

Testo 12

$$1) a) x^2 - 9 > 0 \Leftrightarrow x^2 > 9 \Leftrightarrow x > 3$$

$$D_f =]3, +\infty[$$

$$b) 2x - 6 > 0 \Leftrightarrow 2x > 6 \Leftrightarrow x > \frac{6}{2} \Leftrightarrow x > 3$$

$$D_f =]3, +\infty[$$

$$2) a) \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 5x + 6} = \frac{(x-2)(x+2)}{(x-2)(x-3)} = \cancel{\frac{(x-2)(x+2)}{(x-2)(x-3)}}$$

$$= \frac{(x+2)}{(x-3)} = \frac{2+2}{2-3} = \frac{4}{-1} = -4$$

$$b) \lim_{x \rightarrow +\infty} \frac{4x^3 - 2x + 1}{2x^3 + x^2 - 3} = \frac{4 - \frac{2}{x^2} + \frac{1}{x^3}}{2 + \frac{1}{x} - \frac{3}{x^3}} = \frac{4}{2} = 2$$

$$c) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} \Leftrightarrow \frac{\sqrt{1+x}}{x} - 1 \Leftrightarrow \frac{0}{0} - 1 \Leftrightarrow 1$$

$$3) f(2) = \frac{x^2 - 4}{x - 2} = \frac{(x-2)(x+2)}{x-2} = x+2 = 2+2 = 4$$

$$x = 4$$

$$4) f(0) = x^3 - 3x + 1 = (0)^3 - 3(0) + 1 = 1$$

$$f(1) = x^3 - 3x + 1 = (1)^3 - 3(1) + 1 = 1 - 3 + 1 = -1$$

$$f(0) = 1 > 0$$

$$f(1) = -1 < 0$$

$$5) \quad x+1 > 0 \Leftrightarrow x > -1$$

$$\lim_{x \rightarrow 1^+} f(x) = \frac{2x^2 + 3x - 2}{x+1} = \frac{2(1^+)^2 + 3(1^+) - 2}{1+1} = \frac{2+3-2}{0^+} =$$

$$\frac{3}{0^+} = +\infty$$

$$\lim_{x \rightarrow 1^-} f(x) = \frac{2x^2 + 3x - 2}{x+1} = \frac{2(1^-)^2 + 3(1^-) - 2}{1^-+1} = \frac{3}{0^-} = -\infty$$

$$m = \lim_{x \rightarrow 1} \frac{2x^2 + 3x - 2}{x(x+1)} = \frac{2x^2 + 3x - 2}{x^2 + x} = \frac{2 + 3 - 2}{1 + 1} = \frac{3}{2} = 1.5$$

$$b = \frac{2x^2 + 3x - 2}{x-1} - 2x = \frac{2x^2 + 3x - 2 - 2x(x-1)}{x-1}$$

$$= \frac{2x^2 + 3x - 2 - 2x^2 + 2x}{x-1} = \frac{5x - 2}{x-1}$$

$$= \frac{5x - 2}{x-1} = \frac{x-2}{x-1} = \frac{x}{x} - \frac{2}{1} = 1 - 2 = -1$$

Assintota Vertical = 1

Assintota horizontal = No lim $\frac{\text{grau } 2}{\text{grau } 1}$

Assintota Obliqua = -2

6) a) ~~4(3x^2 - 2x + 1)^3 \cdot (6x - 2)~~

b) $\frac{x^2 - 6x - 1}{(x-3)^2}$

c) $e^{2x} \left(2 \ln(x) + \frac{1}{x} \right)$

7)

$$f(1) = x^3 - 2x = (1)^3 - 2(1) = -1$$

$$f'(1) = 3x^2 - 2 = 3(1)^2 - 2 = 3 - 2 = 1$$

$$y = x + (-1 - 1)$$

$$y = x - 2$$

8) $\int 4x^3 - 6x + 2 \, dx = \frac{4x^4}{4} - \frac{6x^2}{2} + 2x + C, C \in \mathbb{R}$

b) $\int \frac{2x}{x^2+3} \, dx \Leftrightarrow \ln(x^2+3) + C, C \in \mathbb{R}$

c) $\int x \cdot e^{2x} \, dx \Leftrightarrow x \cdot e^{2x} - \int e^{2x} \, dx \Leftrightarrow x \cdot e^{2x} - \frac{1}{2} e^{2x} + C, C \in \mathbb{R}$

d) $\int \ln(x) \, dx = x \ln(x) - \int x \cdot \frac{1}{x} \, dx = x \ln(x) - x + C, C \in \mathbb{R}$

9)

a) $\int_1^3 x^2 - 2x + 1 \, dx = \left[\frac{x^3}{3} - \frac{2x^2}{2} + x \right]_1^3 =$

$$= \frac{3^3}{3} - \frac{2(3)^2}{2} + 3 - \left(\frac{1^3}{3} - \frac{2(1)^2}{2} + 1 \right) = 3 - 9 + 3 - \frac{1}{3} - 1 + 1$$

$$= -3 - \frac{1}{3} = -\frac{9}{3} - \frac{1}{3} = -\frac{8}{3}$$

5) ~~Handwritten scribbles and crossed-out text~~

$$\int_1^2 \frac{2}{x} = \left[2 \ln(x) \right]_1^2 = 2(\ln(2)) - 2(\ln(1)) = 2(1) - 2(0) = 2$$

10) $F(x) = 6x - 4 + C =$

$$F(1) = 6(1) - 4 + 0 = 2$$

11)

12)

$$f(1) = x^2 = 1^2 = 1$$

$$g(1) = 2x = 2 \cdot 1 = 2$$

$$A = \int 2x - x^2 \, dx = \frac{2x^2}{2} - \frac{x^3}{3} + C, C \in \mathbb{R}$$