

## Testo 12

1) a)  $x^2 - 9 > 0 \Leftrightarrow x^2 > 9 \Leftrightarrow x > 3$

$$D_f = ]3, +\infty[$$

b)  $2x - 6 > 0 \Leftrightarrow 2x > 6 \Leftrightarrow x > \frac{6}{2} \Leftrightarrow x > 3$

$$D_f = ]3, +\infty[$$

2) a)  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 5x + 6} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-3)} = \cancel{(x-2)(x-3)}$

$$= \frac{(x+2)}{(x-3)} = \frac{x+2}{x-3} = \frac{4}{-1} = -4$$

b)  $\lim_{x \rightarrow +\infty} \frac{4x^3 - 2x + 1}{2x^3 + x^2 - 3} = \frac{4 - \frac{2}{x^2} + \frac{1}{x^3}}{2 + \frac{1}{x} - \frac{3}{x^3}} = \frac{4}{2} = 2$

c)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} \Leftrightarrow \frac{\sqrt{1+x} - 1}{x} \stackrel{0}{\rightarrow} 1 \Leftrightarrow \frac{0}{0} \rightarrow 1$

3)  $f(2) = \frac{x^2 - 4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = x+2 = 2+2 = 4$

$$\exists x = 4$$

4)  $f(0) = x^3 - 3x + 1 = 0^3 - 3(0) + 1 = 1$

$$f(1) = x^3 - 3x + 1 = 1^3 - 3(1) + 1 = 1 - 3 + 1 = -1$$

$$f(0) = 1 > 0$$

$$f(1) = -1 < 0$$

$$5) x+1 > 0 \Leftrightarrow x > -1$$

$$\cancel{f(x)} = \frac{2x^2 + 3x - 2}{x+1} = \frac{2(-1)^2 + 3(-1) - 2}{-1+1} = \frac{2+3-2}{0} = \frac{3}{0} = +\infty$$

$$f(\infty) = \frac{2x^2 + 3x - 2}{x+1} = \frac{2(1)^2 + 3(1) - 2}{1+1} = \frac{3}{2} = +\infty$$

$$m = \cancel{\frac{2x^2 + 3x - 2}{x+1}} = \frac{2x^2 + 3x - 2}{x(x+1)} = \frac{2x^2 + 3x - 2}{x^2 + x} =$$

$$= \frac{2 + \frac{3}{x} - \frac{2}{x^2}}{1 + \frac{1}{x}} = \frac{2}{1} = 2$$

$$b = \frac{2x^2 + 3x - 2}{x-1} - \frac{2x}{x-1} = \frac{2x^2 + 3x - 2 - 2x(x-1)}{x-1}$$

$$= \frac{2x^2 + 3x - 2 - 2x^2 + 2x}{x-1} = \cancel{\frac{5x - 2}{x-1}}$$

$$= \frac{3x - 2}{x-1} - \frac{2x}{x-1} = \frac{x-2}{x-1} = \frac{x}{x} - \frac{2}{1} = 1 - 2$$

Asintota vertical = 1

Asintota horizontal = Nowiem  $\frac{\text{grau 2}}{\text{grau 1}}$

Asintota obliqua = -2

6) a)

$$4(3x^2 - 2x + 1)^3 \cdot (6x - 2)$$

b)  $\frac{x^2 - 6x - 1}{(x-3)^2}$

c)  $e^{2x} \left( 2\ln(x) + \frac{1}{x} \right)$

d)

$$f(1) = 1^3 - 2 \cdot 1 = (1)^3 - 2(1) = -1$$

$$f'(1) = 3 \cdot 1^2 - 2 = 3(1)^2 - 2 = 3 - 2 = 1$$

$$y = 1 \cdot x + (-1 - i)$$

$$y = x - 2$$

8)  $\int 4x^3 - 6x + 2 \, dx = \frac{4x^4}{4} - \frac{6x^2}{2} + 2x + C, C \in \mathbb{R}$

9)  $\int \frac{2x}{x^2 + 3} \, dx \Leftrightarrow \ln(x^2 + 3) + C, C \in \mathbb{R}$

c)  $\int x \cdot e^{2x} \, dx \Leftrightarrow x \cdot e^{2x} - \int e^{2x} (=) x \cdot e^{2x} - e^{2x} + C, C \in \mathbb{R}$

d)  $\int \ln(x) \, dx = x \ln(x) - \int x \cdot \frac{1}{x} \, dx = x \ln(x) - x + C, C \in \mathbb{R}$

9)

a)  $\int_1^3 x^2 - 2x + 1 \, dx = \left[ \frac{x^3}{3} - \frac{2x^2}{2} + x \right]_1^3 =$

$$= \frac{3^3}{3} - \frac{2(3)^2}{2} + 3 - \frac{1^3}{3} + \frac{2(1)^2}{2} + 1 = 3 - 9 + 3 - \frac{1}{3} + 1 + 1 = \frac{8}{3}$$

$$= -3 - \frac{1}{3} = -\frac{9}{3} - \frac{1}{3} = -\frac{8}{3}$$

~~5)~~

$$\int_1^e \frac{2}{x} = [2\ln(x)]_1^e = 2(\ln(e)) - 2(\ln(1)) = 2(1) - 2(0) = 2$$

10)  $f(x) = 6x - 4 + C =$

$$f(1) = 6(1) - 4 + 0 = 2$$

11)

12)

$$f(1) = x^2 = 1^2 = 1$$

$$g(1) = 2x = 2 \cdot 1 = 2$$

$$A = \int 2x - x^2 \, dx = \frac{2x^2}{2} - \frac{x^3}{3} + C, C \in \mathbb{R}$$