_{f'(x)} \underbrace{\sin(\ln(x))}_{g(x)}\, dx =$

Cálculos Aux.:
$g(x) = \sin(\ln(x))$
$g'(x) = \frac{1}{x} \cos(\ln(x))$
$f(x) = 1$
$f'(x) = x$

$= x \cos(\ln(x)) + x \sin(\ln(x)) - \int \cos(\ln(x)) \frac{1}{\cancel{x}} \times \cancel{x}\, dx$

$= x \cos(\ln(x)) + x \sin(\ln(x)) - \int \cos(\ln(x))\, dx$

Assim, como:

$\int \cos(\ln(x))\, dx = x \cos(\ln(x)) + x \sin(\ln(x)) - \int \cos(\ln(x))\, dx$

$$\Leftrightarrow \quad 2\int \cos(\ln(x))\,dx = x\cos(\ln(x)) + x\,\text{sen}(\ln(x)) + C$$

$$\Leftrightarrow \int \cos(\ln(x))\,dx = \frac{x}{2}\cos(\ln(x)) + \frac{x}{2}\text{sen}(\ln(x)) + \frac{C}{2}$$

$$\Leftrightarrow \int \cos(\ln(x))\,dx = \frac{x}{2}\left[\cos(\ln(x)) + \text{sen}(\ln(x))\right] + K,$$

onde $K = \frac{C}{2}$, $K, C \in \mathbb{R}$ //

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3) $\int (\underbrace{e^{2x}}_{g(x)} \times \underbrace{\text{sen}\, x}_{f'(x)})\,dx =$

Cál. Auxiliares
$g(x) = e^{2x}$
$g'(x) = 2e^{2x}$
$f(x) = -\cos(x)$
$f'(x) = \text{sen}(x)$

$= e^{2x} \times (-\cos(x)) - \int 2e^{2x} \times (-\cos x)\,dx =$

$= -\cos(x)\, e^{2x} + 2\int \underbrace{e^{2x}}_{g(x)} \times \underbrace{\cos x}_{f'(x)}\,dx$

Cál. Auxiliares
$f(x) = \text{sen}(x)$
$f'(x) = \cos(x)$
$g(x) = e^{2x}$
$g'(x) = 2e^{2x}$

$= -e^{2x}\cos(x) + 2\left[e^{2x} \times \text{sen}(x) - \int 2e^{2x} \times \text{sen}(x)\,dx\right]$

$= -e^{2x}\cos(x) + 2e^{2x}\,\text{sen}(x) - 4\int e^{2x}\,\text{sen}(x)\,dx$

Assim, como:

$$\int e^{2x}\,\text{sen}(x)\,dx = -e^{2x}\cos(x) + 2e^{2x}\,\text{sen}(x) - 4\int e^{2x}\,\text{sen}(x)\,dx$$

$$\Leftrightarrow 5\int e^{2x}\,\text{sen}(x)\,dx = -e^{2x}\cos x + 2e^{2x}\,\text{sen}(x) + C$$

$$\Leftrightarrow \int e^{2x}\,\text{sen}(x)\,dx = -\frac{e^{2x}}{5}\left[\cos(x) - 2\,\text{sen}(x)\right] + \frac{C}{5}$$

$$\Leftrightarrow \int e^{2x}\,\text{sen}(x)\,dx = -\frac{e^{2x}}{5}\left[\cos(x) - 2\,\text{sen}(x)\right] + K,$$

onde $K = \frac{C}{5}$, $K \in \mathbb{R}$ //

→ Exercícios (TPC) → Por substituição

a) $\int \frac{\ln(x)}{x\sqrt{1+\ln(x)}}\,dx =$

Mudança variável

$1+\ln(x) = t^2$

$\Leftrightarrow \ln(x) = t^2-1$

$\Leftrightarrow x = e^{(t^2-1)} \in \mathbb{R}^+$

$\varphi(t) = e^{(t^2-1)}$

$\varphi'(t) = 2t\, e^{(t^2-1)}$

$$= \int \frac{t^2-1}{\cancel{e^{(t^2-1)}} \times \cancel{t}} \times 2\cancel{t}\,\cancel{e^{(t^2-1)}}\,dt =$$

$$= 2\int t^2 - 1 \, dt$$

$$= 2\left[\int t^2\, dt - \int 1\, dt\right] = 2\left[\frac{t^3}{3} - t\right] + C$$

$$= 2\left[\frac{1}{3}\left[\sqrt{1+\ln(x)}\right]^3 - \sqrt{1+\ln(x)}\right] + C$$

$$= 2\left[\frac{1}{3}(1+\ln(x))\sqrt{1+\ln(x)} - \sqrt{1+\ln(x)}\right] + C$$

$$= 2\sqrt{1+\ln(x)}\left[\frac{1}{3}[1+\ln(x)] - 1\right] + C$$

$$= 2\sqrt{1+\ln(x)}\left[\frac{1+\ln(x)-3}{3}\right] + C$$

$$= 2\sqrt{1+\ln(x)}\left[\frac{\ln(x)-2}{3}\right] + C$$

$$= \frac{2}{3}\sqrt{1+\ln(x)}\left[\ln(x)-2\right] + C,\ C\in\mathbb{R} \quad //$$

mudança variável
$\sqrt{x} = t$
$\Leftrightarrow x = t^2 \in \mathbb{R}^+$

$\varphi(t) = t^2$
$\varphi'(t) = 2t$

b) $\int \cos(\sqrt{x})\, dx =$

$= \int \cos(t) \times 2t\, dt =$

$= 2 \int \underbrace{t}_{g(x)} \underbrace{\cos(t)}_{f'(x)}\, dt =$

Integração por partes

$g(x) = t$
$g'(x) = 1$
$f(x) = \text{sen}(t)$
$f'(x) = \cos(t)$

$= 2\left[t\,\text{sen}(t) - \int 1 \times \text{sen}(t)\, dt \right] =$

$= 2\left[t\,\text{sen}(t) - (-\cos(t)) \right] + C$

$= 2t\,\text{sen}(t) + 2\cos(t) + C$

$= 2\sqrt{x}\,\text{sen}(\sqrt{x}) + 2\cos(\sqrt{x}) + C,\ C \in \mathbb{R}$ //